Poisson Distribution

Math 122

Random Variables

- A variable x whose value is determined by the outcome of an experiment.
- P(x) = probability of a particular value of x
- Mean/Expected Value: $\mu = \sum xP(x)$
  - If the experiment is repeated many times and the values of x are averaged, the average should be near μ.
- Standard Deviation: σ

Identifying Unusual Values

- Range Rule of Thumb: The usual values are between $\mu - 2\sigma$ and $\mu + 2\sigma$

- Five Percent Rule
  - If $P(x \leq a) \leq 5\%$, then a is unusually low
  - If $P(x \geq a) \leq 5\%$, then a is unusually high

Special Distributions

- We want a few specific, common distributions so that we know what to do when we encounter them.
- Discrete
  - Binomial (counting successes in trials)
  - Poisson (counting events in an interval)
- Continuous
  - Uniform (simple)
  - Normal (pervasive bell curve)
  - t, F, χ^2

Binomial Distribution

- n independent trials.
- Each trial ends in success or failure.
- The probability of success is p
- The probability of failure is q=1-p
- The value of x is the number of successes.
  - P(x = a) = binom**p**df(n, p, a)
  - P(x ≤ a) = binom**c**df(n, p, a)
- $\mu = np$
- $\sigma = \sqrt{npq}$

Poisson Distribution

Poisson Distribution

- The Poisson distribution is a discrete probability distribution that applies to occurrences of some event over a specified interval.
- The interval can be time, distance, area, volume, or some similar unit.
- The random variable x is the number of occurrences of the event in an interval.

Example Poisson Distributions

- The number of major earthquakes during a year.
- The number of births at a hospital in a day.
- The number of emails received in an hour.
- The number of automobile accidents on a given mile of road.
- The number of bug pieces in a tablespoon of peanut butter.
- The number of dandelions on a square foot of dirt.

Poisson Formulas

For a Poisson Distribution with mean μ

$$\sigma = \sqrt{\mu}$$

Know

$$P(x) = \frac{\mu^x \cdot e^{-\mu}}{x!}$$

Where $e \approx 2.718281828459045$

Calculator Functions

- $P(x = a)$ = poisson**p**df(μ, a)
- $P(x \leq a)$ = poisson**c**df(μ, a)
- “c” is for “cumulative”

Earthquakes

- According to the USGS, there were 16,500 earthquakes at or above magnitude 6 in a recent span of 100 years.
- What is the probability that there are 125 or fewer earthquakes at or above magnitude 6 in a given year?

X = # quakes in a year 6.9×10^{-4}

X is Poisson

$P(X \leq 125) = \text{poissoncdf}(16500/100, 125) = .00069$

Earthquakes

- What would be the usual range for the number of earthquakes at or above magnitude 6 during one year (according to the Range Rule of Thumb)?

16,500 quakes over 100 years

$X = \#$ in 1 year

X is poisson

$\mu = 16{,}500/100 = 165$

$\sigma = 12.8$

min usual $= \mu - 2\sigma$

$= 139.3$

max usual $= \mu + 2\sigma$

$= 190.69$

Births

- 120 children are born each year at Seward Memorial Hospital.
- SMH has 2 “birthing rooms.”
- What is the probability on any given day that this is adequate?

X = # births in 1 day

X is Poisson

$\mu = 120/365$

$P(X \leq 2) = \text{Poissoncdf}(120/365, 2) = 0.99$

Births

- 200 children are born each year in a certain hospital.
- How many birthing rooms should the hospital have so that the probability that they have enough rooms on any given day is at least 99.5%?

X = # births in 1 year

X is Poisson

$P(X \le \_\_\_) = .995$

$P(X \le 1) = \text{poissoncdf}(200/365, 1) = .895$

$P(X \le 2) = .98$

$P(X \le 3) = .997$

$P(X \le 4)$

$\vdots$

Bugs in Peanut Butter

- The USDA allows a maximum of 30 “insect parts” in 100g or 3.53oz of peanut butter (twice that for chocolate).
- Suppose that a jar of peanut butter has the maximum allowable number of bug parts.
- If a sandwich is made with one ounce of this peanut butter, then **what is the probability that the peanut butter in the sandwich does not contain any bug parts?**

$\mu = 30/3.53$

X = # parts in 1 oz of p-nut butter $P(X=0) =$

X is poisson

poissonpdf(30/3.53, 0) = 0.0002

Bugs in Peanut Butter

- The USDA allows a maximum of 30 “insect parts” in 100g or 3.53oz of peanut butter (twice that for chocolate).
- Suppose that a jar of peanut butter has the maximum allowable number of bug parts.
- If a sandwich is made with one ounce of this peanut butter, then **what is the probability it contains at least 5 bug parts?**

$\mu = 30/3.53$

$P(X \geq 5) = 1 - P(X < 5) = 1 - P(X \leq 4) = 0.93$

Tornadoes

- There were 7236 tornadoes in Texas in a recent span of 54 years (the most of any state in the USA).
- What is the probability that there are 110 or fewer tornadoes in one year in Texas?
- What is the probability that there are more than 150 tornadoes in one year in Texas?