Introduction to Superconductivity

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Contents

1 Historical overview

1.1 The Basic Phenomena

- perfect conductivity
- perfect diamagnetism, $H_c(T) \approx H_c(0)\left[1-(T/T_c)^2\right]$

1.2 The London Equations

$$\boldsymbol{E} = \frac{\partial}{\partial t}(\Lambda \boldsymbol{J}_s) \tag{1.1}$$

$$\boldsymbol{h} = -c\nabla \times (\Lambda \boldsymbol{J}_s) \tag{1.2}$$

$$\Lambda = \frac{4\pi\lambda^2}{c^2} = \frac{m}{n_s e^2} \tag{1.3}$$

$\boldsymbol{h}$: microscopic flux density

$\boldsymbol{B}$: macroscopic average value

n_s: number density of superconducting electrons

Combining Maxwell equation $\nabla \times \boldsymbol{h} = 4\pi\boldsymbol{J}/c$ leads to $\nabla^2\boldsymbol{h} = h/\lambda^2$, predicting penetration depth λ. Emprically:

$$\lambda(T) \approx \lambda(0)\left[1-(T/T_c)^4\right]^{-1/2} \tag{1.4}$$

Consider a perfect normal conductor in a uniform $\boldsymbol{E}$ field, $d(m\boldsymbol{v})/dt = e\boldsymbol{E}, \boldsymbol{J} = ne\boldsymbol{v}$. But it's not rigorous for spacially nonuniform fields within λ, for which the LEs are most useful. It's because the response of an electron gas to $\boldsymbol{E}$ field is nonlocal.

A more profound motivation for LEs is a quantum one by considering $\boldsymbol{p} = (m\boldsymbol{v} + e\boldsymbol{A}/c)$ to have zero expectancy for ground state, i.e.,

$$\langle \boldsymbol{v}_s \rangle = \frac{-e\boldsymbol{A}}{mc} \tag{1.5}$$

$$\boldsymbol{J}_s = n_s e \langle \boldsymbol{v}_s \rangle = \frac{-\boldsymbol{A}}{\Lambda c} \tag{1.6}$$

which contains the two LEs in a compact form (it's not gauge-invariant, London gauge $\nabla \cdot \boldsymbol{A} = 0$ is required).

1.3 The Pippard Nonlocal Electrodynamics

Introduce the coherence length ξ to propose a nonlocal generalization of the LEs, in analogy to Chamber's nonlocal generalization of Ohm's law:

$$\boldsymbol{J}(\boldsymbol{r}) = \frac{3\sigma}{4\pi l}\int \frac{\boldsymbol{R}[\boldsymbol{R}\cdot\boldsymbol{E}(\boldsymbol{r}')]e^{-R/l}}{R^4}d\boldsymbol{r}'$$

From uncertainty principle, for electrons play a major role in superconductive phenomenon, $E \sim kT_c, \Delta p \approx kT_c/v_F$, thus

$$\xi_0 = a\frac{\hbar v_F}{kT_c} \tag{1.7}$$

hence

$$\boldsymbol{J}_s(\boldsymbol{r}) = -\frac{3}{4\pi\xi_0\Lambda c}\int \frac{\boldsymbol{R}[\boldsymbol{R}\cdot\boldsymbol{A}(\boldsymbol{r}')]e^{-R/\xi}}{R^4}d\boldsymbol{r}' \tag{1.8}$$

instead of $\boldsymbol{J}_s = n_s e\langle \boldsymbol{v}_s\rangle = \frac{-\boldsymbol{A}}{\Lambda c}$, where $1/\xi = 1/\xi_0 + 1/l$.

1.4 The Energy Gap and the BCS Theory

Experimental hints:

Electronic specific heat: $C_{es} \approx \gamma T_c a e^{-bT_c/T}$, while normal state $C_{en} = \gamma T$

Spectroscopic measurement gives minimum energy E_g to create excitations, while thermal one measures $E_g/2$ per statistically independent particle, suggesting pair production.

Key prediction of BCS: $E_g(0) = 2\Delta(0) = 3.528kT_c$ for $T \ll T_c$.

1.5 The Ginzburg-Landau Theory

7 years before BCS, they introduce a complex pseudowavefunction as an order parameter within Landau's general theory of 2nd order phase transitions. Eqt for ψ are obtained from a variational principle and expansion of the free energy in powers of ψ and $\nabla\psi$ with coefficients α and β:

$$n_s = |\psi(x)|^2 \tag{1.9}$$

$$\frac{1}{2m^*}\left(\frac{\hbar}{i}\nabla - \frac{e^*}{c}A\right)^2 \psi + \beta|\psi|^2\psi = -\alpha(T)\psi \tag{1.10}$$

$$\boldsymbol{J}_s = \frac{e^*\hbar}{i2m^*}(\psi^*\nabla\psi - \psi\nabla\psi^*) - \frac{e^{*2}}{m^*c}|\psi|^2\boldsymbol{A} \tag{1.11}$$

GL coherence length: $\xi(T) = \frac{\hbar}{|2m^*\alpha(T)|^{1/2}}$
GL parameter: $\kappa = \frac{\lambda}{\xi}$

1.6 Type II Superconductors

$\xi < \lambda$ leads to a negative surface energy, so that subdivision proceeds until limited by the microscopic length ξ.

Another result: mixed state, or Schubnikov phase. Between H_{c1} and H_{c2}, the flux penetrates in a regular array of flux tubes carrying flux of

$$\Phi_0 = \frac{hc}{2e} = 2.07 \times 10^{-7} G/cm^2 \tag{1.12}$$

1.7 Josephson Tunneling and Flux Quantization

Phase and particle number are conjugate variables:

$$\Delta N \Delta\phi \gtrsim 1 \tag{1.13}$$

Josephson predicted pairs should be able to tunnel between two superconductor even at zero voltage difference:

$$J = J_c \sin(\phi_1 - \phi_2) \tag{1.14}$$

and with a voltage difference V_{12}, the phase difference $\phi = 2eV_{12}t/\hbar$ so that the current would oscillate.

Single-valuedness of $\psi = |\psi|e^{i\phi}$ requires that the fluxoid introduced by F. London:

$$\Phi' = \Phi + \frac{m^*c}{e^*}\oint \frac{\boldsymbol{J}_s \cdot d\boldsymbol{s}}{|\psi|^2} \tag{1.15}$$

to take only integral multiples of $\Phi_0 = hc/2e$, where $\Phi = \oint \boldsymbol{A} \cdot d\boldsymbol{s}$ is the ordinary magnetic flux. For ring with thickness comparable to λ, $J_s = 0$ on the integration path, $\Phi = \Phi' = n\Phi_0$.

1.8 Fluctuation and Nonequilibrium Effects

1.9 High-temperature Superconductivity

2 Introduction to Electrodynamics of Superconductivity

2.1 Screening

From the LEs:

$$d\boldsymbol{J}_s/dt = (c^2/4\pi\lambda^2)\boldsymbol{E} \tag{2.1}$$

Taking the time derivative of the Maxwell eqt $\nabla \times \boldsymbol{h} = 4\pi\boldsymbol{J}/c$ and eliminating $\partial\boldsymbol{h}/\partial t$ using $\nabla \times \boldsymbol{E} = -(1/c)\partial\boldsymbol{h}/\partial t$, we obtain

$$-\nabla \times \nabla \times \boldsymbol{E} = \nabla^2\boldsymbol{E} = \boldsymbol{E}/\lambda^2 \tag{2.2}$$

Taking the curl of the Maxwell eqt $\nabla \times \boldsymbol{h} = (4\pi/c)\boldsymbol{J}$ and by substituting $-\boldsymbol{h}/c\Gamma = \nabla \times \boldsymbol{J}$, we see that

$$\nabla^2\boldsymbol{h} = (1/\lambda^2)\boldsymbol{h} \tag{2.3}$$

where $\lambda^2 = mc^2/4\pi n_s e^2$

2.1.1 E.G. Flat Slab in Parallel Magnetic Field

Boundary: $h = H_a$ at the two surfaces at $x = \pm d/2$
Solution:

$$h = H_a \frac{\cosh(x/\lambda)}{\cosh(d/2\lambda)} \tag{2.4}$$

Average over the sample thickness d, gives

$$B \equiv \bar{h} \equiv H_a + 4\pi M = H_a \frac{2\lambda}{d} \tanh \frac{d}{2\lambda} \tag{2.5}$$

from which when $d \gg \lambda, B \to 0$ and $M \to -H_a/4\pi$. This is **Meissner effect** limit of perfect diamagnetism of bulk superconductors. On the other hand, when $d \ll \lambda$, series expansion shows

$$M \to -(H_a/4\pi)(d^2/12\lambda^2) \tag{2.6}$$

2.1.2 Critical Current of Wire

Consider a long superconducting wire of radius $a \gg \lambda$, carrying a current I. The current generate self-field at the surface of the wire of magnitude $H = 2I/ca \leq H_c$, hence $I_c = caH_c/2$, which scales with the perimeter, suggesting that the current flows only in a surface layer of constant thickness. It can be confirmed analytically by solving the LEs and MEs in this geometry that the thickness of the layer is λ, hence the critical current density

$$J_c = \frac{c}{4\pi}\frac{H_c}{\lambda} \tag{2.7}$$

This J_c also holds for wires thinner than λ, where the current density is nearly uniform and I_c is proportional to the cross-sectional area. J_c is typically of order 10^8A/cm^2, very large.

2.2 The Intermediate State

Such effect depends on the **shape** of the sample.

2.2.1 In Strong Magnetic Fields

2.2.2 Above Critical Current of a superconducting Wire

2.3 High-frequency Electrodynamics

2.3.1 High-frequency Dissipation in Superconductors

3 The BCS Theory