Cavity Optomechanics

Wentao Jiang

Contents

1 Introduction

Historical review:

- Ashkin, focused laser beams can trap and control dielectric particles; Laser cooling;
  Application: optical atomic clocks, precision measurements
- Braginsky, dynamical influence of radiation pressure; quantum fluctuations of it, established the standard quantum limit for continuous position detection
- theoretical: squeezing of light, QND detection of the light intensity, quantum nonlinearities for extremely strong optomechanical coupling, give rise to nonclassical and entangled states of the light field and the mechanics
- experimental: optical feedback cooling; feedback damping, self-induced oscillations
- systems: membranes; nanorods; microdisks; microspheres; optical waveguides; nanomechanical beam inside a superconducting transmission line microwave cavity
- motivations: sensitive optical detection of small forces, displacements, masses and accelerations; interconvert information between solid-state qubits and flying photonic qubits

2 Optical Cavities and Mechanical Resonators

2.1 Optical resonators

F-P resonator (etalon):

- angular frequency: $\omega_{\mathrm{cav},m} \approx m \cdot \pi(c/L)$
- free spectral range (FSR): $\Delta\omega_{\mathrm{FSR}} = \pi\frac{c}{L}$
- optical finesse: $\mathcal{F} \equiv \Delta\omega_{\mathrm{FSR}}/\kappa$
- quality factor: $Q_{\mathrm{opt}} = \omega_{\mathrm{cav}}\tau$
- total cavity loss rate: $\kappa = \kappa_{\mathrm{ex}} + \kappa_0$
  where κ_{ex} refers to the input coupling loss rate
  photons going into the κ_0 decay channel won't be recorded

2.2 Input-output formalism

Master equations: only internal dynamics is of interest
Input-output theory: include the light field being emitted by the cavity, formulated on the level of Heisenberg equations of motion, describing the time evolution of the field amplitude $\hat{a}$ inside the cavity:

$$\dot{\hat{a}} = -\frac{\kappa}{2}\hat{a} + i\Delta\hat{a} + \sqrt{\kappa_{\mathrm{ex}}}\hat{a}_{\mathrm{in}} + \sqrt{\kappa_0}\hat{f}_{\mathrm{in}} \tag{2.1}$$

where laser detuning $\Delta = \omega_L - \omega_{\mathrm{cav}}$, $\hat{a}_{\mathrm{in}}$ should be regarded as a stochastic quantum field. The field is normalized that

$$P = \hbar\omega_L \left\langle \hat{a}_{\mathrm{in}}^{\dagger}\hat{a}_{\mathrm{in}} \right\rangle \tag{2.2}$$

is the input power launched into the cavity. The same kind of description holds for the "unwanted" channel $\hat{f}_{\mathrm{in}}$.

The field reflected from the F-P resonator is given by

$$\hat{a}_{\mathrm{out}} = \hat{a}_{\mathrm{in}} - \sqrt{\kappa_{\mathrm{ex}}}\hat{a} \tag{2.3}$$

After taking average of eqt 2.1, 2.3 and $\left\langle \hat{f}_{\mathrm{in}} \right\rangle = 0$:

$$\langle \hat{a} \rangle = \frac{\sqrt{\kappa_{\mathrm{ex}}}\,\langle \hat{a}_{\mathrm{in}} \rangle}{\kappa/2 - i\Delta} \tag{2.4}$$

$$\chi_{\mathrm{opt}}(\omega) \equiv \frac{1}{-i(\omega + \Delta) + \kappa/2} \tag{2.5}$$

This is adequate as long as $\kappa \ll \Delta\omega_{\mathrm{FSR}}$, i.e., $Q \gg 1$

The steady state cavity population

$$\bar{n}_{\mathrm{cav}} = |\langle \hat{a} \rangle|^2 = \frac{\kappa_{\mathrm{ex}}}{\Delta^2 + (\kappa/2)^2}\frac{P}{\hbar\omega_L} \tag{2.6}$$

Reflection amplitude:

$$\mathcal{R} = \frac{\langle \hat{a}_{\text{out}} \rangle}{\langle \hat{a}_{\text{in}} \rangle} = \frac{(\kappa_0 - \kappa_{\text{ex}})/2 - i\Delta}{(\kappa_0 + \kappa_{\text{ex}})/2 - i\Delta} \tag{2.7}$$

If the external coupling dominates the cavity losses ($\kappa_{\text{ex}} \approx \kappa \gg \kappa_0$), the cavity is called "**overcoupled**" and $|\mathcal{R}|^2 \approx 1$. The case where $\kappa_{\text{ex}} = \kappa_0$ is called "**critical coupling**", where $\mathcal{R}(\Delta = 0) = 0$ on resonance. This implies that the input power is either fully dissipated within the resonator or fully transmitted. The situation $\kappa_{\text{ex}} \ll \kappa_0$ is referred to as "**undercoupling**" and is associated with cavity losses dominated by intrinsic losses, leading to an effective loss of information.

2.3 Mechanical resonators

- displacement field: $\vec{u}_n(\vec{r})$, n denotes the mode
- vibration frequency: Ω_m
- energy damping rate: Γ_m
- mechanical quality factor: $Q_m = 1/\delta\Phi = \Omega_m/\Gamma_m$
- global amplitude: $x(\vec{r})$
- a suitable mode function: $\vec{u}(\vec{r},t) = x(t) \cdot \vec{u}(\vec{r})$

then the temperal evolution of $x(t)$:

$$m_{\text{eff}} \frac{d^2x(t)}{dt^2} + m_{\text{eff}} \frac{dx(t)}{dt} + m_{\text{eff}}\Omega_m^2 x(t) = F_{\text{ex}}(t) \tag{2.8}$$

$F_{\text{ex}}(t)$ denotes the sum of all forces acting on the mechanical oscillator. In the absence of external forces, it is given by the Langevin force.

Eqt 2.8 can be solved easily in frequency space:

$$x(\omega) = \int dt e^{i\omega t} x(t) \tag{2.9}$$

$$x_m(\omega) = \left[m_{\text{eff}}\left(\Omega_m^2 - w^2\right) - i m_{\text{eff}}\Gamma_m\omega\right]^{-1} \tag{2.10}$$

The quantum mechanical treatment:

$$\hat{H} = \hbar\Omega_m \hat{b}^\dagger\hat{b} + \frac{1}{2}\hbar\Omega_m \tag{2.11}$$

$$\hat{x} = x_{\text{XPF}}(\hat{b} + \hat{b}^\dagger) \tag{2.12}$$

$$\hat{p} = -i m_{\text{eff}}\Omega_m x_{\text{XPF}}(\hat{b} - \hat{b}^\dagger) \tag{2.13}$$

where

$$x_{\text{ZPF}} = \sqrt{\frac{\hbar}{2m_{\text{eff}}\Omega_m}} \tag{2.14}$$

is the zero-point fluctuation amplitude.

Coupled to a high-temperature bath:

$$\frac{d}{dt}\bar{n} = -\Gamma_m(\bar{n} - \bar{n}_{\text{th}}) \tag{2.15}$$

$$\frac{d}{dt}\bar{n}(t=0) = \Gamma_m \bar{n}_{\text{th}} \approx \frac{k_B T_{\text{bath}}}{\hbar Q_m} \tag{2.16}$$

2.3.1 Mechanical dissipation

- viscous damping
- clamping losses
- fundamental anharmonic effects
- materials-induced losses

2.3.2 Susceptibility, noise spectra, fluctuation-dissipation theorem

Gated Fourier transform:

$$\tilde{x}(\omega) = \frac{1}{\sqrt{\tau}} \int_0^\tau x(t) e^{i\omega t} dt \tag{2.17}$$

Noise power spectral density:

$$S_{xx} \equiv \int_{-\infty}^{+\infty} \langle x(t)x(0)\rangle e^{i\omega t} dt \tag{2.18}$$

Wiener-Khinchin theorem:

$$\lim_{\tau\to\infty} \left\langle |\tilde{x}(\omega)|^2 \right\rangle = S_{xx}(\omega) \tag{2.19}$$

The experimentally measured mechanical noise spectrum yields the variance of the mechanical displacement $\langle x^2 \rangle$:

$$\int_{-\infty}^{+\infty} S_{xx}(\omega) \frac{d\omega}{2\pi} = \left\langle x^2 \right\rangle \tag{2.20}$$

The fluctuation-dissipation theorem (FDT):

$$S_{xx}(\omega) = 2\frac{k_B T}{\omega} \text{Im}\chi_m(\omega) \tag{2.21}$$

where $\chi_m(\omega)$ denotes the mechanical susceptibility.

The quantum FDT:

$$S_{xx}(\omega) = \frac{\hbar}{1 - \exp(-\hbar\omega/k_B T)} \text{Im}\chi_{xx}(\omega) \tag{2.22}$$

3 Principles of Optomechanical Coupling