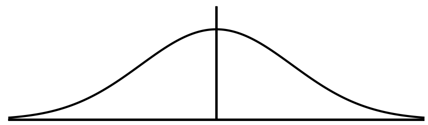
Math 122
Introduction to Statistics
Standard Normal Distribution

A **normal distribution** has a bell shaped density curve such as:



An equation for the density curve for a normal distribution with mean μ and standard deviation σ is

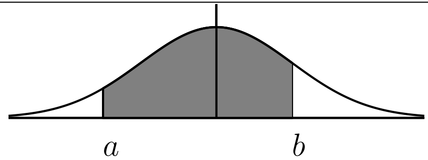
$$y = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}$$

The **standard normal distribution** is the normal distribution with mean $\mu = 0$ and standard deviation $\sigma = 1$. The density curve for the standard normal distribution is

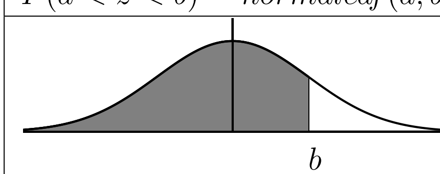
$$y = \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}}$$

The variable z is usually used for a random variable with the standard normal distribution.

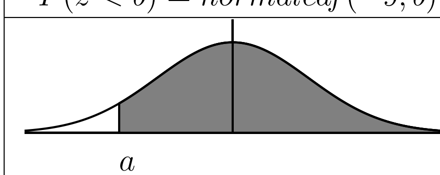
Probabilities: If z has a standard normal distribution, then to find probabilities associated with z, use the *normalcdf* function.



$P(a < z < b) = normalcdf(a, b)$



$P(z < b) = normalcdf(-9, b)$



$P(a < z) = normalcdf(a, 9)$

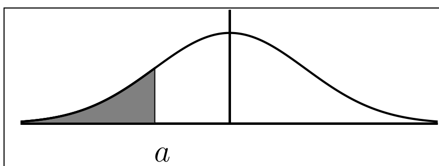
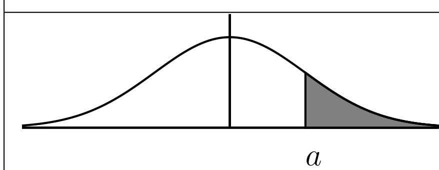
Example: A random variable z has a standard normal distribution.

1. Find $P(-1 < z < 2)$.
2. Find $P(z < 3)$.
3. Find $P(z > 2)$.

Solution

1. $P(-1 < z < 2) = normalcdf(-1, 2) = 0.8186$.
2. $P(z < 3) = normalcdf(-9, 3) = 0.9987$.
3. $P(z > 2) = normalcdf(2, 9) = 0.0228$.

The Inverse Normal Function: To find a value a so that $P(z < a) = \alpha$, use the $invNorm$ function.

	If the shaded region has area α, then $$a = invNorm(\alpha)$$
	If the shaded region has area α, then $$a = invNorm(1-\alpha) = -invnorm(\alpha)$$

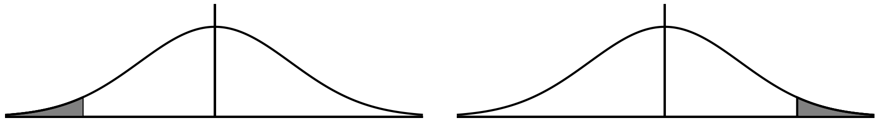
Example: A random variable z has a standard normal distribution.

1. Find a value a so that $P(z < a) = 0.1$.
2. Find a value a so that $P(z > a) = 0.05$.
3. Find values a and b which separate out the middle 95% of values of z.

Solution

1. $a = invnorm(0.1) = -1.2816$
2. $a = -invnorm(0.01) = 1.6449$
3. Here, we will use symmetry. The numbers a and b should be equal in magnitude since the standard normal curve is symmetric. The areas of the tails outside of a and b should each be 0.025. So $b = invnorm(0.025) = -1.96$ and $a = -b = 1.96$.

α **Tails:** Regions such as these



Are called tails. If the shaded area is α, then these are α tails. The lower bound of the right α tail is z_α. That is, z_α is the unique z-value so that $P(z > z_\alpha) = \alpha$. We can use the $invNorm$ function to find z_α.

$$z_\alpha = invNorm(1 - \alpha) = -invnorm(\alpha).$$

Example: A random variable z has a standard normal distribution.

1. Find $z_{0.05}$.
2. Find $z_{0.01}$.

Solution

1. $z_{0.05} = -invnorm(0.05) = 1.6449.$
2. $z_{0.01} = -invnorm(0.01) = 2.3263.$