Math 122
Introduction to Statistics
TWO MEAN T TEST – INDEPENDENT MEANS

DIFFERENCES OF RANDOM VARIABLES

In this section we will test claims relating the means μ_1 and μ_2 of two random variables x_1 and x_2. Let $\bar{x}_1$ and $\bar{x}_2$ be sample means approximating μ_1 and μ_2. If $\bar{x}_1$ and $\bar{x}_2$ have(approximately) t distributions, then so does the difference $\bar{x}_1 - \bar{x}_2$. Moreover, we can express the mean and standard deviation of $\bar{x}_1 - \bar{x}_2$ in terms of the means and standard deviations of $\bar{x}_1$ and $\bar{x}_2$. Now, any claim about the relationship between μ_1 and μ_2 can be expressed as a claim about the difference $\mu_1 - \mu_2$. For example, the claim that $\mu_1 = \mu_2$ is equivalent to $\mu_1 - \mu_2 = 0$. Our approach, then, is to convert any claim about the relationship between μ_1 and μ_2 into a claim about $\mu_1 - \mu_2$ and then to perform a t test for this difference. Of course, the mechanics will be completely performed by technology.

CLAIMS ABOUT TWO TWO INDEPENDENT MEANS

We will consider claims about two means of these forms:

$$\mu_1 = \mu_2 \text{ or } \mu_1 \leq \mu_2 \text{ or } \mu_1 \geq \mu_2 \text{ or } \mu_1 \neq \mu_2 \text{ or } \mu_1 < \mu_2 \text{ or } \mu_1 > \mu_2.$$

The Null Hypothesis H_0 and the Alternative Hypothesis H_1 will always be one of these

H_0	H_1
$\mu_1 \leq \mu_2$	$\mu_1 > \mu_2$
$\mu_1 = \mu_2$	$\mu_1 \neq \mu_2$
$\mu_1 \geq \mu_2$	$\mu_1 < \mu_2$

Example: A sample of 43 male college basketball players had a mean height of 75.5 inches with a standard deviation of 3.1 inches. A sample of 62 male college students had a mean height of 70.1 inches with a standard deviation of 3.7 inches. Treat these samples as simple random samples and use this data to test the claim that male college basketball players are on average taller than other male college students.

1. **Parameters:** Define these symbols:
   - μ_1 is the average height of a male college basketball player.
   - $n_1 = 43$
   - $\bar{x}_1$ is the average height of a simple random sample of 43 male college basketball players.
   - s_1 is the standard deviation of the heights of a simple random sample of 43 male college basketball players.
   - μ_2 is the average height of a male college student.
   - $n_2 = 62$

- $\bar{x}_2$ is the average height of a simple random sample of 62 male college students.
- s_2 is the standard deviation of the heights of a simple random sample of 62 male college students.

Our observed values are

- $\bar{x}_1 = 75.5$ and $s_1 = 3.1$
- $\bar{x}_2 = 70.1$ and $s_2 = 3.7$

2. **Symbolic Claim:** The claim is $\mu_1 > \mu_2$.
3. **Opposite:** The opposite of the claim is $\mu_1 \leq \mu_2$.
4. H_0 **and** H_1**:** H_0 is $\mu_1 \leq \mu_2$ and H_1 is $\mu_1 > \mu_2$.
5. **Significance Level:** We will default to a significance level of $\alpha = 0.05$.
6. **Distribution and Test Statistic:** Our test statistic is a t-score associated to the difference between our sample proportions. Technology gives us a t-score of 8.0904.
7. P**-value:** Technology gives $P = 0$
8. **Formal Conclusion:** Since $P < \alpha$, we reject H_0.
9. **Final Conclusion:** Since our claim is the same as H_1, we first rephrase our formal conclusion to refer to H_1. Since we are rejecting H_0, we are supporting H_1. This means that we are supporting our claim. Our conclusion is:

   > There is enough sample evidence to support the claim that male college basketball players are on average taller than other male college students.

Interpretation: Male college basketball players seem to be on average taller than other male college students.