Hypothesis Testing

Math 122

Rare Event Rule

If an assumption H_0 implies that the probability of an observed event is exceptionally small, then that assumption is probably false.

Rare Event Rule

Suppose that an assumption H_0 implies that the probability of a certain event is 0.0001.

If we observe that event happening then either

1. H_0 is correct and we have observed a highly unlikely event, or
2. H_0 is not correct.

It is more likely that H_0 is false.

In any hypothesis test:

- We define two hypotheses H_0 (called the null hypothesis) and H_1 (called the alternative hypothesis).
- We collect data relevant to the claim.
- We assume H_0 and use this assumption to calculate the probability of seeing data as extreme as our data. This probability is P.
- If P is small, the observations are inconsistent with H_0. We reject H_0 and support H_1.
- If P is large, the observations are consistent with H_0. We do not reject H_0 and do not support H_1.

Steps in a Hypothesis Test

1. Define parameters.
2. State the claim in symbols.
3. State the opposite of the claim.
4. Determine H_0 and H_1.
5. Select a significance level α.
6. Decide on a distribution and test statistics.
7. Find a P-value.
8. Decide on a formal conclusion.
9. Restate the formal conclusion referring to the original claim.

P-value and formal conclusion

- If $\mathbf{P} \leq \boldsymbol{\alpha}$ then H_0 is not consistent with the observations.
  - Reject H_0 and support H_1.

- If $\mathbf{P} > \boldsymbol{\alpha}$ then H_0 is consistent with the observations.
  - Do not reject H_0 and do not support H_1.

Final Conclusion

- If your **claim is H_0** then your conclusion will be
  - There is enough sample evidence to **reject** the claim.
  - There is **not** enough sample evidence to **reject** the claim.
- If your **claim is H_1** then your conclusion will be
  - The evidence **supports** the claim.
  - The sample evidence does not **support** the claim.

Types of Claims We Will Test

(A whirlwind overview of the next 3 weeks)

All depend on some variant of the Central Limit Theorem

One Proportion

- Use
  - one proportion z-test or
  - 1-PropZTest

- **Example**: In a random sample of 87 Grey Forest Glow Worms, 51 were striped. Use this data to test the claim that **most Grey Forest Glow Worms are striped.**

Two Proportions

- Use
  - two proportion z-test or
  - 2-PropZTest

- **Example**: Among 107 male Grey Forest Glow Worms, 72 were striped. Among 96 female GFGWs, 37 were striped. Use this data to test the claim that **the proportion of male GFGWs which are striped is greater than the proportion of female GFGWs which are striped**.

One Mean

- Use
  - t-test or
  - T-Test

- **Example**: A sample of adult GFGWs had the lengths listed below (in inches). Use this data to test the claim that the mean length of an adult GFGW is not 1 inch.

- 0.75, 0,.82, 0.97, 0.99, 1.05, 1.17, 1.28, 1.35

Two Independent Means

- Use
  - two mean t-test or
  - 2-SampTTest

- **Example**: A random sample of 37 GFGWs had an average length of 1.11in with a standard deviation 0.09in. A random sample of 43 blue glow worms had an average length of 1.15in with a standard deviation of 0.12in. Test the claim that **these two types of worms have the same average length**.

Matched Pairs

- Use
  - matched pairs or
  - (a more complex process)

- **Example**: Below are listed the weights of several GFGWs (in oz) along with the weight of food eaten in one day by the same worm. Test the claim that a GFGW on average eats more than its body weight in a day.

Weight	0.09	0.09	0.11	0.12	0.15	0.15	0.16
Eaten	0.07	0.10	0.12	0.13	0.16	0.17	0.16

Linear Correlation

- Use
  - correlation or
  - LinRegTTest
- H_0 is always that there is no linear correlation.
- H_1 is always that there is linear correlation.
- **Example**: Below are listed the weights of several GFGWs (in oz) along with the weight of food eaten in one day by the same worm. Test the claim that **there is a linear correlation between a GFGW's weight and how much the worm eats in a day.**

Weight	0.09	0.09	0.11	0.12	0.15	0.15	0.16
Eaten	0.07	0.10	0.12	0.13	0.16	0.17	0.16

χ^2 Goodness of Fit

- Use
  - chi-squared GOF or
  - $\chi^2 - \text{Test}$
- H_0 is always that the observed frequencies match the expected values.
- H_1 is always that the observed frequencies do not match the expected values.
- **Example**: A candy company claims that 25% of its candy is brown, 35% is red, and 40% is blue. A bag of candy contained the numbers of each color in the table below. **Test the claim that the actual frequencies of colors match the claimed distribution.**
-

Brown	Red	Blue
21	33	37

Contingency Table

- Use
  - contingency table or
  - (a more complex process)
- H_0 is always that the rows and columns are independent.
- H_1 is always that the rows and columns are dependent.
- **Example**: The genders and dietary preferences of a sample of GFGWs were observed with the results below. Test the claim that **dietary preference is dependent on gender.**
-

	Male	Female
Leaves	32	57
Bark	48	41

ANOVA

- Use
  - ANOVA or
  - ANOVA(
- H_0 is always that the samples come from populations with equal means.
- H_1 is always that the samples do not come from populations with equal means.
- **Example**: The table below lists the lengths of samples of three colors of glow worms. Test the claim that **all three colors have the same mean length.**
-

Grey	0.9	0.9	1.11	1.12	1.17	1.18	1.18
Blue	0.78	0.82	0.90	0.99	0.99	1.10	
Green	1.0	1.0	1.1	1.1	1.17	1.2	1.25

Hypothesis Testing Proportions

Math 122

Normal Approximation to Binomial
Central Limit Theorem Variant

Suppose that x has a binomial distribution with n trials and probability of success p.

If $np \geq 5$ and $n(1-p) \geq 5$, then x is approximately normally distributed with

$$\mu = np \text{ and } \sigma = \sqrt{np(1-p)}.$$

Normal Approximation to Binomial

Suppose that x is a binomial random variable with n=1000 and p=0.25. Find P(x≤260).

- P(x≤260) = binomialcdf(1000,0.25,260)=.7791
- Assuming x is normal with μ=np=250 and $\sigma = \sqrt{np(1-p)}$=13.931 gives

$$P(x \leq 260) = P\left(z \leq \frac{260 - 250}{13.931}\right)$$

$$= \text{normalcdf}(-9{,}0.7303) = 0.7674$$

Traditionally, tests about proportions are done using the normal approximation to the binomial rather than the binomial.

One Proportion

Gender Selection

- Among 726 babies born to couples using the XSORT method in an attempt to have a baby girl, 668 of the babies were girls.

- Test the claim: **More than 90% of babies born to couples using the XSORT method are female.**

Technology

- Online: one proportion z-test
- TI: 1-PropZTest

Gender Prediction

- 104 pregnant women were asked to predict the gender of their children. Of these, 57 guessed correctly.
- Test the claim: **Most women can predict the gender of their children.**

Clinical Testing

- Among 724 patients given Tamiu in a clinical trial, 72 experienced nausea.

- Test the claim: **The rate of nausea in patients using Tamiflu is greater than 6%.**

(6% was the rate among patients receiving a placebo.)

Normal Approximation to Binomial Central Limit Theorem Variant

Suppose that x has a binomial distribution with n trials and probability of success p.

If $np \geq 5$ and $n(1-p) \geq 5$, then the sample proportion $\hat{p}$ is approximately normally distributed with

$$\mu = p \text{ and } \sigma = \sqrt{\frac{p(1-p)}{n}}$$

Claims about two means

H_0	H_1
$p_1 \leq p_2$	$p_1 > p_2$
$p_1 = p_2$	$p_1 \neq p_2$
$p_1 \geq p_2$	$p_1 < p_2$

Technology

- Online: two proportion z-test
- TI: 2-PropZTest

Polio Vaccine

- In a famous 1954 experiment to test the effectiveness of the Salk Polio vaccine, 200,000 children were given the vaccine and 200,000 were given a placebo. 33 vaccinated children contracted Polio. 115 non-vaccinated children contracted Polio.

- Test the claim: **The rate of Polio among the vaccinated children is higher than the rate among the non-vaccinated children.**

H1N1 and Obesity

- In a sample of 268 adult H1N1 patients, 47% were found to be obese.
- In a sample of 700 adults, 34% were found to be obese.

- Test the claim: **The rate of obesity among H1N1 patients is greater than the rate of obesity in the general population.**