

# Moisture Transport in a Paperboard Roll

Paperboard is made of wood fibers, which can absorb water. Water absorption changes the mechanical properties of the paperboard, and it might affect the quality of the final product due to moisture-induced deformations. This deformation is caused by the pressure acting on the pore walls. Paperboard is typically modeled as an orthotropic material since the cellulose fibers are mostly aligned along the grain direction, or so-called machine direction. The arrangement of the fibers affects also the transport properties.

This example shows how to model moisture transport in a paperboard roll when the relative humidity in the room changes during 90 days storage.

# Model Definition

The model geometry consists of a hollow paperboard cylinder 70 cm long and 40 cm thick, lying on a 20 cm diameter shaft. A 2D axisymmetric geometry is used, and only half of it is modeled due to symmetry. A roller boundary condition is used on the side lying on the shaft. The temperature is held fixed at 293.15 K and the ambient relative humidity is increased from 5% to 85% in 1 day and kept constant for 90 days. The paperboard absorbs the moisture in the surrounding air, causing hygroscopic swelling. The pore pressure  $p_f$  in the paperboard is computed from:

$$p_{\rm f} = s_{\rm l} p_{\rm l} + (1 - s_{\rm l}) p_{\rm mA}$$

where  $s_l$  is the liquid saturation,  $p_l$  is the liquid pressure,  $s_m = 1 - s_l$  is the moist air saturation, and  $p_{mA}$  is the moist air absolute pressure. The relation between the relative humidity,  $\phi_{\rm w}$ , and the water content,  $w_{\rm c}$ , is described by the sorption isotherm

$$w_{\rm c} = -\frac{1}{B}\log\left(-\frac{RT\log\phi_{\rm w}}{A}\right)$$

here, A and B are fitting parameters (6.232E3 J/mol and 20.56 respectively). The moist air permeability,  $\kappa_m$ , is described using the following model

$$\kappa_{\rm m} = \kappa \left(\frac{\varepsilon_{\rm p} s_{\rm m}}{\varepsilon_{\rm p0} s_{\rm m0}}\right)^a \cdot \frac{1 - \varepsilon_{\rm p0} s_{\rm m0}}{1 - \varepsilon_{\rm p} s_{\rm m}}$$

where  $s_{\rm m}$  is the moist air saturation,  $\varepsilon_{\rm p}$  is the porosity,  $\kappa$  is the reference permeability, and the exponent  $\alpha$  are obtained from experiments. Variables with a 0 subscript refer to their initial value.

Different properties and material parameters are used in the local directions, namely the machine direction (MD), cross-machine direction (CD), and the through-thickness direction (ZD).

The parameters are listed in the tables below:

TABLE I: MODEL PARAMETERS

Parameter	Value	
Porosity	0.5	
Density of paperboard	15000 kg/m <sup>3</sup>	
Ambient pressure	I atm	
Ambient temperature	293.15 K	

table 2: transport parameters (Ref. 1)

PARAMETER	VALUE
Liquid relative permeability	0
Moist air relative permeability	I
Permeability, k (ZD)	4.3646E-15 m <sup>2</sup>
Exponent parameter, a (ZD)	4.18
Permeability, k (CD)	1.9405E-13 m <sup>2</sup>
Exponent parameter, a (CD)	2.2
Vapor diffusion (ZD)	1.2929E-12 s
Vapor diffusion (CD)	7.1295E-11 s

TABLE 3: ELASTIC MATERIAL PROPERTIES (Ref. 2)

PARAMETER	VALUE		
Young's modulus (MD)	3.2 GPa		
Young's modulus (CD)	2 GPa		
Young's modulus (ZD)	0.016 GPa		
Shear modulus (MD-CD)	I GPa		
Shear modulus (CD-ZD)	0.057 GPa		
Shear modulus (MD-ZD)	0.058 GPa		
Poisson's ratio (MD-CD)	0.34		
Poisson's ratio (CD-ZD)	0.01		
Poisson's ratio (MD-ZD)	0.01		

Figure 1 and Figure 2 show the relative humidity after 90 days. Due to the permeability and diffusivity anisotropy, the moisture penetrates faster in the machine direction (vertical diction) than in the through-thickness direction (radial direction).

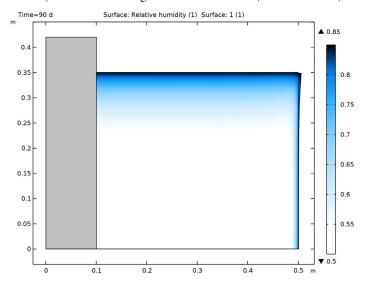


Figure 1: Relative humidity in the deformed configuration after 90 days.

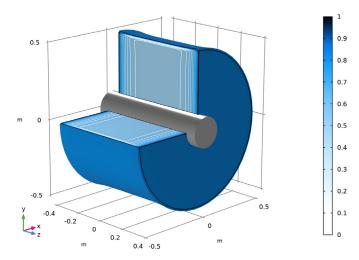


Figure 2: Relative humidity in the deformed configuration after 90 days.

Figure 3 and Figure 4 shows the moisture content through the paperboard in the machine and through-thickness directions at various times.

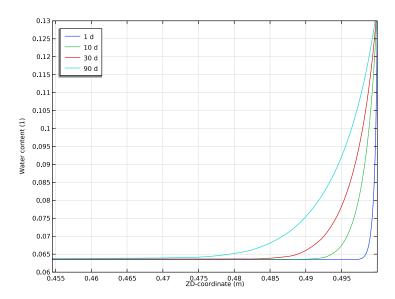


Figure 3: Water content distribution in the paperboard through-thickness direction (radial direction) at various instants.

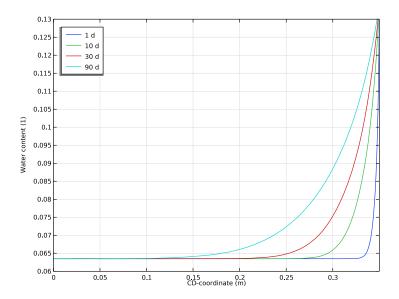


Figure 4: Water content distribution in the paperboard machine direction (vertical, or axial direction) at various instants.

The roll expands mainly in the radial direction because of the lower stiffness of the paperboard in the radial direction. Figure 5 shows that the maximum swelling deformation reaches about 6 mm after 90 days. The corner expands more than the bulk material because of the moisture influx contribution from the top and lateral sides.

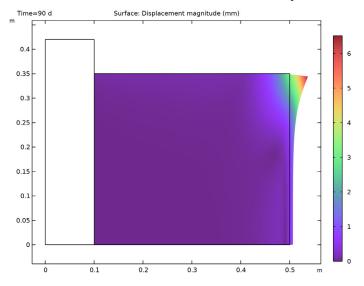


Figure 5: Displacement magnitude and deformation (amplified) after 90 days.

# Notes About the COMSOL Implementation

The ambient relative humidity, pressure, and temperature are defined in the Ambient Properties node which can be linked to the boundary conditions in the Moisture Transport in Solids interface.

The sorption isotherm and the permeability can be defined using analytic functions under the Porous Material node.

The **Unsaturated Poroelasticity** multiphysics coupling is used to model moisture-induced swelling.

# References

1. M. Alexandersson, H. Askfelt, and M. Ristinmaa, "Triphasic Model of Heat and Transport with Internal Mass Exchange in Paperboard," Transp. Porous Med., vol. 112, pp. 381–408, 2016. https://doi.org/10.1007/s11242-016-0651-9.

2. J. Ran and C. Liu, "Modeling of the Stiffness of Corrugated Cardboard Considering Material Nonlinear Effect," J. Phys.: Conf. Ser. 1187, 032069. https://doi.org/ 10.1088/1742-6596/1187/3/032069.

Application Library path: Porous Media Flow Module/Poromechanics/ paperboard roll

# Modeling Instructions

From the File menu, choose New.

#### NEW

In the New window, click Model Wizard.

# MODEL WIZARD

- I In the Model Wizard window, click 2D Axisymmetric.
- 2 In the Select Physics tree, select Structural Mechanics>Poroelasticity> **Unsaturated Poroelasticity.**
- 3 Click Add.
- 4 Click 🗪 Study.
- 5 In the Select Study tree, select General Studies>Time Dependent.
- 6 Click M Done.

#### **GLOBAL DEFINITIONS**

#### Model Parameters

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- 3 Click Load from File.
- 4 Browse to the model's Application Libraries folder and double-click the file paperboard\_roll\_parameters.txt.
- 5 In the Label text field, type Model Parameters.

#### COMPONENT I (COMPI)

You can rename the frame coordinates using the paperboard convention: machine direction (md), cross-machine direction (cd), and through-thickness direction (zd). This can be useful when inserting user-defined material properties.

- I In the Model Builder window, click Component I (compl).
- 2 In the Settings window for Component, locate the Frames section.
- **3** Find the **Spatial frame coordinates** subsection. In the table, enter the following settings:

First	Second	Third
zd	md	cd

**4** Find the **Material frame coordinates** subsection. In the table, enter the following settings:

First	Second	Third
ZD	MD	CD

#### **GEOMETRY I**

#### Paper

- I In the **Geometry** toolbar, click **Rectangle**.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type Ro-Ri.
- 4 In the Height text field, type H/2.
- 5 Locate the **Position** section. In the **zd** text field, type Ri.
- 6 In the Label text field, type Paper.

# Holder

- I In the Geometry toolbar, click Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type Ri.
- 4 In the Height text field, type 1.2\*H/2.
- 5 In the Label text field, type Holder.

#### Form Union (fin)

- I In the Model Builder window, under Component I (compl)>Geometry I click Form Union (fin).
- 2 In the Settings window for Form Union/Assembly, locate the Form Union/Assembly section.

- 3 From the Action list, choose Form an assembly.
- 4 Clear the Create pairs check box.
- 5 In the Geometry toolbar, click **Build All**.

#### DEFINITIONS

**Boundaries Facing Ambient** 

- I In the **Definitions** toolbar, click **\( \frac{1}{2} \) Explicit**.
- 2 In the Settings window for Explicit, locate the Input Entities section.
- 3 From the Geometric entity level list, choose Boundary.
- **4** Select Boundaries 7 and 8 only.
- 5 In the Label text field, type Boundaries Facing Ambient.

Ambient Relative Humidity

- I In the **Definitions** toolbar, click  $\bigwedge$  **Piecewise**.
- 2 In the Settings window for Piecewise, locate the Definition section.
- **3** Find the **Intervals** subsection. In the table, enter the following settings:

Start	End	Function
0	t_r	<pre>phi_init +(phi_end-phi_init)*x/t_r</pre>
t_r	t_end	phi_end

- 4 Locate the Units section. In the Arguments text field, type s.
- 5 In the Function text field, type 1.
- 6 In the Function name text field, type phiw fun.
- 7 In the Label text field, type Ambient Relative Humidity.

Add Ambient Properties node to define the ambient temperature, pressure and relative humidity. Then the boundary conditions can link directly to this node.

Ambient Properties I (ampr I)

- In the Physics toolbar, click **Shared Properties** and choose **Ambient Properties**.
- 2 In the Settings window for Ambient Properties, locate the Ambient Conditions section.
- 3 In the  $T_{amb}$  text field, type T\_amb.
- 4 In the  $\phi_{amb}$  text field, type phiw\_fun(t).

#### MATERIALS

Porous Material I (pmat I)

In the Model Builder window, under Component I (compl) right-click Materials and choose More Materials>Porous Material.

#### Water Content

- I In the Model Builder window, expand the Porous Material I (pmatl) node.
- 2 Right-click Component I (compl)>Materials>Porous Material I (pmatl)>Basic (def) and choose Functions>Analytic.
- 3 In the Settings window for Analytic, locate the Definition section.
- 4 In the Expression text field, type (1/B)\*log(-R const\*T amb/A\*log(phi)).
- 5 In the Arguments text field, type phi.
- 6 Locate the Units section. In the Function text field, type 1.
- 7 In the table, enter the following settings:

Argument	Unit
phi	1

- 8 In the Function name text field, type we int.
- 9 In the Label text field, type Water Content.

# Moist Air Permeability

- I In the Home toolbar, click f(x) Functions and choose Global>Analytic.
- 2 In the Settings window for Analytic, locate the Definition section.
- 3 In the Expression text field, type k\*(epsilonp\*sm/(por\*0.9))^a\*(1-por\*0.9)/(1epsilonp\*sm).
- 4 In the Arguments text field, type k, a, sm, epsilonp.
- **5** Locate the **Units** section. In the **Function** text field, type m^2.
- **6** In the table, enter the following settings:

Argument	Unit
k	m^2
a	1
sm	1
epsilonp	1

7 In the Function name text field, type kappa fun.

8 In the Label text field, type Moist Air Permeability.

Porous Material I (pmat I)

- I In the Model Builder window, under Component I (compl)>Materials click Porous Material I (pmatl).
- 2 Select Domain 2 only.
- 3 In the Settings window for Porous Material, locate the Homogenized Properties section.
- **4** In the table, enter the following settings:

Property	Variable	Value	Unit	Property group Basic	
Water content	<b>w</b> _c	<pre>wc_int(phi )*rho_s* (1-por)</pre>	kg/m³		
Permeability	{kappa   1, kappa   22, kappa   33}; kappaij = 0	{kappa_fun (Km_zd, a_zd, mts.sm, mts.porosi ty), 0, kappa_fun(Km_cd, a_cd, mts.sm, mts.porosi ty)}	m²	Basic	

Solid I (pmat1.solid1)

Right-click Porous Material I (pmat1) and choose Solid.

# MOISTURE TRANSPORT IN SOLIDS (MTS)

Select Domain 2 only.

#### MATERIALS

- I In the Model Builder window, under Component I (compl)>Materials> Porous Material I (pmat1) click Solid I (pmat1.solid1).
- 2 In the Settings window for Solid, locate the Solid Properties section.
- **3** In the  $\theta_s$  text field, type 1-por.

Porous Material I (pmat I)

- I In the Model Builder window, click Basic (def).
- 2 In the Settings window for Basic, locate the Model Inputs section.

- 3 Click Select Quantity.
- 4 In the Physical Quantity dialog box, type relative in the text field.
- 5 Click **Filter**.
- 6 In the tree, select General>Relative humidity (1).
- 7 Click OK.

# MOISTURE TRANSPORT IN SOLIDS (MTS)

# Liquid Water I

- I In the Model Builder window, under Component I (compl)> Moisture Transport in Solids (mts)>Porous Medium I click Liquid Water I.
- 2 In the Settings window for Liquid Water, locate the Liquid Water Properties section.
- **3** In the text field, type krl.

#### Moist Air I

- I In the Model Builder window, click Moist Air I.
- 2 In the Settings window for Moist Air, locate the Moist Air Properties section.
- 3 In the text field, type krm.
- 4 From the list, choose Diagonal.
- **5** In the  $D_n$  table, enter the following settings:

tau_zd*(R_const*T_amb/mts.M_v)	0
0	tau_cd*(R_const*T_amb/mts.M_v)

#### Initial Values 1

- I In the Model Builder window, under Component I (compl)> Moisture Transport in Solids (mts) click Initial Values I.
- 2 In the Settings window for Initial Values, locate the Initial Values section.
- **3** In the  $\phi_w$  text field, type phi init.

#### Moisture Content and Pressure 1

- I In the Physics toolbar, click Boundaries and choose Moisture Content and Pressure. You can link the boundaries condition directly to the Ambient Properties node defined before.
- 2 In the Settings window for Moisture Content and Pressure, locate the Boundary Selection section.
- 3 From the Selection list, choose Boundaries Facing Ambient.

- 4 Locate the Moisture Content and Pressure section. From the  $\phi_{w0}$  list, choose Ambient relative humidity (amprI).
- 5 From the  $p_{m0}$  list, choose Ambient absolute pressure (amprl).

Symmetry I

- I In the Physics toolbar, click 

  Boundaries and choose Symmetry.
- 2 Select Boundary 6 only.

# SOLID MECHANICS (SOLID)

- I In the Model Builder window, under Component I (compl) click Solid Mechanics (solid).
- 2 Select Domain 2 only.

Linear Elastic Material I

- I In the Model Builder window, under Component I (compl)>Solid Mechanics (solid) click Linear Elastic Material I.
- 2 In the Settings window for Linear Elastic Material, locate the Linear Elastic Material section.
- 3 From the Material symmetry list, choose Orthotropic.

Roller I

- I In the Physics toolbar, click Boundaries and choose Roller.
- 2 Select Boundary 5 only.

Symmetry Plane I

- I In the Physics toolbar, click Boundaries and choose Symmetry Plane.
- 2 Select Boundary 6 only.

#### MATERIALS

Porous Material I (pmat I)

Add the material parameters for the deformation. Note that the radial direction is the through-thickness direction and the vertical direction is the cross-machine direction.

- I In the Model Builder window, under Component I (compl)>Materials click Porous Material I (pmat I).
- 2 In the Settings window for Porous Material, locate the Homogenized Properties section.

**3** In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Density	rho	rho_s	kg/m³	Basic
Biot-Willis coefficient	alphaB	alpha_B	1	Poroelastic material
Poisson's ratio	{nuvector1, nuvector2, nuvector3}	<pre>{nu_mz, nu_mc, nu_cz}</pre>	I	Orthotropic
Shear modulus	{Gvector1, Gvector2, Gvector3}	{G_mz, G_mc, G_cz}	N/m²	Orthotropic
Young's modulus	{Evector1, Evector2, Evector3}	{E_zd, E_md, E_cd}	Pa	Orthotropic

#### MULTIPHYSICS

Unsaturated Poroelasticity I (unporo I)

Use the pore pressure computed by the Moisture Transport in Solids interface in order to have no deformation when exposed to the initial ambient conditions.

- I In the Model Builder window, under Component I (compl)>Multiphysics click Unsaturated Poroelasticity I (unporol).
- 2 In the Settings window for Unsaturated Poroelasticity, locate the **Poroelastic Coupling Properties** section.
- 3 From the  $p_{ref}$  list, choose Initial pore pressure (mts/pms1).

#### MESH I

# Mapped I

- I In the Mesh toolbar, click Mapped.
- 2 In the Settings window for Mapped, locate the Domain Selection section.
- 3 From the Geometric entity level list, choose Domain.
- 4 Select Domain 2 only.

#### Distribution 1

- I Right-click Mapped I and choose Distribution.
- 2 In the Settings window for Distribution, locate the Distribution section.
- 3 From the Distribution type list, choose Predefined.

- 4 In the Number of elements text field, type 30.
- **5** Select Boundaries 6 and 7 only.
- **6** In the **Element ratio** text field, type 8.
- 7 From the Growth rate list, choose Exponential.

#### Distribution 2

- I In the Model Builder window, right-click Mapped I and choose Distribution.
- 2 In the Settings window for Distribution, locate the Distribution section.
- 3 From the Distribution type list, choose Predefined.
- 4 In the Number of elements text field, type 30.
- 5 In the Element ratio text field, type 8.
- 6 Select Boundaries 5 and 8 only.
- 7 Select the **Reverse direction** check box.

# Boundary Layers 1

- I In the Mesh toolbar, click Boundary Layers.
- 2 In the Settings window for Boundary Layers, locate the Domain Selection section.
- 3 From the Geometric entity level list, choose Domain.
- **4** Select Domain 2 only.

# Boundary Layer Properties

- I In the Model Builder window, click Boundary Layer Properties.
- 2 In the Settings window for Boundary Layer Properties, locate the Boundary Selection section.
- 3 From the Selection list, choose Boundaries Facing Ambient.

#### Mapped 2

- I In the Mesh toolbar, click Mapped.
- 2 In the Settings window for Mapped, click **Build All**.

#### STUDY I

# Step 1: Time Dependent

- I In the Model Builder window, under Study I click Step I: Time Dependent.
- 2 In the Settings window for Time Dependent, locate the Study Settings section.
- 3 From the Time unit list, choose d.
- 4 In the Output times text field, type range(0,t\_r/20,t\_r) range(t\_r,1[d],t\_end).

5 In the Home toolbar, click **Compute**.

#### RESULTS

Use the **Configuration** node to select which time step to display in 1D Plot Group nodes.

### Multiselect Solution 1

- I In the Results toolbar, click ( ) Configurations and choose Multiselect Solution.
- 2 In the Settings window for Multiselect Solution, locate the Solution section.
- **3** From the Time selection list, choose Interpolated.
- 4 In the Times (d) text field, type 1[d] 10[d] 30[d] 90[d].

# Relative Humidity (mts)

- I In the Model Builder window, under Results click Relative Humidity (mts).
- 2 In the Settings window for 2D Plot Group, locate the Color Legend section.
- 3 Select the Show maximum and minimum values check box.

# Surface 2

- I Right-click Relative Humidity (mts) and choose Surface.
- 2 In the Settings window for Surface, locate the Expression section.
- 3 In the Expression text field, type 1.
- 4 Locate the Coloring and Style section. From the Coloring list, choose Uniform.
- 5 From the Color list, choose Gray.

#### Selection I

- I Right-click Surface 2 and choose Selection.
- 2 Select Domain 1 only.

# Deformation I

- I In the Model Builder window, right-click Surface I and choose Deformation.
- 2 In the Settings window for Deformation, locate the Scale section.
- 3 Select the Scale factor check box. In the associated text field, type 1.
- 4 In the Relative Humidity (mts) toolbar, click Plot.

# Water Content, Cross Machine

- I In the Home toolbar, click Add Plot Group and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, locate the Data section.
- 3 From the Solution parameters list, choose From configuration.

- 4 In the Label text field, type Water Content, Cross Machine.
- 5 Click to expand the **Title** section. From the **Title type** list, choose **None**.
- **6** Locate the **Plot Settings** section.
- 7 Select the y-axis label check box. In the associated text field, type Water content (1).
- 8 Locate the Axis section. Select the Manual axis limits check box.
- **9** In the **x minimum** text field, type **0**.
- **10** In the x maximum text field, type H/2.
- II In the y minimum text field, type 0.06.
- 12 In the y maximum text field, type 0.13.
- 13 Locate the Legend section. From the Position list, choose Upper left.

# Line Graph 1

- I Right-click Water Content, Cross Machine and choose Line Graph.
- **2** Select Boundary 5 only.
- 3 In the Settings window for Line Graph, locate the y-Axis Data section.
- 4 In the Expression text field, type mts.wc/rho s/(1-por).
- 5 Locate the x-Axis Data section. From the Parameter list, choose Expression.
- **6** In the **Expression** text field, type CD.
- 7 Click to expand the **Legends** section. Select the **Show legends** check box.
- 8 In the Water Content, Cross Machine toolbar, click Plot.

# Water Content, Cross Machine

In the Model Builder window, right-click Water Content, Cross Machine and choose Duplicate.

### Water Content, Through Thickness

- I In the Model Builder window, under Results click Water Content, Cross Machine I.
- 2 In the Settings window for ID Plot Group, type Water Content, Through Thickness in the **Label** text field.
- 3 Locate the Axis section. In the x minimum text field, type Ro/1.1.
- 4 In the x maximum text field, type Ro.

#### Line Grabh I

- I In the Model Builder window, expand the Water Content, Through Thickness node, then click Line Graph 1.
- 2 In the Settings window for Line Graph, locate the Selection section.

- **3** Click to select the **Activate Selection** toggle button.
- 4 Select Boundary 6 only.
- **5** Locate the **x-Axis Data** section. In the **Expression** text field, type **ZD**.
- 6 In the Water Content, Through Thickness toolbar, click  **Plot**.

#### ADD PREDEFINED PLOT

- I In the Home toolbar, click Add Predefined Plot to open the Add Predefined Plot window.
- 2 Go to the Add Predefined Plot window.
- 3 In the tree, select Study I/Solution I (soll)>Solid Mechanics>Displacement (solid).
- 4 Click Add Plot in the window toolbar.

# RESULTS

# Surface I

- I In the Model Builder window, expand the Displacement (solid) node, then click Surface I.
- 2 In the Settings window for Surface, locate the Expression section.
- 3 From the Unit list, choose mm.