Created in COMSOL Multiphysics 6.2

Pull-In Voltage for a Biased Resonator — 2D

Introduction

Silicon micromechanical resonators have long been used for designing sensors and are now becoming increasingly important as oscillators in the consumer electronics market. In this sequence of models, a surface micromachined MEMS resonator, designed as part of a micromechanical filter, is analyzed in detail. The resonator is based on that developed in Ref. 1.

This model performs a pull-in analysis of the structure, to predict the point at which the biased system becomes unstable. The analysis begins from the stationary analysis performed in the accompanying model Stationary Analysis of a Biased Resonator — 2D; please review this model first.

Model Definition

The geometry, fabrication, and operation of the device are discussed for the "Stationary Analysis of a Biased Resonator" model.

This model computes the pull-in voltage for the resonator by solving an inverse problem. The y-coordinate of the resonator midpoint is computed using an integration operator (`intop1`). The inverse problem that COMSOL solves computes the DC voltage that must be applied to the beam in order to move the midpoint to a set y-coordinate, `yset`. This is achieved by adding a global equation for the DC voltage, `VdcSP`, applied to the resonator. The equation `intop1(y)-yset=0` is solved to determine the value of `VdcSP`. This means that `VdcSP` is adjusted until the midpoint of the resonator has a y-coordinate given by the set value, `yset`. Essentially COMSOL is being asked to find the voltage that allows the beam to exist in equilibrium (stable or unstable) at a given displacement. Solving the problem in this manner avoids complications with trying to solve a problem with no solution (which is what happens if the voltage is continuously ramped up eventually exceeding the pull-in voltage). The result of the analysis is a displacement versus voltage plot, with a minimum at the pull-in voltage. Note that for a linear spring, the pull-in displacement corresponds to $1/3$ of the gap distance. Although the inclusion of geometric nonlinearities in the solid mechanics solver means that the pull-in displacement changes slightly from this value, it is usually most efficient to search around this point for the pull-in voltage.

Results and Discussion

Figure 1 shows the voltage-displacement curve for the resonator at equilibrium, for y-coordinates that correspond to displacements of around 1/3 of the gap size. The pull-in voltage is 63.3 V.

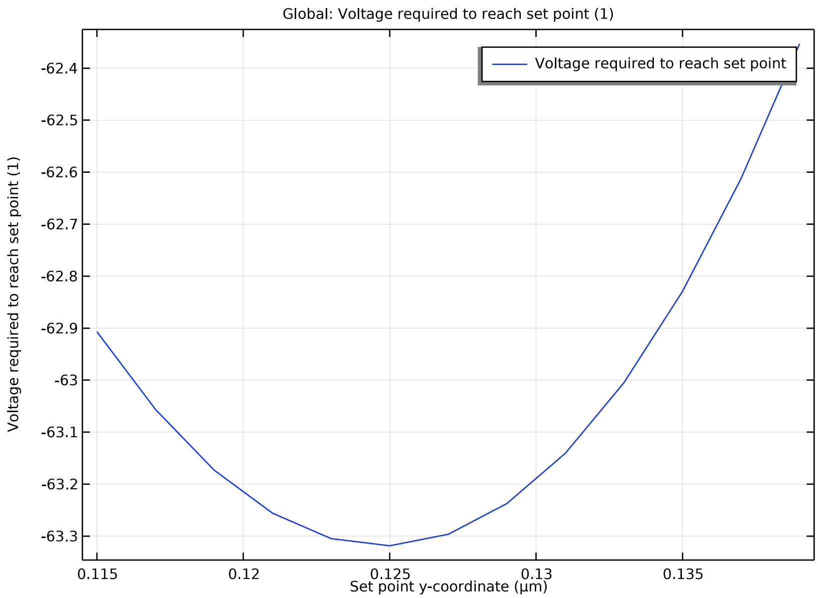

Figure 1: Voltage required to achieve a set displacement versus the target displacement. The pull-in voltage is the minimum of the plot: 63.3 V.

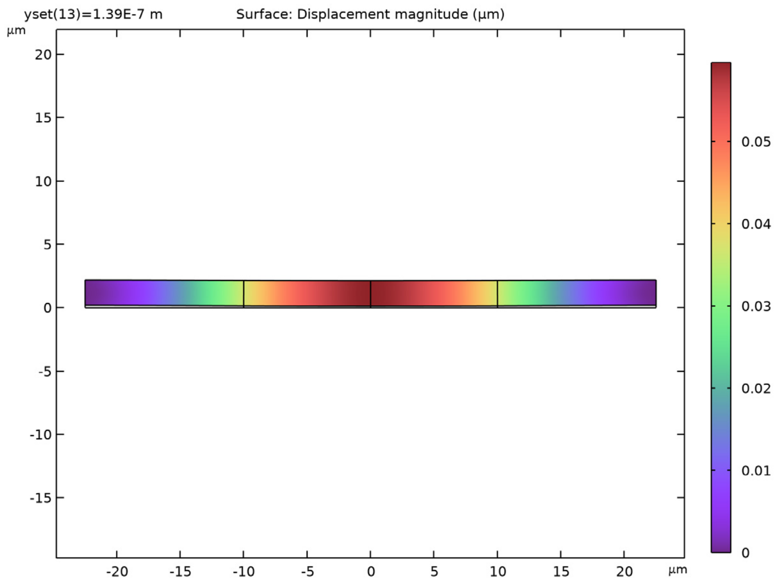

Figure 2: y-displacement of the resonator at pull-in. The displacement at pull-in is 74 nm. For a linear spring the displacement at pull in would be 66 nm.

Figure 2 shows the y-displacement of the resonator at the pull-in voltage. The maximum displacement at pull-in is 74 nm. This is comparable to the (approximate) linear spring value of 66 nm.

Notes About the COMSOL Implementation

To compute the voltage required to generate the desired displacement of the beam, use a global equation. A common use of global equations is for computing the value of a dependent variable based on an ordinary differential equation in the dependent variable itself. However, it is also possible to couple a global equation with the other PDEs in the model as a powerful tool to solve certain kinds of inverse problems. This model uses a global equation to compute the potential applied to the drive electrode. The equation takes the form

$$y_0 = y_{\mathrm{set}}$$

where y_0 is the y-coordinate of the midpoint of the beam's underside and y_{set} is the desired y-coordinate. COMSOL Multiphysics computes the voltage to satisfy the constraint implied by the above equation.

Reference

1. F.D. Bannon III, J.R. Clark and C.T.-C. Nguyen, "High-Q HF Microelectromechanical Filters," *IEEE Journal of Solid State Circuits,* vol. 35, no. 4, pp. 512–526, 2000.

Application Library path: `MEMS_Module/Actuators/biased_resonator_2d_pull_in`

Modeling Instructions

Start from the existing stationary model.

APPLICATION LIBRARIES

1 From the **File** menu, choose **Application Libraries**.

2 In the **Application Libraries** window, select **MEMS Module>Actuators> biased_resonator_2d_basic** in the tree.

3 Click **Open**.

Add a parameter to set the y-coordinate of the midpoint of the resonator.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
yset	100[nm]	1E-7 m	Set point y-coordinate

Add a nonlocal integration coupling to compute the actual displacement.

DEFINITIONS

Integration 1 (intop1)

1 In the **Model Builder** window, expand the **Component 1 (comp1)** node.

2 Right-click **Component 1 (comp1)>Definitions** and choose **Nonlocal Couplings> Integration**.

3 In the **Settings** window for **Integration**, locate the **Source Selection** section.

4 From the **Geometric entity level** list, choose **Point**.

5 Click **Paste Selection**.

6 In the **Paste Selection** dialog box, type `8` in the **Selection** text field.

7 Click **OK**.

Change the drive potential to the value `VdcSP` - which will be solved for in a global equation.

ELECTROSTATICS (ES)

Electric Potential 1

1 In the **Model Builder** window, expand the **Component 1 (comp1)>Electrostatics (es)** node, then click **Electric Potential 1**.

2 In the **Settings** window for **Electric Potential**, locate the **Electric Potential** section.

3 In the V_0 text field, type `VdcSP[V]`.

4 Click the **Show More Options** button in the **Model Builder** toolbar.

5 In the **Show More Options** dialog box, in the tree, select the check box for the node **Physics>Equation-Based Contributions**.

6 Click **OK**.

Add a global equation to compute the voltage for a given displacement, `VdcSP`.

Global Equations 1 (ODE1)

1 In the **Physics** toolbar, click **Global** and choose **Global Equations**.

2 In the **Settings** window for **Global Equations**, locate the **Global Equations** section.

3 In the table, enter the following settings:

Name	f(u,ut,utt,t) (1)	Initial value (u_0) (1)	Initial value (u_t0) (1/s)	Description
VdcSP	(intop1(y)-yset)/yset	0	0	

Set up a parametric sweep over the displacement set point, `yset`.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies>Stationary**.

4 Click **Add Study** in the window toolbar.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

PULL IN

1 In the **Model Builder** window, right-click **Study 2** and choose **Rename**.

2 In the **Rename Study** dialog box, type `Pull In` in the **New label** text field.

3 Click **OK**.

Step 1: Stationary

1 In the **Model Builder** window, under **Pull In** click **Step 1: Stationary**.

2 In the **Settings** window for **Stationary**, click to expand the **Study Extensions** section.

3 Select the **Auxiliary sweep** check box.

4 Click **Add**.

5 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
yset (Set point y-coordinate)	`range(115[nm],2[nm],140[nm])`	`m`

The problem is highly nonlinear due to the presence of the global equation, so the solver settings need to be adjusted accordingly.

Solution 2 (sol2)

1 In the **Study** toolbar, click **Show Default Solver**.

2 In the **Model Builder** window, expand the **Solution 2 (sol2)** node.

3 In the **Model Builder** window, expand the **Pull In>Solver Configurations>Solution 2 (sol2)>Stationary Solver 1** node.

4 Right-click **Pull In>Solver Configurations>Solution 2 (sol2)>Stationary Solver 1** and choose **Fully Coupled**.

5 In the **Settings** window for **Fully Coupled**, click to expand the **Method and Termination** section.

6 From the **Nonlinear method** list, choose **Automatic highly nonlinear (Newton)**.

7 In the **Study** toolbar, click **Compute**.

RESULTS

Pull In Displacement

1 In the **Settings** window for **2D Plot Group**, type `Pull In Displacement` in the **Label** text field.

2 In the **Pull In Displacement** toolbar, click **Plot**.

3 Click the **Zoom Extents** button in the **Graphics** toolbar.

Compare the resulting plot with Figure 2.

Determine the pull-in voltage by plotting `VdcSP` versus `yset`.

Pull In Plot

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 Right-click **1D Plot Group 7** and choose **Rename**.

3 In the **Rename 1D Plot Group** dialog box, type `Pull In Plot` in the **New label** text field.

4 Click **OK**.

Global 1

1 Right-click **Pull In Plot** and choose **Global**.

2 In the **Settings** window for **Global**, locate the **Data** section.

3 From the **Dataset** list, choose **Pull In/Solution 2 (sol2)**.

4 Locate the **y-Axis Data** section. In the table, enter the following settings:

Expression	Unit	Description
`VdcSP`	1	`Voltage required to reach set point`

5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

6 In the **Expression** text field, type `yset`.

7 In the **Pull In Plot** toolbar, click **Plot**.

8 Click the **Zoom Extents** button in the **Graphics** toolbar.

Compare the resulting plot with Figure 1. The pull in voltage is the minimum of the curve: around 63.3 V at yset = 125 nm.