Linear Models for Data Science

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Contents

Preface

Who is this book for?

There are many books on linear models, with various expectations for different levels of familiarity with statistical, mathematical, and coding concepts. These books generally fall into one of two camps:

1. Little to no familiarity with statistical and mathematical concepts, but fairly familiar to coding. These books tend to be written for programmers who want to get into data science. These books tend to explain linear models while trying to avoid statistical and mathematical concepts as much, only covering these concepts if absolutely necessary. These books tend to present linear models in a recipe format giving readers directions on what to do to build their models.

The drawback of such books is that readers do not get much understanding of the underlying concepts of linear models. It is impossible to give directions covering every possible scenario in the real world as real data are messy. Practitioners of data science often have to think outside the box in order to make linear models work for their particular data, and it is difficult to do so without understanding the mathematical framework of linear models.

2. Familiarity with mathematical notation and introductory statistical concepts such as statistical inference, and little to no familiarity with coding. These books tend to be written for mathematicians (or anyone with a strong background in mathematics) who want to get into data science. These books cover the mathematical framework of linear models thoroughly.

The drawback of such books is that readers must be comfortable with mathematical notation. This limits the audience for such books to people with fairly thorough training in mathematics. People without such training will get lost trying to read such books, and do not understand why we need to know the mathematical foundations to use linear models in data science.

This book is meant to be readable by both groups of readers. Some foundational mathematical knowledge will be presented, but will be written so that is

readable by anyone. This book will also explain what these knowledge mean in the context of data science. Practical advice, based on the foundational mathematical knowledge, will also be given.

This book accompanies the course STAT 6021: Linear Models for Data Science, for the Masters of Data Science (MSDS) program at the University of Virginia School of Data Science.

As introductory statistics and introductory programming are pre-requisites for entering the MSDS program, this book assumes basic knowledge of statistical inference and coding. Review materials covering these concepts are provided separately for enrolled students.

Data sets used

I have tried to use as many open source data sets as much as possible so that readers can work on the various examples I have provided on their own. However, some data sets may not be open source and have come from my experience teaching this class since 2019 (and variations of the class since 2013), and have used some data sets that were shared by other statistics and data science educators. It is my goal to eventually use only open source data sets.

Chapters

The chapters for the book is as follows:

Other resources

Some other resources that readers may want to check out:

- *OpenIntro Statistics*, 4th ed. Diez, Cetinkaya-Rundel, Barr, OpenIntro. Get free PDF version at https://leanpub.com/os, just set the price that you want to pay to $0. This is a good book for introductory statistics.
- *Linear Models with R*, 2nd ed. Faraway. This is probably one of the few books that balances between the two camps that I wrote about earlier. It does require familiarity with matrices and linear algebra though.
- *Introduction to Linear Regression Analysis*, 5th or 6th ed. Montgomery, Peck, Vining. You may be able to access an e-version of the book through your university library if you are affiliated with a university. This book is mathematically rigorous so is useful to those who are interested in mathematical proofs that is not covered.
- *Applied Linear Statistical Models* (ALSM), Kutner, Nachtsheim, Neter, Li, 5th ed. This book covers a wide range of topics in linear models and is also mathematically rigorous.

- *Applied Linear Regression Models* (ALRM), Kutner, Nachtsheim, Neter, 4th ed. ALRM is the same as the first 14 chapters of ALSM. The second part of ALSM covers topics in Design of Experiments, which I highly recommend if you are interested in those topics.

Chapter 1

Data Wrangling with R

1.1 Introduction

The data structure that we will be dealing with most often will be data frames. When we read data in to R, they are typically stored as a data frame. A data frame can be viewed like an EXCEL spreadsheet, where data are stored in rows and columns. Before performing any analysis, we want the data frame to have this basic structure:

- Each row of the data frame corresponds to an observation.
- Each column of the data frame corresponds to a variable.

Sometimes, data are not structured in this way, and we will have to transform the data to take on this basic data structure. This process is called data wrangling. The most common and basic operations to transform the data are:

- Selecting a subset of columns of the data frame.
- Selecting a subset of rows of the data frame based on some criteria.
- Change column names.
- Find missing data.
- Create new variables based on existing variables.
- Combine multiple data frames.

We will explore two approaches to data wrangling:

- Using functions that already come pre-loaded with R (sometimes called base R).
- Using functions from the `dplyr` package.

These two approaches are quite different but can achieve the same goals in data wrangling. Each user of R usually ends up with their own preferred way of performing data wrangling operations, but it is important to know both approaches so you are able to work with a broader audience.

1.2 Data Wrangling using Base R Functions

We will use the dataset `ClassDataPrevious.csv` as an example. The data were collected from an introductory statistics class at UVa from a previous semester. Download the dataset from Canvas and read it into R.

```
Data<-read.csv("ClassDataPrevious.csv", header=TRUE)
```

We check the number of rows and columns in this dataframe.

```
dim(Data)
```

```
## [1] 298   8
```

There are 298 rows and 8 columns: so we have 298 students and 8 variables. We can also check the names for each column.

```
colnames(Data)
```

```
## [1] "Year"     "Sleep"    "Sport"    "Courses"  "Major"    "Age"      "Computer"
## [8] "Lunch"
```

The variables are:

1. `Year`: the year the student is in
2. `Sleep`: how much sleep the student averages a night (in hours)
3. `Sport`: the student's favorite sport
4. `Courses`: how many courses the student is taking in the semester
5. `Major`: the student's major
6. `Age`: the student's age (in years)
7. `Computer`: the operating system the student uses (Mac or PC)
8. `Lunch`: how much the student usually spends on lunch (in dollars)

1.2.1 View specific row(s) and/or column(s) of a data frame

We can view specific rows and/or columns of any data frame using the square brackets `[]`, for example:

```
Data[1,2] ##row index first, then column index
```

```
## [1] 8
```

The row index is listed first, then the column index, in the square brackets. This means the first student sleeps 8 hours a night. We can also view multiple rows and columns, for example:

```
Data[c(1,3,4),c(1,5,8)]
```

```
##     Year                            Major Lunch
## 1 Second                         Commerce    11
## 3 Second Cognitive science and psychology    10
```

```
## 4  First                          Pre-Comm     4
```

to view the 1st, 5th, and 8th variables for observations 1, 3, and 4.

There are several ways to view a specific column. For example, to view the 1st column (which is the variable called `Year`):

```
Data$Year ##or
Data[,1] ##or
Data[,-c(2:8)]
```

Note the comma separates the indices for the row and column. An empty value before the comma means we want all the rows, and then the specific column. To view multiple columns, for example the first four columns:

```
Data[,1:4]
Data[,c(1,2,3,4)]
```

To view the values of certain rows, we can use

```
Data[c(1,3),]
```

to view the values for observations 1 and 3. An empty value after the comma means we want all the columns for those specific rows.

1.2.2 Select observations by condition(s)

We may want to only analyze certain subsets of our data, based on some conditions. For example, we may only want to analyze students whose favorite sport is soccer. The `which()` function in R helps us find the indices associated with a condition being met. For example:

```
which(Data$Sport=="Soccer")
```

```
##  [1]   3  20  25  26  31  32  33  38  44  46  48  50  51  64  67  71  87  92  98
## [20]  99 118 122 124 126 128 133 136 137 143 146 153 159 165 174 197 198 207 211
## [39] 214 226 234 241 255 259 260 266 274 278 281 283 294 295
```

informs us which rows belong to observations whose favorite sport is soccer, i.e. the 3rd, 20th, 25th (and so on) students. We can create a new data frame that contains only students whose favorite sport is soccer:

```
SoccerPeeps<-Data[which(Data$Sport=="Soccer"),]
dim(SoccerPeeps)
```

```
## [1] 52  8
```

We are extracting the rows which satisfy the condition, favorite sport being soccer, and storing these rows into a new data frame called `SoccerPeeps`. We can see that this new data frame has 52 observations.

Suppose we want to have a data frame that satisfies two conditions: that the favorite sport is soccer and they are 2nd years at UVa. We can type:

```
SoccerPeeps_2nd<-Data[which(Data$Sport=="Soccer" & Data$Year=="Second"),]
dim(SoccerPeeps_2nd)
```

```
## [1] 25  8
```

This new data frame `SoccerPeeps_2nd` has 25 observations.

We can also set conditions based on numeric variables, for example, we want students who sleep more than eight hours a night

```
Sleepy<-Data[which(Data$Sleep>8),]
```

We can also create a data frame that contains students who satisfy at least one out of two conditions, for example, the favorite sport is soccer or they sleep more than 8 hours a night:

```
Sleepy_or_Soccer<-Data[which(Data$Sport=="Soccer" | Data$Sleep>8),]
```

1.2.3 Change column name(s)

For some datasets, the names of the columns are complicated or do not make sense. We should always give descriptive names to columns that make sense. For this dataset, the names are self-explanatory so we do not really need to change them. As an example, suppose we want to change the name of the 7th column from `Computer` to `Comp`:

```
names(Data)[7]<-"Comp"
```

To change the names of multiples columns (for example, the 1st and 7th columns), type:

```
names(Data)[c(1,7)]<-c("Yr","Computer")
```

1.2.4 Find and remove missing data

There are a few ways to locate missing data. Using the `is.na()` function directly on a data frame produces a lot of output that can be messy to view:

```
is.na(Data)
```

On the other hand, using the `complete.cases()` function is more pleasing to view:

```
Data[!complete.cases(Data),]
```

```
##         Yr Sleep      Sport Courses                                    Major
## 103 Second    NA Basketball       7 psychology and youth and social innovation
## 206 Second     8       None       4                          Cognitive Science
```

```
##     Age Computer Lunch
## 103  19      Mac    10
## 206  19      Mac    NA
```

The code above will extract rows that are not complete cases, in other words, rows that have missing entries. The output informs us observation 103 has a missing value for `Sleep`, and observation 206 has a missing value for `Lunch`.

If you want to remove observations with a missing value, you can use one of the following two lines of code to create new data frames with the rows with missing values removed:

```
Data_nomiss<-na.omit(Data) ##or
Data_nomiss2<-Data[complete.cases(Data),]
```

A word of caution: these lines of code will remove the entire row as long as at least a column has missing entries. As noted earlier, observation 103 has a missing value for only the `Sleep` variable. But this observation still provides information on the other variables, which are now removed.

1.2.5 Summarizing variable(s)

Very often, we want to obtain some characteristics of our data. A common way to summarize a numerical variable is to find its mean. We have four numerical variables in our data frame, which are in columns 2, 4, 6, and 8. To find the mean of all four numerical variables, we can use the `apply()` function:

```
apply(Data[,c(2,4,6,8)],2,mean)
```

```
##     Sleep   Courses       Age     Lunch
##        NA  5.016779 19.573826        NA
```

```
apply(Data[,c(2,4,6,8)],2,mean,na.rm=T)
```

```
##      Sleep    Courses        Age      Lunch
## 155.559259   5.016779  19.573826 156.594175
```

Notice that due to the missing values, the first line has `NA` for some of the variables. The second line includes an optional argument, `na.rm=T`, which will remove the observations with an `NA` value for the variable from the calculation of the mean.

There are at least 3 arguments that are supplied to the `apply()` function:

1. The first argument is a data frame containing all the variables which we want to find the mean of. In this case, we want columns 2, 4, 6, and 8 of the data frame `Data`.
2. The second argument takes on the value `1` or `2`. Since we want to find the mean of columns, rather than rows, we type `2`. If want to mean of a row, we will type `1`.

3. The third argument specifies the name of the function you want to apply to the columns of the supplied data frame. In this case, we want the mean. We can change this to find the median, standard deviation, etc, of these numeric variables if we want to.

We notice the means for some of the variables are suspiciously high, so looking at the medians will be more informative.

```
apply(Data[,c(2,4,6,8)],2,median,na.rm=T)
```

```
##   Sleep Courses     Age   Lunch
##     7.5     5.0    19.0     9.0
```

1.2.6 Summarizing variable by groups

Sometimes we want to summarize a variable by groups. Suppose we want to find the median amount of sleep separately for 1st years, 2nd years, 3rd years, and 4th years get. We can use the `tapply()` function:

```
tapply(Data$Sleep,Data$Yr,median,na.rm=T)
```

```
##  First Fourth Second  Third
##    8.0    7.0    7.5    7.0
```

This informs us the median amount of sleep first years get is 8 hours a night; for fourth years the median amount is 7 hours a night.

There are at least 3 arguments that are supplied to the `tapply()` function:

1. The first argument contains the vector which we want to summarize.
2. The second argument contains the factor which we use to subset our data. In this example, we want to subset according to `Yr`.
3. The third argument is the function which we want to apply to each subset of our data.
4. The fourth argument is optional, in this case, we want to remove observations with missing values from the calculation of the mean.

Notice the output orders the factor levels in alphabetical order. For our context, it is better to rearrange the levels to First, Second, Third, Fourth using the `factor()` function:

```
Data$Yr<-factor(Data$Yr, levels=c("First","Second","Third","Fourth"))

levels(Data$Yr)
```

```
## [1] "First"  "Second" "Third"  "Fourth"
```

```
tapply(Data$Sleep,Data$Yr,median,na.rm=T) ##much nicer
```

```
##  First Second  Third Fourth
##    8.0    7.5    7.0    7.0
```

This output makes a lot more sense for this context.

If we want to summarize a variable on groups formed by more than one variable, we need to adjust the second argument in the `tapply()` function by creating a list. Suppose we want to find the median sleep hour based on the `Yr` and the preferred operating system of the observations,

```
tapply(Data$Sleep,list(Data$Yr,Data$Computer),median,na.rm=T)
```

```
##           Mac   PC
## First  NA 8.0 7.50
## Second  7 7.5 7.50
## Third  NA 7.5 7.00
## Fourth NA 7.0 7.25
```

Interestingly, it looks like there were observations who did not specify which operating system they use, hence the extra column in the output.

1.2.7 Create a new variable based on existing variable(s)

Depending on the context of our analysis, we may need to create new variables based on existing variables. There are a few variations of this task, based on the type of variable you want to create, and the type of variable it is based on.

1.2.7.1 Create a numeric variable based on another numeric variable

The variable `Sleep` is in number of hours. Suppose we need to convert the values of `Sleep` to number of minutes, we can simply perform the following mathematical operation:

```
Sleep_mins<-Data$Sleep * 60
```

and store the transformed variable into a vector called `Sleep_mins`.

1.2.7.2 Create a binary variable based on a numeric variable

Suppose we want to create a binary variable (categorical variable with two levels), called `deprived`. An observation will obtain a value of "yes" if they sleep for less than 7 hours a night, and "no" otherwise. The `ifelse()` function is useful in creating binary variables:

```
deprived<-ifelse(Data$Sleep<7, "yes", "no")
```

There are 3 arguments associated with the `ifelse()` function:

1. The first argument is the condition that we wish to use.

2. The second argument is the value of the observation if the condition is true.

3. The third argument is the value of the observation if the condition if false.

1.2.7.3 Create a categorical variable based on a numeric variable

Suppose we want to create a categorical variable based on the number of courses a student takes. We will call this new variable `CourseLoad`, which takes on the following values:

- `light` if 3 courses or less,
- `regular` if 4 or 5 courses,
- `heavy` if more than 5 courses .

The `cut()` function is used in this situation

```
CourseLoad<-cut(Data$Courses, breaks = c(-Inf, 3, 5, Inf),
                labels = c("light", "regular", "heavy"))
```

There are three arguments that are applied to the `cut()` function:

1. The first argument is the vector which you are basing the new variable on.

2. The argument breaks lists how you want to set the intervals associated with `Data$Courses`. In this case, we are creating three intervals: one from $(-\infty, 3]$, another from $(3, 5]$, and the last interval from $(5, \infty]$.

3. The argument labels gives the label for `CourseLoad` associated with each interval.

1.2.7.4 Collapse levels

Sometimes, a categorical variable has more levels than we need for our analysis, and we want to collapse some levels. For example, the variable `Yr` has four levels: First, Second, Third, and Fourth. Perhaps we are more interested in comparing upperclassmen and underclassmen, so we want to collapse First and Second years into underclassmen, and Third and Fourth years into upperclassmen:

```
levels(Data$Yr)
```

```
## [1] "First"  "Second" "Third"  "Fourth"
```

```
new.levels<-c("und", "und", "up","up")
Year2<-factor(new.levels[Data$Yr])
levels(Year2)
```

```
## [1] "und" "up"
```

The levels associated with the variable `Yr` are ordered as First, Second, Third, Fourth. The character vector new.levels has `under` as the first two characters,

and `upper` as the last two characters to correspond to the original levels in the variable `Yr`. The new variable is called `Year2`.

1.2.8 Combine data frames

We have created four new variables, `Sleep_mins`, `deprived`, `CourseLoad`, and `Year2`, based on previously existing variables. Since these variables are all based on the same observations, we can combine them with an existing data frame using the `data.frame()` function:

```
Data<-data.frame(Data,Sleep_mins,deprived,CourseLoad,Year2)
head(Data)
```

```
##       Yr Sleep      Sport Courses                          Major Age Computer
## 1 Second     8 Basketball       6                       Commerce  19      Mac
## 2 Second     7     Tennis       5                     Psychology  19      Mac
## 3 Second     8     Soccer       5 Cognitive science and psychology  21      Mac
## 4  First     9 Basketball       5                       Pre-Comm  19      Mac
## 5 Second     4 Basketball       6                     Statistics  19       PC
## 6  Third     7       None       4                     Psychology  20       PC
##   Lunch Sleep_mins deprived CourseLoad Year2
## 1    11        480       no      heavy   und
## 2    10        420       no    regular   und
## 3    10        480       no    regular   und
## 4     4        540       no    regular   und
## 5     0        240      yes      heavy   und
## 6    11        420       no    regular    up
```

Notice that since we listed the four new variables after `Data` in the `data.frame()` function, they appear after the original columns in the data frame.

Alternatively, we can use the `cbind()` function which gives the same data frame:

```
Data2<-cbind(Data,Sleep_mins,deprived,CourseLoad,Year2)
```

If you are combining data frames which have different observations but the same columns, we can merge them using `rbind()`:

```
dat1<-Data[1:3,1:3]
dat3<-Data[6:8,1:3]
res.dat2<-rbind(dat1,dat3)
head(res.dat2)
```

```
##       Yr Sleep      Sport
## 1 Second     8 Basketball
## 2 Second     7     Tennis
## 3 Second     8     Soccer
## 6  Third     7       None
## 7 Second     7 Basketball
```

```
## 8  First     7 Basketball
```

1.2.9 Export data frame in R to a .csv file

To export a data frame to a .csv file, type:

```
write.csv(Data, file="newdata.csv", row.names = FALSE)
```

A file newdata.csv will be created in the working directory. Note that by default, the argument `row.names` is set to be `TRUE`. This will add a column in the dataframe which is an index number. I do not find this step useful in most analyses so I almost always set `row.names` to be `FALSE`.

1.2.10 Sort data frame by column values

To sort your data frame in ascending order by `Age`:

```
Data_by_age<-Data[order(Data$Age),]
```

To sort in descending order by `Age`:

```
Data_by_age_des<-Data[order(-Data$Age),]
```

To sort in ascending order by `Age` first, then by `Sleep`:

```
Data_by_age_sleep<-Data[order(Data$Age, Data$Sleep),]
```

1.3 Data Wrangling Using dplyr Functions

In the previous section, we were performing data wrangling operations using functions that are built in with base R. In this module, we will be using functions mostly from a package called `dplyr`, which can perform the same operations as well.

Before performing data wrangling operations, let us clear our environment, so that previously declared objects are removed. This allows us to start with a clean slate, which is often desirable when starting on a new analysis. This is done via:

```
rm(list = ls())
```

The `dplyr` package is a subset of the `tidyverse` package, so we can access these functions after installing and loading either package. After installing the `tidyverse` package, load it by typing:

```
##library(dplyr) or
library(tidyverse)
```

The `dplyr` package was developed to make the syntax more intuitive to a broader range of R users, primarily through the use of pipes. However, the code involved

with functions from `dlpyr` tends to be longer than code involved with base R functions, and there are more functions to learn with `dplyr`.

You will find a lot of articles on the internet by various R users about why each of them believes one approach to be superior to the other. I am not fussy about which approach you use as long as you can perform the necessary operations. It is to your benefit to be familiar with both approaches so you can work with a broader range of R users.

We will continue to use the dataset `ClassDataPrevious.csv` as an example. Download the dataset from Canvas and read it into R:

```
Data<-read.csv("ClassDataPrevious.csv", header=TRUE)
```

In the examples below, we are performing the same operations as in the previous section, but using `dplyr` functions instead of base R functions.

1.3.1 Select specific column(s) of a data frame

The `select()` function is used to select specific columns. There are a couple of ways to use this function. First:

```
select(Data,Year)
```

to select the column `Year` from the data frame called `Data`.

1.3.1.1 Pipes

Alternatively, we can use pipes:

```
Data%>%
  select(Year)
```

Pipes in R are typed using `%>%` or by pressing Ctrl + Shift + M on your keyboard. To think of the operations above, we can read the code as

1. take the data frame called Data
2. and then select the column named Year.

We can interpret a pipe as "and then". Commands after a pipe should be placed on a new line (press enter). Pipes are especially useful if we want to execute several commands in sequence, which we will see in later examples.

1.3.2 Select observations by condition(s)

The `filter()` function allows us to subset our data based on some conditions, for example, to select students whose favorite sport is soccer:

```
filter(Data, Sport=="Soccer")
```

We can create a new data frame called `SoccerPeeps` that contains students whose favorite sport is soccer:

```
SoccerPeeps<-Data%>%
  filter(Sport=="Soccer")
```

Suppose we want to have a data frame, called `SoccerPeeps_2nd`, that satisfies two conditions: that the favorite sport is soccer and they are 2nd years at UVa:

```
SoccerPeeps_2nd<-Data%>%
  filter(Sport=="Soccer" & Year=="Second")
```

We can also set conditions based on numeric variables, for example, we want the students who sleep more than eight hours a night:

```
Sleepy<-Data%>%
  filter(Sleep>8)
```

We can also create a data frame that contains observations as long as they satisfy at least one out of two conditions: the favorite sport is soccer or they sleep more than 8 hours a night:

```
Sleepy_or_Soccer<-Data%>%
  filter(Sport=="Soccer" | Sleep>8)
```

1.3.3 Change column name(s)

It is straightforward to change the names of columns using the `rename()` function. For example:

```
Data<-Data%>%
  rename(Yr=Year, Comp=Computer)
```

allows us to change the name of two columns: from `Year` and `Computer` to `Yr` and `Comp`.

1.3.4 Summarizing variable(s)

The `summarize()` function allows us to summarize a column. Suppose we want to find the mean value of the numeric columns: `Sleep`, `Courses`, `Age`, and `Lunch`:

```
Data%>%
  summarize(mean(Sleep,na.rm = T),mean(Courses),mean(Age),mean(Lunch,na.rm = T))
```

```
##   mean(Sleep, na.rm = T) mean(Courses) mean(Age) mean(Lunch, na.rm = T)
## 1               155.5593      5.016779  19.57383               156.5942
```

The output looks a bit cumbersome. We can give names to each summary

```
Data%>%
  summarize(avgSleep=mean(Sleep,na.rm = T),avgCourse=mean(Courses),avgAge=mean(Age),
            avgLun=mean(Lunch,na.rm = T))
```

```
##   avgSleep avgCourse   avgAge   avgLun
## 1 155.5593  5.016779 19.57383 156.5942
```

As mentioned previously, the means look suspiciously high for a couple of variables, so looking at the medians may be more informative:

```
Data%>%
  summarize(medSleep=median(Sleep,na.rm = T),medCourse=median(Courses),
            medAge=median(Age),medLun=median(Lunch,na.rm = T))
```

```
##   medSleep medCourse medAge medLun
## 1      7.5         5     19      9
```

Note: For a lot of functions in the `dplyr` package, using American spelling or British spelling works. So we can use `summarise()` instead of `summarize()`.

1.3.5 Summarizing variable by groups

Suppose we want to find the median amount of sleep 1st years, 2nd years, 3rd years, and 4th years get. We can use the `group_by()` function:

```
Data%>%
  group_by(Yr)%>%
  summarize(medSleep=median(Sleep,na.rm=T))
```

```
## # A tibble: 4 x 2
##   Yr     medSleep
##   <chr>     <dbl>
## 1 First       8
## 2 Fourth      7
## 3 Second      7.5
## 4 Third       7
```

The way to read the code above is

1. Get the data frame called `Data`,
2. and then group the observations by `Yr`,
3. and then find the median amount of sleep by each `Yr` and store the median in a vector called `medSleep`.

As seen previously, the ordering of the factor levels is in alphabetical order. For our context, it is better to rearrange the levels to First, Second, Third, Fourth. We can use the `mutate()` function whenever we want to transform or create a new variable. In this case, we are transforming the variable `Yr` by reordering the factor levels with the `fct_relevel()` function:

```
Data<- Data%>%
  mutate(Yr = Yr%>%
           fct_relevel(c("First","Second","Third","Fourth")))
```

1. Get the data frame called `Data`,
2. and then transform the variable called `Yr`,
3. and then reorder the factor levels.

Then, we use pipes, the `group_by()`, and `summarize()` functions like before:

```
Data%>%
  group_by(Yr)%>%
  summarize(medSleep=median(Sleep,na.rm=T))
```

```
## # A tibble: 4 x 2
##   Yr     medSleep
##   <fct>     <dbl>
## 1 First       8
## 2 Second      7.5
## 3 Third       7
## 4 Fourth      7
```

This output makes a lot more sense for this context.

To summarize a variable on groups formed by more than one variable, we just add the other variables in the `group_by()` function:

```
Data%>%
  group_by(Yr,Comp)%>%
  summarize(medSleep=median(Sleep,na.rm=T))
```

```
## `summarise()` has grouped output by 'Yr'. You can override using the `.groups`
## argument.
```

```
## # A tibble: 9 x 3
## # Groups:   Yr [4]
##   Yr     Comp  medSleep
##   <fct>  <chr>    <dbl>
## 1 First  "Mac"     8
## 2 First  "PC"      7.5
## 3 Second ""        7
## 4 Second "Mac"     7.5
## 5 Second "PC"      7.5
## 6 Third  "Mac"     7.5
## 7 Third  "PC"      7
## 8 Fourth "Mac"     7
## 9 Fourth "PC"      7.25
```

1.3.6 Create a new variable based on existing variable(s)

As mentioned in the previously, the `mutate()` function is used to transform a variable or to create a new variable. There are a few variations of this task, based on the type of variable you want to create, and the type of variable it is based on.

1.3.6.1 Create a numeric variable based on another numeric variable

The variable `Sleep` is in number of hours. Suppose we need to convert the values of `Sleep` to number of minutes, we can simply perform the following mathematical operation:

```
Data<-Data%>%
  mutate(Sleep_mins = Sleep*60)
```

and store the transformed variable called `Sleep_mins` and add `Sleep_mins` to the data frame called `Data`.

1.3.6.2 Create a binary variable based on a numeric variable

Suppose we want to create a binary variable , called `deprived`. An observation will obtain a value of `yes` if they sleep for less than 7 hours a night, and `no` otherwise. We will then add this variable deprived to the data frame called `Data`:

```
Data<-Data%>%
  mutate(deprived=ifelse(Sleep<7, "yes", "no"))
```

1.3.6.3 Create a categorical variable based on a numeric variable

Suppose we want to create a categorical variable based on the number of courses a student takes. We will call this new variable `CourseLoad`, which takes on the following values:

- `light` if 3 courses or less,
- `regular` if 4 or 5 courses,
- `heavy` if more than 5 courses

and then add `CourseLoad` to the data frame `Data`. We can use the `case_when()` function from the `dplyr` package, instead of the `cut()` function:

```
Data<-Data%>%
  mutate(CourseLoad=case_when(Courses <= 3 ~ "light",
                              Courses >3 & Courses <=5 ~ "regular",
                              Courses > 5 ~ "heavy"))
```

Notice how the names of the categorical variable are supplied after a specific condition is specified.

1.3.6.4 Collapsing levels

Sometimes, a categorical variable has more levels than we need for our analysis, and we want to collapse some levels. For example, the variable Yr has four levels: First, Second, Third, and Fourth. Perhaps we are more interested in comparing between upperclassmen and underclassmen, so we want to collapse First and Second Yrs into underclassmen, and Third and Fourth Yrs into upperclassmen. We will use the `fct_collapse()` function:

```
Data<-Data%>%
  mutate(UpUnder=fct_collapse(Yr,under=c("First","Second"),up=c("Third","Fourth")))
```

We are creating a new variable called `UpUnder`, which is done by collapsing `First` and `Second` into a new factor called `under`, and collapsing `Third` and `Fourth` into a new factor called `up`. `UpUnder` is also added to the data frame `Data`.

1.3.7 Combine data frames

To combine data frames which have different observations but the same columns, we can combine them using `bind_rows()`:

```
dat1<-Data[1:3,1:3]
dat3<-Data[6:8,1:3]
res.dat2<-bind_rows(dat1,dat3)
head(res.dat2)
```

```
##       Yr Sleep      Sport
## 1 Second     8 Basketball
## 2 Second     7     Tennis
## 3 Second     8     Soccer
## 4  Third     7       None
## 5 Second     7 Basketball
## 6  First     7 Basketball
```

`bind_rows()` works the same way as `rbind()`. Likewise, we can use `bind_cols()` instead of `cbind()`.

1.3.8 Sort data frame by column values

To sort your data frame in ascending order by `Age`:

```
Data_by_age<-Data%>%
  arrange(Age)
```

To sort in descending order by `Age`:

```
Data_by_age_des<-Data%>%
  arrange(desc(Age))
```

To sort in ascending order by `Age` first, then by `Sleep`:

```
Data_by_age_sleep<-Data%>%
  arrange(Age,Sleep)
```

Chapter 2

Data Visualization with R Using ggplot2

2.1 Introduction

Data visualizations are tools that summarize data. Consider the visuals from the CDC covid tracker dashboard to an external site. Without actually having access to the actual data, we have a sense of trends associated with hospitalizations and deaths. Good visualizations are easy to interpret for a wide variety of audiences, and are easier to explain than statistical models.

In this module, you will learn how to create common data visualizations. The choice of data visualization is almost always determined by whether the variable(s) involved is categorical or quantitative. Discrete variables are interesting because depending on the circumstance, they can be viewed as either categorical or quantitative in the context of data visualizations.

We will be using functions from the `ggplot2` package to create visualizations. The `ggplot2` package enables users to create various kinds of data visualizations, beyond the visualizations that can be made in base R. The `ggplot2` package is automatically loaded when we load the `tidyverse` package, although we can load `ggplot2` on its own.

```
library(tidyverse)
```

We will use the dataset `ClassDataPrevious.csv` as an example. The data were collected from an introductory statistics class at UVa from a previous semester. Download the dataset from Canvas and read it into R.

```
Data<-read.csv("ClassDataPrevious.csv", header=TRUE)
```

The variables are:

1. `Year`: the year the student is in
2. `Sleep`: how much sleep the student averages a night (in hours)
3. `Sport`: the student's favorite sport
4. `Courses`: how many courses the student is taking in the semester
5. `Major`: the student's major
6. `Age`: the student's age (in years)
7. `Computer`: the operating system the student uses (Mac or PC)
8. `Lunch`: how much the student usually spends on lunch (in dollars)

2.2 Visualizations with a Single Categorical Variable

2.2.1 Frequency tables

Frequency tables are a common tool to summarize categorical variables. These tables give us the number of observations (sometimes called counts) that belong to each class of a categorical variable. These tables are created using the `table()` function. Suppose we want to see the number of students in each year in our data:

```
table(Data$Year)
```

```
##
##  First Fourth Second  Third
##     83     30    139     46
```

Notice the order of the years could be rearranged to make more sense:

```
Data$Year<-factor(Data$Year, levels=c("First","Second","Third","Fourth"))
levels(Data$Year)
```

```
## [1] "First"  "Second" "Third"  "Fourth"
```

```
mytab<-table(Data$Year)
mytab
```

```
##
##  First Second  Third Fourth
##     83    139     46     30
```

So we have 83 first years, 139 second years, 46 third years, and 30 fourth years in our dataset.

We can report these numbers using proportions instead of counts, using `prop.table()`:

```
prop.table(mytab)
```

```
##
```

```
##     First    Second     Third    Fourth
## 0.2785235 0.4664430 0.1543624 0.1006711
```

or percentages by multiplying by 100:

```
prop.table(mytab) * 100
```

```
##
##    First   Second    Third   Fourth
## 27.85235 46.64430 15.43624 10.06711
```

To round the percentages to two decimal places, use the `round()` function:

```
round(prop.table(mytab) * 100, 2)
```

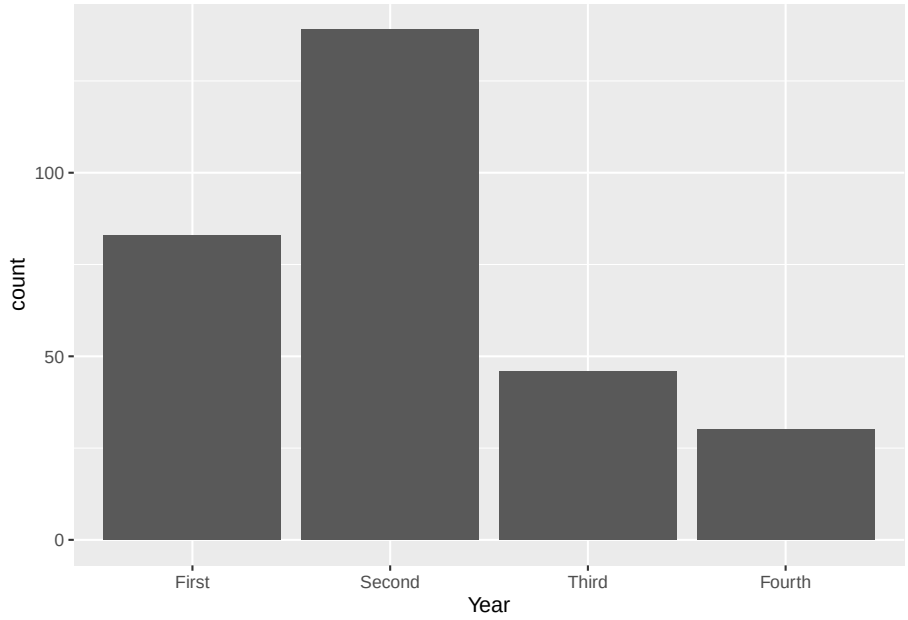
```
##
##  First Second  Third Fourth
##  27.85  46.64  15.44  10.07
```

So about 27.85% of these students are first years, 46.64% are second years, 15.44% are third years, and 10.07% are fourth years.

2.2.2 Bar charts

Bar charts are a simple way to visualize categorical data, and can be viewed as a visual representation of frequency tables. To create a bar chart for the years of these students, we use:

```
ggplot(Data, aes(x=Year))+
  geom_bar()
```



We can read the number of students who are first, second, third, and fourth years by reading off the corresponding value on the vertical axis.

From these two lines we code, we can see the basic structure of creating data visualizations with the `ggplot()` function:

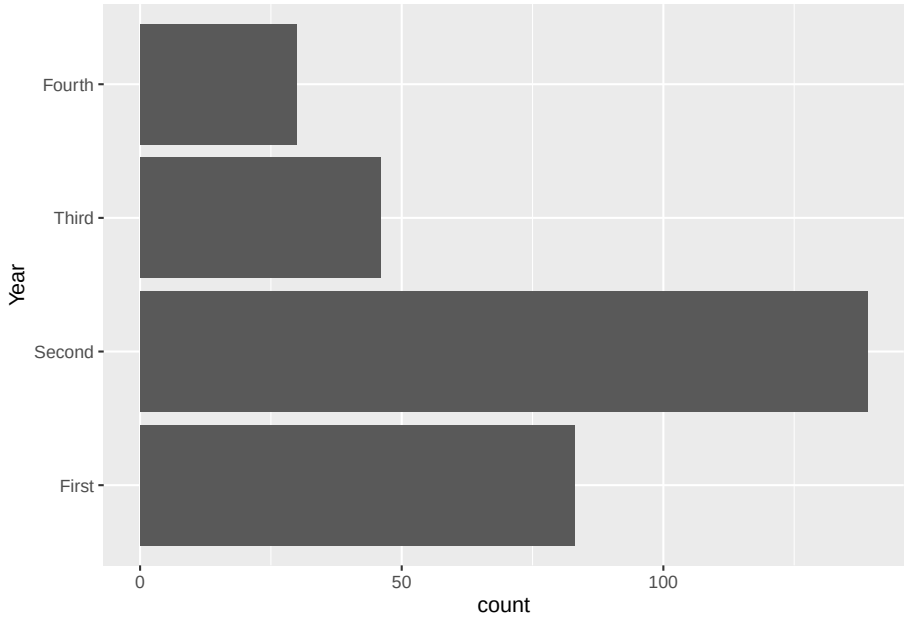
1. Use the `ggplot()` function, and supply the name of the data frame, and the x- and/or y- variables via the aes() function. End this line with a `+` operator, and then press enter.

2. In the next line, specify the type of graph we want to create (called `geoms`). For a bar chart, type `geom_bar()`.

Some describe these lines of code as two layers of code. These two layers must be supplied for all data visualizations with `ggplot()`.

Additional optional layers can be added (these usually deal with the details of the visuals). Suppose we want to change the orientation of this bar chart, we can add an optional line, or layer:

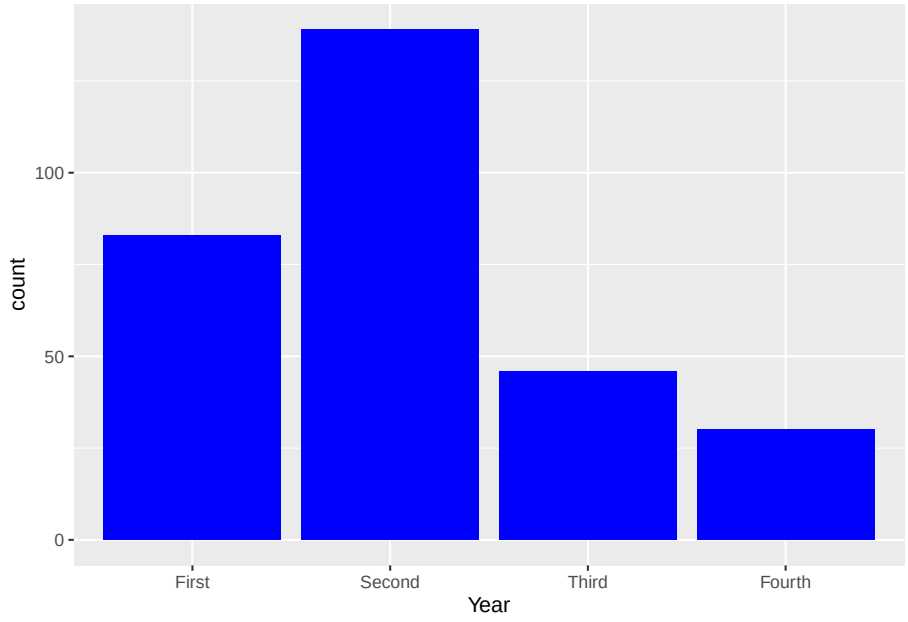
```
ggplot(Data, aes(x=Year))+
  geom_bar()+
  coord_flip()
```



It is recommended that each layer is typed on a line below the previous layer. A + sign is used at the end of each layer to add another layer below.

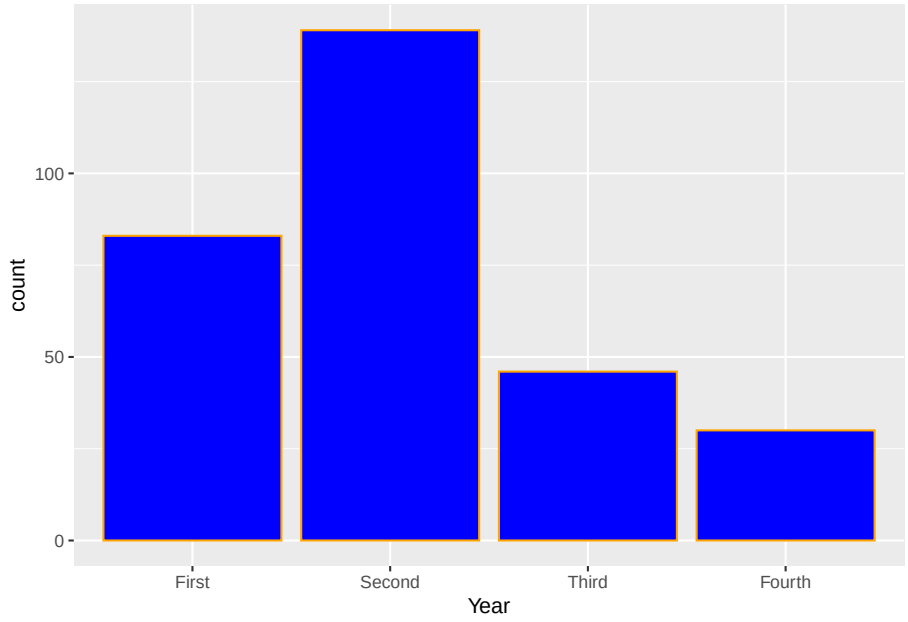
To change the color of the bars:

```
ggplot(Data, aes(x=Year))+
  geom_bar(fill="blue")
```



To have a different color to outline the bars:

```
ggplot(Data, aes(x=Year))+
  geom_bar(fill="blue",color="orange")
```

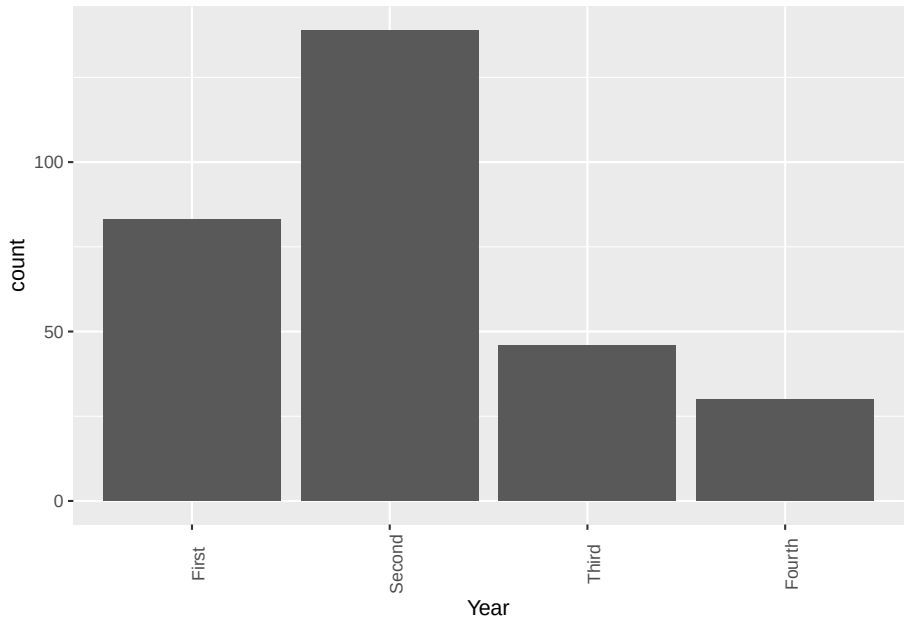


2.2.2.1 Customize title and labels of axes in bar charts

To change the orientation of the labels on the horizontal axis, we add an extra layer called `theme`. This will be useful when we have many classes and/or labels with long names.

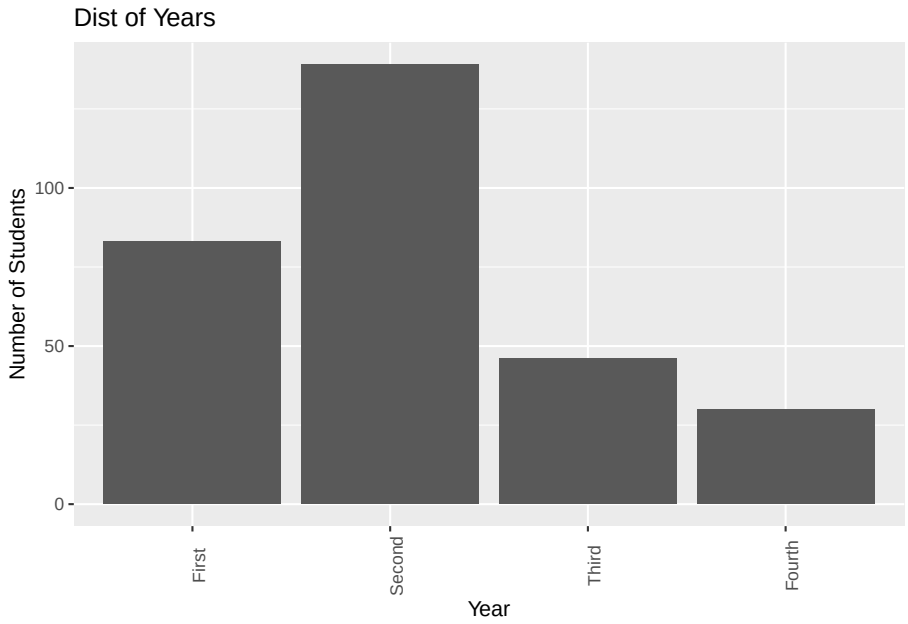
To rotate the labels on the horizontal by 90 degrees:

```
ggplot(Data, aes(x=Year))+
  geom_bar()+
  theme(axis.text.x = element_text(angle = 90))
```



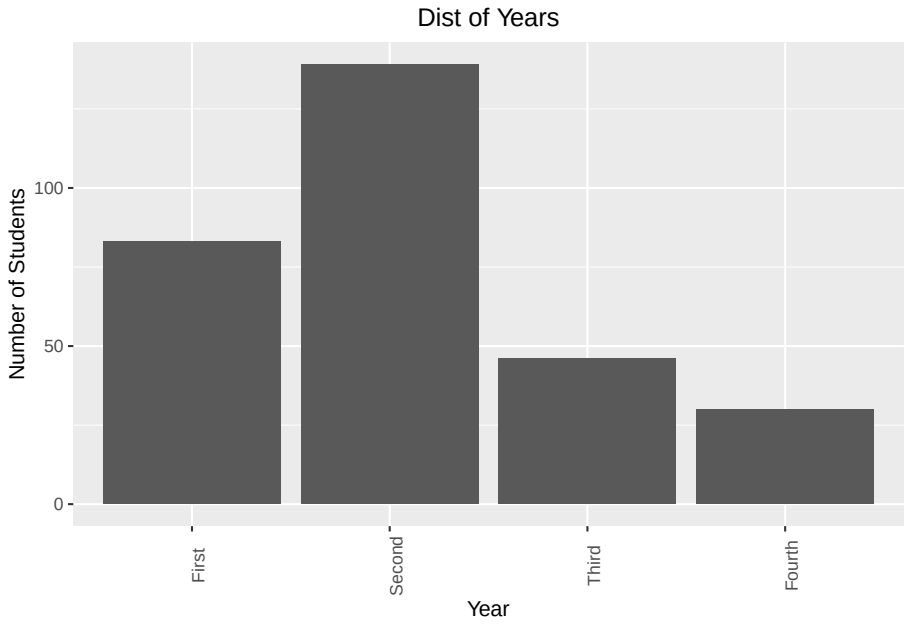
As we create more visualizations, it is good practice to give short but meaningful and descriptive names for each axis and provide a title. We can change the labels of the x- and y- axes, as well as add a title for the bar chart by adding another layer called `labs`:

```
ggplot(Data, aes(x=Year))+
  geom_bar()+
  theme(axis.text.x = element_text(angle = 90))+
  labs(x="Year", y="Number of Students", title="Dist of Years")
```



We can also adjust the position of the title, for example, center-justify it via `theme`:

```
ggplot(Data, aes(x=Year))+
  geom_bar()+
  theme(axis.text.x = element_text(angle = 90),
        plot.title = element_text(hjust = 0.5))+
  labs(x="Year", y="Number of Students", title="Dist of Years")
```



2.2.2.2 Create a bar chart using proportions

Suppose we want to create a bar chart where the y-axis displays the proportions, rather than the counts of each level. There are a few steps to produce such a bar chart. First, we create a new dataframe, where each row represents a year, and we add the proportion of each year into a new column:

```
newData<-Data%>%
  group_by(Year)%>%
  summarize(Counts=n())%>%
  mutate(Percent=Counts/nrow(Data))
```

The code above does the following:

1. Creates a new data frame called `newData` by taking the data frame called `Data`,
2. and then groups the observations by `Year`,
3. and then counts the number of observations in each `Year` and stores these values in a vector called `Counts`,
4. and then creates a new vector called `Percent` by using the mathematical operations as specified in `mutate()`. `Percent` is added to `newData`.

We can take a look at the contents of `newData`:

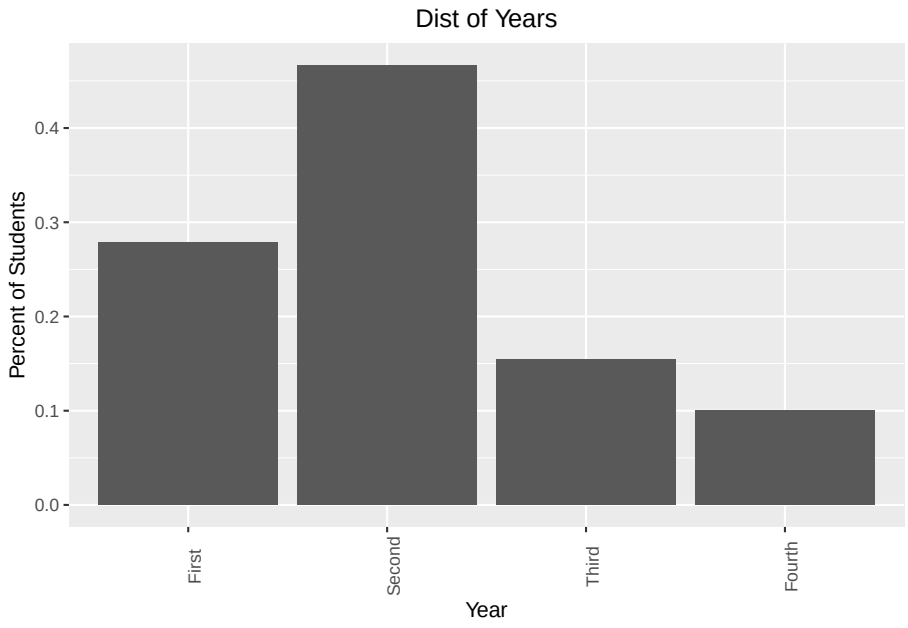
```
newData
```

```
## # A tibble: 4 x 3
##   Year   Counts Percent
```

```
##   <fct>   <int>   <dbl>
## 1 First      83   0.279
## 2 Second    139   0.466
## 3 Third      46   0.154
## 4 Fourth     30   0.101
```

To create a bar chart using proportions:

```
ggplot(newData, aes(x=Year, y=Percent))+
  geom_bar(stat="identity")+
  theme(axis.text.x = element_text(angle = 90),
        plot.title = element_text(hjust = 0.5))+
  labs(x="Year", y="Percent of Students", title="Dist of Years")
```



Note the following:

1. In the first layer, we use `newData` instead of the old data frame. In `aes()`, we specified a y-variable, which we want to be `Percent`.
2. In the second layer, we specified `stat="identity"` inside `geom_bar()`.

2.3 Visualizations with a Single Quantitative Variable

2.3.1 5 number summary

The `summary()` function, when applied to a quantitative variable, produces the 5 number summary: the minimum, the first quartile (25th percentile), the median (50th percentile), the third quartile (75th percentile), and the maximum, as well as the mean. For example, to obtain the 5 number summary of the ages of these students:

```
summary(Data$Age)
```

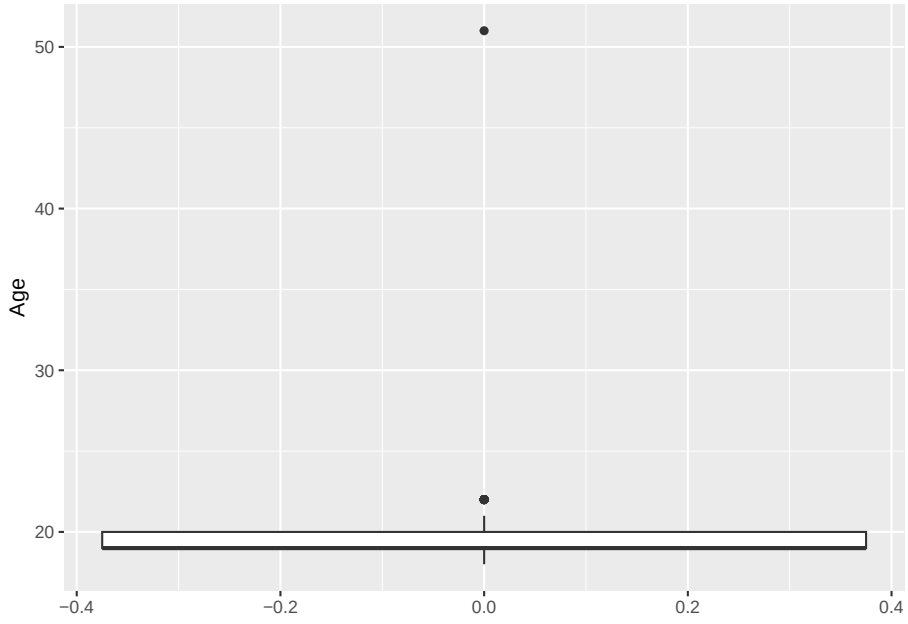
```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   18.00   19.00   19.00   19.57   20.00   51.00
```

The average age of the observations in this dataset is 19.57 years old. Notice the first quartile and the median are both 19 years old, that means at least a quarter of the observations are 19 years old. Also note the maximum of 51 years old, so we have a student who is quite a lot older than the rest.

2.3.2 Boxplots

A boxplot is a graphical representation of the 5 number summary. To create a generic boxplot, we have the following two lines of code when using the `ggplot()` function:

```
ggplot(Data, aes(y=Age))+
  geom_boxplot()
```



Note we are still using the same structure when creating data visualizations with `ggplot()`:

1. Use the `ggplot()` function, and supply the name of the data frame, and the x- and/or y- variables via the aes() function. End this line with a `+` operator, and then press enter.

2. In the next line, specify the type of graph we want to create (called `geoms`). For a boxplot, type `geom_boxplot`.

Notice there are outliers (observations that are a lot older or younger) that are denoted by the dots. One is the 51 year old, and 22 year olds are deemed to be outliers. The rule being used is the $1.5 \times IQR$ rule.

Similar to bar charts, we can change the orientation of boxplots by adding an additional layer as before:

```
ggplot(Data, aes(y=Age))+
  geom_boxplot()+
  coord_flip()
```

We can change the color of the box and the outliers similarly:

```
ggplot(Data, aes(y=Age))+
  geom_boxplot(color="blue", outlier.color = "orange" )
```

2.3.3 Histograms

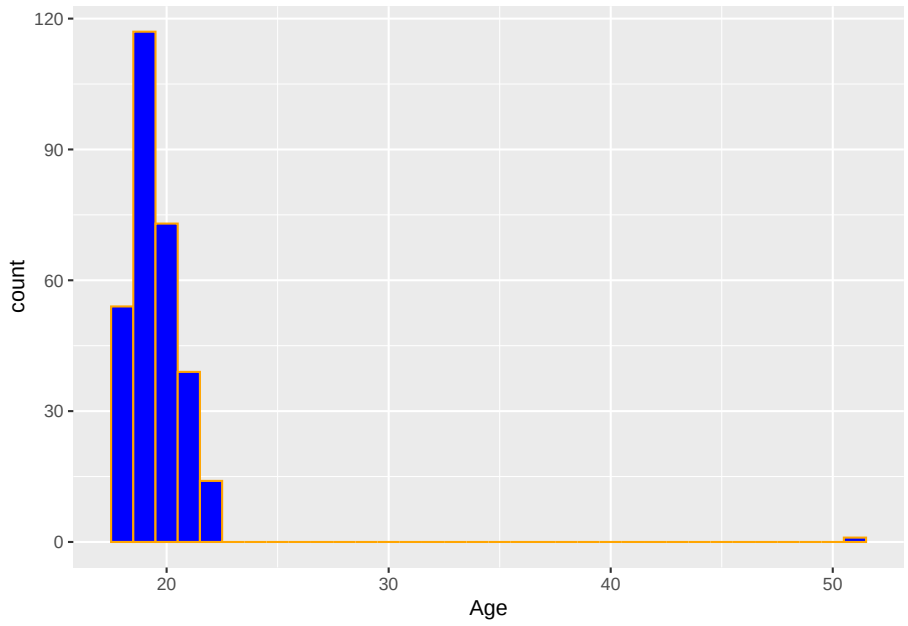
A histogram displays the number of observations within each bin on the x-axis:

```
ggplot(Data,aes(x=Age))+
  geom_histogram()
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

Notice a warning message is displayed when creating a basic histogram. To fix this, we use the binwidth argument within `geom_histogram`. We try `binwidth=1` for now, which means the width of the bin is 1 unit:

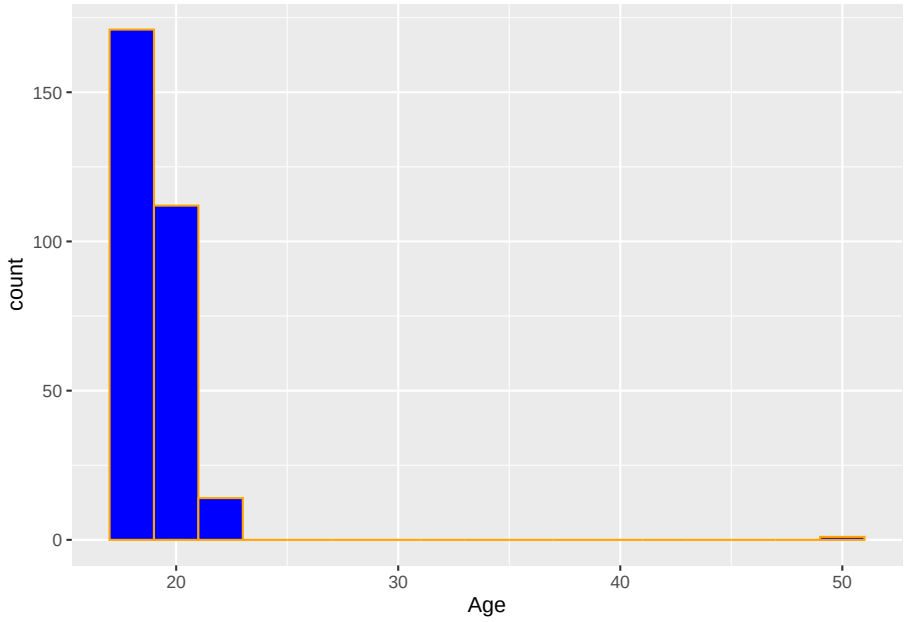
```
ggplot(Data,aes(x=Age))+
  geom_histogram(binwidth = 1,fill="blue",color="orange")
```



The ages of the students are mostly young, with 19 and 20 years olds being the most commonly occuring.

A well-known drawback of histograms is that the width of the bins can drastically affect the visual. For example, suppose we change the binwidth to 2:

```
ggplot(Data,aes(x=Age))+
  geom_histogram(binwidth = 2,fill="blue",color="orange")
```

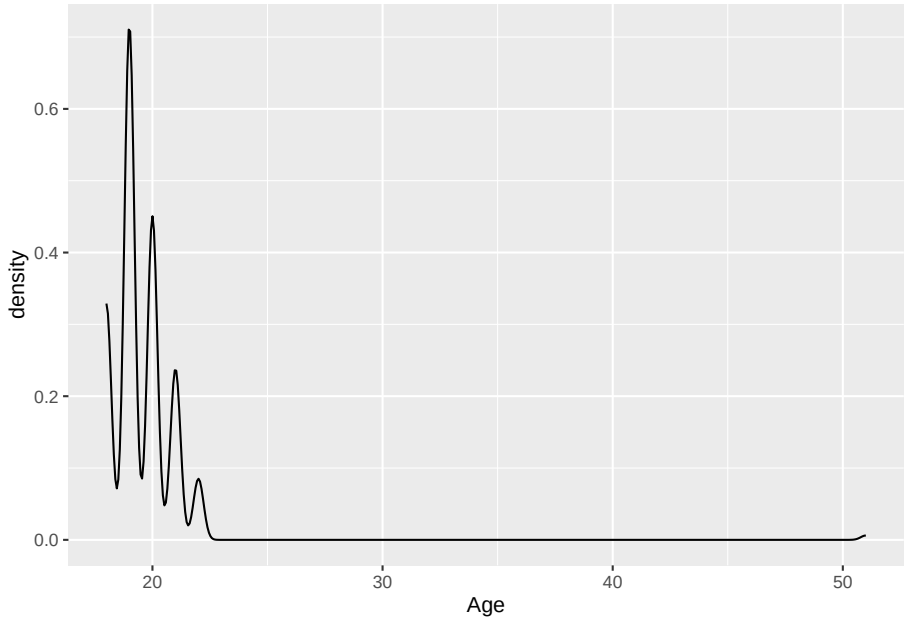


Each bar now contains two ages: the first bar contains the 18 and 19 year olds. Notice how the shape has been changed a little bit from the previous histogram with a different binwidth?

2.3.4 Density plots

Density plots are a variation of histograms, where the plot attempts to use a smooth mathematical function to approximate the shape of the histogram, and is unaffected by binwidth:

```
ggplot(Data,aes(x=Age))+
  geom_density()
```



We can see that 19 and 20 year olds are the most common ages in this data. Be careful in interpreting the values on the veritical axis: these do not represent proportions. A characteristic of density plots is that the area under the plot is always one.

2.4 Bivariate Visualizations

We will now look at visualizations we can create to explore the relationship between two variables. The term bivariate means that we are looking at two variables.

We will be using a new dataset as an example, so we clear the environment:

```
rm(list = ls())
```

We will be using the dataset, `gapminder`, from the `gapminder` package. Install and load the `gapminder` package. Also load the `tidyverse` package (which automatically loads the `ggplot2` package):

```
library(tidyverse)
library(gapminder)
```

We can take a look at the `gapminder` dataset:

```
gapminder[1:15,]
```

```
## # A tibble: 15 x 6
```

```
##    country     continent  year lifeExp      pop gdpPercap
##    <fct>       <fct>     <int>   <dbl>    <int>     <dbl>
##  1 Afghanistan Asia       1952    28.8  8425333      779.
##  2 Afghanistan Asia       1957    30.3  9240934      821.
##  3 Afghanistan Asia       1962    32.0 10267083      853.
##  4 Afghanistan Asia       1967    34.0 11537966      836.
##  5 Afghanistan Asia       1972    36.1 13079460      740.
##  6 Afghanistan Asia       1977    38.4 14880372      786.
##  7 Afghanistan Asia       1982    39.9 12881816      978.
##  8 Afghanistan Asia       1987    40.8 13867957      852.
##  9 Afghanistan Asia       1992    41.7 16317921      649.
## 10 Afghanistan Asia       1997    41.8 22227415      635.
## 11 Afghanistan Asia       2002    42.1 25268405      727.
## 12 Afghanistan Asia       2007    43.8 31889923      975.
## 13 Albania     Europe     1952    55.2  1282697     1601.
## 14 Albania     Europe     1957    59.3  1476505     1942.
## 15 Albania     Europe     1962    64.8  1728137     2313.
```

Per the documentation, the variables are

1. `country`
2. `continent`
3. `year`: from 1952 to 2007 in increments of 5 years
4. `lifeExp`: life expectancy at birth, in years
5. `pop`: population of country
6. `gdpPercap`: GDP per capita in US dollars, adjusted for inflation

We notice that data are collected from each country across a number of different years: 1952 to 2007 in increments of five years. For this example, we will mainly focus on the data for the most recent year, 2007:

```
Data<-gapminder%>%
  filter(year==2007)
```

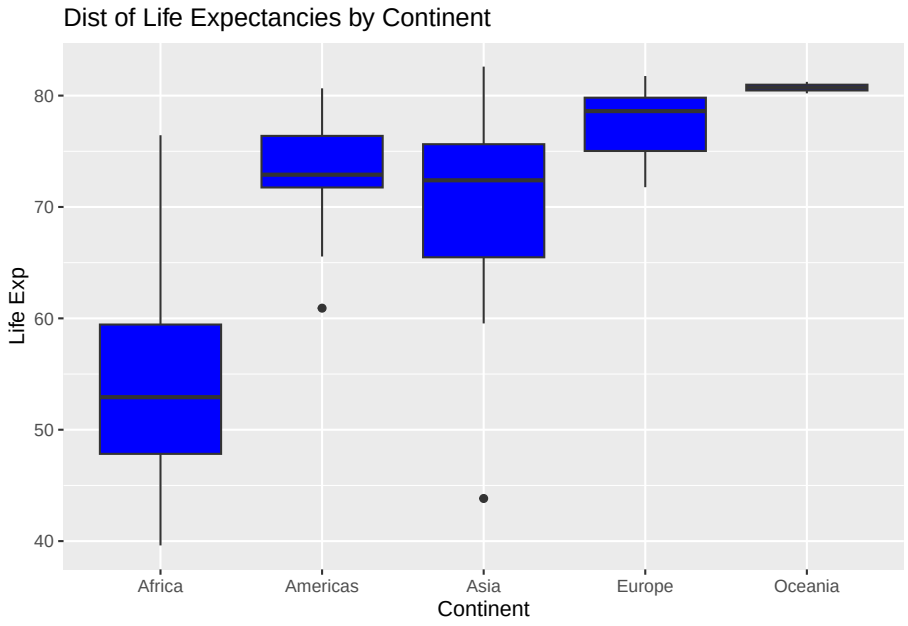
The specific visuals to use will again depend on the type of variables we are using, whether they are categorical or quantitative.

2.4.1 Compare quantitative variable across categories

2.4.1.1 Side by side boxplots

Side by side boxplots are useful to compare a quantitative variable across different classes of a categorical variable. For example, we want to compare life expectancies across the different continents in the year 2007:

```
ggplot(Data, aes(x=continent, y=lifeExp))+
  geom_boxplot(fill="Blue")+
  labs(x="Continent", y="Life Exp", title="Dist of Life Expectancies by Continent")
```

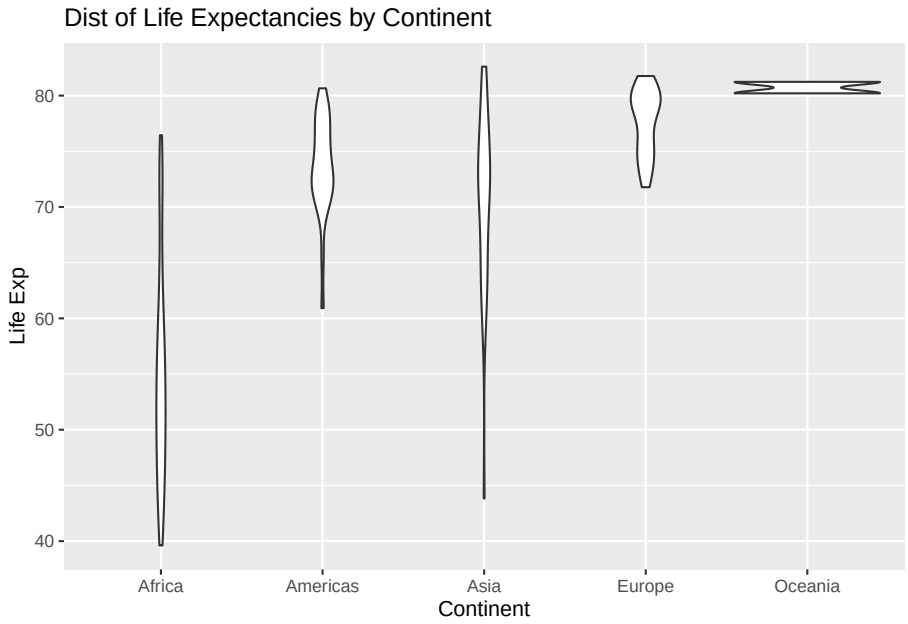


Countries in the Oceania region have long life expectancies with little variation. Comparing the Americas and Asia, the median life expectancies are similar, but the spread is larger for Asia.

2.4.1.2 Violin plots

Violin plots are an alternative to boxplots. To create these plots to compare life expectancies across the different continents in the year 2007:

```
ggplot(Data, aes(x=continent, y=lifeExp))+
  geom_violin()+
  labs(x="Continent", y="Life Exp", title="Dist of Life Expectancies by Continent")
```



The width of the violin informs us which values are more commonly occuring. For example, look at the violin for Europe. The violin is wider at higher life expectancies, so longer life expectancies are more common in European countries.

2.4.2 Summarizing two categorical variables

For this example, we create a new binary variable called `expectancy`, which will be denoted as `low` if the life expectancy in the country is less than 70 years, and `high` otherwise:

```
Data<-Data%>%
  mutate(expectancy=ifelse(lifeExp<70,"Low","High"))
```

2.4.2.1 Two-way tables

Suppose we want to see how `expectancy` varies across the continents. A two-way table can be created for produce counts when two categorical variables are involved:

```
mytab2<-table(Data$continent, Data$expectancy)
##continent in rows, expectancy in columns
mytab2
```

```
##
##            High Low
##   Africa      7  45
```

```
##    Americas   22   3
##    Asia       22  11
##    Europe     30   0
##    Oceania     2   0
```

The first variable in `table()` will be placed in the rows, the second variable will be placed in the columns.

From this table, we can see that 22 countries in the Americas have high life expectancies, while 3 countries in the Americas have low life expectancies.

We may be interested in looking at the proportions, instead of counts, of countries in each continent that have high or low life expectancies. To convert this table to proportions, we can use `prop.table()`:

```
prop.table(mytab2, 1)
```

```
##
##                 High       Low
##    Africa   0.1346154 0.8653846
##    Americas 0.8800000 0.1200000
##    Asia     0.6666667 0.3333333
##    Europe   1.0000000 0.0000000
##    Oceania  1.0000000 0.0000000
```

We want proportions for each continent, so we want proportions in each row to add up to 1. Therefore, the second argument in `prop.table()` is 1. Enter 2 for this argument if we want the proportions in each column to add up to 1.

As before, to convert to percentages and round to 2 decimal places:

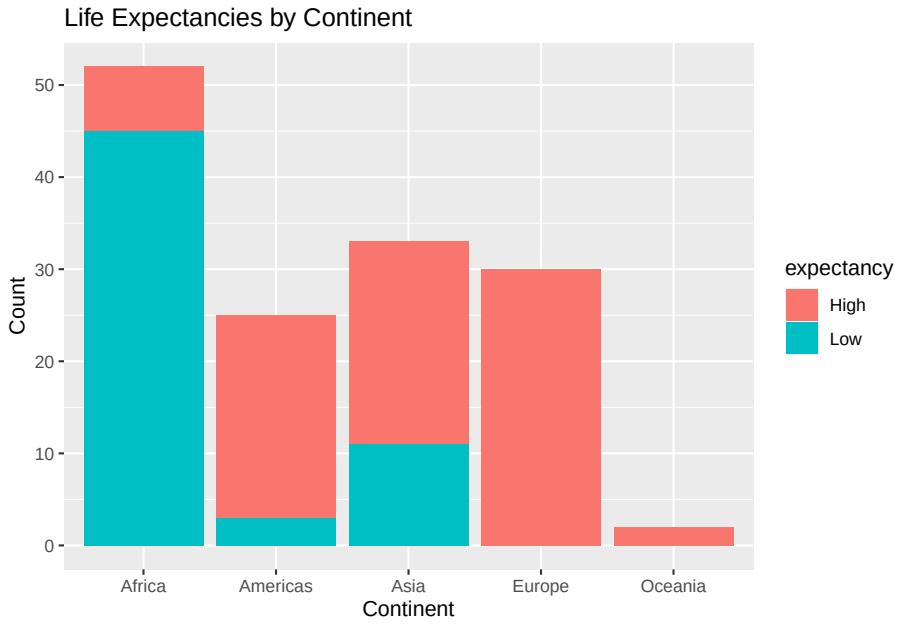
```
round(prop.table(mytab2, 1) * 100, 2)
```

```
##
##               High    Low
##    Africa    13.46  86.54
##    Americas  88.00  12.00
##    Asia      66.67  33.33
##    Europe   100.00   0.00
##    Oceania  100.00   0.00
```

2.4.2.2 Bar charts

A stacked bar chart can be used to display the relationship between the binary variable `expectancy` across continents:

```
ggplot(Data, aes(x=continent, fill=expectancy))+
  geom_bar(position = "stack")+
  labs(x="Continent", y="Count", title="Life Expectancies by Continent")
```

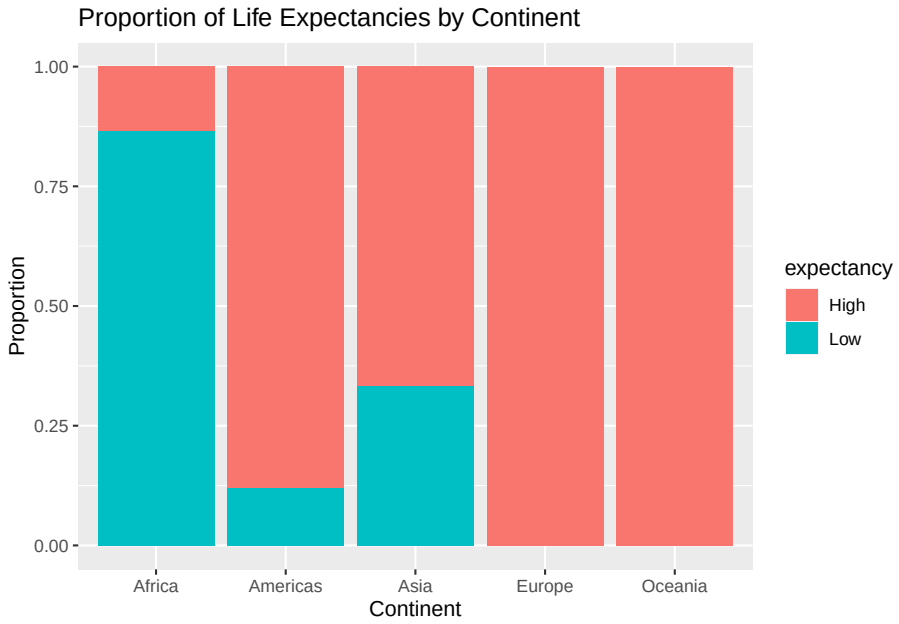


We can see how many countries exist in each continent, and how many of these countries in each continent have high or low life expectancies. For example, there are about 25 countries in the Americas with the majority having high life expectancies.

We can change the way the bar chart is displayed by changing `position` in `geom_bar()` to `position = "dodge"` or `position = "fill"`, the latter being more useful for proportions instead of counts:

```
ggplot(Data, aes(x=continent, fill=expectancy))+
  geom_bar(position = "dodge")
```

```
ggplot(Data, aes(x=continent, fill=expectancy))+
  geom_bar(position = "fill")+
  labs(x="Continent", y="Proportion",
       title="Proportion of Life Expectancies by Continent")
```

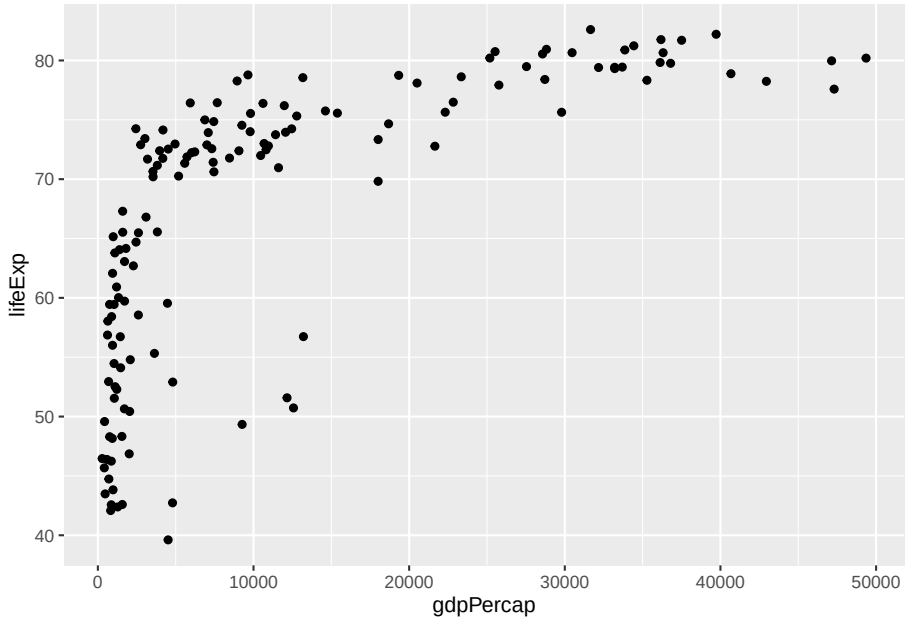


2.4.3 Summarizing two quantitative variables

2.4.3.1 Scatterplots

Scatterplots are the standard visualization when two quantitative variables are involved. To create a scatterplot for life expectancy against GDP per capita:

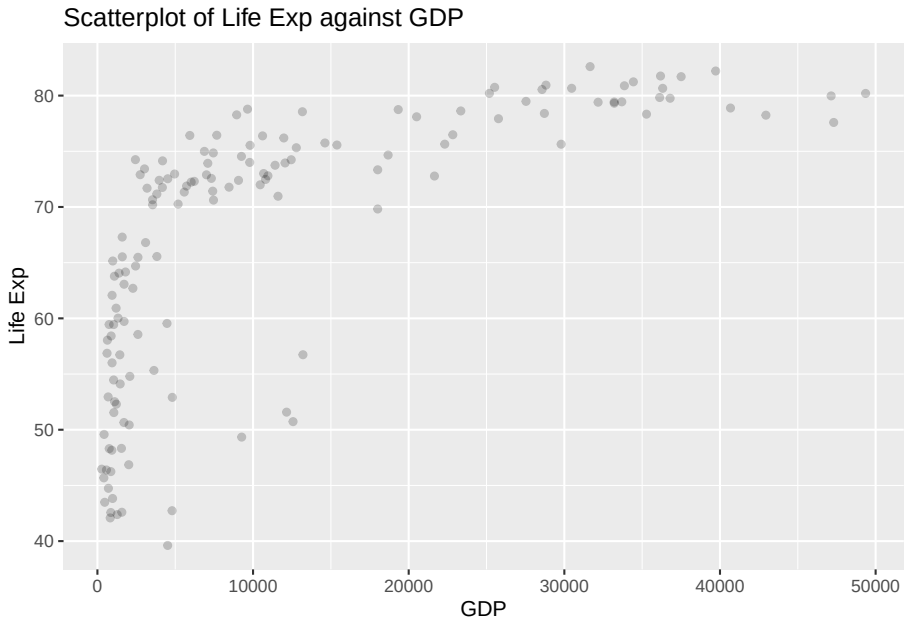
```
ggplot(Data, aes(x=gdpPercap,y=lifeExp))+
  geom_point()
```



We see a curved relationship between life expectancies and GDP per capita. Countries with higher GDP per capita tend to have longer life expectancies.

When there are many observations, plots on the scatterplot may actually overlap each other. To have a sense of how many of these exist, we can add a transparency scale called `alpha=0.2` inside `geom_point()`:

```
ggplot(Data, aes(x=gdpPercap,y=lifeExp))+
  geom_point(alpha=0.2)+
  labs(x="GDP", y="Life Exp",
       title="Scatterplot of Life Exp against GDP")
```



The default value for `alpha` is 1, which means the points are not at all transparent. The closer this value is to 0, the more transparent the points are. A darker point indicates more observations with those specific values on both variables.

2.5 Multivariate Visualizations

We will now look at visualizations we can create to explore the relationship between multiple (more than two) variables. The term multivariate means that we are looking at more than two variables.

2.5.1 Bar charts

Previously, we created a bar chart to look at how `expectancy` varies across the continents. Suppose we want to see how these bar graphs vary across the years, so we use the `year` variable as a third variable via a layer `facet_wrap`:

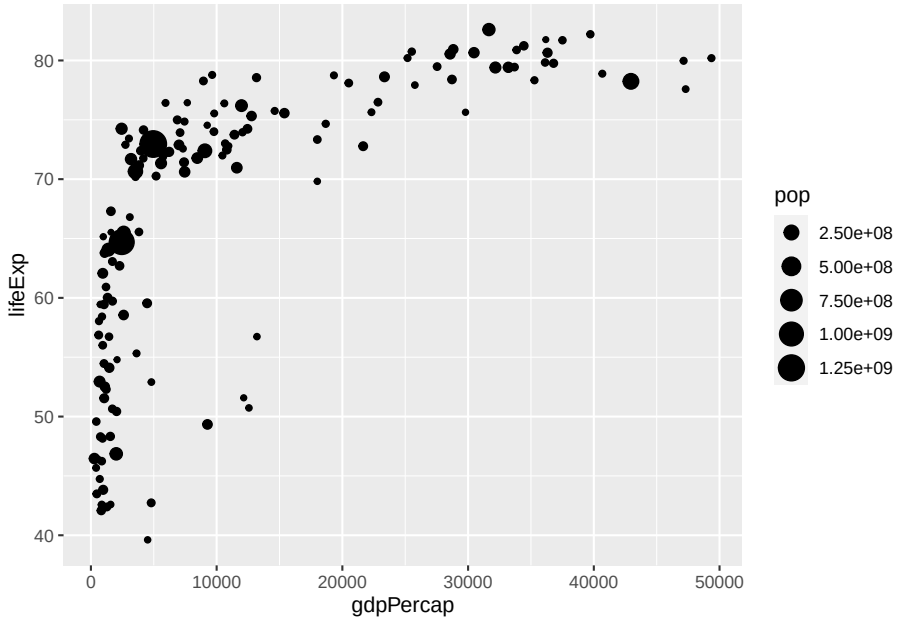
```
##another data frame across all years plus a binary variable
##for expectancy
Data.all<-gapminder%>%
  mutate(expectancy=ifelse(lifeExp<70,"Low","High"))

ggplot(Data.all,aes(x=continent, fill=expectancy))+
  geom_bar(position = "fill")+
  facet_wrap(~year)
```

Notice that three categorical variables are summarized in this bar chart. Is there something that can be done to improve this bar chart? How would you make this improvement?

2.5.2 Scatterplots

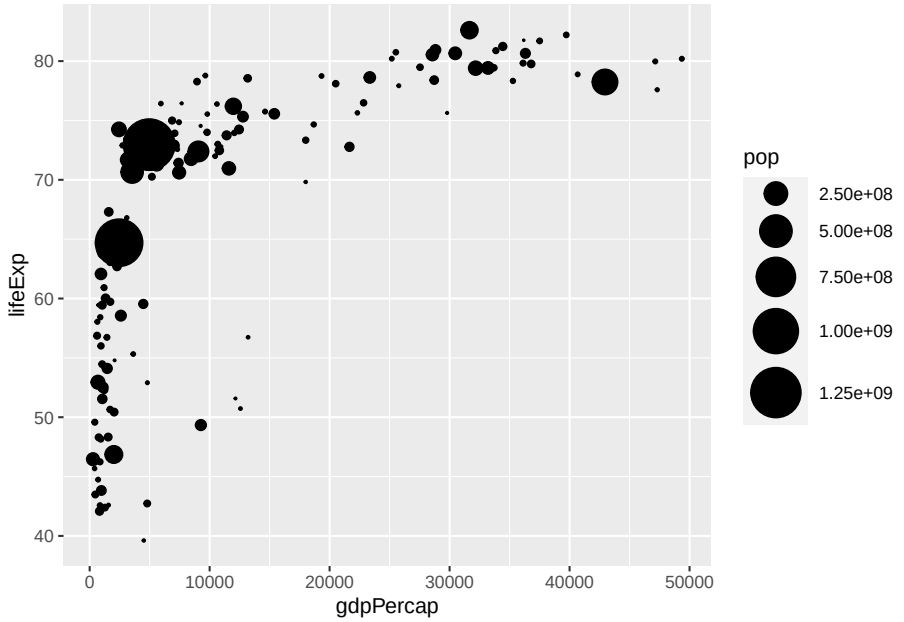
Previously, we created a scatterplot of life expectancy against GDP per capita. We can include another quantitative variable in the scatterplot, by using the size of the plots. We can use the size of the plots to denote the population of the countries. This is supplied via `size` in `aes()`:

```
ggplot(Data, aes(x=gdpPercap, y=lifeExp, size=pop))+
  geom_point()
```



We can adjust the size of the plots by adding a layer `scale_size()`:

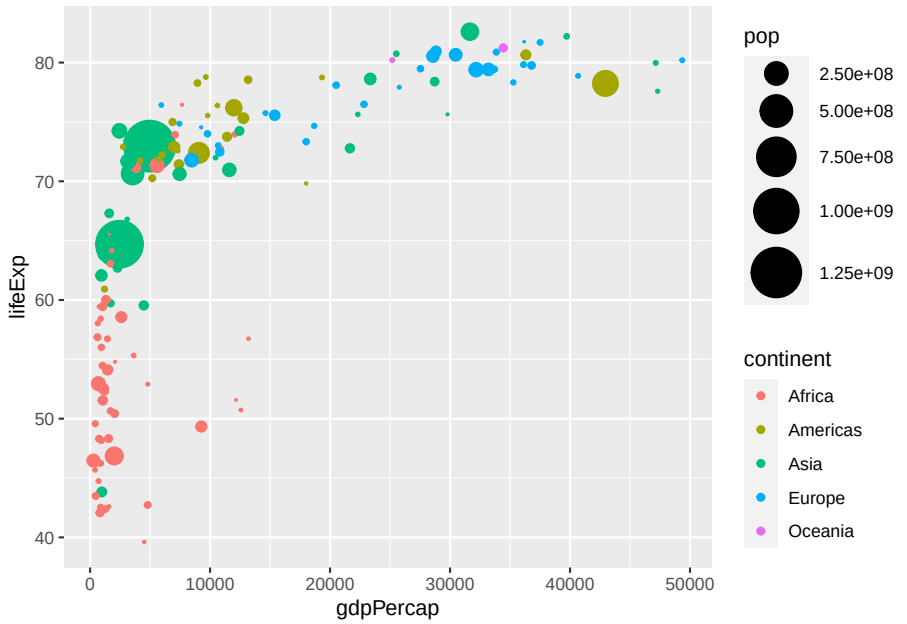
```
ggplot(Data, aes(x=gdpPercap, y=lifeExp, size=pop))+
  geom_point()+
  scale_size(range = c(0.1,12))
```



This scatterplot summarizes three quantitative variables.

We can use different-colored plots to denote which continent each point belongs to:

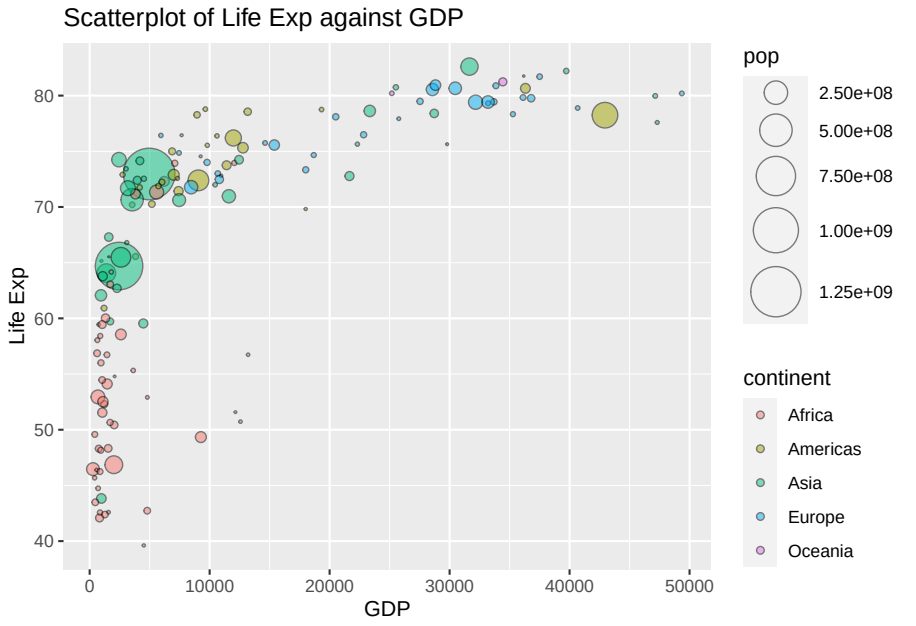
```
ggplot(Data, aes(x=gdpPercap, y=lifeExp, size=pop, color=continent))+
  geom_point()+
  scale_size(range = c(0.1,12))
```



This scatterplot summarizes three quantitative variables and one categorical variable.

We can adjust the plots by changing its shape and making it more translucent via `shape` and `alpha` in `aes()`:

```
ggplot(Data, aes(x=gdpPercap, y=lifeExp, size=pop, fill=continent))+
  geom_point(shape=21, alpha=0.5)+
  scale_size(range = c(0.1,12))+
  labs(x="GDP", y="Life Exp", title="Scatterplot of Life Exp against GDP")
```


Scatterplot of Life Exp against GDP
Life Exp
80
70
60
50
40
0
10000
20000
30000
40000
50000
GDP
pop
2.50e+08
5.00e+08
7.50e+08
1.00e+09
1.25e+09
continent
Africa
Americas
Asia
Europe
Oceania

Chapter 3

Introduction

You can label chapter and section titles using `{#label}` after them, e.g., we can reference Chapter 3. If you do not manually label them, there will be automatic labels anyway, e.g., Chapter 5.

Figures and tables with captions will be placed in `figure` and `table` environments, respectively.

```
par(mar = c(4, 4, .1, .1))
plot(pressure, type = 'b', pch = 19)
```

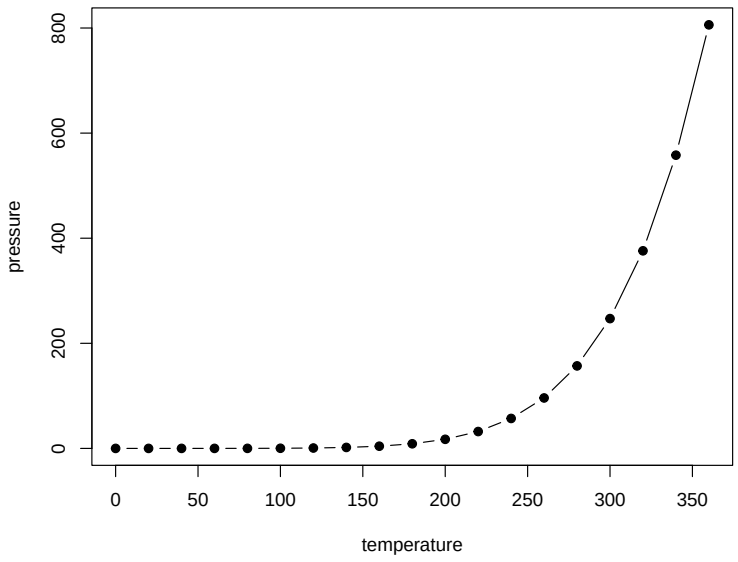


Figure 3.1: Here is a nice figure!

Table 3.1: Here is a nice table!

Sepal.Length	Sepal.Width	Petal.Length	Petal.Width	Species
5.1	3.5	1.4	0.2	setosa
4.9	3.0	1.4	0.2	setosa
4.7	3.2	1.3	0.2	setosa
4.6	3.1	1.5	0.2	setosa
5.0	3.6	1.4	0.2	setosa
5.4	3.9	1.7	0.4	setosa
4.6	3.4	1.4	0.3	setosa
5.0	3.4	1.5	0.2	setosa
4.4	2.9	1.4	0.2	setosa
4.9	3.1	1.5	0.1	setosa
5.4	3.7	1.5	0.2	setosa
4.8	3.4	1.6	0.2	setosa
4.8	3.0	1.4	0.1	setosa
4.3	3.0	1.1	0.1	setosa
5.8	4.0	1.2	0.2	setosa
5.7	4.4	1.5	0.4	setosa
5.4	3.9	1.3	0.4	setosa
5.1	3.5	1.4	0.3	setosa
5.7	3.8	1.7	0.3	setosa
5.1	3.8	1.5	0.3	setosa

Reference a figure by its code chunk label with the `fig:` prefix, e.g., see Figure 3.1. Similarly, you can reference tables generated from `knitr::kable()`, e.g., see Table 3.1.

```
knitr::kable(
  head(iris, 20), caption = 'Here is a nice table!',
  booktabs = TRUE
)
```

You can write citations, too. For example, we are using the **bookdown** package (Xie, 2023) in this sample book, which was built on top of R Markdown and **knitr** (Xie, 2015).

Chapter 4

Literature

Here is a review of existing methods.

4.1 New section

Add some text here. BLAH BLAH

Chapter 5

Methods

We describe our methods in this chapter.

Math can be added in body using usual syntax like this

5.1 math example

p is unknown but expected to be around 1/3. Standard error will be approximated

$$SE = \sqrt{(\frac{p(1-p)}{n})} \approx \sqrt{\frac{1/3(1-1/3)}{300}} = 0.027$$

You can also use math in footnotes like this[1].

We will approximate standard error to 0.027[2]

[1] where we mention $p = \frac{a}{b}$

[2] p is unknown but expected to be around 1/3. Standard error will be approximated

$$SE = \sqrt{(\frac{p(1-p)}{n})} \approx \sqrt{\frac{1/3(1-1/3)}{300}} = 0.027$$

Chapter 6

Applications

Some *significant* applications are demonstrated in this chapter.

6.1 Example one

6.2 Example two

Chapter 7

Final Words

We have finished a nice book.

Bibliography

Xie, Y. (2015). *Dynamic Documents with R and knitr*. Chapman and Hall/CRC, Boca Raton, Florida, 2nd edition. ISBN 978-1498716963.

Xie, Y. (2023). *bookdown: Authoring Books and Technical Documents with R Markdown*. R package version 0.34.