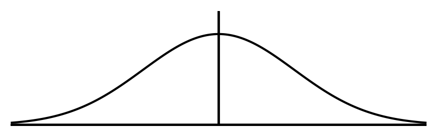
Math 122
Introduction to Statistics
Uniform Distributions

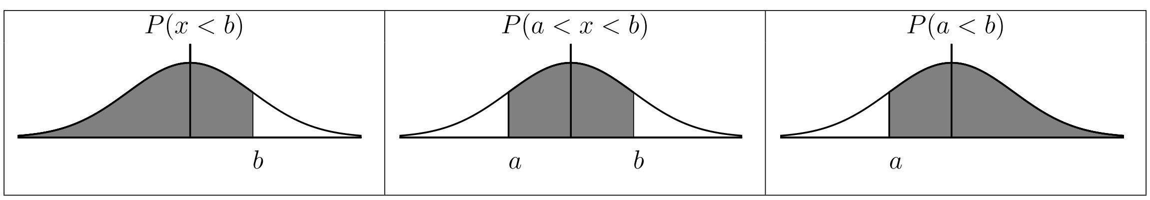
CONTIUOUS RANDOM VARIABLES

Standard Characterisitics:

- Continuous random variables have infinitely many possible values spread out over an interval with no gaps.
- Usually, the probability of any particular value is 0.
- We will usually look for probabilities of the form $P(x < a)$ or $P(a < x < b)$ or $P(a < b)$.
- Note that because the probability of any single value is (usually) 0, it does not matter if we use $<$ or $\leq$.
- Instead of histograms, density curves describe the distribution of probabilities for continuous random variables.

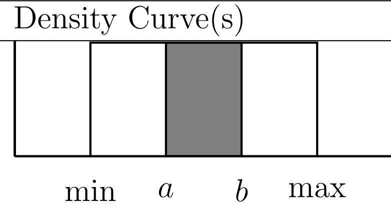
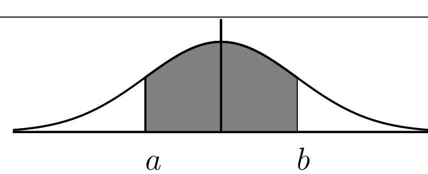


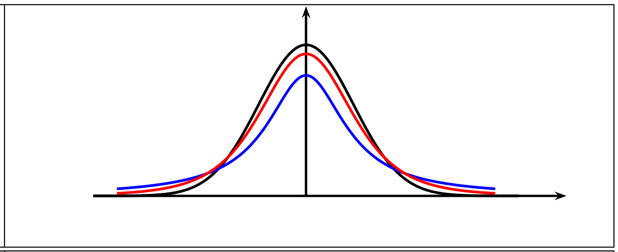
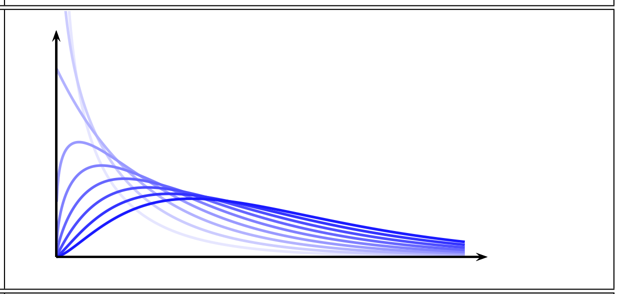
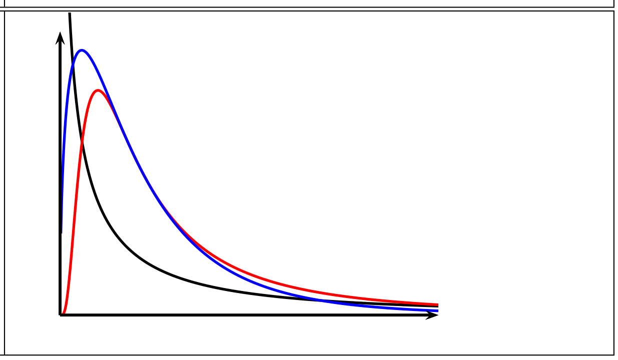
- Probability corresponds to area under the density curve.



- The total area under a density curve is 1.
- How to find these areas depends on the distribution.

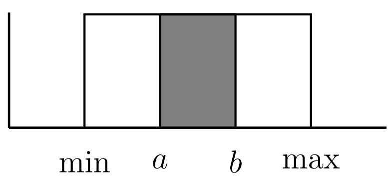
Some Continuous Distributions We Will Encounter:

Distribution	Density Curve(s)
Uniform	
Normal	

Student's t	
χ^2	
F	

UNIFORM DISTRIBUTION

The density function of a uniform distribution is a horizontal line extending from the minimum value of the random variable to the maximum value of the random variable.



The height of the random variable is such that the area of the entire "rectangle" is 1. Here, probabilities are "evenly" distributed across the interval. Finding a probability such as $P(a < x < b)$ amounts to finding the area of a rectangle.

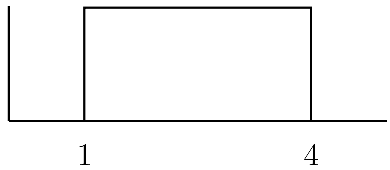
Example: A random variable x is uniformly disstributed from 1 to 4.

1. Find $P(x < 3)$.
2. Find $P(x > 3.5)$.
3. Find $P(2 < x < 3)$.
4. Find a value a so that $P(x < a) = 0.1$.

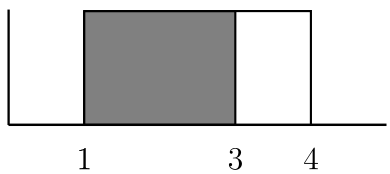
5. Find a value a so that $P(x > a) = 0.05$.

6. Find values a and b which separate out the middle 95% of values of x.

Solution: Since x has a uniform distribution, its density curve is a horizontal line above the interval from 1 to 4. The width of this interval is $4-1 = 3$, so the height of the horizontal line should be $\frac{1}{3}$. Here is the density function:



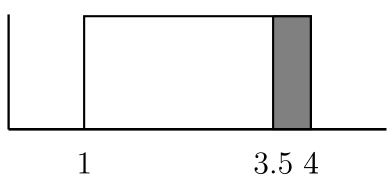
1. Shade the area under the density curve to the left of 3.



We want the area of the shaded rectangle.

$$P(x < 3) = \text{area left of } 3 = (3-1) \times \frac{1}{3} = \frac{2}{3}.$$

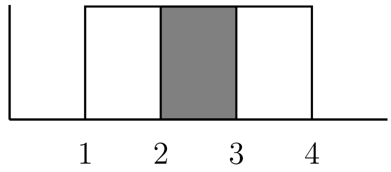
2. Shade the area under the density curve to the right of 3.5.



We want the area of the shaded rectangle.

$$P(x > 3.5) = \text{area right of } 3.4 = (4-3.5) \times \frac{1}{3} = \frac{1}{2} \times 13 = \frac{1}{6}.$$

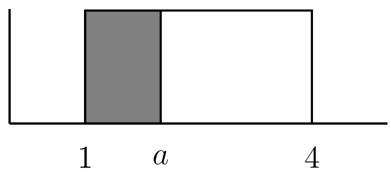
3. Shade the area under the density curve between 2 and 3.



We want the area of the shaded rectangle.

$$P(2 < x < 3) = \text{area between 2 and 3} = (3-2) \times \frac{1}{3} = \frac{1}{3}.$$

4. Shade the area under the density curve to the left of an arbitrary vertical line. Since the desired probability is small, we draw the line to the left of the total region:



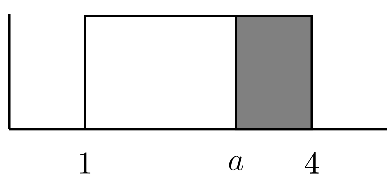
We want to find a so that the area of the rectangle is 0.1. The area of the rectangle is

$$P(x < a) = (a-1) \times \frac{1}{3}.$$

We set this equal to 0.1 and solve:

$$(a-1) \times \frac{1}{3} = 0.1 \text{ gives } a = 1.3.$$

5. Shade the area under the density curve to the right of an arbitrary vertical line. Since the desired probability is small, we draw the line to the right of the total region:



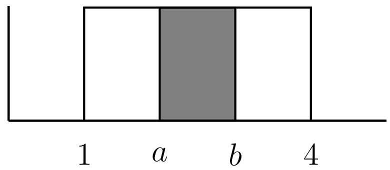
We want to find a so that the area of the rectangle is 0.05. The area of the rectangle is

$$P(x > a) = (4-a) \times \frac{1}{3}.$$

We set this equal to 0.05 and solve:

$$(4-a) \times \frac{1}{3} = 0.05 \text{ gives } a = 3.85.$$

6. The intention of this question is to find a and b as in this picture



so that $P(a < x < b) = 0.95$ and that the areas of the end rectangles are equal. Since the total area is 1, the end rectangles should each be $\frac{1}{2} \times (1-0.95) = 0.025$. Therefore, we mimic the work above to find a so that $P(x < a) = 0.025$. Since

$$P(x < a) = (a-1) \times \frac{1}{3}$$

we want

$$(a-1) \times \frac{1}{3} = 0.025$$

so it has to be that $a = 1.075$. We could find b similarly by focusing on $P(x > b)$. We can also find b using symmetry. It should be that $a-1 = 4-b$, so $b = 4-a+1 = 3.925$.