

# Micromechanics and Macromechanics of a Composite Cylinder

Fiber composites are widely used in industrial applications. Compared to more traditional metallic engineering materials, fiber composites often have superior specific stiffness and strength properties, and they are often more corrosion resistant. Also, properties like strength, stiffness, and toughness can often be tailored to specific applications. A fiber composite consists of load carrying fibers embedded in a polymer resin. The composite material is typically a laminate of individual layers, where the fibers in each layer are unidirectional. This model demonstrates how to perform a stress analysis of a laminated composite cylinder.

Modeling individual fibers in every layer in the laminate is unfeasible. A simplified micromechanics model of a single carbon fiber in epoxy is instead used to estimate the homogenized elastic properties of a single layer. These properties are then used in the macromechanical model of the laminated composite cylinder. Two approaches are used to model the laminate, namely the Layerwise (LW) theory and the Equivalent Single Layer (ESL) theory.

# Model Definition

This model performs different types of analyses of a laminated composite cylinder. The model is divided into three parts:

- · Micromechanics analysis
- Stress analysis using the Layerwise theory
- Stress analysis using the Equivalent Single Layer theory

Eigenfrequencies and mode shapes are computed and compared using both theories.

#### MICROMECHANICS ANALYSIS

A micromechanics analysis of a single layer is performed in order to obtain its homogenized material properties. The composite layer is assumed to be made of carbon fibers unidirectionally embedded in epoxy resin. A repeating unit cell (RUC) having a cylindrical fiber surrounded by resin is shown in Figure 1. The fiber radius is computed assuming a fiber volume fraction of 0.6.

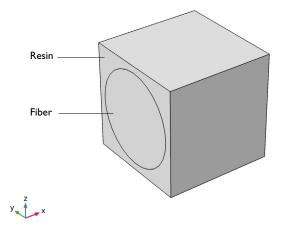


Figure 1: Geometry of the unit cell with a carbon fiber in an epoxy resin.

# Fiber and Resin Properties

The layers of the laminate are made of T300 carbon fiber and 914C epoxy. The carbon fiber is assumed to be transversely isotropic, and the epoxy resin is assumed to be isotropic. The material properties of fiber and resin are given in Table 1 and Table 2, respectively.

TABLE I: CARBON FIBER MATERIAL PROPERTIES.

Material Property	Value
$\overline{\{E_1,E_2\}}$	{230,15} GPa
G <sub>12</sub>	15 GPa
$\{v_{12}, v_{23}\}$	{0.2, 0.07}
ρ	1800 kg/m <sup>3</sup>

TABLE 2: EPOXY RESIN MATERIAL PROPERTIES.

Material Property	Value
E	4 GPa
υ	0.35
ρ	1100 kg/m <sup>3</sup>

# Cell Periodicity

In order to perform a micromechanics analysis, the Cell Periodicity node in the Solid **Mechanics** interface is used. The **Cell Periodicity** node is used to apply periodic boundary conditions to the three pairs of faces of the unit cell.

In order to extract the homogenized elasticity matrix for a layer, the unit cell needs to be analyzed for six different load cases. The Average Strain periodicity type needs to be selected to obtain the homogenized elasticity matrix. This is automatically done with the help of the action buttons in the Cell Periodicity node. The Cell Periodicity node has three action buttons in the tool bar of section called Periodicity Type: Create Load Groups and Study, Create Material by Reference, and Create Material by Value. The action button Create Load Groups and Study generates six different load groups and a stationary study with six load cases. The action button Create Material by Reference generates a Global Material with an elasticity matrix corresponding to that of the homogenized material in terms of variables. The action button Create Material by Value generates a Global Material with an elasticity matrix corresponding to that of the homogenized material in terms of numbers after the study computed. The generated global material can be used to define the properties of individual layers in a composite laminate.

#### STRESS ANALYSIS USING THE LAYERWISE THEORY

# Layerwise (LW) Theory

In the layerwise theory, the degrees of freedom are the displacements (u, v, w) available on the reference surface (or modeled surface) as well as in the through-thickness direction. From a constitutive equation point of view, this theory is similar to 3D solid elasticity. The layerwise theory is useful for detailed modeling of thick composite laminates because it can capture interlaminar shear stresses. It could therefore be used to also study delamination.

# Geometry and Boundary Conditions

The model geometry of a composite cylinder with a length of 0.5 m and a radius of 0.1 m is shown in Figure 2. Boundary conditions and loading are:

- One end of the cylinder is fixed.
- The other end has a roller support.
- A load of 1 kN is applied to a quarter of the cylinder outer surface.

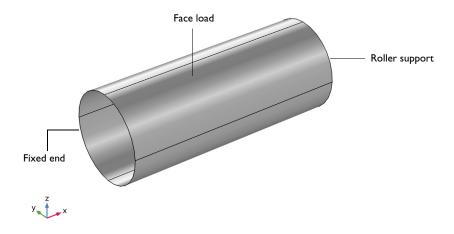


Figure 2: Geometry of the cylinder showing boundary conditions and loading.

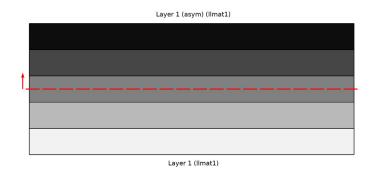


Figure 3: Through-thickness view of the laminated material with five layers.

# Stacking Sequence and Material Properties

The laminate consists of five layers of 1 mm thickness, as shown in Figure 3. The orientations of the layers are different. The orientations, starting from the bottom of the laminate, are taken as 0, 45, 90, -45, and 0 degrees, as shown in Figure 4. The orientation of a layer is specified with respect to the laminate coordinate system as shown in Figure 5. The material properties for each layer are given by the homogenized material computed in the micromechanics analysis.

The first principal material direction showing the fiber orientation in each layer of the physical geometry are shown in Figure 6.

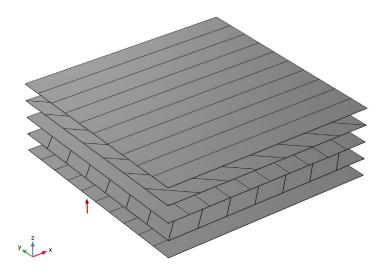


Figure 4: Stacking sequence [0/45/90/-45/0] for the laminate showing the fiber orientation of each layer, from bottom to top.

#### Coordinate system surface: Boundary system

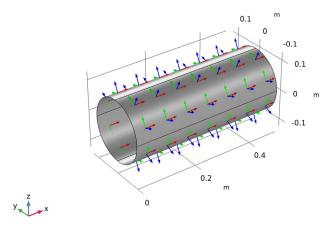


Figure 5: The laminate coordinate system showing the first principal direction along the cylinder axis.

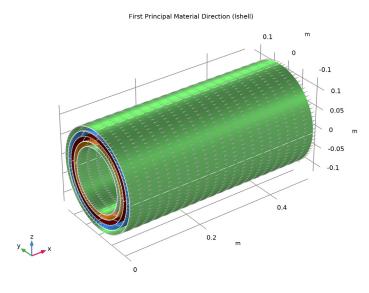


Figure 6: First principal material direction showing the fiber orientation in each layer of the physical geometry. The ply angle is used as a color for each layer.

#### STRESS ANALYSIS USING THE ESL THEORY

Equivalent Single Layer (ESL) Theory

In the equivalent single layer (ESL) theory, the degrees of freedom are the displacements and rotations on the midplane of the laminate. From a constitutive equation point of view, this theory is similar to 3D shell elasticity. Through-thickness homogenized material properties of the laminate are used. It is therefore computationally less expensive than the layerwise theory. It can be used for the modeling of thin to moderately thick laminates with good accuracy. An aim of this analysis is to compare the results to the results obtained from the layerwise theory.

The model setup, including geometry, boundary conditions, material properties, and so on, is the same as described in the previous section.

# Results and Discussion

In the micromechanics analysis, six load cases are used to evaluate the elasticity matrix. The distribution of effective (von Mises) stress for four of the load cases is shown in Figure 7.

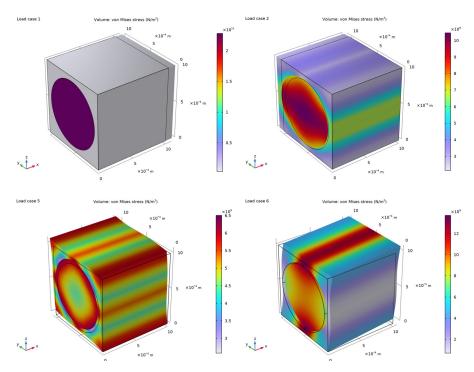


Figure 7: Von Mises stress distribution in the unit cell for four load cases.

The von Mises stress distribution in the composite cylinder obtained from the two theories is presented in Figure 8. Both theories give similar results.

The through-thickness variation of the axial second Piola–Kirchhoff stress at a particular point on the cylinder is shown in Figure 9. We see that the stress variation is discontinuous between layers, but that the two theories predict similar distributions.

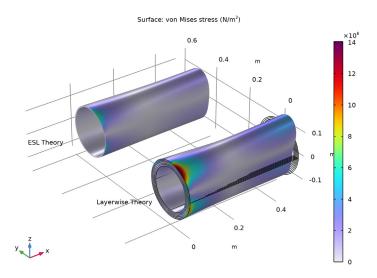


Figure 8: Von Mises stress distribution in the composite cylinder obtained using the Layerwise and ESL theories.

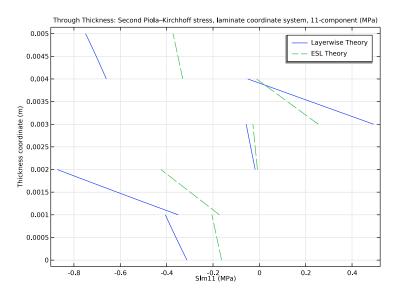


Figure 9: Through-thickness stress variation in the axial direction at a particular point on the cylinder.

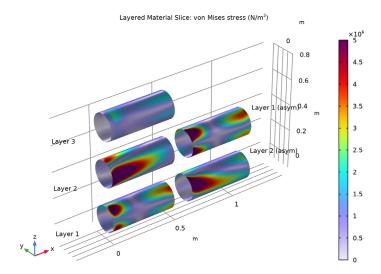


Figure 10: Von Mises stress distribution in the five layers of the laminate.

Figure 10 shows the distribution of von Mises stress in each layer of the laminate using the layerwise theory. The distributions of stress in the individual layers are remarkably different. The middle layer, with fibers perpendicular to the first principal laminate direction, shows the lowest stresses.

The first six eigenfrequencies of the constrained cylinder are shown in Table 3, and the corresponding mode shapes are shown in Figure 11. The eigenfrequencies obtained using the LW and ESL theories match within 0.2%.

TABLE 3: COMPARISON OF EIGENFREQUENCIES.

Eigenfrequencies from layerwise theory (Hz)	Eigenfrequencies from ESL theory (Hz)
486	485
573	572
984	984
999	999
1213	1211
1276	1274

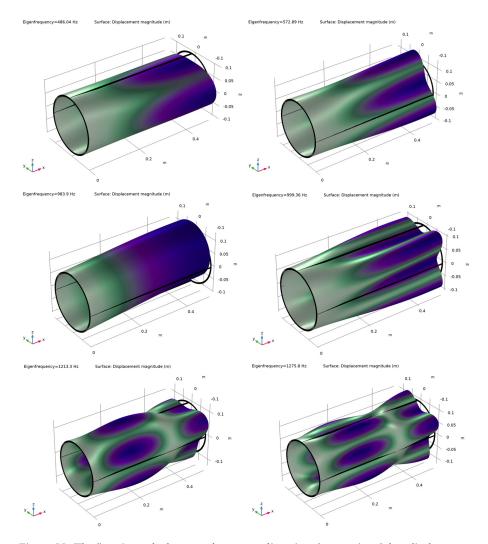


Figure 11: The first six mode shapes and corresponding eigenfrequencies of the cylinder.

# Notes About the COMSOL Implementation

• The micromechanics analysis of a single fiber in a resin can be performed using the **Cell** Periodicity node available in the Solid Mechanics interface. Using this functionality, the

- elasticity matrix of the homogenized material can be computed for the given fiber and resin properties, and the fiber volume fraction.
- The **Cell Periodicity** node has three action buttons on the tool bar of section called Periodicity Type: Create Load Groups and Study, Create Material by Reference, and Create Material by Value. The action button Create Load Groups and Study generates load groups and a stationary study with load cases. The action button Create Material by Reference generates a Global Material with an elasticity matrix corresponding to that of the homogenized material in terms of variables. The action button Create Material by Value generates a Global Material with an elasticity matrix corresponding to that of the homogenized material in terms of numbers after the study computed. The action buttons are active depending on the choices in the Periodicity Type and Calculate Average **Properties** lists.
- Modeling a composite laminated shell requires a surface geometry (2D), in general called a base surface, and a **Layered Material** node which adds an extra dimension (1D) to the base surface geometry in the surface normal direction. Using the Layered Material functionality, you can model several layers of different thickness, material properties, and fiber orientations. You can optionally specify the interface materials between the layers and the control mesh elements in each layer.
- The Lavered Material Link and Lavered Material Stack have an option to transform the given Layered Material into a symmetric or antisymmetric laminate. A repeated laminate can also be constructed using a transform option.
- You can either use the *Layerwise (LW)* theory based **Layered Shell** interface or the Equivalent Single Layer (ESL) theory based Linear Elastic Material, Layered node in Shell interface.
- To analyze the results in a composite shell, you can either create a slice plot using the Layered Material Slice plot for in-plane variation of a quantity, or you can create a Through **Thickness** plot for out-of-plane variation of a quantity at a point. To visualize the results as a 3D solid object, you can use the **Layered Material** dataset which creates a virtual 3D solid object combining the surface geometry (2D) and the extra dimension (1D).

**Application Library path:** Composite Materials Module/Tutorials/ composite cylinder micromechanics and stress analysis

From the File menu, choose New.

#### NEW

In the New window, click Model Wizard.

#### MODEL WIZARD

- I In the Model Wizard window, click **3D**.
- 2 In the Select Physics tree, select Structural Mechanics Solid Mechanics (solid).
- 3 Click Add.
- 4 Click M Done.

#### **GLOBAL DEFINITIONS**

#### Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- 3 Click Load from File.
- 4 Browse to the model's Application Libraries folder and double-click the file composite cylinder micromechanics and stress analysis parameters.txt.

#### GEOMETRY I

Next, create a repeating unit cell (RUC) for a unidirectional fiber composite with square fiber packing. This RUC like many others can be found in the built-in **Part Libraries**.

# PART LIBRARIES

- I In the Home toolbar, click Windows and choose Part Libraries.
- 2 In the Model Builder window, under Component I (compl) click Geometry I.
- 3 In the Part Libraries window, select COMSOL Multiphysics>Unit Cells and RVEs> Fiber Composites>unidirectional\_fiber\_square\_packing in the tree.
- 4 Right-click Component I (compl)>Geometry I and choose Add to Geometry.
- 5 In the Select Part Variant dialog box, select Specify fiber volume fraction in the **Select part variant** list.
- 6 Click OK.

#### **GEOMETRY I**

Unidirectional Fiber Composite, Square Packing I (pil)

- I In the Model Builder window, under Component I (compl)>Geometry I click Unidirectional Fiber Composite, Square Packing I (pil).
- 2 In the Settings window for Part Instance, locate the Input Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description
vf	v_f	0.6	Fiber volume fraction
wm	L	0.001 m	Cell width
dm	L	0.001 m	Cell depth
hm	L	0.001 m	Cell height

# Form Union (fin)

- I In the Model Builder window, click Form Union (fin).
- 2 In the Settings window for Form Union/Assembly, click | Build Selected.

#### SOLID MECHANICS (SOLID)

# Cell Periodicity 1

- I In the Model Builder window, under Component I (compl) right-click Solid Mechanics (solid) and choose the domain setting More>Cell Periodicity.
- 2 In the Settings window for Cell Periodicity, locate the Cell Properties section.
- 3 From the Boundary conditions list, choose Average strain.
- 4 From the Calculate average properties list, choose Elasticity matrix, Standard (XX, YY, ZZ, XY, YZ, XZ).

# Boundary Pair I

- I In the Physics toolbar, click 🖳 Attributes and choose Boundary Pair.
- 2 In the Settings window for Boundary Pair, locate the Boundary Selection section.
- 3 Click Clear Selection.
- 4 Select Boundaries 1, 5, 11, and 12 only.
- 5 Right-click Boundary Pair I and choose Duplicate.

#### Boundary Pair 2

- I In the Model Builder window, click Boundary Pair 2.
- 2 In the Settings window for Boundary Pair, locate the Boundary Selection section.

- 3 Click Clear Selection.
- 4 Select Boundaries 2 and 10 only.
- 5 Right-click Boundary Pair 2 and choose Duplicate.

# Boundary Pair 3

- I In the Model Builder window, click Boundary Pair 3.
- 2 In the Settings window for Boundary Pair, locate the Boundary Selection section.
- 3 Click Clear Selection.
- 4 Select Boundaries 3 and 4 only.

In the upper-right corner of the **Periodicity type** section, you can find three buttons **Create** Load Groups and Study, Create Material by Reference and Create Material by Value. When the Average strain option is selected for the computation of the elasticity matrix, you can automatically generate load groups, a study, and a material by clicking on these buttons.

# Cell Periodicity 1

- I In the Model Builder window, click Cell Periodicity I.
- 2 In the Settings window for Cell Periodicity, click Automated Model Setup in the upperright corner of the Cell Properties section. From the menu, choose **Create Load Groups and Study** to generate load groups and a study node.

#### MATERIALS

#### Material I: Epoxy Resin

- I In the Model Builder window, under Component I (compl) right-click Materials and choose Blank Material.
  - Before adding the fiber material data, set the solid model to transversely isotropic in the Linear Elastic Material node since the fiber material is assumed to have transversely isotropic properties. The isotropic resin material data will be automatically converted into an transversely isotropic format.
- 2 In the Settings window for Material, type Material 1: Epoxy Resin in the Label text field.
- 3 Select Domain 1 only.

**4** Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Young's modulus	E	E_r	Pa	Young's modulus and Poisson's ratio
Poisson's ratio	nu	nu_r	1	Young's modulus and Poisson's ratio
Density	rho	rho_r	kg/m³	Basic

# SOLID MECHANICS (SOLID)

Linear Elastic Material I

- I In the Model Builder window, under Component I (compl)>Solid Mechanics (solid) click Linear Elastic Material I.
- 2 In the Settings window for Linear Elastic Material, locate the Linear Elastic Material section.
- 3 From the Material symmetry list, choose Orthotropic.
- 4 Select the Transversely isotropic check box.

#### MATERIALS

Material 2: Carbon Fiber

- I In the Model Builder window, under Component I (comp I) right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, type Material 2: Carbon Fiber in the Label text
- 3 Locate the Geometric Entity Selection section. Click Paste Selection.
- 4 In the Paste Selection dialog box, type 2 in the Selection text field.
- 5 Click OK.
- 6 In the Settings window for Material, locate the Material Contents section.
- 7 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Young's modulus	{Evect1, Evect2}	{E1_f, E2_f}	Pa	Transversely isotropic
Poisson's ratio	{nuvect1, nuvect2}	{nu12_f , nu23_f}	I	Transversely isotropic

Property	Variable	Value	Unit	Property group
Shear modulus	GvectI	G12_f	N/m²	Transversely isotropic
Density	rho	rho_f	kg/m³	Basic

#### MESH I

Free Triangular I

- I In the Mesh toolbar, click More Generators and choose Free Triangular.
- 2 Select Boundaries 1 and 5 only.
- 3 In the Settings window for Free Triangular, click | Build Selected.

Swept I

- I In the Mesh toolbar, click A Swept.
- 2 In the Settings window for Swept, click **Build Selected**.

#### CELL PERIODICITY STUDY

In the **Home** toolbar, click **Compute**.

#### RESULTS

Stress. Unit Cell

- I In the Settings window for 3D Plot Group, type Stress, Unit Cell in the Label text field.
- 2 In the Stress, Unit Cell toolbar, click Plot.

Before you add another physics, create a homogenized material from the Cell Periodicity feature.

The homogenized material can be created by using either of the two action buttons in the Periodicity type section, Create Material by Reference or Create Material by Value. Choose the second action button in order to generate the material with numbers.

# SOLID MECHANICS (SOLID)

Cell Periodicity 1

- I In the Model Builder window, under Component I (compl)>Solid Mechanics (solid) click Cell Periodicity 1.
- 2 In the Settings window for Cell Periodicity, click Automated Model Setup in the upperright corner of the Cell Properties section. From the menu, choose

**Create Material by Value** to generate a global material node with computed elastic properties.

# Modeling Instructions (Stress Analysis Using the Layerwise (LW) Theory)

This section describes how to model a laminated composite cylinder using the **Layered Shell** interface based on the layerwise theory.

#### ADD COMPONENT

In the Model Builder window, right-click the root node and choose Add Component>3D.

#### **GEOMETRY 2**

Cylinder I (cyll)

- I In the Geometry toolbar, click ( Cylinder.
- 2 In the Settings window for Cylinder, locate the Object Type section.
- 3 From the Type list, choose Surface.
- 4 Locate the Size and Shape section. In the Radius text field, type rc.
- 5 In the Height text field, type hc.
- 6 Locate the Axis section. From the Axis type list, choose x-axis.
- 7 Click **Pail Build Selected**.

# **DEFINITIONS (COMP2)**

Boundary System 2 (sys2)

- I In the Model Builder window, expand the Component 2 (comp2)>Definitions node, then click Boundary System 2 (sys2).
- 2 In the Settings window for Boundary System, locate the Settings section.
- 3 Find the Coordinate names subsection. From the Axis list, choose x.

#### ADD PHYSICS

- I In the Home toolbar, click Add Physics to open the Add Physics window.
- **2** Go to the **Add Physics** window.
- 3 In the tree, select Structural Mechanics>Layered Shell (Ishell).
- 4 Find the Physics interfaces in study subsection. In the table, clear the Solve check box for Cell Periodicity Study.

- 5 Click Add to Component 2 in the window toolbar.
- 6 In the Home toolbar, click Add Physics to close the Add Physics window.

#### **GLOBAL DEFINITIONS**

The laminate is antisymmetric when the middle layer is excluded. Therefore, it is sufficient to define only a part of it in the Layered Material node. The transformation into the full laminate is performed through layered material settings in the Layered Material Link node.

Layered Material: [0/45/90/-45/0]

- I In the Model Builder window, under Global Definitions right-click Materials and choose Layered Material.
- 2 In the Settings window for Layered Material, type Layered Material: [0/45/90/-45/ 0] in the Label text field.
- **3** Locate the **Layer Definition** section. In the table, enter the following settings:

Layer	Material	Rotation (deg)	Thickness	Mesh elements
Layer 1	Homogeneous Material (solidcp   mat)	0	th	1

- 4 Click Add two times.
- **5** In the table, enter the following settings:

Layer	Material	Rotation (deg)	Thickness	Mesh elements
Layer 2	Homogeneous Material (solidcp   mat)	45	th	1
Layer 3	Homogeneous Material (solidcp   mat)	90	th	1

#### MATERIALS

Layered Material Link I (Ilmat I)

I In the Model Builder window, under Component 2 (comp2) right-click Materials and choose Layers>Layered Material Link.

The laminate, partially defined in the Layered Material node, can be transformed into a full laminate using a transform option in the layered material settings. The middle layer is merged in order to create a five layer laminate.

- 2 In the Settings window for Layered Material Link, locate the Layered Material Settings section.
- 3 From the Transform list, choose Antisymmetric.
- 4 Select the Merge middle layers check box.
- 5 Click to expand the Preview Plot Settings section. In the Thickness-to-width ratio text field, type 0.4.
- 6 Locate the Layered Material Settings section. Click Layer Cross-Section Preview in the upper-right corner of the section to enable the through-thickness view of the laminated material as in Figure 3.
- 7 Click Layer Stack Preview in the upper-right corner of the Layered Material Settings section to show the stacking sequence including the fiber orientation as in Figure 4.

#### **GLOBAL DEFINITIONS**

Homogeneous Material (solidcp | mat)

- I In the Model Builder window, under Global Definitions>Materials click Homogeneous Material (solidcplmat).
- 2 In the Settings window for Material, locate the Material Contents section.
- **3** In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Density	rho	rho_l	kg/m³	Basic

#### LAYERED SHELL (LSHELL)

Linear Elastic Material L

- I In the Model Builder window, under Component 2 (comp2)>Layered Shell (Ishell) click Linear Elastic Material I.
- 2 In the Settings window for Linear Elastic Material, locate the Linear Elastic Material section.
- 3 From the Material symmetry list, choose Anisotropic.

Fixed Constraint I

- I In the Physics toolbar, click **Edges** and choose **Fixed Constraint**.
- 2 Select Edges 1, 2, 4, and 6 only.

Roller I

I In the Physics toolbar, click Edges and choose Roller.

2 Select Edges 9–12 only.

# Body Load I

- I In the Physics toolbar, click **Boundaries** and choose **Body Load**.
- 2 Select Boundary 2 only.
- 3 In the Settings window for Body Load, locate the Force section.
- 4 From the Load type list, choose Total force.
- **5** Specify the  $\mathbf{F}_{\text{tot}}$  vector as

0	x
0	у
Ftot	z

#### MESH 2

# Mapped I

- I In the Mesh toolbar, click A More Generators and choose Mapped.
- 2 In the Settings window for Mapped, locate the Boundary Selection section.
- 3 From the Selection list, choose All boundaries.

#### Distribution 1

- I Right-click Mapped I and choose Distribution.
- 2 In the Settings window for Distribution, locate the Edge Selection section.
- 3 From the Selection list, choose All edges.
- 4 Locate the Distribution section. In the Number of elements text field, type 20.
- 5 Click III Build All.

#### ADD STUDY

- I In the Home toolbar, click Add Study to open the Add Study window.
- **2** Go to the **Add Study** window.
- 3 Find the Studies subsection. In the Select Study tree, select General Studies>Stationary.
- **4** Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** check box for Solid Mechanics (solid).
- 5 Click Add Study in the window toolbar.
- 6 In the Home toolbar, click Add Study to close the Add Study window.

#### STUDY I: STATIONARY (LAYERWISE THEORY)

- I In the Model Builder window, click Study I.
- 2 In the Settings window for Study, type Study 1: Stationary (Layerwise Theory) in the Label text field.
- 3 In the Home toolbar, click **Compute**.

#### RESULTS

Layered Material

- I In the Model Builder window, expand the Results>Datasets node, then click Layered Material.
- 2 In the Settings window for Layered Material, locate the Layers section.
- 3 In the Scale text field, type 5.

Cut Point 3D I

- I In the Results toolbar, click Cut Point 3D.
- 2 In the Settings window for Cut Point 3D, locate the Data section.
- 3 From the Dataset list, choose Study I: Stationary (Layerwise Theory)/Solution I (3) (soll).
- 4 Locate the Point Data section. In the X text field, type hc/2.
- 5 In the Y text field, type rc.
- **6** In the **Z** text field, type rc.
- 7 From the Snapping list, choose Snap to closest boundary.

Stress (mises)

- I In the Model Builder window, under Results click Stress (Ishell).
- 2 In the Settings window for 3D Plot Group, type Stress (mises) in the Label text field.
- 3 In the Stress (mises) toolbar, click Plot.

# ADD PREDEFINED PLOT

- I In the Home toolbar, click Add Predefined Plot to open the Add Predefined Plot window.
- 2 Go to the Add Predefined Plot window.
- 3 In the tree, select Study 1: Stationary (Layerwise Theory)/Solution 1 (3) (sol1)> Layered Shell>Stress, Slice (Ishell).
- 4 Click Add Plot in the window toolbar.

#### RESULTS

Stress, Slice (mises)

- I In the Settings window for 3D Plot Group, type Stress, Slice (mises) in the Label text field.
- 2 Locate the Plot Settings section. Clear the Plot dataset edges check box.

Layered Material Slice 1

- I In the Model Builder window, expand the Stress, Slice (mises) node, then click Layered Material Slice 1.
- 2 In the Settings window for Layered Material Slice, locate the Through-Thickness Location section.
- 3 From the Location definition list, choose Layer midplanes.
- 4 Locate the Layout section. From the Displacement list, choose Rectangular.
- 5 From the Orientation list, choose zx.
- 6 In the Relative x-separation text field, type 0.3.
- 7 In the Relative z-separation text field, type 0.7.
- **8** Select the **Show descriptions** check box.
- **9** In the **Relative separation** text field, type 0.6.
- 10 Click to expand the Range section. Select the Manual color range check box.
- II In the Minimum text field, type 0.
- 12 In the Maximum text field, type 5e6.
- 13 Click to expand the Quality section. From the Resolution list, choose No refinement.

#### Deformation

- I In the Model Builder window, expand the Layered Material Slice I node.
- 2 Right-click **Deformation** and choose **Disable**.

Stress, Slice (mises)

- I In the Model Builder window, under Results click Stress, Slice (mises).
- 2 In the Stress, Slice (mises) toolbar, click Plot.

#### ADD PREDEFINED PLOT

- I Go to the Add Predefined Plot window.
- 2 In the tree, select Study I: Stationary (Layerwise Theory)/Solution I (3) (soll)> Layered Shell>Stress, Through Thickness (Ishell).

3 Click Add Plot in the window toolbar.

#### RESULTS

Stress, Through Thickness (SIm I I)

- I In the Settings window for ID Plot Group, type Stress, Through Thickness (Slm11) in the Label text field.
- 2 Locate the Plot Settings section.
- 3 Select the x-axis label check box. In the associated text field, type Slm11 (MPa).

Through Thickness I

- I In the Model Builder window, expand the Stress, Through Thickness (SImII) node, then click Through Thickness I.
- 2 In the Settings window for Through Thickness, locate the Data section.
- 3 From the Dataset list, choose Cut Point 3D 1.
- 4 Locate the x-Axis Data section. In the Expression text field, type 1shell.Slm11.
- 5 From the Unit list, choose MPa.
- 6 Click to expand the Legends section. From the Legends list, choose Manual.
- 7 In the table, enter the following settings:

# Legends Layerwise Theory

# ADD PREDEFINED PLOT

- I Go to the Add Predefined Plot window.
- 2 In the tree, select Study I: Stationary (Layerwise Theory)/Solution I (3) (soll)> Layered Shell>Stress, Through Thickness (Ishell).
- 3 Click Add Plot in the window toolbar.
- 4 In the tree, select Study I: Stationary (Layerwise Theory)/Solution I (3) (soll)> Layered Shell>Geometry and Layup (Ishell)>Thickness and Orientation (Ishell).
- **5** Click **Add Plot** in the window toolbar.
- 6 In the tree, select Study I: Stationary (Layerwise Theory)/Solution I (3) (soll)> Layered Shell>Geometry and Layup (Ishell)>First Principal Material Direction (Ishell).
- 7 Click Add Plot in the window toolbar.

8 In the Home toolbar, click **Add Predefined Plot** to close the Add Predefined Plot window.

#### RESULTS

First Principal Material Direction (Ishell)

- I In the First Principal Material Direction (Ishell) toolbar, click I Plot.
- 2 Click the **Zoom Extents** button in the **Graphics** toolbar.

#### ADD STUDY

- I In the Home toolbar, click Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select General Studies> Eigenfrequency.
- 4 Find the Physics interfaces in study subsection. In the table, clear the Solve check box for Solid Mechanics (solid).
- 5 Click Add Study in the window toolbar.
- 6 In the Home toolbar, click Add Study to close the Add Study window.

# STUDY 2: EIGENFREQUENCY (LAYERWISE THEORY)

- I In the Model Builder window, click Study 2.
- 2 In the Settings window for Study, type Study 2: Eigenfrequency (Layerwise Theory) in the Label text field.

# Step 1: Eigenfrequency

- I In the Model Builder window, under Study 2: Eigenfrequency (Layerwise Theory) click Step 1: Eigenfrequency.
- 2 In the Settings window for Eigenfrequency, locate the Study Settings section.
- **3** Select the **Desired number of eigenfrequencies** check box. In the associated text field, type 12.
- 4 In the Home toolbar, click **Compute**.

#### RESULTS

Mode Shape (Layerwise Theory)

- I In the Settings window for 3D Plot Group, type Mode Shape (Layerwise Theory) in the Label text field.
- 2 Locate the Plot Settings section. From the View list, choose New view.

- 3 Click the **Zoom Extents** button in the **Graphics** toolbar.
- 4 In the Mode Shape (Layerwise Theory) toolbar, click Plot.

Modeling Instructions (Stress Analysis using the Equivalent Single Layer (ESL) Theory)

This section describes how to model a laminated composite cylinder using the **Shell** interface based on the equivalent single layer (ESL) theory.

#### **GEOMETRY 2**

In the Model Builder window, under Component 2 (comp2) click Geometry 2.

Move I (movI)

- I In the Geometry toolbar, click \( \sum\_{\text{transforms}} \) Transforms and choose Move.
- **2** Select the object **cyll** only.
- 3 In the Settings window for Move, locate the Input section.
- 4 Select the **Keep input objects** check box.
- **5** Locate the **Displacement** section. In the **y** text field, type hc.
- 6 Click | Build Selected.

# LAYERED SHELL (LSHELL)

- I In the Model Builder window, under Component 2 (comp2) click Layered Shell (Ishell).
- 2 In the Settings window for Layered Shell, locate the Boundary Selection section.
- 3 In the list, choose 5 (Ilmat1), 6 (Ilmat1), 7 (Ilmat1), and 8 (Ilmat1).
- 4 Click Remove from Selection.
- **5** Select Boundaries 1–4 only.

# ADD PHYSICS

- I In the Home toolbar, click Add Physics to open the Add Physics window.
- 2 Go to the Add Physics window.
- 3 In the tree, select Structural Mechanics>Shell (shell).
- 4 Find the Physics interfaces in study subsection. In the table, clear the Solve check boxes for Cell Periodicity Study, Study 1: Stationary (Layerwise Theory), and Study 2: Eigenfrequency (Layerwise Theory).
- **5** Click **Add to Component 2** in the window toolbar.
- 6 In the Home toolbar, click and Physics to close the Add Physics window.

#### SHELL (SHELL)

- I Click the Show More Options button in the Model Builder toolbar.
- 2 In the Show More Options dialog box, in the tree, select the check box for the node Physics>Advanced Physics Options.
- 3 Click OK.
- 4 In the Settings window for Shell, locate the Boundary Selection section.
- 5 In the list, choose 1, 2, 3, and 4.
- 6 Click Remove from Selection.
- **7** Select Boundaries 5–8 only.
- 8 Click to expand the Advanced Settings section. Clear the Use MITC interpolation check box.

#### Linear Elastic Material, Layered 1

- I Right-click Component 2 (comp2)>Shell (shell) and choose Material Models> Linear Elastic Material, Layered.
- 2 In the Settings window for Linear Elastic Material, Layered, locate the Boundary Selection section.
- 3 From the Selection list, choose All boundaries.
- 4 Locate the Linear Elastic Material section. From the Material symmetry list, choose Anisotropic.

#### Fixed Constraint 1

- I In the Physics toolbar, click **Edges** and choose **Fixed Constraint**.
- **2** Select Edges 9, 10, 12, and 14 only.

#### Symmetry I

- I In the Physics toolbar, click Edges and choose Symmetry.
- **2** Select Edges 21–24 only.

# Face Load 1

- I In the Physics toolbar, click **Boundaries** and choose Face Load.
- 2 Click the Go to Default View button in the Graphics toolbar.
- 3 Select Boundary 6 only.
- 4 In the Settings window for Face Load, locate the Force section.
- 5 From the Load type list, choose Total force.

**6** Specify the  $\mathbf{F}_{tot}$  vector as

0	x
0	у
Ftot	z

#### ADD STUDY

- I In the Home toolbar, click Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select General Studies>Stationary.
- 4 Find the Physics interfaces in study subsection. In the table, clear the Solve check boxes for Solid Mechanics (solid) and Layered Shell (Ishell).
- 5 Click Add Study in the window toolbar.
- 6 In the Home toolbar, click Add Study to close the Add Study window.

# STUDY 3: STATIONARY (ESL THEORY)

- I In the Model Builder window, click Study 3.
- 2 In the Settings window for Study, type Study 3: Stationary (ESL Theory) in the Label text field.
- 3 Locate the Study Settings section. Clear the Generate default plots check box.
- 4 In the Home toolbar, click **Compute**.

#### RESULTS

Layered Material I

- I In the Results toolbar, click More Datasets and choose Layered Material.
- 2 In the Settings window for Layered Material, locate the Data section.
- 3 From the Dataset list, choose Study 3: Stationary (ESL Theory)/Solution 3 (7) (sol3).

Cut Point 3D I

In the Model Builder window, right-click Cut Point 3D I and choose Duplicate.

Cut Point 3D 2

- I In the Model Builder window, click Cut Point 3D 2.
- 2 In the Settings window for Cut Point 3D, locate the Point Data section.
- 3 In the Y text field, type hc+rc.

4 Locate the Data section. From the Dataset list, choose Study 3: Stationary (ESL Theory)/ Solution 3 (7) (sol3).

# Surface I

- I In the Model Builder window, expand the Stress (mises) node.
- 2 Right-click Surface I and choose Duplicate.

#### Surface 2

- I In the Model Builder window, click Surface 2.
- 2 In the Settings window for Surface, locate the Expression section.
- 3 In the Expression text field, type shell.mises.
- 4 Locate the Data section. From the Dataset list, choose Layered Material 1.
- **5** Click to expand the **Title** section. From the **Title type** list, choose **None**.
- 6 Click to expand the Inherit Style section. From the Plot list, choose Surface 1.

# Deformation

- I In the Model Builder window, expand the Surface 2 node, then click Deformation.
- 2 In the Settings window for Deformation, locate the Expression section.
- **3** In the **x-component** text field, type **u3**.
- 4 In the y-component text field, type v3.
- **5** In the **z-component** text field, type w3.

#### Stress (mises)

In the Model Builder window, under Results click Stress (mises).

# Table Annotation I

- I In the Stress (mises) toolbar, click More Plots and choose Table Annotation.
- 2 In the Settings window for Table Annotation, locate the Data section.
- 3 From the Source list, choose Local table.
- **4** In the table, enter the following settings:

x-coordinate	y-coordinate	z-coordinate	Annotation
-2*hc/5	0	0	Layerwise Theory
-2*hc/5	hc	0	ESL Theory

- 5 Locate the Coloring and Style section. Clear the Show point check box.
- 6 From the Anchor point list, choose Lower middle.
- 7 In the Stress (mises) toolbar, click Plot.

#### Through Thickness I

In the Model Builder window, under Results>Stress, Through Thickness (SIm11) right-click Through Thickness I and choose Duplicate.

# Through Thickness 2

- I In the Model Builder window, click Through Thickness 2.
- 2 In the Settings window for Through Thickness, locate the Data section.
- 3 From the Dataset list, choose Cut Point 3D 2.
- 4 Locate the x-Axis Data section. In the Expression text field, type shell.Sl11.
- **5** Click to expand the **Title** section. From the **Title type** list, choose **None**.
- 6 Click to expand the Coloring and Style section. Find the Line style subsection. From the Line list, choose Dashed.
- 7 Locate the **Legends** section. In the table, enter the following settings:

Legends			
ESL	Theory		

8 In the Stress, Through Thickness (SIm I I) toolbar, click Plot.

#### ADD STUDY

- I In the Home toolbar, click Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select General Studies> Eigenfrequency.
- 4 Find the Physics interfaces in study subsection. In the table, clear the Solve check boxes for Solid Mechanics (solid) and Layered Shell (Ishell).
- 5 Click Add Study in the window toolbar.
- 6 In the Home toolbar, click Add Study to close the Add Study window.

# STUDY 4: EIGENFREQUENCY (ESL THEORY)

- I In the Model Builder window, click Study 4.
- 2 In the Settings window for Study, type Study 4: Eigenfrequency (ESL Theory) in the Label text field.
- 3 Locate the Study Settings section. Clear the Generate default plots check box.

# Step 1: Eigenfrequency

- I In the Model Builder window, under Study 4: Eigenfrequency (ESL Theory) click Step 1: Eigenfrequency.
- 2 In the Settings window for Eigenfrequency, locate the Study Settings section.
- 3 Select the **Desired number of eigenfrequencies** check box. In the associated text field, type 12.
- 4 In the Home toolbar, click **Compute**.

#### RESULTS

Layered Material 2a

- I In the Results toolbar, click More Datasets and choose Layered Material.
- 2 In the Settings window for Layered Material, locate the Data section.
- 3 From the Dataset list, choose Study 4: Eigenfrequency (ESL Theory)/Solution 4 (9) (sol4).

#### Selection

- I Right-click Layered Material 2a and choose Selection.
- 2 In the Settings window for Selection, locate the Geometric Entity Selection section.
- 3 From the Geometric entity level list, choose Boundary.
- **4** Select Boundaries 5–8 only.

Mode Shape (Layerwise Theory)

In the Model Builder window, under Results right-click Mode Shape (Layerwise Theory) and choose **Duplicate**.

Mode Shape (ESL Theory)

- I In the Model Builder window, under Results click Mode Shape (Layerwise Theory) I.
- 2 In the Settings window for 3D Plot Group, type Mode Shape (ESL Theory) in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose None.

#### Surface 1

- I In the Model Builder window, expand the Mode Shape (ESL Theory) node, then click Surface 1.
- 2 In the Settings window for Surface, locate the Expression section.
- **3** In the **Expression** text field, type shell.disp.

# Deformation

- I In the Model Builder window, expand the Surface I node, then click Deformation.
- 2 In the Settings window for Deformation, locate the Expression section.
- **3** In the **x-component** text field, type **u3**.
- 4 In the y-component text field, type v3.
- **5** In the **z-component** text field, type w3.

# Mode Shape (ESL Theory)

- I In the Model Builder window, under Results click Mode Shape (ESL Theory).
- 2 In the Settings window for 3D Plot Group, locate the Data section.
- 3 From the Dataset list, choose Layered Material 2a.
- 4 From the Eigenfrequency (Hz) list, choose 1273.8.
- **5** Locate the **Plot Settings** section. From the **View** list, choose **New view**.
- **6** Click the **Zoom Extents** button in the **Graphics** toolbar.
- 7 In the Mode Shape (ESL Theory) toolbar, click  **Plot**.