Random Variables

Math 122

A coin is flipped 4 times.

List the possible outcomes.

Let x be the number of Hs that appear.

What are the possible values of x?

Find the probability of each value.

Average the outcomes.

Draw a bar chart of the probabilities.

In this situation…

- x is a **random variable**.
- This table is a **probability distribution** for x

x	P(x)
0	1/16
1	4/16
2	6/16
3	4/16
4	1/16

- The average 2 is the **mean** or **expected value**

Random Variables

- A **random variable** is a variable whose value is determined by the outcome of an experiment.
- The probability that a random variable x is equal to a number N is written
  - P(x=N) or P(N)
- A table or chart depicting the probability of each value of x is the **probability distribution** of x.

Summary Statistics for a Random Variable x:

- Mean or Expected Value: $\mu = \sum x \cdot P(x)$
- Variance: $\sigma^2 = \sum (x - \mu)^2 \cdot P(x)$
- Standard Deviation: $\sigma = \sqrt{\sum (x - \mu)^2 \cdot P(x)}$

If the procedure which determines the value of x is repeated many times and the outcomes are averaged, then the average should be close to the expected value.

Types of Random Variables

- A **discrete random** variable has gaps between the possible values.

- A **continuous random** variable has no gaps between the possible values.

Examples

- The number of cars that pass a certain intersection during the lunch hour.
- The number which appears when a die is rolled.
- The height of a randomly selected seven year old.
- The number of female children among three children.
- The proportion of patients that get sick after taking a certain drug.
- The salary of a new college graduate.
- Amount of rain in a gauge after a storm.
- The weight lost by a patient on a diet.
- Profit won playing a game.
- Insurance payout.
- Profit from tuition/scholarship.

Let x be the sum of the numbers which appear when two dice are rolled.

Find the probability distribution of x.

11	12	13	14	15	16
21	22	23	24	25	26
31	32	33	34	35	36
41	42	43	44	45	46
51	52	53	54	55	56
61	62	63	64	65	66

Summary Statistics

Online Calculator

- Enter the values of x in List0
- Enter the probabilities in the same order in List1
- Use the command varstat(0,1).

TI Calculator

- Enter the values of x in L1
- Enter the probabilities in L2
- Use 1-VarStats L1,L2

Let x be the sum of the numbers which appear when two dice are rolled.

Find the expected value of x.

11	12	13	14	15	16
21	22	23	24	25	26
31	32	33	34	35	36
41	42	43	44	45	46
51	52	53	54	55	56
61	62	63	64	65	66

Insurance

- Bob buys a $100,000 life insurance policy for $250 for one year. Let x be the amount of money that Bob profits from this policy.
- Find the probability distribution and expected value of x.
- Assume that the probability that Bob lives through the year is 0.9995

Games

In a certain lottery game, the player selects three digits. There is one winning sequence of digits. If the player selects the winning sequence, he wins $5. The game costs $1 to play.

Let x be the profit of a person playing this game. Find the probability distribution and expected value for x.

Odds

- The odds of an event A happening are a ratio

  P(A happens):P(A does not happen)

NE Pick 3 Lottery

Get	Prize	Odds	Winners
Straight	$600	1,000.00	0
Box (6 Way)	$100	166.67	9
Box (3 Way	$200	333.33	0
S/B (Straight) or (3 Way)	$350	1,000.00	7
S/B (Box - 6 Way)	$50	200.00	20
Prize S/B (Box - 3 Way)	$126	500.00	0
CMB (Match 3 Positionally)	$100	1,000.00	1
CMB (Match 2 Positionally)	$8	37.04	12
CMB (Match 1 Positionally)	$1	4.00	210

Expected Value for NE P3 Straight

Expected Value for NE P3 CMB

Range Rule of Thumb

Any value of a random variable above $\mu + 2\sigma$ or below $\mu - 2\sigma$ is considered unusual.

Range Rule of Thumb

Find the unusual values for the sum of two dice using the Range Rule of Thumb.

Probability of a Range

Find $P(3 \leq x \leq 5)$

Find $P(x \geq 5)$

Find $P(x \leq 2)$

x	P(x)
1	0.03
2	0.50
3	0.40
4	0.03
5	0.02
6	0.01
7	0.01

5% Rule

A value N of a random variable x is unusually small or low if $P(x \leq N) \leq 5\%$

A value N of a random variable x is unusually big or high if $P(x \geq N) \leq 5\%$

Find the unusually high and low values according to the 5% rule

x	P(x)
1	0.03
2	0.50
3	0.40
4	0.03
5	0.02
6	0.01
7	0.01

Find the unusually high and low values for the sum of two dice by the 5% rule

Summary

- Random variable
- Probability Distribution
- Expected value or mean
- Finding summary statistics
- Range Rule of Thumb
- 5% Rule