

This task is based on reversible programming languages. Everything you need to know is provided here. Prove *inv-correct* below. Remember that your goal is to complete the proof as fast as possible.

We define expressions, statements, and states:

```

expr := CONST bool
      | GET1
      | GET2

stmt := COND expr stmt stmt expr
      | FLIP
      | SEQ stmt stmt

state := bool

```

We define how expressions evaluate:

```

eval state expr bool

----- E-CONST
eval st (CONST b) b

----- E-GET1
eval st (GET1) st

----- E-GET2
eval st (GET2) (negb st)

```

And how statements execute:

```

exec state stmt state

----- E-COND1
eval st e1 true    exec st s1 st'    eval st' e2 true
-----
exec st (COND e1 s1 s2 e2) st'

----- E-COND2
eval st e1 false    exec st s2 st'    eval st' e2 false
-----
exec st (COND e1 s1 s2 e2) st'

----- E-FLIP
exec st FLIP (negb st)

----- E-SEQ
exec st s1 st'    exec st' s2 st''
-----
exec st (SEQ s1 s2) st''

```

YOUR TASK is to prove the theorem below about the correctness of this function for inverting statements:

$$\text{inv } (s : \text{stmt}) : \text{stmt} := \begin{cases} \text{COND } e_2 \text{ (inv } s_1) \text{ (inv } s_2) e_1 & \text{when } s = \text{COND } e_1 s_1 s_2 e_2 \\ \text{FLIP} & \text{when } s = \text{FLIP} \\ \text{SEQ (inv } s_2) \text{ (inv } s_1) & \text{when } s = \text{SEQ } s_1 s_2 \end{cases}$$

This is the theorem you need to prove:

Theorem (inv-correct).

For all statements s and states st, st' , if $\text{exec } st s st'$, then $\text{exec } st' (\text{inv } s) st$.