Created in COMSOL Multiphysics 6.2

Gravitational Lensing

Introduction

This model demonstrates how the sun causes 1.75 arcseconds of deflection for rays grazing the sun's surface as observed from Earth. Einstein predicted this value after refining his theory of relativity during World War I (Ref. 1).

Model Definition

The gravitational lensing effect is modeled using a refractive index that varies continuously in space, also known as a *graded medium*. The refractive index, n, depends on the gravitational constant G (SI unit: $m^3/(kg \cdot s^2)$), the solar mass m_0 (SI unit: kg), the speed of light c (SI unit: m/s), and the radial distance from the center of the sun r (SI unit: m):

$$n = 1 + \frac{2Gm_0}{c^2 r}$$

The gravitational constant is a built-in physical constant with the name `G_const` and predefined value `6.67384e-11[m^3/(kg*s^2)]`. For a list of all built-in physical constants, see *Physical Constants* in the *COMSOL Multiphysics Reference Manual*.

In this example, two rays are released, which graze the surface of the sun and then continue until a distance of 150 million km is reached. At this point, the deflection angle of the rays from their initial direction is evaluated.

Results and Discussion

The angular change in the direction of the rays is plotted in Figure 1. After release, it takes the rays around 165 s to reach the sun. The rays then begin to deviate from their initial direction due to the gradient in the refractive index. The final value is about 1.75 arcseconds, consistent with Einstein's prediction.

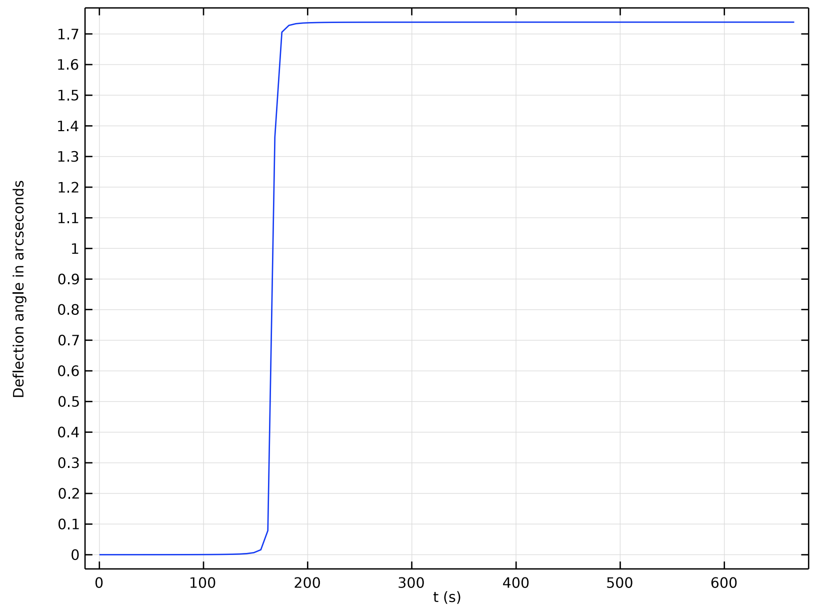

Figure 1: Deflection angle in arcseconds caused by the sun's gravitational field.

Reference

1. https://en.wikipedia.org/wiki/Gravitational_lens

Application Library path: `Ray_Optics_Module/Verification_Examples/gravitational_lensing`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.

2 In the **Select Physics** tree, select **Optics>Ray Optics>Geometrical Optics (gop)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces>Ray Tracing**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
r0	7E5[km]	7E8 m	Radius of the sun
m0	2E30[kg]	2E30 kg	Solar mass

GEOMETRY 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.

2 In the **Settings** window for **Geometry**, locate the **Units** section.

3 From the **Length unit** list, choose **km**.

Sphere 1 (sph1)

1 In the **Geometry** toolbar, click **Sphere**.

2 In the **Settings** window for **Sphere**, locate the **Size** section.

3 In the **Radius** text field, type `r0`.

Block 1 (blk1)

1 In the **Geometry** toolbar, click **Block**.

2 In the **Settings** window for **Block**, locate the **Size and Shape** section.

3 In the **Width** text field, type `2E8`.

4 In the **Depth** text field, type `1E7`.

5 In the **Height** text field, type `1E7`.

6 Locate the **Position** section. In the **x** text field, type `0.5E8`.

7 From the **Base** list, choose **Center**.

8 Click **Build All Objects**.

9 Click the Go to Default View button in the **Graphics** toolbar.

10 Click the **Wireframe Rendering** button in the **Graphics** toolbar.

DEFINITIONS

Variables 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Definitions** and choose **Variables**.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 In the table, enter the following settings:

Name	Expression	Unit	Description
r	sqrt(x^2+y^2+z^2+eps)	m	Radial distance from center of the sun
n	1+2*G_const*m0/(c_const^2*r)		Refractive index

MATERIALS

Material 1 (mat1)

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, locate the **Material Contents** section.

3 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Refractive index, real part	n_iso ; nii = n_iso, nij = 0	n	1	Refractive index

GEOMETRICAL OPTICS (GOP)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometrical Optics (gop)**.

2 In the **Settings** window for **Geometrical Optics**, locate the **Ray Release and Propagation** section.

3 In the **Maximum number of secondary rays** text field, type 0.

Release from Grid 1

1 In the **Physics** toolbar, click **Global** and choose **Release from Grid**.

Release the rays so that they barely avoid contact with the sphere that represents the sun.

2 In the **Settings** window for **Release from Grid**, locate the **Initial Coordinates** section.

3 In the $q_{x,0}$ text field, type `-0.5E8`.

4 In the $q_{y,0}$ text field, type `-7.01E5 7.01E5`.

5 Locate the **Ray Direction Vector** section. Specify the $\mathbf{L}_0$ vector as

1	x
0	y
0	z

MESH 1

Use a **Finer** mesh to improve the mesh resolution in the region surrounding the sun.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2 In the **Settings** window for **Mesh**, locate the **Physics-Controlled Mesh** section.

3 From the **Element size** list, choose **Finer**.

4 Click **Build All**.

STUDY 1

Step 1: Ray Tracing

1 In the **Model Builder** window, under **Study 1** click **Step 1: Ray Tracing**.

2 In the **Settings** window for **Ray Tracing**, locate the **Study Settings** section.

3 From the **Time-step specification** list, choose **Specify maximum path length**.

4 From the **Length unit** list, choose **km**.

5 Click **Range**.

6 In the **Range** dialog box, choose **Number of values** from the **Entry method** list.

7 In the **Stop** text field, type `2E8`.

8 In the **Number of values** text field, type `100`.

9 Click **Replace**.

Solution 1 (sol1)

1 In the **Study** toolbar, click **Show Default Solver**.

2 In the **Model Builder** window, expand the **Solution 1 (sol1)** node, then click **Time-Dependent Solver 1**.

3 In the **Settings** window for **Time-Dependent Solver**, click to expand the **Time Stepping** section.

4 From the **Maximum step constraint** list, choose **Constant**.

5 In the **Maximum step** text field, type `1`.

6 In the **Study** toolbar, click **Compute**.

RESULTS

Ray Trajectories (gop)

In the **Model Builder** window, expand the **Ray Trajectories (gop)** node.

Color Expression 1

1 In the **Model Builder** window, expand the **Results>Ray Trajectories (gop)> Ray Trajectories 1** node, then click **Color Expression 1**.

2 In the **Settings** window for **Color Expression**, locate the **Expression** section.

3 In the **Expression** text field, type `asin(gop.niy)`.

4 From the **Unit** list, choose **arcsec**.

5 In the **Ray Trajectories (gop)** toolbar, click **Plot**.

Ray Evaluation 1

1 In the **Results** toolbar, click **More Derived Values** and choose **Other>Ray Evaluation**.

2 In the **Settings** window for **Ray Evaluation**, locate the **Data** section.

3 From the **Time selection** list, choose **Last**.

4 Locate the **Expression** section. In the **Expression** text field, type `asin(gop.niy)`.

5 From the **Unit** list, choose **arcsec**.

6 Click **Evaluate**.

Compare the resulting values to Einstein's prediction of 1.75 arcseconds.

Deviation from Initial Direction

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Deviation from Initial Direction` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Ray 1**.

4 Click to expand the **Title** section. From the **Title type** list, choose **None**.

5 Locate the **Plot Settings** section.

6 Select the **y-axis label** check box. In the associated text field, type `Deflection angle in arcseconds`.

Ray 1

1 In the **Deviation from Initial Direction** toolbar, click **More Plots** and choose **Ray**.

2 In the **Settings** window for **Ray**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `abs(asin(gop.niy))`.

4 From the **Unit** list, choose **arcsec**.

5 In the **Deviation from Initial Direction** toolbar, click **Plot**.

6 Locate the **Data Series Operation** section. From the **Operation** list, choose **Average**.

7 In the **Deviation from Initial Direction** toolbar, click **Plot**. Compare the resulting plot to Figure 1.