Created in COMSOL Multiphysics 6.2

Torsion of an Isotropic Cosserat Elastic Cylinder

Introduction

The Cosserat theory of elasticity is one of the generalized continuum mechanics theories. It is also known as micropolar theory of elasticity, micropolar continuum mechanics or just micropolar elasticity. The theory incorporates, beside the displacement vector as in classical elasticity, the concept of the local rotation (or local spin) in the continuum, such that each point has six degrees of freedom: three translations and three rotations. The three rotations are also called microrotations. This theory can be used to model inhomogeneous materials, foams, masonries, bones, and so on.

This example demonstrates how to implement an isotropic linear Cosserat elasticity model through the use of the Weak Form PDE interface. A cylindrical bar made out of a Cosserat material is subject to a pure torsion where it is possible to observe the size effect on the response (Ref. 1).

Model Definition

A cylindrical bar with initial radius R_0 and initial length L_0 is axially twisted. It is fully constrained at one end and a prescribed rotation θ is applied on the opposite end. The parameter values are reported in Table 1.

TABLE 1: GEOMETRY AND APPLIED ROTATION.

PARAMETER	VALUE
R_0	1 mm
L_0	10 mm
θ	14 deg

GOVERNING EQUATIONS

In Cosserat media each point has 6 degrees of freedom: three for the displacement vector $\mathbf{u}$, and three microrotations (vector $\mathbf{a}$). The constitutive equation for the Cauchy stress σ is augmented with an additional nonsymmetric term

$$\bar{\sigma} = \sigma + 2\mu_c(W - A) \tag{1}$$

This additional term depends on the difference of the skew-symmetric tensors W and A, and it can be seen as the force stress originated from the difference between the macrorotation and the local microrotation. These tensors are defined as follows:

$$W = \frac{1}{2}(\nabla\mathbf{u} - \nabla\mathbf{u}^T) = \frac{1}{2}\mathrm{anti}(\nabla\times\mathbf{u}) \tag{2}$$

$$A = \mathrm{anti}(\mathbf{a}) = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \tag{3}$$

where a_1, a_2, and a_3 are the components of the microrotation vector $\mathbf{a}$. The proportionality modulus μ_c, called *Cosserat couple modulus* (SI unit: Pa), relates the degree of coupling between the micro and macro rotations. The augmented stress $\bar{\sigma}$ is called *force stress* tensor, and it is used in the balance of linear momentum

$$\nabla \cdot \bar{\sigma} + \mathbf{f} = \mathbf{0} \tag{4}$$

In Cosserat theory, the conservation of angular momentum needs to be included as an additional equation, so that an additional tensor is necessary. It is called the *couple stress* or *stress moment* tensor, and it is defined as a linear combination of the gradient of the microrotation vector:

$$m = \alpha \mathrm{tr}(\nabla \mathbf{a}) I + \beta \nabla \mathbf{a}^T + \gamma \nabla \mathbf{a} \tag{5}$$

Here, α, β, and γ are the so-called first, second, and third microrotation parameters (SI unit: N). The couple stress tensor m will be symmetric, traceless, or nonsymmetric depending on their values. In Ref. 2 these parameters are written in terms of the Cosserat couple modulus μ_c and a length scale parameter. This example considers the following combination (the so-called pointwise positive case):

$$\alpha = \beta = 0 \tag{6}$$

$$\gamma = \mu_c L_c^2 \tag{7}$$

Here, L_c is the length scale that characterizes the Cosserat continuum. The couple stress tensor m is then nonsymmetric, and it can be written as follows:

$$m = \gamma \nabla \mathbf{a} = \mu_c L_c^2 \nabla \mathbf{a} \tag{8}$$

The balance of angular momentum reads:

$$-\nabla \cdot m = -\nabla \cdot \gamma \nabla \mathbf{a} = 4\mu_c axl(W - A) \tag{9}$$

Here, the axial operator reads $axl(A) = \mathbf{a}$. The solution to the problem in terms of $\mathbf{u}$ and $\mathbf{a}$ is obtained by solving Equation 4 and Equation 9 together with the constitutive equations Equation 1 and Equation 8 and additional boundary conditions. In this example, only Dirichlet boundary conditions are considered.

WEAK FORM

Assuming no Neumann boundary condition are applied (no boundary loads), the weak contribution from the momentum conservation (Equation 4) is

$$-\int_V \sigma : \delta\varepsilon dV - \int_V 2\mu_c (W - A) : \delta W dV \tag{10}$$

The first term on the right-hand side is already included in the classical theory of elasticity and the second term is specific for Cosserat theory. The weak contribution coming from the angular momentum conservation (Equation 9) is

$$-\int_V m : \delta(\nabla \mathbf{a}) dV - \int_V 2\mu_c (W - A) : \delta A dV \tag{11}$$

This contribution is not present in the classical theory of elasticity.

MATERIAL PROPERTIES

An isotropic Cosserat medium needs six independent material parameters. Beside the Young's modulus and Poisson's ratio (E,ν), additional parameters include the three microrotation moduli (α, β, γ) and the couple modulus μ_c. Their values are reported in Table 2.

TABLE 2: MATERIAL PROPERTIES.

PARAMETER	VALUE
E	1 MPa
ν	0.3
μ	$E/(2(1+\nu))$
α	0
β	0
γ	μL_c^2
μ_c	0.01μ, μ, 100μ

BOUNDARY CONDITIONS

Dirichlet boundary conditions are applied in this example.

- Displacements are fully constrained on the bottom surface of the cylinder: $\mathbf{u} = 0$

- Microrotations are fully constrained on the bottom surface of the cylinder: $\mathbf{a} = 0$
- On the top surface, the displacement degrees of freedom $\mathbf{u}$ are constrained to reproduce a rigid rotation around the cylinder axis, whereas the microrotation degrees of freedom are free.

Results and Discussion

Figure 1 shows the size effect through the Cosserat length scale L_c on the reaction moment.

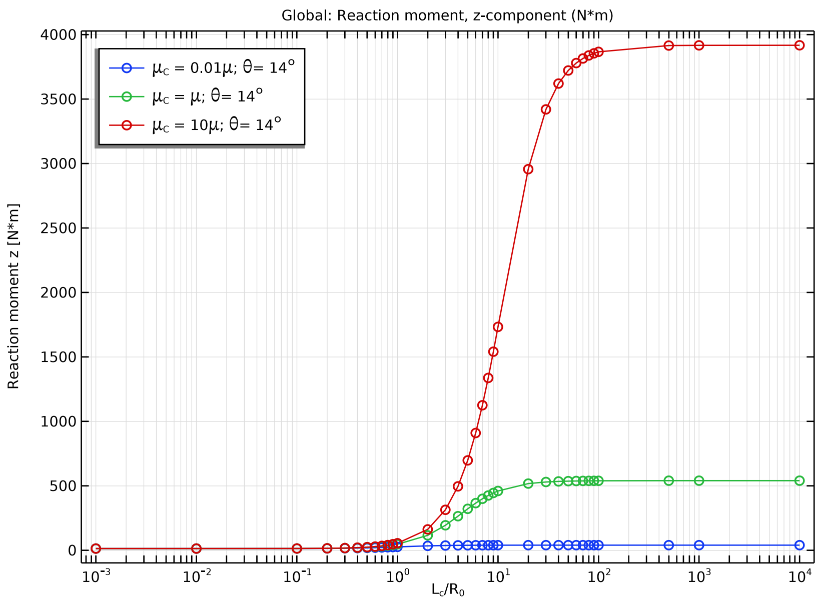

Figure 1: Torque versus scaled Cosserat length scale for different values of the Cosserat couple modulus μ_c *as compared to the macroscopic shear modulus* μ.

Figure 2 shows the reaction moment for the case with the smallest value of Cosserat couple module, $\mu_c = 0.01\mu$. Three different zones can be identified. The first zone, so-called linear Cauchy elasticity, extends up to $L_c < 0.1R_0$. Here there is no size effect and the material behaves as a linear elastic solid (Zone I). There is a transition zone for values between $0.1R_0 < L_c < 10R_0$, where the size effect on the solution (Zone II) is clearly visible. For higher values, $10R_0 < L_c$, the Cosserat effect dominates and the microrotation is nearly constant (Zone III).

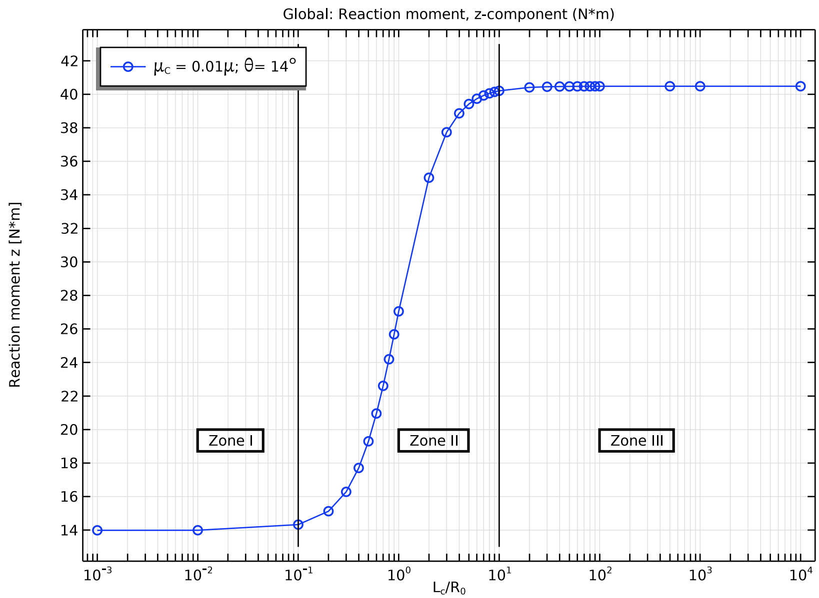

Figure 2: Torque versus scaled Cosserat length scale. Three different zones can be identified.

The bar becomes stiffer as the ratio between the parameter L_c and the cylinder's radius R_0 increases. The bar twists only in the proximity of the constrained end (Figure 3) and the microrotation assume a constant value along the cylinder (Figure 4).

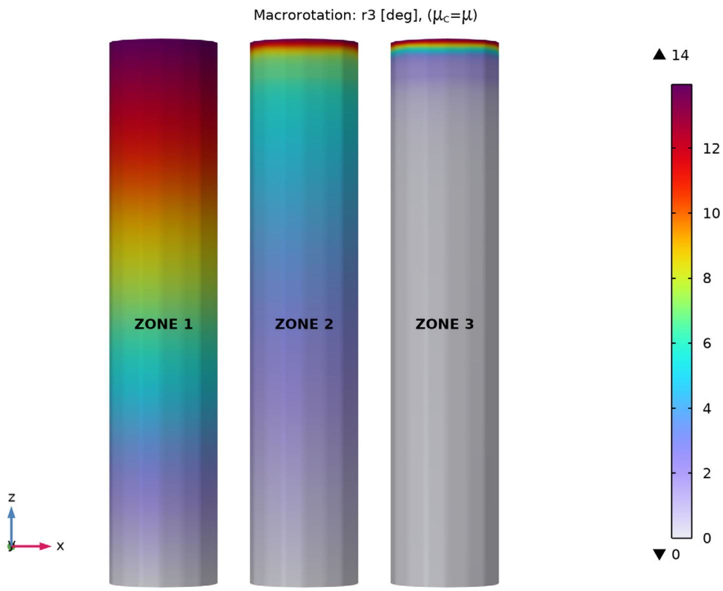

Figure 3: Macrorotation r_3 for $\mu_c = \mu$ and different values of the Cosserat length parameter L_c.

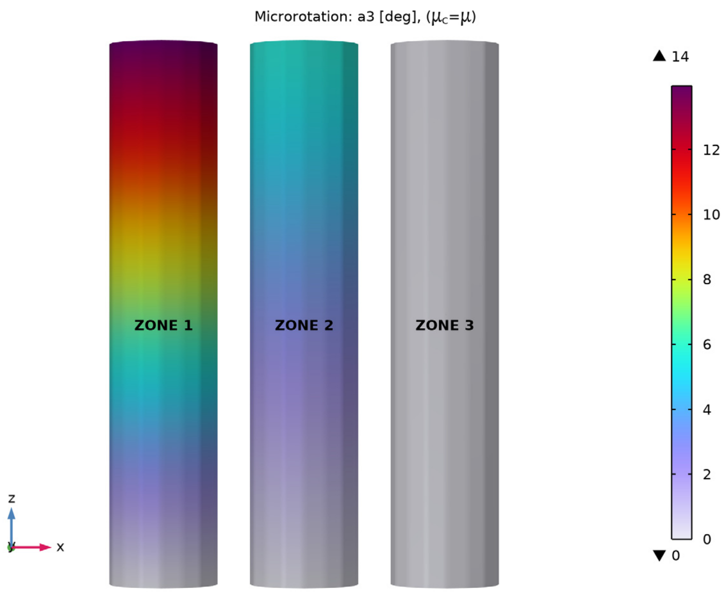

Figure 4: Microrotation, a_3 for $\mu_c = \mu$ and different values of the Cosserat length parameter L_c.

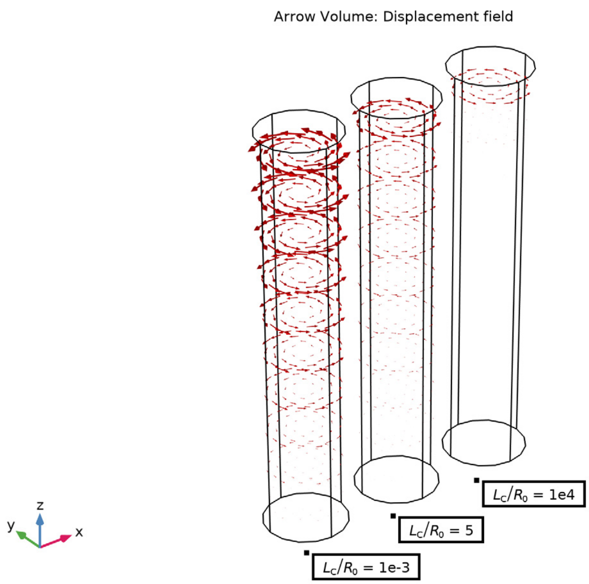

Figure 5: Displacement field before, during, and after transition.

Notes About the COMSOL Implementation

The equation for the balance of angular momentum is added through the use of a **Weak Form PDE** interface, where the three components of the microrotation vector are used as dependent variables.

The **External Stress** feature is used to account for the contribution of the asymmetric stress tensor that multiplies the macrorotation test function.An asymmetric stress can be entered using the **Stress tensor (Nominal)** option under **Stress input**. Alternatively, this contribution can be added directly in the **Weak Form PDE** interface together with the other additional terms.

A **Rigid Connector** is used to apply a rotation at the top of the cylindrical bar. Check the **Evaluate reaction forces** option to automatically compute the torque needed to twist the bar.

Default shape functions are used for the displacement field $\mathbf{u}$. Linear Lagrange shape functions are used for the microrotation vector $\mathbf{a}$.

A parametric study is used to loop over the Cosserat couple modulus and an auxiliary sweep to loop over the Cosserat length scale parameter.

A **Fully Coupled** solver is used to obtain convergence for higher values of the Cosserat couple modulus.

References

1. J. Jeong and H. Ramezani, “Implementation of the finite isotropic linear Cosserat models based on the weak form,” *Proc. COMSOL Conf.*, Hannover, 2008.

2. J. Jeong and others, “A numerical study for linear isotropic Cosserat elasticity with conformally invariant curvature,” *Z. Angew. Math. Mech.* vol. 89, pp. 552–569, 2009.

Application Library path: `Structural_Mechanics_Module/Material_Models/cosserat_torsion`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.

2 In the **Select Physics** tree, select **Structural Mechanics>Solid Mechanics (solid)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Stationary**.

6 Click **Done**.

GLOBAL DEFINITIONS

General Parameters

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, type `General Parameters` in the **Label** text field.

3 Locate the **Parameters** section. In the table, enter the following settings:

Name	Expression	Value	Description
R0	1[mm]	0.001 m	Radius
L0	10[mm]	0.01 m	Length
E0	1e6[MPa]	1E12 Pa	Young's modulus
Nu0	0.3	0.3	Poisson's ratio
Theta0	14[deg]	0.24435 rad	Applied rotation
mu0	E0/(2*(1+Nu0))	3.8462E11 Pa	Shear modulus

Cosserat Parameters

1 In the **Home** toolbar, click **Parameters** and choose **Add>Parameters**.

Insert the parameters related to the Cosserat medium.

2 In the **Settings** window for **Parameters**, type `Cosserat Parameters` in the **Label** text field.

3 Locate the **Parameters** section. In the table, enter the following settings:

Name	Expression	Value	Description
muC	0.01*mu0	3.8462E9 Pa	Cosserat couple modulus
LcR0	100	100	Internal length scale parameter
gammaC	mu0*(LcR0*R0)^2	3.8462E9 N	Third microrotation parameter

GEOMETRY 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.

2 In the **Settings** window for **Geometry**, locate the **Units** section.

3 From the **Length unit** list, choose **mm**.

Cylinder 1 (cyl1)

1 In the **Geometry** toolbar, click **Cylinder**.

2 In the **Settings** window for **Cylinder**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `R0`.

4 In the **Height** text field, type `L0`.

5 Click **Build Selected**.

MATERIALS

Material 1 (mat1)

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, locate the **Material Contents** section.

3 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Young's modulus	E	E0	Pa	Young's modulus and Poisson's ratio
Poisson's ratio	nu	Nu0	1	Young's modulus and Poisson's ratio
Density	rho	1	kg/m³	Basic

ADD PHYSICS

1 In the **Home** toolbar, click **Add Physics** to open the **Add Physics** window.

2 Go to the **Add Physics** window.

3 In the tree, select **Mathematics>PDE Interfaces>Weak Form PDE (w)**.

4 Click to expand the **Dependent Variables** section. In the **Field name (1)** text field, type a.

5 In the **Number of dependent variables** text field, type 3.

6 In the **Dependent variables (1)** table, enter the following settings:

a1

7 Click **Add to Component 1** in the window toolbar.

8 In the **Home** toolbar, click **Add Physics** to close the **Add Physics** window.

MICROROTATION FIELD

1 In the **Settings** window for **Weak Form PDE**, click to expand the **Discretization** section.

2 From the **Element order** list, choose **Linear**.

3 From the **Frame** list, choose **Material**.

4 In the **Label** text field, type `Microrotation Field`.

Define different tensors such as the microrotation tensor.

DEFINITIONS

Macrorotation Vector

1 In the **Model Builder** window, expand the **Component 1 (comp1)>Definitions** node.

2 Right-click **Definitions** and choose **Variables**.

3 In the **Settings** window for **Variables**, type `Macrorotation Vector` in the **Label** text field.

4 Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
w1	solid.curlUX		Curl of displacement, 1 component
w2	solid.curlUY		Curl of displacement, 2 component
w3	solid.curlUZ		Curl of displacement, 3 component
r1	w1/2		Macrorotation vector, 1 component
r2	w2/2		Macrorotation vector, 2 component
r3	w3/2		Macrorotation vector, 3 component

5 Click the **Show More Options** button in the **Model Builder** toolbar.

6 In the **Show More Options** dialog box, in the tree, select the check box for the node **General>Variable Utilities**.

7 Click **OK**.

Macrorotation

1 In the **Home** toolbar, click **Variable Utilities** and choose **Local>Matrix**.

2 In the **Settings** window for **Matrix**, locate the **Input Matrix** section.

3 In the table, enter the following settings:

0	-r3	r2
r3	0	-r1
-r2	r1	0

4 In the **Label** text field, type `Macrorotation`.

5 In the **Name** text field, type `W`.

Microrotation

1 In the **Home** toolbar, click **Variable Utilities** and choose **Local>Matrix**.

2 In the **Settings** window for **Matrix**, type `Microrotation` in the **Label** text field.

3 In the **Name** text field, type `A`.

4 Locate the **Input Matrix** section. In the table, enter the following settings:

0	-a3	a2
a3	0	-a1
-a2	a1	0

Stress Moment Tensor

1 In the **Home** toolbar, click **Variable Utilities** and choose **Local>Matrix**.

2 In the **Settings** window for **Matrix**, locate the **Input Matrix** section.

3 In the table, enter the following settings:

gammaC*a1X	gammaC*a1Y	gammaC*a1Z
gammaC*a2X	gammaC*a2Y	gammaC*a2Z
gammaC*a3X	gammaC*a3Y	gammaC*a3Z

4 In the **Label** text field, type `Stress Moment Tensor`.

5 In the **Name** text field, type `M`.

Asymmetric Stress Tensor

1 In the **Home** toolbar, click **Variable Utilities** and choose **Local>Matrix**.

2 In the **Settings** window for **Matrix**, type `Asymmetric Stress Tensor` in the **Label** text field.

3 In the **Name** text field, type `Pc`.

4 Locate the **Input Matrix** section. In the table, enter the following settings:

2*muC*(W11-A11)	2*muC*(W12-A12)	2*muC*(W13-A13)
2*muC*(W21-A21)	2*muC*(W22-A22)	2*muC*(W23-A23)
2*muC*(W31-A31)	2*muC*(W32-A32)	2*muC*(W33-A33)

The symmetric Cauchy stress tensor is augmented with an asymmetric stress.

SOLID MECHANICS (SOLID)

Linear Elastic Material 1

In the **Model Builder** window, under **Component 1 (comp1)>Solid Mechanics (solid)** click **Linear Elastic Material 1**.

External Stress 1

1 In the **Physics** toolbar, click **Attributes** and choose **External Stress**.

2 In the **Settings** window for **External Stress**, locate the **External Stress** section.

3 From the **Stress input** list, choose **Stress tensor (Nominal)**.

4 In the P_{ext} table, enter the following settings:

Pc11	Pc12	Pc13
Pc21	Pc22	Pc23
Pc31	Pc32	Pc33

Constrain one end of the bar and apply a rigid rotation at the other end.

Fixed Constraint 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Fixed Constraint**.

2 Select Boundary 3 only.

Rigid Connector 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Rigid Connector**.

2 Select Boundary 4 only.

3 In the **Settings** window for **Rigid Connector**, locate the **Center of Rotation** section.

4 From the list, choose **Centroid of selected entities**.

5 Locate the **Prescribed Displacement at Center of Rotation** section. Select the **Prescribed in x direction** check box.

6 Select the **Prescribed in y direction** check box.

7 Select the **Prescribed in z direction** check box.

8 Locate the **Prescribed Rotation** section. From the **By** list, choose **Prescribed rotation**.

9 Specify the Ω vector as

0	x
0	y
1	z

10 In the ϕ_0 text field, type `Theta0`.

11 Click to expand the **Reaction Force Settings** section. Select the **Evaluate reaction forces** check box.

Center of Rotation: Boundary 1

1 In the **Model Builder** window, click **Center of Rotation: Boundary 1**.

2 Select Boundary 4 only.

The Cosserat medium introduces the microrotations as additional degrees of freedom. Their contribution to the weak form are now added.

MICROROTATION FIELD (W)

Weak Form PDE 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Microrotation Field (w)** click **Weak Form PDE 1**.

2 In the **Settings** window for **Weak Form PDE**, locate the **Weak Expressions** section.

3 In the weak text-field array, type `Pc11*test(A11)+Pc12*test(A12)+Pc13*test(A13)-M11*test(a1X)-M12*test(a1Y)-M13*test(a1Z)` on the first row.

4 In the weak text-field array, type `Pc21*test(A21)+Pc22*test(A22)+Pc23*test(A23)-M21*test(a2X)-M22*test(a2Y)-M23*test(a2Z)` on the second row.

5 In the weak text-field array, type `Pc31*test(A31)+Pc32*test(A32)+Pc33*test(A33)-M31*test(a3X)-M32*test(a3Y)-M33*test(a3Z)` on the third row.

Dirichlet Boundary Condition 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Dirichlet Boundary Condition**.

2 Select Boundary 3 only.

MESH 1

Free Quad 1

1 In the **Mesh** toolbar, click **More Generators** and choose **Free Quad**.

2 Select Boundary 3 only.

Swept 1

In the **Mesh** toolbar, click **Swept**.

Distribution 1

1 Right-click **Swept 1** and choose **Distribution**.

2 In the **Settings** window for **Distribution**, locate the **Distribution** section.

3 In the **Number of elements** text field, type `30`.

Size

1 In the **Model Builder** window, under **Component 1 (comp1)>Mesh 1** click **Size**.

2 In the **Settings** window for **Size**, locate the **Element Size** section.

3 From the **Predefined** list, choose **Finer**.

4 Click **Build All**.

Three Cosserat couple parameters are considered through the use of a parametric sweep. For each Cosserat couple parameter the Cosserat length scale is varied through an auxiliary sweep.

STUDY 1

1 In the **Model Builder** window, click **Study 1**.

2 In the **Settings** window for **Study**, locate the **Study Settings** section.

3 Clear the **Generate default plots** check box.

Step 1: Stationary

1 In the **Model Builder** window, under **Study 1** click **Step 1: Stationary**.

2 In the **Settings** window for **Stationary**, click to expand the **Study Extensions** section.

3 Select the **Auxiliary sweep** check box.

4 Click + **Add**.

5 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
LcR0 (Internal length scale parameter)	`1e-3 1e-2 range(0.1,0.1,1) range(2,1,10) range(20,10,100) 500 1e3 1e4`	

Parametric Sweep

1 In the **Study** toolbar, click **Parametric Sweep**.

2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

3 Click + **Add**.

4 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
muC (Cosserat couple modulus)	`0.01*mu0 mu0 10*mu0`	Pa

Solution 1 (sol1)

1 In the **Study** toolbar, click **Show Default Solver**.

2 In the **Model Builder** window, expand the **Solution 1 (sol1)** node.

3 Right-click **Study 1>Solver Configurations>Solution 1 (sol1)>Stationary Solver 1** and choose **Fully Coupled**.

4 In the **Study** toolbar, click = **Compute**.

RESULTS

Torque vs. Lc

1. In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.
2. In the **Settings** window for **1D Plot Group**, type Torque vs. Lc in the **Label** text field.
3. Locate the **Data** section. From the **Dataset** list, choose **Study 1/ Parametric Solutions 1 (sol2)**.
4. Locate the **Plot Settings** section.
5. Select the **x-axis label** check box. In the associated text field, type L_c/ R₀.
6. Select the **y-axis label** check box. In the associated text field, type Reaction moment z [N*m].
7. Locate the **Axis** section. Select the **x-axis log scale** check box.
8. Locate the **Legend** section. From the **Position** list, choose **Upper left**.

Global 1

1. Right-click **Torque vs. Lc** and choose **Global**.
2. In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)>Solid Mechanics> Rigid connectors>Rigid Connector 1>Reaction moment (spatial frame) - N·m> solid.rig1.RMz - Reaction moment, z-component**.
3. Click to expand the **Legends** section. From the **Legends** list, choose **Manual**.
4. In the table, enter the following settings:

Legends
\mu_C = 0.01\mu; \theta = 14\deg
\mu_C = \mu; \theta = 14\deg
\mu_C = 10\mu; \theta = 14\deg

5. Click to expand the **Coloring and Style** section. Find the **Line markers** subsection. From the **Marker** list, choose **Circle**.
6. In the **Torque vs. Lc** toolbar, click **Plot**.

Torque vs. Lc

In the **Model Builder** window, right-click **Torque vs. Lc** and choose **Duplicate**.

Torque vs. Lc, muC = 0.01mu

1. In the **Model Builder** window, under **Results** click **Torque vs. Lc 1**.

2 In the **Settings** window for **1D Plot Group**, type `Torque vs. Lc, muC = 0.01mu` in the **Label** text field.

3 Locate the **Data** section. From the **Parameter selection (muC)** list, choose **First**.

4 In the **Torque vs. Lc, muC = 0.01mu** toolbar, click **Plot**.

Point Graph 1

1 Right-click **Torque vs. Lc, muC = 0.01mu** and choose **Point Graph**.

2 Select Point 4 only.

3 In the **Settings** window for **Point Graph**, locate the **y-Axis Data** section.

4 In the **Expression** text field, type `LcR0*0.003+13`.

5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

6 In the **Expression** text field, type `0.1`.

7 Click to expand the **Coloring and Style** section. From the **Color** list, choose **Black**.

8 Click to expand the **Title** section. From the **Title type** list, choose **None**.

9 Right-click **Point Graph 1** and choose **Duplicate**.

Point Graph 2

1 In the **Model Builder** window, click **Point Graph 2**.

2 In the **Settings** window for **Point Graph**, locate the **x-Axis Data** section.

3 In the **Expression** text field, type `10`.

4 In the **Torque vs. Lc, muC = 0.01mu** toolbar, click **Plot**.

Torque vs. Lc, muC = 0.01mu

In the **Torque vs. Lc, muC = 0.01mu** toolbar, click **More Plots** and choose **Table Annotation**.

Table Annotation 1

1 In the **Settings** window for **Table Annotation**, locate the **Data** section.

2 From the **Source** list, choose **Local table**.

3 In the table, enter the following settings:

x-coordinate	y-coordinate	Annotation
0.01	20	Zone I
1	20	Zone II
100	20	Zone III

4 Locate the **Coloring and Style** section. Clear the **Show point** check box.

5 Select the **Show frame** check box.

6 In the **Torque vs. Lc, muC = 0.01mu** toolbar, click **Plot**.

Macrorotation, r3

1 In the **Home** toolbar, click **Add Plot Group** and choose **3D Plot Group**.

2 In the **Settings** window for **3D Plot Group**, type Macrorotation, r3 in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **None**.

4 Click to expand the **Title** section. From the **Title type** list, choose **Manual**.

5 In the **Title** text area, type Macrorotation: r3 [deg], (\mu_C=\mu).

6 Clear the **Parameter indicator** text field.

7 Locate the **Color Legend** section. Select the **Show maximum and minimum values** check box.

8 Click to expand the **Plot Array** section. Select the **Enable** check box.

Volume 1

1 Right-click **Macrorotation, r3** and choose **Volume**.

2 In the **Settings** window for **Volume**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1/Parametric Solutions 1 (sol2)**.

4 From the **Parameter value (LcR0)** list, choose **0.001**.

5 From the **Parameter value (muC (Pa))** list, choose **3.8462E11**.

6 Locate the **Expression** section. In the **Expression** text field, type r3.

7 In the **Unit** field, type deg.

8 Locate the **Coloring and Style** section. Click **Change Color Table**.

9 In the **Color Table** dialog box, select **Rainbow>Prism** in the tree.

10 Click **OK**.

11 Right-click **Volume 1** and choose **Duplicate**.

Volume 2

1 In the **Model Builder** window, click **Volume 2**.

2 In the **Settings** window for **Volume**, locate the **Data** section.

3 From the **Parameter value (LcR0)** list, choose **5**.

4 Click to expand the **Title** section. From the **Title type** list, choose **None**.

5 Click to expand the **Inherit Style** section. From the **Plot** list, choose **Volume 1**.

6 Right-click **Volume 2** and choose **Duplicate**.

Volume 3

1 In the **Model Builder** window, click **Volume 3**.

2 In the **Settings** window for **Volume**, locate the **Data** section.

3 From the **Parameter value (LcR0)** list, choose **10000**.

4 In the **Macrorotation, r3** toolbar, click **Plot**.

5 Click the **Go to XZ View** button in the **Graphics** toolbar.

Annotation 1

1 In the **Model Builder** window, right-click **Macrorotation, r3** and choose **Annotation**.

2 In the **Settings** window for **Annotation**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1/Parametric Solutions 1 (sol2)**.

4 Locate the **Position** section. In the **Z** text field, type `L0/2`.

5 Locate the **Annotation** section. Select the **LaTeX markup** check box.

6 In the **Text** text field, type `\textbf{ZONE 1}`.

7 Locate the **Coloring and Style** section. Clear the **Show point** check box.

8 From the **Anchor point** list, choose **Upper middle**.

9 Click to expand the **Plot Array** section. Select the **Manual indexing** check box.

10 Right-click **Annotation 1** and choose **Duplicate**.

Annotation 2

1 In the **Model Builder** window, click **Annotation 2**.

2 In the **Settings** window for **Annotation**, locate the **Annotation** section.

3 In the **Text** text field, type `\textbf{ZONE 2}`.

4 Locate the **Plot Array** section. In the **Index** text field, type `1`.

5 Right-click **Annotation 2** and choose **Duplicate**.

Annotation 3

1 In the **Model Builder** window, click **Annotation 3**.

2 In the **Settings** window for **Annotation**, locate the **Annotation** section.

3 In the **Text** text field, type `\textbf{ZONE 3}`.

4 Locate the **Plot Array** section. In the **Index** text field, type `2`.

5 In the **Macrorotation, r3** toolbar, click **Plot**.

6 Click the **Show Grid** button in the **Graphics** toolbar.

Macrorotation, r3

In the **Model Builder** window, right-click **Macrorotation, r3** and choose **Duplicate**.

Microrotation, a3

1 In the **Model Builder** window, expand the **Results>Macrorotation, r3.1** node, then click **Macrorotation, r3.1**.

2 In the **Settings** window for **3D Plot Group**, type Microrotation, a3 in the **Label** text field.

3 Locate the **Title** section. In the **Title** text area, type Microrotation: a3 [deg], (\mu_C=\mu).

Volume 1

1 In the **Model Builder** window, click **Volume 1**.

2 In the **Settings** window for **Volume**, locate the **Expression** section.

3 In the **Expression** text field, type a3.

Volume 2

1 In the **Model Builder** window, click **Volume 2**.

2 In the **Settings** window for **Volume**, locate the **Expression** section.

3 In the **Expression** text field, type a3.

Volume 3

1 In the **Model Builder** window, click **Volume 3**.

2 In the **Settings** window for **Volume**, locate the **Expression** section.

3 In the **Expression** text field, type a3.

4 In the **Microrotation, a3** toolbar, click **Plot**.

5 Click the **Go to XZ View** button in the **Graphics** toolbar.

6 Click the **Show Grid** button in the **Graphics** toolbar.

Displacement, Arrow Plot

1 In the **Home** toolbar, click **Add Plot Group** and choose **3D Plot Group**.

2 In the **Settings** window for **3D Plot Group**, type Displacement, Arrow Plot in the **Label** text field.

3 Locate the **Title** section. From the **Title type** list, choose **Custom**.

4 Find the **Solution** subsection. Clear the **Solution** check box.

Arrow Volume 1

Right-click **Displacement, Arrow Plot** and choose **Arrow Volume**.

Displacement, Arrow Plot

Locate the **Plot Array** section. Select the **Enable** check box.

Arrow Volume 1

1 In the **Model Builder** window, click **Arrow Volume 1**.

2 In the **Settings** window for **Arrow Volume**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1/Parametric Solutions 1 (sol2)**.

4 From the **Parameter value (muC (Pa))** list, choose **3.8462E11**.

5 From the **Parameter value (LcR0)** list, choose **0.001**.

6 Locate the **Arrow Positioning** section. Find the **Z grid points** subsection. In the **Points** text field, type 10.

7 Locate the **Coloring and Style** section.

8 Select the **Scale factor** check box. In the associated text field, type 3.

9 In the **Displacement, Arrow Plot** toolbar, click **Plot**.

10 Right-click **Arrow Volume 1** and choose **Duplicate**.

Arrow Volume 2

1 In the **Model Builder** window, click **Arrow Volume 2**.

2 In the **Settings** window for **Arrow Volume**, locate the **Data** section.

3 From the **Parameter value (LcR0)** list, choose **5**.

4 Click to expand the **Title** section. From the **Title type** list, choose **None**.

5 Click to expand the **Inherit Style** section. From the **Plot** list, choose **Arrow Volume 1**.

6 Right-click **Arrow Volume 2** and choose **Duplicate**.

Arrow Volume 3

1 In the **Model Builder** window, click **Arrow Volume 3**.

2 In the **Settings** window for **Arrow Volume**, locate the **Data** section.

3 From the **Parameter value (LcR0)** list, choose **10000**.

4 In the **Displacement, Arrow Plot** toolbar, click **Plot**.

Displacement, Arrow Plot

In the **Displacement, Arrow Plot** toolbar, click **More Plots** and choose **Table Annotation**.

Table Annotation 1

1 In the **Settings** window for **Table Annotation**, locate the **Data** section.

2 Select the **LaTeX markup** check box.

3 From the **Source** list, choose **Local table**.

4 In the table, enter the following settings:

x-coordinate	y-coordinate	z-coordinate	Annotation
0	0	-L0/10	$L_\textrm{C}/ R_\textrm{0}$ = 1e-3
2.5*R0	0	-L0/10	$L_\textrm{C}/ R_\textrm{0}$ = 5
5*R0	0	-L0/10	$L_\textrm{C}/ R_\textrm{0}$ = 1e4

5 Locate the **Coloring and Style** section. From the **Anchor point** list, choose **Upper left**.

6 Select the **Show frame** check box.

7 In the **Displacement, Arrow Plot** toolbar, click **Plot**.

8 Click the **Show Grid** button in the **Graphics** toolbar.