Principles of Lasers

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Contents

1 Concepts

Spontaneous and stimulated emission, absorption

Spontaneous (or radiative) emission:

$$\nu_0 = (E_2 - E_1)/h \tag{1.1}$$

$$\left(\frac{dN_2}{dt}\right)_{sp} = -AN_2 = -\frac{N_2}{\tau_{sp}} \tag{1.2}$$

Non-radiative decay:

$$\left(\frac{dN_2}{dt}\right)_{nr} = -\frac{N_2}{\tau_{nr}} \tag{1.3}$$

Stimulated emission & absorption:

$$\left(\frac{dN_2}{dt}\right)_{st} = -W_{21}N_2 \tag{1.4}$$

$$W_{21} = \sigma_{21}F \tag{1.5}$$

$$\left(\frac{dN_1}{dt}\right)_{a} = -W_{12}N_1 \tag{1.6}$$

$$W_{12} = \sigma_{12}F \tag{1.7}$$

σ_{21}: stimulated emission cross section.
σ_{12}: absorption cross section.

Shown by Einstein:

$$g_2W_{21} = g_1W_{12} \tag{1.8}$$

$$g_2\sigma_{21} = g_1\sigma_{12} \tag{1.9}$$

Increase of photon flux F:

$$dF = \sigma_{21}F[N_2 - (g_2N_1/g_1)]dz \tag{1.10}$$

At thermal equilibrium:

$$\frac{N_2^e}{N_1^e} = \frac{g_2}{g_1}\exp\left(-\frac{E_2 - E_1}{kT}\right) \tag{1.11}$$

1.1 The Laser

To generate laser, population inversion: $N_2 > g_2N_1/g_1$
Gain per pass: $\exp\{\sigma[N_2 - g_2N_1/g_1]l\}$
Critical inversion:

$$N_c = -[\ln R_1R_2 + 2\ln(1 - L_i)]/2\sigma l \tag{1.12}$$

R_1, R_2: power reflectivity of the two mirrors
L_i: internal loss per pass
Define:

$$\gamma_1 = -\ln R_1 = -\ln(1 - T_1) \tag{1.13}$$

$$\gamma_2 = -\ln R_2 = -\ln(1 - T_2) \tag{1.14}$$

$$\gamma_i = -\ln(1 - L_i) \tag{1.15}$$

$$\therefore N_c = \gamma/\sigma l \tag{1.16}$$

$$\gamma = \gamma_i + (\gamma_1 + \gamma_2)/2 \tag{1.17}$$

1.2 Pumping Schemes

- two-level: impossible to produce inversion
- three-level scheme
- four-level scheme: easier to realize inversion
- quasi-three-level: the ground level consists of sublevels

For most four-level and quasi-three-level lasers, the depletion of the ground level can be neglected, hence:

$$(dN_2/dt)_p = R_p \tag{1.18}$$

R_p: pump rate. To achieve threshold, R_p must reach R_{cp}.

1.3 Properties of Laser Beams

1. Monochromaticity: much narrower (ten orders) than linewidth of spontaneous emission
2. Coherence: temporal & spatial coherence
3. Directionality: $\theta_d = \beta\lambda/D = \beta\lambda/[S_c]^{1/2}$
   S_c: coherence area
   β: numerical coefficient
   λ: wavelength
4. Brightness: $B = 4P/(\pi D\theta)^2$ for $\theta \ll 1$
   with $\theta = \theta_d$, $B = \left(\frac{2}{\beta\pi\lambda}\right)^2 P$
5. Short time duration

2 Interaction of Radiation with Atoms and Ions