

Radiative Cooling of a Glass Plate

In glass production, the glass melt is cooled down mainly by radiation. To avoid stresses it is important to evenly cool the glass body.

Numerical treatment of radiative heat transfer helps to optimize this cooling process. The governing equation — the Radiative Transfer Equation (RTE) — is an integro-differential equation that requires a lot of computational resources to be solved. Therefore, COMSOL Multiphysics offers three common methods to solve the RTE along with the heat transfer equation.

This tutorial is intended to show the typical set up of all methods computing the heat transfer by radiation inside a gray medium.

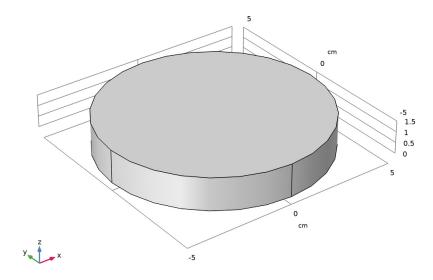


Figure 1: Cylindrical glass plate.

Model Definition

The model geometry is shown in Figure 1. It is a cylinder of radius r = 5 cm and height h = 1.5 cm. The radiative cooling starts from an initial temperature of 600°C due to radiation into an ambient surrounding at 20°C. Convective cooling is neglected, which is reasonable for high temperatures as in this model.

The material properties for the glass body are summarized in Table 1.

TABLE I: MATERIAL PROPERTIES FOR GLASS.

MATERIAL PROPERTY	VALUE
Thermal conductivity	1.2 W/(m·K)
Density	2200 kg/m ³
Heat capacity at constant pressure	850 J/(kg·K)
Scattering coefficient	0

Scattering effects are neglected. For the absorption coefficient, a parametric sweep is used to get results for k = 5, 70, 120. The boundaries of the glass body have a surface emissivity, ε , equal to 1.

THERMAL ANALYSIS

A detailed explanation about the discrete ordinates method and the P1 method can be found in Radiative Heat Transfer in Finite Cylindrical Media or Radiative Heat Transfer in Finite Cylindrical Media — P1 Method respectively.

The Rosseland approximation is a simplified method that results in an additional nonlinear term for the thermal conductivity. Hence, this method has almost no impact on the computational cost.

For large optical thickness where the integral of the absorption coefficient along a typical path is large, radiation effects only spread at its close surrounding and does not travel far through the medium before being absorbed or scattered. This leads to a diffusion-like equation for the radiative heat flux (Ref. 1):

$$q_{\mathrm{r},\,\lambda} = -\frac{4\pi}{3\beta_{\lambda}} \nabla I_{\mathrm{b},\,\lambda}$$

For a gray medium (after integration over all wave numbers) the radiative heat flux depends on the temperature gradient and can be expressed as:

$$q_{\mathbf{r}} = -k_{\mathbf{r}} \nabla T$$

where $k_{\rm r}$ is a highly nonlinear coefficient, considered as a conductivity, for radiative transfer of the form

$$k_{\rm r} = \frac{16n^2 \sigma T^3}{3\beta_{\rm r}}$$

with β_r , the Rosseland-mean extinction coefficient, σ the scattering coefficient and n the refractive index. Thus the Rosseland approximation method is also called the diffusion method.

Results and Discussion

The figures below compare the results for a low and a high absorption coefficient. The P1 method provides a very good approximation for lower absorption coefficients (Figure 2), but with increasing absorption coefficients the results differ increasingly.

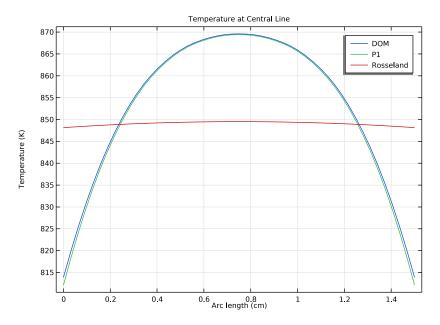


Figure 2: Vertical temperature distribution at the center of the cylinder for k = 5.

The Rosseland approximation provides a fast and satisfying solution for the temperature field when a very high absorption coefficient or a rather high optical thickness is considered (Figure 3).

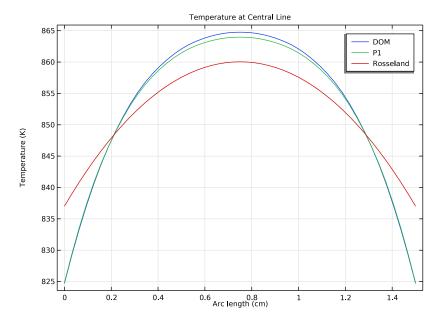


Figure 3: Vertical temperature distribution at the center of the cylinder for k = 120.

Both methods need to be used carefully and it is recommended to validate their applicability. If they can be used, they provide a very fast solution compared to the highly accurate discrete ordinates method.

Reference

1. M.F. Modest, Radiative Heat Transfer, 2nd ed., Academic Press, 2003.

Application Library path: Heat Transfer Module/Thermal Radiation/ glass_plate

From the File menu, choose New.

NEW

In the New window, click Model Wizard.

MODEL WIZARD

- I In the Model Wizard window, click **3D**.
- 2 In the Select Physics tree, select Heat Transfer>Radiation> Heat Transfer with Radiation in Participating Media.
- 3 Click Add.
- 4 Click 🔁 Study.
- 5 In the Select Study tree, select General Studies>Time Dependent.
- 6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description
nr	1.45	1.45	Refractive index
T0	600[degC]	873.15 K	Initial temperature
T_amb	20[degC]	293.15 K	Ambient temperature
k	5	5	Absorption coefficient

GEOMETRY I

- I In the Model Builder window, under Component I (compl) click Geometry I.
- 2 In the Settings window for Geometry, locate the Units section.
- **3** From the **Length unit** list, choose **cm**.

Cylinder I (cyl1)

- I In the Geometry toolbar, click Cylinder.
- 2 In the Settings window for Cylinder, locate the Size and Shape section.

- 3 In the Radius text field, type 5.
- 4 In the Height text field, type 1.5.
- 5 In the Geometry toolbar, click **Build All**.

Create a user-defined material for the glass body.

GLOBAL DEFINITIONS

Glass

- I In the Model Builder window, under Global Definitions right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, type Glass in the Label text field.

MATERIALS

Material Link I (matlnk I)

In the Model Builder window, under Component I (compl) right-click Materials and choose More Materials>Material Link.

Material Link 2 (matlnk2)

- I Right-click Materials and choose More Materials>Material Link.
- 2 In the Settings window for Material Link, locate the Geometric Entity Selection section.
- 3 From the Geometric entity level list, choose Boundary.
- 4 From the Selection list, choose All boundaries.

GLOBAL DEFINITIONS

Glass (mat I)

- I In the Model Builder window, under Global Definitions>Materials click Glass (mat1).
- 2 In the Settings window for Material, locate the Material Contents section.

3 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Thermal conductivity	k_iso ; kii = k_iso, kij = 0	1.2	W/(m·K)	Basic
Density	rho	2200	kg/m³	Basic
Heat capacity at constant pressure	Ср	850	J/(kg·K)	Basic

For radiative heat transfer, the absorption and scattering coefficients are also needed. Add these properties to the material.

4 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Absorption coefficient	kappaR	k	I/m	Basic
Scattering coefficient	sigmaS	0	I/m	Basic

The boundary condition for radiative cooling requires a surface emissivity. One way to define this coefficient is to add a material for the boundaries.

5 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Surface emissivity	epsilon_rad	1	I	Basic

Start with the discrete ordinate method to compute the radiative cooling of the glass plate. This method provides the most accurate solution for arbitrary radiation models and is therefore the default method for radiation in participating media. To reduce the memory requirement, you can adjust the performance index for the discrete ordinate method. The method computes the radiative intensities for a number of directions (24 directions by default) and the segregated solver only computes a few directions at once. The performance index determines the number of directions which are calculated at one segregated step and the number of segregated steps accordingly.

RADIATION IN PARTICIPATING MEDIA (RPM)

- I In the Model Builder window, under Component I (compl) click Radiation in Participating Media (rpm).
- 2 In the Settings window for Radiation in Participating Media, locate the Participating Media Settings section.

- **3** Find the **Radiation settings** subsection. From the $P_{\rm index}$ list, choose **0.6**.
- **4** In the n_r text field, type nr.

HEAT TRANSFER IN SOLIDS (HT)

Initial Values 1

- I In the Model Builder window, under Component I (compl)>Heat Transfer in Solids (ht) click Initial Values 1.
- 2 In the Settings window for Initial Values, locate the Initial Values section.
- **3** In the *T* text field, type T0.

Set the initial temperature to 600°C.

Surface-to-Ambient Radiation 1

- In the Physics toolbar, click **Boundaries** and choose Surface-to-Ambient Radiation.
- 2 In the Settings window for Surface-to-Ambient Radiation, locate the Boundary Selection section.
- 3 From the Selection list, choose All boundaries.
- 4 Locate the Surface-to-Ambient Radiation section. In the T_{amb} text field, type T_amb.

MESH I

Build a suitable mesh manually. First, mesh the surface with a free triangular mesh and then add a swept mesh.

Free Triangular 1

- I In the Mesh toolbar, click \times More Generators and choose Free Triangular.
- 2 Select Boundary 4 only.

Size 1

- I In the Mesh toolbar, click Size Attribute and choose Extra Fine.
- 2 In the Settings window for Size, locate the Geometric Entity Selection section.
- 3 From the Geometric entity level list, choose Edge.
- 4 Select Edges 4, 5, 8, and 11 only.

Size 2

- I In the Mesh toolbar, click Size Attribute and choose Fine.
- 2 In the Settings window for Size, click Build Selected.

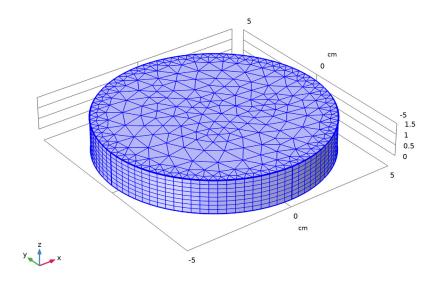
Swebt I

In the Mesh toolbar, click Swept.

Distribution I

- I Right-click **Swept I** and choose **Distribution**.
- 2 In the Settings window for Distribution, locate the Distribution section.
- 3 In the Number of elements text field, type 8.
- 4 Click Pauld Selected.

The mesh should look like that in the figure below.



STUDY I: DOM

Next, rename the study node to identify the studies and the related solutions easily.

- I In the Model Builder window, click Study I.
- 2 In the Settings window for Study, type Study 1: DOM in the Label text field.

Step 1: Time Dependent

The model only compares the results after 10 seconds. To keep the file size small, let COMSOL Multiphysics store only this time step in the file. The computational time step is chosen automatically.

- I In the Model Builder window, under Study I: DOM click Step I: Time Dependent.
- 2 In the Settings window for Time Dependent, locate the Study Settings section.

3 In the Output times text field, type 10.

Parametric Sweep

- I In the Study toolbar, click Parametric Sweep.
- 2 In the Settings window for Parametric Sweep, locate the Study Settings section.
- 3 Click + Add.
- **4** In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
k (Absorption coefficient)	5 70 120	

5 In the Study toolbar, click **Compute**.

RESULTS

Temperature (ht)

Next, run the same model but with the P1 method. The only modification to do is to change the Radiation discretization method in the Heat Transfer with Radiation in Participating Media settings window.

ADD COMPONENT

In the Model Builder window, right-click the root node and choose Add Component>3D.

GEOMETRY 2

- I In the Settings window for Geometry, locate the Units section.
- **2** From the **Length unit** list, choose **cm**.

MESH 2

Import I

- I In the Mesh toolbar, click Import.
- 2 In the Settings window for Import, locate the Import section.
- 3 From the Source list, choose Meshing sequence.
- 4 Click Import.

MATERIALS

Material Link 3 (matlnk3)

In the Model Builder window, under Component 2 (comp2) right-click Materials and choose More Materials>Material Link.

Material Link 4 (matlnk4)

- I Right-click Materials and choose More Materials>Material Link.
- 2 In the Settings window for Material Link, locate the Geometric Entity Selection section.
- 3 From the Geometric entity level list, choose Boundary.
- 4 From the Selection list, choose All boundaries.

HEAT TRANSFER IN SOLIDS (HT), RADIATION IN PARTICIPATING MEDIA (RPM)

- I In the Model Builder window, under Component I (compl), Ctrl-click to select Heat Transfer in Solids (ht) and Radiation in Participating Media (rpm).
- 2 Right-click and choose Copy.

COMPONENT 2 (COMP2)

In the Model Builder window, right-click Component 2 (comp2) and choose Paste Multiple Items.

HEAT TRANSFER IN SOLIDS (HT2), RADIATION IN PARTICIPATING MEDIA (RPM2)

- I In the Model Builder window, under Component 2 (comp2), Ctrl-click to select Heat Transfer in Solids (ht2) and Radiation in Participating Media (rpm2).
- 2 In the Messages from Paste dialog box, click **OK**.

MULTIPHYSICS

Heat Transfer with Radiation in Participating Media 2 (htrpm2)

- I In the Physics toolbar, click Multiphysics Couplings and choose Domain> Heat Transfer with Radiation in Participating Media.
- 2 In the Settings window for Heat Transfer with Radiation in Participating Media, locate the **Domain Selection** section.
- 3 From the Selection list, choose All domains.

RADIATION IN PARTICIPATING MEDIA (RPM2)

- I In the Model Builder window, under Component 2 (comp2) click Radiation in Participating Media (rpm2).
- 2 In the Settings window for Radiation in Participating Media, locate the Participating Media Settings section.

3 Find the Radiation settings subsection. From the Radiation discretization method list, choose PI approximation.

To compare the results, a second study is used to compute the same setup with the P1 method. Add an empty study and copy the settings from the first one.

ADD STUDY

- I In the Home toolbar, click Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select Empty Study.
- 4 Click Add Study in the window toolbar.

STUDY 2: PI

In the Settings window for Study, type Study 2: P1 in the Label text field.

STUDY I: DOM

Parametric Sweep, Step 1: Time Dependent

- I In the Model Builder window, under Study I: DOM, Ctrl-click to select Parametric Sweep and Step 1: Time Dependent.
- 2 Right-click and choose Copy.

STUDY 2: PI

In the Model Builder window, right-click Study 2: PI and choose Paste Multiple Items.

Steb 1: Time Dependent

- I In the Model Builder window, under Study 2: PI click Step I: Time Dependent.
- 2 In the Settings window for Time Dependent, locate the Physics and Variables Selection section.
- 3 In the table, clear the Solve for check boxes for Heat Transfer in Solids (ht) and Radiation in Participating Media (rpm).
- 4 In the table, clear the **Solve for** check box for Heat Transfer with Radiation in Participating Media I (htrpmI).

STUDY I: DOM

- I In the Model Builder window, under Study I: DOM click Step I: Time Dependent.
- 2 In the Settings window for Time Dependent, locate the Physics and Variables Selection section.

- 3 In the table, clear the Solve for check boxes for Heat Transfer in Solids (ht2) and Radiation in Participating Media (rpm2).
- 4 In the table, clear the **Solve for** check box for Heat Transfer with Radiation in Participating Media 2 (htrpm2).

STUDY 2: PI

I In the Home toolbar, click **Compute**.

The same procedure applies for solving with the Rosseland approximation.

ADD COMPONENT

In the Model Builder window, right-click the root node and choose Add Component>3D.

GEOMETRY 3

- I In the Settings window for Geometry, locate the Units section.
- **2** From the **Length unit** list, choose **cm**.

MESH 3

Import I

- I In the Mesh toolbar, click Import.
- 2 In the Settings window for Import, locate the Import section.
- **3** From the **Source** list, choose **Meshing sequence**.
- 4 Click Import.

MATERIALS

Material Link 5 (matlnk5)

In the Model Builder window, under Component 3 (comp3) right-click Materials and choose More Materials>Material Link.

Material Link 6 (matlnk6)

- I Right-click Materials and choose More Materials>Material Link.
- 2 In the Settings window for Material Link, locate the Geometric Entity Selection section.
- 3 From the Geometric entity level list, choose Boundary.
- 4 From the Selection list, choose All boundaries.

HEAT TRANSFER IN SOLIDS (HT2)

In the Model Builder window, under Component 2 (comp2) right-click Heat Transfer in Solids (ht2) and choose Copy.

COMPONENT 3 (COMP3)

In the Model Builder window, right-click Component 3 (comp3) and choose Paste Heat Transfer in Solids.

HEAT TRANSFER IN SOLIDS (HT2)

In the Messages from Paste dialog box, click **OK**.

HEAT TRANSFER IN SOLIDS (HT3)

Solid 1

In the Model Builder window, under Component 3 (comp3)>Heat Transfer in Solids (ht3) click Solid 1.

Optically Thick Participating Medium 1

- I In the Physics toolbar, click 🖳 Attributes and choose **Optically Thick Participating Medium.**
- 2 In the Settings window for Optically Thick Participating Medium, locate the Optically Thick Participating Medium section.
- 3 In the n_r text field, type nr.

ADD STUDY

- I Go to the Add Study window.
- 2 Find the Studies subsection. In the Select Study tree, select Empty Study.
- 3 Click Add Study in the window toolbar.
- 4 In the Model Builder window, under Component I (compl) click Heat Transfer in Solids (ht).
- 5 In the Home toolbar, click Add Study to close the Add Study window.

STUDY 3: ROSSELAND

In the Settings window for Study, type Study 3: Rosseland in the Label text field.

STUDY 2: PI

Parametric Sweep, Step 1: Time Dependent

- I In the Model Builder window, under Study 2: PI, Ctrl-click to select Parametric Sweep and Step 1: Time Dependent.
- 2 Right-click and choose Copy.

STUDY 3: ROSSELAND

In the Model Builder window, right-click Study 3: Rosseland and choose Paste Multiple Items.

Step 1: Time Dependent

- I In the Model Builder window, under Study 3: Rosseland click Step I: Time Dependent.
- 2 In the Settings window for Time Dependent, locate the Physics and Variables Selection section.
- 3 In the table, clear the Solve for check boxes for Heat Transfer in Solids (ht), Radiation in Participating Media (rpm), Heat Transfer in Solids (ht2), and Radiation in Participating Media (rpm2).
- 4 In the table, clear the **Solve for** check boxes for Heat Transfer with Radiation in Participating Media I (htrpmI) and Heat Transfer with Radiation in Participating Media 2 (htrpm2).

STUDY I: DOM

- I In the Model Builder window, under Study I: DOM click Step I: Time Dependent.
- 2 In the Settings window for Time Dependent, locate the Physics and Variables Selection section.
- 3 In the table, clear the Solve for check box for Heat Transfer in Solids (ht3).

STUDY 2: PI

- I In the Model Builder window, under Study 2: PI click Step I: Time Dependent.
- 2 In the Settings window for Time Dependent, locate the Physics and Variables Selection section.
- 3 In the table, clear the Solve for check box for Heat Transfer in Solids (ht3).

STUDY 3: ROSSELAND

In the **Home** toolbar, click **Compute**.

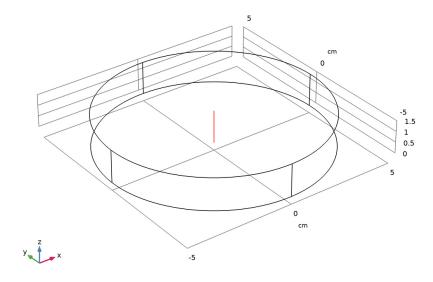
RESULTS

Incident Radiation (rpm2)

To compare the results, add a temperature plot along the centerline of the glass plate. Therefore create a cut line dataset for each parametric solution.

Cut Line 3D I

- I In the Model Builder window, expand the Results>Datasets node.
- 2 Right-click Results>Datasets and choose Cut Line 3D.
- 3 In the Settings window for Cut Line 3D, locate the Data section.
- 4 From the Dataset list, choose Study 1: DOM/Parametric Solutions 1 (sol2).
- 5 Locate the Line Data section. In row Point 2, set X to 0, and z to 1.5.
- 6 Click Plot.



Copy the dataset twice and assign the solutions from the other studies.

7 Right-click Cut Line 3D I and choose Duplicate.

Cut Line 3D 2

- I In the Model Builder window, click Cut Line 3D 2.
- 2 In the Settings window for Cut Line 3D, locate the Data section.

- 3 From the Dataset list, choose Study 2: PI/Parametric Solutions 2 (6) (sol7).
- 4 Right-click Cut Line 3D 2 and choose Duplicate.

Cut Line 3D 3

- I In the Model Builder window, click Cut Line 3D 3.
- 2 In the Settings window for Cut Line 3D, locate the Data section.
- 3 From the Dataset list, choose Study 3: Rosseland/Parametric Solutions 3 (12) (sol12).

Now, add a 1D plot group for the temperature.

Temperature at Central Line for k = 5

- I In the Results toolbar, click \to ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Temperature at Central Line for k = 5 in the Label text field.
- 3 Click to expand the Title section. From the Title type list, choose Manual.
- 4 In the **Title** text area, type Temperature at Central Line.

Line Graph 1

- I In the Temperature at Central Line for k = 5 toolbar, click \bigwedge Line Graph.
- 2 In the Settings window for Line Graph, locate the Data section.
- 3 From the Dataset list, choose Cut Line 3D 1.
- 4 From the Parameter selection (k) list, choose First.
- **5** From the **Time selection** list, choose **Last**.
- 6 Click to expand the Legends section. From the Legends list, choose Manual.
- **7** Select the **Show legends** check box.
- **8** In the table, enter the following settings:

Legends DOM

- 9 In the Temperature at Central Line for k = 5 toolbar, click Plot.
- **10** Right-click **Line Graph I** and choose **Duplicate**.

Line Graph 2

- I In the Model Builder window, click Line Graph 2.
- 2 In the Settings window for Line Graph, locate the Data section.
- 3 From the Dataset list, choose Cut Line 3D 2.

4 Locate the **Legends** section. In the table, enter the following settings:

Legends Р1

5 Right-click **Line Graph 2** and choose **Duplicate**.

Line Graph 3

- I In the Model Builder window, click Line Graph 3.
- 2 In the Settings window for Line Graph, locate the Data section.
- 3 From the Dataset list, choose Cut Line 3D 3.
- **4** Locate the **Legends** section. In the table, enter the following settings:

Legends Rosseland

5 In the Temperature at Central Line for k = 5 toolbar, click Plot.

Temperature at Central Line for k = 5

Duplicate the plot group and change the dataset to k = 120.

I In the Model Builder window, right-click Temperature at Central Line for k = 5 and choose **Duplicate**.

Temperature at Central Line for k = 120

- I In the Model Builder window, under Results click Temperature at Central Line for k = 5.1.
- 2 In the Settings window for ID Plot Group, type Temperature at Central Line for k = 120 in the Label text field.

Line Graph 1

- I In the Model Builder window, expand the Temperature at Central Line for k = 120 node, then click Line Graph 1.
- 2 In the Settings window for Line Graph, locate the Data section.
- 3 From the Parameter selection (k) list, choose Last.

Line Graph 2

- I In the Model Builder window, click Line Graph 2.
- 2 In the Settings window for Line Graph, locate the Data section.
- 3 From the Parameter selection (k) list, choose Last.

Line Graph 3

- I In the Model Builder window, click Line Graph 3.
- 2 In the Settings window for Line Graph, locate the Data section.
- 3 From the Parameter selection (k) list, choose Last.

The plots are shown in Figure 2 and Figure 3. For high optical thicknesses, the Rosseland approximation better represents the overall temperature distribution, whereas the P1 approximation is appropriate for lower optical thicknesses.