

Mechanical Multiport System: Elastic Wave Propagation in a Small Aluminum Plate

In this tutorial, the vibrational behavior of a small aluminum plate with four waveguide structures is analyzed. This is an example of a structural component located in any device where elastic waves are propagating, like a smart speaker, an electric motor, or a MEMS device. The plate can be thought of as a mechanical multiport system. The model uses the port boundary condition at the inlet/outlet of the waveguide structures. The port conditions consistently capture and treat the different propagating elastic modes, like longitudinal, transverse, and torsional waves. The transmission and reflection of the various modes is characterized through the scattering matrix of the system, which is automatically computed. This allows the vibrational behavior of the component to be characterized in great detail, for example, for subsequent use in a system simulation.

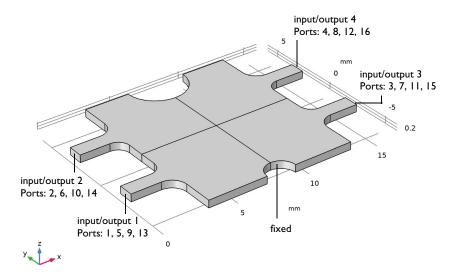


Figure 1: Model geometry layout, location of the inlet/outlets of the four waveguides, and the port numbers associated with each inlet/outlet.

Model Definition

The model consists of a small 16 mm by 12 mm by 0.5 mm aluminum piece that has four waveguide structures with an inlet/outlet (connection to other components), see Figure 1. The plate is fixed on one surface. The structure is meant to represent a small subcomponent in a device or system where vibrations are generated, for example, by a transducer. In order to control and understand transmission, such as feedback phenomena, it is necessary to understand how elastic waves (vibrations) propagate through the structure when excited. A compact lumped representation is given by the scattering or Smatrix of the system. The scattering matrix relates the incident and reflected mode amplitudes at all the inlet/outlets. Component $S_{
m ii}$, represents the amplitude of the outgoing mode at port i when the system is excited at port j. The system is excited at 10 kHz.

In this model, each inlet/outlet has four **Port** features added. Each port describes one specific propagating mode, either the longitudinal, the torsional, the transverse 1, or the transverse 2 mode. The modes are depicted in Figure 2. The scattering matrix elements are automatically computed when solving a model with a port feature; all the elements can be computed when the **Port Sweep** functionality is used. Using a combination of ports (representing all propagating modes) at an elastic waveguide outlet will also act as a perfect nonreflecting condition that is superior to a perfectly matched layer (PML) configuration.

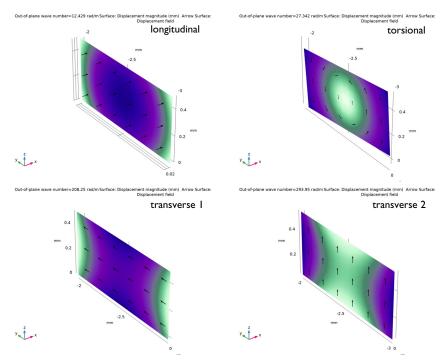


Figure 2: The four propagating modes captured by the ports located at the four inlet/outlets of the mechanical system.

The displacement of the multiport aluminum structure, when the four different modes are excited at inlet/outlet 1, is depicted in Figure 3, Figure 4, Figure 5, and Figure 6.

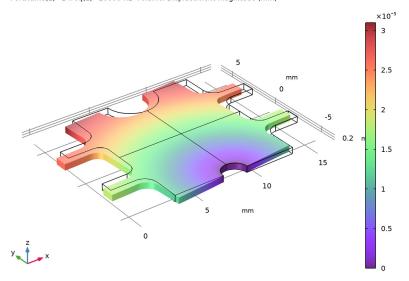


Figure 3: Displacement when the longitudinal mode is excited at the inlet/outlet 1.

PortName(2)=5 freq(1)=10000 Hz Volume: Displacement magnitude (mm)

×10⁻³ 1.5 0.5

Figure 4: Displacement when the torsional mode is excited at the inlet/outlet 1.

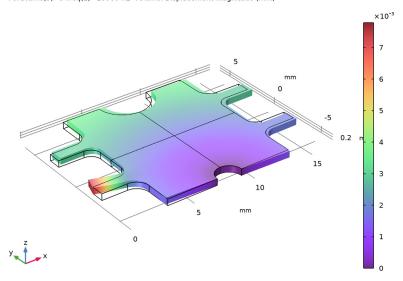


Figure 5: Displacement when the traverse 1 mode is excited at the inlet/outlet 1.

PortName(4)=13 freq(1)=10000 Hz Volume: Displacement magnitude (mm)

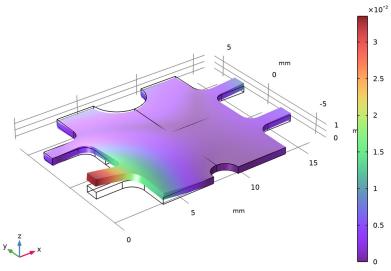


Figure 6: Displacement when the traverse 2 mode is excited at the inlet/outlet 1.

The power transmitted or reflected by the excitation of a given port can be assessed by plotting the absolute value of the scattering parameters squared $|S_{ii}|^2$. Note that the mode shapes are always scaled to unit power in the mechanical port formulation. For the acoustic ports, both amplitude and power scaling options exist.

The power of the various outgoing modes at all inlet/outlets, when the system is excited with a specific port, is depicted in Figure 7. The graphs show how energy is transmitted and reflected in the system and how energy is converted from one propagating mode to another.

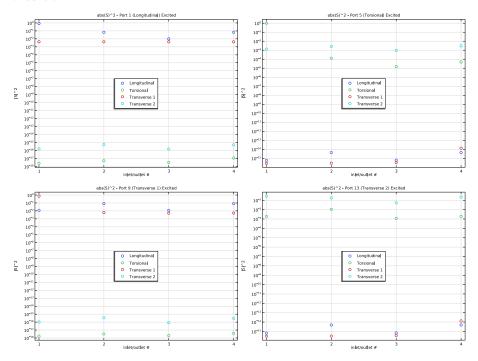


Figure 7: Power of the various outgoing modes at all inlet/outlets when the system is excited with a specific port.

Application Library path: Structural Mechanics Module/Elastic Waves/ mechanical_multiport_system

From the File menu, choose New.

NEW

In the New window, click Model Wizard.

MODEL WIZARD

- I In the Model Wizard window, click **3D**.
- 2 In the Select Physics tree, select Acoustics>Elastic Waves> Solid Mechanics (Elastic Waves) (solid).
- 3 Click Add.
- 4 Click 🗪 Study.
- 5 In the Select Study tree, select Empty Study.
- 6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- 3 Click Load from File.
- **4** Browse to the model's Application Libraries folder and double-click the file mechanical multiport system parameters.txt.

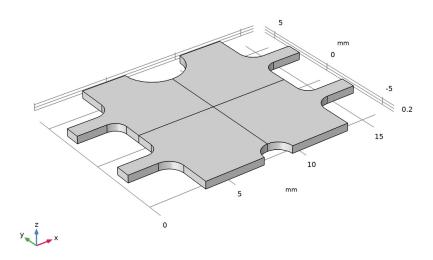
GEOMETRY I

- I In the Model Builder window, under Component I (compl) click Geometry I.
- 2 In the Settings window for Geometry, locate the Units section.
- 3 From the Length unit list, choose mm.

Import I (impl)

- I In the Home toolbar, click Import.
- 2 In the Settings window for Import, locate the Import section.
- 3 Click **Browse**.
- **4** Browse to the model's Application Libraries folder and double-click the file mechanical_multiport_system.mphbin.
- 5 Click Import.

6 In the Home toolbar, click **Build All**.



Create selections for the inlet/outlet boundary of the four waveguide structures. On each waveguide inlet/outlet boundary several Port features will be added, each capturing one specific propagating mode.

DEFINITIONS

inbut/outbut |

- I In the **Definitions** toolbar, click **\(\frac{1}{2} \) Explicit**.
- 2 In the Settings window for Explicit, type input/output 1 in the Label text field.
- 3 Locate the Input Entities section. From the Geometric entity level list, choose Boundary.
- 4 Select Boundary 1 only.

inbut/outbut 2

- I In the **Definitions** toolbar, click **\(\frac{1}{2} \) Explicit**.
- 2 In the Settings window for Explicit, type input/output 2 in the Label text field.
- 3 Locate the Input Entities section. From the Geometric entity level list, choose Boundary.
- 4 Select Boundary 6 only.

input/output 3

- I In the **Definitions** toolbar, click 🗣 **Explicit**.
- 2 In the Settings window for Explicit, type input/output 3 in the Label text field.
- 3 Locate the Input Entities section. From the Geometric entity level list, choose Boundary.
- **4** Select Boundary 47 only.

inbut/outbut 4

- I In the **Definitions** toolbar, click **\(\) Explicit**.
- 2 In the Settings window for Explicit, type input/output 4 in the Label text field.
- 3 Locate the Input Entities section. From the Geometric entity level list, choose Boundary.
- 4 Select Boundary 48 only.

ADD MATERIAL

- I In the Home toolbar, click **Add Material** to open the **Add Material** window.
- 2 Go to the Add Material window.
- 3 In the tree, select Built-in>Aluminum 6063-T83.
- 4 Click Add to Component in the window toolbar.
- 5 In the Home toolbar, click Radd Material to close the Add Material window.

SOLID MECHANICS (SOLID)

Fixed Constraint I

- I In the Model Builder window, under Component I (compl) right-click Solid Mechanics (solid) and choose Fixed Constraint.
- 2 Select Boundaries 23 and 25 only.

Now, proceed and add four port conditions per input/output boundary. First add the ports that represent the longitudinal modes, group them, and then duplicate them. Then add the ports that represent the torsional and the two translational modes.

Port 1

- I In the Physics toolbar, click **Boundaries** and choose Port.
- 2 In the Settings window for Port, locate the Boundary Selection section.
- 3 From the Selection list, choose input/output 1.

4 Locate the Incident Mode Settings section. From the Incident wave excitation at this port list, choose Off.

Note that by default the first port added has the excitation turned on. Here, turn it off; you can then select any port as source and run the model and use a port sweep to excite the desired ports. When a port sweep is performed, all ports are (automatically) one by one used as the excitation source. If all ports are excited, the full scattering matrix is computed.

Port 2

- I In the Physics toolbar, click **Boundaries** and choose Port.
- 2 In the Settings window for Port, locate the Boundary Selection section.
- 3 From the Selection list, choose input/output 2.

Port 3

- I In the Physics toolbar, click **Boundaries** and choose Port.
- 2 In the Settings window for Port, locate the Boundary Selection section.
- 3 From the Selection list, choose input/output 3.

Port 4

- I In the Physics toolbar, click **Boundaries** and choose Port.
- 2 In the Settings window for Port, locate the Boundary Selection section.
- 3 From the Selection list, choose input/output 4.

Port 1, Port 2, Port 3, Port 4

- I In the Model Builder window, under Component I (compl)>Solid Mechanics (solid), Ctrlclick to select Port I, Port 2, Port 3, and Port 4.
- **2** Right-click and choose **Group**.

Longitudinal Mode

- I In the Settings window for Group, type Longitudinal Mode in the Label text field.
- 2 Right-click Longitudinal Mode and choose Duplicate.

Torsional Mode

- I In the Model Builder window, under Component I (compl)>Solid Mechanics (solid) click Longitudinal Mode 1.
- 2 In the Settings window for Group, type Torsional Mode in the Label text field.
- 3 Right-click Torsional Mode and choose Duplicate.

Translational Mode 1

- I In the Model Builder window, under Component I (compl)>Solid Mechanics (solid) click Torsional Mode 1.
- 2 In the Settings window for Group, type Translational Mode 1 in the Label text field.
- 3 Right-click Translational Mode I and choose Duplicate.

Translational Mode 2

- I In the Model Builder window, under Component I (compl)>Solid Mechanics (solid) click Translational Mode 1.1.
- 2 In the Settings window for Group, type Translational Mode 2 in the Label text field.

Torsional Mode

In the Model Builder window, expand the Component I (compl)>Solid Mechanics (solid)> **Torsional Mode** node.

Translational Mode 1

In the Model Builder window, expand the Component I (compl)>Solid Mechanics (solid)> Translational Mode I node.

MESH I

- I In the Model Builder window, expand the Component I (compl)>Solid Mechanics (solid)> Translational Mode 2 node, then click Component I (compl)>Mesh I.
- 2 In the Settings window for Mesh, locate the Sequence Type section.
- 3 From the list, choose User-controlled mesh.

Size

- I In the Model Builder window, under Component I (compl)>Mesh I click Size.
- 2 In the Settings window for Size, locate the Element Size section.
- Click the Custom button.
- 4 Locate the Element Size Parameters section. In the Maximum element size text field, type 1.
- 5 In the Minimum element size text field, type 0.05.
- **6** In the **Curvature factor** text field, type **0.3**.
- 7 In the Resolution of narrow regions text field, type 5.

Free Tetrahedral I

In the Model Builder window, right-click Free Tetrahedral I and choose Delete.

Free Triangular I

- I In the Mesh toolbar, click \times More Generators and choose Free Triangular.
- 2 Select Boundaries 4 and 9 only.
- 3 In the Settings window for Free Triangular, click 📗 Build Selected.

Copy Face I

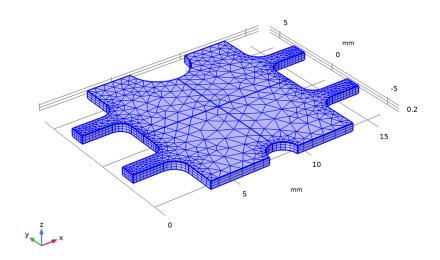
- I In the Mesh toolbar, click Copy and choose Copy Face.
- **2** Select Boundaries 4 and 9 only.
- 3 In the Settings window for Copy Face, locate the Destination Boundaries section.
- **4** Click to select the **Activate Selection** toggle button.
- **5** Select Boundaries 27 and 31 only.
- 6 Click | Build Selected.

Swebt I

In the Mesh toolbar, click A Swept.

Distribution I

- I Right-click Swept I and choose Distribution.
- 2 In the Settings window for Distribution, locate the Distribution section.
- **3** In the **Number of elements** text field, type **3**.



Before solving the full model, perform a single Boundary Mode Analysis study to compute the propagating modes and identify their mode wave numbers. Note that the values for this specific setup are already found in the Parameters list.

STUDY I - MODES

- I In the Model Builder window, click Study I.
- 2 In the Settings window for Study, type Study 1 Modes in the Label text field.
- 3 Locate the Study Settings section. Clear the Generate default plots check box.

Step 1: Boundary Mode Analysis

- I In the Study toolbar, click Study Steps and choose Other>Boundary Mode Analysis.
- 2 In the Settings window for Boundary Mode Analysis, locate the Study Settings section.
- 3 In the Mode analysis frequency text field, type f0.
- 4 Select the Desired number of modes check box. In the associated text field, type 4.
- 5 Select the Search for modes around shift check box. In the associated text field, type 0.8*
- 6 From the Search method around shift list, choose Larger real part.
- 7 In the Study toolbar, click **Compute**.

ADD PREDEFINED PLOT

- I In the Home toolbar, click Add Predefined Plot to open the Add Predefined Plot window.
- 2 Go to the Add Predefined Plot window.
- 3 In the tree, select Study I Modes/Solution I (soll)>Solid Mechanics>Mode Shape (solid).
- 4 Click **Add Plot** in the window toolbar.
- 5 In the Home toolbar, click Add Predefined Plot to close the Add Predefined Plot window.

RESULTS

Arrow Surface 1

- I Right-click Mode Shape (solid) and choose Arrow Surface.
- 2 In the Settings window for Arrow Surface, locate the Arrow Positioning section.
- 3 In the Number of arrows text field, type 20.
- 4 Locate the Coloring and Style section. From the Arrow length list, choose Logarithmic.
- 5 From the Color list, choose Black.

Selection I

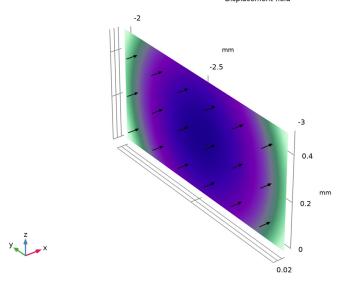
- I Right-click Arrow Surface I and choose Selection.
- 2 Select Boundary 1 only.

Mode Shape (solid)

- I In the Model Builder window, under Results click Mode Shape (solid).
- 2 In the Settings window for 3D Plot Group, locate the Plot Settings section.
- **3** Clear the **Plot dataset edges** check box.
- 4 In the Mode Shape (solid) toolbar, click Plot.

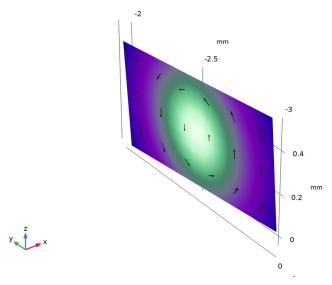
5 Click the **Zoom Extents** button in the **Graphics** toolbar.

Out-of-plane wave number=12.429 rad/m Surface: Displacement magnitude (mm) Arrow Surface:
Displacement field



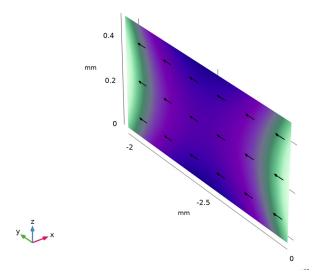
6 Locate the Data section. From the Out-of-plane wave number (rad/m) list, choose 27.342.

Out-of-plane wave number=27.342 rad/m Surface: Displacement magnitude (mm) Arrow Surface:
Displacement field



8 From the Out-of-plane wave number (rad/m) list, choose 208.25.

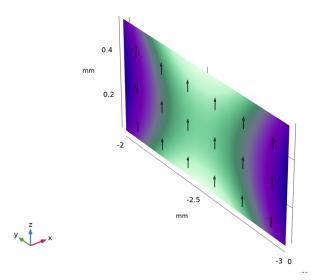
Out-of-plane wave number=208.25 rad/m Surface: Displacement magnitude (mm) Arrow Surface:
Displacement field



10 From the Out-of-plane wave number (rad/m) list, choose 293.95.

II In the Mode Shape (solid) toolbar, click **Plot**.

Out-of-plane wave number=293.95 rad/m Surface: Displacement magnitude (mm) Arrow Surface: Displacement field



Proceed to do the full system analysis. This requires setting up a study with one **Boundary** Mode Analysis per port feature. In each step, it should be ensured that the correct mode is computed (longitudinal, torsional, transverse). The search uses the values of the mode wave numbers computed in the previous study (also found in the Parameters list).

For simplicity, one inlet/outlet boundary will be selected as the source of excitations (here the inlet/outlet 1). The desired modes will be activated one at a time using the port sweep functionality. It is activated in the **Port Sweep Settings** section and ports 1, 5, 9, and 13 are used as sources.

ADD STUDY

- I In the Home toolbar, click Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select Empty Study.
- 4 Click Add Study in the window toolbar.
- 5 In the Home toolbar, click Add Study to close the Add Study window.

STUDY 2 - FREQUENCY DOMAIN (PORT SWEEP)

- I In the **Settings** window for **Study**, type Study 2 Frequency Domain (Port Sweep) in the Label text field.
- 2 Locate the Study Settings section. Clear the Generate default plots check box.
- 3 Clear the Generate convergence plots check box.

Step 1: Boundary Mode Analysis

- I In the Study toolbar, click Study Steps and choose Other>Boundary Mode Analysis.
- 2 In the Settings window for Boundary Mode Analysis, locate the Study Settings section.
- 3 In the Mode analysis frequency text field, type f0.
- 4 Select the **Desired number of modes** check box.
- 5 Select the Search for modes around shift check box. In the associated text field, type 0.9* k lo.
- 6 From the Search method around shift list, choose Larger real part.
- 7 Right-click Step 1: Boundary Mode Analysis and choose Duplicate.

Step 2: Boundary Mode Analysis I

- I In the Model Builder window, click Step 2: Boundary Mode Analysis I.
- 2 In the Settings window for Boundary Mode Analysis, locate the Study Settings section.
- 3 In the Port name text field, type 2.
- 4 Right-click Step 2: Boundary Mode Analysis I and choose Duplicate.

Step 3: Boundary Mode Analysis 2

- I In the Model Builder window, click Step 3: Boundary Mode Analysis 2.
- 2 In the Settings window for Boundary Mode Analysis, locate the Study Settings section.
- **3** In the **Port name** text field, type **3**.
- 4 Right-click Step 3: Boundary Mode Analysis 2 and choose Duplicate.

Step 4: Boundary Mode Analysis 3

- I In the Model Builder window, click Step 4: Boundary Mode Analysis 3.
- 2 In the Settings window for Boundary Mode Analysis, locate the Study Settings section.
- 3 In the Port name text field, type 4.
- 4 Right-click Step 4: Boundary Mode Analysis 3 and choose Duplicate.

Step 5: Boundary Mode Analysis 4

I In the Model Builder window, click Step 5: Boundary Mode Analysis 4.

- 2 In the Settings window for Boundary Mode Analysis, locate the Study Settings section.
- 3 In the Port name text field, type 5.
- 4 In the Search for modes around shift text field, type 0.9*k to.
- 5 Right-click Step 5: Boundary Mode Analysis 4 and choose Duplicate.

Step 6: Boundary Mode Analysis 5

- I In the Model Builder window, click Step 6: Boundary Mode Analysis 5.
- 2 In the Settings window for Boundary Mode Analysis, locate the Study Settings section.
- 3 In the Port name text field, type 6.
- 4 Right-click Step 6: Boundary Mode Analysis 5 and choose Duplicate.

Step 7: Boundary Mode Analysis 6

- I In the Model Builder window, click Step 7: Boundary Mode Analysis 6.
- 2 In the Settings window for Boundary Mode Analysis, locate the Study Settings section.
- **3** In the **Port name** text field, type 7.
- 4 Right-click Step 7: Boundary Mode Analysis 6 and choose Duplicate.

Step 8: Boundary Mode Analysis 7

- I In the Model Builder window, click Step 8: Boundary Mode Analysis 7.
- 2 In the Settings window for Boundary Mode Analysis, locate the Study Settings section.
- 3 In the Port name text field, type 8.
- 4 Right-click Step 8: Boundary Mode Analysis 7 and choose Duplicate.

Step 9: Boundary Mode Analysis 8

- I In the Model Builder window, click Step 9: Boundary Mode Analysis 8.
- 2 In the Settings window for Boundary Mode Analysis, locate the Study Settings section.
- **3** In the **Port name** text field, type **9**.
- 4 In the Search for modes around shift text field, type 0.9*k tr1.
- 5 Right-click Step 9: Boundary Mode Analysis 8 and choose Duplicate.

Step 10: Boundary Mode Analysis 9

- I In the Model Builder window, click Step 10: Boundary Mode Analysis 9.
- 2 In the Settings window for Boundary Mode Analysis, locate the Study Settings section.
- 3 In the Port name text field, type 10.
- 4 Right-click Step 10: Boundary Mode Analysis 9 and choose Duplicate.

Step 11: Boundary Mode Analysis 10

- I In the Model Builder window, click Step II: Boundary Mode Analysis 10.
- 2 In the Settings window for Boundary Mode Analysis, locate the Study Settings section.
- 3 In the Port name text field, type 11.
- 4 Right-click Step 11: Boundary Mode Analysis 10 and choose Duplicate.

Step 12: Boundary Mode Analysis 11

- I In the Model Builder window, click Step 12: Boundary Mode Analysis 11.
- 2 In the Settings window for Boundary Mode Analysis, locate the Study Settings section.
- 3 In the Port name text field, type 12.
- 4 Right-click Step 12: Boundary Mode Analysis 11 and choose Duplicate.

Step 13: Boundary Mode Analysis 12

- I In the Model Builder window, click Step 13: Boundary Mode Analysis 12.
- 2 In the Settings window for Boundary Mode Analysis, locate the Study Settings section.
- 3 In the Port name text field, type 13.
- 4 In the Search for modes around shift text field, type 0.9*k_tr2.
- 5 Right-click Step 13: Boundary Mode Analysis 12 and choose Duplicate.

Step 14: Boundary Mode Analysis 13

- I In the Model Builder window, click Step 14: Boundary Mode Analysis 13.
- 2 In the Settings window for Boundary Mode Analysis, locate the Study Settings section.
- 3 In the Port name text field, type 14.
- 4 Right-click Step 14: Boundary Mode Analysis 13 and choose Duplicate.

Step 15: Boundary Mode Analysis 14

- I In the Model Builder window, click Step 15: Boundary Mode Analysis 14.
- 2 In the Settings window for Boundary Mode Analysis, locate the Study Settings section.
- 3 In the Port name text field, type 15.
- 4 Right-click Step 15: Boundary Mode Analysis 14 and choose Duplicate.

Step 16: Boundary Mode Analysis 15

- I In the Model Builder window, click Step 16: Boundary Mode Analysis 15.
- 2 In the Settings window for Boundary Mode Analysis, locate the Study Settings section.
- 3 In the Port name text field, type 16.

Step 17: Frequency Domain

- I In the Study toolbar, click Study Steps and choose Frequency Domain> Frequency Domain.
- 2 In the Settings window for Frequency Domain, locate the Study Settings section.
- **3** In the **Frequencies** text field, type **f**0.

SOLID MECHANICS (SOLID)

- I In the Model Builder window, under Component I (compl) click Solid Mechanics (solid).
- 2 In the Settings window for Solid Mechanics, click to collapse the Port Sweep Settings section.
- 3 Click to expand the Port Sweep Settings section. From the list, choose Activate port sweep.

Remember to add the PortName parameter to the **Parameters** list.

GLOBAL DEFINITIONS

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description
PortName	1	1	Port name parameter

STUDY 2 - FREQUENCY DOMAIN (PORT SWEEP)

Parametric Sweep

- I In the Study toolbar, click Parametric Sweep.
- 2 In the Settings window for Parametric Sweep, locate the Study Settings section.
- 3 Click + Add.
- **4** In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
PortName (Port name	1, 5, 9, 13	
parameter)		

5 In the Study toolbar, click **Compute**.

ADD PREDEFINED PLOT

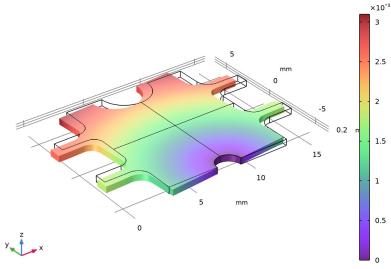
- I In the Home toolbar, click Add Predefined Plot to open the Add Predefined Plot window.
- 2 Go to the Add Predefined Plot window.
- 3 In the tree, select Study 2 Frequency Domain (Port Sweep)/ Parametric Solutions I (sol19)>Solid Mechanics>Displacement (solid).
- 4 Click Add Plot in the window toolbar.
- 5 In the Home toolbar, click **Add Predefined Plot** to close the **Add Predefined Plot** window.

RESULTS

Displacement (solid)

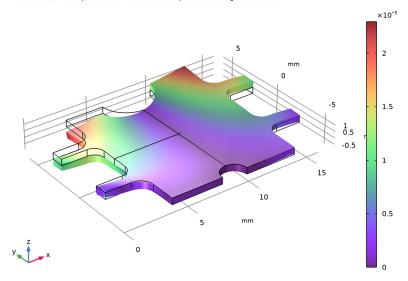
- I Click the Zoom Extents button in the Graphics toolbar.
- 2 In the Settings window for 3D Plot Group, locate the Data section.
- 3 From the Parameter value (PortName) list, choose 1.
- 4 In the Displacement (solid) toolbar, click Plot.

PortName(1)=1 freq(1)=10000 Hz Volume: Displacement magnitude (mm)



5 From the Parameter value (PortName) list, choose 5.

PortName(2)=5 freq(1)=10000 Hz Volume: Displacement magnitude (mm)



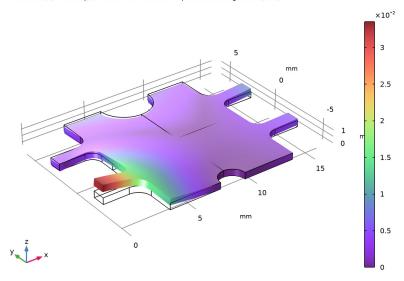
7 From the Parameter value (PortName) list, choose 9.

PortName(3)=9 freq(1)=10000 Hz Volume: Displacement magnitude (mm) ×10⁻³ 1

9 From the Parameter value (PortName) list, choose 13.

10 In the **Displacement (solid)** toolbar, click **Plot**.





Included in the solution, when using the port feature, is the automatic computation of the scattering parameter S_ij. This represents the amplitude of the outgoing wave at port i when excited at port j. The variable S11 is, for example, called solid. Smatrix11, if the port number has two digits the format is, for example, solid. Smatrix1 12. Now, set up 4 plots that show the outgoing energy of the various modes (proportional to abs(S_ij)^2) at the four inlet/outlets. Each plot corresponds to exciting a specific mode at the inlet/outlet number 1. Setting up the plots is a bit time consuming but can be simplified using **Duplicate** as well as copy/paste of expressions. The instructions are omitted here as they are long. For details, open the model and inspect the plots.

