Circuit Quantum Electrodynamics

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1 Introduction

Requirements of a quantum computer:

1. isolated from sources of noise
2. strongly coupled to each other
3. refer to [1] for more and detailed requirements

Cavity QED system: an atom modeled as a two-level system coupled to a harmonic oscillator, whose excitations are photons.

cQED-like systems:

- alkali atoms above an optical cavity formed by two mirrors, readout by transmission of a laser through the cavity
- Rydberg atoms with 3-D microwave cavities, require cooling to ∼1K, use atoms to probe the cavity photons by selective ionization

1.1 Quantum Circuits

the only known dissipationless non-linear circuit element

High frequencies (GHz) LC circuit at low temperatures (<100mK) will have resolvable energy levels corresponding to microwave photons. However, its harmonicity makes it impossible to observe the discrete nature of these photons.

Josephson junction, cooper pair box (CPB): enhance nonlinearity

Atoms: quantum purity for 10^{14} periods, hard to tune

Artificial atoms (quantum dots, CPB): more tunable, more decoherence

1.2 Circuit Quantum Electrodynamics

Superconductive circuits $\Longleftrightarrow$ artificial atoms

Key to cQED readout: use 1-D coplanar waveguide (CPW) resonator as a cavity

Benefits:

- travel length of MW photon in CPW: 10km
- stronger coupling (10000 times larger than ordinary alkali atom, 10 times larger than Rydberg atom)
- 1-D transmission line cavity increasing the energy density by 10^6 over 3-D MW cavities, further increase dipole coupling by 1000

2 Cavity Quantum Electrodynamics

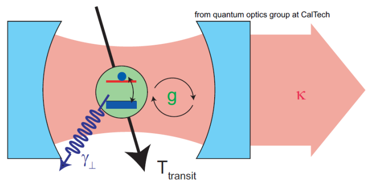


Figure 1: A two level atom interacts with the field inside of a high finesse cavity. [2]

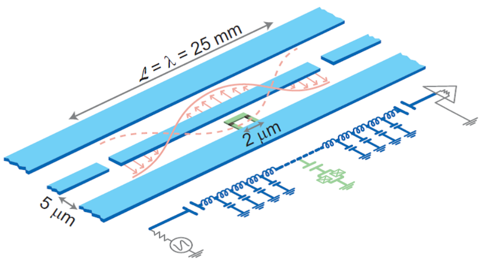


Figure 2: Schematic representation of cavity QED with superconducting circuits. [2]

Jaynes-Cummings Hamiltonian:

$$H_{\mathrm{JC}} = \hbar\omega_r(a^\dagger a + 1/2) + \hbar\frac{\omega_a}{2}\sigma_z + \hbar g(a^\dagger\sigma^- + a\sigma^+) \quad (2.1)$$

g: coupling rate

κ: photon decay rate

$Q = \omega_r/\kappa$: quality factor

$\gamma_\perp$: atom decay rate

T_{transit}: atom transit time before leaving cavity

When the atom is resonant with the cavity, the two system can freely exchange energy. When many oscillations can be completed before the atom decays, it's called the **strong coupling** limit: $g > \gamma, \kappa, 1/T_{\text{transit}}$.

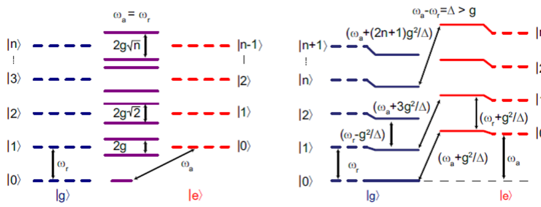


Figure 3: Energy level diagram of the J-C Hamiltonian. [2]

Eigenstates: $|\psi_\pm\rangle = (|g\rangle |n\rangle \pm |e\rangle |n-1\rangle)/\sqrt{2}$, whose energies split by $2g\sqrt{n}$, i.e., energy levels are anharmonic even for single photon in the strong coupling limit.

Because excitations are equally shared between atoms and the cavity, they decay at $\Gamma_{\text{eff}} = (\gamma_\perp + \kappa)/2$

Purcell effect(?)

2.1 Dispersive Limit

Resonant limit: $\Delta = \omega_a - \omega_r \ll g$

Dispersive limit: $\Delta \gg g$

Using perturbation to the interaction part and expand in powers of g/Δ to second order:

$$H \approx \hbar \left(\omega_r + \frac{g^2}{\Delta}\sigma_z\right)(a^\dagger a + 1/2) + \hbar\omega_a \sigma_z/2 \tag{2.2}$$

The interaction is through the dispersive shift term proportional to g^2/Δ, which commutes with the rest of the Hamiltonian, conserves both the photon number and the atom state.

Result: the effective frequency of the cavity $\omega_r' = \omega_r \pm g^2/\Delta$ depends on the state of the atom.

Rewrite the Hamiltonian to

$$H \approx \hbar(a^\dagger a + 1/2) + \frac{\hbar}{2}\left(\omega_a + \frac{2g^2}{\Delta}a^\dagger a + \frac{g^2}{\Delta}\right)\sigma_z/2 \tag{2.3}$$

Result: the interaction gives the atom frequency a "light" shift consisting of a photon number-dependent "Stark" shift $2ng^2/\Delta$ and vacuum noise induced "Lamb" shift g^2/Δ

Accurate general solution:

$$E_{\pm,n} = \hbar\omega_r \pm \frac{\hbar}{2}\sqrt{4ng^2 + \Delta^2} \tag{2.4}$$

$$E_{g,0} = -\frac{\hbar\Delta}{2} \tag{2.5}$$

where n is the total number of excitations, **not** the number of photons and $\pm$ refer to the higher energy or lower energy state in the n excitation manifold, **not** the atom state.

Eigenstates:

$$|-,n\rangle = \cos\theta_n |g,n\rangle - \sin\theta_n |e,n-1\rangle \tag{2.6}$$

$$|+,n\rangle = \sin\theta_n |g,n\rangle + \cos\theta_n |e,n-1\rangle \tag{2.7}$$

$$\theta_n = \frac{1}{2}\arctan\left(\frac{2g\sqrt{n}}{\Delta}\right) \tag{2.8}$$

When dispersive limit is reached:

$$|-,n\rangle = |g,n\rangle - \frac{g\sqrt{n}}{\Delta}|e,n-1\rangle \tag{2.9}$$

$$|+,n\rangle = \frac{g\sqrt{n}}{\Delta}|g,n\rangle + |e,n-1\rangle \tag{2.10}$$

i.e., qubit states have some photon component, which can be used to create an "entanglement" bus when multiple qubits are strongly coupled to the same cavity with overlapping photonic part of their wavefunctions.

Instead of $\Gamma_{\text{eff}} = (\gamma_\perp + \kappa)/2$ as in resonant limit, the qubit has some probability to decay as a photon at a rate γ_κ, because it has a photonic aspect, and vice versa photons can be emitted by the qubit into non-radiative modes at a rate κ_γ:

$$\gamma_\kappa \approx \left(\frac{g}{\Delta}\right)^2 \kappa \tag{2.11}$$

$$\kappa_\gamma \approx \left(\frac{g}{\Delta}\right)^2 \gamma \tag{2.12}$$

The non-radiative ssource of decay $\gamma_\perp$ will begin to dominate before the limit set by suppressed radiative decay.

2.2 Strong Dispersive Interactions

Dispersive regime: $\Delta \gg g$

Strong dispersive regime: $\Delta \gg g$, $\chi = g^2/\Delta > \gamma, \kappa$

i.e., the qubit spectrum resolves into individual photon number peaks, Cavity shift is also larger than cavity linewidth.

For measuring the state of a single atom using the cavity, one can use many photons to realize it for $\chi < \kappa$. For single photon precision, dispersive strong coupling is required.

If $\chi > \gamma$ is reached, the atom is shifted by more than a linewidth for each photon, making it possible to measure the exact photon number distribution. A CNOT gate can also be realized if the qubit is made only to respond when a single photon is present.

3 Cavity QED with Superconducting Circuits

3.1 The Parallel LCR Oscillator

Equation of motion:

$$\frac{d^2q}{dt^2} - \frac{1}{RC}\frac{dq}{dt} + \frac{q}{LC} = 0 \tag{3.1}$$

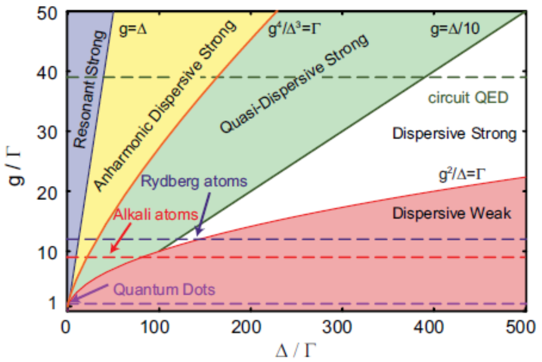


Figure 4: A phase diagram for cavity QED, described by the atom-photon coupling strength g and the detuning Δ, normalized to the rates of decay $\Gamma = \max[\gamma, \kappa, 1/T]$.[2]

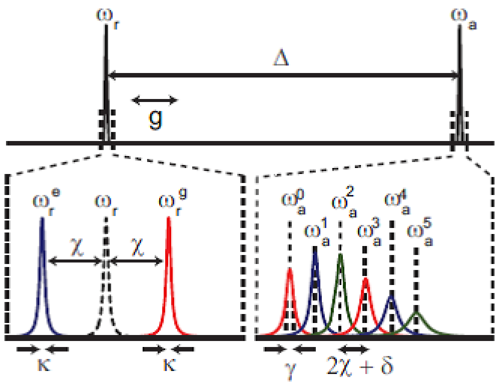


Figure 5: Spectrum of coupled cavity and atom system in strong dispersive limit. [2]

Solution:

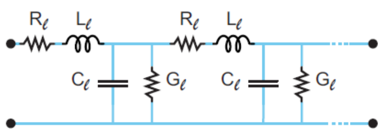
$$q(t) = q_0 \exp\left[i(\omega_0 + i\frac{\kappa}{2})t + \phi\right] \tag{3.2}$$

where charge oscillation frequency $\omega_0 = 1/\sqrt{RC}$ and decay rate $\kappa = 2/RC$.
Impedance:

$$Z_{LCR}(\omega) = \left(j\omega C + \frac{1}{j\omega L} + \frac{1}{R}\right)^{-1} = \frac{R}{1 + 2jQ\delta\omega/\omega_0} \tag{3.3}$$

where the quality factor $Q = \omega_0 RC$

3.2 Transmission Line as Series of LC Circuits



Impedance of one small section:

$$Z_0 = \sqrt{\frac{R_l + j\omega L_l}{G_l + j\omega C_l}} \tag{3.4}$$

Small loss: $Z_0 = \sqrt{L_l/C_l}$
Propagation coefficient: $\gamma = \sqrt{(R_l + j\omega L_l)(G_l + j\omega C_l)}$
$\beta = \mathrm{Im}\gamma$ describes the phase, for given frequency, it defines a phase velocity $v = \omega/\beta$
The attenuation $\alpha = \mathrm{Re}\gamma \approx R_l/Z_0 + G_l Z_0$
Effective input impedance of an arbituary load Z_l:

$$Z_{\rm in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \tag{3.5}$$

where l is the length.

For termination with high impedances, there will be two types of resonance:

- high impedance resonance with $l = n\lambda/2$
- high admittance resonance with $l = (2n + 1)\lambda/4$

References

[1] David P DiVincenzo. The physical implementation of quantum computation. *arXiv preprint quant-ph/0002077*, 2000.

[2] David Isaac Schuster. *Circuit quantum electrodynamics.* Thesis, 2007.