Elastic Cloaking with Polar Material

Introduction

An invisibility cloak aims at making inclusions neutral to probing incident wave fields. This can be achieved by surrounding the inclusion with a coating, which is designed so that outside of it the field produced by external sources remains exactly the same as that obtained in the case of free-field propagation. In that way, the cloak makes the inclusion impossible to be detected from measurements done outside of the cloak itself. In the context of linear elastodynamics, this implies that, in the most general case, the cloak should work for both P and S waves at the same time.

This example demonstrates how to implement the anisotropic linear elastic material model comprising non symmetric stresses, which is theoretically needed to obtain exact cloaking of an infinite cylinder when probed with the field generated by a point source (Ref. 1).

Model Definition

An infinite cylinder with radius r_0 is covered by a cloak of radius r_1. A unitary point load produces an oscillating force in the x direction at a circular frequency of 40 rad/s, producing a dipole excitation for P waves aligned with the x-axis, and at the same time a dipole excitation for S waves aligned with the y-axis. The geometrical parameters used in the model are listed in Table 1.

TABLE 1: GEOMETRICAL PARAMETERS.

PARAMETER	VALUE
r_0	20 cm
r_1	40 cm

GOVERNING EQUATIONS

The material properties required to obtain exact cloaking can be computed with the so-called transformation approach (Ref. 2). With this method, one starts by considering an unbounded domain occupied by a homogeneous isotropic linear elastic solid, where the displacement field $\mathbf{u}$ is governed by the elastodynamic Navier equations:

$$\nabla \cdot (C\nabla\mathbf{u}) = -\omega^2\rho\mathbf{u} \tag{1}$$

where a harmonic time dependence has been assumed. A mapping $\mathbf{x}' = \chi(\mathbf{x})$ can be then introduced for points with radial coordinate $r \le r_1$ as

$$r' = r_0 + \frac{r_1 - r_0}{r_1}r \tag{2}$$

This maps the origin onto a circle with radius r_0 (the inner of the cloak) and points on a circle of radius r_1 (the outer of the cloak) onto themselves. This means that the region of space inside the circle of radius r_1 is radially compressed to fit into the annular region between r_0 and r_1. This map is applied then as a coordinate transformation to Equation 1, and the resulting equation can be shown to be equivalent to

$$\nabla \cdot (C^{\mathrm{cloak}} \nabla \mathbf{u}') = -\omega^2 \rho^{\mathrm{cloak}} \mathbf{u}' \tag{3}$$

where $\mathbf{u}'$ is used for

$$\mathbf{u}'(\mathbf{x}') = \mathbf{u}(\chi^{-1}(\mathbf{x}')) \tag{4}$$

and

$$C^{\mathrm{cloak}}{}_{ijkl} = J^{-1} F_{iI} C_{IjKl} F_{kK} \tag{5}$$

$$\rho^{\mathrm{cloak}} = J^{-1} \rho \tag{6}$$

Here, F is the deformation gradient associated with the map χ, and J is its determinant, that is, the Jacobian of the transformation. Thus, the coefficients associated to the metric change induced by the change of coordinates are reinterpreted here as new material properties. If the annular region between r_0 and r_1 is filled with a linear elastic material whose properties are given by Equation 5 and Equation 6, then the solution outside the cloak will be exactly the same as the original free field propagation solution of Equation 1, while inside the cloak the displacement field will follow from Equation 4. Note that the same mapping χ equipped with a different definition of $\mathbf{u}'$ can lead to cloaks with different constitutive equations (Ref. 2). Here, the gauge transformation expressed by Equation 4 is chosen because it leads to a scalar density. Note, however, that the elasticity tensor defined in Equation 5 possesses the major symmetries

$$C^{\mathrm{cloak}}{}_{ijkl} = C^{\mathrm{cloak}}{}_{klji} \tag{7}$$

but not the minor ones

$$C^{\mathrm{cloak}}{}_{ijkl} \neq C^{\mathrm{cloak}}{}_{jikl} \tag{8}$$

$$C^{\mathrm{cloak}}{}_{ijkl} \neq C^{\mathrm{cloak}}{}_{ijlk}$$

implying that the stress in the cloak is not symmetric.

MATERIAL PROPERTIES

The material properties of the cloak computed in cylindrical coordinates result in the eight non-null elastic moduli (plane strain) are listed in Table 2, along with the properties of the background isotropic hosting solid, whose elasticity tensor is specified through the two Lamé parameters λ and μ. Following Ref. 1, these are parameters that result in the same ratio between P- and S-wave speeds as that of fused silica.

TABLE 2: MATERIAL PROPERTIES.

Parameter	Value
λ	2.3 Pa
μ	1 Pa
ρ	1 kg/m^3
C^{cloak}_{rrrr}	(l+2μ)(r-r0)/r
$C^{\text{cloak}}_{\theta\theta\theta\theta}$	(λ+2μ)r/(r-r0)
$C^{\text{cloak}}_{rr\theta\theta}$	λ
$C^{\text{cloak}}_{\theta\theta rr}$	λ
$C^{\text{cloak}}_{r\theta\theta r}$	μ
$C^{\text{cloak}}_{\theta rr\theta}$	μ
$C^{\text{cloak}}_{r\theta r\theta}$	μ(r-r0)/r
$C^{\text{cloak}}_{\theta r\theta r}$	μr/(r-r0)
ρ^{cloak}	ρ(r1/(r1-r0))^2(r-r0)/r

The material properties of the cloak are implemented with the aid of the **External Stress** attribute under the **Linear Elastic Material** node. This allows to add a non-symmetric stress component to the one computed via Hooke's Law in the **Linear Elastic Material**. The total stress in the cloak is thus additively split into a part that can be obtained setting a standard symmetric elasticity tensor, plus a nonsymmetric stress that directly depends on the elements of the gradient of displacements.

This task is more easily performed when the equations are written in Cartesian coordinates. Equation 3 is thus firstly rewritten in component form in a Cartesian frame as

$$\frac{\partial}{\partial x_i}\left(C^{\text{cloak}}_{ijkl}\frac{\partial u'_l}{\partial x_k}\right) + \rho^{\text{cloak}}\omega^2 u'_j = 0$$

or, explicitly

$$\frac{\partial}{\partial x}\left(C^{\text{cloak}}_{1111}\frac{\partial u'}{\partial x} + C^{\text{cloak}}_{1121}\frac{\partial u'}{\partial y} + C^{\text{cloak}}_{1112}\frac{\partial v'}{\partial x} + C^{\text{cloak}}_{1122}\frac{\partial v'}{\partial y}\right) + \dots$$

$$\dots + \frac{\partial}{\partial y}\left(C^{\text{cloak}}_{2111}\frac{\partial u'}{\partial x} + C^{\text{cloak}}_{2121}\frac{\partial u'}{\partial y} + C^{\text{cloak}}_{2112}\frac{\partial v'}{\partial x} + C^{\text{cloak}}_{2122}\frac{\partial v'}{\partial y}\right) + \rho^{\text{cloak}}\omega^2 u' = 0$$

$$\frac{\partial}{\partial x}\left(C^{\text{cloak}}_{1211}\frac{\partial u'}{\partial x} + C^{\text{cloak}}_{1221}\frac{\partial u'}{\partial y} + C^{\text{cloak}}_{1212}\frac{\partial v'}{\partial x} + C^{\text{cloak}}_{1222}\frac{\partial v'}{\partial y}\right) + \dots$$

$$\dots + \frac{\partial}{\partial y}\left(C^{\text{cloak}}_{2211}\frac{\partial u'}{\partial x} + C^{\text{cloak}}_{2221}\frac{\partial u'}{\partial y} + C^{\text{cloak}}_{2212}\frac{\partial v'}{\partial x} + C^{\text{cloak}}_{2222}\frac{\partial v'}{\partial y}\right) + \rho^{\text{cloak}}\omega^2 v' = 0$$

Note indeed that the eight non-null moduli in cylindrical coordinates give rise in general to 16 non-null moduli in Cartesian coordinates. The four distinct elements of the stress tensor can be thus represented as

$$\sigma_{11} = C^{\text{cloak}}_{1111}\frac{\partial u'}{\partial x} + C^{\text{cloak}}_{1121}\frac{\partial u'}{\partial y} + C^{\text{cloak}}_{1112}\frac{\partial v'}{\partial x} + C^{\text{cloak}}_{1122}\frac{\partial v'}{\partial y}$$

$$\sigma_{12} = C^{\text{cloak}}_{2111}\frac{\partial u'}{\partial x} + C^{\text{cloak}}_{2121}\frac{\partial u'}{\partial y} + C^{\text{cloak}}_{2112}\frac{\partial v'}{\partial x} + C^{\text{cloak}}_{2122}\frac{\partial v'}{\partial y}$$

$$\sigma_{21} = C^{\text{cloak}}_{1211}\frac{\partial u'}{\partial x} + C^{\text{cloak}}_{1221}\frac{\partial u'}{\partial y} + C^{\text{cloak}}_{1212}\frac{\partial v'}{\partial x} + C^{\text{cloak}}_{1222}\frac{\partial v'}{\partial y}$$

$$\sigma_{22} = C^{\text{cloak}}_{2211}\frac{\partial u'}{\partial x} + C^{\text{cloak}}_{2221}\frac{\partial u'}{\partial y} + C^{\text{cloak}}_{2212}\frac{\partial v'}{\partial x} + C^{\text{cloak}}_{2222}\frac{\partial v'}{\partial y}$$

or equivalently as

$$\sigma = \sigma^{\text{sym}} + \sigma^{\text{ext}}$$

where σ^{sym} can be written in Voigt notation as

$$
\sigma^{\text{sym}} = \begin{bmatrix} C_{1111}^{\text{cloak}} & C_{1122}^{\text{cloak}} & 0 \\ C_{1122}^{\text{cloak}} & C_{2222}^{\text{cloak}} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial u'}{\partial x} \\ \frac{\partial v'}{\partial y} \\ \frac{\left(\frac{\partial u'}{\partial y} + \frac{\partial v'}{\partial x}\right)}{2} \end{bmatrix}
$$

and the external stress is

$$
\sigma^{\text{ext}} = \begin{bmatrix} C_{1121}^{\text{cloak}} \frac{\partial u'}{\partial y} + C_{1112}^{\text{cloak}} \frac{\partial v'}{\partial x} & \sigma_{12} \\ \sigma_{21} & C_{2221}^{\text{cloak}} \frac{\partial u'}{\partial y} + C_{2212}^{\text{cloak}} \frac{\partial v'}{\partial x} \end{bmatrix}
$$

Results and Discussion

Figure 1 shows the displacement field for the free field case and the cloak case. It can be seen how the waves do not interact with the region inside the cloak, thus not producing scattering. At the same time, the cloaked region is protected from the probing incident radiation. Interference between P and S waves emitted from the point source can be observed.

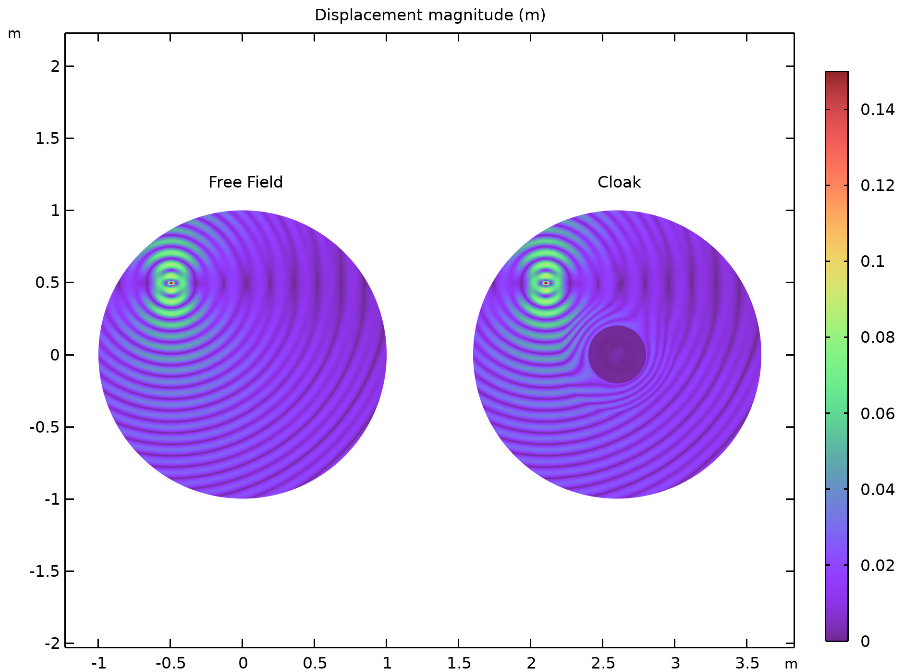

Figure 1: The displacement magnitude in the free field scenario compared with the cloaked case.

Figure 2 shows the volumetric strain, highlighting the P wave part of the field. The point force applied in the horizontal direction produces a dipolar excitation for P waves aligned with the horizontal axis.

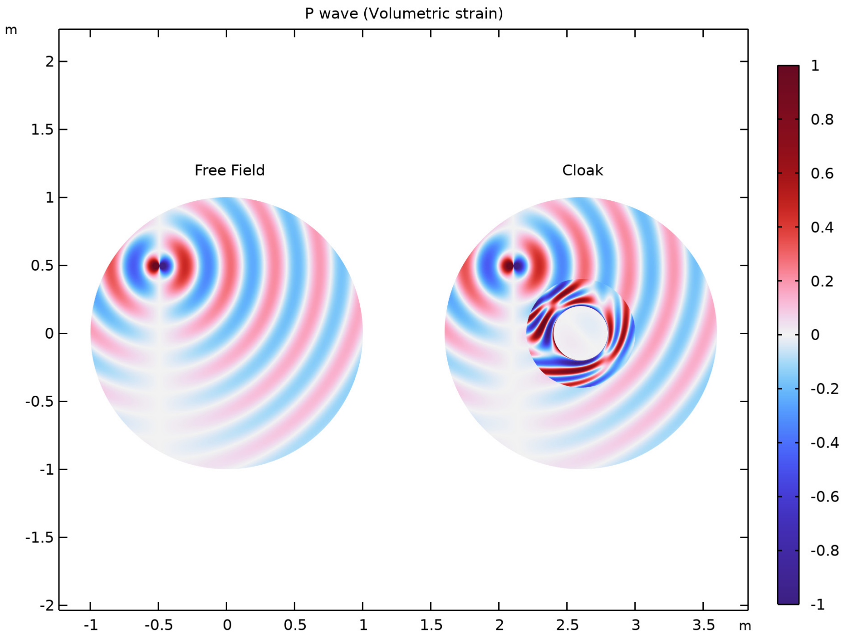

Figure 2: P wave emitted by the source in the free field and in the cloaked case scenario.

Figure 3 shows instead the S wave, via the local rotation expressed as the out-of-plane component of the curl of the displacement field. The point force in this case produces a dipolar excitation aligned with the vertical axis. Figure 2 together with Figure 3 clearly show how the cloak is capable of steering both P and S waves around the shielded area at the same time.

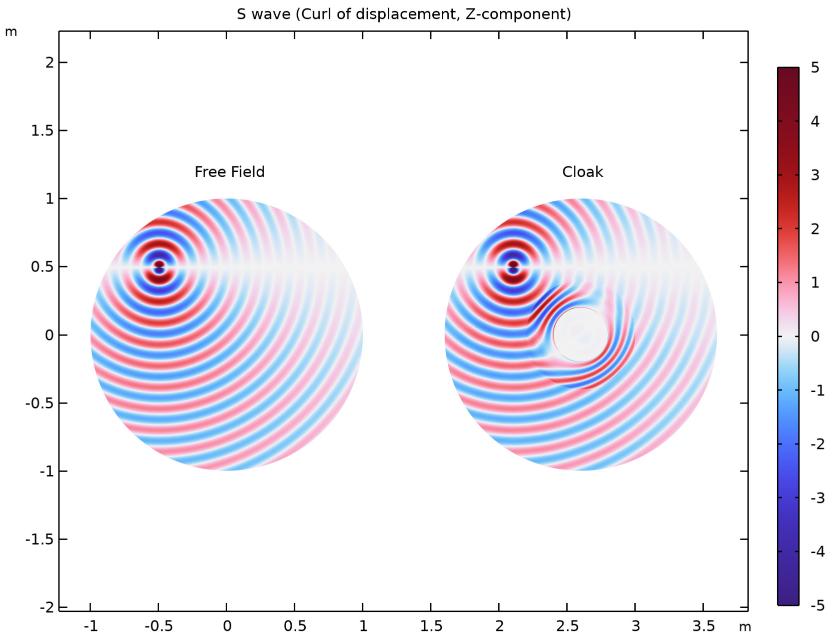

Figure 3: S wave emitted by the source in the free field and in the cloaked case scenario.

Notes About the COMSOL Implementation

- The computational domain is truncated with a cylindrical **Perfectly Matched Layer** to represent propagation in an unbounded domain. A mapped mesh is used in the PML to make it work properly.
- Note that the elastic moduli $C_{\theta\theta\theta\theta}^{\text{cloak}}$ and $C_{\theta r\theta r}^{\text{cloak}}$ tend to infinite near the inner boundary of the cloak, while the density vanishes along with $C_{r\theta r\theta}^{\text{cloak}}$ and C_{rrrr}^{cloak}. This is due to the fact that P and S waves traveling along the tangential direction should travel with infinite velocity to exhibit the zero phase change implied by the singular mapping χ (all the points on the inner of the cloak are mapped from one single point). High order shape functions are used, and **Boundary Layers** are adopted when setting up the mesh, in order to get as accurate numerical solutions as possible.

References

1. M. Brun, S. Guenneau, and A.B. Movchan, “Achieving control of in-plane elastic waves,” *Appl. Phys. Lett.*, vol. 94, no. 6, p. 061903, 2009.

2. A.N. Norris, and A. Shuvalov, “Elastic cloaking theory,” *Wave Motion*, vol. 48, no. 6, pp. 525–538, 2011.

Application Library path: `Acoustics_Module/Elastic_Waves/elastic_cloaking`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1. In the **Model Wizard** window, click **2D**.
2. In the **Select Physics** tree, select **Structural Mechanics>Solid Mechanics (solid)**.
3. Click **Add**.
4. Click **Study**.
5. In the **Select Study** tree, select **General Studies>Frequency Domain**.
6. Click **Done**.

GLOBAL DEFINITIONS

Geometrical Parameters

1. In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
2. In the **Settings** window for **Parameters**, type `Geometrical Parameters` in the **Label** text field.
3. Locate the **Parameters** section. In the table, enter the following settings:

Name	Expression	Value	Description
`r0`	`0.2 [m]`	0.2 m	`Inner radius of cloak`
`r1`	`0.4 [m]`	0.4 m	`Outer radius of cloak`

Name	Expression	Value	Description
Dpml	0.2 [m]	0.2 m	Width of PML
r2	1 [m] +Dpml	1.2 m	Outer radius of computational domain

Material Properties and Simulation Parameters

1 In the **Home** toolbar, click **Parameters** and choose **Add>Parameters**.

2 In the **Settings** window for **Parameters**, type `Material Properties and Simulation Parameters` in the **Label** text field.

3 Locate the **Parameters** section. In the table, enter the following settings:

Name	Expression	Value	Description
lambda	2.3 [Pa]	2.3 Pa	First Lamé constant
mu	1 [Pa]	1 Pa	Second Lamé constant
rho	1 [kg/m^3]	1 kg/m³	Density
cP	sqrt((lambda+2*mu)/rho)	2.0736 m/s	Speed P waves
cS	sqrt(mu/rho)	1 m/s	Speed S waves
omega	40 [rad/s]	40 rad/s	Circular frequency
kappaP	omega/cP	19.29 rad/m	Wavenumber P aves
kappaS	omega/cS	40 rad/m	Wavenumber S waves
wlengthP	2*pi/kappaP	0.32573 m	Wavelength P waves
wlengthS	2*pi/kappaS	0.15708 m	Wavelength S waves

GEOMETRY 1

Circle 1 (c1)

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `r2`.

4 Click to expand the **Layers** section. In the table, enter the following settings:

Layer name	Thickness (m)
Layer 1	Dpml
Layer 2	r2-Dpml-r1
Layer 3	r1-r0

5 Click **Build Selected**.

Point 1 (pt1)

1 In the **Geometry** toolbar, click **Point**.

2 In the **Settings** window for **Point**, locate the **Point** section.

3 In the **x** text field, type `-((r2-Dpml-r1)/2+r1)*cos(pi/4)`.

4 In the **y** text field, type `((r2-Dpml-r1)/2+r1)*sin(pi/4)`.

5 Click **Build Selected**.

DEFINITIONS

PML

1 In the **Model Builder** window, expand the **Component 1 (comp1)>Definitions** node.

2 Right-click **Definitions** and choose **Selections>Explicit**.

3 In the **Settings** window for **Explicit**, type `PML` in the **Label** text field.

4 Select Domains 1, 2, 7, and 12 only.

Cloak

1 In the **Definitions** toolbar, click **Explicit**.

2 In the **Settings** window for **Explicit**, type `Cloak` in the **Label** text field.

3 Select Domains 5, 6, 9, and 10 only.

Background Solid

1 In the **Definitions** toolbar, click **Complement**.

2 In the **Settings** window for **Complement**, type `Background Solid` in the **Label** text field.

3 Locate the **Input Entities** section. Under **Selections to invert**, click **Add**.

4 In the **Add** dialog box, in the **Selections to invert** list, choose **PML** and **Cloak**.

5 Click **OK**.

Background and Cloak

1 In the **Definitions** toolbar, click **Union**.

2 In the **Settings** window for **Union**, type `Background and Cloak` in the **Label** text field.

3 Locate the **Input Entities** section. Under **Selections to add**, click **Add**.

4 In the **Add** dialog box, in the **Selections to add** list, choose **Cloak** and **Background Solid**.

5 Click **OK**.

Perfectly Matched Layer 1 (pml1)

1 In the **Definitions** toolbar, click **Perfectly Matched Layer**.
2 In the **Settings** window for **Perfectly Matched Layer**, locate the **Domain Selection** section.
3 From the **Selection** list, choose **PML**.
4 Locate the **Geometry** section. From the **Type** list, choose **Cylindrical**.

First, set up the simulation for obtaining the free-field solution.

SOLID MECHANICS (SOLID)

Linear Elastic Material 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Solid Mechanics (solid)** click **Linear Elastic Material 1**.
2 In the **Settings** window for **Linear Elastic Material**, locate the **Linear Elastic Material** section.
3 From the **Specify** list, choose **Lamé parameters**.
4 From the λ list, choose **User defined**. In the associated text field, type `lambda`.
5 From the μ list, choose **User defined**. In the associated text field, type `mu`.
6 From the ρ list, choose **User defined**. In the associated text field, type `rho`.

Point Load 1

1 In the **Physics** toolbar, click **Points** and choose **Point Load**.
2 Select Point 3 only.
3 In the **Settings** window for **Point Load**, locate the **Force** section.
4 Specify the $\mathbf{F}_\mathrm{P}$ vector as

1	x
0	y

The mesh and the interpolation order are set according to the requirements for the cloak scenario, where the material properties that tend to infinity need to be properly resolved.

MESH 1

Mapped 1

1 In the **Mesh** toolbar, click **Mapped**.
2 In the **Settings** window for **Mapped**, locate the **Domain Selection** section.
3 From the **Geometric entity level** list, choose **Domain**.

4 From the **Selection** list, choose **PML**.

Size

1 In the **Model Builder** window, click **Size**.

2 In the **Settings** window for **Size**, locate the **Element Size** section.

3 Click the **Custom** button.

4 Locate the **Element Size Parameters** section. In the **Maximum element size** text field, type `wlengthS/12`.

Distribution 1

1 In the **Model Builder** window, right-click **Mapped 1** and choose **Distribution**.

2 Select Boundary 1 only.

3 In the **Settings** window for **Distribution**, locate the **Distribution** section.

4 In the **Number of elements** text field, type `7`.

5 Click **Build Selected**.

Mapped 2

1 In the **Mesh** toolbar, click **Mapped**.

2 In the **Settings** window for **Mapped**, locate the **Domain Selection** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 From the **Selection** list, choose **Cloak**.

Size 1

1 Right-click **Mapped 2** and choose **Size**.

2 In the **Settings** window for **Size**, locate the **Element Size** section.

3 Click the **Custom** button.

4 Locate the **Element Size Parameters** section.

5 Select the **Maximum element size** check box. In the associated text field, type `wlengthS/25`.

Boundary Layers 1

1 In the **Mesh** toolbar, click **Boundary Layers**.

2 In the **Settings** window for **Boundary Layers**, locate the **Domain Selection** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 From the **Selection** list, choose **Cloak**.

Boundary Layer Properties

1 In the **Model Builder** window, click **Boundary Layer Properties**.
2 Select Boundaries 19, 20, 24, and 25 only.
3 In the **Settings** window for **Boundary Layer Properties**, locate the **Layers** section.
4 From the **Thickness specification** list, choose **All layers**.
5 In the **Number of layers** text field, type `20`.
6 Click **Build Selected**.

Free Triangular 1

1 In the **Mesh** toolbar, click **Free Triangular**.
2 In the **Settings** window for **Free Triangular**, click **Build All**.

SOLID MECHANICS (SOLID)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Solid Mechanics (solid)**.
2 In the **Settings** window for **Solid Mechanics**, click to expand the **Discretization** section.
3 From the **Displacement field** list, choose **Quintic Lagrange**.

FREE FIELD

1 In the **Model Builder** window, click **Study 1**.
2 In the **Settings** window for **Study**, type `Free Field` in the **Label** text field.
3 Locate the **Study Settings** section. Clear the **Generate default plots** check box.

Step 1: Frequency Domain

1 In the **Model Builder** window, under **Free Field** click **Step 1: Frequency Domain**.
2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.
3 In the **Frequencies** text field, type `omega/2/pi`.
4 In the **Home** toolbar, click **Compute**.

RESULTS

Displacement Field

1 In the **Home** toolbar, click **Add Plot Group** and choose **2D Plot Group**.
2 In the **Settings** window for **2D Plot Group**, type `Displacement Field` in the **Label** text field.
3 Click to expand the **Title** section. From the **Title type** list, choose **Manual**.
4 Clear the **Parameter indicator** text field.

5 In the **Title** text area, type `Displacement magnitude (m)`.

6 Locate the **Plot Settings** section. Clear the **Plot dataset edges** check box.

Free Field

1 Right-click **Displacement Field** and choose **Surface**.

2 In the **Settings** window for **Surface**, type `Free Field` in the **Label** text field.

3 Locate the **Coloring and Style** section. Click **Change Color Table**.

4 In the **Color Table** dialog box, select **Rainbow>SpectrumLight** in the tree.

5 Click **OK**.

6 In the **Displacement Field** toolbar, click **Plot**.

Selection 1

1 Right-click **Free Field** and choose **Selection**.

2 In the **Settings** window for **Selection**, locate the **Selection** section.

3 From the **Selection** list, choose **Background and Cloak**.

Free Field

1 In the **Model Builder** window, click **Free Field**.

2 In the **Settings** window for **Surface**, click to expand the **Range** section.

3 Select the **Manual color range** check box.

4 In the **Minimum** text field, type `0`.

5 In the **Maximum** text field, type `0.15`.

6 In the **Displacement Field** toolbar, click **Plot**.

7 In the **Model Builder** window, collapse the **Free Field** node.

P Wave

1 In the **Home** toolbar, click **Add Plot Group** and choose **2D Plot Group**.

2 In the **Settings** window for **2D Plot Group**, type `P Wave` in the **Label** text field.

3 Locate the **Title** section. From the **Title type** list, choose **Manual**.

4 Clear the **Parameter indicator** text field.

5 In the **Title** text area, type `P wave (Volumetric strain)`.

6 Locate the **Plot Settings** section. Clear the **Plot dataset edges** check box.

Free Field

1 Right-click **P Wave** and choose **Surface**.

2 In the **Settings** window for **Surface**, type `Free Field` in the **Label** text field.

3 Click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)>Solid Mechanics>Strain>solid.evol - Volumetric strain - 1**.

4 Locate the **Coloring and Style** section. Click **Change Color Table**.

5 In the **Color Table** dialog box, select **Wave>Wave** in the tree.

6 Click **OK**.

Selection 1

1 Right-click **Free Field** and choose **Selection**.

2 In the **Settings** window for **Selection**, locate the **Selection** section.

3 From the **Selection** list, choose **Background and Cloak**.

Free Field

1 In the **Model Builder** window, click **Free Field**.

2 In the **Settings** window for **Surface**, locate the **Range** section.

3 Select the **Manual color range** check box.

4 In the **Minimum** text field, type `-1`.

5 In the **Maximum** text field, type `1`.

6 In the **P Wave** toolbar, click **Plot**.

7 In the **Model Builder** window, collapse the **Free Field** node.

S wave

1 In the **Home** toolbar, click **Add Plot Group** and choose **2D Plot Group**.

2 In the **Settings** window for **2D Plot Group**, type `S wave` in the **Label** text field.

3 Locate the **Title** section. From the **Title type** list, choose **Manual**.

4 Clear the **Parameter indicator** text field.

5 In the **Title** text area, type `S wave (Curl of displacement, Z-component)`.

6 Locate the **Plot Settings** section. Clear the **Plot dataset edges** check box.

Surface 1

1 Right-click **S wave** and choose **Surface**.

2 In the **Settings** window for **Surface**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)>Solid Mechanics>Displacement>Curl of displacement (material and geometry frames)>solid.curlUZ - Curl of displacement, Z-component**.

3 Locate the **Coloring and Style** section. Click **Change Color Table**.

4 In the **Color Table** dialog box, select **Wave>Wave** in the tree.

5 Click **OK**.

Selection 1

1 Right-click **Surface 1** and choose **Selection**.

2 In the **Settings** window for **Selection**, locate the **Selection** section.

3 From the **Selection** list, choose **Background and Cloak**.

Surface 1

1 In the **Model Builder** window, click **Surface 1**.

2 In the **Settings** window for **Surface**, locate the **Range** section.

3 Select the **Manual color range** check box.

4 In the **Minimum** text field, type `-5`.

5 In the **Maximum** text field, type `5`.

6 In the **S wave** toolbar, click **Plot**.

7 In the **Model Builder** window, collapse the **Surface 1** node.

Now, set up the simulation for the cloak. First, add a cylindrical coordinate system to define the radial dependence of the material properties.

DEFINITIONS

Cylindrical System 2 (sys2)

1 In the **Definitions** toolbar, click **Coordinate Systems** and choose **Cylindrical System**.

2 In the **Settings** window for **Cylindrical System**, locate the **Coordinate Names** section.

3 From the **Frame** list, choose **Material (X, Y, Z)**.

Material Properties Cloak

1 In the **Model Builder** window, right-click **Definitions** and choose **Variables**.

The material properties of the cloak are defined in the cylindrical coordinate system. The elastic moduli are transformed to the global Cartesian coordinates. The computations have already been done, so you can directly import the final values.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 Click **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file `elastic_cloaking_variables.txt`.

5 In the **Label** text field, type `Material Properties Cloak`.

SOLID MECHANICS (SOLID)

Linear Elastic Material Cloak

1 In the **Physics** toolbar, click **Domains** and choose **Linear Elastic Material**.

2 In the **Settings** window for **Linear Elastic Material**, type `Linear Elastic Material Cloak` in the **Label** text field.

3 Locate the **Domain Selection** section. From the **Selection** list, choose **Cloak**.

4 Locate the **Linear Elastic Material** section. From the **Material symmetry** list, choose **Anisotropic**.

5 From the **D** list, choose **User defined**. In the **associated** table, enter the following settings:

Ccl_1111	Ccl_1122	0	0	0	0
Ccl_1122	Ccl_2222	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

6 From the ρ list, choose **User defined**. In the associated text field, type `rhocl`.

External Stress 1

1 In the **Physics** toolbar, click **Attributes** and choose **External Stress**.

2 In the **Settings** window for **External Stress**, locate the **External Stress** section.

3 From the **Stress input** list, choose **Stress tensor (Nominal)**.

4 In the P_{ext} table, enter the following settings:

Ccl_1121*solid.gradUxY+ Ccl_1112*solid.gradUyX	Ccl_2111*solid.gradUxX+ Ccl_2121*solid.gradUxY+ Ccl_2112*solid.gradUyX+ Ccl_2122*solid.gradUyY	0
Ccl_1211*solid.gradUxX+ Ccl_1221*solid.gradUxY+ Ccl_1212*solid.gradUyX+ Ccl_1222*solid.gradUyY	Ccl_2221*solid.gradUxY+ Ccl_2212*solid.gradUyX	0
0	0	0

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies>Frequency Domain**.

4 Click **Add Study** in the window toolbar.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

CLOAK

1 In the **Model Builder** window, click **Study 2**.

2 In the **Settings** window for **Study**, type `Cloak` in the **Label** text field.

3 Locate the **Study Settings** section. Clear the **Generate default plots** check box.

Step 1: Frequency Domain

1 In the **Model Builder** window, under **Cloak** click **Step 1: Frequency Domain**.

2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

3 In the **Frequencies** text field, type `omega/2/pi`.

4 In the **Home** toolbar, click **Compute**.

RESULTS

Displacement Field

1 In the **Model Builder** window, under **Results** click **Displacement Field**.

2 In the **Settings** window for **2D Plot Group**, click to expand the **Plot Array** section.

3 Select the **Enable** check box.

Free Field

In the **Model Builder** window, right-click **Free Field** and choose **Duplicate**.

Cloak

1 In the **Model Builder** window, under **Results>Displacement Field** click **Free Field 1**.

2 In the **Settings** window for **Surface**, type `Cloak` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Cloak/Solution 2 (sol2)**.

4 Click to expand the **Title** section. From the **Title type** list, choose **None**.

5 Click to expand the **Inherit Style** section. From the **Plot** list, choose **Free Field**.

6 In the **Displacement Field** toolbar, click **Plot**.

7 Click the Zoom Extents button in the **Graphics** toolbar.

Annotation Free Field

1 In the **Model Builder** window, right-click **Displacement Field** and choose **Annotation**.

2 In the **Settings** window for **Annotation**, type `Annotation Free Field` in the **Label** text field.

3 Locate the **Annotation** section. In the **Text** text field, type `Free Field`.

4 Locate the **Position** section. In the **X** text field, type `0.02`.

5 In the **Y** text field, type `1.2`.

6 Locate the **Coloring and Style** section. Clear the **Show point** check box.

7 From the **Anchor point** list, choose **Center**.

8 Click to expand the **Plot Array** section. Select the **Manual indexing** check box.

9 In the **Displacement Field** toolbar, click **Plot**.

10 Right-click **Annotation Free Field** and choose **Duplicate**.

Annotation Cloak

1 In the **Model Builder** window, under **Results>Displacement Field** click **Annotation Free Field 1**.

2 In the **Settings** window for **Annotation**, type `Annotation Cloak` in the **Label** text field.

3 Locate the **Annotation** section. In the **Text** text field, type `Cloak`.

4 Locate the **Plot Array** section. In the **Index** text field, type `1`.

5 In the **Displacement Field** toolbar, click **Plot**.

P Wave

1 In the **Model Builder** window, under **Results** click **P Wave**.

2 In the **Settings** window for **2D Plot Group**, locate the **Plot Array** section.

3 Select the **Enable** check box.

Free Field

In the **Model Builder** window, right-click **Free Field** and choose **Duplicate**.

Cloak

1 In the **Model Builder** window, under **Results>P Wave** click **Free Field 1**.

2 In the **Settings** window for **Surface**, type `Cloak` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Cloak/Solution 2 (sol2)**.

4 Locate the **Title** section. From the **Title type** list, choose **None**.

5 Locate the **Inherit Style** section. From the **Plot** list, choose **Free Field**.

6 In the **P Wave** toolbar, click **Plot**.

7 Click the **Zoom Extents** button in the **Graphics** toolbar.

Annotation Cloak, Annotation Free Field

1 In the **Model Builder** window, under **Results>Displacement Field**, Ctrl-click to select **Annotation Free Field** and **Annotation Cloak**.

2 Right-click and choose **Copy**.

P Wave

In the **Model Builder** window, under **Results** right-click **P Wave** and choose **Paste Multiple Items**.

Annotation Cloak, Annotation Free Field

1 In the **Model Builder** window, under **Results>P Wave**, Ctrl-click to select **Annotation Free Field** and **Annotation Cloak**.

2 In the **P Wave** toolbar, click **Plot**.

S wave

1 In the **Model Builder** window, under **Results** click **S wave**.

2 In the **Settings** window for **2D Plot Group**, locate the **Plot Array** section.

3 Select the **Enable** check box.

Free Field

1 In the **Model Builder** window, under **Results>S wave** click **Surface 1**.

2 In the **Settings** window for **Surface**, type `Free Field` in the **Label** text field.

3 Right-click **Free Field** and choose **Duplicate**.

Cloak

1 In the **Model Builder** window, under **Results>S wave** click **Free Field 1**.

2 In the **Settings** window for **Surface**, type `Cloak` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Cloak/Solution 2 (sol2)**.

4 Locate the **Title** section. From the **Title type** list, choose **None**.

5 Locate the **Inherit Style** section. From the **Plot** list, choose **Free Field**.

6 In the **S wave** toolbar, click **Plot**.

Annotation Cloak, Annotation Free Field

1 In the **Model Builder** window, under **Results>P Wave**, Ctrl-click to select **Annotation Free Field** and **Annotation Cloak**.

2 Right-click and choose **Copy**.

S wave

In the **Model Builder** window, under **Results** right-click **S wave** and choose **Paste Multiple Items**.

Annotation Cloak, Annotation Free Field

1 In the **Model Builder** window, under **Results>S wave**, Ctrl-click to select **Annotation Free Field** and **Annotation Cloak**.

2 In the **S wave** toolbar, click **Plot**.

Modify the Free Field study to not include the cloak for future reruns.

FREE FIELD

Step 1: Frequency Domain

1 In the **Model Builder** window, under **Free Field** click **Step 1: Frequency Domain**.

2 In the **Settings** window for **Frequency Domain**, locate the **Physics and Variables Selection** section.

3 Select the **Modify model configuration for study step** check box.

4 In the tree, select **Component 1 (comp1)>Solid Mechanics (solid)> Linear Elastic Material Cloak**.

5 Right-click and choose **Disable**.