

Precipitation of Barium Sulfate

Introduction

Crystallization is an important separation process in the chemical industry. It is used for the production of pharmaceuticals and industrial chemicals. It can also be used in resource recovery as a way of separating valuable materials from waste. An example of this is the recovery of metals from batteries.

The driving force for crystallization is the amount of solute in excess of its solubility. This is referred to as the supersaturation. Different methods of generating supersaturation are typically used to categorize crystallization processes. These include cooling, evaporation, addition of a secondary solvent (antisolvent), and by chemical reaction (precipitation).

The two main mechanisms that occur during crystallization is nucleation and growth. Nucleation is the formation of new crystal particles in the form of nuclei from solute molecules. Crystal growth is the incorporation of solute molecules into an already existing crystal lattice.

The model is based on the work by Ölander (Ref. 1), Schwarzer and Peukert (Ref. 2), and Tang and others (Ref. 3). The model aims to simulate the evolution of a crystal particle population by solving the population balance equation. The system is modeled both as a perfectly mixed 0D batch reactor and a 3D T-mixer.

Model Definition

The precipitation in 0D is modeled as a time-dependent system where barium chloride and sulphuric acid are assumed perfectly mixed at initial concentrations of 500 and 330 mol/m³. The 3D barium sulfate precipitation takes place in a tubular T-mixer consisting of a 10 mm long mixing pipe with a diameter of 1 mm. Two oppositely placed inlets each with a diameter of 0.5 mm are located at the top of the mixing pipe. An overview of the geometry can be seen in Figure 1. In 3D, barium chloride and sulphuric acid enter from the oppositely placed inlets at concentrations of 500 and 330 mol/m³. The inlet velocity is adjusted to achieve a Reynolds number of 255 based on the diameter of the mixing pipe. The flow field used for species and particle transport is solved for by the Laminar Flow interface. The T-mixer is modeled at steady state.



Figure 1: T-mixer geometry consisting of two inlet pipes and one main mixing pipe.

In both scenarios, the dissociation of barium chloride and the first step dissociation of sulphuric acid are assumed complete. The second step dissociation of sulphuric acid and formation of aqueous barium sulfate take place according to

$$\mathrm{HSO}_4^- \leftrightarrow \mathrm{SO}_4^{2-} + \mathrm{H}^+$$

$$Ba^{2+} + SO_4^{2-} \leftrightarrow BaSO_4(aq)$$

while the formation of crystalline barium sulfate takes place according to

$$Ba^{2+} + SO_4^{2-} \leftrightarrow BaSO_4(s)$$

The reactions are set up with the Reaction Engineering interface in 0D and Chemistry interface coupled with the Transport of Diluted Species interface in 3D. The supersaturation is defined from the reactant concentrations using the mean activity coefficient, γ , and the solubility product $K_{\rm SP}$ (mol²/m⁶) according to

$$S = \gamma_{\pm} \sqrt{\frac{c_{\text{Ba}^{2+}} c_{\text{SO}_{4}^{2-}}}{K_{\text{SP}}}}$$
 (1)

Multiple models are available to describe the activity coefficient. Here, the extended Debye–Hückel model presented by Bromley (Ref. 4) is used. The activity coefficient is calculated as

$$\gamma_{\pm} = \frac{-A \left| Z_{Ba^{2+}} Z_{SO_4^{2-}} \right| \sqrt{I_m}}{1 + \sqrt{I_m}} + \frac{\left| Z_{Ba^{2+}} Z_{SO_4^{2-}} \right|}{\left| Z_{Ba^{2+}} \right| + \left| Z_{SO_4^{2-}} \right|} \left(\frac{F_{Ba^{2+}}}{\left| Z_{Ba^{2+}} \right|} + \frac{F_{SO_4^{2-}}}{\left| Z_{SO_4^{2-}} \right|} \right)$$
(2)

and

$$F_{Ba^{2+}} = \sum_{a} \left(\frac{(0.06 + 0.6B_{Ba^{2+},a}) \left| Z_{Ba^{2+}} Z_a \right|}{\left(1 + \frac{1.5}{\left| Z_{Ba^{2+}} Z_a \right|} I_m \right)^2} \left(\frac{\left| Z_{Ba^{2+}} \right| + \left| Z_a \right|}{2} \right)^2 m_a \right)$$
(3)

$$F_{SO_4^{2^-}} = \sum_{c} \left(\frac{(0.06 + 0.6B_{SO_4^{2^-},c}) \left| Z_{SO_4^{2^-}} Z_c \right|}{\left(1 + \frac{1.5}{\left| Z_{SO_2^{2^-}} Z_c \right|} I_m \right)^2} \left(\frac{\left| Z_{SO_4^{2^-}} \right| + \left| Z_c \right|}{2} \right)^2 m_c \right)$$
(4)

where a and c are the anions and cations in the solution. The constants $B_{a,c}$ are calculated from tabulated values from Bromley (Ref. 4) according to

$$B_{a,c} = B_a + B_c + \delta_a \delta_c \tag{5}$$

For interactions with hydrogen sulfate ions the values are instead taken from and calculations done according to Ref. 3.

THE POPULATION BALANCE EQUATION

To model the population distribution as a function of particle size, the particle number density, $n (1/m^4)$, is defined as:

$$n = \frac{N}{L} \tag{6}$$

Where L describes the particle size expressed as particle diameter (m) and N is the number of particles per unit volume $(1/m^3)$. The change in particle number density with respect to time in a crystal particle population can be described using the population balance equation

$$\frac{\partial n}{\partial t} + \nabla \cdot (\mathbf{u}n) + \frac{\partial (G(S, L)n(L))}{\partial L} = \frac{v}{S_c} \nabla^2 n + B_0 \delta(L - L_{c0})$$
 (7)

where

- G is the crystal growth rate (m/s)
- v is the kinematic viscosity (m^2/s)
- S_c is the Schmidt number
- B_0 the nucleation rate as a source of number density $(1/m^4/s)$
- ullet $L_{
 m c0}$ the smallest stable crystal size (m)

The terms on the left-hand side represent the change of population density with respect to time, the convective crystal transport, and crystal growth, respectively. The terms on the right-hand side represent the diffusive crystal transport according to Fick's second law and the birth rate from crystal nucleation. Here, the nucleation rate as a source of number density, B_0 , correlates to the nucleation rate as a source of particle number, B_N ($1/m^3/s$), as $B_0 = B_N/L$, where B_N is defined according to classical nucleation theory as (Ref. 1, Ref. 3).

$$B_{\rm N} = \frac{3}{2} D_{\rm AB} (\sqrt{K_{\rm SP} S N_{\rm A}})^{7/3} \sqrt{\frac{\gamma_{\rm CL}}{k_{\rm B} T}} V_{\rm m} \exp \left(\frac{-16\pi}{3} \frac{\gamma_{\rm CL}^3}{T^3 k_{\rm p}^3 (\ln S)^2} \right)$$
(8)

where

- D_{AB} is the apparent diffusion coefficient (m²/s)
- N_A is the Avogadro number (1/mol)
- γ_{CL} is the interfacial energy (J/m^2)
- $k_{\rm B}$ is the Boltzmann constant (J/K)
- $V_{\rm m}$ is the molecular volume (m³)
- T is the temperature (K)

Nucleation occurs at the smallest stable crystal size $L_{
m c0}$ defined as

$$L_{c0} = \frac{4\gamma_{\rm CL} V_{\rm m}}{v_{d} k_{\rm B} T \ln S} \tag{9}$$

with the dissociation number v_d . In this model the smallest crystal size at which nucleation occurs is taken as constant. The transport controlled growth rate can be described as (Ref. 3)

$$G = \frac{k_a \operatorname{Sh} D_{AB} \sqrt{K_{SP}} M_{W,C} (S-1)}{\rho_C L}$$
(10)

Here k_a and k_v are the area and volume shape factors relating the particle area and volume to the size, while Sh is the Sherwood number, $M_{W,C}$ is the crystal molar mass (kg/mol) and ρ_C the crystal density (kg/m³).

CRYSTAL SIZE DISCRETIZATION

The continuous population balance equation (Equation 7) is discretized using a number of intervals (bins). Each interval, i = 1, 2, 3, ..., I, corresponds to a crystal-size interval

spanning from size L_{i-1} to L_i with an interval length ΔL_i . The smallest crystal size, L_0 , is taken as the smallest stable crystal size, L_{c0} . Thus, the nucleation source term becomes the particle flux at the left-hand side face of the interval i = 1. The size points, L_i , are defined geometrically such that the interval length increases with the interval index according to (Ref. 1)

$$L_i = \frac{L_0 - 2^{(-I/q)} L_I}{1 - 2^{(-I/q)}} + 2^{(i-I)/q} \left(L_I - \frac{L_0 - 2^{(-I/q)} L_I}{1 - 2^{(-I/q)}} \right) \tag{11}$$

The model implements 100 size intervals in the perfectly mixed reactor and 30 for the Tmixer scenario covering crystal sizes from 0.82 to 500 nm. The discretized population balance equations for each crystal size interval is modeled with a Global ODEs and DAEs interface in 0D and separate Stabilized Convection-Diffusion Equation interfaces in 3D with nucleation and growth added as source terms.

The growth-rate term in the PBE (Equation 7) is approximated with a total variation diminishing scheme blending central and backward differences according to (Ref. 5)

$$\frac{\partial (Gn)_i}{\partial L} = \frac{(Gn)_i - (Gn)_{i-1}}{\Delta L_i} \tag{12}$$

where

$$(Gn)_{i} = G_{i} \left(n_{i} + \frac{1}{2} \Phi \left(\frac{n_{i+1} - n_{i} + \varepsilon}{n_{i} - n_{i-1} + \varepsilon} \right) (n_{i} - n_{i-1}) \right), \tag{13}$$

Here, Φ is a limiting function to reduce numerical fluctuations and ε is a built-in nonzero constant to avoid division by zero. The limiting function, initially introduced by Koren in Ref. 6, is implemented according to

$$\Phi(r) = \max\left(0, \min\left(\frac{1-\kappa}{4} + \frac{1+\kappa}{4} + \frac{\frac{1+\kappa}{4}}{\frac{1-\kappa}{4} + \frac{1+\kappa}{4}}r, 2\right)\right)$$
(14)

where $\kappa \in [-1, 1]$ is the blending constant. At $\kappa = -1$, the approximation takes on the form of a second-order backward finite-element scheme and at $\kappa = 1$ a first-order central scheme. The number density of all intervals smaller than the smallest defined interval is assumed to be zero while the number density of intervals at crystal sizes greater than the largest defined size is assumed to be equal to the largest defined interval. Here, Equation 13 represents the growth-induced particle flux at the right-hand side interval

face. The left-hand side particle flux, $(Gn)_{i-1}$, is similarly written by reducing the interval index by 1.

REACTION RATE

The reaction rate, r, resulting from the formation of crystalline barium sulfate, in units of mol/m³/s, is expressed using the nucleation and growth rates according to

$$r = \frac{\rho_c}{M_{\rm W,C}} \left(B_{\rm nuc} L_0^{\ 3} k_{\rm v} \Delta L_1 + \sum_i \left(\frac{L_i + L_{i-1}}{2} \right)^3 k_{\rm v} \frac{\partial (Gn)_i}{\partial L} \Delta L_i \right) \tag{15}$$

The reaction rate is added as a consumption term to the barium and sulfate species in the Chemistry interface. Assuming that aqueous barium sulfate is in equilibrium with barium and sulfate ions, the consumption term can instead be added to aqueous barium sulfate. The discretized equations along with the required sum expressions are generated with the Crystal PBE Discretization add-in, available in the Chemical Reaction Engineering Module Add-in Library.

MESH

The mesh is constructed as a tradeoff between computation time and accuracy. The resolution is the highest between the oppositely placed inlets to resolve the mixing of streams. The mesh along the length of the mixing pipe is focused on resolving gradients in the cross-flow direction. Using meshes finer than what this model implements will give more accurate flow fields and crystal growth but require longer times to find a solution.

Results and Discussion

The crystal size distribution for both the perfectly mixed batch reactor and T-mixer scenarios are presented along with complementing data.

PERFECTLY MIXED BATCH REACTOR

The supersaturation simultaneously acts as the driving force for the precipitation and represents the amount of available reactant in the solution. The change in supersaturation with time in a perfectly mixed batch reactor is seen in Figure 2. The initial supersaturation is consumed as barium sulfate precipitates until reaching an equilibrium where the ion concentrations are equal to the solubility.

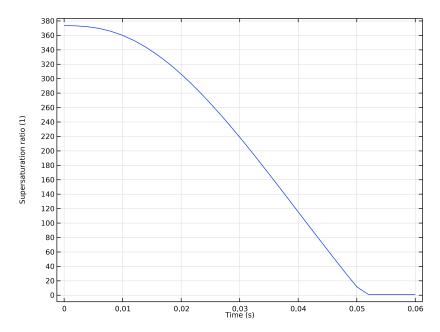


Figure 2: Supersaturation as a function of time in a perfectly mixed batch reactor.

The size distribution of crystals formed from the supersaturation over time is shown in Figure 3. The density distribution is normalized by the total amount of crystal particles per unit volume at each time.

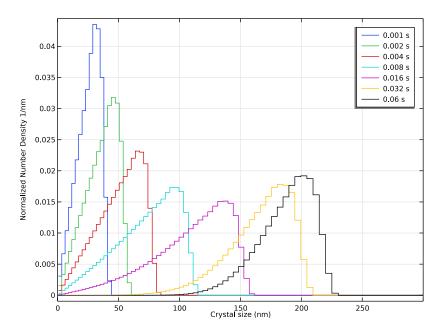


Figure 3: Normalized density distribution at various times in a perfectly mixed batch reactor.

The transition from solute reactants to crystalline particles can be seen from Figure 4, which shows the mass concentrations of the reacting species and the crystalline product over time. The total concentration remains constant, verifying the conservation of mass in the system.

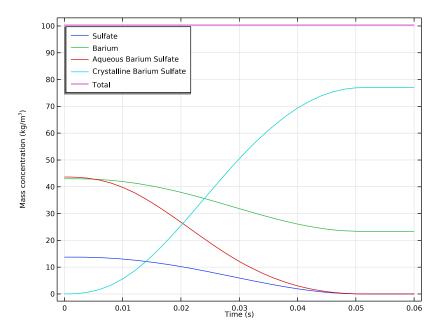


Figure 4: Mass concentrations of reacting species and crystalline product over time.

T-MIXER

The computed velocity field and supersaturation in the T-mixer at steady state is shown in Figure 5. The two separate inlet flows mix at the top of the mixing pipe. Along the length of the mixing pipe the combined flow becomes increasingly uniform. The supersaturated state occurs in areas where barium and sulfate ions interact. This is initially at the top part of the mixing pipe. The supersaturated decreases throughout the mixing pipe, indicating nucleation and crystal growth.

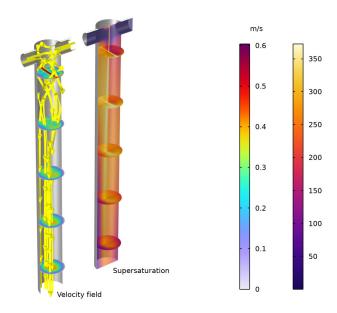


Figure 5: Velocity field and supersaturation in the T-mixer.

The density distribution at different points along the main mixing channel is shown in Figure 6. The distribution at z = 9.375 mm and z = 8.75 mm represent a very small number of crystals. At these crystal sizes the distribution is highly dependent on the mesh. To a lesser degree this is also true for distributions at larger crystal sizes.

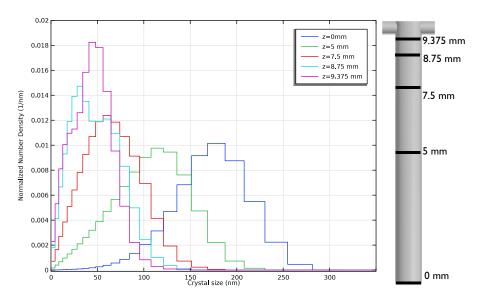


Figure 6: Number density distribution at various points along the T-mixer main mixing

References

- 1. M. Ölander, Numerical Simulations for Battery Recycling, master's thesis, KTH, Royal Institute of Technology, 2023.
- 2. H.C. Schwarzer and W. Peukert, "Combined experimental/numerical study on the precipitation of nanoparticles," AICHE J., vol. 50, no. 12, pp. 3234–3247, 2004.
- 3. H. Y. Tang, S. Rigopoulos, and G. Papadakis, "On the interaction of turbulence with nucleation and growth in reaction crystallisation," J. Fluid Mech, vol. 944, p. A48, 2022.
- 4. L. A. Bromley, "Thermodynamic properties of strong electrolytes in aqueous solution", AICHE J., vol. 19, no. 2, pp. 313-320, 1973.
- 5. S. Qamar, M.P. Elsner, I.A. Angelov, G. Warnecke and A. Seidel-Morgenstern, "A comparative study of high resolution schemes for solving population balances in crystallization," Comput. Chem. Eng., vol. 30, no. 6-7, pp. 1119-1131, 2006.
- 6. B. Koren, "A robust upwind discretization method for advection, diffusion and source terms," Notes Numer. Fluid Mech., vol. 45, pp. 117-138, 1993.

Application Library path: Chemical_Reaction_Engineering_Module/ Mixing_and_Separation/barium_sulfate_precipitation

Modeling Instructions

From the File menu, choose New.

NEW

- I In the New window, click Blank Model.
- 2 In the Home toolbar, click Windows and choose Add-in Libraries.

ADD-IN LIBRARIES

- I In the Add-in Libraries window, select Chemical Reaction Engineering Module> population_balance_equation_discretization in the tree.
- 2 In the tree, clear the check box for the node Chemical Reaction Engineering Module> population_balance_equation_discretization.
- 3 In the tree, select the check box for the node Chemical Reaction Engineering Module> population_balance_equation_discretization.
- 4 Click Mone.
- 5 In the Developer toolbar, click Add-ins and choose
 Population Balance Equation Discretization>Population Balance Equation Discretization.

Import the parameters for the model from a separate file.

GLOBAL DEFINITIONS

Parameters I

- I In the Model Builder window, click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- 3 Click Load from File.
- **4** Browse to the model's Application Libraries folder and double-click the file barium_sulfate_precipitation_parameters.txt.

Import the constants for the Debye-Hückel model.

Debye Hückel constants

- I In the Home toolbar, click Pi Parameters and choose Add>Parameters.
- 2 In the Settings window for Parameters, type Debye Hückel constants in the Label text field.
- 3 Locate the Parameters section. Click **Load from File.**
- **4** Browse to the model's Application Libraries folder and double-click the file barium_sulfate_precipitation_debye_huckel_constants.txt.

Define a function to be used for the Debye-Hückel model.

Ion interaction

- I In the Home toolbar, click f(x) Functions and choose Global>Analytic.
- 2 In the Settings window for Analytic, type Ion interaction in the Label text field.
- 3 In the Function name text field, type B dot.
- 4 Locate the **Definition** section. In the **Expression** text field, type ((0.06+0.6*B)*(Z1* Z2))/((1+I*1.5/(Z1*Z2))^2)+B.
- 5 In the Arguments text field, type B, Z1, Z2, I.
- **6** Locate the **Units** section. In the **Function** text field, type kg/mol.
- 7 In the table, enter the following settings:

Argument	Unit
В	kg/mol
1	mol/kg

ADD COMPONENT

In the **Home** toolbar, click **Add Component** and choose **OD**.

Import the required variable expressions for the Debye-Hückel model and the nucleation rate from separate files.

DEFINITIONS

Activity coefficient (Debye Hückel model)

- I In the Model Builder window, under Component I (compl) right-click Definitions and choose Variables.
- 2 In the Settings window for Variables, type Activity coefficient (Debye Hückel model) in the Label text field.
- 3 Locate the Variables section. Click **Load from File.**

4 Browse to the model's Application Libraries folder and double-click the file barium_sulfate_precipitation_debye_huckel_model1.txt.

Nucleation rate

- I In the Model Builder window, right-click Definitions and choose Variables.
- 2 In the Settings window for Variables, type Nucleation rate in the Label text field.
- 3 Locate the Variables section. Click **Load from File.**
- **4** Browse to the model's Application Libraries folder and double-click the file barium_sulfate_precipitation_nucleation_rate.txt.

ADD PHYSICS

- I In the Home toolbar, click Windows and choose Add Physics.
- 2 Go to the Add Physics window.
- 3 In the tree, select Chemical Species Transport>Reaction Engineering (re).
- 4 Click Add to Component I in the window toolbar.

REACTION ENGINEERING (RE)

- I In the Settings window for Reaction Engineering, locate the Energy Balance section.
- **2** In the *T* text field, type T.

Reaction I

- I Right-click Component I (compl)>Reaction Engineering (re) and choose Reaction.
- 2 In the Settings window for Reaction, locate the Reaction Formula section.
- 3 In the Formula text field, type HS04(-)=H(+)+S04(2-).
- 4 Click Apply.
- **5** Locate the **Equilibrium Settings** section. In the K_i text field, type K_dis.

Reaction 2

- I In the Reaction Engineering toolbar, click \angle Reaction.
- 2 In the Settings window for Reaction, locate the Reaction Formula section.
- 3 In the Formula text field, type BaS04 = Ba(++) + S04(2-).
- 4 Click Apply.
- **5** Locate the **Equilibrium Settings** section. In the K_j text field, type K_i p.

Species 1

- I In the Reaction Engineering toolbar, click 📩 Species.
- 2 In the Settings window for Species, locate the Name section.

3 In the text field, type C1(-).

Species: SO4(2-)

- I In the Model Builder window, click Species: SO4(2-).
- 2 In the Settings window for Species, click to expand the Reaction Rate section.
- 3 From the list, choose User defined.
- 4 In the R_i text field, type re.r_1+re.r_2-r_c_sum.

Species: Ba(++)

- I In the Model Builder window, click Species: Ba(++).
- 2 In the Settings window for Species, locate the Reaction Rate section.
- 3 From the list, choose User defined.
- 4 In the R_i text field, type re.r_2-r_c_sum.

Initial Values 1

- I In the Model Builder window, click Initial Values I.
- 2 In the Settings window for Initial Values, locate the Volumetric Species Initial Values section.
- **3** In the table, enter the following settings:

Species	Concentration (mol/m^3)
Ba(++)	cBa_2p_0
CI(-)	cCl_1m_0
H(+)	cH_1p_0
HSO4(-)	cHS04_1m_0
SO4(2-)	cS04_2m_0

ADD COMPONENT

In the Model Builder window, right-click the root node and choose Add Component>3D.

Import variable expressions for the Debye-Hückel model and nucleation rate from separate files.

DEFINITIONS (COMP2)

Activity coefficient (Debye Hückel model)

I In the Model Builder window, under Component 2 (comp2) right-click Definitions and choose Variables.

- 2 In the Settings window for Variables, type Activity coefficient (Debye Hückel model) in the Label text field.
- 3 Locate the Variables section. Click Load from File.
- **4** Browse to the model's Application Libraries folder and double-click the file barium_sulfate_precipitation_debye_huckel_model2.txt.

Nucleation rate

- I Right-click **Definitions** and choose **Variables**.
- 2 In the Settings window for Variables, type Nucleation rate in the Label text field.
- 3 Locate the Variables section. Click **Load from File.**
- **4** Browse to the model's Application Libraries folder and double-click the file barium_sulfate_precipitation_nucleation_rate.txt.

GEOMETRY I

Cylinder I (cyll)

- I In the Geometry toolbar, click (Cylinder.
- 2 In the Settings window for Cylinder, locate the Size and Shape section.
- 3 In the Radius text field, type D/2.
- 4 In the Height text field, type Lc.

Cylinder 2 (cyl2)

- I In the Geometry toolbar, click Cylinder.
- 2 In the Settings window for Cylinder, locate the Size and Shape section.
- 3 In the Radius text field, type D/4.
- 4 In the Height text field, type 2*D.
- **5** Locate the **Position** section. In the **x** text field, type -D.
- 6 In the z text field, type Lc-D/4.
- 7 Locate the Axis section. From the Axis type list, choose x-axis.

Work Plane I (wbl)

- I In the Geometry toolbar, click Work Plane.
- 2 In the Settings window for Work Plane, locate the Plane Definition section.
- 3 In the z-coordinate text field, type 9[mm].

Work Plane I (wp I)>Plane Geometry

In the Model Builder window, click Plane Geometry.

Work Plane I (wp I)>Circle I (c1)

- I In the Work Plane toolbar, click (Circle.
- 2 In the Settings window for Circle, locate the Size and Shape section.
- 3 In the Radius text field, type D/2.

Work Plane 2 (wp2)

- I In the Model Builder window, right-click Geometry I and choose Work Plane.
- 2 In the Settings window for Work Plane, locate the Plane Definition section.
- 3 From the Plane list, choose yz-plane.

Work Plane 2 (wp2)>Plane Geometry

In the Model Builder window, click Plane Geometry.

Work Plane 2 (wp2)>Rectangle 1 (r1)

- I In the Work Plane toolbar, click Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type D.
- 4 In the **Height** text field, type Lc.
- **5** Locate the **Position** section. In the **xw** text field, type -D/2.

Ignore Faces I (igf1)

- I In the Model Builder window, under Component 2 (comp2)>Geometry I right-click Work Plane 2 (wp2) and choose Virtual Operations>Ignore Faces.
- 2 On the object fin, select Boundaries 12–17, 28–31, 35, and 38 only.
- 3 In the Geometry toolbar, click Build All.

ADD MATERIAL FROM LIBRARY

In the Home toolbar, click Windows and choose Add Material from Library.

ADD MATERIAL

- I Go to the Add Material window.
- 2 In the tree, select Built-in>Water, liquid.
- **3** Click **Add to Component** in the window toolbar.

ADD PHYSICS

- I In the Home toolbar, click Windows and choose Add Physics.
- **2** Go to the **Add Physics** window.

- 3 In the tree, select Chemical Species Transport>Chemistry (chem).
- 4 Click Add to Component 2 in the window toolbar.

CHEMISTRY (CHEM)

Reaction I

- 1 Right-click Component 2 (comp2)>Chemistry (chem) and choose Reaction.
- 2 In the Settings window for Reaction, locate the Reaction Formula section.
- 3 In the Formula text field, type $HSO4(-) \le H(+) + SO4(2-)$.
- 4 Click Apply.
- **5** Locate the **Rate Constants** section. In the k^{f} text field, type 1e5.
- 6 Select the Specify equilibrium constant check box.
- 7 Locate the Equilibrium Settings section. From the Equilibrium constant list, choose User defined.
- **8** In the K_i text field, type K_dis.

Reaction 2

- I In the Physics toolbar, click Domains and choose Reaction.
- 2 In the Settings window for Reaction, locate the Reaction Formula section.
- **3** In the **Formula** text field, type Ba(++)+S04(2-)<=>BaS04.
- 4 Click Apply.
- **5** Locate the **Rate Constants** section. In the $k^{\rm f}$ text field, type 1e5.
- 6 Select the Specify equilibrium constant check box.
- 7 Locate the Equilibrium Settings section. From the Equilibrium constant list, choose User defined.
- **8** In the K_j text field, type 1/K_ip.

Species 1

- I In the Physics toolbar, click **Domains** and choose Species.
- 2 In the Settings window for Species, locate the Name section.
- 3 In the text field, type C1(-).
- **4** In the **Physics** toolbar, click **Domains** and choose **Species**.
- I In the Settings window for Species, locate the Name section.
- 2 In the text field, type H20.
- 3 Locate the Type section. From the list, choose Solvent.

Species: SO4(2-)

- I In the Model Builder window, click Species: SO4(2-).
- 2 In the Settings window for Species, click to expand the Reaction Rate section.
- **3** From the list, choose **User defined**.
- **4** In the R_i text field, type chem.r_1-chem.r_2-r_c_sum.

Species: Ba(++)

- I In the Model Builder window, click Species: Ba(++).
- 2 In the Settings window for Species, locate the Reaction Rate section.
- **3** From the list, choose **User defined**.
- **4** In the R_i text field, type -chem.r_2-r_c_sum.
- 5 In the Model Builder window, click Chemistry (chem).
- 6 In the Settings window for Chemistry, locate the Mixture Properties section.
- 7 From the Phase list, choose Liquid.

ADD PHYSICS

- I In the Home toolbar, click Windows and choose Add Physics.
- 2 Go to the Add Physics window.
- 3 In the tree, select Fluid Flow>Single-Phase Flow>Laminar Flow (spf).
- 4 Click Add to Component 2 in the window toolbar.

LAMINAR FLOW (SPF)

Inlet 1

- I Right-click Component 2 (comp2)>Laminar Flow (spf) and choose Inlet.
- 2 In the Settings window for Inlet, locate the Boundary Condition section.
- 3 From the list, choose Fully developed flow.
- **4** Locate the **Fully Developed Flow** section. In the $U_{\rm av}$ text field, type u_in.
- **5** Select Boundary 1 only.

Inlet 2

- I In the Physics toolbar, click **Boundaries** and choose Inlet.
- 2 In the Settings window for Inlet, locate the Boundary Condition section.
- 3 From the list, choose Fully developed flow.
- **4** Locate the **Fully Developed Flow** section. In the $U_{\rm av}$ text field, type u_in.

5 Select Boundary 26 only.

Outlet 1

- I In the Physics toolbar, click **Boundaries** and choose **Outlet**.
- **2** Select Boundaries 8 and 15 only.
- 3 In the Model Builder window, click Laminar Flow (spf).
- 4 In the Settings window for Laminar Flow, locate the Physical Model section.
- **5** In the T_{ref} text field, type T.

GLOBAL DEFINITIONS

Default Model Inputs

- I In the Model Builder window, under Global Definitions click Default Model Inputs.
- 2 In the Settings window for Default Model Inputs, locate the Browse Model Inputs section.
- 3 In the tree, select General>Temperature (K) minput.T.
- 4 Find the Expression for remaining selection subsection. In the Temperature text field, type Τ.

ADD PHYSICS

- I In the Home toolbar, click Windows and choose Add Physics.
- 2 Go to the Add Physics window.
- 3 In the tree, select Chemical Species Transport>Transport of Diluted Species (tds).
- 4 Click Add to Component 2 in the window toolbar.

TRANSPORT OF DILUTED SPECIES (TDS)

- I In the Settings window for Transport of Diluted Species, click to expand the **Dependent Variables** section.
- 2 In the Number of species text field, type 6.
- 3 In the Concentrations (mol/m³) table, enter the following settings:

cBa_2p
cS04_2m
cHS04_1m
cH_1p
cCl_1m
cBaS04

Transport Properties 1

- I In the Model Builder window, under Component 2 (comp2)> Transport of Diluted Species (tds) click Transport Properties 1.
- 2 In the Settings window for Transport Properties, locate the Convection section.
- 3 From the **u** list, choose **Velocity field (spf)**.
- **4** Locate the **Diffusion** section. In the D_{cBa2p} text field, type D_species.
- **5** In the $D_{\rm cSO42m}$ text field, type D_species.
- **6** In the D_{cHSO41m} text field, type D_species.
- 7 In the $D_{\rm cH1p}$ text field, type D_species.
- **8** In the $D_{
 m cCl1m}$ text field, type D_species.
- **9** In the D_{cBaSO4} text field, type D_species.

Initial Values 1

- I In the Model Builder window, click Initial Values I.
- 2 In the Settings window for Initial Values, locate the Initial Values section.
- **3** In the cBa_{2p} text field, type cBa_2p_0.
- 4 In the $cSO4_{2m}$ text field, type cS04_2m_0.
- **5** In the $cHSO4_{1m}$ text field, type cHS04_1m_0.
- **6** In the cH_{1p} text field, type cH_{1p} 0.
- **7** In the cCl_{1m} text field, type cCl_1m_0.

Inflow I

- I In the Physics toolbar, click **Boundaries** and choose Inflow.
- **2** Select Boundary 26 only.
- 3 In the Settings window for Inflow, locate the Concentration section.
- **4** In the $c_{0,cBa2p}$ text field, type cBa_2p_0.
- **5** In the $c_{0,\mathrm{cCl1m}}$ text field, type cCl_1m_0.

Inflow 2

- I In the Physics toolbar, click **Boundaries** and choose Inflow.
- 2 Select Boundary 1 only.
- 3 In the Settings window for Inflow, locate the Concentration section.
- **4** In the $c_{0,cSO42m}$ text field, type cSO4_2m_0.
- **5** In the $c_{0.{
 m cHSO41m}}$ text field, type cHS04_1m_0.

6 In the $c_{0,\text{cH1p}}$ text field, type cH_1p_0.

Reactions 1

- I In the Physics toolbar, click **Domains** and choose Reactions.
- 2 In the Settings window for Reactions, locate the Reaction Rates section.
- 3 From the $R_{\rm cBa2p}$ list, choose Reaction rate for species Ba_2p (chem).
- 4 From the $R_{\rm cSO42m}$ list, choose Reaction rate for species SO4_2m (chem).
- 5 From the $R_{\rm cHSO41m}$ list, choose Reaction rate for species HSO4_Im (chem).
- 6 From the $R_{\rm cH1p}$ list, choose Reaction rate for species H_Ip (chem).
- 7 From the $R_{\rm cCl1m}$ list, choose Reaction rate for species Cl_Im (chem).
- 8 From the R_{cBaSO4} list, choose Reaction rate for species BaSO4 (chem).
- **9** Click to expand the **Equation** section. Locate the **Domain Selection** section. From the **Selection** list, choose **All domains**.

Outflow I

- I In the Physics toolbar, click **Boundaries** and choose **Outflow**.
- **2** Select Boundaries 8 and 15 only.

CHEMISTRY (CHEM)

- I In the Model Builder window, under Component 2 (comp2) click Chemistry (chem).
- 2 In the Settings window for Chemistry, locate the Species Matching section.
- 3 From the Species solved for list, choose Transport of Diluted Species.
- 4 Find the Bulk species subsection. In the table, enter the following settings:

Species	Туре	Molar concentration	Value (mol/m^3)
Ba(++)	Variable	cBa_2p	Solved for
BaSO4	Variable	cBaSO4	Solved for
CI(-)	Free species	cCl_Im	Solved for
H(+)	Variable	cH_Ip	Solved for
H2O	Solvent	User defined	c_sol
HSO4(-)	Variable	cHSO4_Im	Solved for
SO4(2-)	Variable	cSO4_2m	Solved for

MESH I

- I In the Model Builder window, under Component 2 (comp2) click Mesh I.
- 2 In the Settings window for Mesh, locate the Physics-Controlled Mesh section.

3 From the Element size list, choose Normal.

Information I

In the Model Builder window, right-click Mesh I and choose Build All.

MESH 2

- I In the Mesh toolbar, click Add Mesh and choose Add Mesh.
- 2 In the Settings window for Mesh, locate the Sequence Type section.
- 3 From the list, choose User-controlled mesh.

Size

- I In the Model Builder window, under Component 2 (comp2)>Meshes>Mesh 2 click Size.
- 2 In the Settings window for Size, locate the Element Size section.
- **3** Click the **Custom** button.
- 4 Locate the Element Size Parameters section. In the Maximum element size text field, type 5F-5.
- 5 In the Minimum element size text field, type 2E-5.
- **6** In the **Curvature factor** text field, type **0.6**.
- 7 In the Resolution of narrow regions text field, type 0.5.

Size 1

- I In the Model Builder window, click Size I.
- 2 In the Settings window for Size, locate the Element Size section.
- **3** Click the **Custom** button.
- 4 Locate the Element Size Parameters section.
- 5 Select the Maximum element size check box. In the associated text field, type 4E-5.
- 6 In the Model Builder window, right-click Size I and choose Disable.

Corner Refinement I

- I In the Model Builder window, click Corner Refinement I.
- 2 In the Settings window for Corner Refinement, locate the Refinement section.
- 3 In the Element size scaling factor text field, type 0.5.

Free Tetrahedral I

- I In the Model Builder window, click Free Tetrahedral I.
- 2 In the Settings window for Free Tetrahedral, locate the Domain Selection section.
- 3 From the Geometric entity level list, choose Domain.

4 Select Domains 1 and 4 only.

Swept I

- I In the Model Builder window, expand the Boundary Layers I node.
- 2 Right-click Mesh 2 and choose Swept.
- 3 In the Settings window for Swept, locate the Domain Selection section.
- 4 From the Geometric entity level list, choose Domain.
- 5 Click Paste Selection.
- 6 In the Paste Selection dialog box, type 2 3 in the Selection text field.
- 7 Click OK.

Distribution 1

- I Right-click Swept I and choose Distribution.
- 2 In the Settings window for Distribution, locate the Distribution section.
- **3** From the **Distribution type** list, choose **Predefined**.
- 4 In the Number of elements text field, type 50.
- 5 In the Element ratio text field, type 10.

Boundary Layers 2

In the Mesh toolbar, click Boundary Layers.

Boundary Layer Properties

- I In the Model Builder window, click Boundary Layer Properties.
- 2 Select Boundaries 13 and 16 only.
- 3 In the Settings window for Boundary Layer Properties, locate the Layers section.
- 4 In the Number of layers text field, type 10.
- 5 In the Stretching factor text field, type 1.
- 6 From the Thickness specification list, choose First layer.
- 7 In the Thickness text field, type 0.8E-5.

Boundary Layers 2

- I In the Model Builder window, click Boundary Layers 2.
- 2 Drag and drop below Free Tetrahedral I.

Swebt I

- I In the Model Builder window, click Swept I.
- 2 Drag and drop below Free Tetrahedral I.

3 In the Settings window for Swept, click Build All.

The discretized population balance equation will be described using the imported add-in.

GLOBAL DEFINITIONS

Population Balance Equation Discretization I

- I In the Model Builder window, under Global Definitions click Population Balance Equation Discretization 1.
- 2 In the Smallest particle size text field, type 0.82.
- 3 In the Largest particle size text field, type 500.
- 4 In the Settings window for Population Balance Equation Discretization, select the 0D **component** check box.
- 5 Choose OD Component (compl) from the OD component list.
- **6** Select the **3D** component check box.
- 7 Choose 3D Component (comp2) from the 3D component list.
- **8** Locate the **0D** Component section. In the text field **Number of intervals**, type 100.
- **9** Click the **Geometric** button.
- 10 In the Geometric constant text field, type 60.
- II Locate the 3D Component section. In the text field Number of intervals, type 30.
- 12 Click the Geometric button.
- **I3** In the **Geometric constant** text field, type 8.
- 14 Locate the Growth Rate Approximation section.
- 15 Click the Total variation diminishing button.
- **16** In the text field **Blending constant**, type 1/3.
- 17 Locate the Generate Default Plots section.
- 18 Select the Generate default plots check box.
- 19 At the top of the settings window, click Create/Refresh.

After running the add-in, a growth rate function, crystal parameters and supersaturation and nucleation expressions need to be added.

GLOBAL DEFINITIONS

Growth Rate (GL)

I In the Model Builder window, under Global Definitions click Growth Rate (GL).

- 2 In the Settings window for Analytic, locate the Definition section.
- 3 In the Expression text field, type k_a/(3*k_v)*Sh*D_AB*sqrt(K_sp)*M_C*(S-1)/ rho_C/L.
- **4** Locate the **Units** section. In the table, enter the following settings:

Argument	Unit
L	m
S	(mol/m^3)*(mol/m^3)/(mol^2/m^6)

Crystal properties

- I In the Model Builder window, click Crystal properties.
- 2 In the Settings window for Parameters, locate the Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description
k_v	pi/6	0.5236	Crystal volume shape factor
rho_C	rho_BaSO4	4480 kg/m³	Crystal density
M_C	M_BaSO4	0.23338 kg/mol	Crystal molecular weight
nstart	1e10	IEI0	Initial number density

DEFINITIONS (COMPI)

Crystallization variables

- I In the Model Builder window, under Component I (compl)>Definitions click Crystallization variables.
- 2 In the Settings window for Variables, locate the Variables section.
- **3** In the table, enter the following settings:

Name	Expression	Unit	Description
S	gamma* sqrt(max(eps, re.c_Ba_2p* re.c_S04_2m/ K_sp))		Supersaturation ratio
В0	B_0	I/(m ⁴ ·s)	Nucleation rate as source of number density

DEFINITIONS (COMP2)

- I In the Model Builder window, under Component 2 (comp2)>Definitions click Crystallization variables.
- 2 In the Settings window for Variables, locate the Variables section.
- **3** In the table, enter the following settings:

Name	Expression	Unit	Description
S	gamma* sqrt(max(eps, cBa_2p* cSO4_2m/ K_sp))		Supersaturation ratio
В0	B_0	I/(m ⁴ ·s)	Nucleation rate as source of number density

Select the inlet of the T-mixer.

Inlet selection

- I In the Model Builder window, expand the Component 2 (comp2)>Definitions>Selections node, then click Inlet selection.
- 2 Select Boundaries 1 and 26 only.

ADD STUDY

- I In the Home toolbar, click Windows and choose Add Study.
- 2 Go to the Add Study window.
- **3** Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** check boxes for all interfaces apart from Reaction Engineering (RE) and Global ODEs and DAEs (ge).
- 4 Find the Studies subsection. In the Select Study tree, select General Studies> Time Dependent.
- 5 Click Add Study in the window toolbar.

STUDY I

- I In the Model Builder window, click Study I.
- 2 In the Settings window for Study, locate the Study Settings section.
- 3 Clear the Generate default plots check box.

Solution I (soll)

In the Study toolbar, click Show Default Solver.

COMPONENT I (COMPI)

In the Model Builder window, expand the Solution I (soll) node.

STUDY I

In the Model Builder window, expand the Study I node.

Solver Configurations

In the Model Builder window, expand the Study I>Solver Configurations node.

Solution I (soll)

- I In the Model Builder window, expand the Study I>Solver Configurations>Solution I (soll) node, then click Time-Dependent Solver I.
- 2 In the Settings window for Time-Dependent Solver, locate the General section.
- 3 From the Times to store list, choose Output times by interpolation.

Step 1: Time Dependent

- I In the Model Builder window, under Study I click Step I: Time Dependent.
- 2 In the Settings window for Time Dependent, locate the Study Settings section.
- 3 In the Output times text field, type 0 0.001 range(0.002,2.0e-3,0.06).
- 4 In the Study toolbar, click **Compute**.

RESULTS

Particle Number Density Distribution 1

- I In the Model Builder window, expand the Results node, then click Particle Number Density Distribution I.
- 2 In the Settings window for ID Plot Group, locate the Data section.
- 3 From the Dataset list, choose Study I/Solution I (soll).
- 4 From the Time selection list, choose From list.
- 5 In the Times (s) list, choose 0.001, 0.002, 0.004, 0.008, 0.016, 0.032, and 0.06.
- 6 In the Particle Number Density Distribution I toolbar, click **Tool** Plot.
- 7 Click to expand the **Title** section. From the **Title type** list, choose **None**.
- 8 Locate the Plot Settings section.
- 9 Select the x-axis label check box. In the associated text field, type Crystal size (nm).
- 10 Select the y-axis label check box. In the associated text field, type Normalized Number Density 1/nm.
- II Locate the Axis section. Select the Manual axis limits check box.

12 In the **x minimum** text field, type 0.

13 In the x maximum text field, type 300.

14 In the Model Builder window, expand the Particle Number Density Distribution I node.

Line Segments 1

- I In the Model Builder window, expand the Particle Number Density Distribution I node, then click Line Segments 1.
- 2 In the Settings window for Line Segments, click to expand the Legends section.
- 3 Select the Show legends check box.
- 4 In the Particle Number Density Distribution I toolbar, click **Plot**.

Discretized points 1

- I In the Model Builder window, under Results click Discretized points I.
- 2 In the Settings window for ID Plot Group, locate the Data section.
- 3 From the Dataset list, choose Study I/Solution I (soll).
- 4 From the Time selection list, choose Last.
- 5 In the Discretized points I toolbar, click **Plot**.

Supersaturation

- I In the Home toolbar, click In Add Plot Group and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, locate the Data section.
- 3 From the Dataset list, choose Study I/Solution I (soll).
- **4** Click to expand the **Title** section. From the **Title type** list, choose **None**.
- 5 In the Label text field, type Supersaturation.

Global I

- I Right-click Supersaturation and choose Global.
- 2 In the Settings window for Global, locate the y-Axis Data section.
- 3 Click Clear Table.
- **4** In the table, enter the following settings:

Expression	Unit	Description	
comp1.S	1	Supersaturation ratio	

- **5** Click to expand the **Legends** section. Clear the **Show legends** check box.
- 6 In the Supersaturation toolbar, click Plot.

Mass Balance

- I In the Home toolbar, click Add Plot Group and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Mass Balance in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Study I/Solution I (soll).
- 4 Click to expand the **Title** section. From the **Title type** list, choose **None**.
- 5 Locate the Plot Settings section.
- 6 Select the x-axis label check box. In the associated text field, type Time (s).
- 7 Select the y-axis label check box. In the associated text field, type Mass concentration (kg/m < SUP > 3 < /SUP >).
- 8 Locate the Legend section. From the Position list, choose Upper left.

Global I

- I Right-click Mass Balance and choose Global.
- 2 In the Settings window for Global, locate the y-Axis Data section.
- 3 Click \ Clear Table.
- **4** In the table, enter the following settings:

Expression	Unit	Description
(comp1.re.c_HS04_1m+ comp1.re.c_S04_2m)* M_S04_2m	kg/m^3	Sulfate mass concentration
M_Ba_2p*comp1.re.c_Ba_2p	kg/m^3	Barium mass concentration
M_C*comp1.re.c_BaSO4	kg/m^3	Barium sulfate mass concentration
comp1.M_c_sum	kg/m^3	Total crystal mass concentration
M_S04_2m* (comp1.re.c_S04_2m+ comp1.re.c_HS04_1m)+ M_Ba_2p*comp1.re.c_Ba_2p+ M_C*comp1.re.c_BaS04+ comp1.M_c_sum	kg/m^3	Total mass concentration

- 5 Click to expand the Legends section. From the Legends list, choose Manual.
- **6** In the table, enter the following settings:

Legends	
Sulfate	
Barium	

Legends

Aqueous Barium Sulfate Crystalline Barium Sulfate Total

7 In the Mass Balance toolbar, click Plot.

Discretized points I, Mass Balance, Particle Number Density Distribution I, Supersaturation

- I In the Model Builder window, under Results, Ctrl-click to select Particle Number Density Distribution 1, Discretized points 1, Supersaturation, and Mass Balance.
- 2 Right-click and choose **Group**.

RESULTS

Perfectly Mixed Batch Reactor

- I In the Model Builder window, under Results click Group I.
- 2 In the Settings window for Group, type Perfectly Mixed Batch Reactor in the Label text field.

ADD STUDY

The study used to solve for crystallization in the T-mixer is generated by a model method added to the Application Builder. Note that the method editor is only available in the Windows® version of the COMSOL desktop. To see the method, please refer to the method generateStudy under methods in the existing model file barium_sulfate_precipitation.

STUDY 2

- I In the Model Builder window, click Study 2.
- 2 In the Settings window for Study, locate the Study Settings section.
- 3 Clear the Generate default plots check box.
- 4 In the Home toolbar, click **Compute**.

RESULTS

Create Evaluation Groups

To evaluate the crystal number density distribution at various points along the T-mixer, additional evaluation groups with a set normal (in z-direction) are required. This is done with a model method, similar to how **Study 2** was set up.

NEW METHOD

- I In the Developer toolbar, click New Method.
- 2 In the New Method dialog box, type generate Evaluation Groups in the Name text field.
- 3 Click OK.

APPLICATION BUILDER

generateEvaluationGroups

- I In the Application Builder window, under Methods click generate Evaluation Groups.
- **2** Copy the following code into the **generateEvaluationGroups** method editor window:

```
int N = 30:
for (int i3 = 1; i3 <= 4; i3++) {
model.result().evaluationGroup().create("eg densdistr"+toString(i3),
"EvaluationGroup");
model.result().evaluationGroup("eg densdistr"+toString(i3)).create("gev1",
"EvalGlobal");
model.result().evaluationGroup("eg densdistr"+
toString(i3)).feature("gev1").set("data", "dset2");
model.result().evaluationGroup("eg densdistr"+toString(i3)).create("int1",
"IntSurface"):
model.result().evaluationGroup("eg densdistr"+
toString(i3)).feature("int1").set("intvolume", true);
model.result().evaluationGroup("eg densdistr"+toString(i3)).create("int2",
"IntSurface");
model.result().evaluationGroup("eg densdistr"+
toString(i3)).feature("int2").set("intvolume", true);
model.result().evaluationGroup("eg densdistr"+toString(i3)).create("int3",
"IntSurface");
model.result().evaluationGroup("eg densdistr"+
toString(i3)).feature("int3").set("intvolume", true);
model.result().evaluationGroup("eg_densdistr"+
toString(i3)).feature("gev1").set("expr", new String[]{});
model.result().evaluationGroup("eg densdistr"+
toString(i3)).feature("gev1").set("descr", new String[]{});
model.result().evaluationGroup("eg densdistr"+
toString(i3)).feature("gev1").setIndex("expr", "comp2.L0", 0);
model.result().evaluationGroup("eg_densdistr"+
toString(i3)).feature("int1").setIndex("expr", "comp2.n1"+"*w+(scdeq"+
".dfluxz)", 0);
model.result().evaluationGroup("eg_densdistr"+
```

```
toString(i3)).feature("int1").setIndex("expr", "comp2.n1"+"*w+(scdeq"+
".dfluxz)", 1);
int i2 = 1;
for (int i = 1; i \le N; i++) {
model.result().evaluationGroup("eg densdistr"+
toString(i3)).feature("gev1").setIndex("expr", "comp2.L"+toString(i), i2);
model.result().evaluationGroup("eg densdistr"+
toString(i3)).feature("int1").setIndex("expr", "comp2.n"+toString(i)+"*w+
(scdeq"+toString(i)+".dfluxz)", i2-1);
}
model.result().evaluationGroup("eg densdistr"+
toString(i3)).feature("int2").setIndex("expr", "w", i2-1);
model.result().evaluationGroup("eg densdistr"+
toString(i3)).feature("int3").setIndex("expr", "comp2.N_c*w", i2-1);
i2 = i2+1;
if (i > 1) {
model.result().evaluationGroup("eg densdistr"+
toString(i3)).feature("int1").setIndex("expr", "comp2.n"+toString(i)+"*w+
(scdeq"+toString(i)+".dfluxz)", i2-1);
model.result().evaluationGroup("eg densdistr"+
toString(i3)).feature("int2").setIndex("expr", "w", i2-1);
model.result().evaluationGroup("eg densdistr"+
toString(i3)).feature("int3").setIndex("expr", "comp2.N_c*w", i2-1);
if (i < N) {
model.result().evaluationGroup("eg_densdistr"+
toString(i3)).feature("gev1").setIndex("expr", "comp2.L"+toString(i), i2);
}
i2 = i2+1;
}
for (int i = 0; i \le i2-2; i++) {
model.result().evaluationGroup("eg_densdistr"+
toString(i3)).feature("gev1").setIndex("unit", "nm", i);
}
with(model.result().evaluationGroup("eg densdistr"+toString(i3)));
set("type", "general");
set("generalexpr", "(int1/int2)/(int3/int2)*1e-9");
set("keepchildnodes", true);
set("includeparameters", false);
set("concatenation", "vertical");
endwith();
}
```

METHODS

In the **Home** toolbar, click **Model Builder**.

GLOBAL DEFINITIONS

Click Method Call and choose generateEvaluationGroups.

GenerateEvaluationGroups 1

In the Model Builder window, under Global Definitions right-click GenerateEvaluationGroups I and choose Run.

RESULTS

In the Model Builder window, expand the Results node.

Cut Plane 1

- I In the Model Builder window, expand the Results>Datasets node.
- 2 Right-click Results>Datasets and choose Cut Plane.
- 3 In the Settings window for Cut Plane, locate the Data section.
- 4 From the Dataset list, choose Study 2/Solution 2 (sol2).
- 5 Locate the Plane Data section. From the Plane type list, choose General.
- 6 From the Plane entry method list, choose Point and normal vector.
- 7 Find the Point subsection. In the z text field, type 5e-3.
- 8 Right-click Cut Plane I and choose Duplicate.

Cut Plane 2

- I In the Model Builder window, click Cut Plane 2.
- 2 In the Settings window for Cut Plane, locate the Plane Data section.
- 3 Find the Point subsection. In the z text field, type 7.5e-3.
- 4 Right-click Cut Plane 2 and choose Duplicate.

Cut Plane 3

- I In the Model Builder window, click Cut Plane 3.
- 2 In the Settings window for Cut Plane, locate the Plane Data section.
- 3 Find the Point subsection. In the z text field, type 8.75e-3.
- 4 Right-click Cut Plane 3 and choose Duplicate.

Cut Plane 4

- I In the Model Builder window, click Cut Plane 4.
- 2 In the Settings window for Cut Plane, locate the Plane Data section.
- 3 Find the **Point** subsection. In the **z** text field, type 9.375e-3.

Density Distribution Evaluation Group

- I In the Model Builder window, under Results click Density Distribution Evaluation Group.
- 2 In the Settings window for Evaluation Group, locate the Data section.

3 From the Dataset list, choose Study 2/Solution 2 (sol2).

Surface Integration 1

- I In the Model Builder window, expand the Density Distribution Evaluation Group node, then click Surface Integration 1.
- 2 Select Boundaries 8 and 15 only.

Surface Integration 2

- I In the Model Builder window, click Surface Integration 2.
- **2** Select Boundaries 8 and 15 only.

Surface Integration 3

- I In the Model Builder window, click Surface Integration 3.
- 2 Select Boundaries 8 and 15 only.

Density Distribution Evaluation Group

- I In the Model Builder window, click Density Distribution Evaluation Group.
- 2 In the Density Distribution Evaluation Group toolbar, click **= Evaluate**.

Evaluation Group 1

- I In the Model Builder window, click Evaluation Group I.
- 2 In the Settings window for Evaluation Group, locate the Data section.
- 3 From the Dataset list, choose Cut Plane I.
- 4 In the Evaluation Group I toolbar, click **= Evaluate**.

Evaluation Group 2

- I In the Model Builder window, click Evaluation Group 2.
- 2 In the Settings window for Evaluation Group, locate the Data section.
- 3 From the Dataset list, choose Cut Plane 2.
- 4 In the Evaluation Group 2 toolbar, click **= Evaluate**.

Evaluation Group 3

- I In the Model Builder window, click Evaluation Group 3.
- 2 In the Settings window for Evaluation Group, locate the Data section.
- 3 From the Dataset list, choose Cut Plane 3.
- 4 In the Evaluation Group 3 toolbar, click **= Evaluate**.

Evaluation Group 4

I In the Model Builder window, click Evaluation Group 4.

- 2 In the Settings window for Evaluation Group, locate the Data section.
- 3 From the Dataset list, choose Cut Plane 4.
- 4 In the Evaluation Group 4 toolbar, click **= Evaluate**.

Particle Number Density Distribution 2

- I In the Model Builder window, click Particle Number Density Distribution 2.
- 2 In the Settings window for ID Plot Group, click to expand the Title section.
- **3** From the **Title type** list, choose **None**.
- 4 Locate the Plot Settings section. Select the x-axis label check box.
- 5 Select the y-axis label check box.
- 6 In the x-axis label text field, type Crystal size (nm).
- 7 In the y-axis label text field, type Normalized Number Density (1/nm).

Table Graph 1

- I In the Model Builder window, expand the Particle Number Density Distribution 2 node, then click Table Graph 1.
- 2 In the Settings window for Table Graph, locate the Data section.
- 3 Select the Row-based check box.
- 4 From the x-axis data list, choose Row 2.
- 5 From the Plot rows list, choose Manual.
- 6 In the Rows list, select Row 1.
- 7 Click to expand the Legends section. From the Legends list, choose Manual.
- 8 Select the **Show legends** check box.
- **9** In the table, enter the following settings:

Legends z=0mm

10 Right-click **Table Graph I** and choose **Duplicate**.

Table Graph 2

- I In the Model Builder window, click Table Graph 2.
- 2 In the Settings window for Table Graph, locate the Data section.
- 3 From the Evaluation group list, choose Evaluation Group 1.

4 Locate the **Legends** section. In the table, enter the following settings:

Legends		
z=5	mm	

5 Right-click Table Graph 2 and choose Duplicate.

Table Graph 3

- I In the Model Builder window, click Table Graph 3.
- 2 In the Settings window for Table Graph, locate the Data section.
- 3 From the Evaluation group list, choose Evaluation Group 2.
- **4** Locate the **Legends** section. In the table, enter the following settings:

Legends			
z=7.5	mm		

5 Right-click **Table Graph 3** and choose **Duplicate**.

Table Graph 4

- I In the Model Builder window, click Table Graph 4.
- 2 In the Settings window for Table Graph, locate the Data section.
- 3 From the Evaluation group list, choose Evaluation Group 3.
- **4** Locate the **Legends** section. In the table, enter the following settings:

Legends			
z=8.75	mm		

5 Right-click **Table Graph 4** and choose **Duplicate**.

Table Graph 5

- I In the Model Builder window, click Table Graph 5.
- 2 In the Settings window for Table Graph, locate the Legends section.
- **3** In the table, enter the following settings:

Legends z=9.375 mm

4 Locate the Data section. From the Evaluation group list, choose Evaluation Group 4.

Particle Number Density Distribution 2

I In the Model Builder window, click Particle Number Density Distribution 2.

- 2 In the Settings window for ID Plot Group, locate the Axis section.
- 3 Select the Manual axis limits check box.
- 4 In the x minimum text field, type 0.
- 5 In the x maximum text field, type 350.
- 6 In the y minimum text field, type -1e-4.
- 7 In the y maximum text field, type 0.02.
- 8 In the Particle Number Density Distribution 2 toolbar, click **Plot**.

Velocity Field. Supersaturation and Concentrations

- I In the Home toolbar, click Add Plot Group and choose 3D Plot Group.
- 2 In the Settings window for 3D Plot Group, locate the Data section.
- 3 From the Dataset list, choose Study 2/Solution 2 (sol2).
- 4 In the Label text field, type Velocity Field, Supersaturation and Concentrations
- 5 Click to expand the **Title** section. From the **Title type** list, choose **None**.
- **6** Locate the **Plot Settings** section. Clear the **Plot dataset edges** check box.
- 7 Locate the Color Legend section. Select the Show units check box.
- 8 From the Position list, choose Right double.
- **9** Click to expand the **Plot Array** section. Select the **Enable** check box.

Slice 1

- I Right-click Velocity Field, Supersaturation and Concentrations and choose Slice.
- 2 In the Settings window for Slice, locate the Plane Data section.
- 3 From the Plane list, choose xy-planes.
- 4 Click to expand the Coloring and Style section. Click Change Color Table.
- 5 In the Color Table dialog box, select Rainbow>Prism in the tree.
- 6 Click OK.
- 7 In the Settings window for Slice, click to expand the Plot Array section.
- 8 Select the Manual indexing check box.

Surface I

- I In the Model Builder window, right-click Velocity Field, **Supersaturation and Concentrations** and choose **Surface**.
- 2 In the Settings window for Surface, locate the Expression section.

- **3** In the **Expression** text field, type 1.
- 4 Click to expand the **Plot Array** section. Select the **Manual indexing** check box.

Selection 1

- I Right-click Surface I and choose Selection.
- **2** Select Boundaries 4, 5, 7, 10, 20, 21, 24, and 25 only.

Material Appearance 1

- I In the Model Builder window, right-click Surface I and choose Material Appearance.
- 2 In the Settings window for Material Appearance, locate the Appearance section.
- 3 From the Appearance list, choose Custom.
- 4 From the Material type list, choose Steel (anodized).

Streamline 1

- I In the Model Builder window, right-click Velocity Field, **Supersaturation and Concentrations** and choose **Streamline**.
- **2** Select Boundary 1 only.
- 3 In the Settings window for Streamline, locate the Streamline Positioning section.
- 4 In the Number text field, type 3.
- 5 Click to expand the Coloring and Style section. Find the Line style subsection. From the Type list, choose Ribbon.
- 6 Find the Point style subsection. From the Type list, choose Arrow.
- 7 Select the Scale factor check box. In the associated text field, type 0.0016.
- 8 From the Color list, choose Yellow.
- 9 Click to expand the Plot Array section. Select the Manual indexing check box.
- 10 Right-click Streamline I and choose Duplicate.

Streamline 2

- I In the Model Builder window, click Streamline 2.
- 2 In the Settings window for Streamline, locate the Selection section.
- **3** Click to select the **Activate Selection** toggle button.
- 4 Select Boundary 26 only.

Slice 2

I In the Model Builder window, right-click Velocity Field, **Supersaturation and Concentrations** and choose **Slice**.

- 2 In the Settings window for Slice, locate the Expression section.
- **3** In the **Expression** text field, type S.
- 4 Locate the Plane Data section. From the Plane list, choose zx-planes.
- 5 In the Planes text field, type 1.
- 6 Click to expand the Coloring and Style section. Click Change Color Table.
- 7 In the Color Table dialog box, select Thermal>HeatCamera in the tree.
- 8 Click OK.
- 9 In the Settings window for Slice, locate the Plot Array section.
- 10 Select the Manual indexing check box.
- II In the **Index** text field, type 1.

Slice I, Surface I

- I In the Model Builder window, under Results>Velocity Field, Supersaturation and Concentrations, Ctrl-click to select Slice I and Surface I.
- 2 Right-click and choose **Duplicate**.

Transparency 1

- I In the Model Builder window, right-click Slice 2 and choose Transparency.
- 2 In the Settings window for Transparency, locate the Transparency section.
- **3** Set the **Transparency** value to **0.4**.

Slice 3

- I In the Model Builder window, under Results>Velocity Field, **Supersaturation and Concentrations** click **Slice 3**.
- 2 In the Settings window for Slice, locate the Expression section.
- **3** In the **Expression** text field, type S.
- 4 Click to expand the Inherit Style section. From the Plot list, choose Slice 2.
- 5 Locate the Plot Array section. In the Index text field, type 1.

Surface 2

- I In the Model Builder window, click Surface 2.
- 2 In the Settings window for Surface, locate the Plot Array section.
- **3** In the **Index** text field, type 1.

Slice 2, Slice 3, Surface 2

- I In the Model Builder window, under Results>Velocity Field, Supersaturation and Concentrations, Ctrl-click to select Slice 2, Slice 3, and Surface 2.
- 2 Right-click and choose **Duplicate**.

Slice 4, Slice 5, Surface 3

- I In the Model Builder window, under Results>Velocity Field, Supersaturation and Concentrations, Ctrl-click to select Slice 4, Slice 5, and Surface 3.
- 2 In the Settings window for Slice, locate the Expression section.
- 3 In the Expression text field, type cBaS04.
- 4 Locate the Coloring and Style section. Click | Change Color Table.
- 5 In the Color Table dialog box, select Rainbow>Prism in the tree.
- 6 Click OK.
- 7 In the Settings window for Slice, locate the Plot Array section.
- 8 In the Index text field, type 2.

Slice 5

- I In the Model Builder window, click Slice 5.
- 2 In the Settings window for Slice, locate the Expression section.
- 3 In the Expression text field, type cBaS04.
- 4 Locate the Inherit Style section. From the Plot list, choose Slice 4.
- **5** Locate the **Plot Array** section. In the **Index** text field, type 2.

Surface 3

- I In the Model Builder window, click Surface 3.
- 2 In the Settings window for Surface, locate the Plot Array section.
- 3 In the Index text field, type 2.

Slice 4, Slice 5, Surface 3

- I In the Model Builder window, under Results>Velocity Field, Supersaturation and Concentrations, Ctrl-click to select Slice 4, Slice 5, and Surface 3.
- 2 Right-click and choose **Duplicate**.

Slice 6, Slice 7, Surface 4

- I In the Model Builder window, under Results>Velocity Field, Supersaturation and Concentrations, Ctrl-click to select Slice 6, Slice 7, and Surface 4.
- 2 In the Settings window for Slice, locate the Expression section.

- 3 In the Expression text field, type cS04 2m.
- 4 Locate the Plot Array section. In the Index text field, type 3.

Slice 7

- I In the Model Builder window, click Slice 7.
- 2 In the Settings window for Slice, locate the Expression section.
- 3 In the Expression text field, type cS04 2m.
- 4 Locate the Inherit Style section. From the Plot list, choose Slice 6.
- **5** Locate the **Plot Array** section. In the **Index** text field, type **3**.

Surface 4

- I In the Model Builder window, click Surface 4.
- 2 In the Settings window for Surface, locate the Plot Array section.
- 3 In the **Index** text field, type 3.

Slice 6, Slice 7, Surface 4

- I In the Model Builder window, under Results>Velocity Field, Supersaturation and Concentrations, Ctrl-click to select Slice 6, Slice 7, and Surface 4.
- 2 Right-click and choose **Duplicate**.

Slice 8. Slice 9. Surface 5

- I In the Model Builder window, under Results>Velocity Field, Supersaturation and Concentrations, Ctrl-click to select Slice 8, Slice 9, and Surface 5.
- 2 In the Settings window for Slice, locate the Expression section.
- 3 In the Expression text field, type cBa 2p.
- 4 Locate the **Plot Array** section. In the **Index** text field, type 4.

Slice 9

- I In the Model Builder window, click Slice 9.
- 2 In the Settings window for Slice, locate the Expression section.
- 3 In the Expression text field, type cBa 2p.
- 4 Locate the Inherit Style section. From the Plot list, choose Slice 8.
- **5** Locate the **Plot Array** section. In the **Index** text field, type 4.

Surface 5

- I In the Model Builder window, click Surface 5.
- 2 In the Settings window for Surface, locate the Plot Array section.

3 In the Index text field, type 4.

Annotation I

- I In the Model Builder window, right-click Velocity Field, **Supersaturation and Concentrations** and choose **Annotation**.
- 2 In the Settings window for Annotation, locate the Annotation section.
- 3 In the **Text** text field, type Velocity field.
- **4** Click to expand the **Coloring and Style** section. Clear the **Show point** check box.
- **5** Right-click **Annotation I** and choose **Duplicate**.

Annotation 2

- I In the Model Builder window, click Annotation 2.
- 2 In the Settings window for Annotation, locate the Annotation section.
- **3** In the **Text** text field, type **Supersaturation**.
- 4 Right-click Annotation 2 and choose Duplicate.

Annotation 3

- I In the Model Builder window, click Annotation 3.
- 2 In the Settings window for Annotation, locate the Annotation section.
- 3 In the **Text** text field, type Aqueous barium sulfate.
- 4 Right-click Annotation 3 and choose Duplicate.

Annotation 4

- I In the Model Builder window, click Annotation 4.
- 2 In the Settings window for Annotation, locate the Annotation section.
- 3 In the **Text** text field, type Sulfate.
- 4 Right-click Annotation 4 and choose Duplicate.

Annotation 5

- I In the Model Builder window, click Annotation 5.
- 2 In the Settings window for Annotation, locate the Annotation section.
- 3 In the **Text** text field, type Barium.

Size bins

- I In the Home toolbar, click **Add Plot Group** and choose **3D Plot Group**.
- 2 In the Settings window for 3D Plot Group, locate the Data section.

- 3 From the Dataset list, choose Study 2/Solution 2 (sol2).
- 4 In the Label text field, type Size bins.
- **5** Locate the **Color Legend** section. From the **Position** list, choose **Right double**.
- 6 Locate the Plot Array section. Select the Enable check box.
- 7 Locate the Plot Settings section. Clear the Plot dataset edges check box.

Slice 1

- I Right-click Size bins and choose Slice.
- 2 In the Settings window for Slice, locate the Expression section.
- **3** In the **Expression** text field, type n5.
- 4 Click to expand the Coloring and Style section. Click | Change Color Table.
- 5 In the Color Table dialog box, select Rainbow>Prism in the tree.
- 6 Click OK.
- 7 In the Settings window for Slice, click to expand the Title section.
- 8 From the Title type list, choose Manual.
- 9 In the **Title** text area, type n5 (1/m⁴).
- 10 Locate the Plane Data section. From the Plane list, choose xy-planes.
- II Locate the Plot Array section. Select the Manual indexing check box.

Slice 2

- I In the Model Builder window, right-click Size bins and choose Slice.
- 2 In the Settings window for Slice, locate the Expression section.
- **3** In the **Expression** text field, type n5.
- **4** Locate the **Title** section. From the **Title type** list, choose **None**.
- 5 Locate the Plane Data section. From the Plane list, choose zx-planes.
- 6 In the Planes text field, type 1.
- 7 Click to expand the Inherit Style section. From the Plot list, choose Slice 1.
- 8 Locate the Plot Array section. Select the Manual indexing check box.

Transbarency I

- I Right-click Slice 2 and choose Transparency.
- 2 In the Settings window for Transparency, locate the Transparency section.
- **3** Set the **Transparency** value to **0.4**.

Surface I

- I In the Model Builder window, right-click Size bins and choose Surface.
- 2 In the Settings window for Surface, click to expand the Title section.
- **3** Locate the **Expression** section. In the **Expression** text field, type 1.
- **4** Locate the **Title** section. From the **Title type** list, choose **None**.
- 5 Locate the Plot Array section. Select the Manual indexing check box.

Selection 1

- I Right-click Surface I and choose Selection.
- **2** Select Boundaries 4, 5, 7, 10, 20, 21, 24, and 25 only.

Material Appearance 1

- I In the Model Builder window, right-click Surface I and choose Material Appearance.
- 2 In the Settings window for Material Appearance, locate the Appearance section.
- 3 From the Appearance list, choose Custom.
- 4 From the Material type list, choose Steel (anodized).

Slice I, Slice 2, Surface I

- I In the Model Builder window, under Results>Size bins, Ctrl-click to select Slice I, Slice 2, and Surface 1.
- 2 Right-click and choose **Duplicate**.

Slice 3, Slice 4, Surface 2

- I In the Model Builder window, under Results>Size bins, Ctrl-click to select Slice 3, Slice 4, and Surface 2.
- 2 In the Settings window for Slice, locate the Expression section.
- **3** In the **Expression** text field, type n10.
- 4 Locate the Inherit Style section. From the Plot list, choose Slice 1.
- 5 Locate the **Title** section. In the **Title** text area, type n10 (1/m⁴).
- 6 Locate the Plot Array section. In the Index text field, type 1.

Slice 4

- I In the Model Builder window, click Slice 4.
- 2 In the Settings window for Slice, locate the Expression section.
- **3** In the **Expression** text field, type n10.
- 4 Locate the Plot Array section. In the Index text field, type 1.

Surface 2

- I In the Model Builder window, click Surface 2.
- 2 In the Settings window for Surface, locate the Plot Array section.
- 3 In the Index text field, type 1.

Slice 3, Slice 4, Surface 2

- I In the Model Builder window, under Results>Size bins, Ctrl-click to select Slice 3, Slice 4, and Surface 2.
- 2 Right-click and choose **Duplicate**.

Slice 5

- I In the Model Builder window, click Slice 5.
- 2 In the Settings window for Slice, locate the Expression section.
- **3** In the **Expression** text field, type n15.
- **4** Locate the **Title** section. In the **Title** text area, type n15 (1/m⁴).
- 5 Locate the Plot Array section. In the Index text field, type 2.

Slice 6

- I In the Model Builder window, click Slice 6.
- 2 In the Settings window for Slice, locate the Expression section.
- **3** In the **Expression** text field, type n15.
- 4 Locate the **Plot Array** section. In the **Index** text field, type 2.

Surface 3

- I In the Model Builder window, click Surface 3.
- 2 In the Settings window for Surface, locate the Plot Array section.
- **3** In the **Index** text field, type 2.

Slice 5, Slice 6, Surface 3

- I In the Model Builder window, under Results>Size bins, Ctrl-click to select Slice 5, Slice 6, and Surface 3.
- 2 Right-click and choose **Duplicate**.

Slice 7, Slice 8, Surface 4

- I In the Model Builder window, under Results>Size bins, Ctrl-click to select Slice 7, Slice 8, and Surface 4.
- 2 In the Settings window for Slice, locate the Expression section.
- **3** In the **Expression** text field, type n20.

- 4 Locate the **Title** section. In the **Title** text area, type n20 (1/m⁴).
- **5** Locate the **Plot Array** section. In the **Index** text field, type **3**.

Slice 8

- I In the Model Builder window, click Slice 8.
- 2 In the Settings window for Slice, locate the Expression section.
- **3** In the **Expression** text field, type n20.
- 4 Locate the **Plot Array** section. In the **Index** text field, type 3.

Surface 4

- I In the Model Builder window, click Surface 4.
- 2 In the Settings window for Surface, locate the Plot Array section.
- 3 In the Index text field, type 3.

Annotation I

- I In the Model Builder window, right-click Size bins and choose Annotation.
- 2 In the Settings window for Annotation, locate the Annotation section.
- 3 In the **Text** text field, type Interval 5.
- 4 Locate the Coloring and Style section. Clear the Show point check box.
- **5** Right-click **Annotation I** and choose **Duplicate**.

Annotation 2

- I In the Model Builder window, click Annotation 2.
- 2 In the Settings window for Annotation, locate the Annotation section.
- **3** In the **Text** text field, type Interval 10.
- 4 Right-click Annotation 2 and choose Duplicate.

Annotation 3

- I In the Model Builder window, click Annotation 3.
- 2 In the Settings window for Annotation, locate the Annotation section.
- 3 In the **Text** text field, type Interval 15.
- 4 Right-click Annotation 3 and choose Duplicate.

Annotation 4

- I In the Model Builder window, click Annotation 4.
- 2 In the Settings window for Annotation, locate the Annotation section.
- 3 In the **Text** text field, type Interval 20.

Discretized points 2

- I In the Model Builder window, under Results click Discretized points 2.
- 2 In the Settings window for ID Plot Group, locate the Data section.
- 3 From the Dataset list, choose Study I/Solution I (soll).
- 4 In the Discretized points 2 toolbar, click Plot.

Discretized points 2, Particle Number Density Distribution 2, Size bins, Velocity Field, Supersaturation and Concentrations

- I In the Model Builder window, under Results, Ctrl-click to select Particle Number Density Distribution 2, Discretized points 2, Velocity Field, Supersaturation and Concentrations, and Size bins.
- 2 Right-click and choose Group.

T-mixer

In the Settings window for Group, type T-mixer in the Label text field.