Homework 5

SUBMISSION INSTRUCTIONS. **Please pay attention to these as they will make marking your assignments less burdensome.**

1. Put all of your code files in a folder called <your first name>_<your last name>_homework_5. For example I would call mine

andrew_osborne_homework_5

2. Compress your folder using the zip tool of your choice.

3. Upload your HW to Canvas.

INTRODUCTION. The aims of this assignment are:

1) Allow you to practice modularization with a piece of code that performs computation on nuclear data.
2) Develop some intuition with ENDF data and fundamentals of nuclear cross sections.

In this assignment you will reconstruct resonances in the capture and fission cross sections of ^{238}Pu at a temperature of 0K. You will then compute the Doppler broadened resonances at 30,000K.

Part 1. Energy-dependent cross sections in ^{238}Pu.

BACKGROUND. The ENDF libraries provided by LANL include resonance parameters for each nuclide. These parameters are found in MFMT 2151. An excerpt relevant to the first few resonances is shown below.

```
 9.423800+4 2.360046+2          0          0          1          09434 2151    1
 9.423800+4 1.000000+0          0          0          2          09434 2151    2
 1.000000-5 5.000000+2          1          2          0          09434 2151    3
 0.000000+0 1.150000+0          0          0          1          09434 2151    4
 2.360046+2 0.000000+0          0          0        312        529434 2151    5
-2.100000-1 5.000000-1 3.443000-3 1.150000-3 2.200000-3 9.300000-59434 2151    6
 2.885000+0 5.000000-1 3.859970-2 7.470000-5 3.800000-2 5.250000-49434 2151    7
 9.975000+0 5.000000-1 4.142800-2 2.080000-4 3.700000-2 4.220000-39434 2151    8
 1.860000+1 5.000000-1 3.897100-2 4.140000-3 3.400000-2 8.310000-49434 2151    9
 3.220000+1 5.000000-1 3.886800-2 6.800000-5 3.400000-2 4.800000-39434 2151   10
 3.660000+1 5.000000-1 3.992400-2 2.400000-5 3.400000-2 5.900000-39434 2151   11
 5.980000+1 5.000000-1 3.598000-2 1.310000-3 3.400000-2 6.700000-49434 2151   12
 ...
```

This is copied and pasted directly from the ENDF library for ^{238}Pu. The first line gives the ZZAAA number and the atomic mass in amu. The 3rd line indicates that this resonance range spans 10^{-5} to 200 eV. The first entry on the fourth line says that the target spin is zero (I=0). The data for the resonances themselves start on the 6th line, the first being a negative-energy (relative to binding energy) resonance. On the 7th line, the data shows that there is a positive energy resonance at 2.885 eV, with angular momentum J=0.5, a total width of 0.0386 eV, a scattering width of 7.47 x 10^{-5} eV, a capture width of 0.038 eV, and a fission width of 5.25 x 10^{-4} eV.

The energy-dependent cross sections can be computed using the Breit-Wigner formula:

$$\sigma_x(E_c) = \sigma_0 \frac{\Gamma_x}{\Gamma} \sqrt{\frac{E_0}{E_c}} \frac{\Gamma^2}{4(E_c - E_0)^2 + \Gamma^2} \qquad (1)$$

where

$$\sigma_0(E_c) = 4\pi \lambda\!\!\!^{-2} g \frac{\Gamma_n}{\Gamma} \qquad (2)$$

and the remaining symbols can be found in the scattering lecture notes.

INSTRUCTIONS:

The instructor has provided a version of this data that is easier for Matlab to read, and a set of stub Matlab functions for you to fill out. The goal of this exercise is to produce a code which plots the energy-dependent capture and fission cross sections in the given resonance energy range, computed using the Single-Level Breit-Wigner (SLBW) approach.

a) Using the provided stub functions as a guide, write out a high-level algorithm for the code.
b) The function that reconstructs the resonances is called resonance_reconstruct(). Write out an algorithm for this function using the attached notes on Breit-Wigner scattering (BreitWigner.pdf) as a guide.
c) Implement your algorithm in the provided stub function files.
d) Plot the energy dependent fission and capture cross sections from 1 to 70 eV (center of mass energy), reconstructing only the six positive-energy resonances given in the data file.
e) Write down the peak capture and fission cross section values, in barns, for each resonance.

TIPS:

1) Make sure you are using the correct units in your BW equations (MKS units).
2) The cross sections you get will also be in MKS units - you need to convert to barns for plotting.
3) Have a look at http://t2.lanl.gov/nis/data/endf/endfvii.1-n-pdf/pu238.pdf to see what the reconstructed resonances should look like. In this case since we are dealing with a heavy nucleus, the lab frame spectrum should look almost the same as the center of mass spectrum that you compute.
4) In this example, I=0 and J=1/2, which means your g-factor will only have a single value.

5) Pay careful attention to quantities that are functions of center of mass energy as opposed to energy in the lab frame
6) Use the Matlab import wizard to easily import the data from the data file.

Part 2. Doppler Broadening of Resonances in ^{238}Pu.

BACKGROUND. In Part 1 you computed the cross sections at a temperature of 0K. In this part you will recompute these cross sections at 30,000K using the Doppler Broadening method you learned in class.

Given an energy dependent cross section at 0K $\sigma(E_c)$, one can compute the Doppler broadened cross section $\sigma(E_c,T)$ using the formula:

$$\sigma\left(E_c, T\right) = \frac{1}{v} \int v_r \sigma\left(\left|v - V\right|\right) \left(\frac{M}{2\pi kT}\right)^{\frac{1}{2}} e^{-\frac{MV^2}{2kT}} dV \qquad (3)$$

where v is the incoming neutron energy in the lab frame, V is the velocity of the nucleus in the lab frame [m/s], v_r = v - V is the relative velocity between the neutron and the target nucleus [m/s], M is the mass of the nucleus [kg], k is Boltzmann's constant [m^2-kg-s^2/K], and T is the temperature [K].

INSTRUCTIONS:

a) Write an algorithm for a function called doppler_broaden(), using Eq. (3) and your lecture notes as a guide.
b) Implement your algorithm in the doppler_broaden() function.
c) Plot out the doppler broadened fission and capture resonance cross sections.

TIPS:

1) Your algorithm should consist of a loop over each of the energies E_c of Part 1. In the body of the loop is where you should implement Eq. (3).
2) In each loop, you will need to recompute σ(|v-V|) for the range of V (the nucleus velocity) that you are integrating over. One way to do this is to use your function from Part 1 to recompute the cross section for a finer resolution grid. An easier way to do this is to write a function for σ(v) (given you know $\sigma(E_c)$), and interpolate σ(v) over the |v-V| range of interest.