Created in COMSOL Multiphysics 6.2

Frequency Response of a Biased Resonator — 2D

Introduction

Silicon micromechanical resonators have long been used for designing sensors and are now becoming increasingly important as oscillators in the consumer electronics market. In this sequence of models, a surface micromachined MEMS resonator, designed as part of a micromechanical filter, is analyzed in detail. The resonator is based on that developed in Ref. 1.

This model performs a frequency-domain analysis of the structure, which is also biased with its operating DC offset. The analysis begins from the stationary analysis performed in the accompanying model Stationary Analysis of a Biased Resonator — 2D; please review this model first.

Model Definition

The geometry, fabrication, and operation of the device are discussed for the Stationary Analysis of a Biased Resonator — 2D model.

For the frequency-domain analysis of the structure, consider an applied drive voltage consisting of a 35 V DC offset with a 100 mV drive signal added as a harmonic perturbation. Solve the linearized problem to compute the response of the system.

DAMPING

To obtain the response of the system, you need to add damping to the model. For this study, assume that the damping mechanism is Rayleigh damping or material damping.

To specify the damping, two material constants are required (α_{dM} and β_{dK}). For a system with a single degree of freedom (a mass-spring-damper system) the equation of motion with viscous damping is given by

$$m\frac{d^2u}{dt^2}+c\frac{du}{dt}+ku = f(t)$$

where c is the damping coefficient, m is the mass, k is the spring constant, u is the displacement, t is the time, and $f(t)$ is a driving force.

In the Rayleigh damping model, the parameter c is related to the mass, m, and the stiffness, k, by the equation:

$$c = \alpha_{dM}m+\beta_{dK}k$$

The Rayleigh damping term in COMSOL Multiphysics is proportional to the mass and stiffness matrices and is added to the static weak term.

The damping coefficient, c, is frequently defined as a damping ratio or factor, expressed as a fraction of the critical damping, c_0, for the system such that

$$\xi = \frac{c}{c_0}$$

where for a system with one degree of freedom

$$c_0 = 2\sqrt{km}$$

Finally note that for large values of the quality factor, Q,

$$\xi \cong \frac{1}{2Q}$$

The material parameters α_{dM} and β_{dK} are usually not available in the literature. Often the damping ratio is available, typically expressed as a percentage of the critical damping. It is possible to transform damping factors to Rayleigh damping parameters. The damping factor, ξ, for a specified pair of Rayleigh parameters, α_{dM} and β_{dK}, at the frequency, f, is

$$\xi = \frac{1}{2}\left(\frac{\alpha_{dM}}{2\pi f} + \beta_{dK} 2\pi f\right)$$

Using this relationship at two frequencies, f_1 and f_2, with different damping factors, ξ_1 and ξ_2, results in an equation system that can be solved for α_{dM} and β_{dK}:

$$\begin{bmatrix} \frac{1}{4\pi f_1} & \pi f_1 \\ \frac{1}{4\pi f_2} & \pi f_2 \end{bmatrix} \begin{bmatrix} \alpha_{dM} \\ \beta_{dK} \end{bmatrix} = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}$$

The damping factors for this model are provided as α_{dM} = 4189 Hz and β_{dK} = $8.29 \cdot 10^{-13}$ s, consistent with the observed Quality factor of 8000 for the fundamental mode.

Results and Discussion

Figure 1 shows the frequency response of the resonator. A clear anti-resonance structure for the frequency response is observable. This response can be compared to that shown in

Figure 15 (a) in Ref. 1. Although the experimental results are from a pair of coupled resonators in this instance, the two resonances are sufficiently separate in frequency space that it is possible to distinguish the two modes. If the details of the external circuits were available, a terminal boundary condition with an attached circuit could be used to compute the electrical response of the system for a more direct comparison with the experimental results

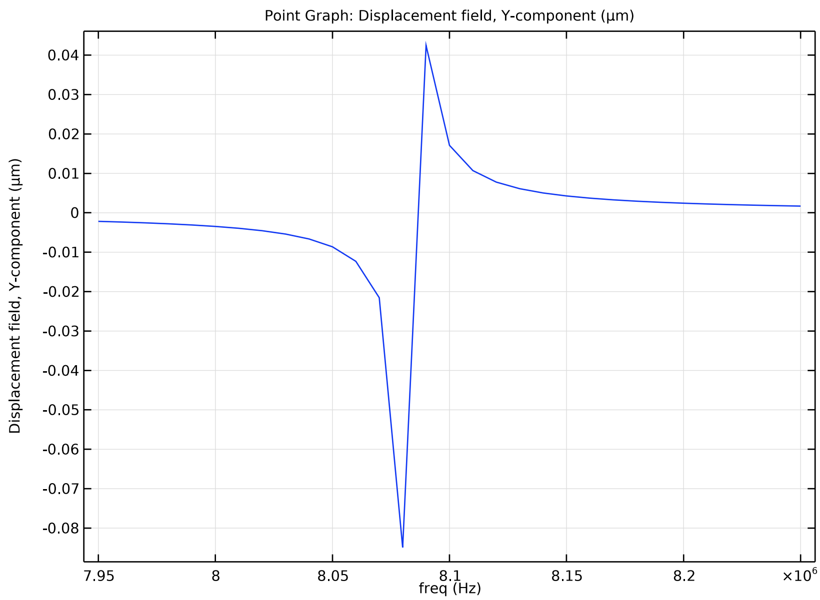

Figure 1: Frequency response of the fundamental mode of the resonator.

Reference

1. F.D. Bannon III, J.R. Clark and C.T.-C. Nguyen, "High-Q HF Microelectromechanical Filters," *IEEE Journal of Solid State Circuits*, vol. 35, no. 4, pp. 512–526, 2000.

Application Library path: `MEMS_Module/Actuators/biased_resonator_2d_freq`

Modeling Instructions

Start from the existing stationary model.

APPLICATION LIBRARIES

1 From the **File** menu, choose **Application Libraries**.

2 In the **Application Libraries** window, select **MEMS Module>Actuators> biased_resonator_2d_basic** in the tree.

3 Click **Open**.

Create parameters for the material damping factors.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
alpha	4189[Hz]	4189 Hz	Damping parameter - alpha
beta	8.29e-13[s]	8.29E-13 s	Damping parameter - beta

COMPONENT 1 (COMP1)

In the **Model Builder** window, expand the **Component 1 (comp1)** node.

SOLID MECHANICS (SOLID)

Add damping to the physics settings.

Linear Elastic Material 1

In the **Model Builder** window, expand the **Component 1 (comp1)>Solid Mechanics (solid)** node, then click **Linear Elastic Material 1**.

Damping 1

1 In the **Physics** toolbar, click **Attributes** and choose **Damping**.

2 In the **Settings** window for **Damping**, locate the **Damping Settings** section.

3 In the α_{dM} text field, type alpha.

4 In the β_{dK} text field, type beta.

Add a **Harmonic Perturbation** to the DC bias term, to represent the AC drive voltage.

ELECTROSTATICS (ES)

Electric Potential 1

In the **Model Builder** window, expand the **Component 1 (comp1)>Electrostatics (es)** node, then click **Electric Potential 1**.

Harmonic Perturbation 1

1. In the **Physics** toolbar, click **Attributes** and choose **Harmonic Perturbation**.
2. In the **Settings** window for **Harmonic Perturbation**, locate the **Electric Potential** section.
3. In the V_0 text field, type `0.1`.

   Set up the frequency domain study.

ADD STUDY

1. In the **Home** toolbar, click **Add Study** to open the **Add Study** window.
2. Go to the **Add Study** window.
3. Find the **Studies** subsection. In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces>Solid Mechanics>Frequency Domain, Prestressed**.
4. Click **Add Study** in the window toolbar.
5. In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 2

Step 2: Frequency-Domain Perturbation

1. In the **Model Builder** window, under **Study 2** click **Step 2: Frequency-Domain Perturbation**.
2. In the **Settings** window for **Frequency-Domain Perturbation**, locate the **Study Settings** section.
3. In the **Frequencies** text field, type `range(7.95[MHz],0.01[MHz],8.25[MHz])`.
4. In the **Model Builder** window, click **Study 2**.
5. In the **Settings** window for **Study**, type `Frequency domain` in the **Label** text field.
6. Locate the **Study Settings** section. Clear the **Generate default plots** check box.
7. In the **Home** toolbar, click **Compute**.

   Produce a plot of the frequency response of the system.

RESULTS

Frequency Domain

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.
2 In the **Settings** window for **1D Plot Group**, locate the **Data** section.
3 From the **Dataset** list, choose **Frequency domain/Solution 2 (sol2)**.
4 In the **Label** text field, type `Frequency Domain`.

Point Graph 1

1 Right-click **Frequency Domain** and choose **Point Graph**.
2 In the **Settings** window for **Point Graph**, locate the **Selection** section.
3 Click **Paste Selection**.
4 In the **Paste Selection** dialog box, type `9` in the **Selection** text field.
5 Click **OK**.
6 In the **Settings** window for **Point Graph**, locate the **y-Axis Data** section.
7 In the **Expression** text field, type `v`.
8 In the **Frequency Domain** toolbar, click **Plot**.

Compare the resulting plot with Figure 1.