Introduction to Superconductivity

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1 Historical overview

1.1 The Basic Phenomena

- perfect conductivity
- perfect diamagnetism, $H_c(T) \approx H_c(0)\left[1-(T/T_c)^2\right]$

1.2 The London Equations

$$\boldsymbol{E} = \frac{\partial}{\partial t}(\Lambda \boldsymbol{J}_S) \tag{1.1}$$

$$\boldsymbol{h} = -c\nabla \times (\Lambda \boldsymbol{J}_S) \tag{1.2}$$

$$\Lambda = \frac{4\pi\lambda^2}{c^2} = \frac{m}{n_S e^2} \tag{1.3}$$

$\boldsymbol{h}$: microscopic flux density

$\boldsymbol{B}$: macroscopic average value

n_S: number density of superconducting electrons

Combining Maxwell equation $\nabla \times \boldsymbol{h} = 4\pi \boldsymbol{J}/c$ leads to $\nabla^2 \boldsymbol{h} = h/\lambda^2$, predicting penetration depth λ. Emprically:

$$\lambda(T) \approx \lambda(0)\left[1-(T/T_c)^4\right]^{-1/2} \tag{1.4}$$

Consider a perfect normal conductor in a uniform $\boldsymbol{E}$ field, $d(m\boldsymbol{v})/dt = e\boldsymbol{E}, \boldsymbol{J} = ne\boldsymbol{v}$. But it's not rigorous for spacially nonuniform fields within λ, for which the LEs are most useful. It's because the response of an electron gas to $\boldsymbol{E}$ field is nonlocal.

A more profound motivation for LEs is a quantum one by considering $\boldsymbol{p} = (m\boldsymbol{v} + e\boldsymbol{A}/c)$ to have zero expectancy for ground state, i.e.,

$$\langle \boldsymbol{v}_S \rangle = \frac{-e\boldsymbol{A}}{mc} \tag{1.5}$$

$$\boldsymbol{J}_S = n_S e \langle \boldsymbol{v}_S \rangle = \frac{-\boldsymbol{A}}{\Lambda c} \tag{1.6}$$

which contains the two LEs in a compact form (it's not gauge-invariant, London gauge $\nabla \cdot \boldsymbol{A} = 0$ is required).

1.3 The Pippard Nonlocal Electrodynamics

Introduce the coherence length ξ to propose a nonlocal generalization of the LEs, in analogy to Chamber's nonlocal generalization of Ohm's law:

$$\boldsymbol{J}(\boldsymbol{r}) = \frac{3\sigma}{4\pi l}\int \frac{\boldsymbol{R}[\boldsymbol{R}\cdot \boldsymbol{E}(\boldsymbol{r}')]e^{-R/l}}{R^4} d\boldsymbol{r}'$$

From uncertainty principle, for electrons play a major role in superconductive phenomenon, $E \sim kT_c, \Delta p \approx kT_c/v_F$, thus

$$\xi_0 = a\frac{\hbar v_F}{kT_c} \tag{1.7}$$

hence

$$\boldsymbol{J}_S(\boldsymbol{r}) = -\frac{3}{4\pi\xi_0\Lambda c}\int \frac{\boldsymbol{R}[\boldsymbol{R}\cdot \boldsymbol{A}(\boldsymbol{r}')]e^{-R/\xi}}{R^4} d\boldsymbol{r}' \tag{1.8}$$

instead of $\boldsymbol{J}_S = n_S e \langle \boldsymbol{v}_S \rangle = \frac{-\boldsymbol{A}}{\Lambda c}$, where $1/\xi = 1/\xi_0 + 1/l$.

1.4 The Energy Gap and the BCS Theory

Experimental hints:

Electronic specific heat: $C_{es} \approx \gamma T_c a e^{-bT_c/T}$, while normal state $C_{en} = \gamma T$

Spectroscopic measurement gives minimum energy E_g to create excitations, while thermal one measures $E_g/2$ per statistically independent particle, suggesting pair production.

Key prediction of BCS: $E_g(0) = 2\Delta(0) = 3.528kT_c$ for $T \ll T_c$.

1.5 The Ginzburg-Landau Theory

7 years before BCS, they introduce a complex pseudowavefunction as an order parameter within Landau's general theory of 2nd order phase transitions. Eqt for ψ are obtained

from a variational principle and expansion of the free energy in powers of ψ and $\nabla\psi$ with coefficients α and β:

$$n_S = |\psi(x)|^2 \tag{1.9}$$

$$\frac{1}{2m^*}\left(\frac{\hbar}{i}\nabla - \frac{e^*}{c}A\right)^2\psi + \beta|\psi|^2\psi = -\alpha(T)\psi \tag{1.10}$$

$$\boldsymbol{J}_S = \frac{e^*\hbar}{i2m^*}(\psi^*\nabla\psi - \psi\nabla\psi^*) - \frac{e^{*2}}{m^*c}|\psi|^2\boldsymbol{A} \tag{1.11}$$

GL coherence length: $\xi(T) = \frac{\hbar}{|2m^*\alpha(T)|^{1/2}}$
GL parameter: $\kappa = \frac{\lambda}{\xi}$

1.6 Type II Superconductors

$\xi < \lambda$ leads to a negative surface energy, so that subdivision proceeds until limited by the microscopic length ξ.

Another result: mixed state, or Schubnikov phase. Between H_{c1} and H_{c2}, the flux penetrates in a regular array of flux tubes carrying flux of

$$\Phi_0 = \frac{hc}{2e} = 2.07 \times 10^{-7} G/cm^2 \tag{1.12}$$

1.7 Josephson Tunneling and Flux Quantization

Phase and particle number are conjugate variables:

$$\Delta N \Delta\phi \gtrsim 1 \tag{1.13}$$

Josephson predicted pairs should be able to tunnel between two superconductor even at zero voltage difference:

$$J = J_c \sin(\phi_1 - \phi_2) \tag{1.14}$$

and with a voltage difference V_{12}, the phase difference $\phi = 2eV_{12}t/\hbar$ so that the current would oscillate.

Single-valuedness of $\psi = |\psi|e^{i\phi}$ requires that the fluxoid introduced by F. London:

$$\Phi' = \Phi + \frac{m^*c}{e^*}\oint \frac{\boldsymbol{J}_S \cdot d\boldsymbol{s}}{|\psi|^2} \tag{1.15}$$

to take only integral multiples of $\Phi_0 = hc/2e$, where $\Phi = \oint \boldsymbol{A}\cdot d\boldsymbol{s}$ is the ordinary magnetic flux. For ring with thickness comparable to λ, $J_S = 0$ on the integration path, $\Phi = \Phi' = n\Phi_0$.

1.8 Fluctuation and Nonequilibrium Effects

1.9 High-temperature Superconductivity

2 Introduction to Electrodynamics of Superconductivity