

This task is based on skew binary numbers. Everything you need to know is provided here.
 Prove incr-correct below. Remember that your goal is to complete the proof as fast as possible.

A **num** is a skew binary number. It consists of a list of digits 0 (represented by N0), 1 (N1), and 2 (N2), with the least significant (“ones”) digit at the start of the list.

```
digit := N0 | N1 | N2
num := list digit
```

To convert a skew binary number to a decimal number, we multiply the value of each digit by its weight, where the weight of the i th digit (zero-indexed) is $2^{i+1} - 1$. For example, [N1; N2] converts to $1 \times 1 + 2 \times 3 = 7$, while [N2; N1] converts to $2 \times 1 + 1 \times 3 = 5$. The conv function does this conversion formally:

```
weight (i : nat) : nat := 2i+1 - 1

value (n : digit) : nat := { 0 when n = N0
                             1 when n = N1
                             2 when n = N2

convi (ns : num) (i : nat) : nat := { 0 when ns = nil
                                       value n × weight i + convi ns (1 + i) when ns = n :: ns'

conv (ns : num) : nat := convi ns 0
```

A skew binary number is *canonical* if there is at most one N2, and it is to the left of any N1s. For example, [N0; N2] and [N2; N1] are canonical, while [N1; N2] is not.

```
no-twos (ns : num) : bool := { true when ns = nil
                               no-twos ns' when ns = N0 :: ns'
                               no-twos ns' when ns = N1 :: ns'
                               false when ns = N2 :: ns'

canon (ns : num) : bool := { true when ns = nil
                             canon ns' when ns = N0 :: ns'
                             no-twos ns' when ns = N1 :: ns'
                             no-twos ns' when ns = N2 :: ns'
```

YOUR TASK is to prove the theorem below about the correctness of this function for incrementing skew binary numbers:

```
incr1 (ns : num) : num := { [N1] when ns = nil
                             N1 :: ns' when ns = N0 :: ns'
                             N2 :: ns' when ns = N1 :: ns'
                             nil otherwise

incr2 (ns : num) : num := { N0 :: incr2 ns' when ns = N0 :: ns'
                             N0 :: incr1 ns' when ns = N2 :: ns'
                             nil otherwise

incr (ns : num) : num := { incr1 ns when no-twos ns
                           incr2 ns otherwise
```

We also provide this useful lemma:

Lemma (weight-S).

For all natural numbers i , we have $\text{weight } (1 + i) = 1 + 2 \times \text{weight } i$.

This is the theorem you need to prove:

Theorem (incr-correct).

For all nums ns , if $\text{canon } ns = \text{true}$, then $\text{conv } (\text{incr } ns) = 1 + \text{conv } ns$.