

# Eigenfrequency Analysis of a Free Cylinder

### Introduction

In the following example you compute the eigenfrequencies of a free circular pipe using three different approaches:

- An axisymmetric model using the Solid Mechanics interface.
- An axisymmetric model using the Shell interface.
- A sector of a 3D model using cyclic symmetry in the Solid Mechanics interface.

The example is taken from NAFEMS Free Vibration Benchmarks (Ref. 1). The eigenfrequencies are compared with the values given in the benchmark report.

As an extension, you will also compute eigenfrequencies with twisting deformation.

# Model Definition

The model is NAFEMS Test No 41, "Free Cylinder" described on page 41 in NAFEMS Free Vibration Benchmarks, vol. 3 (Ref. 1). The Benchmark tests the capability to handle rigid body modes and eigenfrequencies.

The cylinder is 10 m tall with an inner radius of 1.8 m and a thickness of 0.4 m.

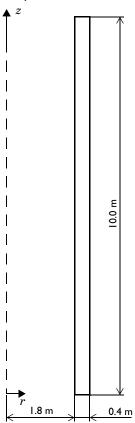


Figure 1: Model geometry in the rz-plane.

In the axisymmetric solid model, the geometry consists of this rectangle.

In the axisymmetric shell interface, the mesh is placed on the line representing the inner boundary of the cylinder, and an offset property is used in order to account for the fact that the shell model should represent the midsurface.

In the 3D solid model, the rectangle is swept around the axis of revolution, so that a  $15^{\circ}$  sector is formed. As long as  $360^{\circ}$  is as an exact multiple of the sector angle, any angle could have been used.

# MATERIAL

The material is isotropic linear elastic with  $E = 2.0 \cdot 10^{11}$  Pa, v = 0.3, and  $\rho = 8000$  kg/m<sup>3</sup>.

#### LOADS

In an eigenfrequency analysis loads are not needed.

#### CONSTRAINTS

In the axisymmetric models, no constraints are applied because the cylinder is free. In the 3D solid model, cyclic symmetry constraints are applied to the cuts in the azimuthal direction.

### Results

For structural mechanics, there are two possible interpretations of axisymmetry. The most common one is that there are no displacements out of the RZ-plane. Another interpretation, which also allows twisting motion, is that all derivatives of the displacements with respect to the azimuthal coordinate is zero. Such an extension is available when using the Solid Mechanics interface.

The original NAFEMS example does not contain out-of-plane displacements, in which case there is one rigid body mode. The rigid body mode with an eigenvalue close to zero is found in all physics interfaces. The corresponding shape is a pure axial rigid body translation without any radial displacement. The eigenfrequencies are in close agreement with the target values from the NAFEMS Free Vibration Benchmarks (Ref. 1); see below.

EIGENFREQUENCY	SOLID MECHANICS, AXISYMMETRY	SHELL, AXISYMMETRY	SOLID MECHANICS, 3D	TARGET (Ref. 1)
$f_2$	243.50	243.64	243.50	243.53
$f_3$	377.39	378.16	377.39	377.41
$f_4$	394.21	394.11	394.22	394.11
$f_5$	397.84	397.36	397.84	397.72
$f_6$	405.36	407.43	405.36	405.28

The analytical solution for twisting vibration of a free cylindrical pipe is

$$f_n = \frac{n}{2L} \sqrt{\frac{G}{\rho}} \tag{1}$$

Here, G is the shear modulus,

$$G = \frac{E}{2(1+v)} \tag{2}$$

In this case, there is one more rigid body mode: pure rotation around the axis of revolution. The computed nontrivial eigenfrequencies have a very good agreement with the analytical solution:

EIGENFREQUENCY	SOLID MECHANICS, AXISYMMETRY	SOLID MECHANICS, 3D	TARGET (ANALYTICAL)
$f_1$	155.04	155.04	155.04
$f_2$	310.09	310.09	310.09

Figure 2 shows the shape of the second eigenmode in the axisymmetric solid model. In Figure 3, the same plot is shown for the axisymmetric shell interface. In both cases, Revolution 2D datasets have been used for extending the axisymmetric model into 3D space.

Eigenfrequency=243.5 Hz Surface: Displacement magnitude (m)



Figure 2: The second nonrigid eigenmode, computed using an axisymmetric solid mechanics interface.

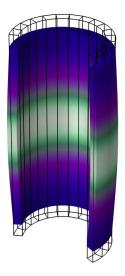




Figure 3: The second nonrigid eigenmode, computed using an axisymmetric shell interface. Due to the offset property, the shell is modeled at the true midsurface, even though the mesh is at the inner boundary of the cylinder.

In Figure 4 and Figure 5, two eigenmodes from the 3D solid model are shown. A Sector 3D dataset has been used for expanding the results from the original 15° sector. Eigenfrequency=243.5 Hz

Surface: Displacement magnitude (m)





 $\label{lem:condition} \emph{Figure 4: The second nonrigid eigenmode, computed using a 3D solid mechanics interface with cyclic symmetry boundary conditions.}$ 

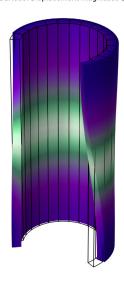




Figure 5: The first nonrigid eigenmode, computed using a 3D solid mechanics interface with cyclic symmetry boundary conditions.

# Notes About the COMSOL Implementation

In the 3D solid model, you could have used ordinary **Symmetry** boundary conditions instead of the **Periodic Condition**. The effect would have been that only the in-plane modes were computed.

In a real pipe, there are however also other eigenmodes, which are not axially symmetric. You can find such modes by using azimuthal mode numbers other than zero in the settings for the cyclic symmetry condition (3D) and Solid Mechanics interface settings (2D axisymmetry). Such modes can be visualized by setting the azimuthal mode number to the corresponding value in the Advanced section in the settings for the Revolution 2D and **Sector 3D** datasets.

# Reference

1. F. Abassian, D.J. Dawswell, and N.C. Knowles, Free Vibration Benchmarks, vol.3, NAFEMS, Glasgow, 1987.

# Application Library path: Structural Mechanics Module/

Verification\_Examples/free\_cylinder

# Modeling Instructions

From the File menu, choose New.

#### NEW

In the New window, click Model Wizard.

#### MODEL WIZARD

- I In the Model Wizard window, click 2D Axisymmetric.
- 2 In the Select Physics tree, select Structural Mechanics>Solid Mechanics (solid).
- 3 Click Add.
- 4 Click 🔵 Study.
- 5 In the Select Study tree, select General Studies>Eigenfrequency.
- 6 Click M Done.

#### **GLOBAL DEFINITIONS**

#### Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- **3** In the table, enter the following settings:

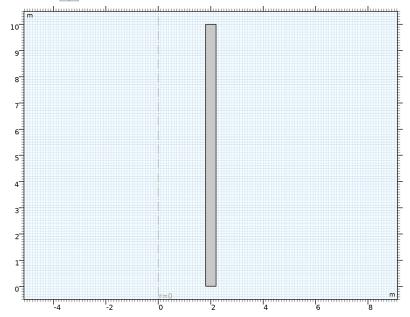
Name	Expression	Value	Description
height	10[m]	10 m	Height of cylinder
thic	0.4[m]	0.4 m	Thickness of cylinder
r_in	1.8[m]	1.8 m	Inner radius

#### **GEOMETRY I**

### Rectangle I (rI)

- I In the Geometry toolbar, click Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.

- 3 In the Width text field, type thic.
- 4 In the **Height** text field, type height.
- 5 Locate the **Position** section. In the **r** text field, type **r** in.
- 6 Click Build All Objects.
- 7 Click the **Zoom Extents** button in the **Graphics** toolbar.



# **GLOBAL DEFINITIONS**

In this example, the same material data will be referenced from several physics interfaces, so it is convenient to define a global material.

### Material I (mat I)

- I In the Model Builder window, under Global Definitions right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, click to expand the Material Properties section.
- 3 In the Material properties tree, select Basic Properties>Density.
- 4 Click + Add to Material.
- 5 In the Material properties tree, select Solid Mechanics>Linear Elastic Material> Young's Modulus and Poisson's Ratio.
- 6 Click + Add to Material.

7 Locate the Material Contents section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Density	rho	8000	kg/m³	Basic
Young's modulus	E	2e11	Pa	Young's modulus and Poisson's ratio
Poisson's ratio	nu	0.3	I	Young's modulus and Poisson's ratio

#### MATERIALS

Material Link I (matlnk I)

In the Model Builder window, under Component I (compl) right-click Materials and choose More Materials>Material Link.

#### MESH I

#### Mapped I

In the Mesh toolbar, click Mapped.

#### Distribution I

- I In the Model Builder window, right-click Mapped I and choose Distribution.
- 2 In the Settings window for Distribution, locate the Distribution section.
- 3 In the Number of elements text field, type 20.
- 4 Select Boundary 1 only.

#### Distribution 2

- I In the Model Builder window, right-click Mapped I and choose Distribution.
- 2 In the Settings window for Distribution, locate the Distribution section.
- 3 In the Number of elements text field, type 2.
- 4 Select Boundary 2 only.
- 5 Click **Build All**.

#### STUDY I, 2D AXISYMMETRIC SOLID

- I In the Model Builder window, click Study I.
- 2 In the Settings window for Study, type Study 1, 2D Axisymmetric Solid in the Label text field.
- 3 In the Home toolbar, click **Compute**.

#### RESULTS

Mode Shape (solid)

Visualize an eigenmode in 3D.

Mode Shape, 3D (solid)

- I In the Model Builder window, click Mode Shape, 3D (solid).
- 2 In the Settings window for 3D Plot Group, locate the Data section.
- 3 From the Eigenfrequency (Hz) list, choose 243.5.
- 4 Click the Show Grid button in the Graphics toolbar.
- 5 In the Mode Shape, 3D (solid) toolbar, click Plot.
- **6** Click the **Zoom Extents** button in the **Graphics** toolbar.

# COMPONENT I (COMPI)

Add a 2D axisymmetry Shell interface with the same data, and compute the eigenfrequencies.

#### ADD PHYSICS

- I In the Home toolbar, click Add Physics to open the Add Physics window.
- 2 Go to the Add Physics window.
- 3 In the tree, select Structural Mechanics>Shell (shell).
- 4 Click Add to Component I in the window toolbar.
- 5 In the Home toolbar, click and Physics to close the Add Physics window.

# SHELL (SHELL)

Select Boundary 1 only.

Thickness and Offset I

Since the inner boundary of the cylinder is used as geometry for the shell interface, and shell normal is pointing inward, set top surface of shell on the boundary.

- I In the Model Builder window, under Component I (compl)>Shell (shell) click Thickness and Offset I.
- 2 In the Settings window for Thickness and Offset, locate the Thickness and Offset section.
- **3** In the  $d_0$  text field, type thic.
- 4 From the Position list, choose Top surface on boundary.

#### MATERIALS

Material Link 2 (matlnk2)

- I In the Model Builder window, under Component I (compl) right-click Materials and choose More Materials>Material Link.
- 2 In the Settings window for Material Link, locate the Geometric Entity Selection section.
- 3 From the Geometric entity level list, choose Boundary.
- 4 Select Boundary 1 only.

#### ADD STUDY

- I In the Home toolbar, click Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select General Studies> Eigenfrequency.
- 4 Find the Physics interfaces in study subsection. In the table, clear the Solve check box for Solid Mechanics (solid).
- 5 Click Add Study in the window toolbar.
- 6 In the Home toolbar, click Add Study to close the Add Study window.

# STUDY 2, 2D AXISYMMETRIC SHELL

- I In the Model Builder window, click Study 2.
- 2 In the Settings window for Study, type Study 2, 2D Axisymmetric Shell in the Label text field.
- 3 In the Home toolbar, click **Compute**.

#### RESULTS

Mode Shape, 3D (shell)

- I In the Model Builder window, under Results click Mode Shape, 3D (shell).
- 2 In the Settings window for 3D Plot Group, locate the Data section.
- 3 From the Eigenfrequency (Hz) list, choose 243.64.
- 4 Click the Show Grid button in the Graphics toolbar.
- 5 In the Mode Shape, 3D (shell) toolbar, click Plot.

#### ROOT

Now, add a 3D solid sector with cyclic symmetry boundary conditions and compute the eigenfrequencies.

#### ADD COMPONENT

In the Model Builder window, right-click the root node and choose Add Component>3D.

#### **GEOMETRY 2**

Work Plane I (wbl)

- I In the Geometry toolbar, click Work Plane.
- 2 In the Settings window for Work Plane, locate the Plane Definition section.
- 3 From the Plane list, choose xz-plane.

#### **GEOMETRY I**

Rectangle I (rI)

In the Model Builder window, under Component I (compl)>Geometry I right-click Rectangle I (rI) and choose Copy.

#### **GEOMETRY 2**

Work Plane I (wb I)>Plane Geometry

- I In the Model Builder window, under Component 2 (comp2)>Geometry 2> Work Plane I (wpI) click Plane Geometry.
- 2 Right-click Component 2 (comp2)>Geometry 2>Work Plane I (wpI)>Plane Geometry and choose Paste Rectangle.

Revolve I (rev1)

- I In the Model Builder window, under Component 2 (comp2)>Geometry 2 right-click Work Plane I (wpl) and choose Revolve.
- 2 In the Settings window for Revolve, locate the Revolution Angles section.
- 3 Click the Angles button.
- 4 In the End angle text field, type 15.
- 5 Click Build All Objects.

#### ADD PHYSICS

- I In the Home toolbar, click and Physics to open the Add Physics window.
- 2 Go to the Add Physics window.

- 3 In the tree, select Structural Mechanics>Solid Mechanics (solid).
- 4 Click Add to Component 2 in the window toolbar.
- 5 In the Home toolbar, click and Physics to close the Add Physics window.

#### SOLID MECHANICS 2 (SOLID2)

Periodic Condition I

- I Right-click Component 2 (comp2)>Solid Mechanics 2 (solid2) and choose Connections> **Periodic Condition.**
- 2 Select Boundaries 2 and 5 only.
- 3 In the Settings window for Periodic Condition, locate the Periodicity Settings section.
- **4** From the **Type of periodicity** list, choose **Cyclic symmetry**.

#### MESH 2

# Mapped I

- I In the Mesh toolbar, click A More Generators and choose Mapped.
- 2 Select Boundary 3 only.

#### Distribution 1

- I Right-click Mapped I and choose Distribution.
- 2 In the Settings window for Distribution, locate the Distribution section.
- **3** In the **Number of elements** text field, type 2.
- 4 Select Edges 2 and 7 only.

#### Mapped I

In the Model Builder window, right-click Mapped I and choose Build Selected.

In the Mesh toolbar, click A Swept.

#### Distribution I

- I Right-click Swept I and choose Distribution.
- 2 In the Settings window for Distribution, locate the Distribution section.
- 3 In the Number of elements text field, type 20.
- 4 Click III Build All.

#### MATERIALS

Material Link 3 (matlnk3)

In the Model Builder window, under Component 2 (comp2) right-click Materials and choose More Materials>Material Link.

#### ADD STUDY

- I In the Home toolbar, click Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select General Studies> Eigenfrequency.
- 4 Find the Physics interfaces in study subsection. In the table, clear the Solve check boxes for Solid Mechanics (solid) and Shell (shell).
- 5 Click Add Study in the window toolbar.
- 6 In the Home toolbar, click Add Study to close the Add Study window.

#### STUDY 3, 3D SOLID SECTOR

- I In the Model Builder window, click Study 3.
- 2 In the Settings window for Study, type Study 3, 3D Solid Sector in the Label text field.

### Step 1: Eigenfrequency

- I In the Model Builder window, under Study 3, 3D Solid Sector click Step 1: Eigenfrequency.
- 2 In the Settings window for Eigenfrequency, locate the Study Settings section.
- 3 Select the **Desired number of eigenfrequencies** check box. In the associated text field, type 10.
- 4 In the Model Builder window, collapse the Study 3, 3D Solid Sector node.
- 5 In the Home toolbar, click **Compute**.

#### RESULTS

Mode Shape (solid2)

- I In the Settings window for 3D Plot Group, locate the Data section.
- 2 From the Eigenfrequency (Hz) list, choose 243.5.
- 3 In the Mode Shape (solid2) toolbar, click Plot.

#### Sector 3D I

I In the Results toolbar, click More Datasets and choose Sector 3D.

- 2 In the Settings window for Sector 3D, locate the Symmetry section.
- 3 In the Number of sectors text field, type 360/15.
- 4 From the Sectors to include list, choose Manual.
- 5 In the Start sector text field, type 18.
- 6 In the Number of sectors to include text field, type 15.

#### Mode Shape (solid2)

- I In the Model Builder window, under Results click Mode Shape (solid2).
- 2 In the Settings window for 3D Plot Group, locate the Data section.
- 3 From the Dataset list, choose Sector 3D 1.
- **4** Click the **Zoom Extents** button in the **Graphics** toolbar.
- 5 In the Mode Shape (solid2) toolbar, click Plot.
- **6** Click the Show Grid button in the Graphics toolbar. Also twisting modes can be displayed.
- 7 From the Eigenfrequency (Hz) list, choose 155.04.
- 8 In the Mode Shape (solid2) toolbar, click Plot.

#### COMPONENT 2 (COMP2)

Add a 3D **Shell** interface with cyclic symmetry and same data, and compute the eigenfrequencies.

I In the Model Builder window, click Component 2 (comp2).

#### ADD PHYSICS

- I In the Home toolbar, click Add Physics to open the Add Physics window.
- 2 Go to the Add Physics window.
- 3 In the tree, select Structural Mechanics>Shell (shell).
- 4 Click Add to Component 2 in the window toolbar.
- 5 In the Home toolbar, click 🎇 Add Physics to close the Add Physics window.

#### SHELL 2 (SHELL2)

- I In the Settings window for Shell, locate the Boundary Selection section.
- 2 Click Clear Selection.
- **3** Select Boundary 1 only.

Linear Elastic Material I

In the Model Builder window, collapse the Component 2 (comp2)>Shell 2 (shell2)> Linear Elastic Material I node.

Thickness and Offset I

- I In the Model Builder window, click Thickness and Offset I.
- 2 In the Settings window for Thickness and Offset, locate the Thickness and Offset section.
- **3** In the  $d_0$  text field, type thic.
- 4 From the Position list, choose Top surface on boundary.

To enforce cyclic symmetry boundary conditions in shell interface a cylindrical coordinate system is needed to get proper orientation of the source.

### **DEFINITIONS (COMP2)**

Cylindrical System 3 (sys3)

In the Definitions toolbar, click  $\bigvee_{x}^{z}$  Coordinate Systems and choose Cylindrical System.

### SHELL 2 (SHELL2)

Periodic Condition I

- I In the Physics toolbar, click **Edges** and choose **Periodic Condition**.
- **2** Select Edges 1 and 6 only.
- 3 In the Settings window for Periodic Condition, locate the Periodicity Settings section.
- **4** From the Type of periodicity list, choose Cyclic symmetry.
- **5** In the  $\theta_S$  text field, type 15[deg].
- 6 Click to expand the Orientation of Source section. From the Transform to intermediate map list, choose Cylindrical System 3 (sys3).

# MATERIALS

Material Link 4 (matlnk4)

- I In the Model Builder window, under Component 2 (comp2) right-click Materials and choose More Materials>Material Link.
- 2 In the Settings window for Material Link, locate the Geometric Entity Selection section.
- 3 From the Geometric entity level list, choose Boundary.
- 4 Select Boundary 1 only.

#### ADD STUDY

- I In the Home toolbar, click Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select General Studies> Eigenfrequency.
- 4 Find the Physics interfaces in study subsection. In the table, clear the Solve check boxes for Solid Mechanics (solid), Shell (shell), and Solid Mechanics 2 (solid2).
- **5** Click **Add Study** in the window toolbar.
- 6 In the Model Builder window, click the root node.
- 7 In the Home toolbar, click Add Study to close the Add Study window.

#### STUDY 4, 3D SHELL SECTOR

In the Settings window for Study, type Study 4, 3D Shell Sector in the Label text field.

# Step 1: Eigenfrequency

- I In the Model Builder window, under Study 4, 3D Shell Sector click Step 1: Eigenfrequency.
- 2 In the Settings window for Eigenfrequency, locate the Study Settings section.
- **3** Select the **Desired number of eigenfrequencies** check box. In the associated text field, type 10.
- 4 In the Model Builder window, collapse the Study 4, 3D Shell Sector node.
- 5 In the Home toolbar, click **Compute**.

# RESULTS

Mode Shape (shell2)

- I In the Settings window for 3D Plot Group, locate the Data section.
- 2 From the Eigenfrequency (Hz) list, choose 243.64.
- 3 In the Mode Shape (shell2) toolbar, click Plot.

#### Sector 3D 2

- I In the Results toolbar, click More Datasets and choose Sector 3D.
- 2 In the Settings window for Sector 3D, locate the Data section.
- 3 From the Dataset list, choose Shell 2.
- 4 Locate the Symmetry section. In the Number of sectors text field, type 360/15.
- 5 From the Sectors to include list, choose Manual.
- 6 In the Start sector text field, type 18.

7 In the Number of sectors to include text field, type 15.

Mode Shape (shell2)

- I In the Model Builder window, under Results click Mode Shape (shell2).
- 2 In the Settings window for 3D Plot Group, locate the Data section.
- 3 From the Dataset list, choose Sector 3D 2.
- 4 Click the **Zoom Extents** button in the **Graphics** toolbar.
- 6 Click the Show Grid button in the Graphics toolbar. Also twisting modes can be displayed.
- 7 From the Eigenfrequency (Hz) list, choose 155.81.

#### SOLID MECHANICS (SOLID)

The twisting modes can also be computed using the axisymmetric Solid Mechanics and Shell interfaces. To do that, use circumferential mode extension.

- I In the Model Builder window, under Component I (compl) click Solid Mechanics (solid).
- 2 In the Settings window for Solid Mechanics, locate the Axial Symmetry Approximation section.
- **3** Find the **Time-harmonic** subsection. Select the **Circumferential mode extension** check box.

#### STUDY I, 2D AXISYMMETRIC SOLID

Step 1: Eigenfrequency

- I In the Model Builder window, under Study I, 2D Axisymmetric Solid click Step 1: Eigenfrequency.
- 2 In the Settings window for Eigenfrequency, locate the Physics and Variables Selection section.
- 3 In the table, clear the Solve for check boxes for Shell (shell), Solid Mechanics 2 (solid2), and Shell 2 (shell2).
- 4 Locate the Study Settings section.
- 5 Select the **Desired number of eigenfrequencies** check box. In the associated text field, type 10.
- **6** In the **Home** toolbar, click **Compute**.

#### RESULTS

Mode Shape, 3D (solid)

Display the first twist mode.

- I In the Model Builder window, under Results click Mode Shape, 3D (solid).
- 2 In the Settings window for 3D Plot Group, locate the Data section.
- 3 From the Eigenfrequency (Hz) list, choose 155.04.
- 4 In the Mode Shape, 3D (solid) toolbar, click Plot.
- 5 From the Eigenfrequency (Hz) list, choose 243.5.

### SHELL (SHELL)

- I In the Model Builder window, under Component I (compl) click Shell (shell).
- 2 In the Settings window for Shell, locate the Axial Symmetry Approximation section.
- **3** Find the **Time-harmonic** subsection. Select the **Circumferential mode extension** check box.

### STUDY 2, 2D AXISYMMETRIC SHELL

Steb 1: Eigenfrequency

- I In the Model Builder window, under Study 2, 2D Axisymmetric Shell click Step 1: Eigenfrequency.
- 2 In the Settings window for Eigenfrequency, locate the Physics and Variables Selection section.
- 3 In the table, clear the Solve for check boxes for Solid Mechanics (solid), Solid Mechanics 2 (solid2), and Shell 2 (shell2).
- 4 Locate the Study Settings section.
- 5 Select the **Desired number of eigenfrequencies** check box. In the associated text field, type 10.
- **6** In the **Home** toolbar, click **Compute**.

#### RESULTS

Mode Shape, 3D (shell)

Display the first twist mode.

- I In the Model Builder window, under Results click Mode Shape, 3D (shell).
- 2 In the Settings window for 3D Plot Group, locate the Data section.

- 3 From the Eigenfrequency (Hz) list, choose 155.81.
- 4 In the Mode Shape, 3D (shell) toolbar, click Plot.
- 5 From the Eigenfrequency (Hz) list, choose 243.64.
- 6 In the Mode Shape, 3D (shell) toolbar, click  **Plot**.

# STUDY 3, 3D SOLID SECTOR

# Step 1: Eigenfrequency

- I In the Model Builder window, expand the Study 3, 3D Solid Sector node, then click Step 1: Eigenfrequency.
- 2 In the Settings window for Eigenfrequency, locate the Physics and Variables Selection section.
- 3 In the table, clear the Solve for check box for Shell 2 (shell2).