Homework #1 Answer Key

1. a) If a^2 is not irrational or $2a$ is not irrational, then a is not irrational.

   b) If r^2 is not a rational number, then r is not a rational number.

   c) If the candidate cannot parallel park, then the candidate does not pass the driver's test.

2. a) Hypothesis: n is an odd integer.
   Conclusion: n^2 is odd.

   b) Hypothesis: n is a positive integer
   Conclusion: the sum of the first n positive integers is $\frac{n(n+1)}{2}$.

   c) Hypothesis: the candidate can read the line Q Z S P M W 4.
   Conclusion: the candidate can pass thie vision test.

3. a) If $n=1$, then $(n-1)(n-2)=0$.

b) If $n=2$, then $n^2-n-2=0$.

c) If $n^2-4n+4=0$, then $n=2$.

4. a) Converse: If $2<5$, then $\sqrt{2}<\sqrt{5}$.

Contrapositive: If $2\geq 5$, then $\sqrt{2}\geq\sqrt{5}$.

b) Converse: If $\sqrt{2}\geq\sqrt{5}$, then $2\geq 5$.

Contrapositive: If $\sqrt{2}<\sqrt{5}$, then $2<5$.

5. For example (many possible solutions):

① A is true, B is true, then "A or B" is a true statement.

EXAMPLE A: $2+3=5$ (true)

B: $4-1=3$ (true)

$\boxed{2+3=5 \quad \text{or} \quad 4-1=3}$ is a true statement.

② A is true, B is false, then "A or B" is a true statement.

EXAMPLE A: Some dogs are brown. (true!)

B: Josh doesn't want dinner. (false!)

$\boxed{\text{Some dogs are brown or Josh doesn't want dinner}}$

is a true statement.

(3) A is false, B is true, then "A or B" is a true statement.

<u>EXAMPLE</u> A: $2 \cdot 3 = 7$. (false)
B: 4 divides 96. (true)

$\boxed{2 \cdot 3 = 7 \text{ or } 4 \text{ divides } 96}$ is a true statement.

(4) A is false, B is false, then "A or B" is a false statement.

<u>EXAMPLE</u> A: $2 \cdot 3 = 5$ (false)
B: 4 divides 101 (false)

$\boxed{2 \cdot 3 = 5 \text{ or } 4 \text{ divides } 101}$ is a false statement.

6. (1) If $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$, then the hypothesis of L'Hopital's rule is satisfied (i.e. the hypothesis evaluates to a true statement)

Then: $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = "\frac{0}{0}".$

Thus, for this scenario which makes L'Hopital's rule true, the corresponding indeterminant form is "$\frac{0}{0}$".

(2) If $\lim_{x\to a} f(x) = +\infty$ and $\lim_{x\to a} g(x) = +\infty$,

then the hypothesis is satisfied, i.e. evaluates to true. As before,

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \text{"}\frac{+\infty}{+\infty}\text{"},$$

So we see that the corresponding indeterminate form is "$\frac{+\infty}{+\infty}$".

(3) If $\lim_{x\to a} f(x) = +\infty$ and $\lim_{x\to a} g(x) = -\infty$,

then, again, the hypothesis evaluates to true.

As before,

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \text{"}\frac{+\infty}{-\infty}\text{"}.$$

So the indeterminate form corresponding to this scenario is "$\frac{+\infty}{-\infty}$".

(4) If $\lim_{x\to a} f(x) = -\infty$ and $\lim_{x\to a} g(x) = +\infty$,

the hypothesis is again satisfied. As before,

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \text{"}\frac{-\infty}{+\infty}\text{"}.$$

Thus, the corresponding indeterminate form is "$\frac{-\infty}{-\infty}$".

⑤ If $\lim_{x\to a} f(x) = -\infty$ and $\lim_{x\to a} g(x) = -\infty$,

the hypothesis is satisfied. So since

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \text{"}\frac{-\infty}{-\infty}\text{"},$$

the corresponding indeterminate form is "$\frac{-\infty}{-\infty}$".

7. a) 2

b) 4

c) 3

d) 4

e) 2

f) 1