Papers

An Assessment of Finite Difference Beam Propagation Method

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Abstract—**A finite difference beam propagation method (FD-BPM) is outlined and assessed in comparison with a conventional beam propagation method (FFT-BPM) which uses fast Fourier transformation. In the comparative study three straight waveguides with different index profiles that are frequently encountered in integrated optics are utilized. Using both methods normalized effective index values of the eigenmodes of these waveguides are calculated and compared with the exact values obtained from analytical expressions. As a further accuracy criteria, the power loss due to numerical errors, when an eigenmode of a waveguide is excited, is evaluated. Based on this comparison accuracy, computational efficiency, and stability of the FD-BPM are assessed.**

I. Introduction

Accurate analysis of guided-wave structures such as intersecting, branching, or coupled waveguides is essential for the development of photonic integrated circuits. For such analysis, which could be quite involved and complicated, several different approaches have been developed [1]–[6]. The so-called beam propagation method (BPM) has been successfully used to analyze a wide spectrum of guided-wave structures [1]. To utilize this method, usually the problem is reduced to a one-dimensional cross-sectional index profile by defining effective indexes to various parts of the structure. Then the paraxial wave equation is solved in the resulting one-dimensional effective index profile using an algorithm that involves fast Fourier transformation (FFT). Even though the effective index approximation is not adequate for certain cases, it can still be applied to get qualitative predictions about the behavior of a specific guided-wave structure. It is of course, possible to analyze a particular structure using the two-dimensional cross-sectional index profile, but this requires extensive computational effort [1]. The accuracy and applicability of the BPM have been studied extensively [7], [8].

It is also possible to solve the paraxial wave equation using alternate numerical techniques. One such technique uses finite elements to solve the variational functional that represents the paraxial wave equation [6]. An alternate numerical scheme to solve the paraxial wave equation is to use a finite difference approximation. We will refer to this technique (which uses an FD approximation rather than FFT) as FD-BPM. FD-BPM has been successfully applied to the analysis of nonlinear propagation in a radially symmetric structure [5]. However, as far as the authors know, no attempt has been made to analyze the integrated-optical structures using this technique. Furthermore, the accuracy and the computational efficiency have not been studied. The purpose of this paper is to assess the potential of FD-BPM. This is done through a comparative study of FFT-BPM and FD-BPM for certain representative structures that are frequently encountered in guided-wave optics.

In Section II, FD-BPM is outlined and compared with FFT-BPM. In the next section, several representative cases are numerically analyzed using both techniques and results are compared as far as the accuracy and computational efficiency are concerned. Finally, general conclusions are drawn.

II. Finite Difference Beam Propagation Method (FD-BPM)

In the presence of the one-dimensional cross-sectional index profile $n(x, z)$ and in the paraxial limit, Helmholtz equation can be reduced to the paraxial wave equation [9] which is

$$2jk_0n_0 \frac{\partial E_y}{\partial z} = \frac{\partial^2 E_y}{\partial x^2} + k_0^2[n^2(x, z) - n_0^2]E_y \quad (1)$$

where E_y is the only electric field component of the TE mode of the slab waveguide geometry whose index profile is represented by $n(x, z)$. In the FFT-BPM, the electric field is found at each step by applying the following operator algorithm

$$E_y(z + \Delta z) = PQPE_y(z) \quad (2)$$

where

$$P = \exp\left(-j \frac{\Delta z}{4k_0n_0} \frac{\partial^2}{\partial x^2}\right) \quad (3)$$

$$Q = \exp\left\{-j \frac{k_0 n_0 \Delta z}{2}\left[\frac{n^2(x, z + \Delta z/2)}{n_0^2} - 1\right]\right\}. \quad (4)$$

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In FD-BPM, the partial differential equation is replaced by the finite difference approximation, which yields

$$2jk_0 n_0 \frac{\partial E_i}{\partial z} = \frac{E_{i-1} - 2E_i + E_{i+1}}{\Delta x^2} + k_0^2[n_i^2(z) - n_0^2]E_i \tag{5}$$

where E_i is the electric field at $(i\Delta x, z)$ with $i = 0, 1, 2, \cdots, N-1$. If we integrate (5) in the interval $[z, z + \Delta z]$ and approximate the integration of the right-hand term using trapezoidal rule, we can relate the electric field at $z + \Delta z$, i.e., $E_i(z + \Delta z)$, to the electric field at z, i.e., $E_i(z)$ by the following expression:

$$-aE_{i-1}(z + \Delta z) + bE_i(z + \Delta z) - aE_{i+1}(z + \Delta z) = aE_{i-1}(z) + cE_i(z) + aE_{i+1}(z) \tag{6}$$

where

$$a = \frac{\Delta z}{2\Delta x^2} \tag{6a}$$

$$b = \frac{\Delta z}{\Delta x^2} - \frac{\Delta z}{2}(n_i^2(z + \Delta z) - n_0^2) + 2jk_0 n_0 \tag{6b}$$

$$c = -\frac{\Delta z}{\Delta x^2} + \frac{\Delta z}{2}(n_i^2(z) - n_0^2) + 2jk_0 n_0. \tag{6c}$$

This results in a tridiagonal system of linear equations, which can be solved very efficiently [11]. The solution to this system of equations can be also shown to be stable.

III. Numerical Results

To compare these two techniques, two computer programs were developed employing both algorithms and various simulations were performed on a SUN–SPARC workstation. Required FFT program and the program to solve a tridiagonal linear system of equations were taken from [11] and [12], respectively. In the numerical simulations three representative cases were studied. All these cases are straight waveguides, i.e., they are all uniform in z direction. In the first case the waveguide has a slowly varying index distribution which is given as

$$n(x) = n_b + \Delta n \cosh^{-2}\left(\frac{2x}{w}\right). \tag{7}$$

This index profile approximates that of a Ti diffused $LiNbO_3$ waveguide. In the simulation the parameters were chosen as $n_b = 2.15$, $\Delta n = 0.003$, $w = 4$ μm. The resulting waveguide is single-moded and its normalized effective index is 0.4556 at $\lambda = 1.15$ μm. The exact value of the propagation constant is obtained using the analytical formula given in [13].

In the second and third cases waveguide has a step index profile, i.e., the refractive index is uniform in the core and cladding regions, and abruptly changes at the core and cladding interfaces. Such an index profile is typical of semiconductor slab waveguides and two-dimensional semiconducting waveguides under effective index approximation. In the second case slab waveguide is symmetrical and the core thickness is 4 μm. The refractive indexes of the core and cladding are 3.38 and 3.377, respectively. In the third case the slab waveguide has an asymmetrical step index profile. One of the claddings is air and the core index and thickness and other cladding indexes are the same as the second case. For the second and third cases the waveguides are single-moded and the accurate normalized effective indexes are 0.6426 and 0.4609, respectively, at $\lambda = 1.15$ μm. These values are obtained from the well-known eigenvalue equation for the three-layer slab waveguide.

For both FFT-BPM and FD-BPM calculations the propagation constants of the modes of these three cases are found from the modal power spectra which is obtained by correlating the propagating fields with the initial field as described in [10]. A Gaussian profile whose full width at half maximum is 4 μm is used as the initial field profile. The computational window is 40 μm for all the simulations. For both cases the accuracy of the results depend on the number of grid points N in the transverse, i.e., x-direction, and the size of the propagation steps, Δz in the direction of propagation, i.e., in z-direction. To assess the accuracy we calculated the percentage error of the normalized effective index as a function of Δz for different N values. These results are shown in Fig. 1(a), (b), (c) for the three cases described earlier, respectively. For all cases as N gets larger and Δz gets smaller, accuracy improves. For the slowly varying index case the accuracy of both methods is not very sensitive to the actual N value as long as it is not too small. As Δz increases, however, accuracy degrades for both schemes. Accuracy of the FFT-BPM starts to degrade at smaller Δz values compared to FD-BPM. For the symmetrical step index case one needs to use a substantially large N value to get an accurate answer. Again as Δz increases accuracy of both schemes degrades. Degradation in the FD-BPM result is slower and well behaved. On the other hand FFT-BPM results show an oscillatory behavior. For the case of asymmetrical step index guide, the higher the N value, the better the accuracy. But the behavior of both schemes as Δz increases are drastically different. FD-BPM results degrade smoothly and slowly as Δz increases, which indicates that propagating steps approaching 20 μm can be used. FFT-BPM results, however, show an oscillatory degradation as Δz increases. This behavior is expected, because it is well known that for an accurate analysis of a waveguide structure with large refractive index change very small propagating steps are required [7], [8]. For this case, the magnitude of the FFT-BPM accuracy and its behavior as a function of Δz becomes comparable to that of FD-BPM only for $\Delta z < 0.2$ μm.

As another test on the accuracy, we calculated the power loss of the eigenmode of a waveguide as it propagates. If the numerical calculations are accurate we do not expect any power loss as the eigenmode propagates. However, due to the approximate nature of both techniques certain amount of loss is observed in the numerical calculations. We refer to this loss as the loss due to numerical errors, and it is plotted in Fig. 2(a)–(c) as a function of Δz for different N values. An increase in the loss due to numerical errors clearly indicates a loss in the accuracy. For the slowly varying index case, numerical loss

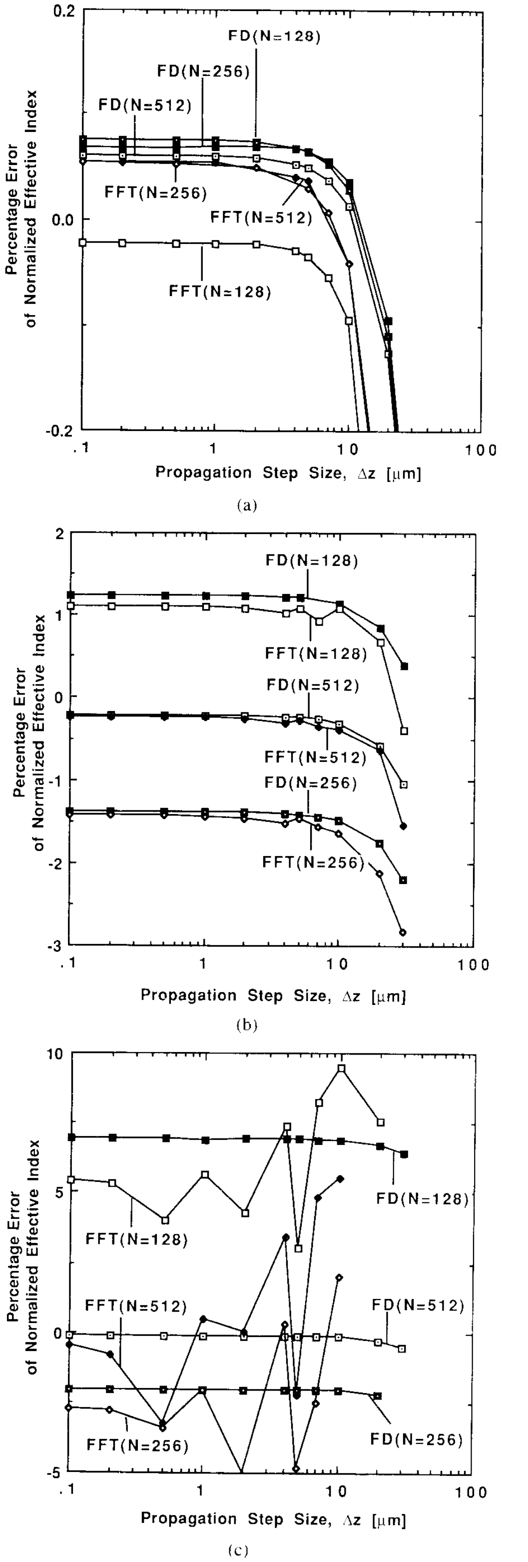

Fig. 1. Percentage of error of the normalized effective index as a function of propagation step size Δz for a waveguide (a) with a slowly varying index profile (case 1), (b) with a symmetric step index profile (case 2), and (c) with an asymmetric step index profile (case 3). The parameter N is a number of grid points used in the calculation.

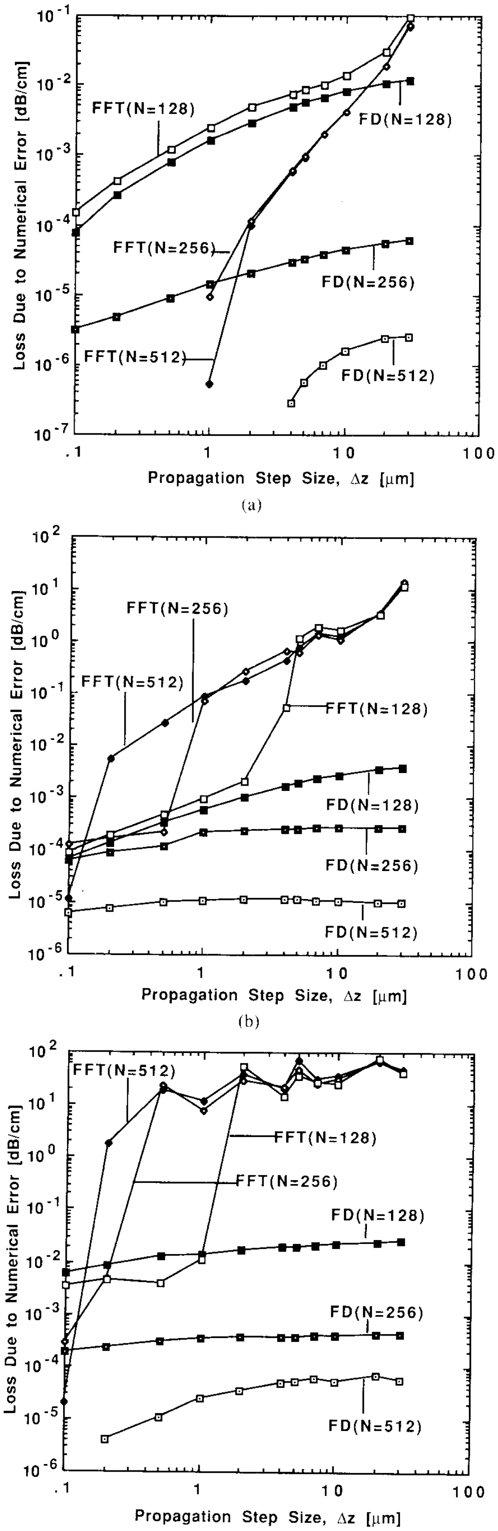

Fig. 2. Loss due to numerical error as a function of propagation step size Δz for a waveguide (a) with a slowly varying index profile (case 1), (b) with a symmetric step index profile (case 2), and (c) with an asymmetric step index profile (case 3). The parameter N is a number of grid points used in the calculation. The loss is evaluated when an eigenmode of a waveguide is launched.

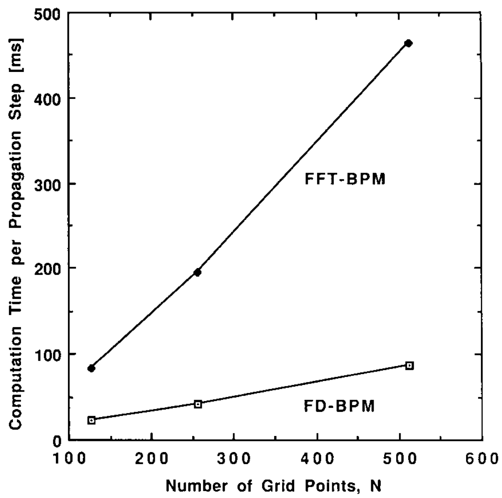

Fig. 3. CPU time per propagation step as function of a number of grid points N. All the CPU times are evaluated on a SUN-SPARC workstation.

of both schemes increases as Δz increases and N decreases. The degradation is gradual and well behaved in the case of FD-BPM. For FFT-BPM, however, loss increases very sharply with increasing Δz. Furthermore, numerical loss is quite sensitive to N values for both schemes. As N increases numerical loss decreases by orders of magnitude for the FD-BPM. For FFT-BPM an improvement in numerical loss with increasing N is observed only for Δz values less than 1 μm. For $\Delta z > 1$ μm, numerical loss for $N = 256$ and 512 becomes almost identical. For $\Delta z > 30$ μm losses for all N values increase sharply, indicating a sharp degradation in the accuracy of FFT-BPM which agrees with the previous conclusions based on Fig. 1(a). For the symmetrical and asymmetrical step index cases the same features are again observed. Numerical loss of the FD-BPM is not a very sensitive function of Δz, but is a very sensitive function of N. On the other hand, with FFT-BPM one has to use very small propagating steps to increase accuracy and get the benefit of increasing N. For example, for the asymmetric step index case increasing Δz from 0.1 to 0.2 μm for $N = 512$ increases the loss due to numerical errors from 2×10^{-5} dB/cm to 2 dB/cm. This result indicates that $\Delta z = 0.2$ μm is unacceptable in the simulations, because it results in an unrealistic loss. On the other hand, a corresponding change in the percentage error of the normalized effective index as observed from Fig. 1(c) is about 0.5%, which is acceptable. This observation indicates that loss due to numerical error could be used as a more sensitive indicator of the accuracy of a beam propagation method.

Although it is possible to get the same accuracy using both methods, due to the need for a small propagating step, the required computation time effort for FFT-BPM can be drastically higher than that of FD-BPM. To compare the computational speed of both methods we compared the CPU time required per propagation step as a function of N in the range from 128 to 512. This is the time it takes to find the field profile Δz away from a known field profile and is independent of the structure under consideration. As shown in Fig. 3, as the number of grid points increases, CPU time per step for both techniques increases. The increase in FFT-BPM is more rapid and over the N values considered FD-BPM is 4 to 6 times faster than FFT-BPM. This is a direct indication of the fact that computation time required to solve a tridiagonal system of N linear equations increases as N, whereas time required to obtain the FFT of a function using N grid points increases as $N \log N$. Comparison of the accuracy of both methods shows that one can get accurate results with larger propagating step sizes in FD-BPM. Combined with the CPU time improvement per step, this indicates that FD-BPM can be much more efficient than FFT-BPM.

IV. Conclusions

In this paper, a beam propagation method employing a finite difference approximation is studied in comparison with that using fast Fourier transformation. In the study, three different one-dimensional index profiles, whose effective index values can be determined accurately using well-known analytical expressions, were considered. Using both methods the effective indexes of the eigenmodes of these three different cases were calculated. It is found that the computation time per propagation step for FD-BPM is 4–6 times less than that of FFT-BPM when grid points ranges from 128–512. Furthermore, FD-BPM is much more stable with respect to propagation step size, Δz, and number of grid points N variations. For comparable accuracy one needs much smaller propagation step sizes in the FFT-BPM than the FD-BPM especially in the analysis of step index waveguides. This indicates that combined with the CPU time improvement per step FD-BPM can be much more efficient than FFT-BPM. As a further test on the accuracy of both methods, loss due to numerical errors, when eigenmode of the waveguide is launched, is calculated. Results indicate that this loss value could be a more sensitive indicator of the accuracy. One can obtain unacceptably high loss even though the accuracy of the propagation constant could be acceptable. Calculated loss values also indicate that FD-BPM is more efficient and stable compared to FFT-BPM.

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