

# Michelson Interferometer

A Michelson interferometer is composed of five elements: two mirrors, one beam splitter, an imaging device (screen), and a coherent light source. Figure 1 shows and describes a basic Michelson interferometer arrangement, the interference pattern is generated when the rays reflected by mirrors M1 and M2 arrive at the screen with different optical path lengths.

Changing, even slightly, the optical path length of either beam results in a change of the interference pattern at the screen. This change of optical path length can be accomplished by moving one of the mirrors. Unexpected changes in optical path length can also occur if any of the device's optical components undergo deformation, such as thermal expansion. This model illustrates the effects of thermal expansion on the interference pattern obtained at the screen of the interferometer.

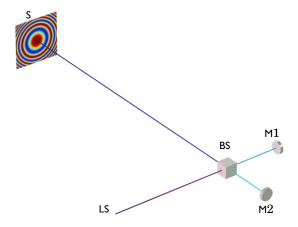


Figure 1: An illuminated Michelson interferometer arrangement with the resulting interference pattern displayed on the screen (S). The light comes from the light source(LS), hits the beam splitter (BS) before being equally diverted toward mirror 1 (M1) and mirror 2 (M2). Once reflected at the mirrors, the two composing beams return to the beam splitter (BS) where they recombine to travel toward the screen (S) or the light source (LS).

In this model the rays propagate through the beam splitter and the air enclosing the optical components. The mirrors are treated as Wall boundary conditions at which specular reflection occurs. The beam splitter (BS) is composed of two BK7 glass prisms separated by a thin dielectric film (BS interface). The beam splitter is surrounded by an anti-reflective (AR) coating. The AR coating and the BS interface are modeled by applying the Thin Dielectric Film subnode to the Material Discontinuity node.

The Settings window for the Material Discontinuity node includes an option to automatically set up a single-layer coating with the desired reflectance. In this case, a reflectance of 0.5 is desired for the interior boundary of the beam splitter so that an incoming ray is divided into two rays of equal intensity.

The light source used in the model is a He-Ne laser. For sake of simplicity only one ray is initially released from a point on a face of the enclosure. The initial ray is polarized such that it forms an s-polarized wave at the beam splitter surface. For the interference pattern to be accurate, it must correspond to a wavefront that subtends a very small solid angle. This can be accomplished by setting initial radius of curvature of the wavefront to -1 m and setting the initial optical path length difference between the beam splitter and the two mirrors to a large multiple of the free-space wavelength.

The interference pattern obtained at the screen depends on the radii of curvature, phase, and angle of incidence of the two rays as they arrive at the screen. For a spherical wavefront with principal radius of curvature  $r_{1,0}$  with normal incidence at a surface, the change in phase corresponding to a shift  $x_p$  in position on the screen is

$$\Delta \Psi_1 \, = \, k \bigg( \sqrt{x_p^2 + r_{1,\,0}^2} - r_{1,\,0} \bigg)$$

where k is the wave number. Similarly, the change in phase for a wavefront with principal radius of curvature  $r_{2,0}$  is

$$\Delta \Psi_2 = k \left( \sqrt{x_p^2 + r_{2,0}^2} - r_{2,0} \right)$$

For small values of  $x_p$ , an approximate solution for the difference in phase between the two wavefronts can be obtained by taking a Taylor series expansion of the expressions for  $\Delta \Psi_1$  and  $\Delta \Psi_2$  and retaining terms of up to second order in  $x_p$ ,

$$\Delta \Psi_1 - \Delta \Psi_2 \cong kx_p^2 \left(\frac{1}{2r_{1,0}} - \frac{1}{2r_{2,0}}\right)$$

If the two rays interfere constructively at  $x_p = 0$ , they also interfere constructively where the phase difference between the two wavefronts is an integer multiple of  $2\pi$ . The first such point occurs where

$$x_p = \sqrt{\lambda_0 \left(\frac{1}{2r_{1,0}} - \frac{1}{2r_{2,0}}\right)^{-1}}$$
 (1)

which is the distance from the center of the interference pattern to a point of maximum intensity on the first fringe.

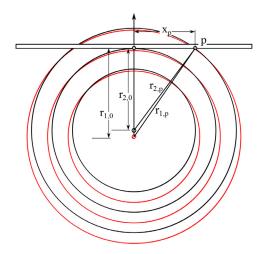


Figure 2: Interference of coherent light emitted from two point sources separated by a small distance.

# Results and Discussion

Figure 3 shows the interference pattern resulting from the combination of two rays having an optical path length difference of  $\delta_d$  = 8000  $\lambda_0$ . As expected for spherical waves, the interference fringes are circular. On the figure it is possible to approximate the distance

from the center of the screen to the radial position of greatest intensity on the first circular fringe.

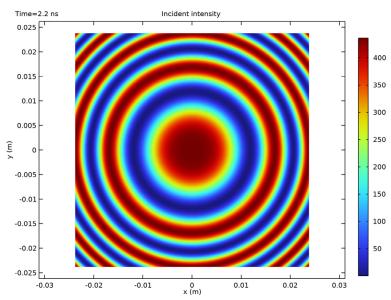


Figure 3: Interference pattern obtained for an optical path length difference of  $\delta_d$  = 8000  $\lambda_0$ .

# Reference

1. M. Born and E. Wolf, *Principle of Optics*, 7th ed., Cambridge University Press, 2011.

# Application Library path: Ray\_Optics\_Module/

 ${\tt Spectrometers\_and\_Monochromators/michelson\_interferometer}$ 

# Modeling Instructions

From the File menu, choose New.

#### NEW

In the New window, click Model Wizard.

#### MODEL WIZARD

- I In the Model Wizard window, click 1 3D.
- 2 In the Select Physics tree, select Optics>Ray Optics>Geometrical Optics (gop).
- 3 Click Add.
- 4 Click Study.
- 5 In the Select Study tree, select Preset Studies for Selected Physics Interfaces>Ray Tracing.
- 6 Click **Done**.

#### **GLOBAL DEFINITIONS**

#### Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description
lam	632.8[nm]	6.328E-7 m	Wavelength of He-Ne laser
delta_d	8000*lam	0.0050624 m	Optical path length difference
th	0.12[in]	0.003048 m	Thickness of mirrors
dia	0.5[in]	0.0127 m	Diameter of mirrors
d1	2.335[in]	0.059309 m	Distance between mirror M1 and the center of the beam splitter
d2	d1-delta_d	0.054247 m	Distance between mirror M2 and the center of the beam splitter
dE	12[in]	0.3048 m	Distance between the beam splitter and the screen
n_int	3.9641	3.9641	Refractive index of the beam- splitter interface
n_coat	1.2354	1.2354	Refractive index of the anti- reflective coating

# PART LIBRARIES

- I In the Home toolbar, click Part Libraries.
- 2 In the Part Libraries window, select Ray Optics Module>3D>Beam Splitters> beam\_splitter\_cube in the tree.

3 Click Add to Geometry.

#### GEOMETRY I

Beam Splitter Cube I (pil)

- I In the Model Builder window, under Component I (compl)>Geometry I click
  Beam Splitter Cube I (pil).
- 2 In the Settings window for Part Instance, locate the Input Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description
d	dia	0.0127 m	Side length

### Cylinder I (cyl1)

- I In the Geometry toolbar, click Cylinder.
- 2 In the Settings window for Cylinder, locate the Size and Shape section.
- 3 In the Radius text field, type dia/2.
- 4 In the Height text field, type th.
- 5 Locate the Position section. In the y text field, type -d2-th.
- 6 Locate the Axis section. From the Axis type list, choose y-axis.

#### Cylinder 2 (cyl2)

- I In the Geometry toolbar, click ( Cylinder.
- 2 In the Settings window for Cylinder, locate the Size and Shape section.
- 3 In the Radius text field, type dia/2.
- 4 In the Height text field, type th.
- **5** Locate the **Position** section. In the **x** text field, type d1.
- 6 Locate the Axis section. From the Axis type list, choose x-axis.
- 7 Click | Build Selected.
- 8 Click the Zoom Extents button in the Graphics toolbar.

# Block I (blk I)

- I In the Geometry toolbar, click Block.
- 2 In the Settings window for Block, locate the Size and Shape section.
- 3 In the Width text field, type 4\*dia.
- 4 In the Depth text field, type th.

- 5 In the Height text field, type 4\*dia.
- 6 Locate the Position section. From the Base list, choose Center.
- 7 In the y text field, type dE+th/2.

Block 2 (blk2)

- I In the Geometry toolbar, click Block.
- 2 In the Settings window for Block, locate the Size and Shape section.
- 3 In the Width text field, type 200[mm].
- 4 In the Depth text field, type 450[mm].
- 5 In the Height text field, type 100[mm].
- 6 Locate the Position section. From the Base list, choose Center.
- 7 In the y text field, type 100[mm].

Form Union (fin)

- I In the Geometry toolbar, click | Build All.
- 2 Click the Go to Default View button in the Graphics toolbar.

#### **DEFINITIONS**

View 1

In the Model Builder window, expand the Component I (compl)>Definitions node, then click View I.

Hide for Geometry 1

- I In the View I toolbar, click w Hide.
- 2 In the Settings window for Hide for Geometry, locate the Selection section.
- 3 From the Geometric entity level list, choose Boundary.
- 4 Click the Transparency button in the Graphics toolbar.
- **5** On the object **fin**, select Boundaries 1–5 and 33 only.

#### ADD MATERIAL

- I In the Home toolbar, click 🤼 Add Material to open the Add Material window.
- 2 Go to the Add Material window.
- 3 In the tree, select Built-in>Air.
- 4 Click Add to Component in the window toolbar.
- 5 In the tree, select Optical>Schott Glass>Schott N-BK7 Glass.

- 6 Click Add to Component in the window toolbar.
- 7 In the Home toolbar, click **Add Material** to close the Add Material window.

# MATERIALS

Schott N-BK7 Glass (mat2) Select Domains 4 and 5 only.

GEOMETRICAL OPTICS (GOP)

- I In the Model Builder window, under Component I (compl) click Geometrical Optics (gop).
- 2 In the Settings window for Geometrical Optics, locate the Intensity Computation section.
- 3 From the Intensity computation list, choose Compute intensity and power.
- 4 Select the Compute phase check box.

Material Discontinuity I

Apply an anti-reflective coating to the outside of the beam splitter.

- In the Model Builder window, under Component I (compl)>Geometrical Optics (gop) click Material Discontinuity I.
- 2 In the Settings window for Material Discontinuity, locate the Coatings section.
- 3 From the Thin dielectric films on boundary list, choose Anti-reflective coating.
- 4 Select the Treat as single layer dielectric film check box.
- **5** In the  $\lambda_0$  text field, type 1am.

Ray Properties 1

- I In the Model Builder window, click Ray Properties I.
- 2 In the Settings window for Ray Properties, locate the Ray Properties section.
- **3** In the  $\lambda_0$  text field, type 1am.

Wall I

- I In the Physics toolbar, click **Boundaries** and choose **Wall**.
- 2 Select Boundary 7 only.

Mirror I

- I In the Physics toolbar, click **Boundaries** and choose Mirror.
- 2 Select Boundaries 14 and 27 only.

Beam Splitter

Use a second Material Discontinuity node for the interior boundary of the beam splitter.

- I In the Physics toolbar, click **Boundaries** and choose Material Discontinuity.
- 2 In the Settings window for Material Discontinuity, type Beam Splitter in the Label text field.
- **3** Locate the **Coatings** section. From the **Thin dielectric films on boundary** list, choose **Specify reflectance**.
- 4 In the R text field, type 0.5.
- 5 Select the Treat as single layer dielectric film check box.
- **6** In the *n* text field, type n\_int.
- 7 In the  $\lambda_0$  text field, type lam.
- **8** In the  $\theta_i$  text field, type 45[deg].
- 9 Select Boundary 19 only.

#### Release from Grid 1

Specify how the ray is going to be released. Release a single ray from the face of the enclosure facing mirror M1. Generate a fully polarized, -1 m radius of curvature spherical wave.

- I In the Physics toolbar, click A Global and choose Release from Grid.
- ${\bf 2} \ \ {\bf In \ the \ Settings \ window \ for \ Release \ from \ Grid, \ locate \ the \ Initial \ Coordinates \ section.}$
- 3 In the  $q_{x,0}$  text field, type -100[mm].
- 4 Locate the Ray Direction Vector section. Specify the  $\boldsymbol{L}_0$  vector as

1	x
0	у
0	z

- 5 Locate the Initial Radii of Curvature section. From the Wavefront shape list, choose Spherical wave.
- **6** In the  $r_0$  text field, type -1[m].
- 7 Locate the Initial Polarization section. From the Initial polarization type list, choose Fully polarized.
- 8 From the Initial polarization list, choose User defined.
- **9** Specify the **u** vector as
- 0 x

0	у
1	z

Ray Termination I

- I In the Physics toolbar, click A Global and choose Ray Termination.
- 2 In the Settings window for Ray Termination, locate the Termination Criteria section.
- 3 From the Spatial extents of ray propagation list, choose Bounding box, from geometry.

#### STUDY I

Step 1: Ray Tracing

- I In the Model Builder window, under Study I click Step I: Ray Tracing.
- 2 In the Settings window for Ray Tracing, locate the Study Settings section.
- 3 Click Range.
- 4 In the Range dialog box, type 0.1 in the Step text field.
- 5 In the Stop text field, type 2.2.
- 6 Click Replace.

Step 1: Ray Tracing

- I In the Model Builder window, click Step I: Ray Tracing.
- 2 In the Home toolbar, click **Compute**.

#### RESULTS

Create a dataset to display the interference pattern on the screen.

Cut Plane I

- I In the Results toolbar, click Cut Plane.
- 2 In the Settings window for Cut Plane, locate the Data section.
- 3 From the Dataset list, choose Ray 1.
- 4 Locate the Plane Data section. From the Plane list, choose xz-planes.
- 5 In the y-coordinate text field, type dE.

Interference Pattern

- I In the Results toolbar, click 2D Plot Group.
- 2 In the Settings window for 2D Plot Group, type Interference Pattern in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Cut Plane 1.

# Interference Pattern 1

- I In the Interference Pattern toolbar, click More Plots and choose Interference Pattern.
- 2 In the Settings window for Interference Pattern, locate the Coordinate Range section.
- 3 From the Origin location specification list, choose At ray of greatest intensity.
- **4** In the **Interference Pattern** toolbar, click **Plot**. The plot should look like Figure 3.