

# Uniaxial Loading of a Shape Memory Alloy Using the Souza—Auricchio Model

Shape memory alloys (SMAs) are used in a wide range of applications due to their ability to recover their initial shape when heated, and to the hysteresis they show during a loading-unloading cycle before recovering their initial state. These two properties, called the shape memory effect and pseudoelasticity, make these alloys interesting for many applications in, for example, the aerospace, transportation, and medical industries.

This example model shows the SMA behavior under uniaxial loading. Three studies are performed:

- A boundary-load sweep shows the pseudoelasticity effect at different fixed temperatures
- A prescribed-displacement sweep shows the pseudoelasticity effect with a partial unloading-partial loading loop
- The shape memory effect is portrayed after increasing the temperature

This model is similar to the Uniaxial Loading of a Shape Memory Alloy model, but it uses the Souza-Auricchio model instead of the Lagoudas model.

The Souza-Auricchio model introduces a transformation strain tensor calculated from the limit function  $F = |\sigma_{tr}| - \sigma_0$ , where the conjugated thermodynamic stress associated with the transformation strain has the following expression (Ref. 1-3):

$$\sigma_{\rm tr} = \operatorname{dev}(\sigma) - H_k \varepsilon_{\rm tr} - \beta \langle T - T^* \rangle \frac{\varepsilon_{\rm tr}}{|\varepsilon_{\rm tr}|} - \frac{\partial I}{\partial \varepsilon_{\rm tr}}$$

The transformation strain,  $\varepsilon_{tr}$ , is computed from the flow rule

$$\dot{\varepsilon}_{\rm tr} = \lambda_{\rm p} \frac{\partial F}{\partial \sigma_{\rm tr}}$$

and the Kuhn-Tucker condition, where  $\lambda_{\rm p}$  is the plastic multiplier.

# Model Definition

A cylinder made of nickel-titanium (NiTi) alloy is submitted to axial tension. The cylinder is modeled by a rectangle in a 2D axisymmetric geometry. The Shape Memory Alloy node is set to use the Souza-Auricchio model.

The phase transformation parameters are calculated from the parameters used for the Lagoudas model portrayed in the Uniaxial Loading of a Shape Memory Alloy model (Ref. 4).

TABLE I: MATERIAL PARAMETERS, LAGOUDAS MODEL.

Material property	Value
Young's modulus, $E_{ m A}$	55 GPa
Young's modulus, martensite $E_{ m M}$	46 GPa
Poisson's ratio	0.33
Heat capacity at constant pressure	400 J/(kg.K)
Density	6500 kg/m <sup>3</sup>
Martensite start temperature, $M_{\scriptscriptstyle m S}$	245 K
Martensite finish temperature, $M_{ m f}$	230 K
Austenite start temperature, $A_{ m S}$	270 K
Austenite finish temperature, $A_{ m f}$	280 K
Slope of martensite limit curve, $C_{ m M}$	7.4 MPa/K
Slope of austenite limit curve, $C_{ m A}$	7.4 MPa/K
Maximum transformation strain	0.056

Note that the Lagoudas model contains more parameters than the Souza-Auricchio model. The latter model assumes that the slope of limit curves to martensite and austenite domains are equal, that is,  $C_{\rm M}$  =  $C_{\rm A}$ . In this example, the slope of limit curve  $\beta$  equals the slope of martensite limit curve  $C_{\rm M}$ .

Similarly, the Souza-Auricchio model assumes that the width of the transition zones are equal, that is, the temperature difference  $A_f - A_s$  and  $M_s - M_f$  are the same, which is not the case in the Lagoudas model. The reference temperature is then defined as

$$T^* = \frac{M_s + A_f}{2} = T_\sigma - \frac{\sigma_{Ms} + \sigma_{Af}}{2\beta}$$

In this example, the hardening modulus  $H_{\mathbf{k}}$  is derived from the difference in martensite start and finish temperatures,  $M_{\rm s}$  –  $M_{\rm f}$ , and the slope of martensite limit curve  $C_{\rm M}$ .

$$H_{\rm k} = \frac{C_{\rm M}(M_{\rm s} - M_{\rm f})}{\varepsilon_{\rm tr,\,max}} = \frac{\sigma_{\rm Mf} - \sigma_{\rm Ms}}{\varepsilon_{\rm tr,\,max}}$$

where  $\varepsilon_{tr.max}$  is the maximum transformation strain.

It is also possible to map Souza-Auricchio parameters to the stresses at the start and finish transformations, see Ref. 3. The material properties are listed in Table 2.

TABLE 2: MATERIAL PROPERTIES OF PHASE CHANGE.

Material Parameter	Symbol	Lagoudas parameters	Ref. 4	Value
Maximum transformation strain	ε <sub>tr,max</sub>	$\varepsilon_{ m tr,max}$	$\epsilon_{ m L}$	0.056
Slope of limit curve	β	$C_{ m M}$	$\frac{\Delta\sigma}{\Delta T}$	7.4 MPa/K
Reference temperature	$T^*$	$\frac{M_{\rm s}+A_{\rm f}}{2}$	$T_{\sigma} - \frac{\sigma_{\rm Ms} + \sigma_{\rm Af}}{2\beta}$	265.5 K
Elastic domain radius	$\sigma_0$	$\frac{C_{\rm M}(A_{\rm f}\!-\!M_{\rm s})}{2}$	$\frac{\sigma_{ m Ms} - \sigma_{ m Af}}{2}$	129.5 MPa
Hardening modulus	$H_{ m k}$	$\frac{C_{\rm M}(M_{\rm s}-M_{\rm f})}{\varepsilon_{\rm tr,max}}$	$\frac{\sigma_{Mf} - \sigma_{Ms}}{\epsilon_{tr,max}}$	1.982 GPa

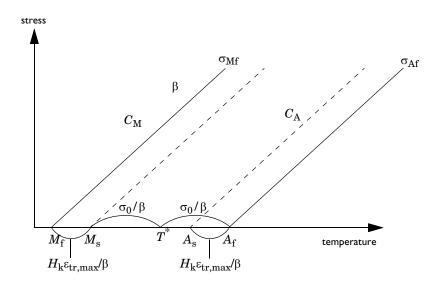


Figure 1: Phase diagram showing phase transformation parameters for the Lagoudas and Souza-Auricchio models.

The first study runs a parametric sweep for four different prescribed temperatures. Two of the temperatures (328 K, 308 K) are above the austenite finish temperature, one temperature (276 K) lies between the austenite start and finish temperatures, and one temperature (260 K) is below the austenite start temperature. The boundary load is a

loading-unloading cycle up to 800 MPa for first study and up to 300 MPa for the third study, see Figure 2.

The second study runs a parametric sweep on the axial displacement for a constant temperature of 298 K. The prescribed displacement is applied on one face in order to reach an axial strain value of 0.07. After reaching this maximum value, the axial displacement is decreased down to 40% of its value, then increased up to 80% the maximum, to finally decrease to 0, see Figure 3.

The third study runs at a constant temperature of 260 K, which is below the austenite start temperature. After one mechanical cycle according to Figure 2, the temperature is uniformly increased to 300 K to achieve the reverse transformation to austenite following Figure 4.

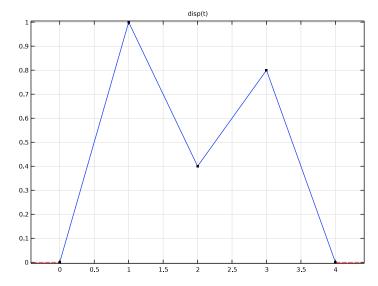


Figure 2: Boundary load for the first and third studies.

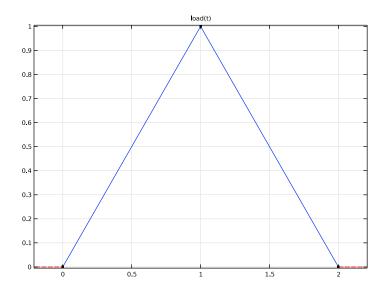


Figure 3: Prescribed displacement for the second study.

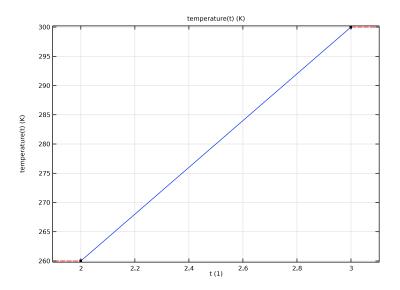


Figure 4: Temperature increase in the third study.

#### PSEUDOELASTICITY — SINGLE LOADING CYCLE

In Figure 5, the curves for high temperatures (328 K and 308 K) show the pseudoelasticity effect: the stress-strain relation is linear up to a temperature-dependent stress limit. Above this limit, the martensite transformation begins, resulting in a material with lower stiffness. The transformation is complete when reaching the maximum strain, at which the microstructure is 100% martensite and the tangent stiffness is taken from the Young's modulus of the martensite. During unloading, the reverse transformation occurs at a lower stress level than the stress limit for the forward transformation.

The axial stress-strain curve for a prescribed temperature of 276 K (between the austenite start and finish temperatures) shows that the forward and reverse transformations occur at lower stress levels. Also, the reverse transformation is not complete when the stress is completely released. This can be seen in Figure 6 where the forward transformation occurs sooner, that is, at lower stress level. The backward transformation starts later, and it is not complete at the end of the cycle. At a lower temperature, the forward transformation occurs at a lower stress level, and the revers transformation does not even start, resulting in residual strains.

Figure 7 sums up all these observations on a phase diagram: at higher temperatures the forward transformation occurs at higher stress, and the reverse transformation is complete. At lower temperatures the forward transformation occurs at lower stress and the reverse transformation does not start, or it is incomplete.

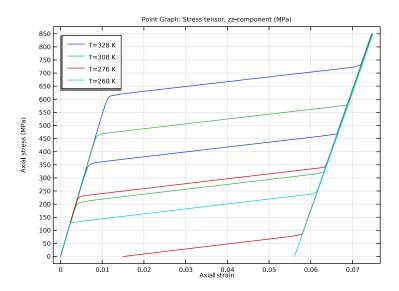


Figure 5: Stress vs strain curve at several temperatures.

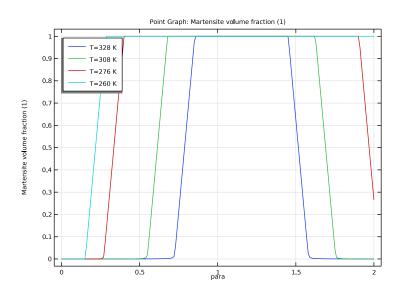


Figure 6: Evolution of martensite volume fraction at several temperatures.

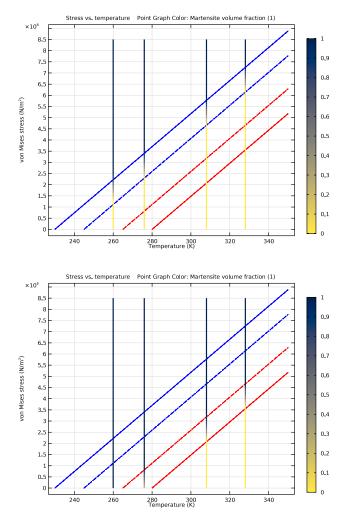


Figure 7: Stress-temperature paths reported on the phase diagram for the four temperatures. Top: loading. Bottom: unloading.

# PSEUDOELASTICITY — MULTIPLE LOADING CYCLES

The first loading step and the first unloading step are similar to the ones in the previous study. At a strain level of 0.028, the material is loaded again, which leads to a stress increase with high stiffness, see Figure 8. This part of the curve is represented by a plateau in Figure 9. When the yield stress limit is reached, the forward transformation continues and

follows the same path as the first load. When the material is unloaded, the stress decreases and the material undergoes the reverse transformation to pure austenite. Finally the stress is decreased to zero. The four stages can be seen in Figure 10.

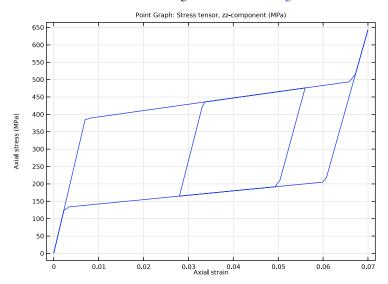


Figure 8: Stress versus strain curve for loading-unloading cycle with an internal loop.

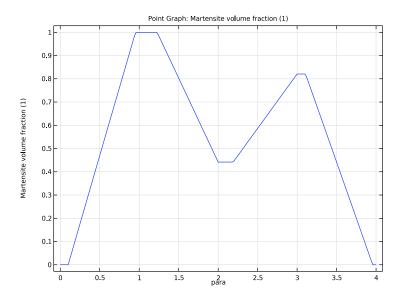


Figure 9: Evolution of martensite during the loading-unloading cycle with an internal loop.

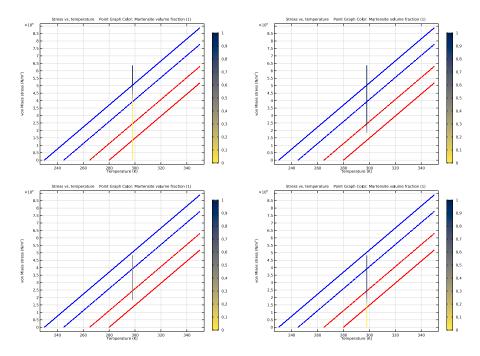


Figure 10: Stress-temperature paths reported on the phase diagram. Top left: loading. Top right: partial unloading. Bottom left: partial loading. Bottom right: unloading.

# SHAPE MEMORY EFFECT

The mechanical loading-unloading cycle is the same as in the first study (temperature of 260 K). At this point the residual strain is 0.056. Increasing the temperature from 260 K to 300 K decreases the residual strain to 0 as shown in Figure 11. This is also visible on

the phase diagram, Figure 12: after unloading the material does not reach the austenite transition zone, then temperature increase is needed to recover the austenite phase.

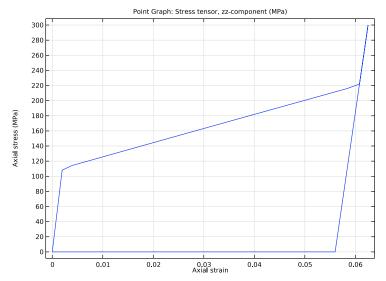


Figure 11: Stress versus strain curve showing the shape memory effect.

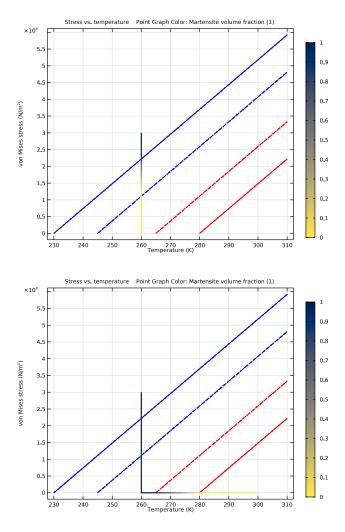


Figure 12: Stress-temperature paths reported on the phase diagram. Top: loading. Bottom: unloading and heating.

# References

1. A. Souza and others, "Three-dimensional model for solids undergoing stress-induced phase transformations," European J. Mech. A Solids, vol. 17, pp. 789-806, 1988.

- 2. F. Auricchio and L. Petrini, "A three-dimensional model describing stress-temperature induced solid phase transformations. Part I: solution algorithm and boundary value problems," *Int. J. Numer. Meth. Eng.*, vol. 61, pp. 807–836. 2004.
- 3. F. Auricchio and others, "SMA numerical modeling versus experimental results: Parameter identification and model prediction capabilities," *J. Mater. Eng. Perform.*, vol. 18, pp. 649–654, 2009
- 4. D. Lagoudas (ed.), Shape Memory Alloys: Modeling and Engineering Applications, Springer, 2008.

**Application Library path:** Nonlinear\_Structural\_Materials\_Module/Shape\_Memory\_Alloys/uniaxial\_loading\_of\_shape\_memory\_alloy\_souza\_auricchio

# Modeling Instructions

From the File menu, choose New.

#### NEW

In the New window, click Model Wizard.

#### MODEL WIZARD

- I In the Model Wizard window, click 2D Axisymmetric.
- 2 In the Select Physics tree, select Structural Mechanics>Solid Mechanics (solid).
- 3 Click Add.
- 4 Click 🔵 Study.
- 5 In the Select Study tree, select General Studies>Stationary.
- 6 Click M Done.

### **GLOBAL DEFINITIONS**

#### Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.

**3** In the table, enter the following settings:

Name	Expression	Value	Description
T	298[K]	298 K	Applied temperature
para	0	0	Continuation parameter

#### **GEOMETRY I**

- I In the Model Builder window, under Component I (compl) click Geometry I.
- 2 In the Settings window for Geometry, locate the Units section.
- 3 From the Length unit list, choose cm.

# Rectangle I (rI)

- I In the Geometry toolbar, click Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type 6.
- 4 In the Height text field, type 20.

# SOLID MECHANICS (SOLID)

Shape Memory Alloy I

- I In the Model Builder window, under Component I (compl) right-click Solid Mechanics (solid) and choose Material Models>Shape Memory Alloy.
- 2 In the Settings window for Shape Memory Alloy, locate the Domain Selection section.
- 3 From the Selection list, choose All domains.
- **4** Locate the **Model Input** section. From the *T* list, choose **User defined**. In the associated text field, type T.
- 5 Locate the Shape Memory Alloy section. From the Material model list, choose Souza-Auricchio.
- 6 Find the Phase transformation subsection. From the Specify list, choose Temperature.

#### Roller I

- I In the Physics toolbar, click Boundaries and choose Roller.
- 2 Select Boundary 2 only.

# Boundary Load I

In the Physics toolbar, click — Boundaries and choose Boundary Load.

Create an interpolation function to apply the load cycle.

#### DEFINITIONS

#### Load

- I In the Home toolbar, click f(x) Functions and choose Local>Interpolation.
- 2 In the Settings window for Interpolation, type Load in the Label text field.
- 3 Locate the **Definition** section. In the **Function name** text field, type load.
- **4** In the table, enter the following settings:

t	f(t)
0	0
1	1
2	0

5 Click Plot.

# SOLID MECHANICS (SOLID)

# Boundary Load 1

- I In the Model Builder window, under Component I (compl)>Solid Mechanics (solid) click Boundary Load I.
- 2 Select Boundary 3 only.
- 3 In the Settings window for Boundary Load, locate the Force section.
- **4** Specify the  $\mathbf{F}_A$  vector as

0	r
850[MPa]*load(para)	z

#### MATERIALS

# NiTi Allov

- I In the Materials toolbar, click **Blank Material**.
- 2 In the Settings window for Material, type NiTi Alloy in the Label text field.
- **3** Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Poisson's ratio	nu	0.33	1	Basic
Young's modulus	E_A	55[GPa]	Pa	Austenite phase

Property	Variable	Value	Unit	Property group
Young's modulus	E_M	46[GPa]	Pa	Martensite phase
Slope of limit curve	beta	7.4[MPa/K]	Pa/K	Souza- Auricchio model
Maximum transformation strain	etrmaxAuricchio	0.056	I	Souza- Auricchio model
Martensite start temperature	TMs_SA	245[K]	K	Souza- Auricchio model
Martensite finish temperature	TMf_SA	230[K]	K	Souza- Auricchio model
Austenite finish temperature	TAf_SA	280[K]	K	Souza- Auricchio model
Density	rho	6500	kg/m³	Basic

#### MESH I

# Mapped I

In the Mesh toolbar, click Mapped.

# Size

- I In the Model Builder window, click Size.
- 2 In the Settings window for Size, locate the Element Size section.
- 3 From the Predefined list, choose Coarser.
- 4 Click Build All.

# STUDY: PSEUDOELASTICITY, SINGLE LOADING CYCLE

- I In the Model Builder window, click Study I.
- 2 In the Settings window for Study, type Study: Pseudoelasticity, Single Loading Cycle in the Label text field.

# Parametric Sweep

- I In the Study toolbar, click Parametric Sweep.
- 2 In the Settings window for Parametric Sweep, locate the Study Settings section.

- 3 Click + Add.
- **4** In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
T (Applied temperature)	328 308 276 260	K

# Step 1: Stationary

- I In the Model Builder window, click Step 1: Stationary.
- 2 In the Settings window for Stationary, click to expand the Study Extensions section.
- **3** Select the **Auxiliary sweep** check box.
- 4 Click + Add.
- 5 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
para (Continuation parameter)	range(0,0.01,2)	

# Solution I (soll)

- I In the Study toolbar, click Show Default Solver.
- 2 In the Model Builder window, expand the Solution I (soll) node.
- 3 In the Model Builder window, expand the Study: Pseudoelasticity, Single Loading Cycle>
  Solver Configurations>Solution I (sol1)>Stationary Solver I node, then click Parametric I.
- 4 In the Settings window for Parametric, click to expand the Continuation section.
- **5** Select the **Tuning of step size** check box.
- **6** In the **Minimum step size** text field, type 0.0001.
- 7 In the Model Builder window, click Study: Pseudoelasticity, Single Loading Cycle.
- 8 In the Settings window for Study, locate the Study Settings section.
- 9 Clear the Generate default plots check box.
- 10 In the Study toolbar, click **Compute**.

#### RESULTS

#### Stress vs. Strain

- I In the Home toolbar, click <a> Add Plot Group</a> and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Stress vs. Strain in the Label text field.

3 Locate the Data section. From the Dataset list, choose Study: Pseudoelasticity, Single Loading Cycle/Parametric Solutions I (sol2).

Point Graph 1

- I Right-click Stress vs. Strain and choose Point Graph.
- **2** Select Point 1 only.
- 3 In the Settings window for Point Graph, click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (compl)> Solid Mechanics>Stress>Stress tensor (spatial frame) - N/m2>solid.sGpzz - Stress tensor, zz-component.
- 4 Locate the y-Axis Data section. From the Unit list, choose MPa.
- 5 Click Replace Expression in the upper-right corner of the x-Axis Data section. From the menu, choose Component I (compl)>Solid Mechanics>Strain> Strain tensor (material and geometry frames)>solid.eZZ - Strain tensor, ZZ-component.
- 6 Click to expand the **Legends** section. Find the **Include** subsection. Clear the **Point** check box.
- 7 Select the **Show legends** check box.

Stress vs. Strain

- I In the Model Builder window, click Stress vs. Strain.
- 2 In the Settings window for ID Plot Group, locate the Plot Settings section.
- 3 Select the x-axis label check box.
- 4 Select the y-axis label check box.
- 5 In the x-axis label text field, type Axial strain.
- 6 In the y-axis label text field, type Axial stress (MPa).
- 7 Locate the Legend section. From the Position list, choose Upper left.
- 8 In the Stress vs. Strain toolbar, click  **Plot**.

The resulting plot should look like Figure 5.

Martensite Volume Fraction

- I In the Home toolbar, click Add Plot Group and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Martensite Volume Fraction in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Study: Pseudoelasticity, Single Loading Cycle/Parametric Solutions I (sol2).

# Point Graph 1

- I Right-click Martensite Volume Fraction and choose Point Graph.
- **2** Select Point 1 only.
- 3 In the Settings window for Point Graph, click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (compl)> Solid Mechanics>Shape memory alloy>solid.xiGp\_M Martensite volume fraction 1.
- **4** Locate the **Legends** section. Select the **Show legends** check box.
- 5 Find the Include subsection. Clear the Point check box.

#### Martensite Volume Fraction

- I In the Model Builder window, click Martensite Volume Fraction.
- 2 In the Settings window for ID Plot Group, locate the Legend section.
- 3 From the Position list, choose Upper left.

#### ADD PREDEFINED PLOT

- I In the Home toolbar, click Add Predefined Plot to open the Add Predefined Plot window.
- 2 Go to the Add Predefined Plot window.
- 3 In the tree, select Study: Pseudoelasticity, Single Loading Cycle/
  Parametric Solutions I (sol2)>Solid Mechanics>
  Shape Memory Alloy Phase Diagram (Shape Memory Alloy I).
- 4 Click Add Plot in the window toolbar.

#### RESULTS

Shape Memory Alloy Phase Diagram (Shape Memory Alloy I)

- I In the Settings window for ID Plot Group, locate the Data section.
- 2 From the Parameter selection (para) list, choose From list.
- **3** In the parameter values list, select all solution steps between 0 and 1.
- 4 In the Shape Memory Alloy Phase Diagram (Shape Memory Alloy I) toolbar, click Plot to get the paths during stress loading (Figure 7, top).
- **5** In the parameter values list, select all solution steps between 1 and 2.
- 6 In the Shape Memory Alloy Phase Diagram (Shape Memory Alloy I) toolbar, click Plot to get the paths during stress unloading (Figure 7, bottom).

7 From the Parameter selection (para) list, choose All.

Now perform a study with a partial unloading-loading cycle. First define the load function.

#### DEFINITIONS

# Displacement

- I In the Home toolbar, click f(x) Functions and choose Global>Interpolation.
- 2 In the Settings window for Interpolation, type Displacement in the Label text field.
- 3 Locate the **Definition** section. In the **Function name** text field, type disp.
- **4** In the table, enter the following settings:

t	f(t)
0	0
1	1
2	0.4
3	0.8
4	0

5 Click om Plot.

# SOLID MECHANICS (SOLID)

# Prescribed Displacement I

- I In the Physics toolbar, click Boundaries and choose Prescribed Displacement.
- 2 Select Boundary 3 only.
- 3 In the Settings window for Prescribed Displacement, locate the Prescribed Displacement section.
- 4 From the Displacement in z direction list, choose Prescribed.
- **5** In the  $u_{0z}$  text field, type 20[cm]\*0.07\*disp(para).

#### ADD STUDY

- I In the Home toolbar, click Add Study to open the Add Study window.
- **2** Go to the **Add Study** window.
- 3 Find the Studies subsection. In the Select Study tree, select General Studies>Stationary.
- 4 Click Add Study in the window toolbar.
- 5 In the Home toolbar, click Add Study to close the Add Study window.

#### STUDY: PSEUDOELASTICITY, MULTIPLE LOADING CYCLES

- I In the Model Builder window, click Study 2.
- 2 In the Settings window for Study, type Study: Pseudoelasticity, Multiple Loading Cycles in the Label text field.

# Step 1: Stationary

- I In the Model Builder window, under Study: Pseudoelasticity, Multiple Loading Cycles click Step 1: Stationary.
- 2 In the Settings window for Stationary, locate the Study Extensions section.
- 3 Select the Auxiliary sweep check box.
- 4 Click + Add.
- **5** In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
para (Continuation parameter)	range(0,0.02,4)	

- 6 Locate the Physics and Variables Selection section. Select the Modify model configuration for study step check box.
- 7 In the tree, select Component I (compl)>Solid Mechanics (solid)>Boundary Load I.
- 8 Click O Disable.
- 9 In the Model Builder window, click Study: Pseudoelasticity, Multiple Loading Cycles.
- 10 In the Settings window for Study, locate the Study Settings section.
- II Clear the Generate default plots check box.

#### Solution 7 (sol7)

- I In the Study toolbar, click Show Default Solver.
- 2 In the Model Builder window, expand the Solution 7 (sol7) node.
- 3 In the Model Builder window, expand the Study: Pseudoelasticity, Multiple Loading Cycles>Solver Configurations>Solution 7 (sol7)>Stationary Solver I node, then click Parametric I.
- 4 In the Settings window for Parametric, locate the Continuation section.
- **5** Select the **Tuning of step size** check box.
- **6** In the **Minimum step size** text field, type 0.0001.
- 7 In the Study toolbar, click **Compute**.

Duplicate the two first plots to reproduce Figure 8 and Figure 9.

#### RESULTS

Martensite Volume Fraction, Stress vs. Strain

- I In the Model Builder window, under Results, Ctrl-click to select Stress vs. Strain and Martensite Volume Fraction.
- 2 Right-click and choose **Duplicate**.

Stress vs. Strain I

- I In the Model Builder window, click Stress vs. Strain I.
- 2 In the Settings window for ID Plot Group, locate the Data section.
- 3 From the Dataset list, choose Study: Pseudoelasticity, Multiple Loading Cycles/ Solution 7 (sol7).
- 4 Locate the Legend section. Clear the Show legends check box.
- 5 In the Stress vs. Strain I toolbar, click Plot.

Martensite Volume Fraction 1

- I In the Model Builder window, click Martensite Volume Fraction I.
- 2 In the Settings window for ID Plot Group, locate the Data section.
- 3 From the Dataset list, choose Study: Pseudoelasticity, Multiple Loading Cycles/ Solution 7 (sol7).
- 4 Locate the Legend section. Clear the Show legends check box.
- 5 In the Martensite Volume Fraction I toolbar, click **Plot**.

#### ADD PREDEFINED PLOT

- I Go to the Add Predefined Plot window.
- 2 In the tree, select Study: Pseudoelasticity, Multiple Loading Cycles/Solution 7 (sol7)> Solid Mechanics>Shape Memory Alloy Phase Diagram (Shape Memory Alloy I).
- 3 Click Add Plot in the window toolbar.

#### RESULTS

Shape Memory Alloy Phase Diagram (Shape Memory Alloy I) I

- I In the Settings window for ID Plot Group, locate the Data section.
- 2 From the Parameter selection (para) list, choose From list.
- **3** In the parameter values list, select all solution steps between 0 and 1.
- 4 In the Shape Memory Alloy Phase Diagram (Shape Memory Alloy I) I toolbar, click Plot to get the path during stress loading (Figure 10, top).

- 5 You can do the same three times, selecting all solution steps between 1 and 2, between 2 and 3, and between 3 and 4 to plot path during partial unloading, partial loading, and total unloading, respectively.
- 6 From the Parameter selection (para) list, choose All.

To show the shape memory effect you need to apply a sweep on the mechanical loading, then on the temperature.

#### DEFINITIONS

# Temberature

- I In the Home toolbar, click f(x) Functions and choose Global>Interpolation.
- 2 In the Settings window for Interpolation, type Temperature in the Label text field.
- **3** Locate the **Definition** section. In the **Function name** text field, type temperature.
- **4** In the table, enter the following settings:

t	f(t)
2	260
3	300

**5** Locate the **Units** section. In the **Function** table, enter the following settings:

Function	Unit
temperature	K

**6** In the **Argument** table, enter the following settings:

Argument	Unit
t	1

7 Click Plot.

# SOLID MECHANICS (SOLID)

Duplicate the **Shape Memory Alloy I** node to apply a temperature defined by the interpolation function.

Shape Memory Alloy I

In the Model Builder window, under Component I (compl)>Solid Mechanics (solid) rightclick Shape Memory Alloy I and choose Duplicate.

# Shape Memory Alloy 2

- I In the Model Builder window, click Shape Memory Alloy 2.
- 2 In the Settings window for Shape Memory Alloy, locate the Model Input section.
- **3** In the T text field, type temperature(para).

# Boundary Load 2

- I In the Physics toolbar, click Boundaries and choose Boundary Load.
- 2 Select Boundary 3 only.
- 3 In the Settings window for Boundary Load, locate the Force section.
- **4** Specify the  $\mathbf{F}_{\mathsf{A}}$  vector as

0	r
300[MPa]*load(para)	z

#### ADD STUDY

- I In the Home toolbar, click Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select General Studies>Stationary.
- 4 Click Add Study in the window toolbar.
- 5 In the Home toolbar, click Add Study to close the Add Study window.

#### STUDY: SHAPE MEMORY EFFECT

- I In the Model Builder window, click Study 3.
- 2 In the Settings window for Study, type Study: Shape Memory Effect in the Label text field.

#### Steb 1: Stationary

- I In the Model Builder window, under Study: Shape Memory Effect click Step 1: Stationary.
- 2 In the Settings window for Stationary, locate the Study Extensions section.
- 3 Select the Auxiliary sweep check box.
- 4 Click + Add.
- **5** In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
para (Continuation parameter)	range(0,0.02,2) range(2.05,0.05,3)	

- 6 Locate the Physics and Variables Selection section. Select the Modify model configuration for study step check box.
- 7 In the tree, select Component I (compl)>Solid Mechanics (solid)>Boundary Load I.
- 8 Click O Disable.
- 9 In the tree, select Component I (compl)>Solid Mechanics (solid)> Prescribed Displacement I.
- 10 Click ODisable.
- II In the Model Builder window, click Study: Shape Memory Effect.
- 12 In the Settings window for Study, locate the Study Settings section.
- **13** Clear the **Generate default plots** check box.

Solution 8 (sol8)

- I In the Study toolbar, click Show Default Solver.
- 2 In the Model Builder window, expand the Solution 8 (sol8) node.
- 3 In the Model Builder window, expand the Study: Shape Memory Effect>
  Solver Configurations>Solution 8 (sol8)>Stationary Solver 1 node, then click Parametric 1.
- 4 In the Settings window for Parametric, locate the Continuation section.
- **5** Select the **Tuning of step size** check box.
- 6 In the Minimum step size text field, type 0.0001.
- 7 In the Study toolbar, click **Compute**.

Duplicate the second stress-strain plot to reproduce Figure 11.

# RESULTS

Stress vs. Strain 1

In the Model Builder window, under Results right-click Stress vs. Strain I and choose Duplicate.

Stress vs. Strain 1.1

- I In the Model Builder window, click Stress vs. Strain I.I.
- 2 In the Settings window for ID Plot Group, locate the Data section.
- 3 From the Dataset list, choose Study: Shape Memory Effect/Solution 8 (sol8).
- 4 In the Stress vs. Strain 1.1 toolbar, click  **Plot**.

#### ADD PREDEFINED PLOT

I Go to the Add Predefined Plot window.

- 2 In the tree, select Study: Shape Memory Effect/Solution 8 (sol8)>Solid Mechanics> Shape Memory Alloy Phase Diagram (Shape Memory Alloy 2).
- 3 Click Add Plot in the window toolbar.

#### RESULTS

#### **Parameters**

- I In the Model Builder window, under Results click Parameters.
- 2 In the Settings window for Parameters, locate the Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description
dset4_solid_sma2_Tm ax	310[K]	310 K	Maximum temperature of phase diagram

Shape Memory Alloy Phase Diagram (Shape Memory Alloy 2)

- I In the Model Builder window, click Shape Memory Alloy Phase Diagram (Shape Memory Alloy 2).
- 2 In the Settings window for ID Plot Group, locate the Data section.
- 3 From the Parameter selection (para) list, choose From list.
- **4** In the parameter values list, select all solution steps between 0 and 1.
- 5 In the Shape Memory Alloy Phase Diagram (Shape Memory Alloy 2) toolbar, click Plot to get the path during stress loading (Figure 12, top).
- **6** In the parameter values list, select all solution steps between 2 and 3.
- 7 In the Shape Memory Alloy Phase Diagram (Shape Memory Alloy 2) toolbar, click **Plot** to get the path during stress loading (Figure 12, bottom).

Martensite Volume Fraction, Shape Memory Alloy Phase Diagram (Shape Memory Alloy 1), Stress vs. Strain

- I In the Model Builder window, under Results, Ctrl-click to select Stress vs. Strain, Martensite Volume Fraction, and Shape Memory Alloy Phase Diagram (Shape Memory Alloy I).
- 2 Right-click and choose **Group**.

Pseudoelasticity, Single Loading Cycle

In the Settings window for Group, type Pseudoelasticity, Single Loading Cycle in the Label text field.

Martensite Volume Fraction I, Shape Memory Alloy Phase Diagram (Shape Memory Alloy I) I, Stress vs. Strain I

- In the Model Builder window, under Results, Ctrl-click to select Stress vs. Strain 1, Martensite Volume Fraction 1, and
   Shape Memory Alloy Phase Diagram (Shape Memory Alloy 1) 1.
- 2 Right-click and choose **Group**.

Pseudoelasticity, Multiple Loading Cycles

In the **Settings** window for **Group**, type Pseudoelasticity, Multiple Loading Cycles in the **Label** text field.

Shape Memory Alloy Phase Diagram (Shape Memory Alloy 2), Stress vs. Strain 1.1

- I In the Model Builder window, under Results, Ctrl-click to select Stress vs. Strain I.I and Shape Memory Alloy Phase Diagram (Shape Memory Alloy 2).
- 2 Right-click and choose **Group**.

Shape Memory Effect

In the Settings window for Group, type Shape Memory Effect in the Label text field.

You need to disable the following nodes in the studies if you want to run them again.

#### STUDY: PSEUDOELASTICITY, SINGLE LOADING CYCLE

Steb 1: Stationary

- I In the Model Builder window, under Study: Pseudoelasticity, Single Loading Cycle click Step 1: Stationary.
- 2 In the Settings window for Stationary, locate the Physics and Variables Selection section.
- 3 Select the Modify model configuration for study step check box.
- 4 In the tree, select Component I (compl)>Solid Mechanics (solid)> Prescribed Displacement I.
- 5 Click ODisable.
- 6 In the tree, select Component I (compl)>Solid Mechanics (solid)>Shape Memory Alloy 2.
- 7 Click O Disable.
- 8 In the tree, select Component I (compl)>Solid Mechanics (solid)>Boundary Load 2.
- 9 Click O Disable.

# STUDY: PSEUDOELASTICITY, MULTIPLE LOADING CYCLES

I In the Model Builder window, under Study: Pseudoelasticity, Multiple Loading Cycles click Step I: Stationary.

- 2 In the Settings window for Stationary, locate the Physics and Variables Selection section.
- 3 In the tree, select Component I (compl)>Solid Mechanics (solid)>Shape Memory Alloy 2.
- 4 Click O Disable.
- 5 In the tree, select Component I (compl)>Solid Mechanics (solid)>Boundary Load 2.
- 6 Click O Disable.