Created in COMSOL Multiphysics 6.2

Optimization of a Porous Microchannel Heat Sink

Introduction

In the model Performance of a Porous Microchannel Heat Sink, a parameter study is performed to find the optimum value for the thickness of the porous material layer within a microchannel heat sink (MCHS). This example extends the abovementioned model by another study, in which the parametric sweep is replaced by an Optimization study node. The figure of merit is a measure of performance of the heat sink (in this case relative to an MCHS without porous material) and is therefore used as an objective function that can be maximized. The results agree with those found in the original model version.

Model Definition

The design and operating conditions are described in the original model, Performance of a Porous Microchannel Heat Sink.

Inside the porous domains, the governing equation is the Brinkman equation with a Forchheimer correction term (also known as the Brinkman–Forchheimer or Darcy–Brinkman–Forchheimer equation). The pressure drop depends on the velocity field **u** as

$$-\nabla p = \frac{\mu}{\kappa}\mathbf{u} + \frac{c_F}{\sqrt{\kappa}}\rho\mathbf{u}|\mathbf{u}| \tag{1}$$

where μ (SI unit: Pa·s) is the fluid viscosity, ρ (SI unit: kg/m^3) the density, and κ (SI unit: m^2) the permeability of the porous substrate.

As the performance of the MCHS should be optimized, the performance parameters are recapitulated here:

- The pressure drop, that is the pressure difference between inlet and outlet of the porous MCHS
- The average heat transfer coefficient of the MCHS, given by

$$h_{\text{mchs}} = \frac{q_{\text{in}}}{\overline{T}_{\text{w}} - T_{\text{in}}} \tag{2}$$

  with the average wall temperature at the bottom centerline $\overline{T}_{\text{w}}$.
- The Reynolds number is defined as

$$\mathrm{Re} = \frac{\rho u_{\mathrm{in}} D_{\mathrm{h}}}{\mu} \tag{3}$$

with the hydraulic diameter D_h (m) that is defined based on the length and width of the free flow channel, l_f and w_f respectively, as follows:

$$D_{\mathrm{h}} = \frac{2 l_{\mathrm{f}} w_{\mathrm{f}}}{l_{\mathrm{f}} + w_{\mathrm{f}}}$$

- The Nusselt number describes the ratio of convective to conductive heat transfer according to

$$\mathrm{Nu} = \frac{D_{\mathrm{h}} h_{\mathrm{mchs}}}{k_{\mathrm{f}}} \tag{4}$$

  where k_f is the fluids thermal conductivity.

- The Figure of Merit (FOM) compares the performance of two different designs with the following expression:

$$\mathrm{FOM} = \frac{h_{\mathrm{mchs}} / h_{\mathrm{mchs,\ base}}}{(\Omega / \Omega_{\mathrm{base}})^{1/3}} \tag{5}$$

  The index `base` refers to the values for the MCHS without the porous structure and $\Omega = u_{in} l_f w_f \Delta p$ is the pumping power.

Equation 1 is valid for $1 \le \mathrm{Re} \le 1000$. An estimation of the Reynolds number (Equation 3) results in $\mathrm{Re} \sim 300$ such that the choice of the Brinkman-Forchheimer equation is valid.

When the FOM reaches its maximum value, the performance of the porous MCHS is best compared to a standard MCHS without any porous layer. Therefore, the easiest way to use the Optimization study feature is to use the FOM as the objective function and to use **Maximization** as the objective type.

Results and Discussion

The result of the optimization study matches the results of the parametric study in the original model. The best performance compared to a standard MCHS without any porous layer is reached for a porous layer thickness of 0.1 mm. To compare the results of the optimization study with the parameter study performed in the model Performance of a Porous Microchannel Heat Sink, the results for the average pressure drop, the average heat transfer coefficient, and the dimensionless Reynolds and Nusselt numbers are plotted as extra points in the already existing table plots. The extra points are marked with an asterisk. Figure 1 shows that the values for pressure drop and heat transfer coefficient lie well on the curve showing both as a functions of the porous layer thickness.

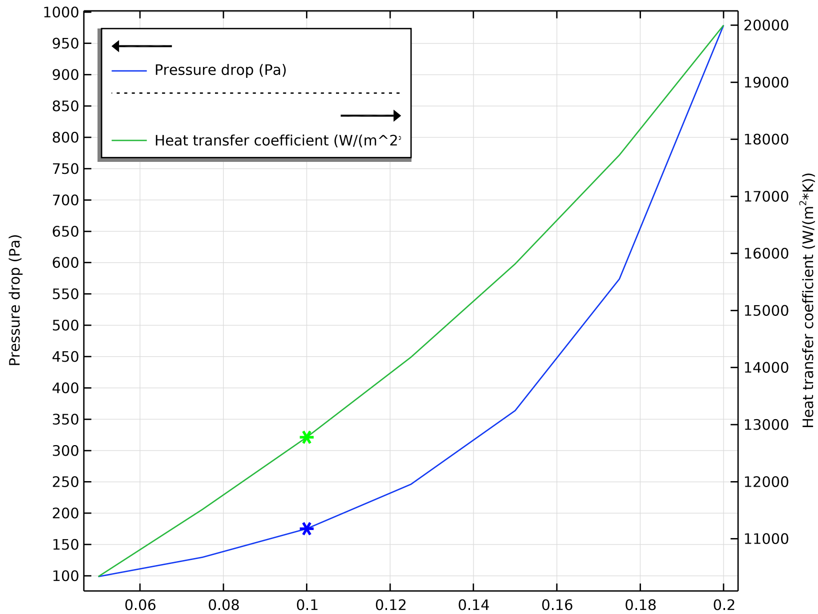

Figure 1: Pressure drop and average heat transfer coefficient. The results from the parameter study where the porous layer thickness varies from 0.05 to 0.2 mm are taken from the original model "Performance of a Porous Microchannel Heat Sink". The results of the optimization study are added with an asterisk.

The dimensionless numbers — the Reynolds number and the Nusselt number — also agree well, as can be seen in Figure 2. Figure 3 shows the Figure of Merit and again the optimized value fits perfectly well to the values of the parameter study.

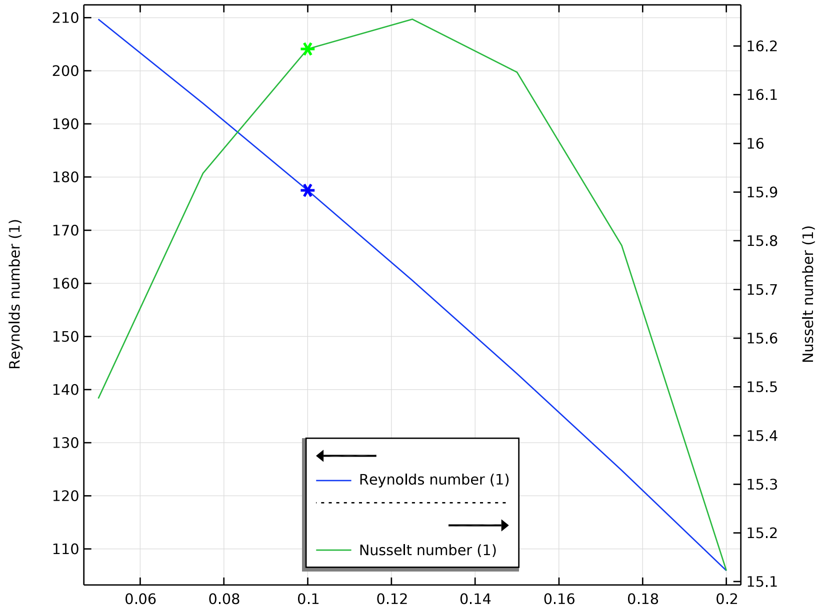

Figure 2: Reynolds number and Nusselt number as a function of porous layer thickness. The results are taken from the original model "Performance of a Porous Microchannel Heat Sink". The results of the optimization study are added with an asterisk.

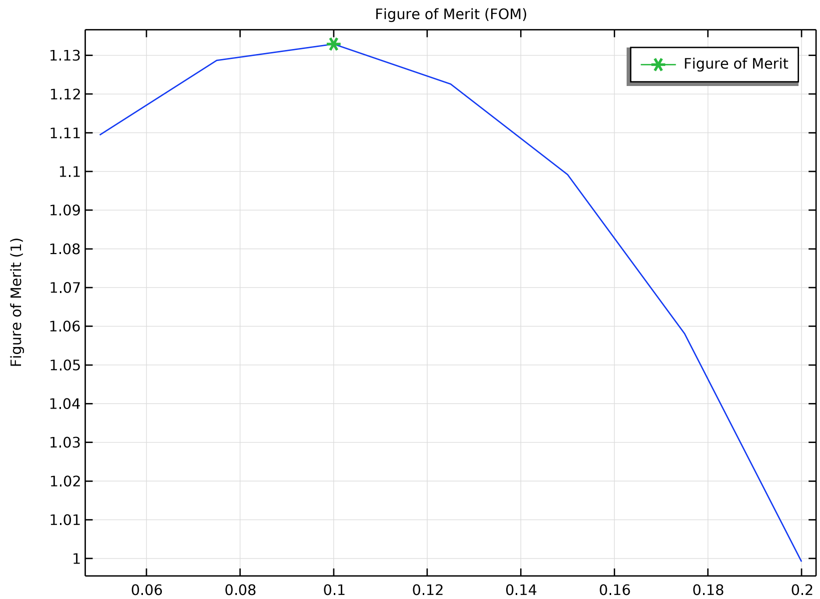

Figure 3: The Figure of Merit comparing the performances of the porous and the conventional MCHS. The result of the optimization study was added as an asterisk.

Notes About the COMSOL Implementation

Note that in COMSOL Multiphysics it is not only possible to use minimization of an objective function, which is the usual way, but also maximization. This makes it possible to just enter the Figure of Merit as the objective function.

Application Library path: `Porous_Media_Flow_Module/Heat_Transfer/porous_microchannel_heat_sink_optimization`

Modeling Instructions

ROOT

Start by loading the model file that contains the setup of the microchannel heat sink (MCHS) model with and without porous layer.

APPLICATION LIBRARIES

1 From the **File** menu, choose **Application Libraries**.

2 In the **Application Libraries** window, select **Porous Media Flow Module>Heat Transfer>porous_microchannel_heat_sink** in the tree.

3 Click **Open**.

To keep the results of the original model, add a new study including optimization.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **Preset Studies for Selected Multiphysics>Stationary, One-Way NITF**.

4 Right-click and choose **Add Study**.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 3: OPTIMIZATION

In the **Settings** window for **Study**, type `Study 3: Optimization` in the **Label** text field.

Optimization

1 In the **Study** toolbar, click **Optimization** and choose **Optimization**.

2 In the **Settings** window for **Optimization**, locate the **Optimization Solver** section.

3 From the **Method** list, choose **BOBYQA**, because the BOBYQA solver is generally the fastest of the derivative-free solvers when the objective function is smooth.

4 Locate the **Objective Function** section. In the table, enter the following settings:

Expression	Description	Evaluate for
`comp1.FOM`		Stationary 2

5 From the **Type** list, choose **Maximization**.

6 Locate the **Control Variables and Parameters** section. Click **Add**.

7 In the table, enter the following settings:

Parameter name	Initial value	Scale	Lower bound	Upper bound
th_porous (Porous structure thickness)	`0.1[mm]`	`0.1[mm]`	`0.05[mm]`	`0.2[mm]`

According to the parameter study, the optimum value must be around 0.1mm. Using this value as the initial value is helpful because the optimization algorithm uses this as the starting point for finding the optimized solution.

8 In the **Study** toolbar, click **Compute**.

The optimization study takes about the same time as the parametric study. Note, that the tolerance for the optimization is 0.01 which corresponds to an accuracy of 0.01mm whereas the parametric study uses 0.025mm steps. The fact that the parameter study calculated the optimal value is rather a coincidence.

RESULTS

To analyze the performance of the optimized porous MCHS and to compare it to the solutions of the model 'Performance of a Porous Microchannel Heat Sink', duplicate the **Global Evaluation 2** node and apply the new dataset.

1 In the **Model Builder** window, expand the **Results>Datasets** node.

Global Evaluation 1

1 In the **Model Builder** window, expand the **Results>Derived Values** node.

2 Right-click **Results>Derived Values>Global Evaluation 1** and choose **Duplicate**.

Global Evaluation 3

1 In the **Model Builder** window, click **Global Evaluation 3**.

2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 3: Optimization/Parametric Solutions 2 (sol15)**.

4 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
`FOM`	`1`	`Figure of Merit`

5 Click next to **Evaluate**, then choose **New Table**.

Heat-Transfer Coefficient and Pressure Drop

Add the values for the heat transfer coefficient, the pressure drop, the dimensionless Reynolds number, the Nusselt number, and the figure of merit as points in the existing table graphs.

Table Graph 3

1. In the **Model Builder** window, right-click **Heat-Transfer Coefficient and Pressure Drop** and choose **Table Graph**.
2. In the **Settings** window for **Table Graph**, locate the **Data** section.
3. From the **Table** list, choose **Table 5**.
4. From the **Plot columns** list, choose **Manual**.
5. In the **Columns** list, select **Pressure drop (Pa)**.
6. Click to expand the **Preprocessing** section. Find the **x-axis column** subsection. From the **Transformation** list, choose **Linear**.
7. In the **Scaling** text field, type 1000. This is necessary because the porous layer thickness as control parameter of the optimization is saved in m whereas in the existing table it is plotted in mm.
8. Locate the **Coloring and Style** section. Find the **Line markers** subsection. From the **Marker** list, choose **Asterisk**.
9. From the **Positioning** list, choose **Interpolated**.
10. From the **Color** list, choose **Blue**.
11. Right-click **Table Graph 3** and choose **Duplicate**.

Table Graph 4

1. In the **Model Builder** window, click **Table Graph 4**.
2. In the **Settings** window for **Table Graph**, locate the **Data** section.
3. In the **Columns** list, select **Heat transfer coefficient (W/(m^2*K))**.
4. Locate the **Coloring and Style** section. From the **Color** list, choose **Green**.
5. Locate the **y-Axis** section. Select the **Plot on secondary y-axis** check box.
6. In the **Heat-Transfer Coefficient and Pressure Drop** toolbar, click **Plot**. Compare the plot with Figure 1.

Global 1

1. In the **Model Builder** window, right-click **FOM** and choose **Global**.
2. In the **Settings** window for **Global**, locate the **Data** section.
3. From the **Dataset** list, choose **Study 3: Optimization/Solution 13 (sol13)**.

4 Locate the **y-Axis Data** section. In the table, enter the following settings:

Expression	Unit	Description
FOM	1	Figure of Merit

5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

6 In the **Expression** text field, type th_porous.

7 Click to expand the **Coloring and Style** section. Find the **Line markers** subsection. From the **Marker** list, choose **Asterisk**.

8 In the **FOM** toolbar, click **Plot**.

Table Graph 3

1 In the **Model Builder** window, expand the **Results>Reynolds and Nusselt Numbers** node.

2 Right-click **Reynolds and Nusselt Numbers** and choose **Table Graph**.

3 In the **Settings** window for **Table Graph**, locate the **Data** section.

4 From the **Table** list, choose **Table 5**.

5 From the **Plot columns** list, choose **Manual**.

6 In the **Columns** list, select **Reynolds number (1)**.

7 Locate the **Preprocessing** section. Find the **x-axis column** subsection. From the **Transformation** list, choose **Linear**.

8 In the **Scaling** text field, type 1000.

9 Locate the **Coloring and Style** section. From the **Color** list, choose **Blue**.

10 Find the **Line markers** subsection. From the **Marker** list, choose **Asterisk**.

11 From the **Positioning** list, choose **Interpolated**.

12 Click to expand the **Legends** section. In the **Reynolds and Nusselt Numbers** toolbar, click **Plot**.

13 Right-click **Table Graph 3** and choose **Duplicate**.

Table Graph 4

1 In the **Model Builder** window, click **Table Graph 4**.

2 In the **Settings** window for **Table Graph**, locate the **Data** section.

3 In the **Columns** list, select **Nusselt number (1)**.

4 Locate the **y-Axis** section. Select the **Plot on secondary y-axis** check box.

5 Locate the **Coloring and Style** section. From the **Color** list, choose **Green**.

6 In the **Reynolds and Nusselt Numbers** toolbar, click **Plot** and compare with Figure 2.

Pressure (spf) 2, Temperature (ht) 2, Temperature and Fluid Flow (nitf1) 2, Velocity (spf) 2

Finally, the plot groups can be grouped.

1. In the **Model Builder** window, under **Results**, Ctrl-click to select **Temperature (ht) 2**, **Velocity (spf) 2**, **Pressure (spf) 2**, and **Temperature and Fluid Flow (nitf1) 2**.
2. Right-click and choose **Group**.

Optimization

In the **Settings** window for **Group**, type `Optimization` in the **Label** text field.