Test Next Wednesday

Sections 1-6

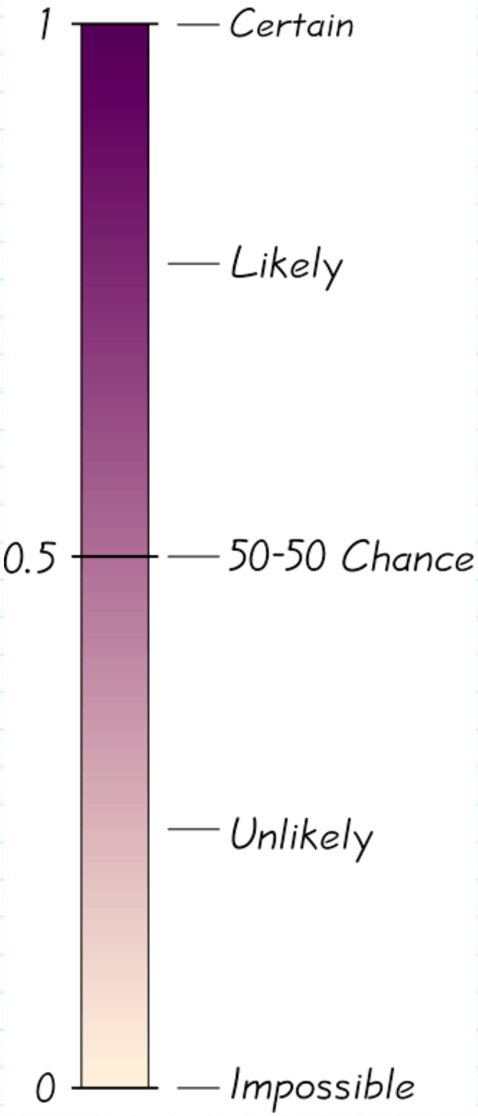
Competencies 1-5

Probability

Math 122 – Introduction to Statistics and Probability

Probability -- A Measure of Likelihood

- Every event has a probability which is a number from 0 to 1.
- Events with probabilities near 1 are likely to happen.
- Events with probabilities near 0 are unlikely to happen.



What is an event?

- We perform **experiments** or **procedures**
- An **event** is a collection of outcomes from a procedure
- A **simple event** is a single outcome of the procedure
- A **compound event** is made up of multiple simple events
- The set of all simple events for a procedure is the **sample space**

Flip 2 coins

- What is the sample space (all simple events)?

HH, HT, TT, TH

- What are some compound events?

At least one H is flipped.
The first flip is an H.
The coins land on different sides.

Roll a "Number Cube"

- What is the sample space?

1, 2, 3, 4, 5, 6

- What are some compound events?

Roll an even #.

Roll an odd #

Roll a # greater than 2

Flip One Coin and Roll a Number Cube

- What is the sample space?

Probability of an event

- Events are given names like A, B, C,…
- The probability of A is P(A).

  P(2 heads)
- The event that A does not happen is the **complement** of A and is denoted $\bar{A}$
- The probability that A does not happen is $P(\bar{A})=1-P(A)$.

Finding Probabilities
The Classical Approach

Only use when simple events are equally likely

Classical Approach

If all simple events are equally likely, then

$$P(A) = \frac{\text{Number of ways } A \text{ can happen}}{\text{Number of simple events}}$$

Flip 2 Coins

- Sample Space: {HH, HT, TH, TT} – 4
- What is the probability of two H's?

  $P(2\ Hs) = \frac{1}{4}$
- What is the probability of at least 1 H?

  $P(\text{at least } 1\ H) = 3/4$
- What is the probability of no H's?

  $P(\text{no } Hs) = 1/4$

GGG, GGB, GBG, GBB, BGG, BGB, BBG, ~~BBB~~

A family has three children.

If there is at least one G, what is the prob. there are at least 2? 4/7

- When considering genders of the children, what is the sample space?

GGG, GGB, GBG, GBB, BGG, BGB, BBG, BBB

- What is the probability that the family has no girls?

$P(\text{no girls}) = \frac{1}{8}$

- What is the probability that the family has at least one girl?

$P(\text{at least one G}) = \frac{7}{8}$

Relative Frequency $\boxed{\text{Approximation}}$

Relative Frequency Approximation

- To approximate the probability of an event A:
  - Repeat your procedure many times
  - Count the number of times A happens

$$P(A) \approx \frac{\text{number of times A happens}}{\text{number of times procedure repeated}}$$

Probability that a randomly selected car in the US is involved in a crash over the period of one year.

- In a recent year:
  - 135,670,000 cars registered in the US
  - 6,511,100 cars involved in crashes

$$P(\text{car in crash}) = \frac{6{,}511{,}100}{135{,}670{,}000} \approx 0.048$$

Law of Large Numbers

If a procedure is repeated many times, the relative frequency approximation of the probability of an event will tend to be close to the actual probability of the event.

Disjoint

- If two events A and B cannot happen at the same time, then they are **disjoint**.

- If A and B are disjoint, then

$$P(A \text{ or } B)=P(A) + P(B)$$

- If A and B are not disjoint, then

$$P(A \text{ or } B)=P(A) + P(B) - P(A \text{ and } B)$$

14 Students were asked if they colored their hair. These were the results:

	Color	No Color
Male	1 A	5 B
Female	6 C	2 D

$P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) = \frac{1}{14} + \frac{5}{14} + \frac{6}{14} = \frac{12}{14}$

What is the probability a randomly selected student here is either male or colors his/her hair?

$P(\text{Male or Color}) = P(\text{Male}) + P(\text{Color}) - P(\text{Male \underline{and} Color})$

$\frac{6}{14} + \frac{7}{14} - \frac{1}{14} = \frac{12}{14}$

In a family with three children, what is the probability that the first child is a girl or the last child is a girl?

$P(1^{st}\,G \text{ or } 3^{rd}\,G) = 6/8$

(List possibilities)

$P(1^{st}\,G \text{ or } 3^{rd}\,G) = P(1^{st}\,G) + P(3^{rd}\,G) - P(1^{st}\,G \text{ and } 3^{rd}\,G)$

$= \frac{1}{2} + \frac{1}{2} - \frac{2}{8}$

$= \frac{6}{8}$

G ___ G

Independent Events

Two events A and B are independent if the occurrence of one event does not affect the probability of the occurrence of the other.

Independent or Dependent?

- The two numbers which appear when a number cube is rolled twice.

Independent

- The two results when a penny and a quarter are flipped.

Independent

- Two coins are flipped. A is the event that no heads appear. B is the event that at least one head appears.

Dependent

Independent or Dependent?

- A bowl is full of colored marbles. A marble is selected. Its color is noted, and it is **replaced**. A second marble is selected, and its color is noted.
- Are the two colors independent or dependent? Independent
- This is selection **with replacement**.

Independent or Dependent?

- A bowl is full of colored marbles. A marble is selected. Its color is noted, and it is **not replaced**. A second marble is selected, and its color is noted.

- Are the two colors independent or dependent? Dependent

- This is selection **without replacement**.

Multiplication Rule

- If A and B are independent events, then

$$P(A \text{ } \underline{\underline{\text{and}}} \text{ } B) = P(A) \underset{\text{multiply}}{\uparrow} P(B)$$

R R R R R
B B B

Marbles

- A bowl contains 5 red and 3 blue marbles.
- Two marbles are selected from the bowl with replacement.
- What is the probability that the first is red and the second is blue?

$P(1^{st} \text{ red and } 2^{nd} \text{ Blue}) = P(1^{st} \text{ Red}) \times P(2^{nd} \text{ Blue})$

$= \frac{5}{8} \times \frac{3}{8} = \frac{15}{64}$

Multiplication Rule

- If A and B are independent events, then

$$P(A \text{ and } B) = P(A)\, P(B)$$

- **Informal**: To find P(A and B), use the equation above, but when P(B) is calculated, assume that A has already occurred.
- **More Formal**: If A and B are dependent

$$P(A \text{ and } B) = P(A)\, P(B|A)$$

- $P(B|A)$ = "the probability of B, given A"

R R R R ~~R~~
B B B

Marbles

- A bowl contains 5 red and 3 blue marbles.
- Two marbles are selected from the bowl **without** replacement.
- What is the probability that the first is red and the second is blue?

$P(1^{st} R \text{ and } 2^{nd} B) = P(1^{st} R) \times P(2^{nd} B)$

$= \frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$

In a batch of 1000 stereos 5 are defective. If two of the stereos are selected with replacement, what is the probability that **neither** is defective?

$P(\text{neither defective}) = P(1^{st} \text{ Good } \underline{\text{and}} \text{ } 2^{nd} \text{ Good})$

$= P(1^{st} \text{ Good}) \times P(2^{nd} \text{ Good})$

$= \frac{995}{1000} \times \frac{995}{1000}$

$= 0.990$

In a batch of 1000 stereos 5 are defective. If two of the stereos are selected with replacement, what is the probability that **at least one** is defective?

$$P(\text{at least } 1) = 1 - P(\text{none})$$

$$= 1 - 0.99$$

$$= 0.01$$

Redundancy

- A certain type of battery operated alarm clock works 80% of the time. Bob buys two of these clocks.
- What is the probability on a given morning that both clocks fail?

$P(\text{Both Fail}) = P(\text{clock 1 fails } \underline{\text{and}} \text{ clock 2 fails})$

$= P(\text{clock 1 fails}) \times P(\text{clock 2 fails})$

$= 0.2 \times 0.2$

$= 0.04$

Redundancy

- A certain type of battery operated alarm clock works 80% of the time. Bob buys two of these clocks.
- What is the probability that at least one works?

P(at least one works) = 1 − P(both fail)

$= 1 - .04$

$= 0.96$