Created in COMSOL Multiphysics 6.2

Time-to-Frequency Fast Fourier Transform of a Coaxial Low-Pass Filter

Introduction

A very wideband coaxial low-pass filter is designed using a 2D axisymmetric model. To address the wideband frequency response with a fine frequency resolution, the model is built with a transient physics interface first and then S-parameters are calculated using a time-to-frequency fast Fourier transform (FFT). The computed S-parameters show a low-pass frequency response with a cutoff frequency around 24.5 GHz.

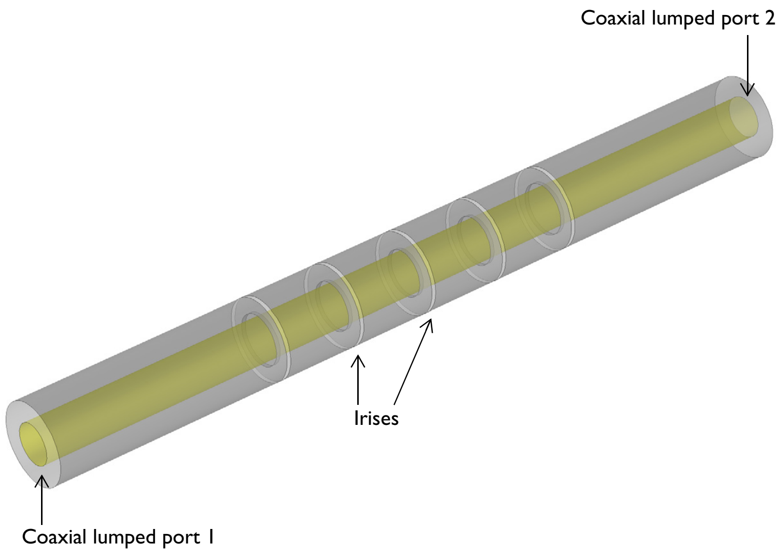


Figure 1: Coaxial structure with irises that is symmetric around the axis of the center conductor. The example is efficiently modeled using a 2D axisymmetric configuration.

Model Definition

To achieve a low-pass frequency response, an air-filled coaxial cable is tuned with five annular rings (irises) that are added to the outer conductor wall in this example model. The coaxial cable walls and iris parts are modeled as perfect electric conductor (PEC). The volume inside the center conductor of the coaxial cable is removed and only the conducting metal surfaces are modeled. The space between the center and outer conductor is set to vacuum. Each end of the coaxial cable is terminated with a 41.56 Ω lumped port. The characteristic impedance for a lossless coaxial cable is calculated using the radius of the center and outer conductor parts:

$$Z = \sqrt{\frac{L}{C}} = \sqrt{\frac{\frac{\mu_0\mu_r}{2\pi}\ln\frac{R_{\mathrm{coax}}}{r_{\mathrm{coax}}}}{(2\pi\varepsilon_0\varepsilon_r)/\left(\ln\frac{R_{\mathrm{coax}}}{r_{\mathrm{coax}}}\right)}} = \frac{Z_0}{2\pi\sqrt{\varepsilon_r}}\ln\left(\frac{R_{\mathrm{coax}}}{r_{\mathrm{coax}}}\right)$$

where Z_0 is the characteristic impedance of free space, ε_r is the relative permittivity of the material between the center and outer conductors, R_{coax} is the radius of the outer conductor, and r_{coax} is the radius of the inner conductor.

In the lumped port setting window, by clicking the check box "Calculate S-parameter" on the excitation port, the voltage excitation type is set to the modulated Gaussian and the center frequency (f_0) of the modulating sinusoidal function can be specified. The excitation voltage is defined as:

$$\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{\left(t-\frac{2}{f_0}\right)^2}{2\sigma^2}\right)\sin(2\pi f_0(1+\eta_f)t)$$

where σ is the standard deviation $1/2f_0$, f_0 is the center frequency and η_f is the modulating frequency shift ratio.

The frequency here has to be matched to the center frequency of the S-parameter calculation used in the Time to Frequency FFT study step (see below).

The end time of the Time Dependent study step is set to 100 times of the period of the modulating sinusoidal function, which is long enough in this model to ensure that the input energy is fully decayed. This would work for a typical passive circuit except for closed cavity type devices, where the energy decay time can be much longer. The stop condition is automatically added under the Time-Dependent solver (the **Calculate S-parameter** check box activates this stop condition in the solver settings). When the sum of total electric and magnetic energy in the modeling domain is below 70 dB compared to the input energy, the Time Dependent study is terminated by the stop condition and all time-domain data will be passed to the FFT step. To generate the frequency-domain data without significant distortion in the frequency range between 0 and $2f_0$, the time step, satisfying the Nyquist criterion, is set to $1/4f_0 = 1/2B$, where B is the bandwidth $2f_0$.

To provide a fine frequency resolution, the end time of the FFT study step is much longer than that of the Time Dependent study. Zero-padding is applied between the end time of the Time Dependent study and that of the FFT study step.

Results and Discussion

The computed S-parameters (Figure 2) shows the low-pass frequency response of the coaxial filter. The –3 dB cutoff is observed at 24.5 GHz. The last 15% close to the highest frequency is noisy, so the current simulation settings are not appropriate for the analysis close to the high-frequency limit of the FFT. Figure 3 shows the contour plot of the electric field norm distribution and the arrow plot of the time-averaged power flow at 10 GHz, which is within the passband where the electric field is confined on the irises. The power flow is straight toward the observation port from the excitation port.

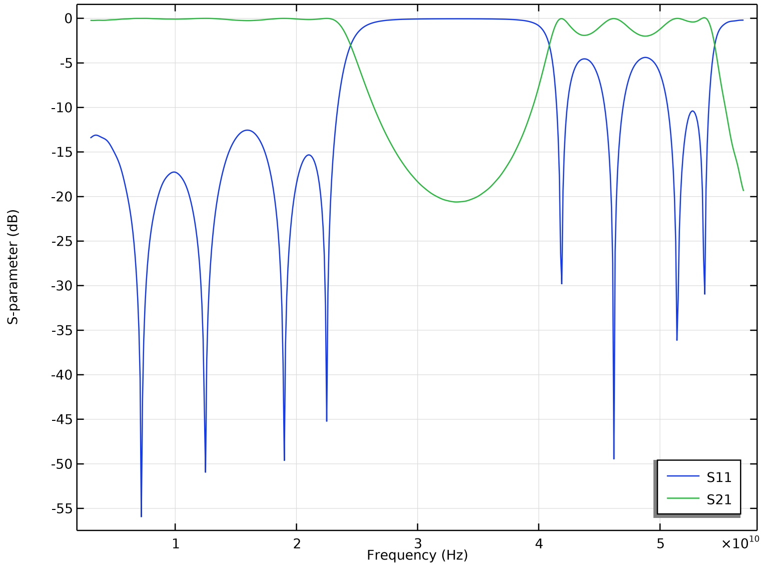


Figure 2: Time-to-frequency fast Fourier transform of a transient simulation calculates frequency responses. The -3 dB cutoff is observed at 24.5 GHz.

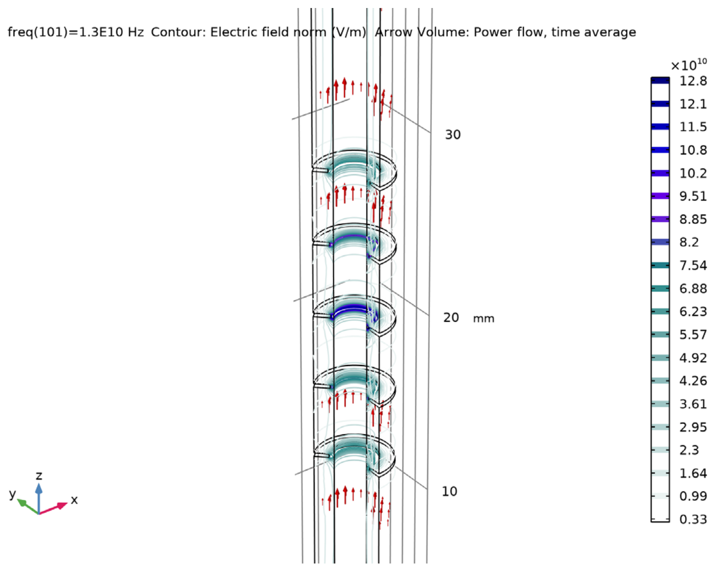


Figure 3: A 3D result plot from a 2D axisymmetric model. The electric field norm distribution and time-averaged power flow at 10 GHz are visualized.

Application Library path: `RF_Module/Filters/coaxial_low_pass_filter_transient`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D Axisymmetric**.

2 In the **Select Physics** tree, select **Radio Frequency>Electromagnetic Waves, Transient (temw)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces>Time Dependent with FFT**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
r_coax	1[mm]	0.001 m	Inner radius, coaxial cable
R_coax	2[mm]	0.002 m	Outer radius, coaxial cable
f0	30[GHz]	3E10 Hz	Center frequency
L	c_const/f0	0.0099931 m	Wavelength, free space
T0	1/f0	3.3333E-11 s	Period
h_max	L/10	9.9931E-4 m	Maximum mesh element size
Tend	100*T0	3.3333E-9 s	End time
Z0	(Z0_const/2/pi)*log(R_coax/r_coax)	41.56 Ω	Characteristic impedance

Here, `c_const` used in the free space wavelength is a predefined COMSOL constant for the speed of light in vacuum.

GEOMETRY 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.

2 In the **Settings** window for **Geometry**, locate the **Units** section.

3 From the **Length unit** list, choose **mm**.

Create the air-filled part of a coaxial cable.

Rectangle 1 (r1)

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Width** text field, type `R_coax-r_coax`.

4 In the **Height** text field, type `40`.

5 Locate the **Position** section. In the **r** text field, type `r_coax`.

Add a few irises inside the coaxial cable to convert the cable to a low-pass filter.

Rectangle 2 (r2)

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Width** text field, type `0.82`.

4 In the **Height** text field, type `0.2`.

5 Locate the **Position** section. In the **r** text field, type `R_coax-0.82`.

6 In the **z** text field, type `23.9`.

Rectangle 3 (r3)

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Width** text field, type `0.75`.

4 In the **Height** text field, type `0.2`.

5 Locate the **Position** section. In the **r** text field, type `R_coax-0.75`.

6 In the **z** text field, type `27.9`.

Mirror 1 (mir1)

1 In the **Geometry** toolbar, click **Transforms** and choose **Mirror**.

2 Select the objects **r2** and **r3** only.

3 In the **Settings** window for **Mirror**, locate the **Input** section.

4 Select the **Keep input objects** check box.

5 Locate the **Point on Line of Reflection** section. In the **z** text field, type `20`.

6 Locate the **Normal Vector to Line of Reflection** section. In the **r** text field, type `0`.

7 In the **z** text field, type `1`.

Rectangle 4 (r4)

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Width** text field, type `0.86`.

4 In the **Height** text field, type `0.2`.

5 Locate the **Position** section. In the **r** text field, type `R_coax-0.86`.

6 In the **z** text field, type `19.9`.

Difference 1 (dif1)

1 In the **Geometry** toolbar, click **Booleans and Partitions** and choose **Difference**.

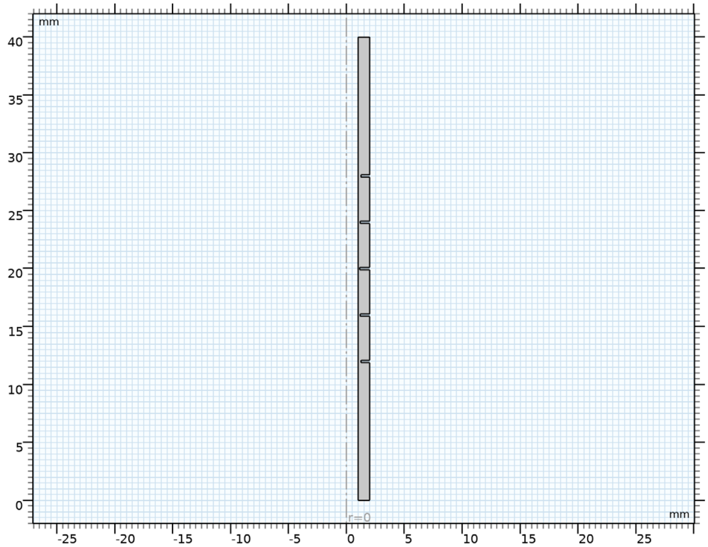
2 Select the object **r1** only.

3 In the **Settings** window for **Difference**, locate the **Difference** section.

4 Click to select the **Activate Selection** toggle button for **Objects to subtract**.

5 Select the objects **mir1(1)**, **mir1(2)**, **r2**, **r3**, and **r4** only.

6 Click **Build All Objects**.



Next, set up the physics using appropriate boundary conditions.

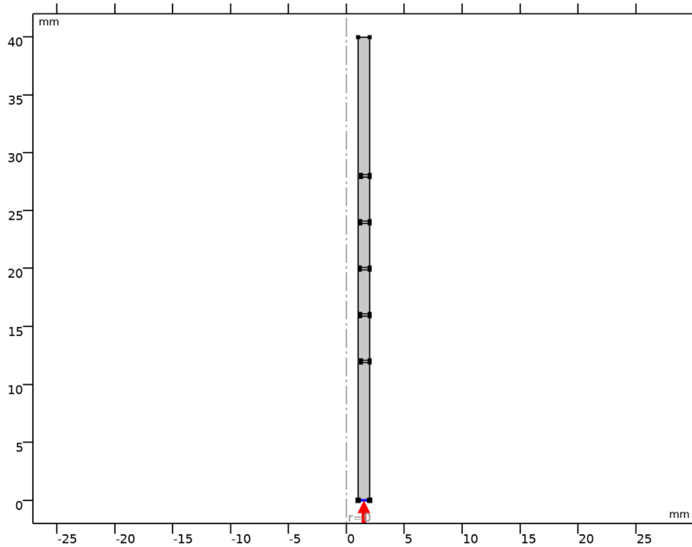
ELECTROMAGNETIC WAVES, TRANSIENT (TEMW)

Lumped Port 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Electromagnetic Waves, Transient (temw)** and choose **Lumped Port**.

2 Select Boundary 2 only.

This is the input port of the filter.



3 In the **Settings** window for **Lumped Port**, locate the **Settings** section.

4 Select the **Calculate S-parameter** check box.

5 In the f_0 text field, type `f0`.

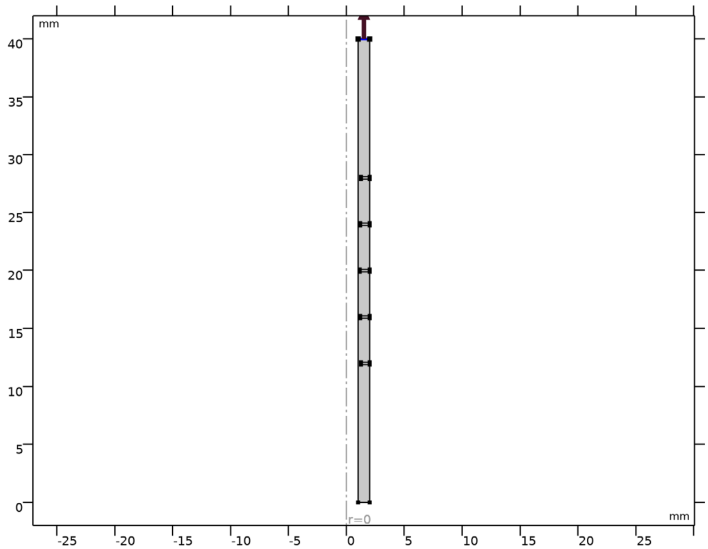
6 In the Z_{ref} text field, type `Z0`.

Lumped Port 2

1 In the **Physics** toolbar, click **Boundaries** and choose **Lumped Port**.

2 Select Boundary 3 only.

This is the output port of the filter.



3 In the **Settings** window for **Lumped Port**, locate the **Settings** section.

4 In the Z_{ref} text field, type `Z0`.

MATERIALS

Material 1 (mat1)

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, locate the **Material Contents** section.

3 In the table, enter the following settings:

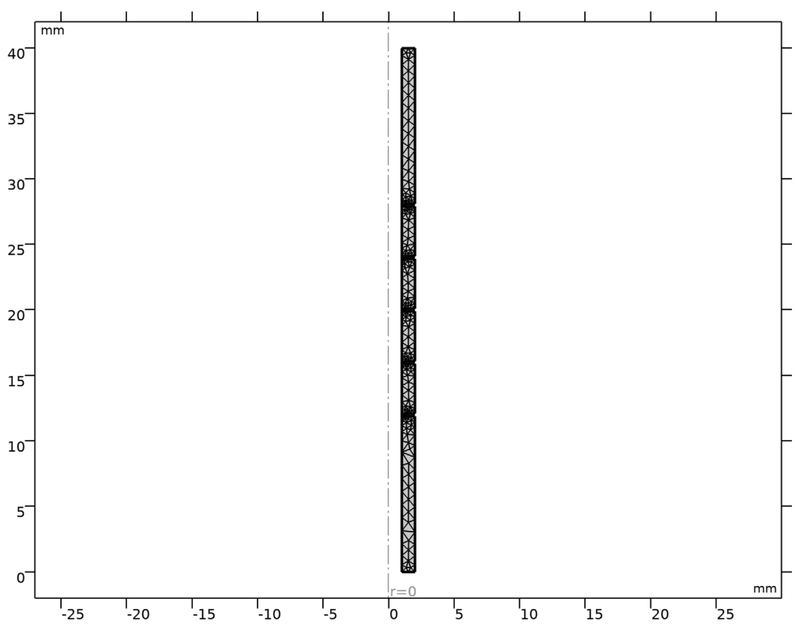
Property	**Variable**	**Value**	**Unit**	**Property group**
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	1	1	Basic

Property	Variable	Value	Unit	Property group
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	1	1	Basic
Electrical conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	0	S/m	Basic

Set the maximum mesh size smaller than 0.2 wavelengths.

MESH 1

1. In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.
2. In the **Settings** window for **Mesh**, locate the **Sequence Type** section.
3. From the list, choose **Physics-controlled mesh**.
4. Locate the **Electromagnetic Waves, Transient (temw)** section. In the **Maximum element size in free space** text field, type `h_max`.
5. Click **Build All**.



6 In the **Model Builder** window, right-click **Mesh 1** and choose **Edit Physics-Induced Sequence**.

STUDY 1

Step 1: Time Dependent

1 In the **Model Builder** window, under **Study 1** click **Step 1: Time Dependent**.

2 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.

3 In the **Output times** text field, type `range(0,1/(4*f0),Tend)`. The Sampling rate `4*f0` satisfies the Nyquist condition for the time to frequency fast Fourier transform (FFT) where its bandwidth is 2*f0 excluding negative frequencies.

Step 2: Time to Frequency FFT

1 In the **Model Builder** window, click **Step 2: Time to Frequency FFT**.

2 In the **Settings** window for **Time to Frequency FFT**, locate the **Study Settings** section.

3 In the **End time** text field, type `Tend*3`. This makes sure that the FFT end time is longer than the simulation time so zero-padding can be applied during the time to frequency FFT. This will generate a finer frequency resolution in the resulting frequency response.

4 In the **Maximum output frequency** text field, type `f0*2`.

Step 3: Combine Solutions

1 In the **Model Builder** window, click **Step 3: Combine Solutions**.

2 In the **Settings** window for **Combine Solutions**, locate the **Combine Solutions Settings** section.

3 In the **Excluded if** text field, type `freq<0.1*f0 || freq>2*f0-0.1*f0`. This excludes the first 5% and last 5% of the frequency response after FFT.

4 In the **Home** toolbar, click **Compute**.

RESULTS

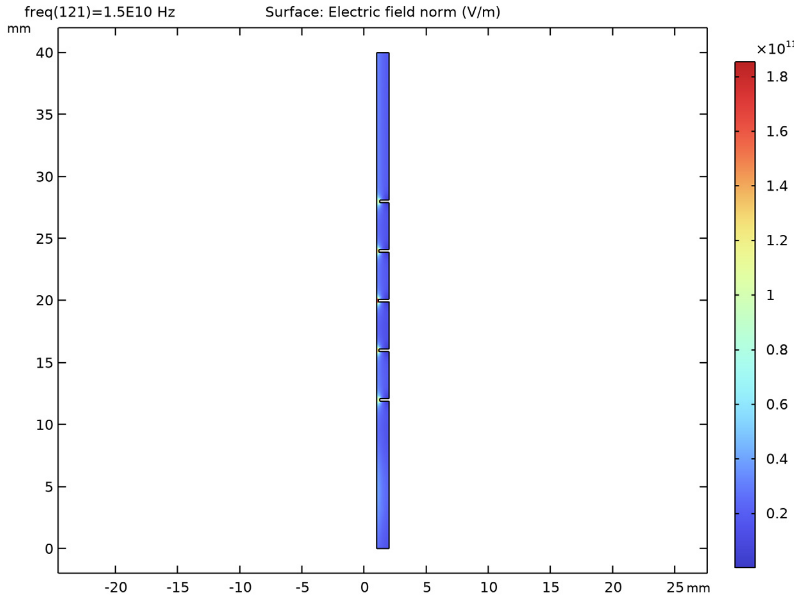
2D Plot Group 1

Choose one frequency in the passband.

1 In the **Settings** window for **2D Plot Group**, locate the **Data** section.

2 From the **Parameter value (freq (Hz))** list, choose **1.5E10**.

3 In the **2D Plot Group 1** toolbar, click **Plot**.



Strong electric fields are confined around the irises when the frequency is within the range of passband.

S-parameter (temw)

1 In the **Model Builder** window, click **S-parameter (temw)**.

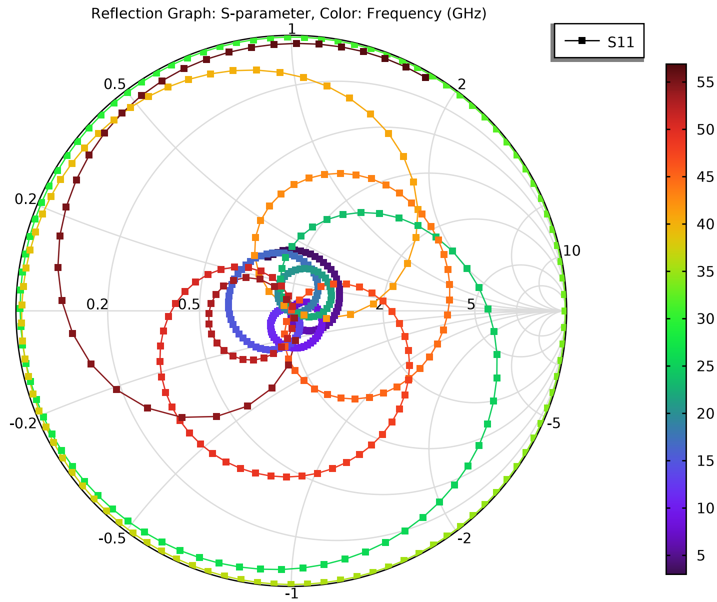
2 In the **Settings** window for **1D Plot Group**, locate the **Legend** section.

3 From the **Position** list, choose **Lower right**.

The S-parameters as a function of frequency calculated from a transient simulation of the coaxial filter is shown in Figure 2.

Smith Plot (temw)

1 In the **Model Builder** window, click **Smith Plot (temw)**.



The input matching properties is described with the Smith plot.

3D Plot Group 4

1 In the **Home** toolbar, click **Add Plot Group** and choose **3D Plot Group**.

2 In the **Settings** window for **3D Plot Group**, locate the **Data** section.

3 From the **Parameter value (freq (Hz))** list, choose **1.3E10**.

Contour 1

1 Right-click **3D Plot Group 4** and choose **Contour**.

2 In the **Settings** window for **Contour**, locate the **Coloring and Style** section.

3 Click **Change Color Table**.

4 In the **Color Table** dialog box, select **Aurora>AuroraAustralis** in the tree.

5 Click **OK**.

6 In the **3D Plot Group 4** toolbar, click **Plot**.

Arrow Volume 1

1 In the **Model Builder** window, right-click **3D Plot Group 4** and choose **Arrow Volume**.

2 In the **Settings** window for **Arrow Volume**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)>Electromagnetic Waves, Transient>Energy and power>temw.Poavr,...,temw.Poavz - Power flow, time average**.

3 In the **3D Plot Group 4** toolbar, click **Plot**.

4 Click the **Zoom In** button in the **Graphics** toolbar.

Figure 3 shows the 3D result plot out of a 2D axisymmetric model.