Created in COMSOL Multiphysics 6.2

Electrical Signals in a Heart

This example was kindly provided by Prof. Simonetta Filippi and Dr Christian Cherubini from Università Campus Biomedico di Roma, Italy.

Introduction

Modeling the electrical activity in cardiac tissue is an important step in understanding the patterns of contractions and dilations in the heart. The heart produces rhythmic electrical pulses, initiated from a point known as the sinus node. The electrical pulses, in turn, trigger the mechanical contractions of the muscle. In a healthy heart these electrical pulses are damped out, but a number of heart conditions involve an elevated risk of re-entry of the signals. This means that the normal steady pulse is disturbed, a severe and acute condition often referred to as arrhythmia.

In this example, different aspects of electrical signal propagation in cardiac tissue are studied using the *FitzHugh–Nagumo* equations and the *Ginzburg–Landau* equations, both of which are solved on the same geometry. Interesting patterns emerging from these types of models are, for example, spiral waves, which, in the context of cardiac electrical signals, can produce effects similar to those observed in cardiac arrhythmia.

EXCITABLE MEDIA AND THE FITZHUGH–NAGUMO EQUATIONS

It has been shown that many important characteristics of electrical signal propagation in cardiac tissue can be reproduced by a class of equations which describe *excitable media*, that is, materials consisting of elementary segments or cells with the following basic characteristics:

- Well-defined rest state
- Threshold for excitation
- A diffusive-type coupling to its nearest neighbors

Excitable media is a rather general concept, which is useful for modeling of a number of different phenomena, including nerve pulses, the spreading of forest fires, and certain types of chemical reactions, in addition to the electrical signals in cardiac tissue. One of the most important qualitative characteristics displayed by excitable media, and equally a common denominator between the diversity of phenomena mentioned above, is the almost immediate damping out of signals below a certain threshold. On the other hand, signals exceeding this threshold propagate without damping.

The heart works by passing ionic current inside the muscle, thus triggering the rhythmic contractions that pump blood in and out. The ions move through small pores or *gates* in the cellular membrane, which can be either open (excitation state) or closed (rest state).

In nerve cells and cardiac cells the three abstract characteristics of excitable media are manifested as:

- Rest cell membrane potential
- Threshold for opening the ionic gates in the cellular membrane
- The diffusive spreading of the electrical signals

The state of the membrane gates is random on a microscopic scale, but the probability of a given state can be modeled as a continuous function of the voltage, thus allowing an averaged macroscopic continuum description of the current flow.

The *FitzHugh–Nagumo* equations for excitable media describe the simplest physiological model with two variables, an *activator* and an *inhibitor*. In these heart models the activator variable corresponds to the electric potential, and the inhibitor is a variable that describes the voltage-dependent probability of the pores in the membrane being open and ready to transmit ionic current.

CHAOTIC DYNAMICS AND THE GINZBURG–LANDAU EQUATIONS

The *Ginzburg–Landau* equations provide a relatively simple way of modeling some aspects of the transition, displayed by many dynamical systems under the influence of strong external stimulus, from periodic oscillatory behavior into a chaotic state with gradually increasing amplitude of oscillations and decreasing periodicity.

Although their first use was to describe the theory of superconductivity, the Ginzburg–Landau equations are also generic in their nature (as are the FitzHugh–Nagumo equations), and examples of dynamical systems that you can model successfully using these equations are:

- The formation of vortices behind a slender obstacle in transversal fluid flow
- Oscillating chemical reactions of the Belousov-Zhabotinsky type

In this example, the Ginzburg–Landau equations simulate the dynamics of the spiral waves in excitable media.

Model Definition

The geometry here is a simplified 3D model of a heart with two chambers, represented with semispherical cavities[1].

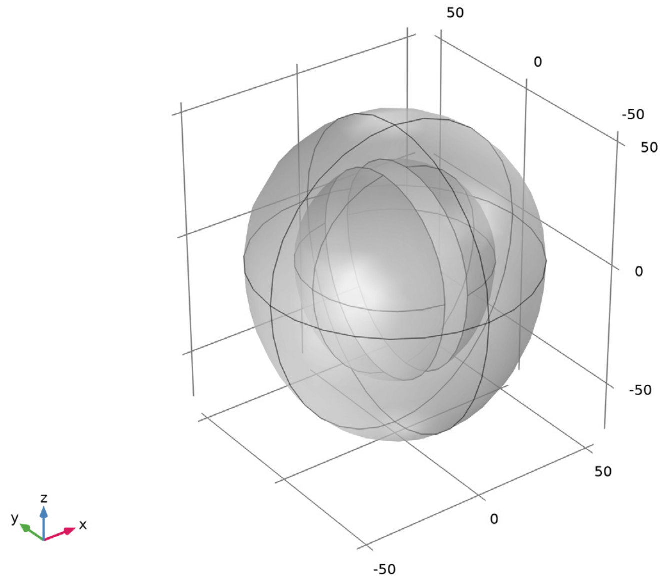


Figure 1: Model geometry.

THE FITZHUGH–NAGUMO EQUATIONS

The equations are the following:

$$\frac{\partial u_1}{\partial t} = \Delta u + (\alpha - u_1)(u_1 - 1)u_1 + (-u_2)$$

$$\frac{\partial u_2}{\partial t} = \varepsilon(\beta u_1 - \gamma u_2 - \delta)$$

Here u_1 is an action potential (the activator variable), and u_2 is a gate variable (the inhibitor variable). The parameter α represents the threshold for excitation, ε represents the excitability, and β, γ, and δ are parameters that affect the rest state and dynamics of the system.

1. Note that it is possible to import more realistic human/animal heart geometries, with anisotropies and inhomogeneities as well as proper dimensions, into COMSOL Multiphysics using the CAD import capabilities.

The boundary conditions for u_1 are insulating, using the assumption that no current is flowing into or out of the heart. The initial condition defines an initial potential distribution u_1 where one quadrant of the heart is at a constant, elevated potential V_0, while the rest remains at zero. The adjacent quadrant has instead an elevated value v_0 for the inhibitor u_2. It is convenient to implement this initial distribution using the following logical expressions, where TRUE evaluates to 1 and FALSE to 0:

$$u_1(0, x, y, z) = V_0((x+d) > 0) \cdot ((z+d) > 0)$$
$$u_2(0, x, y, z) = v_2((-x+d) > 0) \cdot ((z+d) > 0)$$

Here d is equal to 10^{-5}, and it is included in the expressions to shift the elevated potential slightly off the main axes.

THE GINZBURG–LANDAU EQUATIONS

The Ginzburg–Landau equations are:

$$\frac{\partial v_1}{\partial t} - \Delta(v_1 - c_1 v_2) = v_1 - (v_1 - c_3 v_2)(v_1^2 + v_2^2)$$
$$\frac{\partial v_2}{\partial t} - \Delta(c_1 v_1 + v_2) = v_2 - (c_3 v_1 + v_2)(v_1^2 + v_2^2)$$

The two variables v_1 and v_2 are the activator and inhibitor, respectively. The constants c_1 and c_3 are parameters reflecting the properties of the material. These constants also determine the existence and nature of the stable solutions.

As in the previous model, the boundary conditions are kept insulating. The initial condition, which gives a smooth transition step near $z = 0$, are the following:

$$v_1(0, x, y, z) = \tanh(z)$$
$$v_2(0, x, y, z) = -\tanh(z)$$

Notes About the COMSOL Implementation

The simplified geometry is quite straightforward to create using the drawing tools in COMSOL Multiphysics. The FitzHugh–Nagumo and Ginzburg–Landau equations are also readily entered in one of the PDE interfaces[2].

2. With this stage completed, it would also be straightforward to replace the rather simple equations with a physiologically more realistic mathematical model.

It is important to note that these equations are strongly nonlinear. It is therefore necessary (especially in full 3D models like these) to use a much finer mesh or use higher element order than in this example to get results with some degree of reliability for the time intervals of interest. This is particularly important in solving the Ginzburg–Landau equations, which describe inherently chaotic phenomena. They are highly sensitive to perturbations in the initial value and similarly to numerical errors during the course of the time-dependent solution. We recommend the use of the fourth-order Hermite element for the Ginzburg–Landau equation.

For the reasons above, the results presented here are only intended as a first rough estimate of the qualitative behavior that you can expect the system to show under a given stimulus. Consequently, higher-order elements, finer meshing, and smaller relative and absolute time dependent tolerances clearly give quantitatively more correct simulation results. The implementation of these improvements leads to longer computational time than that for the rough model described here which takes a few minutes to compute on a standard PC. When attempting these types of large models we strongly recommend the use of 64-bit platforms.

To avoid warnings about inconsistent units, turn off unit support.

Results and Discussion

THE FITZHUGH–NAGUMO EQUATIONS

The plots in Figure 2 below show the action potential u_1. To visualize the solution on the inside, a quarter of the outside shell of the heart and one of the chamber surfaces are suppressed in the plot.

The parameters used in the model along with the initial pulse lead to a reentrant wave which travels around the tissue without damping in a characteristic spiral pattern.

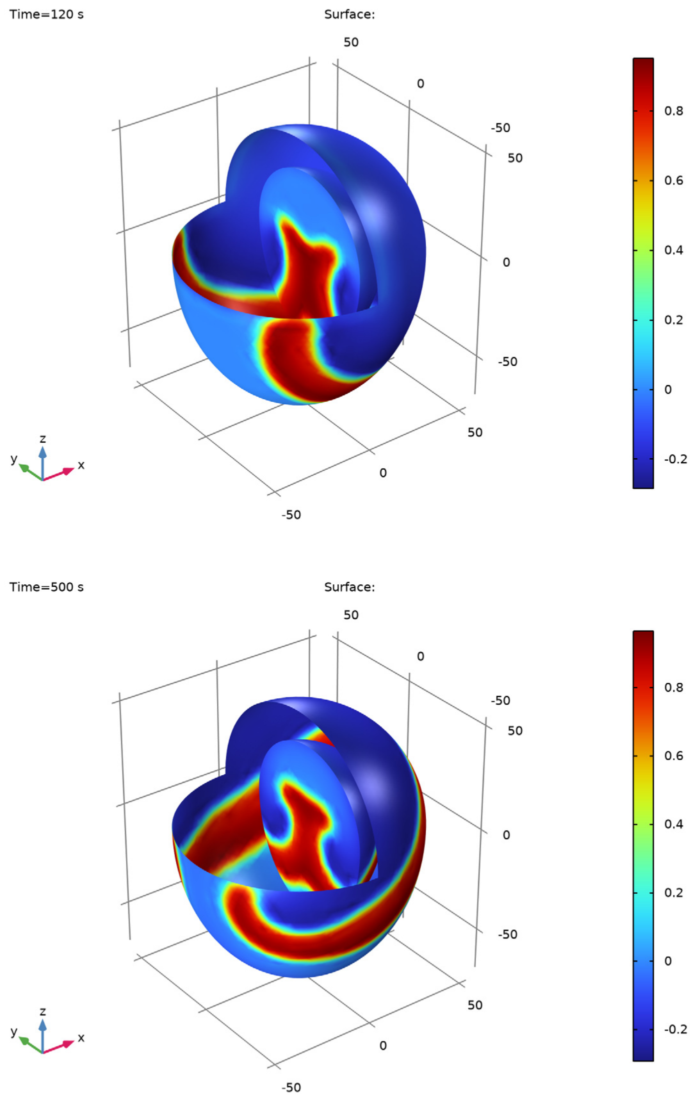
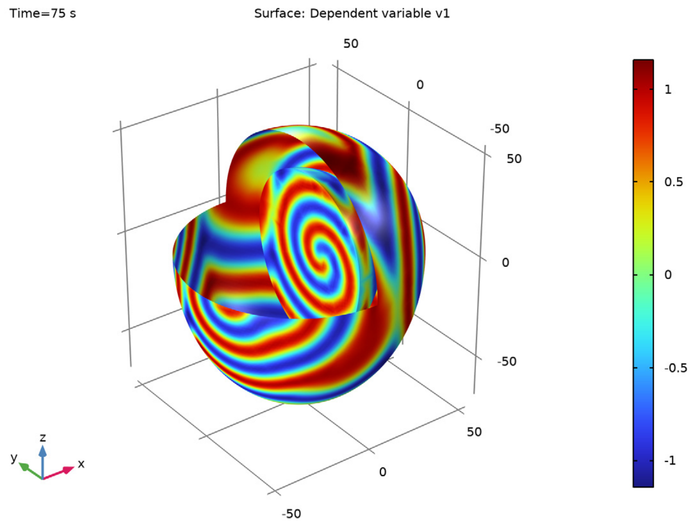


Figure 2: Solution to the FitzHugh–Nagumo equations at times t = 120 s (top) and t = 500 s (bottom).

The Ginzburg–Landau Equations

Figure 3 below shows the species v_1 at different times.



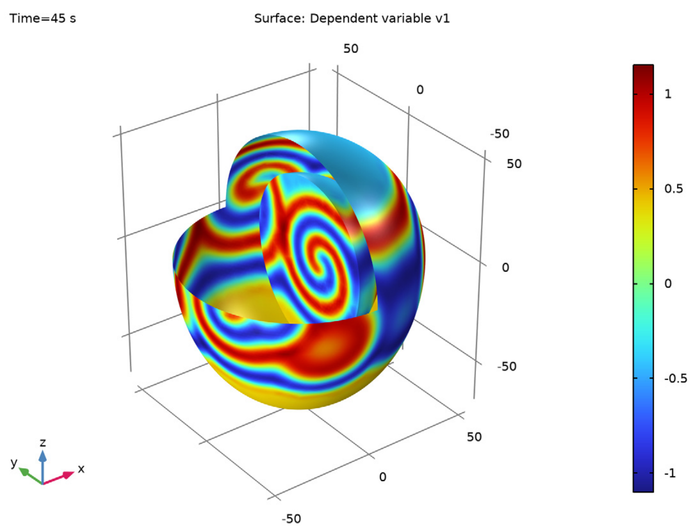


Figure 3: Solution to the Ginzburg–Landau equations at times t = 75 s (top) and t = 45 s (bottom).

The equation parameters and initial condition used here lead the diffusing species (v_1) to display characteristic spiral patterns with growing complexity over time.

References

1. F.H. Fenton, E.M. Cherry, H.M. Hastings, and S.J. Evans, "Real-time computer simulations of excitable media: JAVA as a scientific language and as a wrapper for C and FORTRAN programs," *BioSystems*, vol. 64, pp. 73–96, 2002.

2. Y. Kuramoto, *Chemical Oscillations, Waves and Turbulence*, Dover Publications, 2003.

3. J. Keener and J. Sneyd, *Mathematical Physiology*, Springer, 1998.

Application Library path: `COMSOL_Multiphysics/Equation_Based/heart_electrical`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.

2 In the **Select Physics** tree, select **Mathematics>PDE Interfaces>General Form PDE (g)**.

3 Click **Add**.

4 In the **Number of dependent variables** text field, type 2.

5 Click **Study**.

6 In the **Select Study** tree, select **General Studies>Time Dependent**.

7 Click **Done**.

ROOT

1 In the **Model Builder** window, click the root node.

2 In the root node's **Settings** window, locate the **Unit System** section.

3 From the **Unit system** list, choose **None**.

The equations in this model are given in dimensionless form. Turning off unit support avoids warnings about inconsistent use of units.

Geometry 1

Follow the instructions below to recreate the geometry illustrated in Figure 1.

Sphere 1 (sph1)

1 In the **Geometry** toolbar, click **Sphere**.

2 In the **Settings** window for **Sphere**, locate the **Size** section.

3 In the **Radius** text field, type 54.

4 Click **Build Selected**.

Ellipsoid 1 (elp1)

1 In the **Geometry** toolbar, click **More Primitives** and choose **Ellipsoid**.

2 In the **Settings** window for **Ellipsoid**, locate the **Size and Shape** section.

3 In the **a-semiaxis** text field, type 54.

4 In the **b-semiaxis** text field, type 54.

5 In the **c-semiaxis** text field, type 70.

6 Click **Build Selected**.

Next create the egg-shaped solid by fusing the top half of the sphere with the bottom half of the ellipsoid. Use blocks to remove the bottom half of sphere and the top half of ellipsoid.

Block 1 (blk1)

1 In the **Geometry** toolbar, click **Block**.

2 In the **Settings** window for **Block**, locate the **Size and Shape** section.

3 In the **Width** text field, type 110.

4 In the **Depth** text field, type 110.

5 In the **Height** text field, type 110.

6 Locate the **Position** section. From the **Base** list, choose **Center**.

7 In the **z** text field, type 55.

8 Click **Build Selected**.

Difference 1 (dif1)

1 In the **Geometry** toolbar, click **Booleans and Partitions** and choose **Difference**.

2 Select the object **elp1** only.

3 In the **Settings** window for **Difference**, locate the **Difference** section.

4 Click to select the **Activate Selection** toggle button for **Objects to subtract**.

5 Select the object **blk1** only.

6 Click **Build Selected**.

Block 2 (blk2)

1 In the **Geometry** toolbar, click **Block**.

2 In the **Settings** window for **Block**, locate the **Size and Shape** section.

3 In the **Width** text field, type `110`.

4 In the **Depth** text field, type `110`.

5 In the **Height** text field, type `110`.

6 Locate the **Position** section. From the **Base** list, choose **Center**.

7 In the **z** text field, type `-55`.

8 Click **Build Selected**.

Difference 2 (dif2)

1 In the **Geometry** toolbar, click **Booleans and Partitions** and choose **Difference**.

2 Select the object **sph1** only.

3 In the **Settings** window for **Difference**, locate the **Difference** section.

4 Click to select the **Activate Selection** toggle button for **Objects to subtract**.

5 Select the object **blk2** only.

6 Click **Build Selected**.

Union 1 (uni1)

1 In the **Geometry** toolbar, click **Booleans and Partitions** and choose **Union**.

2 Click in the **Graphics** window and then press Ctrl+A to select both objects.

3 In the **Settings** window for **Union**, click **Build All Objects**.

Now create the cavity inside the heart.

Scale 1 (sca1)

1 In the **Geometry** toolbar, click **Transforms** and choose **Scale**.

2 In the **Settings** window for **Scale**, locate the **Input** section.

3 Select the **Keep input objects** check box.

4 Select the object **uni1** only.

5 Locate the **Scale Factor** section. In the **Factor** text field, type `2/3`.

6 Click **Build Selected**.

7 Click the **Transparency** button in the **Graphics** toolbar.

Difference 3 (dif3)

1 In the **Geometry** toolbar, click **Booleans and Partitions** and choose **Difference**.

2 Select the object **uni1** only.

3 In the **Settings** window for **Difference**, locate the **Difference** section.

4 Click to select the **Activate Selection** toggle button for **Objects to subtract**.

5 Select the object **sca1** only.

6 Click **Build Selected**.

The next step is to create the wall separating the two chambers.

Cylinder 1 (cyl1)

1 In the **Geometry** toolbar, click **Cylinder**.

2 In the **Settings** window for **Cylinder**, locate the **Size and Shape** section.

3 In the **Radius** text field, type 45.

4 In the **Height** text field, type 10.

5 Locate the **Position** section. In the **x** text field, type -5.

6 In the **z** text field, type -5.

7 Locate the **Axis** section. From the **Axis type** list, choose **Cartesian**.

8 In the **x** text field, type 1.

9 In the **z** text field, type 0.

10 Click **Build Selected**.

Union 2 (uni2)

1 In the **Geometry** toolbar, click **Booleans and Partitions** and choose **Union**.

2 Click in the **Graphics** window and then press Ctrl+A to select both objects.

3 In the **Settings** window for **Union**, locate the **Union** section.

4 Clear the **Keep interior boundaries** check box.

5 Click **Build All Objects**.

This completes the geometry.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
alpha	0.1	0.1	Excitation threshold
epsilon	0.01	0.01	Excitability
beta	0.5	0.5	System parameter
gamma	1	1	System parameter
delta	0	0	System parameter
V0	1	1	Elevated potential value
nu0	0.3	0.3	Elevated inhibitor value
d	1e-5	1E-5	Off-axis shift distance

Use the default Neumann boundary condition at all the boundaries.

GENERAL FORM PDE (G)

General Form PDE 1

1 In the **Model Builder** window, under **Component 1 (comp1)>General Form PDE (g)** click **General Form PDE 1**.

2 In the **Settings** window for **General Form PDE**, locate the **Conservative Flux** section.

3 Specify the second Γ vector as

0	x
0	y
0	z

4 Locate the **Source Term** section. In the f text-field array, type `(alpha-u1)*(u1-1)*u1-u2` on the first row.

5 In the f text-field array, type `epsilon*(beta*u1-gamma*u2-delta)` on the second row.

Initial Values 1

1 In the **Model Builder** window, click **Initial Values 1**.

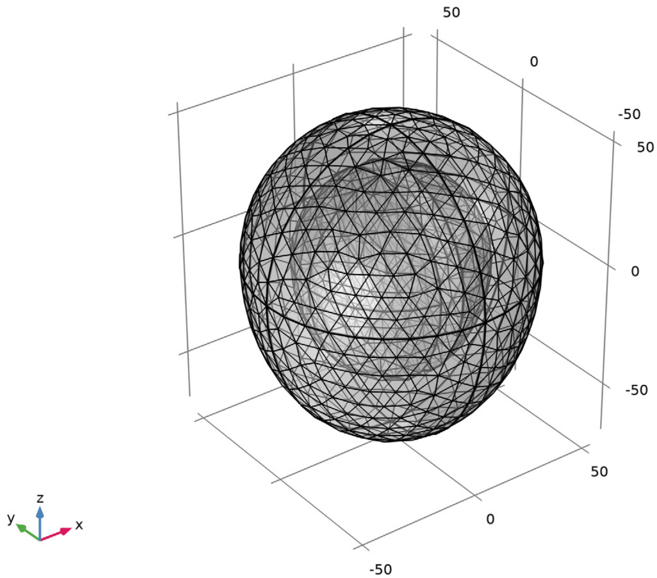
2 In the **Settings** window for **Initial Values**, locate the **Initial Values** section.

3 In the $u1$ text field, type `V0*((x+d)>0)*((z+d)>0)`.

4 In the $u2$ text field, type `nu0*((-x+d)>0)*((z+d)>0)`.

MESH 1

1. In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.
2. In the **Settings** window for **Mesh**, locate the **Physics-Controlled Mesh** section.
3. From the **Element size** list, choose **Fine**.
4. Click **Build All**.



STUDY 1

Step 1: Time Dependent

1. In the **Model Builder** window, under **Study 1** click **Step 1: Time Dependent**.
2. In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.
3. In the **Output times** text field, type `range(0,5,500)`.
4. In the **Home** toolbar, click **Compute**.

RESULTS

General Form PDE

The default plot shows a slice plot of the dependent variable. To create plots given in Figure 2, you need to suppress some of the boundaries.

Study 1/Solution 1 (sol1)

In the **Model Builder** window, expand the **Results>Datasets** node, then click **Study 1/ Solution 1 (sol1)**.

Selection

1 In the **Results** toolbar, click **Attributes** and choose **Selection**.
2 In the **Settings** window for **Selection**, locate the **Geometric Entity Selection** section.
3 From the **Geometric entity level** list, choose **Boundary**.
4 Click **Paste Selection**.
5 In the **Paste Selection** dialog box, type `1 3 10-18` in the **Selection** text field.
6 Click **OK**.
7 In the **Settings** window for **Selection**, locate the **Geometric Entity Selection** section.
8 Click **Create Selection**.
9 In the **Create Selection** dialog box, click **OK**.

3D Plot Group 2

1 In the **Results** toolbar, click **3D Plot Group**.
2 In the **Settings** window for **3D Plot Group**, locate the **Plot Settings** section.
3 Clear the **Plot dataset edges** check box.

Surface 1

1 Right-click **3D Plot Group 2** and choose **Surface**.
2 In the **3D Plot Group 2** toolbar, click **Plot**.
3 Click the **Transparency** button in the **Graphics** toolbar.

3D Plot Group 2

1 In the **Model Builder** window, click **3D Plot Group 2**.
2 In the **Settings** window for **3D Plot Group**, locate the **Data** section.
3 From the **Time (s)** list, choose **120**.
4 In the **3D Plot Group 2** toolbar, click **Plot**.

   This completes the modeling of the FitzHugh-Nagumo equations. To model the Complex Landau-Ginzburg equations use the following set of instructions.

ADD PHYSICS

1 In the **Home** toolbar, click **Add Physics** to open the **Add Physics** window.
2 Go to the **Add Physics** window.

3 In the tree, select **Mathematics>PDE Interfaces>General Form PDE (g)**.

4 Click to expand the **Dependent Variables** section. In the **Field name (1)** text field, type v.

5 In the **Number of dependent variables** text field, type 2.

6 In the **Dependent variables (1)** table, enter the following settings:

v1

7 Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** check box for **Study 1**.

8 Click **Add to Component 1** in the window toolbar.

9 In the **Home** toolbar, click **Add Physics** to close the **Add Physics** window.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** check box for **General Form PDE (g)**.

4 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies>Time Dependent**.

5 Click **Add Study** in the window toolbar.

6 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
c1	2	2	PDE parameter
c3	-0.2	-0.2	PDE parameter

GENERAL FORM PDE 2 (G2)

The accuracy of the solution can be improved by using cubic Hermite elements instead of the default quadratic Lagrange elements.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **General Form PDE 2 (g2)**.

2 In the **Settings** window for **General Form PDE**, click to expand the **Discretization** section.

3 From the **Shape function type** list, choose **Hermite**.

General Form PDE 1

Use the default Neumann boundary condition at all the boundaries also in this case.

1 In the **Model Builder** window, under **Component 1 (comp1)>General Form PDE 2 (g2)** click **General Form PDE 1**.

2 In the **Settings** window for **General Form PDE**, locate the **Conservative Flux** section.

3 Specify the first Γ vector as

`-v1x+c1*v2x`	x
`-v1y+c1*v2y`	y
`-v1z+c1*v2z`	z

4 Specify the second Γ vector as

`-c1*v1x-v2x`	x
`-c1*v1y-v2y`	y
`-c1*v1z-v2z`	z

5 Locate the **Source Term** section. In the *f* text-field array, type `v1-(v1-c3*v2)*(v1^2+v2^2)` on the first row.

6 In the *f* text-field array, type `v2-(c3*v1+v2)*(v1^2+v2^2)` on the second row.

Initial Values 1

1 In the **Model Builder** window, click **Initial Values 1**.

2 In the **Settings** window for **Initial Values**, locate the **Initial Values** section.

3 In the $v1$ text field, type `tanh(z[1/m])`.

4 In the $v2$ text field, type `-tanh(z[1/m])`.

Tighten the relative and absolute tolerances by a factor of 10.

STUDY 2

Step 1: Time Dependent

1 In the **Model Builder** window, under **Study 2** click **Step 1: Time Dependent**.

2 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.

3 In the **Output times** text field, type `range(0,5,75)`.

4 From the **Tolerance** list, choose **User controlled**.

5 In the **Relative tolerance** text field, type `0.001`.

Solution 2 (sol2)

1 In the **Study** toolbar, click **Show Default Solver**.

2 In the **Model Builder** window, expand the **Solution 2 (sol2)** node, then click **Time-Dependent Solver 1**.

3 In the **Settings** window for **Time-Dependent Solver**, click to expand the **Absolute Tolerance** section.

4 From the **Tolerance method** list, choose **Manual**.

5 In the **Absolute tolerance** text field, type `0.0001`.

6 In the **Study** toolbar, click **Compute**.

RESULTS

Follow the instructions to create plots given in Figure 3.

Study 2/Solution 2 (sol2)

In the **Model Builder** window, under **Results>Datasets** click **Study 2/Solution 2 (sol2)**.

Selection

1 In the **Results** toolbar, click **Attributes** and choose **Selection**.

2 In the **Settings** window for **Selection**, locate the **Geometric Entity Selection** section.

3 From the **Geometric entity level** list, choose **Boundary**.

4 From the **Selection** list, choose **Explicit 1**.

3D Plot Group 4

1 In the **Results** toolbar, click **3D Plot Group**.

2 In the **Settings** window for **3D Plot Group**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 2/Solution 2 (sol2)**.

4 Locate the **Plot Settings** section. Clear the **Plot dataset edges** check box.

Surface 1

1 Right-click **3D Plot Group 4** and choose **Surface**.

2 In the **Settings** window for **Surface**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)> General Form PDE 2>v1 - Dependent variable v1**.

3 In the **3D Plot Group 4** toolbar, click **Plot**.

3D Plot Group 4

1 In the **Model Builder** window, click **3D Plot Group 4**.

2 In the **Settings** window for **3D Plot Group**, locate the **Data** section.

3 From the **Time (s)** list, choose **45**.

4 In the **3D Plot Group 4** toolbar, click **Plot**.