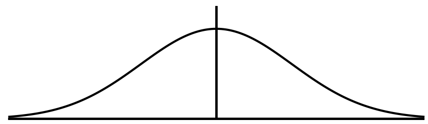
Math 122
Introduction to Statistics
Normal Distributions

A **normal distribution** has a bell shaped density curve such as:



An equation for the density curve for a normal distribution with mean μ and standard deviation σ is

$$y = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}$$

The **standard normal distribution** is the normal distribution with mean $\mu = 0$ and standard deviation $\sigma = 1$. The density curve for the standard normal distribution is

$$y = \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}}$$

The variable z is usually used for a random variable with the standard normal distribution.

We can do calculations for the standard normal distribution using the *normalcdf* and *invnorm* functions.

Any normal distribution for which $\mu \neq 0$ or $\sigma \neq 1$ is a nonstandard normal distribution.

z **Scores:** If x has any normal distribution with mean μ and standard deviation σ, then

$$z = \frac{x - \mu}{\sigma}$$

has the standard normal distribution. To convert from a z score to a value of x, we can use

$$x = \mu + z\sigma.$$

To answer questions about any normal distribution, we can convert to z scores and use a standard normal distribution.

Examples: Heights of college age males are normally distributed with a mean of 69.6 inches and a standard deviation of 3.2 inches.

1. What is the probability that a random college age male is between five and six feet tall?

2. What is the probability that a random college age male is shorter than 5 feet 6 inches?
3. What is the probability that a random college age male is taller than six feet?
4. Find the height which separates the shortest 10% of the population of college age males from the rest.
5. Find the height which separates the tallest 3% of the population of college age males from the rest.
6. Find Q_3 for the heights of college age males.
7. Find P_{53} for the heights of college age males.
8. Find the heights which separate out the middle 95% of college age males by height.

Solutions: For each of these problems, let x be the height of a random college age male in inches. Then x has a normal distribution with mean 69.6 and standard deviation 3.2, so

$$z = \frac{x - 69.6}{3.2}$$

has the standard normal distribution.

1. First, we convert the heights five feet and six feet to 60 inches and 72 inches. Then

$$\begin{aligned} P(60 < x < 72) &= P\left(\frac{60 - 69.6}{3.2} < \frac{x - 69.6}{3.2} < \frac{72 - 69.6}{3.2}\right) \\ &= P\left(-3 < z < 0.75\right) \\ &= \mathit{normalcdf}(-3, 0.75) \\ &= 0.7720 \end{aligned}$$

2. First, we convert the height to 66 inches. Then

$$\begin{aligned} P(x < 66) &= P\left(\frac{x - 69.6}{3.2} < \frac{66 - 69.6}{3.2}\right) \\ &= P\left(z < -1.125\right) \\ &= \mathit{normalcdf}(-9, -1.125) \\ &= 0.1303 \end{aligned}$$

3. Converting the height to 72 inches gives:

$$\begin{aligned} P(x > 72) &= P\left(\frac{x - 69.6}{3.2} > \frac{72 - 69.6}{3.2}\right) \\ &= P\left(z > 0.75\right) \\ &= \mathit{normalcdf}(0.75, 9) \\ &= 0.2266 \end{aligned}$$

4. For this problem, we first find the z score that separates out the bottom 10% of z values. This is
$$z = invnorm(0.1) = -1.28$$
We then convert this to a height:
$$x = \mu + z\sigma = 69.6 - 1.28 \times 3.2 = 65.5 inches.$$

5. As with the last example, we first find a z score. Since we want the *tallest*, we have to adjust the result of *invnorm*:
$$z = -invnorm(0.03) = 1.88.$$
We then convert to a height:
$$x = \mu + z\sigma = 69.6 + 1.88 \times 3.2 = 75.6 inches.$$

6. Recall that Q_3 is a number which separates the bottom 75% of data values from the top 25%. We can find this with *invnorm* (finding the z first):
$$z = invnorm(0.75) = 0.6745$$
Then
$$x = \mu + z\sigma = 69.6 + 0.6745 \times 3.2 = 71.8 inches.$$

7. Recall that P_{53} separates the bottom 53% of data values from the top 47% of data values. We can find this with *invnorm* (finding the z first):
$$z = invnorm(0.53) = 0.0753$$
Then
$$x = \mu + z\sigma = 69.6 + 0.0753 \times 3.2 = 69.8 inches.$$

8. First, we find z scores separating the top and bottom 2.5% from the middle 95%. We do this with *invnorm*:
$$z = \pm invnorm(0.025) = \pm 1.96.$$
Then we can find the heights:
$$69.6 - 1.96 \times 3.2 = 63.3 inches$$
$$69.6 + 1.96 \times 3.2 = 75.9 inches$$
About 95% of college age males are between 63.3 inches and 75.9 inches.