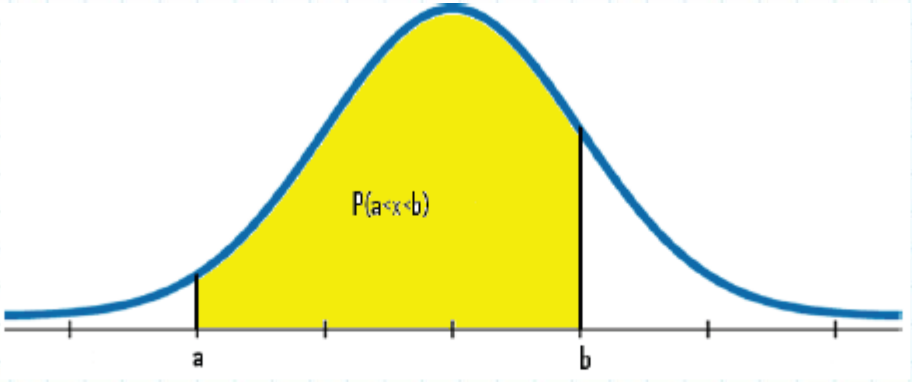
Standard Normal Distribution

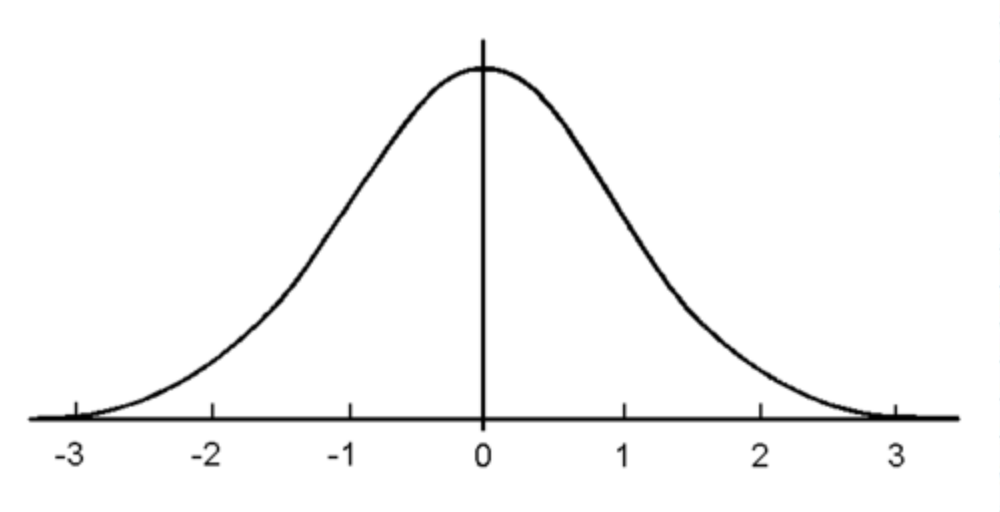
Math 122

Density Functions

- Every continuous random variable has a density function
- The total area under the function is 1
- To find P(a<x<b), we find the area under the function between a and b

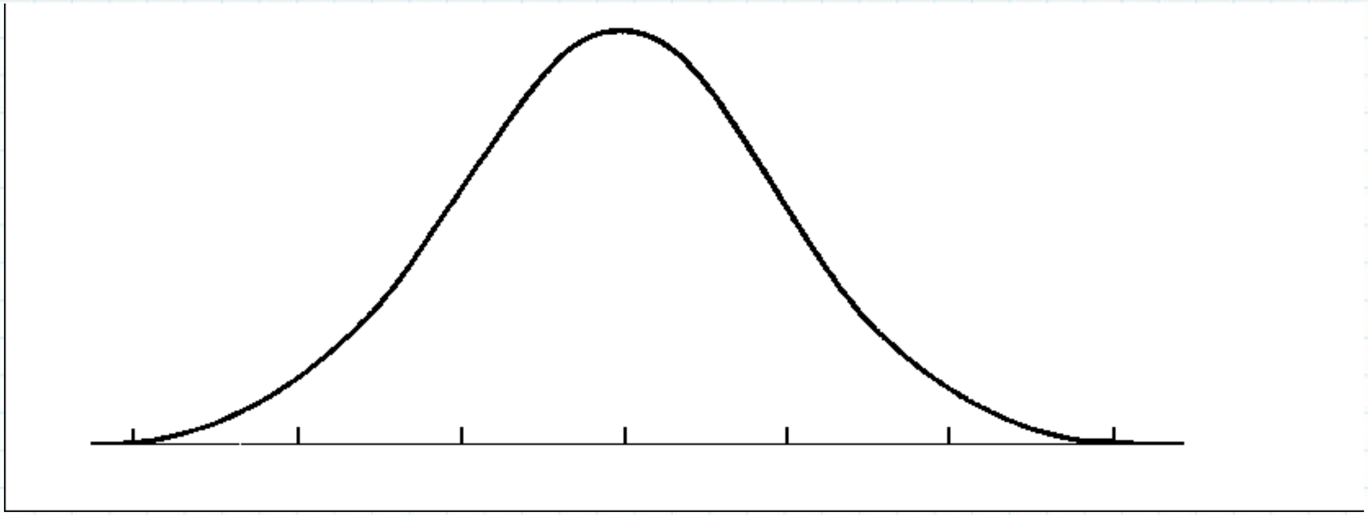


Normal Distribution



- The most important distribution for statistics.

Normal Distribution with mean μ and standard deviation σ

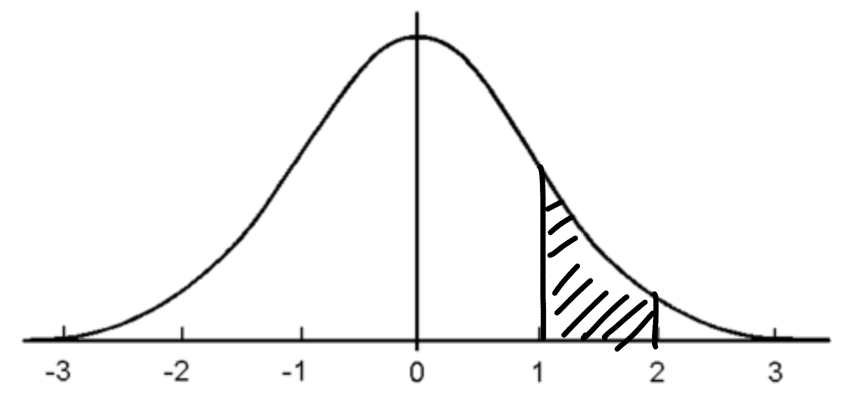


$$y = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}$$

Standard Normal Distribution

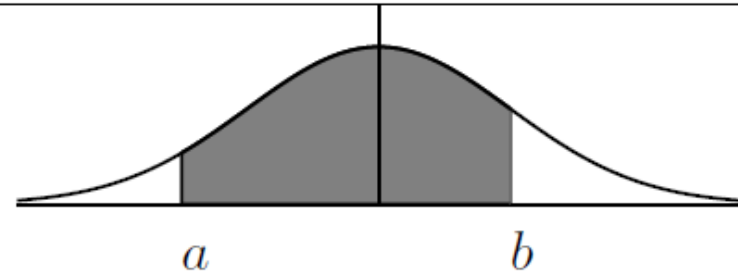
- Has mean μ=0 and standard deviation σ=1

$$y = \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}}$$

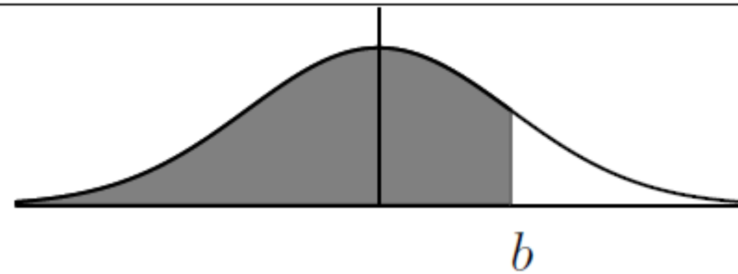


- Usually use z for a standard normal distribution

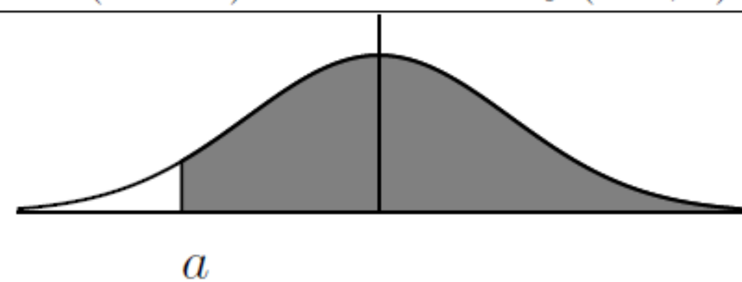
Standard Normal Distribution



$$P(a < z < b) = normalcdf(a, b)$$



$$P(z < b) = normalcdf(-9, b)$$



$$P(a < z) = normalcdf(a, 9)$$

Suppose that z has a standard normal distribution

$P(-1 < z < 1) = 0.68$

$P(-2 < z < 2) = 0.954$

$P(-3 < z < 3) = 0.997$

Empirical Rule

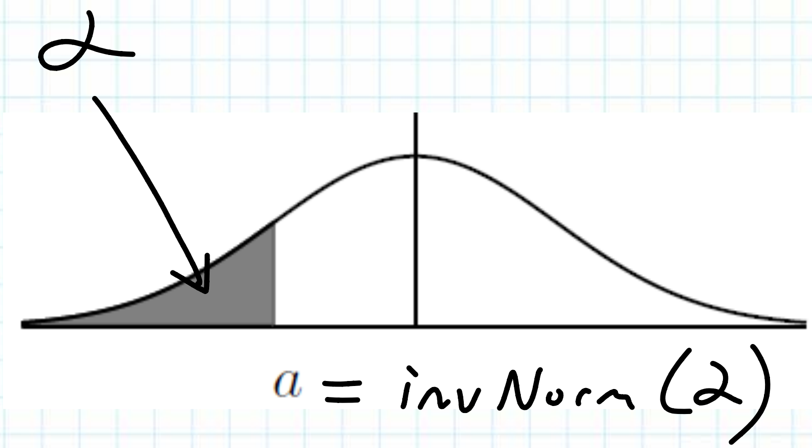
$P(z < 3) = \text{normalcdf}(-9, 3) = 0.9986$

$P(z > 1.5) = \text{normalcdf}(1.5, 9) = 0.0668$

Inverse Normal Function

- To find a number a so that P(z<a)=α use

  invnorm(α)

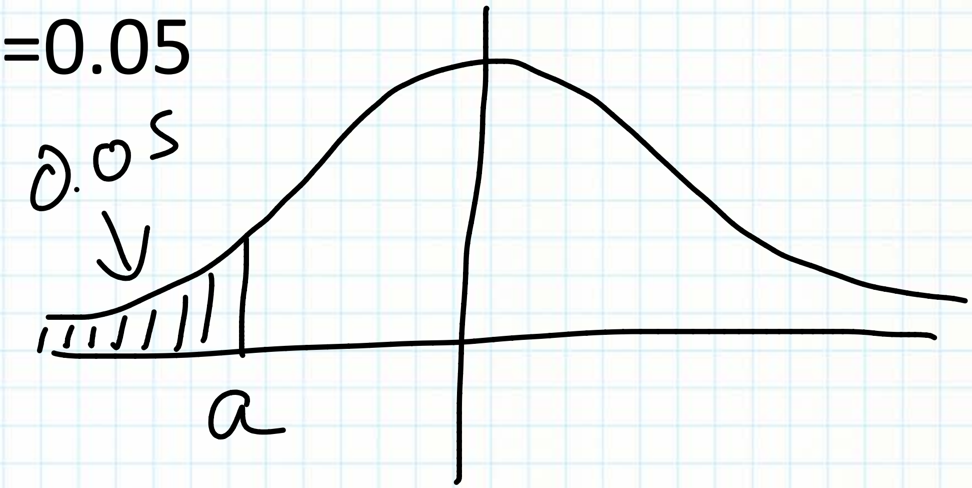


Suppose that z has a standard normal distribution

- Find a so that $P(z<a)=0.05$

$a = \text{invnorm}(0.05)$

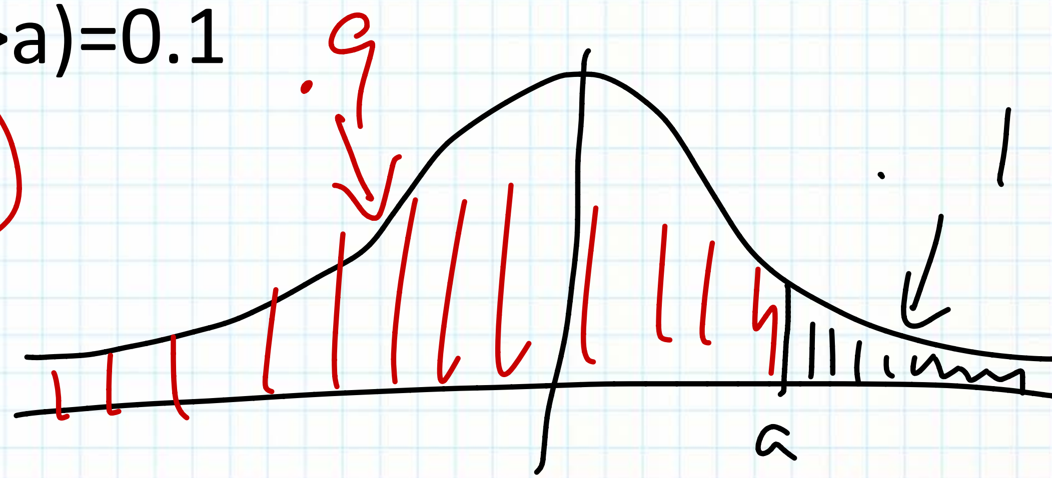
$= -1.64$



- Find a so that $P(z>a)=0.1$

$a = \text{invnorm}(0.9)$

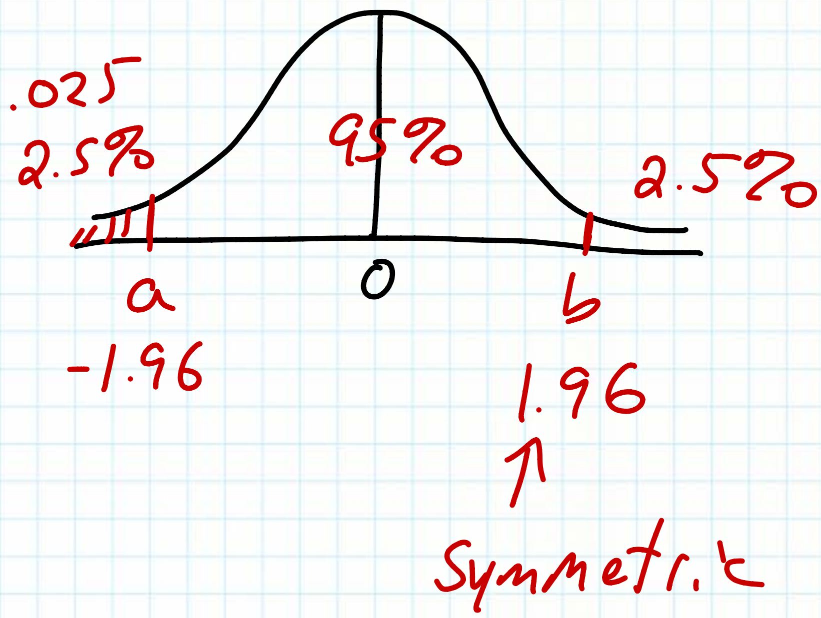
$= 1.28$



Suppose that z has a standard normal distribution

- Find a and b which separate the middle 95% of values of z from the rest.

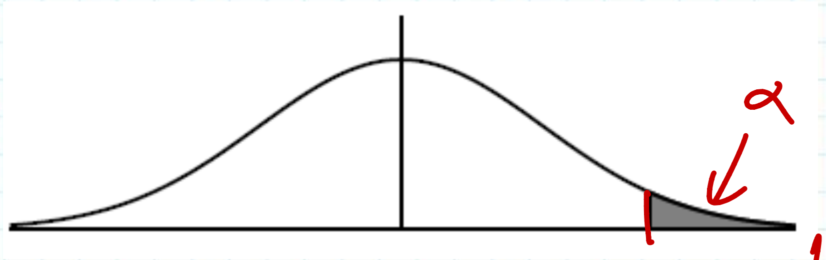
$a = \text{invnorm}(.025)$
$= -1.96$



α Tails

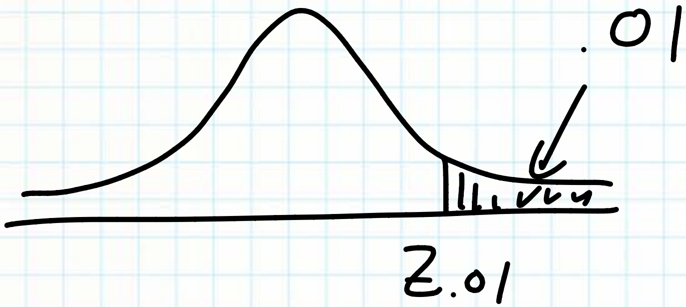
- The number z_α is the unique number so that

$$P(z > z_\alpha) = \alpha$$



$z_\alpha = |\text{invnorm}(\alpha)|$

- Find $z_{0.01} = |\text{invnorm}(0.01)| = 2.33$



- Find $z_{0.005} = 2.58$

Z - standard normal

$P(1.2 < Z < 2.2) = 0.101$

$P(Z < 0.5) = 0.691$ normalcdf$(-9, 0.5)$

Find a # a so that $P(Z > a) = 0.75$

$a = $ invnorm$(0.25) = -0.67$