Created in COMSOL Multiphysics 6.2

Pull-In Voltage for a Biased Resonator — 3D

Introduction

Silicon micromechanical resonators have long been used for designing sensors and are now becoming increasingly important as oscillators in the consumer electronics market. In this sequence of models, a surface micromachined MEMS resonator, designed as part of a micromechanical filter, is analyzed in detail. The resonator is based on that developed in Ref. 1.

This model performs a pull-in analysis of the structure, to predict the point at which the biased system becomes unstable. The analysis begins from the stationary analysis performed in the accompanying model Stationary Analysis of a Biased Resonator — 3D; please review this model first.

Model Definition

The geometry, fabrication, and operation of the device are discussed for the Stationary Analysis of a Biased Resonator — 3D model.

This model computes the pull-in voltage for the resonator by solving an inverse problem. The z-coordinate of the resonator midpoint is computed using an integration operator (`intop1`). The inverse problem that COMSOL solves computes the DC voltage that must be applied to the beam in order to move the midpoint to a set z-coordinate, `zset`. This is achieved by adding a global equation for the DC voltage, `VdcSP`, applied to the resonator. The equation `intop1(z)-zset=0` is solved to determine the value of `VdcSP`. This means that `VdcSP` is adjusted until the midpoint of the resonator has a z-coordinate given by the set value, `zset`. Essentially COMSOL is being asked to find the voltage that allows the beam to exist in equilibrium (stable or unstable) at a given displacement. Solving the problem in this way avoids complications with trying to solve a problem with no solution (which is what happens if the voltage is continuously ramped up eventually exceeding the pull-in voltage). The result of the analysis is a displacement versus voltage plot, with a minimum at the pull-in voltage. Note that for a linear spring, the pull-in displacement corresponds to 1/3 of the gap distance. Although the inclusion of geometric nonlinearities in the solid mechanics solver means that the pull-in displacement changes slightly from this value, it is usually most efficient to search around this point for the pull-in voltage.

Results and Discussion

Figure 1 shows the voltage-displacement curve for the resonator at equilibrium. The y-coordinate at which the pull in occurs corresponds to a displacements around 1/3 of the gap size. The pull-in voltage is 59.1 V.

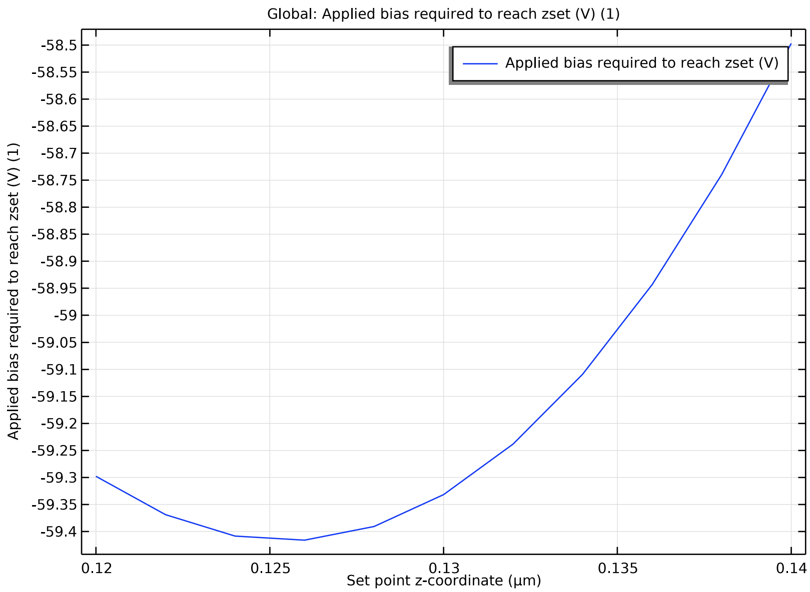

Figure 1: Voltage required to achieve a set displacement versus the target displacement. The pull-in voltage is the minimum of the plot: 59.4 V.

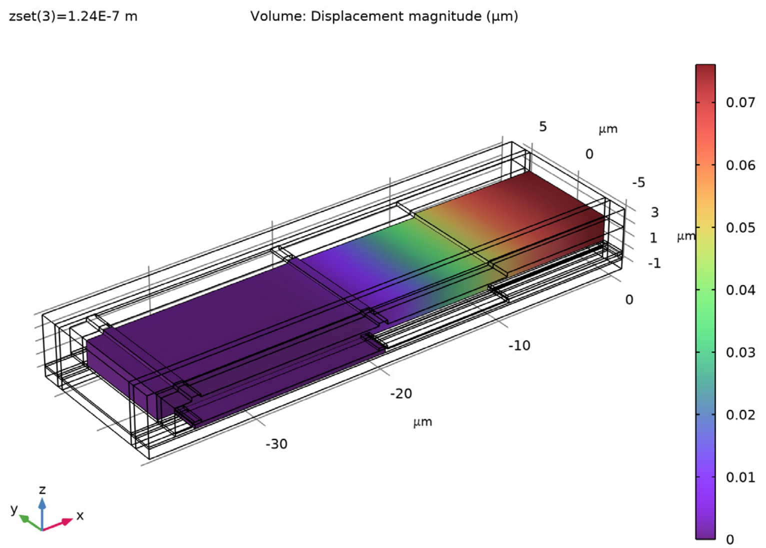

Figure 2: z-displacement of the resonator at pull-in. The displacement at pull-in is 75 nm. For a linear spring the displacement at pull in would be 66 nm.

Figure 2 shows the z-displacement of the resonator at the pull-in voltage. The maximum displacement at pull-in is 75 nm. This is comparable to the linear spring value of 66 nm.

Notes About the COMSOL Implementation

To compute the voltage required to generate the desired displacement of the beam, use a global equation. A common use of global equations is for computing the value of a dependent variable based on an ordinary differential equation in the dependent variable itself. However, it is also possible to couple a global equation with the other PDEs in the model as a powerful tool to solve certain kinds of inverse problems. This model uses a global equation to compute the potential applied to the drive electrode. The equation takes the form

$$z_0 = z_{\mathrm{set}}$$

where z_0 is the z-coordinate of the midpoint of the beam's underside and z_{set} is the desired z-coordinate. COMSOL Multiphysics computes the voltage to satisfy the constraint implied by the above equation. Note that the large difference in scale between the set-point displacement (10^{-7} m) and the applied voltages (10^{2} V) means that care must be taken with the dependent variable scaling in the solver settings.

Reference

1. F.D. Bannon III, J.R. Clark and C.T.-C. Nguyen, “High-Q HF Microelectromechanical Filters,” *IEEE Journal of Solid State Circuits*, vol. 35, no. 4, pp. 512–526, 2000.

Application Library path: `MEMS_Module/Actuators/biased_resonator_3d_pull_in`

Modeling Instructions

Open the existing stationary study (filename: biased_resonator_3d_basic.mph).

APPLICATION LIBRARIES

1 From the **File** menu, choose **Application Libraries**.

2 In the **Application Libraries** window, select **MEMS Module>Actuators>biased_resonator_3d_basic** in the tree.

3 Click **Open**.

Add a parameter to set the z-coordinate of the midpoint of the resonator.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
`zset`	`100[nm]`	1E-7 m	`Set point z-coordinate`

Add a nonlocal integration coupling to compute the actual displacement.

DEFINITIONS

Integration 1 (intop1)

1 In the **Model Builder** window, expand the **Component 1 (comp1)** node.

2 Right-click **Component 1 (comp1)>Definitions** and choose **Nonlocal Couplings>Integration**.

3 In the **Settings** window for **Integration**, locate the **Source Selection** section.

4 From the **Geometric entity level** list, choose **Point**.

5 Select Point 253 only.

Change the drive potential to the value `VdcSP` - which will be solved for in a global equation.

ELECTROSTATICS (ES)

Terminal 2

1 In the **Model Builder** window, expand the **Component 1 (comp1)>Electrostatics (es)** node, then click **Terminal 2**.

2 In the **Settings** window for **Terminal**, locate the **Terminal** section.

3 In the V_0 text field, type `VdcSP`.

Add a global equation to compute the voltage for a given displacement, `VdcSP`.

4 Click the **Show More Options** button in the **Model Builder** toolbar.

5 In the **Show More Options** dialog box, in the tree, select the check box for the node **Physics>Equation-Based Contributions**.

6 Click **OK**.

Global Equations 1 (ODE1)

1 In the **Physics** toolbar, click **Global** and choose **Global Equations**.

2 In the **Settings** window for **Global Equations**, locate the **Global Equations** section.

3 In the table, enter the following settings:

Name	f(u,ut,utt,t) (1)	Initial value (u_0) (1)	Initial value (u_t0) (1/s)	Description
VdcSP	intop1(z)-zset	0	0	Applied bias required to reach zset (V)

Set up a parametric sweep over `zset`.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.
2 Go to the **Add Study** window.
3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies>Stationary**.
4 Click **Add Study** in the window toolbar.
5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 2

Step 1: Stationary

1 In the **Settings** window for **Stationary**, click to expand the **Study Extensions** section.
2 Select the **Auxiliary sweep** check box.
3 Click **Add**.
4 From the list in the **Parameter name** column, choose **zset (Set point z-coordinate)**.
5 Click **Range**.
6 In the **Range** dialog box, type `120e-9` in the **Start** text field.
7 In the **Step** text field, type `2e-9`.
8 In the **Stop** text field, type `140e-9`.
9 Click **Replace**.

The dependent variables require scaling correctly in order to assist the solver.

Solution 2 (sol2)

1 In the **Study** toolbar, click **Show Default Solver**.
2 In the **Model Builder** window, expand the **Solution 2 (sol2)** node, then click **Dependent Variables 1**.
3 In the **Settings** window for **Dependent Variables**, locate the **General** section.
4 From the **Defined by study step** list, choose **User defined**.
5 In the **Model Builder** window, expand the **Study 2>Solver Configurations>Solution 2 (sol2)>Dependent Variables 1** node, then click **Spatial mesh displacement (comp1.spatial.disp)**.
6 In the **Settings** window for **Field**, locate the **Scaling** section.
7 From the **Method** list, choose **Automatic**.
8 In the **Model Builder** window, under **Study 2>Solver Configurations>Solution 2 (sol2)>Dependent Variables 1** click **Displacement field (comp1.u)**.

9 In the **Settings** window for **Field**, locate the **Scaling** section.

10 From the **Method** list, choose **Automatic**.

11 In the **Model Builder** window, under **Study 2>Solver Configurations>Solution 2 (sol2)> Dependent Variables 1** click **Electric potential (comp1.V)**.

12 In the **Settings** window for **Field**, locate the **Scaling** section.

13 From the **Method** list, choose **Manual**.

14 In the **Scale** text field, type `100`.

15 In the **Model Builder** window, under **Study 2>Solver Configurations>Solution 2 (sol2)> Dependent Variables 1** click **Global Equations 1 (comp1.ODE1)**.

16 In the **Settings** window for **State**, locate the **Scaling** section.

17 From the **Method** list, choose **Manual**.

18 In the **Scale** text field, type `100`.

The problem is highly nonlinear, so the solver settings need to be adjusted accordingly.

19 In the **Model Builder** window, expand the **Study 2>Solver Configurations> Solution 2 (sol2)>Stationary Solver 1** node.

20 Right-click **Study 2>Solver Configurations>Solution 2 (sol2)>Stationary Solver 1** and choose **Fully Coupled**.

21 In the **Settings** window for **Fully Coupled**, locate the **General** section.

22 From the **Linear solver** list, choose **Direct**.

23 Click to expand the **Method and Termination** section. From the **Nonlinear method** list, choose **Automatic highly nonlinear (Newton)**.

24 In the **Model Builder** window, right-click **Study 2** and choose **Rename**.

25 In the **Rename Study** dialog box, type `Pull In` in the **New label** text field.

26 Click **OK**.

27 In the **Study** toolbar, click **Compute**.

RESULTS

Displacement (solid) 1

Determine the pull-in voltage by plotting `VdcSP` versus `zset`.

1D Plot Group 9

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, locate the **Data** section.

3 From the **Dataset** list, choose **Pull In/Solution 2 (sol2)**.

Global 1

1 Right-click **1D Plot Group 9** and choose **Global**.

2 In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)>Electrostatics> VdcSP - Applied bias required to reach zset (V) - 1**.

3 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

4 In the **Expression** text field, type `zset`.

Pull-In Plot

1 In the **Model Builder** window, under **Results** click **1D Plot Group 9**.

2 In the **Settings** window for **1D Plot Group**, type `Pull-In Plot` in the **Label** text field.

3 In the **Pull-In Plot** toolbar, click **Plot**.

4 Click the **Zoom Extents** button in the **Graphics** toolbar.

Compare the resulting plot with Figure 1. The pull in voltage is the minimum of the curve: 59.4 V at zset = 126 nm.

Now look at the displacement at pull in. The default plot group can be used.

Displacement (solid) 1

1 In the **Model Builder** window, click **Displacement (solid) 1**.

2 In the **Settings** window for **3D Plot Group**, locate the **Data** section.

3 From the **Parameter value (zset (m))** list, choose **1.24E-7**.

4 In the **Displacement (solid) 1** toolbar, click **Plot**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar.

Compare the resulting plot with Figure 2. The displacement at pull-in is 75 nm.