Central Limit Theorem

Math 122

Normal Distribution Calculations

- normalcdf(lower, upper, mean, st. deviation)
- invNorm(area to left, mean, st. deviation)

Heights of college age males are normally distributed with mean 69.6 in and standard deviation 3.2 in.

What is the probability that a randomly chosen college age male is between 5’10 and 6’2?

Find a height taller than 95% of college age males.

Parameter vs. Statistic

	Population	Sample
Size	N	n
Mean	μ	$\bar{x}$
Proportion	p	$\hat{p}$
Standard Deviation	σ	s
Variance	σ^2	s^2

We want to use sample statistics to estimate population parameters.

To do so, we need to know the distributions of sample statistics.

Unbiased Estimators

These sample statistics tend to be close to the population parameters and do not over- or under-estimate the population parameter systematically .

$$\bar{x},\ s^2,\ \text{and}\ \hat{p}$$

Biased Estimators

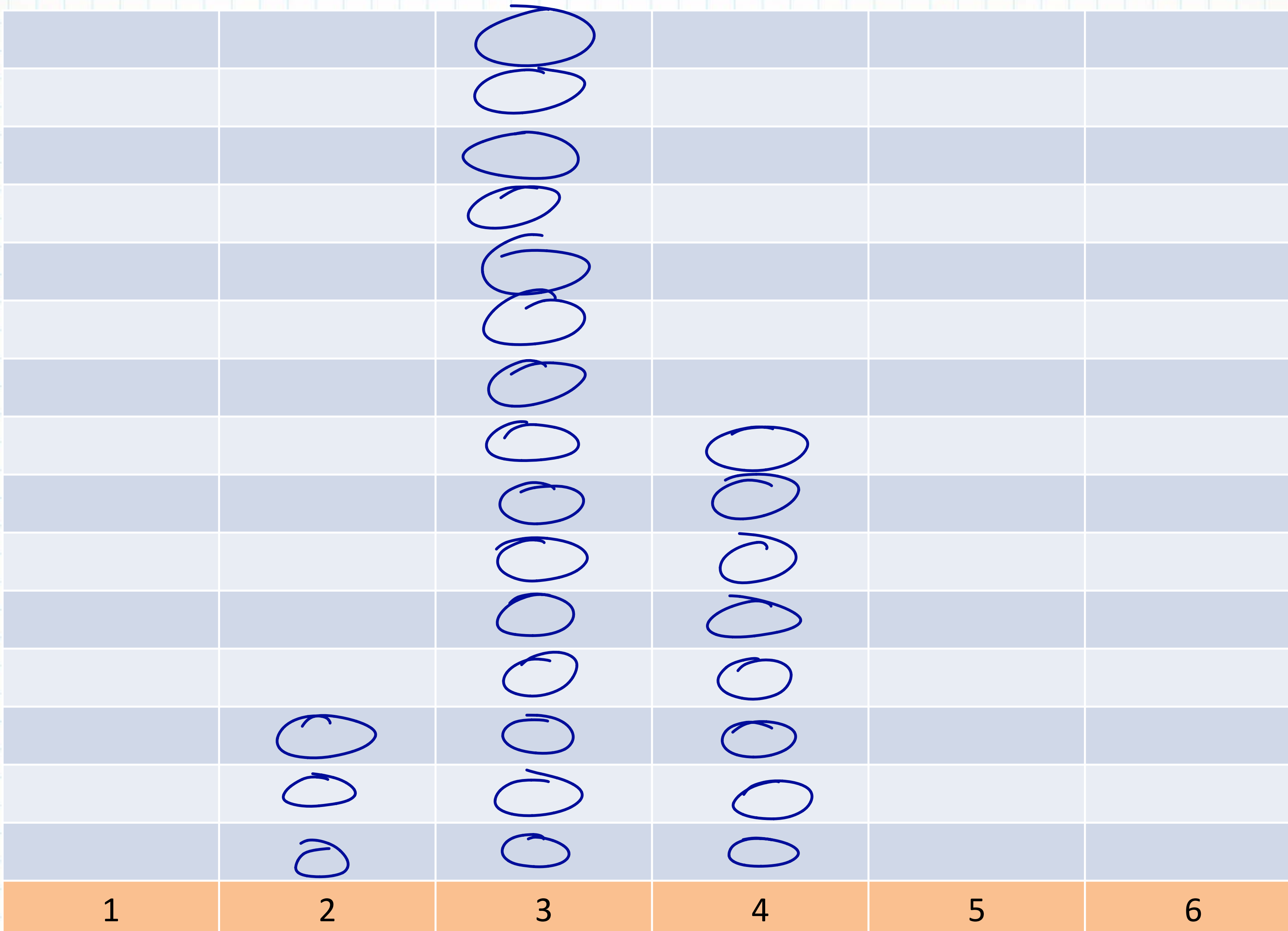
These sample statistics may not be close to the population parameters, and these may systematically over- or under-estimate the population parameter.

$$\text{median, range, } s$$

Dice

- Let x be the number that appears when a die is rolled.
- Roll the die 10 times. Let $\bar{x}$ be the average value of x for the 10 rolls.

- Roll the die 10 times. Add the numbers. Divide by 10.


1
2
3
4
5
6

Central Limit Theorem

Suppose that x is a random variable with mean μ and standard deviation σ.

Suppose that $\bar{x}$ is the mean of simple random samples of size n of values of x.

If x is normally distributed or if $n > 30$

then

$\bar{x}$ is approximately normally distributed with

$\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$.

Central Limit Theorem

When working with sample averages use:

$$\mu_{\bar{x}} = \mu \text{ and } \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}.$$

The eight females in my Number Theory class have an average height of 67.5 inches.

Heights of college age females are normally distributed with mean 63.8in and standard deviation 2.7in.

What is the probability that **a randomly chosen college age female** is taller than 67.5"?

X = height of random female

$\mu = 63.8$

$\sigma = 2.7$

$$P(X > 67.5) = \text{normalcdf}(67.5, \underbrace{120}_{\text{Really Big \#}}, \overset{\mu}{63.8}, \overset{\sigma}{2.7}) = 0.0853$$

Heights of college age females are normally distributed with mean 63.8in and standard deviation 2.7in.

What is the probability that the **average height of a simple random sample of 8 college age females** is taller than 67.5 inches?

$\bar{X}$ = avg. of simp. rand. sample of 8 female heights

$\mu_{\bar{X}} = \mu_X = 63.8$

$\sigma_{\bar{X}} = \frac{2.7}{\sqrt{8}}$

$P(\bar{X} > 67.5) = \text{normalcdf}(67.5, 120, 63.8, 2.7/\sqrt{8}) = 5.3116 \times 10^{-5}$

$= .000053116$

IQs are normally distributed with a mean of 100 and a standard deviation of 15.

What is the probability that a **randomly selected person** has an IQ over 105?

X = IQ of random person

$\mu = 100$

$\sigma = 15$

$P(x > 105) = \text{normalcdf}(105, 1000, 100, 15) = 0.3694$

IQs are normally distributed with a mean of 100 and a standard deviation of 15.

What is the probability that a **simple random sample of 25** people has an average IQ over 105?

X = IQ of random person

$\mu = 100$

$\sigma = 15$

$\bar{X}$ = avg of 25 IQ

$\mu_{\bar{X}} = 100$

$\sigma_{\bar{X}} = \frac{15}{\sqrt{25}}$

$P(\bar{X} > 105) = \text{normalcdf}(105, 1000, 100, \frac{15}{\sqrt{25}}) = .0478$

College age males have an average weight of 179.7lb with a standard deviation of 47.7lb.

Find a weight which is greater than the average weight of 95% of all simple random samples of 50 college age males.

Inverse

CLT

X = weight of individual

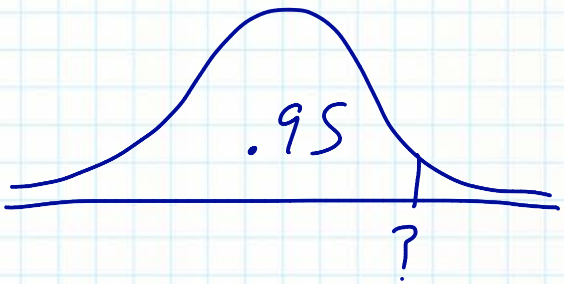
$\mu = 179.7$

$\sigma = 47.7$

$\bar{X}$ = avg. weight of 50

$\mu_{\bar{x}} = 179.7$

$\sigma_{\bar{x}} = \dfrac{47.7}{\sqrt{50}}$



Weight = invNorm$\left(.95, 179.7, \dfrac{47.7}{\sqrt{50}}\right) = 190.8$ lb

A tour boat is to be built to accommodate up to 50 college age males. How much weight should the boat be designed to carry to accommodate 95% of all such groups?

$50 \times 190.8 = 9540$ lb

Central Limit Theorem Version 2

Suppose that x is a random variable with mean μ and standard deviation σ.

Suppose that $\bar{x}$ is the mean of simple random samples of size n of values of x.

If x is normally distributed or if $n > 30$
then the distribution of

$$\frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

is approximately a standard normal distribution.

Central Limit Theorem
Version 3

Suppose that x is a random variable with mean μ and standard deviation σ.

Suppose that $\bar{x}$ is the mean of simple random samples of size n of values of x.

If x is normally distributed or if $n > 30$ then the distribution of

$$\frac{\bar{x}-\mu}{s/\sqrt{n}}$$

is approximately a Student t distribution with $n - 1$ degrees of freedom.

Proportions
Version 1

Suppose that x is the number of successes out of simple random samples of size n from a population in which the probability of success is p.

If $np \geq 5$ and $n(1-p) \geq 5$, then the sample successes x are approximately normally distributed with

$$\mu = np \text{ and } \sigma = \sqrt{np(1-p)}.$$

Proportions Version 2

Suppose that x is the number of successes out of simple random samples of size n from a population in which the probability of success is p.

If $np \geq 5$ and $n(1-p) \geq 5$, then the sample proportion $\hat{p}$ is approximately normally distributed with

$$\mu = p \text{ and } \sigma = \sqrt{\frac{p(1-p)}{n}}$$