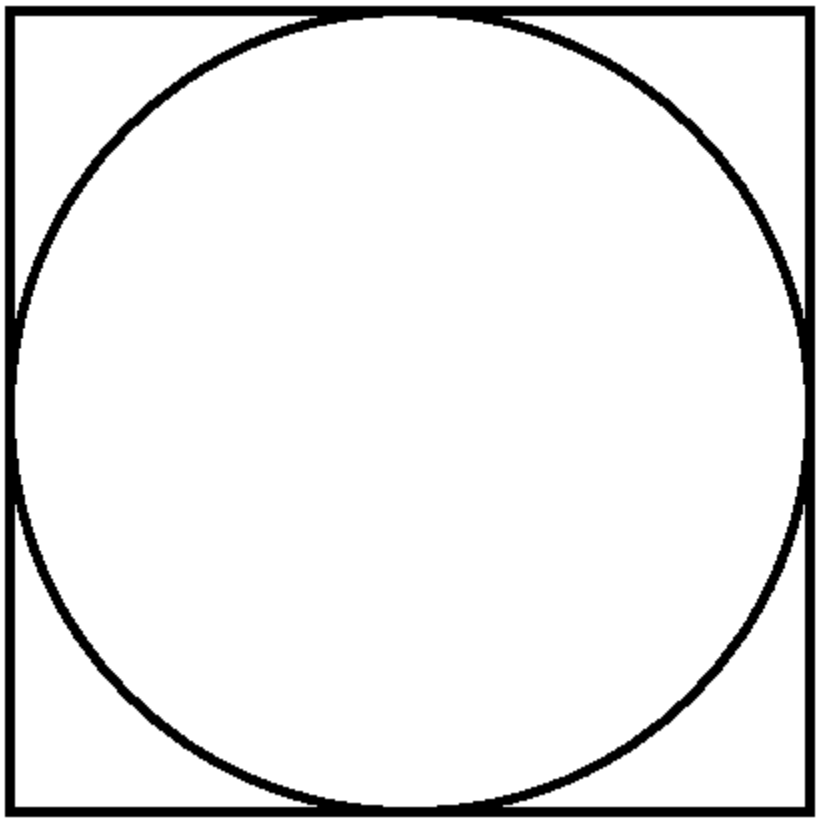
Continuous Distributions

Math 122

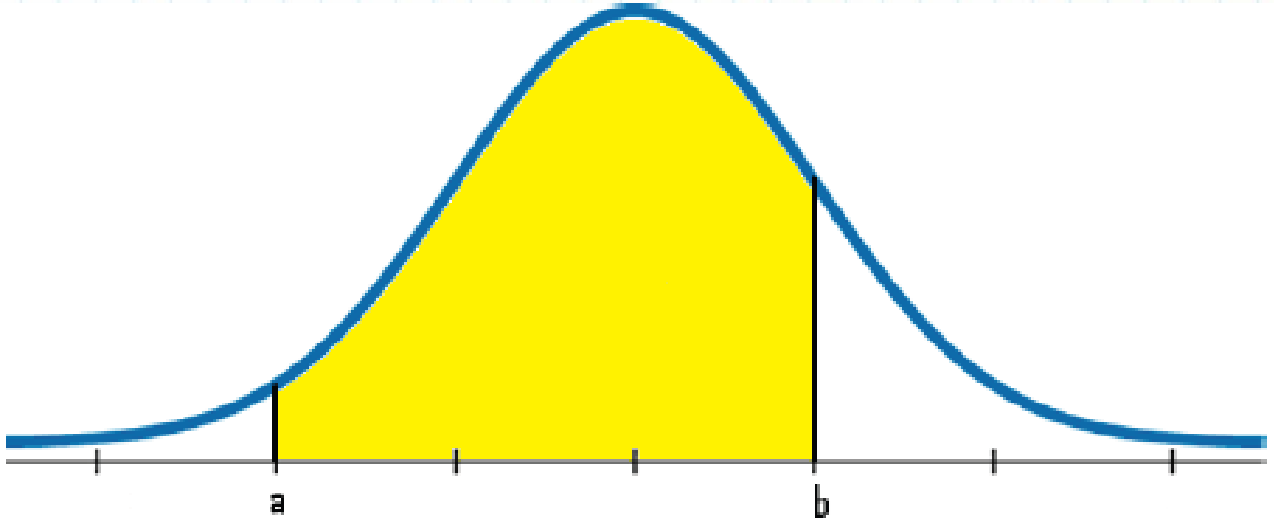
Binomial and Poisson

- Discrete Distributions
- Gaps between possible values
  - 0, 1, 2, 3, 4…
- Each possible value has nonzero probability

A marker is thrown at the square below. How could we find the probability that the marker hits the circle?



The total area under this curve is 1. A marker is thrown at the curve, what is the probability it hits the yellow region?

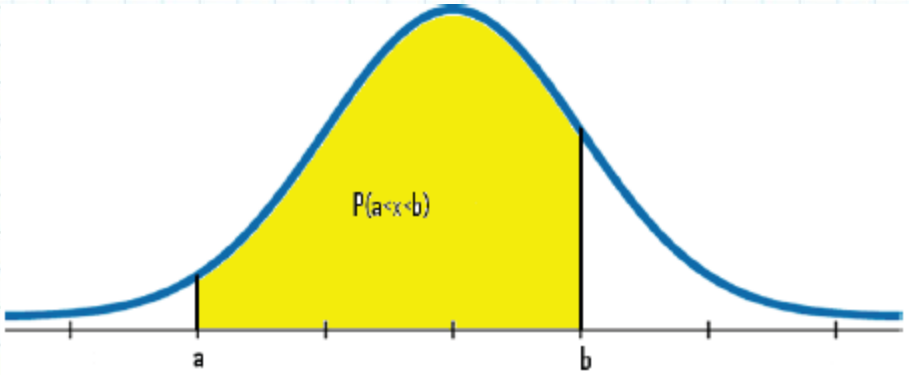


Continuous Random Variables

- Infinitely many possible values
- Possible values spread over a range with no gaps
- Probability of any single value 0
- Probability corresponds to area
- Consider $P(x<b)$ or $P(a<x)$ or $P(a<x<b)$

Probability = Area

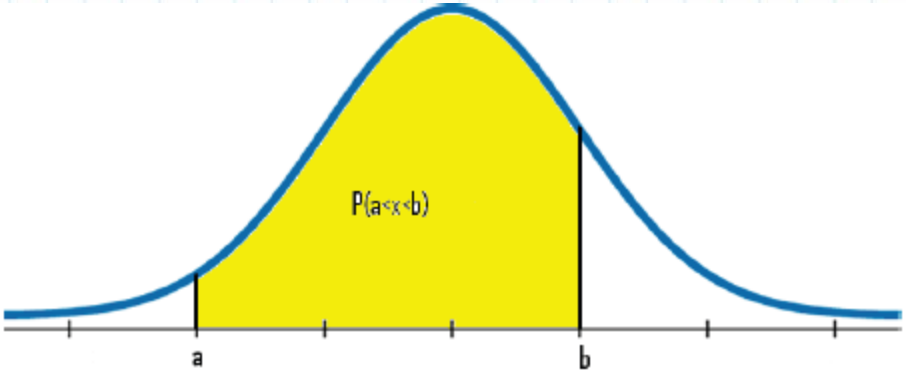
- Finding probabilities corresponds to finding areas under smooth curves such as



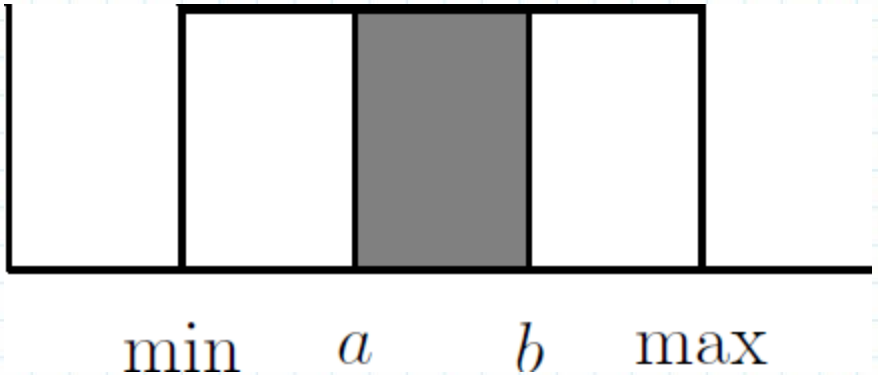
- The curve is called the **density curve** or **density function**

Density Functions

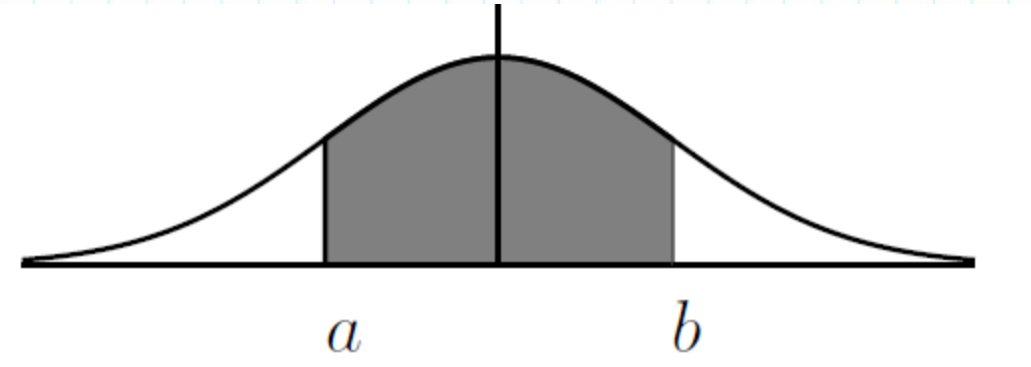
- Every continuous random variable has a density function
- The total area under the function is 1
- To find P(a<x<b), we find the area under the function between a and b



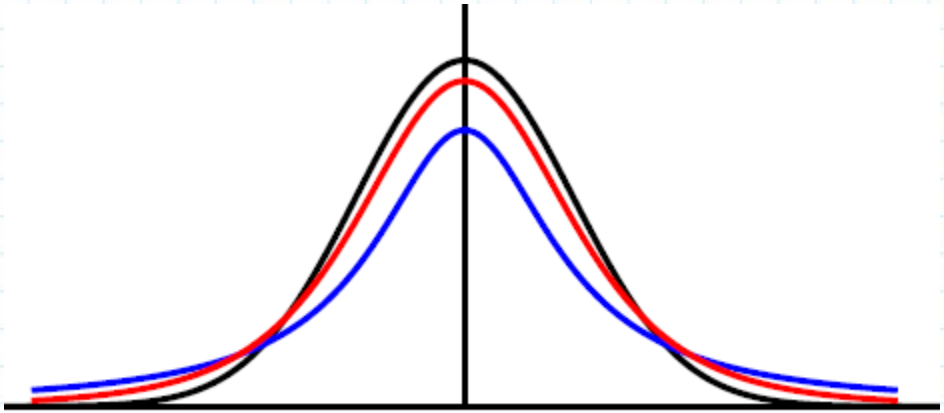
Uniform Distribution



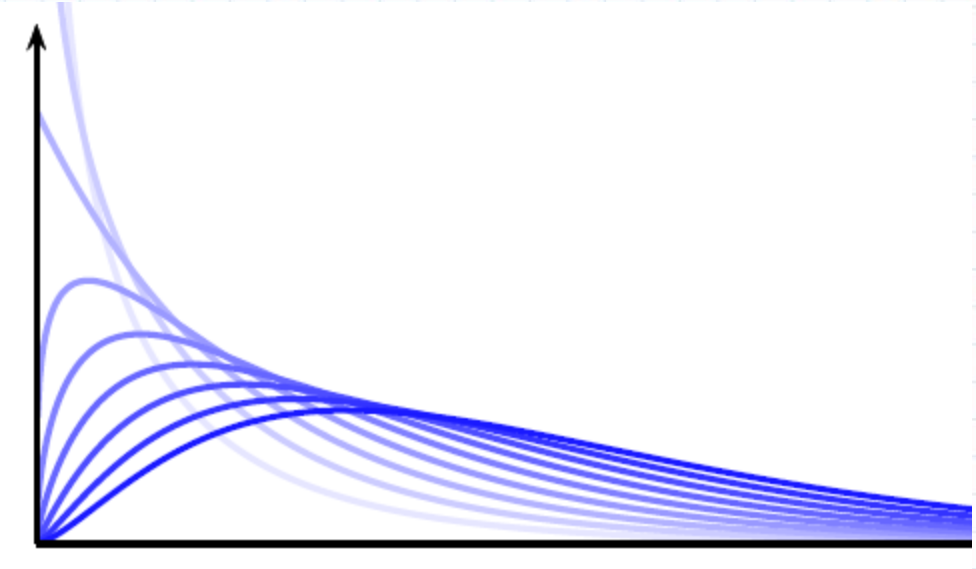
Normal Distribution



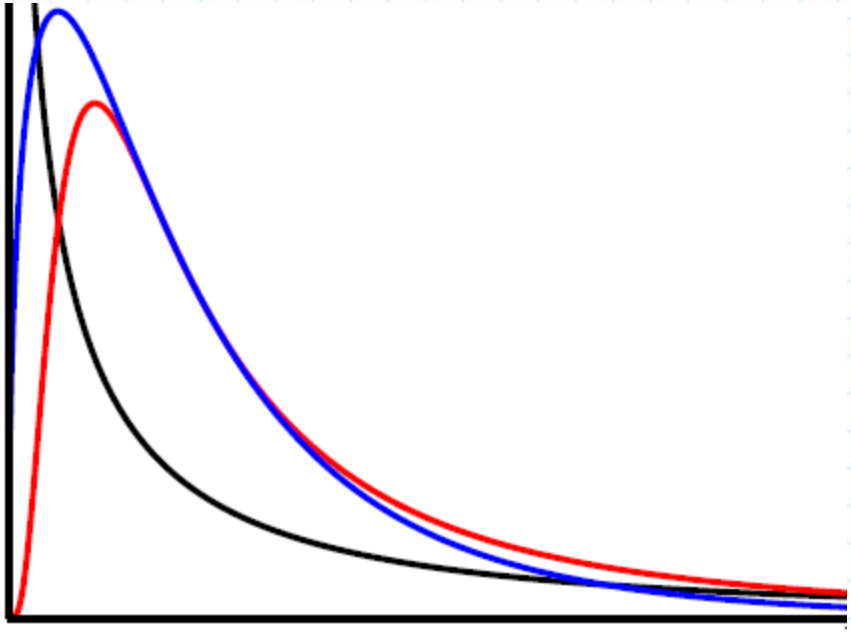
Student t Distributions



Chi-Squared Distributions

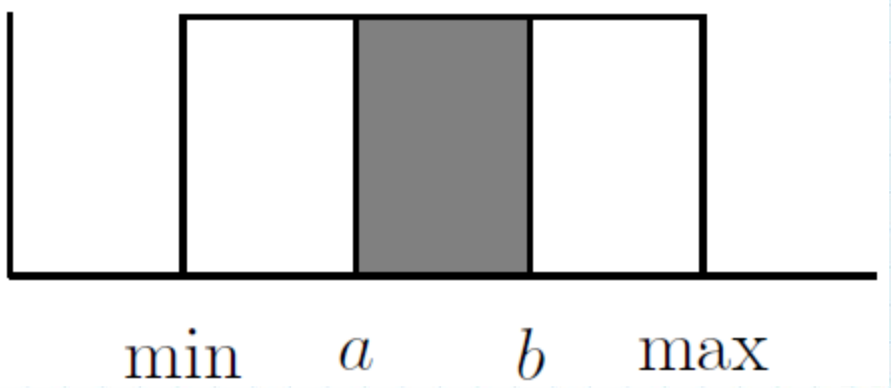


F Distributions



Uniform Distribution

- Density function is a horizontal line from the minimum value of the random variable to the maximum value.
- The height of the line is selected so that the total area under the curve is 1



A random variable x is uniformly distributed from 1 to 4.

- Find $P(x<3)$

- Find $P(1<x<2)$

A random variable x is uniformly distributed from 1 to 4.

- Find a so that $P(x<a)=0.1$

A random variable x is uniformly distributed from 1 to 4.

- Find a so that $P(x>a)=0.01$

A random variable x is uniformly distributed from 1 to 4.

- Find a and b which separate the middle 95% of values of x from the highest and lowest values.

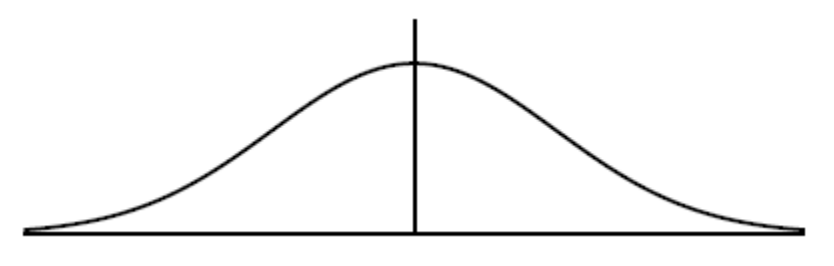
- The Newport Power and Light Company provides electricity with voltages uniformly distributed between 123 and 125 volts.

- Find the probability that a random voltage is greater than 124.5

- The Newport Power and Light Company provides electricity with voltages uniformly distributed between 123 and 125 volts.

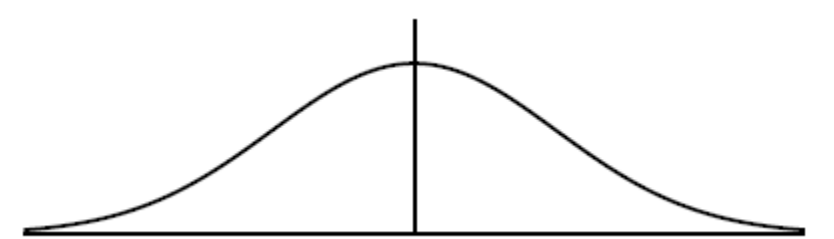
- Find a voltage which is greater than 99% of all voltages.

Normal Distributions



- The most important distribution for statistics.

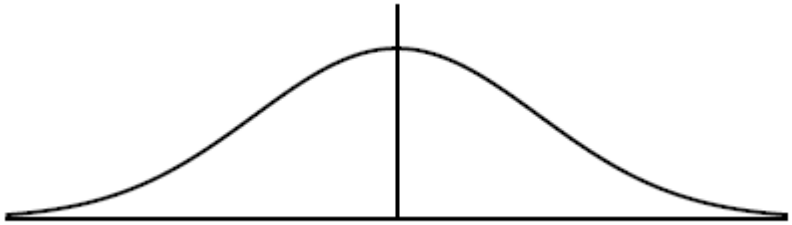
Normal Distribution with mean μ and standard deviation σ



$$y = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}$$

Standard Normal Distribution

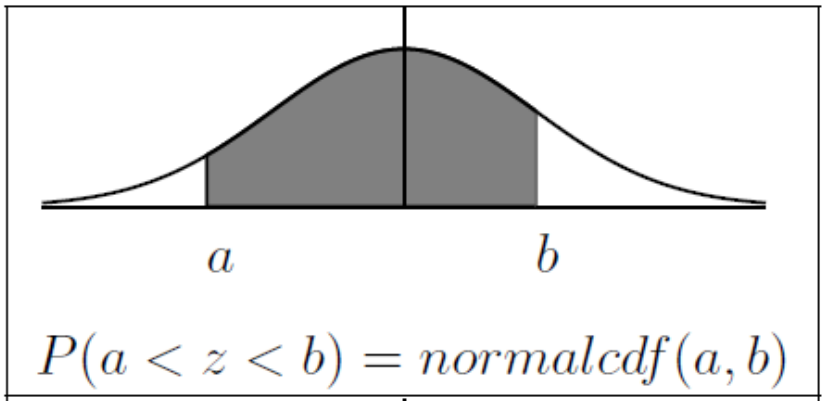
- Has mean μ=0 and standard deviation σ=1



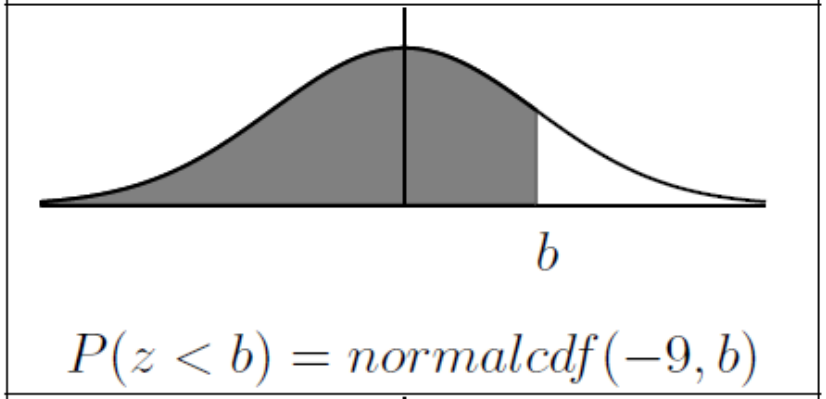
$$y = \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}}$$

- Usually use z for a standard normal distribution

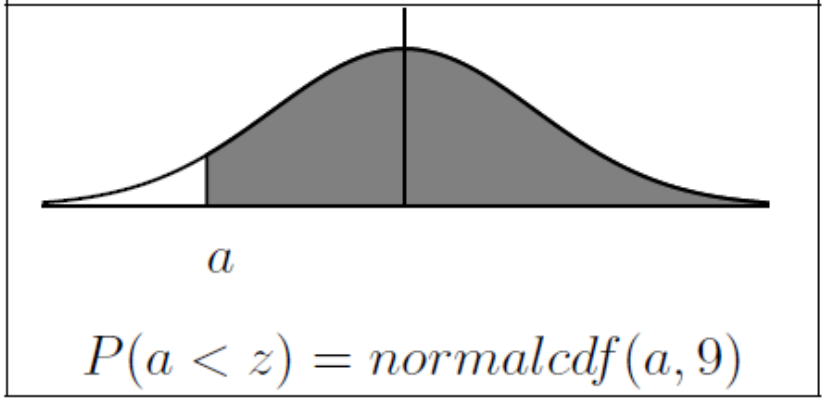
Standard Normal Distribution



$$P(a < z < b) = normalcdf(a, b)$$



$$P(z < b) = normalcdf(-9, b)$$

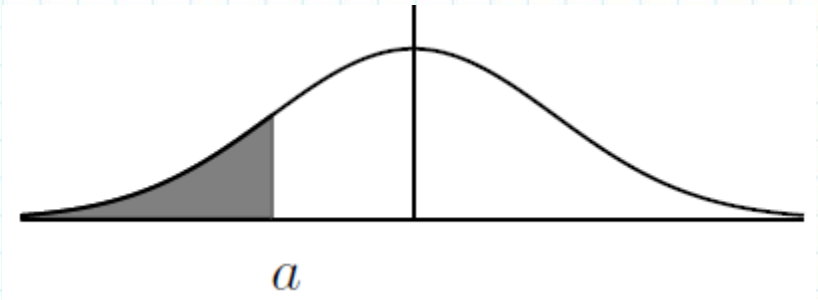


$$P(a < z) = normalcdf(a, 9)$$

Suppose that z has a standard normal distribution

Inverse Normal Function

- To find a number a so that P(x<a)=α use invnorm(α)



Suppose that z has a standard normal distribution

- Find a so that $P(z<a)=0.05$

- Find a so that $P(z>a)=0.1$

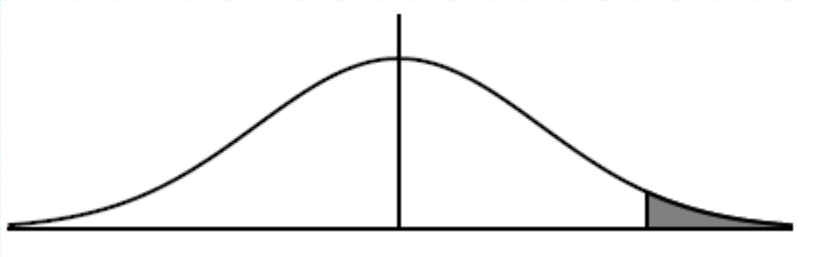
Suppose that z has a standard normal distribution

- Find a and b which separate the middle 95% of values of z from the rest.

α Tails

- The number z_α is the unique number so that

$$P(z > z_\alpha) = \alpha$$



- Find $z_{0.01}$

- Find $z_{0.005}$

Summary

- Probabilities for continuous random variables correspond to areas under density curves.
- The density curve of a uniform distribution is a horizontal line.
- The density curve of a normal distribution is a special bell curve.
- A standard normal distribution has mean 0 and standard deviation 1.
- Use normalcdf(a,b) to find P(a<z<b)