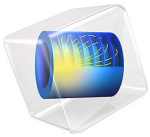

Positive and Negative Corona Discharges

Introduction

This tutorial presents a study of positive and negative corona discharges in dry air at atmospheric pressure. The discharges are sustained within two electrodes in a coaxial configuration by a high voltage DC source applied to the inner electrode. Two different types of models are used: a full self-consistent plasma model and a simplified model. The plasma model solves the electron and ion continuity and momentum equations in the drift-diffusion approximation, self-consistently coupled with Poisson's equation. The local field approximation is used, which means that transport and source coefficients are assumed to be well parameterized through the reduced electric field. The simplified model is based on the conservation of current transported by the charged carriers and the electric field at the corona electrode needs to be specified. The space charge density and the electric potential obtained with the simplified model are in good agreement with the simulation results obtained with the self-consistent plasma model.

Modeling corona discharges with a plasma self-consistent model is very computational expensive. One of the reasons is that ionization waves can develop which increases the numerical difficulty of the problem and increases the computational time since very small time steps are needed to describe its physics. This makes higher dimensional models unpractical for industrial applications.

In a corona discharge at steady state the ionization of the background gas occurs in a region very close to the corona electrode in what is called the ionizing region. Between the ionizing region and the large electrode (normally grounded) there is only one type of charge carrier: positive or negative ions depending on the polarity of the corona. This region is called the transport region.

For some applications involving corona discharges it is not needed all the complexity of a plasma model and simplified models can be used to obtain the space charge density and the electric potential. Here, it is used a simplified model that describes the transport of one charge carrier type in the transport region. The plasma physics responsible to the discharge maintenance is condensed in the given electric field at the corona electrode. Evidently, the value of the provided electric field at the corona electrode is critical to obtain accurate values for the potential and space charge density.

Plasma Model Definition

The model is one dimensional in the radial direction between the electrodes and describes the behavior of charged species using fluid-type equations. The discharge is assumed to be diffuse and uniform in the radial direction.

The model solves the electron and ions continuity and momentum equations, in the drift-diffusion approximation, self-consistently coupled with Poisson's equation. The local field approximation is used, which means that transport and source coefficients are assumed to be well parameterized through the reduced electric field (E/N). In the local field approximation the fluid equation for the mean electron energy is not solved, which reduces significantly the complexity of the numerical problem.

The local field approximation is valid in a situation where the rate of electron energy gain from the electric field is locally balanced by the energy loss rate. When this condition is met the electrons are said to be in local equilibrium with the electric field and the electron mean properties can be expressed as a function the reduced electric field.

The model presented in the following section is used to simulate the ionization of the neutral gas as well as the transport of charged particles when the electric potential is applied at the inner conductor. The high electric field generated by the combination of high potential and small conductor curvature radius causes the ionization of the neutral gas surrounding the corona wire.

DOMAIN EQUATIONS

The electron density is computed by solving the drift-diffusion equation for the electron density:

$$\frac{\partial}{\partial t}(n_e) + \nabla \cdot [-n_e(\mu_e \bullet \mathbf{E}) - \mathbf{D}_e \bullet \nabla n_e] = R_e \tag{1}$$

Convection of electrons due to fluid motion is neglected. For more detailed information on electron transport see the section *Theory for the Drift Diffusion Interface* in the *Plasma Module User's Guide*.

When using the local field approximation the electron density equation is not solved, and the transport and source coefficients are mapped by the reduced electric field. In practice, when using the local field approximation or the local energy approximation, the transport and source coefficients are still given as a function of the mean electron energy. When using the local field approximation however, it must be provided a function that relates the mean electron energy and the reduced electric field

$$\varepsilon = F(E/N)\,. \tag{2}$$

The electron source R_e is defined later. The electron diffusivity is computed from the electron mobility using the relation

$$\mathbf{D}_e = \mu_e T_e \tag{3}$$

The source coefficients in the above equation is determined by the plasma chemistry using rate coefficients. Suppose that there are M reactions that contribute to the growth or decay of electron density and P inelastic electron-neutral collisions. In general, $P \gg M$. In the case of rate coefficients, the electron source term is given by

$$R_e = \sum_{j=1}^{M} x_j k_j N_n n_e \tag{4}$$

where x_j is the mole fraction of the target species for reaction j, k_j is the rate coefficient for reaction j (SI unit: m^3/s), and N_n is the total neutral number density (SI unit: $1/\mathrm{m}^3$). For DC discharges it is better practice to use Townsend coefficients instead of rate coefficients to define reaction rates. Townsend coefficients provide a better description of what happens in the cathode fall region Ref. 1. When Townsend coefficients are used, the electron source term is given by:

$$R_e = \sum_{j=1}^{M} x_j \alpha_j N_n |\Gamma_e| \tag{5}$$

where α_j is the Townsend coefficient for reaction j (SI unit: m^2), and Γ_e is the electron flux as defined above (SI unit: $1/(\mathrm{m}^2 \cdot \mathrm{s})$). Townsend coefficients can increase the stability of the numerical scheme when the electron flux is field driven as is the case with DC discharges.

For nonelectron species, the following equation is solved for the mass fraction of each species:

$$\rho \frac{\partial}{\partial t}(w_k) + \rho(\mathbf{u} \cdot \nabla) w_k = \nabla \cdot \mathbf{j}_k + R_k \tag{6}$$

For detailed information on the transport of the nonelectron species see the section *Theory for the Heavy Species Transport Interface* in the *Plasma Module User's Guide*.

The electrostatic field is computed using the equation

$$-\nabla \cdot \varepsilon_0 \varepsilon_r \nabla V = \rho \tag{7}$$

The space charge density ρ is automatically computed based on the plasma chemistry specified in the model using the formula

$$\rho = q\left(\sum_{k=1}^{N} Z_k n_k - n_e\right) \tag{8}$$

For detailed information about electrostatics see *Theory for the Electrostatics Interface* in the *Plasma Module User's Guide*.

BOUNDARY CONDITIONS

Electrons are lost to the wall due to random motion within a few mean free paths of the wall and gained due to secondary emission effects, resulting in the boundary condition for the electron flux

$$\mathbf{n} \cdot \Gamma_e = \left(\frac{1}{2} v_{e,\mathrm{th}} n_e\right) - \sum_p \gamma_p (\Gamma_p \cdot \mathbf{n}) \tag{9}$$

The second term on the right-hand side of Equation 9 is the gain of electrons due to secondary emission effects, γ_p being the secondary emission coefficient. For the heavy species, ions are lost to the wall due to surface reactions and the fact that the electric field is directed toward the wall:

$$\mathbf{n} \cdot \mathbf{j}_k = M_w R_k + M_w c_k Z \mu_k (\mathbf{E} \cdot \mathbf{n})[Z_k \mu_k (\mathbf{E} \cdot \mathbf{n}) > 0] \tag{10}$$

The discharge is driven by a DC electric potential (V_0) applied to the inner conductor of the coaxial geometry (at coordinate $r = r_\mathrm{i}$). The other boundary (at coordinate $r = r_\mathrm{o}$) is grounded. To facilitate the beginning of the numerical simulation a step function is used to modulate V_0 with the transient applied potential assuming the form

$$V = V_0 \tanh\left(\frac{t}{\tau}\right). \tag{11}$$

This numeric technique does not interfere with the results at steady state, which are the ones of interest in this work.

PLASMA CHEMISTRY

The chemistry of a plasma sustained in air can be very complex and a detailed study of the main excited states could easily have hundreds of reactions. The main goal of this model is to study the charge particle density profiles and currents. With that in mind it is used a simplified set of reactions that describes correctly the creation and destruction of charged species in a background of dry air.

Table 1 lists the chemical reactions considered Ref. 2. In the fluid equations nitrogen and oxygen are not treated separately as in a detailed chemistry. Instead a general species A is used for the background gas. A can be ionized forming positive ions p, and A can attach electrons forming negative ions n.

The creation and destruction of electrons in the volume is described by ionization and attachment Townsend coefficients and by a rate constants for the three-body attachment. The rate coefficients for the electron-ion recombination and ion-ion recombination are obtained from Ref. 2. The Townsend coefficients are obtained as a function of the mean electron energy by suitably averaging over the electron energy distribution computed using a Boltzmann solver with a consistent set of electron scattering collisions cross sections of nitrogen and oxygen Ref. 3. It is used a mixture of 80% nitrogen and 20% oxygen. The relation between the mean electron energy and the reduced electric field is also obtain from the Boltzmann solver and is given in Figure 1.

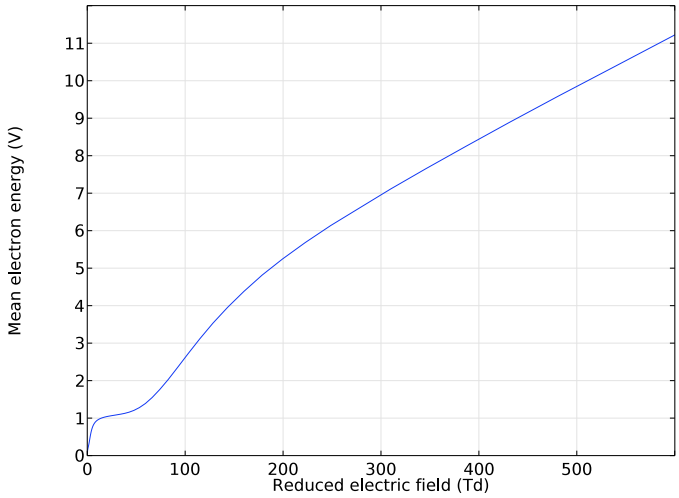

Figure 1: Mean electron energy as a function of the reduced electric field for a mixture of 80% nitrogen and 20% oxygen.

For detailed information on how to compute source coefficients from a Boltzmann solver see the section *The Boltzmann Equation, Two-Term Approximation Interface* in the *Plasma Module User's Guide*.

TABLE 1: TABLE OF COLLISIONS AND REACTIONS MODELED.

Reaction	Formula	Type	$\Delta\varepsilon$ (eV)	k_f (m^3/s)
1	e+A=>p+2e	Ionization	15	-
2	e+A=>n	Attachment	-	-
3	e+2A=>n+A	Attachment	-	-
4	e+p=>A	Reaction	-	$5 \cdot 10^{-14}$
5	n+p=>2A	Reaction	-	$5 \cdot 10^{-12}$

At steady state, the plasma main charged species are ions. For this reason, the initial conditions have an equal density of positive and negative ions and a small density of electrons. These initial conditions preserve charge neutrality as it is important for numerical reasons.

In addition to the volumetric reactions, the following surface reactions are implemented:

TABLE 2: TABLE OF SURFACE REACTIONS.

Reaction	Formula	Sticking coefficient
1	p=>A	1
2	n=>A	1

When the ions reach the wall, they are assumed to change back to neutral atoms. Note that the secondary emission coefficient for positive ions is set to 0.05 at the corona electrode boundary and to 0 at the outer electrode. The mean electron energy of the secondary electron is set to 4 eV. When using the local field approximation the mean energy of the secondary electron is only used in postprocessing.

Simplified Corona Model

The simplified model uses the same geometry as the plasma model presented in Figure 1. The simplified model is based on the conservation of current transported by the charged carriers. It should be emphasized that the model is not self-consistent in the sense that the both potential and the electric field need to be given at the corona electrode. In other words, the electric field necessary to sustain the discharge is not obtained from first principles: electron and ion transport, electrons gaining energy from the electric field, and electrons losing energy in collisions with the background gas.

DOMAIN EQUATIONS

The simplified model for the corona solves the transport of a charge carrier using the charge conservation equation coupled with Poisson's equation. When only the drift in the electric field is included the domain equations are

$$\nabla \bullet \mathbf{J} = S \tag{12}$$

$$\mathbf{J} = z\mu\rho\mathbf{E} \tag{13}$$

$$\varepsilon_0 \nabla^2 V = -\rho \tag{14}$$

where $\mathbf{J}$ (SI unit: A/m^2) is the current density, and S (SI unit: A/m^3) is a current source, z is the charge number, μ (SI unit: $m^2/(V{\cdot}s)$) is the mobility, ρ (SI unit: C/m^3) is the space charge number density, $\mathbf{E}$ is the electric field, V is electric potential, and ε_0 is the vacuum permittivity. These set of equations can be manipulated to obtain the following transport equation

$$\mu\left(\frac{\rho^2}{\varepsilon_0} - \nabla V \bullet \nabla\rho\right) = S \tag{15}$$

where it is assumed that the mobility is constant. Without the source term this equation assumes a form that it is widely used to model the space charge density in electrostatic precipitators

$$\frac{\rho^2}{\varepsilon_0} - \nabla V \bullet \nabla\rho = 0\,. \tag{16}$$

It is interesting to note that in this simplified form the domain equations, Equation 14 and Equation 16, do not depend in any properties of the charge carrier, and the solutions for V and ρ in positive and negative corona are symmetric. Also interesting to note is that the domain equations do not contain any information related to plasma creation and maintenance. All plasma physics is condensed in the boundary conditions for the inner electrode.

BOUNDARY CONDITIONS

The normal component of the electric field at the corona electrode is used as a boundary condition for Poisson's equation

$$\mathbf{n} \bullet \mathbf{E} = E_0\,. \tag{17}$$

The other boundary conditions for Poisson's equation is $V = 0$ at the outer electrode. The boundary condition for Equation 15 involves in finding the space charge density ρ_q at the corona electrode, using a Lagrange multiplier, so that the imposed potential V_0 is verified

$$V - V_0 = 0 . \tag{18}$$

In this model both potential and electric field are imposed at the corona electrode. To obtain predictive physical results the value of the electric field at the wire needs to be close enough to the real one. Here, it is used Peek's law

$$E_0 = 3\times10^6\delta\left(1+\frac{0.03}{\sqrt{\delta r_i}}\right) \tag{19}$$

where E_0 (SI unit: V/m) is the breakdown electric field, δ is the gas number density normalized to the gas density at 760 torr and 293.15 K, r_i is the radius of the corona electrode.

When changing the polarity of the corona, make sure that the electric field and potential at the electrode (defined in the Electrode multiphysics feature), the sign of the charge number (defined in the Transport Properties feature in the Charge Transport interface), and the sign of the initial value of the space charge density (defined in the Initial Values feature of the Charge Transport interface) are coherent.

Results and Discussion

In this section, the results of the plasma self-consistent model for the positive and negative coronas are presented first. After, the electric potential and space charge density obtained from both models are compared and discussed.

The results are for a DC negative and positive corona sustained with –/+30 kV applied to the inner electrode. The background gas is kept at a constant density as obtained by the ideal gas law at atmospheric pressure and at a temperature of 600 K. All results presented and discussed correspond to a steady-state operation.

When comparing with corona discharges sustained in noble gases, air corona discharges need much higher voltage to breakdown the background gas and to sustain a discharge. There are two main reasons for this: (i) the electron collision frequency in air is higher (in part due to rotational and vibrational interactions) making it more difficult to accelerate electrons; and (ii) oxygen is electronegative.

Figure 2 and Figure 3 present the spatial profiles of the charged species for the positive and negative coronas obtained with the plasma model. The discharges can be separated in two regions: (a) corresponding to a region of less than 1 mm near the cathode where most of the ionization occurs; and (b) the rest of the volume that reaches to the ground electrode, which is called the transport region. The profiles are distinctly different. In the positive corona the positive ion density dominates over all domain while in the negative corona the positive ion is only dominant in the cathode fall region and in the transport region it is the negative ion the dominant species.

Figure 4 shows the rate expressions for the dominant charge carriers obtained with the self-consistent model for each polarity: the positive ions for the positive corona, and the negative ions for the negative corona. In the positive corona positive ions are created by ionization by electron impact from the ground sate in a narrow region near the corona electrode. In the negative corona negative ions are created by electron attachment in the ionization and transport regions. For the present conditions in the negative polarity the transport carriers have an important source in the transport region that cannot be neglected.

Figure 5 and Figure 6 present the spatial distribution of the potential and space charge density obtained with the self-consistent and the simplified models. In the simplified model of the positive corona it was used Peek's law for the electric field at the electrode and no source term was added. The potential and space charge density have a reasonable agreement with those obtained with the plasma model. The differences can be mostly attributed to the less intense electric field obtained with Peek's law. This point is proven in the negative corona model for which simulation results from both models are in excellent agreement. In the simplified negative corona two important changes were made to improve agreement between models: the electric field at the inner electrode is the one obtained from plasma model, and it is added a source for the charge carriers from the plasma model since the electron attachment is important in the transport region.

In conclusion, the simplified model can reproduced the electric potential and the space charge density obtained with the plasma self-consistent model with good agreement provided that the electric field at the corona electrode and the rate expressions of the charge carriers are close enough. The same models were used with different applied voltages in a range from +/–20 kV to +/–50 kV and the same conclusions can be drawn. This information is of great interest for modeling electrostatic precipitators where a good estimation of the electric potential and the space charge density is needed to compute the charge accumulation in the particles and consequently the precipitator efficiency.

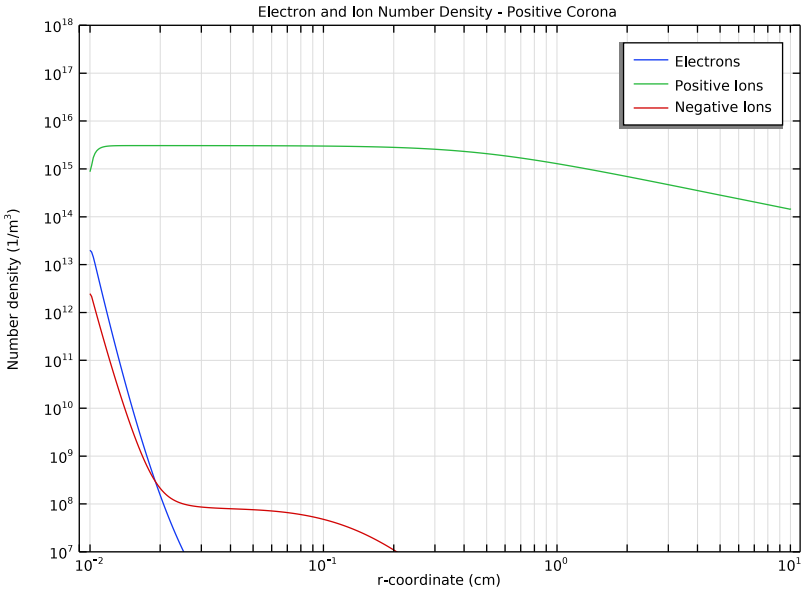

Figure 2: Number density of the charged species for the positive corona obtained with the self-consistent model.

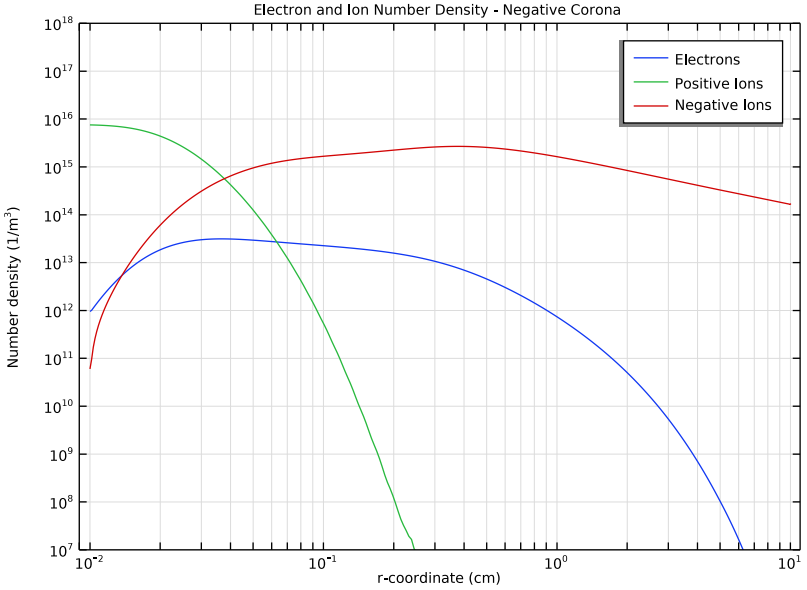

Figure 3: Number density of the charged species for the negative corona obtained with the self-consistent model.

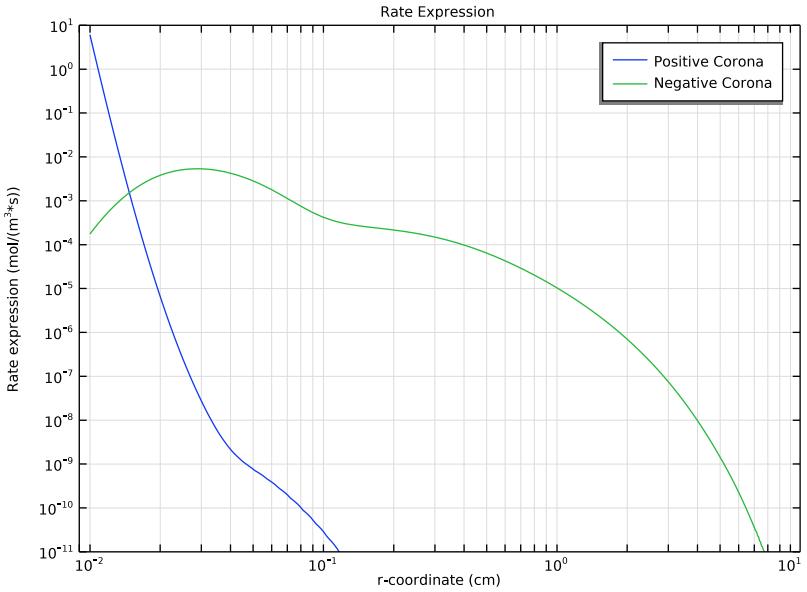

Figure 4: Rate expressions of the positive ions for the positive corona and of the negative ions for the negative corona obtained with the self-consistent model.

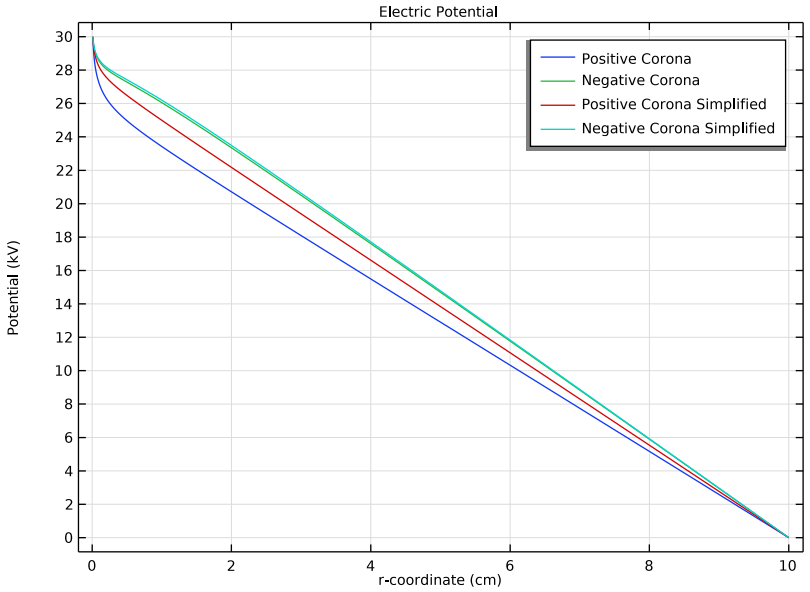

Figure 5: Electric potential of the positive and negative coronas obtained with the self-consistent and simplified models. The potential of the negative corona is multiplied by −1 so that the curves shapes can be compared.

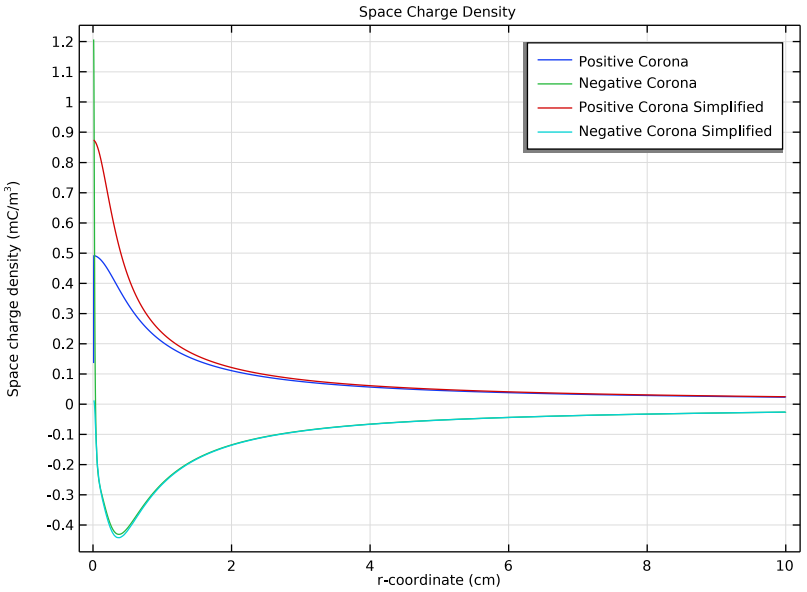

Figure 6: Space charge density of the positive and negative coronas obtained with the self-consistent and simplified models.

References

1. M.A. Lieberman and A.J. Lichtenberg, *Principles of Plasma Discharges and Materials Processing*, John Wiley & Sons, 2005.

2. A.A. Kulikovsky, "Positive streamer between parallel plate electrode in atmospheric pressure air," *J. Phys. D: Appl. Phys.*, vol. 30, pp. 441–450, 1997.

3. LXCAT, see https://fr.lxcat.net/ for Phelps database, 2016.

Application Library path: `Plasma_Module/Corona_Discharges/positive_and_negative_corona_discharges`

Modeling Instructions

The following instructions show how to create models for a positive and negative corona discharges. Two different types of models are used. In the first, the **Plasma** interface is used to solve the problem self-consistently. In the second, a simplified model is used. In this model the electric field at the corona electrode is given and assumed constant.

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

First, prepare the model using the **Plasma** interface.

MODEL WIZARD

1 In the **Model Wizard** window, click **1D Axisymmetric**.

2 In the **Select Physics** tree, select **Plasma>Plasma (plas)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Time Dependent**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 Click **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file `corona_parameters.txt`.

DEFINITIONS

Variables 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Definitions** and choose **Variables**.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 In the table, enter the following settings:

Name	Expression	Unit	Description
mueN	3.74e24[1/(V*s*m)]*(plas.Erd*1e21[1/(V*m^2)])^-0.22	1/(V·m·s)	Reduced electron mobility
Vapp	V0*ramp	V	Applied Voltage with ramp function
ramp	tanh(1e5*t[1/s])		Ramp function

GEOMETRY 1

Follow the steps below to create the model geometry: a simple 1D geometry consisting of a single domain bounded by the corona electrode (left, inner conductor) and the ground (right, outer conductor).

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.

2 In the **Settings** window for **Geometry**, locate the **Units** section.

3 From the **Length unit** list, choose **cm**.

Interval 1 (i1)

1 Right-click **Component 1 (comp1)>Geometry 1** and choose **Interval**.

2 In the **Settings** window for **Interval**, locate the **Interval** section.

3 In the table, enter the following settings:

Coordinates (cm)
rin
10

4 Click **Build All Objects**.

PLASMA (PLAS)

This model uses the local field approximation to parameterize in space source terms and transport coefficients.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Plasma (plas)**.

2 In the **Settings** window for **Plasma**, locate the **Plasma Properties** section.

3 Select the **Use reduced electron transport properties** check box.

4 From the **Mean electron energy** list, choose **Local field approximation**.

Solve this model using the finite volume method and a Scharfetter-Gummel scheme. You can switch from the finite element method to the finite volume method by suitable choices in the **Discretization** section.

5 Click to expand the **Discretization** section. From the **Formulation** list, choose **Finite volume (constant shape function)**.

Electron Impact Reaction 1

1 In the **Physics** toolbar, click — **Domains** and choose **Electron Impact Reaction**.

Add an ionization reaction.

2 In the **Settings** window for **Electron Impact Reaction**, locate the **Reaction Formula** section.

3 In the **Formula** text field, type `A+e=>p+2e`.

4 Locate the **Collision Type** section. From the **Collision type** list, choose **Ionization**.

5 In the Δε text field, type `15`.

Import the ionization Townsend coefficient.

6 Locate the **Collision** section. From the **Specify reaction using** list, choose **Use lookup table**.

7 Locate the **Reaction Parameters** section. From the **Rate constant form** list, choose **Townsend coefficient**.

8 Find the **Townsend coefficient data** subsection. Click **Load from File**.

9 Browse to the model's Application Libraries folder and double-click the file `alpha.txt`.

Electron Impact Reaction 2

1 In the **Physics** toolbar, click — **Domains** and choose **Electron Impact Reaction**.

Add an attachment reaction.

2 In the **Settings** window for **Electron Impact Reaction**, locate the **Reaction Formula** section.

3 In the **Formula** text field, type `A+e=>n`.

4 Locate the **Collision Type** section. From the **Collision type** list, choose **Attachment**.

Import the attachment Townsend coefficient.

5 Locate the **Collision** section. From the **Specify reaction using** list, choose **Use lookup table**.

6 Locate the **Reaction Parameters** section. From the **Rate constant form** list, choose **Townsend coefficient**.

7 Find the **Townsend coefficient data** subsection. Click **Load from File**.

8 Browse to the model's Application Libraries folder and double-click the file `eta.txt`.

Electron Impact Reaction 3

1 In the **Physics** toolbar, click — **Domains** and choose **Electron Impact Reaction**.

Add a 3-body attachment reaction.

2 In the **Settings** window for **Electron Impact Reaction**, locate the **Reaction Formula** section.

3 In the **Formula** text field, type `A + A + e => n + A`.

4 Locate the **Collision Type** section. From the **Collision type** list, choose **Attachment**.

5 Locate the **Reaction Parameters** section. In the k^{f} text field, type `1.4e-41*(0.026/plas.Te)*exp(100/T0-0.061/plas.Te)*N_A_const^2*0.1`.

Reaction 1

1 In the **Physics** toolbar, click — **Domains** and choose **Reaction**.

Add electron-ion recombination.

2 In the **Settings** window for **Reaction**, locate the **Reaction Formula** section.

3 In the **Formula** text field, type `e+p=>A`.

4 Locate the **Reaction Parameters** section. In the k^{f} text field, type `rei*N_A_const`.

Reaction 2

1 In the **Physics** toolbar, click — **Domains** and choose **Reaction**.

Add ion-ion recombination.

2 In the **Settings** window for **Reaction**, locate the **Reaction Formula** section.

3 In the **Formula** text field, type `n+p=>A+A`.

4 Locate the **Reaction Parameters** section. In the k^{f} text field, type `rnp*N_A_const`.

Species: A

When solving any type of reacting flow problem one species must always be chosen to fulfill the mass constraint. This should be taken as the species with the largest mass fraction.

1 In the **Model Builder** window, click **Species: A**.

2 In the **Settings** window for **Species**, locate the **Species Formula** section.

3 Select the **From mass constraint** check box.

4 Locate the **General Parameters** section. From the **Preset species data** list, choose **N2**.

Species: p

1 In the **Model Builder** window, click **Species: p**.

2 In the **Settings** window for **Species**, locate the **Species Formula** section.

3 From the **Species type** list, choose **Ion**.

4 Locate the **General Parameters** section. From the **Preset species data** list, choose **N2**.

5 In the z text field, type `1`.

6 In the n_0 text field, type `ni0`.

7 Locate the **Mobility and Diffusivity Expressions** section. From the **Specification** list, choose **Specify mobility, compute diffusivity**.

8 Locate the **Mobility Specification** section. In the u_m text field, type `muiN/plas.Nn`.

Species: n

1 In the **Model Builder** window, click **Species: n**.

2 In the **Settings** window for **Species**, locate the **Species Formula** section.

3 From the **Species type** list, choose **Ion**.

4 Locate the **General Parameters** section. From the **Preset species data** list, choose **N2**.

5 In the z text field, type `-1`.

6 In the n_0 text field, type `ni0`.

7 Locate the **Mobility and Diffusivity Expressions** section. From the **Specification** list, choose **Specify mobility, compute diffusivity**.

8 Locate the **Mobility Specification** section. In the u_m text field, type `muiN/plas.Nn`.

Surface Reaction 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Surface Reaction**.

Surface reactions must always be included in a plasma model because they describe how ions, excited, and radical species interact with the wall.

2 In the **Settings** window for **Surface Reaction**, locate the **Reaction Formula** section.

3 In the **Formula** text field, type `p=>A`.

4 Locate the **Secondary Emission Parameters** section. In the γ_i text field, type `0.05`.

5 In the ε_i text field, type `4`.

6 Select Boundary 1 only.

2: p=>A

1 Right-click **1: p=>A** and choose **Duplicate**.

2 In the **Settings** window for **Surface Reaction**, locate the **Secondary Emission Parameters** section.

3 In the γ_i text field, type `0`.

4 In the ε_i text field, type `0`.

5 Locate the **Boundary Selection** section. Click **Clear Selection**.

6 Select Boundary 2 only.

3: p=>A

1 Right-click **2: p=>A** and choose **Duplicate**.

2 In the **Settings** window for **Surface Reaction**, locate the **Reaction Formula** section.

3 In the **Formula** text field, type `n=>A`.

4 Locate the **Boundary Selection** section. Click  **Clear Selection**.

5 From the **Selection** list, choose **All boundaries**.

Plasma Model 1

Set up the background gas information, the electron mobility and import the table that gives the relation between the mean electron energy and the reduced electric field.

1 In the **Model Builder** window, click **Plasma Model 1**.

2 In the **Settings** window for **Plasma Model**, locate the **Model Inputs** section.

3 In the T text field, type `T0`.

4 In the p_A text field, type `P0`.

5 Locate the **Electron Density and Energy** section. In the $\mu_e N_n$ text field, type `mueN`.

6 Locate the **Mean Electron Energy Specification** section. From the **Specify using** list, choose **Use lookup table**.

7 Find the **Mean electron energy** subsection. Click  **Load from File**.

8 Browse to the model's Application Libraries folder and double-click the file `EN_to_Nrg.txt`.

Initial Values 1

1 In the **Model Builder** window, click **Initial Values 1**.

2 In the **Settings** window for **Initial Values**, locate the **Initial Values** section.

3 In the $n_{e,0}$ text field, type `ne0`.

Ground 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Ground**.

2 Select Boundary 2 only.

Metal Contact 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Metal Contact**.

2 In the **Settings** window for **Metal Contact**, locate the **Terminal** section.

3 In the V_0 text field, type `Vapp`.

4 Select Boundary 1 only.

Wall 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Wall**.

2 In the **Settings** window for **Wall**, locate the **Boundary Selection** section.

3 From the **Selection** list, choose **All boundaries**.

MESH 1

Meshing is a critical step in any plasma model. A fine mesh is needed close to the electrodes to capture the separation of space charge between the electrons and ions close to the wall.

Edge 1

In the **Mesh** toolbar, click **Edge**.

Distribution 1

1 Right-click **Edge 1** and choose **Distribution**.

2 In the **Settings** window for **Distribution**, locate the **Distribution** section.

3 From the **Distribution type** list, choose **Predefined**.

4 In the **Number of elements** text field, type `500`.

5 In the **Element ratio** text field, type `800`.

6 From the **Growth rate** list, choose **Exponential**.

7 Select the **Symmetric distribution** check box.

8 Click **Build All**.

POSITIVE CORONA

1 In the **Model Builder** window, click **Study 1**.

2 In the **Settings** window for **Study**, type `Positive Corona` in the **Label** text field.

3 Locate the **Study Settings** section. Clear the **Generate default plots** check box.

Solve for positive corona first.

Step 1: Time Dependent

1 In the **Model Builder** window, under **Positive Corona** click **Step 1: Time Dependent**.

2 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.

3 In the **Output times** text field, type `0 10^{range(-8,8/49,0)}`.

4 In the **Home** toolbar, click **Compute**.

Prepare plots that show the charge particle number densities, the electric potential, the space charge density, and the rate expression.

RESULTS

Electron and Ion Number Density - Positive Corona

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Electron and Ion Number Density - Positive Corona` in the **Label** text field.

3 Locate the **Data** section. From the **Time selection** list, choose **Last**.

4 Click to expand the **Title** section. From the **Title type** list, choose **Label**.

5 Locate the **Plot Settings** section.

6 Select the **x-axis label** check box. In the associated text field, type `r-coordinate (cm)`.

7 Select the **y-axis label** check box. In the associated text field, type `Number density (1/m<sup>3</sup>)`.

8 Locate the **Axis** section. Select the **y-axis log scale** check box.

9 Select the **x-axis log scale** check box.

10 Select the **Manual axis limits** check box.

11 In the **x minimum** text field, type `0.009`.

12 In the **x maximum** text field, type `11`.

13 In the **y minimum** text field, type `1e7`.

14 In the **y maximum** text field, type `1e18`.

Electrons

1 Right-click **Electron and Ion Number Density - Positive Corona** and choose **Line Graph**.

2 In the **Settings** window for **Line Graph**, type `Electrons` in the **Label** text field.

3 Locate the **Selection** section. From the **Selection** list, choose **All domains**.

4 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

5 In the **Expression** text field, type `r`.

6 Click to expand the **Legends** section. Select the **Show legends** check box.

7 Find the **Include** subsection. Clear the **Solution** check box.

8 Select the **Label** check box.

9 In the **Electron and Ion Number Density - Positive Corona** toolbar, click **Plot**.

Positive Ions

1 Right-click **Electrons** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, type `Positive Ions` in the **Label** text field.

3 Locate the **y-Axis Data** section. In the **Expression** text field, type `plas.n_wp`.

Negative Ions

1 Right-click **Positive Ions** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, type `Negative Ions` in the **Label** text field.

3 Locate the **y-Axis Data** section. In the **Expression** text field, type `plas.n_wn`.

4 In the **Electron and Ion Number Density - Positive Corona** toolbar, click **Plot**.

Electric Potential

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Electric Potential` in the **Label** text field.

3 Click to expand the **Title** section. From the **Title type** list, choose **Label**.

4 Locate the **Data** section. From the **Time selection** list, choose **Last**.

5 Locate the **Plot Settings** section.

6 Select the **x-axis label** check box. In the associated text field, type `r-coordinate (cm)`.

7 Select the **y-axis label** check box. In the associated text field, type `Potential (kV)`.

Positive Corona

1 Right-click **Electric Potential** and choose **Line Graph**.

2 In the **Settings** window for **Line Graph**, type `Positive Corona` in the **Label** text field.

3 Locate the **Selection** section. From the **Selection** list, choose **All domains**.

4 Locate the **y-Axis Data** section. In the **Expression** text field, type `V`.

5 From the **Unit** list, choose **kV**.

6 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

7 In the **Expression** text field, type `r`.

8 Locate the **Legends** section. Select the **Show legends** check box.

9 Find the **Include** subsection. Clear the **Solution** check box.

10 Select the **Label** check box.

11 In the **Electric Potential** toolbar, click **Plot**.

Space Charge Density

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Space Charge Density` in the **Label** text field.

3 Locate the **Data** section. From the **Time selection** list, choose **Last**.

4 Locate the **Title** section. From the **Title type** list, choose **Label**.

5 Locate the **Plot Settings** section.

6 Select the **x-axis label** check box. In the associated text field, type `r-coordinate (cm)`.

7 Select the **y-axis label** check box. In the associated text field, type `Space charge density (mC/m<sup>3</sup>)`.

Positive Corona

1 Right-click **Space Charge Density** and choose **Line Graph**.

2 In the **Settings** window for **Line Graph**, type `Positive Corona` in the **Label** text field.

3 Locate the **Selection** section. From the **Selection** list, choose **All domains**.

4 Locate the **y-Axis Data** section. In the **Expression** text field, type `plas.scharge`.

5 From the **Unit** list, choose **mC/m^3**.

6 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

7 In the **Expression** text field, type `r`.

8 Locate the **Legends** section. Select the **Show legends** check box.

9 Find the **Include** subsection. Clear the **Solution** check box.

10 Select the **Label** check box.

11 In the **Space Charge Density** toolbar, click **Plot**.

Rate Expression

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Rate Expression` in the **Label** text field.

3 Click to expand the **Title** section. From the **Title type** list, choose **Label**.

4 Locate the **Plot Settings** section.

5 Select the **x-axis label** check box. In the associated text field, type `r-coordinate (cm)`.

6 Select the **y-axis label** check box. In the associated text field, type `Rate expression (mol/(m<sup>3</sup>*s))`.

7 Locate the **Axis** section. Select the **x-axis log scale** check box.

8 Select the **y-axis log scale** check box.

9 Select the **Manual axis limits** check box.

10 In the **x minimum** text field, type `0.009`.

11 In the **x maximum** text field, type `11`.

12 In the **y minimum** text field, type `1e-11`.

13 In the **y maximum** text field, type `10`.

Positive Corona

1 Right-click **Rate Expression** and choose **Line Graph**.

2 In the **Settings** window for **Line Graph**, type `Positive Corona` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Positive Corona/Solution 1 (sol1)**.

4 From the **Time selection** list, choose **Last**.

5 Locate the **Selection** section. From the **Selection** list, choose **All domains**.

6 Locate the **y-Axis Data** section. In the **Expression** text field, type `plas.R_wp`.

7 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

8 In the **Expression** text field, type `r`.

9 Click to expand the **Legends** section. Select the **Show legends** check box.

10 Find the **Include** subsection. Clear the **Solution** check box.

11 Select the **Label** check box.

12 In the **Rate Expression** toolbar, click **Plot**.

Solve for the negative corona in a new study. Use the variable cp to change the polarity of the discharge.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
`cp`	`-1`	`-1`	`Corona polarity`

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies> Time Dependent**.

4 Click **Add Study** in the window toolbar.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 2

Step 1: Time Dependent

1 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.

2 In the **Output times** text field, type `0 10^{range(-8,8/49,0)}`.

3 In the **Model Builder** window, click **Study 2**.

4 In the **Settings** window for **Study**, locate the **Study Settings** section.

5 Clear the **Generate default plots** check box.

6 In the **Label** text field, type `Negative Corona`.

7 In the **Home** toolbar, click **Compute**.

Prepare the plots for the results of the negative corona.

RESULTS

Electron and Ion Number Density - Negative Corona

1 In the **Model Builder** window, expand the **Results** node.

2 Right-click **Electron and Ion Number Density - Positive Corona** and choose **Duplicate**.

3 Right-click **Electron and Ion Number Density - Positive Corona 1** and choose **Move Up**.

4 Right-click **Electron and Ion Number Density - Positive Corona 1** and choose **Move Up**.

5 Right-click **Electron and Ion Number Density - Positive Corona 1** and choose **Move Up**.

6 In the **Model Builder** window, click **Electron and Ion Number Density - Positive Corona 1**.

7 In the **Settings** window for **1D Plot Group**, type `Electron and Ion Number Density - Negative Corona` in the **Label** text field.

8 Locate the **Data** section. From the **Dataset** list, choose **Negative Corona/Solution 2 (sol2)**.

9 In the **Electron and Ion Number Density - Negative Corona** toolbar, click **Plot**.

Plot the electric potential of the negative corona multiplied by -1 so that the curves can be easily compared.

Negative Corona

1 In the **Model Builder** window, expand the **Electric Potential** node.

2 Right-click **Positive Corona** and choose **Duplicate**.

3 In the **Settings** window for **Line Graph**, type `Negative Corona` in the **Label** text field.

4 Locate the **Data** section. From the **Dataset** list, choose **Negative Corona/Solution 2 (sol2)**.

5 From the **Time selection** list, choose **Last**.

6 Locate the **y-Axis Data** section. In the **Expression** text field, type `-V`.

7 Click to expand the **Legends** section. In the **Electric Potential** toolbar, click **Plot**.

Negative Corona

1 In the **Model Builder** window, expand the **Space Charge Density** node.

2 Right-click **Positive Corona** and choose **Duplicate**.

3 In the **Settings** window for **Line Graph**, type `Negative Corona` in the **Label** text field.

4 Locate the **Data** section. From the **Dataset** list, choose **Negative Corona/Solution 2 (sol2)**.

5 From the **Time selection** list, choose **Last**.

6 Click to expand the **Legends** section. In the **Space Charge Density** toolbar, click **Plot**.

Negative Corona

1 In the **Model Builder** window, expand the **Results>Rate Expression** node.

2 Right-click **Positive Corona** and choose **Duplicate**.

3 In the **Settings** window for **Line Graph**, type `Negative Corona` in the **Label** text field.

4 Locate the **Data** section. From the **Dataset** list, choose **Negative Corona/Solution 2 (sol2)**.

5 Locate the **y-Axis Data** section. In the **Expression** text field, type `plas.R_wn`.

6 Click to expand the **Legends** section. In the **Rate Expression** toolbar, click **Plot**.

Evaluate the electric field at the corona electrode.

Point Evaluation 1

1 In the **Results** toolbar, click **Point Evaluation**.

2 In the **Settings** window for **Point Evaluation**, locate the **Data** section.

3 From the **Time selection** list, choose **Last**.

4 Select Boundary 1 only.

5 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
`plas.Er`	`V/m`	`Electric field, r component`

6 Click **Evaluate**.

Point Evaluation 2

1 Right-click **Point Evaluation 1** and choose **Duplicate**.

2 In the **Settings** window for **Point Evaluation**, locate the **Data** section.

3 From the **Dataset** list, choose **Negative Corona/Solution 2 (sol2)**.

4 Click **Evaluate**.

Prepare the simplified model for the corona discharge.

ADD PHYSICS

1 In the **Home** toolbar, click **Add Physics** to open the **Add Physics** window.

2 Go to the **Add Physics** window.

3 Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** check boxes for **Positive Corona** and **Negative Corona**.

4 In the tree, select **Plasma>Electric Discharges>Corona Discharge**.

5 Click **Add to Component 1** in the window toolbar.

6 In the **Home** toolbar, click **Add Physics** to close the **Add Physics** window.

ELECTROSTATICS (ES)

Charge Conservation 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Electrostatics (es)** click **Charge Conservation 1**.

2 In the **Settings** window for **Charge Conservation**, locate the **Constitutive Relation D-E** section.

3 From the ε_r list, choose **User defined**.

Ground 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Ground**.

2 Select Boundary 2 only.

CHARGE TRANSPORT (CT)

Transport Properties 1

The variable cp is used to change the polarity of the discharge in a practical way.

1 In the **Model Builder** window, under **Component 1 (comp1)>Charge Transport (ct)** click **Transport Properties 1**.

2 In the **Settings** window for **Transport Properties**, locate the **Model Inputs** section.

3 In the T text field, type `T0`.

4 In the P_A text field, type `P0`.

5 Locate the **Migration in Electric Field** section. In the $\mu_i N$ text field, type `muiN`.

6 In the z_q text field, type `cp`.

Initial Values 1

1 In the **Model Builder** window, click **Initial Values 1**.

2 In the **Settings** window for **Initial Values**, locate the **Initial Values** section.

3 In the *rhoq* text field, type `cp*1.0E-4[C/m^3]`.

Add an electrode feature to define the electric field and the electric potential at the corona electrode.

MULTIPHYSICS

Electrode 1 (el1)

1 In the **Physics** toolbar, click  **Multiphysics Couplings** and choose **Boundary>Electrode**.

2 Select Boundary 1 only.

3 In the **Settings** window for **Electrode**, locate the **Electric Potential** section.

4 In the V_0 text field, type `V0`.

5 Locate the **Electric Field** section. In the E_c text field, type `cp*3e6[V/m]`.

6 In the r_c text field, type `rin`.

This model does not need a mesh as refined as the one used before. Create a new coarser mesh.

MESH 1

In the **Model Builder** window, under **Component 1 (comp1)** right-click **Mesh 1** and choose **Duplicate**.

MESH 2

In the **Model Builder** window, expand the **Mesh 2** node.

Distribution 1

1 In the **Model Builder** window, expand the **Component 1 (comp1)>Meshes>Mesh 2>Edge 1** node, then click **Distribution 1**.

2 In the **Settings** window for **Distribution**, locate the **Distribution** section.

3 In the **Number of elements** text field, type `300`.

4 In the **Element ratio** text field, type `100`.

5 Click  **Build All**.

Add a new study to solve for the positive corona.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** check box for **Plasma (plas)**.

4 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies>Stationary**.

5 Click **Add Study** in the window toolbar.

6 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 3

Step 1: Stationary

1 In the **Settings** window for **Stationary**, click to expand the **Mesh Selection** section.

2 In the **Model Builder** window, click **Study 3**.

3 In the **Settings** window for **Study**, type `Positive Corona Simplified` in the **Label** text field.

4 Locate the **Study Settings** section. Clear the **Generate default plots** check box.

5 Clear the **Generate convergence plots** check box.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
cp	+1	1	Corona polarity

POSITIVE CORONA SIMPLIFIED

1 In the **Home** toolbar, click **Compute**.

Plot the results for the simplified positive corona model.

RESULTS

Electric Potential

In the **Model Builder** window, expand the **Results** node.

Positive Corona Simplified

1 In the **Model Builder** window, expand the **Electric Potential** node.

2 Right-click **Positive Corona** and choose **Duplicate**.

3 In the **Settings** window for **Line Graph**, type `Positive Corona Simplified` in the **Label** text field.

4 Locate the **Data** section. From the **Dataset** list, choose **Positive Corona Simplified/ Solution 3 (sol3)**.

5 Locate the **y-Axis Data** section. In the **Expression** text field, type `V2`.

6 Click to expand the **Legends** section. In the **Electric Potential** toolbar, click **Plot**.

Positive Corona Simplified

1 In the **Model Builder** window, expand the **Space Charge Density** node.

2 Right-click **Positive Corona** and choose **Duplicate**.

3 In the **Settings** window for **Line Graph**, type `Positive Corona Simplified` in the **Label** text field.

4 Locate the **Data** section. From the **Dataset** list, choose **Positive Corona Simplified/ Solution 3 (sol3)**.

5 Locate the **y-Axis Data** section. In the **Expression** text field, type `rhoq`.

6 In the **Space Charge Density** toolbar, click **Plot**.

Prepare the model to solve for the negative corona.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
`cp`	`-1`	`-1`	`Corona polarity`

CHARGE TRANSPORT (CT)

Add a source term and set it to use the rate expression of the negative ions obtained from the self-consistent model.

Source 1

1 In the **Physics** toolbar, click **Domains** and choose **Source**.

2 In the **Settings** window for **Source**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **All domains**.

4 Locate the **Source** section. From the **Specify source using** list, choose **Reaction rate**.

5 In the R text field, type `-plas.R_wn`.

Use the electric field obtained from the self-consistent model.

MULTIPHYSICS

Electrode 1 (el1)

1 In the **Model Builder** window, expand the **Multiphysics** node, then click **Electrode 1 (el1)**.

2 In the **Settings** window for **Electrode**, locate the **Electric Field** section.

3 From the **Electrode electric field** list, choose **User defined**. In the E_0 text field, type `-5.9536E6`.

Add a new study to solve for the negative corona.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** check box for **Plasma (plas)**.

4 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies>Stationary**.

5 Click **Add Study** in the window toolbar.

6 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 4

Step 1: Stationary

1 In the **Settings** window for **Stationary**, click to expand the **Values of Dependent Variables** section.

2 Find the **Values of variables not solved for** subsection. From the **Settings** list, choose **User controlled**.

3 From the **Method** list, choose **Solution**.

4 From the **Study** list, choose **Negative Corona, Time Dependent**.

5 In the **Model Builder** window, click **Study 4**.

6 In the **Settings** window for **Study**, type `Negative Corona Simplified` in the **Label** text field.

7 Locate the **Study Settings** section. Clear the **Generate default plots** check box.

8 Clear the **Generate convergence plots** check box.

9 In the **Home** toolbar, click **Compute**.

Prepare the plots for the negative corona.

RESULTS

Electric Potential

In the **Model Builder** window, expand the **Results** node.

Negative Corona Simplified

1 In the **Model Builder** window, expand the **Electric Potential** node.

2 Right-click **Positive Corona Simplified** and choose **Duplicate**.

3 In the **Settings** window for **Line Graph**, type `Negative Corona Simplified` in the **Label** text field.

4 Locate the **Data** section. From the **Dataset** list, choose **Negative Corona Simplified/Solution 4 (sol4)**.

5 Locate the **y-Axis Data** section. In the **Expression** text field, type `-V2`.

6 Click to expand the **Legends** section. In the **Electric Potential** toolbar, click **Plot**.

Negative Corona Simplified

1 In the **Model Builder** window, expand the **Space Charge Density** node.

2 Right-click **Positive Corona Simplified** and choose **Duplicate**.

3 In the **Settings** window for **Line Graph**, type `Negative Corona Simplified` in the **Label** text field.

4 Locate the **Data** section. From the **Dataset** list, choose **Negative Corona Simplified/Solution 4 (sol4)**.

5 In the **Space Charge Density** toolbar, click **Plot**.

Evaluate the electric field obtained with Peek's law with the one obtained with the self-consistent model.

Point Evaluation 3

1 In the **Model Builder** window, expand the **Results>Derived Values** node.

2 Right-click **Results>Derived Values>Point Evaluation 2** and choose **Duplicate**.

3 In the **Settings** window for **Point Evaluation**, locate the **Data** section.

4 From the **Dataset** list, choose **Positive Corona Simplified/Solution 3 (sol3)**.

5 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
ct.Er	V/m	Electric field, r component

6 Click ▼ next to = **Evaluate**, then choose **New Table**.