Created in COMSOL Multiphysics 6.2

Dynamics of a Hopping Hoop

Introduction

In this conceptual example, a hopping hoop is modeled using the Multibody Dynamics interface. The model setup is based on a problem described in Ref. 1, with slight modifications. The original problem discusses the dynamics of an ideal, massless, rigid ring with a point mass attached to its perimeter. In this example, the different types of motion are investigated, which a thin rolling ring-point mass system can exhibit. It is shown, that under certain conditions, the rolling ring can disconnect from the ground and lift into the air.

Model Definition

As shown in Figure 1, the 2D model geometry consists of a thin ring placed at one end of a fixed, flat surface.

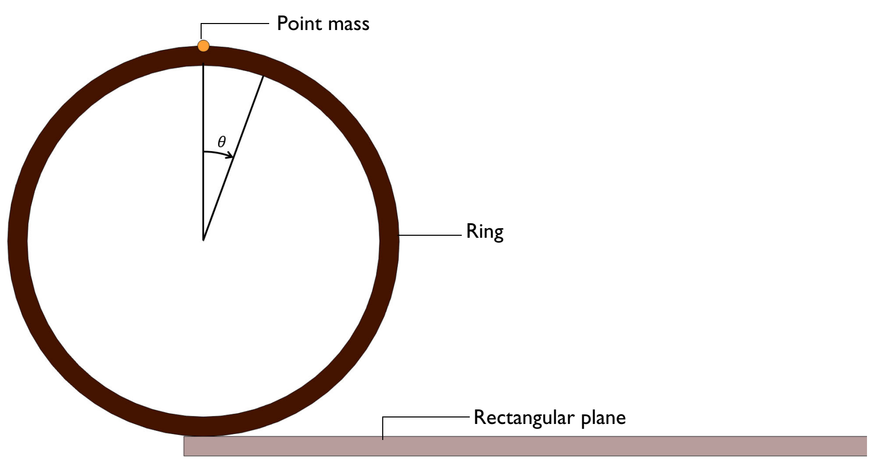

Figure 1: Geometry of the ring with a point mass at the top, placed on a rectangular block.

Both the ring and the plane are modeled as rigid bodies. A point mass is attached to the top point of the ring. Unlike the original problem, where the ring is considered as massless, a small fraction of the ring's total mass is uniformly distributed along its perimeter. The mass ratio of the system is γ. Thus, if m is the total mass of the system, a mass γm is assigned to the point mass and the remaining mass, $(1-\gamma)m$, is assigned to the ring. Using the special case $\gamma = 1$, the configuration of the original problem can be recovered. Gravity is acting in the vertical direction.

The physical parameters used in the model are listed in Table 1. The fundamental model parameters are set to unity, as the exact numerical values are irrelevant for the general dynamics of the system.

TABLE 1: PHYSICAL PROPERTIES.

Property	Symbol	Unit	Value
Radius of ring	R_c	m	1
Total mass	m	kg	1
Mass ratio	γ	1	0.75

The contact between the rolling ring and the plane is modeled using the **Rigid Body Contact** node, and frictional forces are included with the **Friction** subnode. The coefficient of friction between the ring and the plane is set to $\mu = 1$.

The motion starts with the ring being in an unstable equilibrium, where the point mass is at the top of the ring. To nudge the ring into a rolling motion, initial linear and angular velocities are applied. A time-dependent study is run for different initial velocities.

Results and Discussion

When the ring is pushed forward with an initial velocity, it begins to roll. The mass at the top of the ring moves toward the ground plane accelerated by gravity. The translational velocity of the system along the plane increases. With the continued rotation of the ring, the point mass reaches the bottom most position and then moves up. While the point mass moves upward, the ring's translational and rotational velocity decreases. When the point mass reaches top again, the ring velocity is at a minimum. Thus, during the revolution of the ring, the rotational speed of the system varies substantially.

Figure 2 shows the velocity of the ring for four different initial velocities. The cycloid path of the point mass and the center of gravity of the system are also plotted. For different initial velocities, the system exhibits different types of motions. For a small initial velocity ($v_0 = 0.1$ m/s), the ring exhibits a pure rolling motion without any slipping or hopping. For a slightly increased initial velocity ($v_0 = 2.8$ m/s), the ring rolls on the plane with some slipping. As the initial velocity is increased further ($v_0 = 3.1$ m/s and $v_0 = 3.5$ m/s), the ring initially rolls and then starts to hop. This is illustrated in Figure 2, where the trace of the center of gravity is colored red when the hoop is airborne. This part of the curve forms a projectile motion parabola.

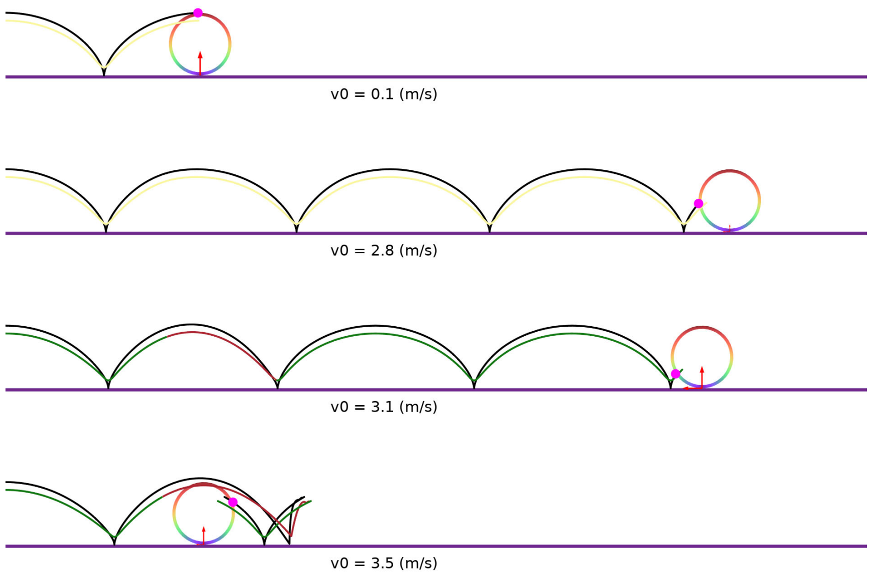

Figure 2: Velocity of the ring at $t = 5.2$ s for four different initial velocities. The path of the point mass and center of gravity are also shown. The arrows proportional to contact forces.

Below, we look at some key features of the system for the different types of motions.

PURE ROLLING

For a small initial velocity, the ring-mass system undergoes pure rolling without slipping or hopping. Figure 3 plots the variation of translational velocity of the ring center and the normalized angular velocity for two different initial velocities. As evident from the plot, the translational velocity of the center is equal to the angular velocity of the system for pure rolling. As the initial velocity increases, the velocity of the system increases.

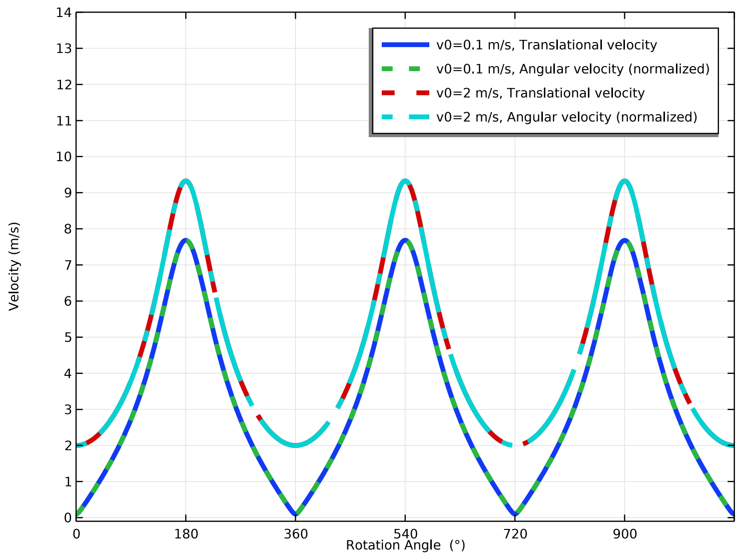

Figure 3: Velocity as a function of rotation angle for $v_0 = 0.1$ m/s and 2 m/s.

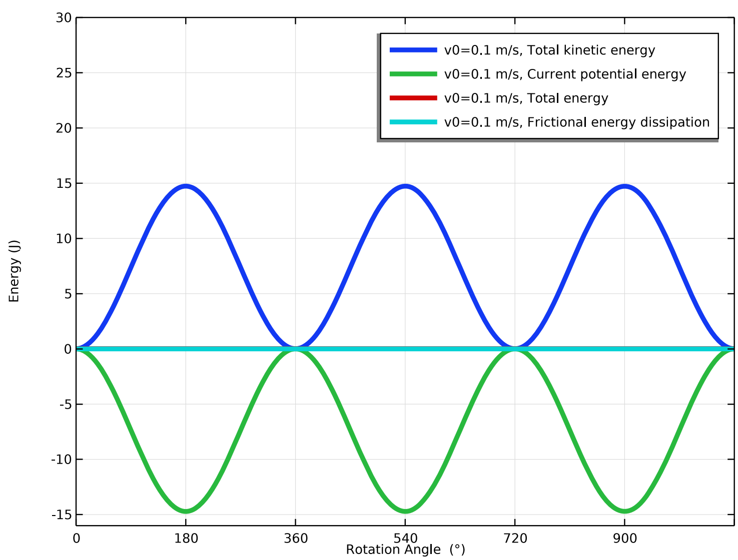

Figure 4: Energy as a function of rotation angle for $v_0 = 0.1$ m/s.

In the case of pure rolling, the potential and kinetic energies of the system vary harmonically as shown in Figure 4. As the ring does not hop for low of initial velocities, the height of the center of mass of the ring does not change. Hence, the vertical position of the point mass is the only contributing factor for the harmonic variation of the energies.

The variations of normal and frictional contact forces are plotted in Figure 5. The motion of the system is driven by the horizontal force from friction, which causes the system to accelerate or decelerate. When the ring is in contact with the surface, the vertical contact force would be positive. When contact is lost, the normal contact force becomes zero. You can also see that as the initial velocity increases, the minimum contact force decreases.

Another condition for pure rolling is that the friction force cannot exceed the normal force. The ratio of friction force to normal force, also called friction force utilization factor, is plotted in Figure 6. The figure shows that for $v_0 = 0.1$ m/s, only 38 % of the available friction is utilized in the pure rolling case. This can also be interpreted as the minimum friction coefficient needed to maintain pure rolling. As the initial velocity increases, the friction force utilization factor also increases reaching a maximum value of 1, thus utilizing a larger fraction of the available friction force.

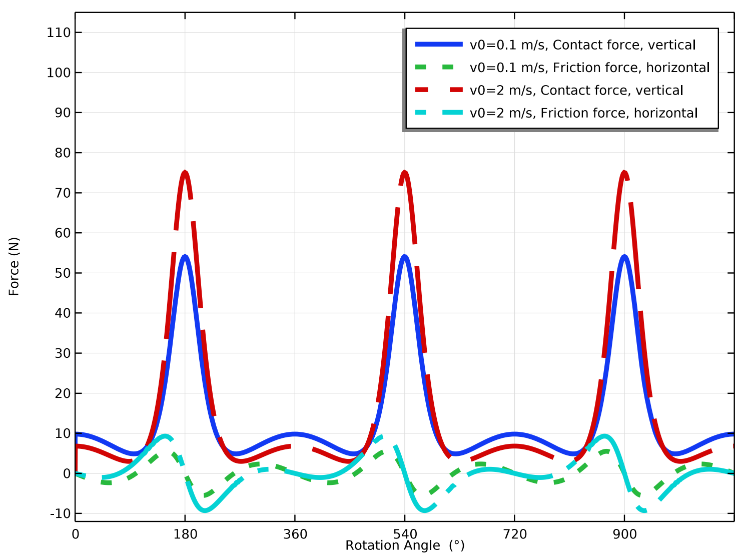

Figure 5: Contact and friction forces as a function of rotation angle for $v_0 = 0.1$ *m/s and* 2 *m/s.*

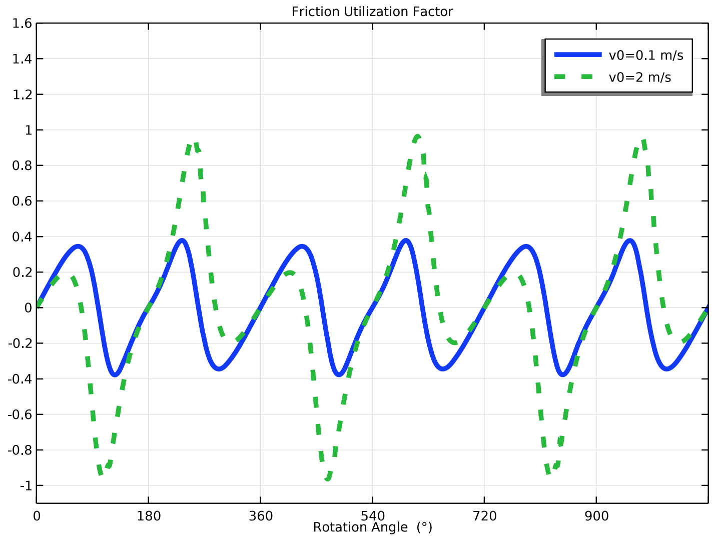

Figure 6: Friction utilization factor as a function of rotation angle for $v_0 = 0.1$ *m/s and* 2 *m/s.*

ROLLING WITH SLIPPING

As the initial velocity is increased further, the ring starts to slip. The translational and angular velocities for $v_0 = 2.8$ m/s are plotted in Figure 7. As shown in figure, the translational and angular velocities are no longer identical for this case. This is also evident from the friction utilization factor shown in Figure 10, which levels out at 1. As shown in Figure 7, the peak velocity is decreasing with each cycle. This system could reach a pure rolling state, when enough energy has dissipated. The system energy as a function of rotation angle is plotted in Figure 8 (note that the total energy in this plot is defined as the sum of potential and kinetic energies).

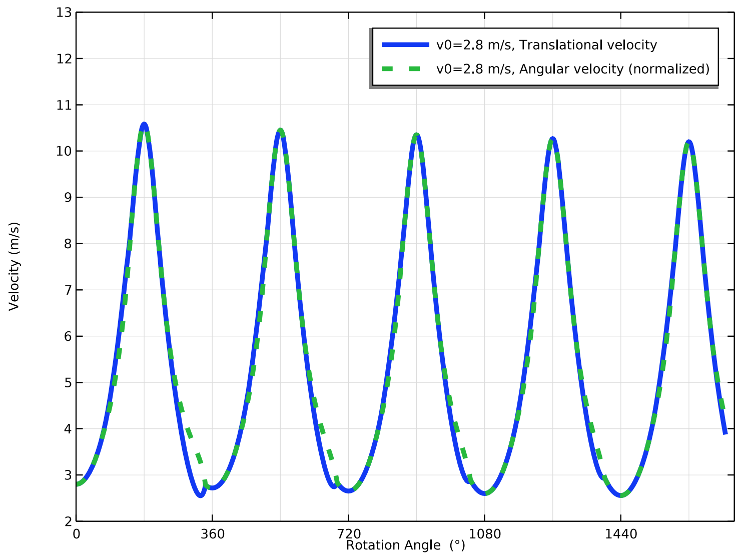

Figure 7: Velocity as a function of rotation angle for $v_0 = 2.8$ *m/s.*

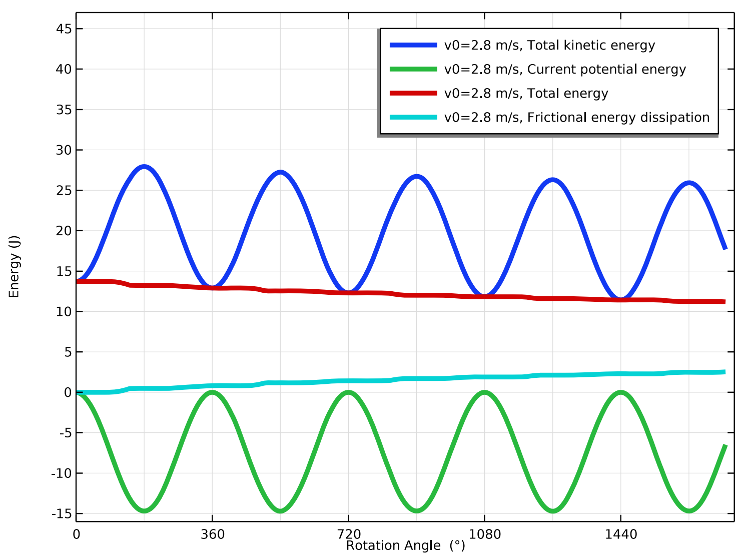

Figure 8: Energy as a function of rotation angle for $v_0 = 2.8$ *m/s.*

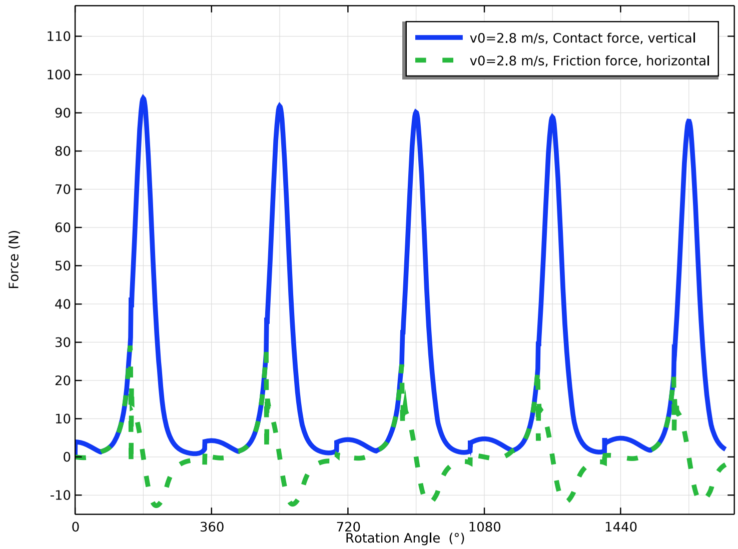

Figure 9: Contact and friction forces as a function of rotation angle for $v_0 = 2.8$ m/s.

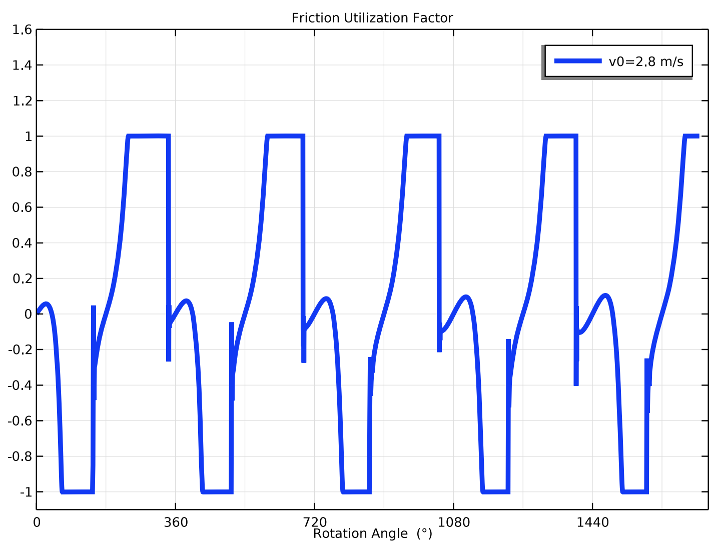

Figure 10: Friction utilization factor as a function of rotation angle for $v_0 = 2.8$ m/s.

ROLLING WITH HOPPING

For the initial velocity of $v_0 = 3.1$ m/s, the ring hops just before completing one revolution. The plot of translational and angular velocity (Figure 11) shows that the angular velocity is constant when the hoop is airborne. The system energy as a function of rotation angle is plotted in Figure 12. Between 350° and 390°, the potential energy of the system is larger than zero, causing the kinetic energy to be larger than the total energy. This is one consequence of the upward motion of the ring's center.

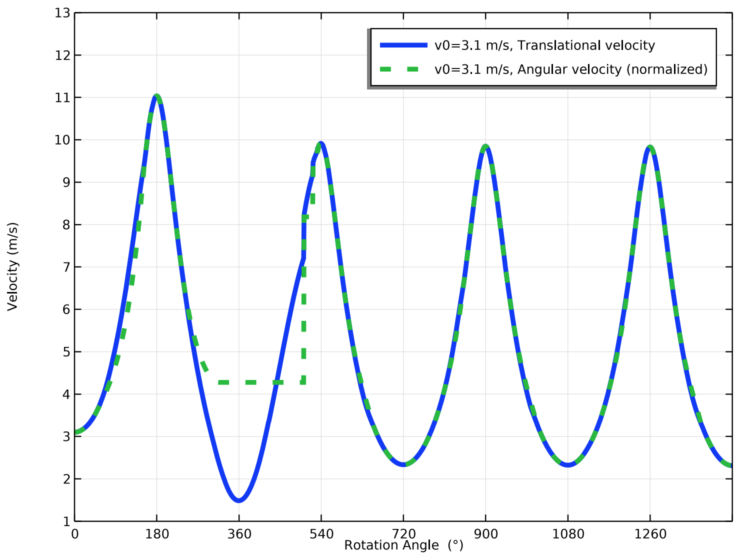

Figure 11: Velocity as a function of rotation angle for $v_0 = 3.1$ m/s.

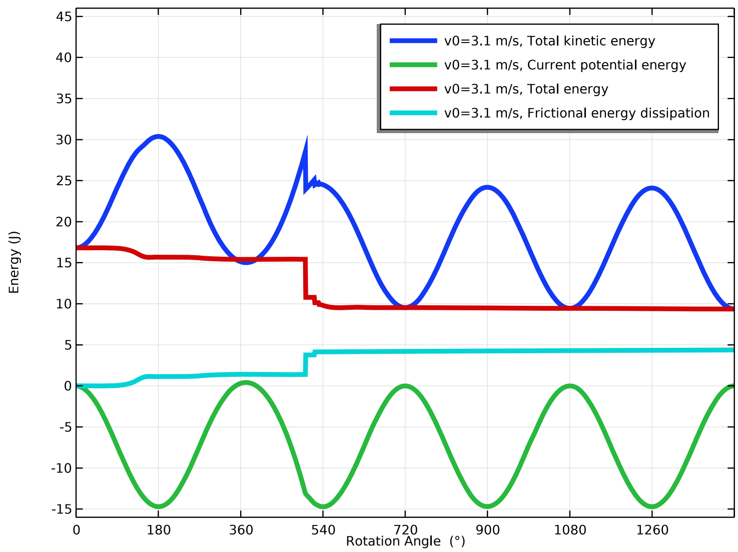

Figure 12: Energy as a function of rotation angle for $v_0 = 3.1$ *m/s.*

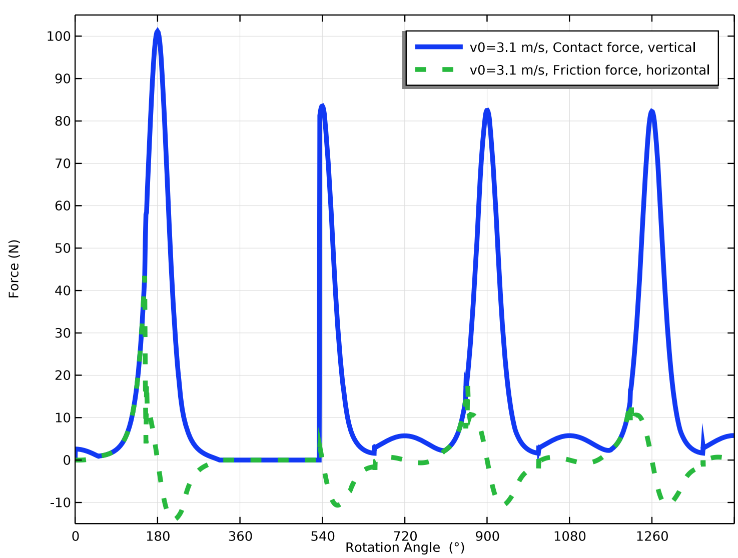

Figure 13: Contact and friction forces as a function of rotation angle for $v_0 = 3.1$ *m/s.*

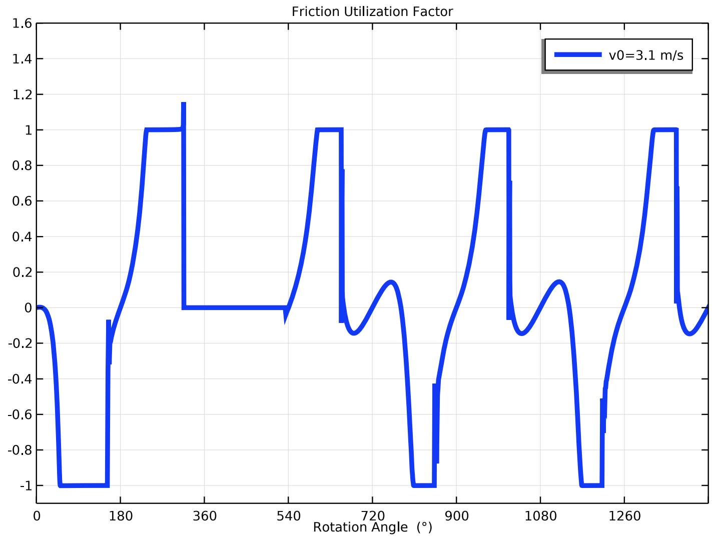

Figure 14: Friction utilization factor as a function of rotation angle for $v_0 = 3.1$ *m/s.*

Reference

1. J.E. Littlewood, *A Mathematician's Miscellany*, MR 087285, Methuen, London, 1953.

Application Library path: `Multibody_Dynamics_Module/Verification_Examples/hopping_hoop`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.

2 In the **Select Physics** tree, select **Structural Mechanics>Multibody Dynamics (mbd)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Time Dependent**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 Click **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file `hopping_hoop_parameters.txt`.

GEOMETRY 1

Circle 1 (c1)

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `Rc`.

4 Click to expand the **Layers** section. In the table, enter the following settings:

Layer name	Thickness (m)
Layer 1	0.1*Rc

5 Click **Build Selected**.

Delete Entities 1 (del1)

1 In the **Model Builder** window, right-click **Geometry 1** and choose **Delete Entities**.

2 In the **Settings** window for **Delete Entities**, locate the **Entities or Objects to Delete** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 On the object **c1**, select Domain 5 only.

5 Click **Build Selected**.

Rectangle 1 (r1)

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Width** text field, type `Lbase`.

4 In the **Height** text field, type `0.1`.

5 Locate the **Position** section. In the **x** text field, type `-0.1`.

6 In the **y** text field, type `-1.1`.

7 Click **Build Selected**.

Form Union (fin)

1 In the **Model Builder** window, under **Component 1 (comp1)>Geometry 1** click **Form Union (fin)**.

2 In the **Settings** window for **Form Union/Assembly**, locate the **Form Union/Assembly** section.

3 From the **Action** list, choose **Form an assembly**.

4 Clear the **Create pairs** check box.

5 Click **Build Selected**.

DEFINITIONS

Point Mass

1 In the **Definitions** toolbar, click **Explicit**.

2 In the **Settings** window for **Explicit**, type `Point Mass` in the **Label** text field.

3 Locate the **Input Entities** section. From the **Geometric entity level** list, choose **Point**.

4 Select Point 6 only.

Ring Edges

1 In the **Definitions** toolbar, click **Explicit**.

2 In the **Settings** window for **Explicit**, type `Ring Edges` in the **Label** text field.

3 Locate the **Input Entities** section. From the **Geometric entity level** list, choose **Boundary**.

4 Select Boundary 5 only.

5 Select the **Group by continuous tangent** check box.

Average: Point Mass

1 In the **Definitions** toolbar, click **Nonlocal Couplings** and choose **Average**.

2 In the **Settings** window for **Average**, type `Average: Point Mass` in the **Label** text field.

3 Locate the **Source Selection** section. From the **Geometric entity level** list, choose **Point**.

4 From the **Selection** list, choose **Point Mass**.

5 Right-click **Average: Point Mass** and choose **Duplicate**.

Average: Ring Edges

1 In the **Model Builder** window, under **Component 1 (comp1)>Definitions** click **Average: Point Mass 1 (aveop2)**.

2 In the **Settings** window for **Average**, type `Average: Ring Edges` in the **Label** text field.

3 Locate the **Source Selection** section. From the **Geometric entity level** list, choose **Boundary**.

4 From the **Selection** list, choose **Ring Edges**.

Variables 1

1 In the **Model Builder** window, right-click **Definitions** and choose **Variables**.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 In the table, enter the following settings:

Name	Expression	Unit	Description
Wg	g_const*mtot*((aveop1(y)-Rc)*gamma+aveop2(y)*(1-gamma))	J	Current potential energy

MULTIBODY DYNAMICS (MBD)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Multibody Dynamics (mbd)**.

2 In the **Settings** window for **Multibody Dynamics**, click to expand the **Initial Values** section.

3 Specify the d**u**/dt vector as

4 In the table, enter the following settings:

v0	X

5 In the dϕ/dt text field, type `-omega0`.

Rigid Material: Plane

1 In the **Physics** toolbar, click **Domains** and choose **Rigid Material**.

2 In the **Settings** window for **Rigid Material**, type `Rigid Material: Plane` in the **Label** text field.

3 Select Domain 5 only.

4 Locate the **Density** section. From the ρ list, choose **User defined**.

Fixed Constraint 1

In the **Physics** toolbar, click **Attributes** and choose **Fixed Constraint**.

Rigid Material: Ring

1 In the **Physics** toolbar, click **Domains** and choose **Rigid Material**.

2 In the **Settings** window for **Rigid Material**, type `Rigid Material: Ring` in the **Label** text field.

3 Select Domains 1–4 only.

4 Locate the **Density** section. From the ρ list, choose **User defined**.

Mass and Moment of Inertia 1

1 In the **Physics** toolbar, click **Attributes** and choose **Mass and Moment of Inertia**.

2 In the **Settings** window for **Mass and Moment of Inertia**, locate the **Center of Mass** section.

3 From the list, choose **Centroid of selected entities**.

4 From the **Entity level** list, choose **Point**.

5 Locate the **Mass and Moment of Inertia** section. In the m text field, type `mtot*gamma`.

Center of Mass: Point 1

1 In the **Model Builder** window, click **Center of Mass: Point 1**.

2 In the **Settings** window for **Center of Mass: Point**, locate the **Point Selection** section.

3 From the **Selection** list, choose **Point Mass**.

Added Mass: Ring

1 In the **Physics** toolbar, click **Boundaries** and choose **Added Mass**.

2 In the **Settings** window for **Added Mass**, type `Added Mass: Ring` in the **Label** text field.

3 Locate the **Boundary Selection** section. From the **Selection** list, choose **Ring Edges**.

4 Locate the **Added Mass** section. From the **Mass type** list, choose **Total mass**.

5 In the **m** text field, type `mtot*(1-gamma)`.

Gravity 1

In the **Physics** toolbar, click **Global** and choose **Gravity**.

Rigid Body Contact 1

1 In the **Physics** toolbar, click **Global** and choose **Rigid Body Contact**.

2 In the **Settings** window for **Rigid Body Contact**, locate the **Source** section.

3 From the **Source** list, choose **Rigid Material: Ring**.

4 Locate the **Destination** section. From the **Shape** list, choose **Planar**.

5 Locate the **Boundary Selection, Destination** section. Click to select the **Activate Selection** toggle button.

6 Select Boundary 15 only.

7 Locate the **Contact Settings** section. In the f_p text field, type `0.001`.

8 In the τ_n text field, type `0.05[ms]`.

9 Select the **Compute viscous contact dissipation** check box.

Friction 1

1 In the **Physics** toolbar, click **Attributes** and choose **Friction**.

2 In the **Settings** window for **Friction**, locate the **Friction** section.

3 In the μ text field, type `mu`.

4 In the v_0 text field, type `1e-4`.

5 Click to expand the **Advanced** section. Select the **Compute frictional dissipation** check box.

MESH 1

Edge 1

1 In the **Mesh** toolbar, click **More Generators** and choose **Edge**.

2 In the **Settings** window for **Edge**, locate the **Boundary Selection** section.

3 From the **Selection** list, choose **Ring Edges**.

Distribution 1

1 Right-click **Edge 1** and choose **Distribution**.

2 In the **Settings** window for **Distribution**, locate the **Distribution** section.

3 In the **Number of elements** text field, type `10`.

Mapped 1

In the **Mesh** toolbar, click **Mapped**.

Distribution 1

1 Right-click **Mapped 1** and choose **Distribution**.

2 Select Boundary 15 only.

3 In the **Settings** window for **Distribution**, locate the **Distribution** section.

4 In the **Number of elements** text field, type `1`.

5 Click **Build All**.

STUDY 1

Parametric Sweep

1 In the **Study** toolbar, click **Parametric Sweep**.

2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

3 Click **Add**.

4 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
v0 (Initial velocity)	`0.1 1 2 2.8 3.1 3.5`	`m/s`

Step 1: Time Dependent

1 In the **Model Builder** window, click **Step 1: Time Dependent**.

2 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.

3 In the **Output times** text field, type `range(0,dt,t_end)`.

4 From the **Tolerance** list, choose **User controlled**.

5 In the **Relative tolerance** text field, type `0.0001`.

Solution 1 (sol1)

1 In the **Study** toolbar, click **Show Default Solver**.

2 In the **Model Builder** window, expand the **Solution 1 (sol1)** node, then click **Time-Dependent Solver 1**.

3 In the **Settings** window for **Time-Dependent Solver**, click to expand the **Time Stepping** section.

4 From the **Maximum BDF order** list, choose **3**.

5 Find the **Algebraic variable settings** subsection. From the **Consistent initialization** list, choose **Off**.

6 In the **Study** toolbar, click **Compute**.

Follow the instructions below to plot the velocity of the ring and the path of the point mass and center of gravity as shown in Figure 2.

RESULTS

Velocity (mbd)

1 In the **Model Builder** window, under **Results** click **Velocity (mbd)**.

2 In the **Settings** window for **2D Plot Group**, click to expand the **Title** section.

3 From the **Title type** list, choose **None**.

4 Locate the **Color Legend** section. Clear the **Show legends** check box.

Surface

1 In the **Model Builder** window, expand the **Velocity (mbd)** node, then click **Surface**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `mbd.vel`.

4 Locate the **Coloring and Style** section. Click **Change Color Table**.

5 In the **Color Table** dialog box, select **Rainbow>SpectrumLight** in the tree.

6 Click **OK**.

Arrow Line

In the **Model Builder** window, right-click **Arrow Line** and choose **Delete**.

Velocity (mbd)

In the **Model Builder** window, under **Results** click **Velocity (mbd)**.

Point Trajectories: Point Mass

1 In the **Velocity (mbd)** toolbar, click **More Plots** and choose **Point Trajectories**.

2 In the **Settings** window for **Point Trajectories**, type `Point Trajectories: Point Mass` in the **Label** text field.

3 Locate the **Selection** section. From the **Selection** list, choose **Point Mass**.

4 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Type** list, choose **Tube**.

5 Select the **Radius scale factor** check box. In the associated text field, type `0.03`.

6 Find the **Point style** subsection. From the **Type** list, choose **Point**.

7 In the **Point radius expression** text field, type `0.15`.

8 Select the **Radius scale factor** check box.

9 From the **Color** list, choose **Magenta**.

10 Right-click **Point Trajectories: Point Mass** and choose **Duplicate**.

Point Trajectories: Center of Gravity

1 In the **Model Builder** window, under **Results>Velocity (mbd)** click **Point Trajectories: Point Mass 1**.

2 In the **Settings** window for **Point Trajectories**, type `Point Trajectories: Center of Gravity` in the **Label** text field.

3 Locate the **Trajectory Data** section. From the **Plot data** list, choose **Global**.

4 In the **X-expression** text field, type `aveop1(x)*gamma+aveop2(x)*(1-gamma)`.

5 In the **Y-expression** text field, type `aveop1(y)*gamma+aveop2(y)*(1-gamma)`.

6 Locate the **Coloring and Style** section. Find the **Point style** subsection. From the **Type** list, choose **None**.

Color Expression 1

1 Right-click **Point Trajectories: Center of Gravity** and choose **Color Expression**.

2 In the **Settings** window for **Color Expression**, locate the **Expression** section.

3 In the **Expression** text field, type `aveop2(y)>1e-3`.

4 Locate the **Coloring and Style** section. Click **Change Color Table**.

5 In the **Color Table** dialog box, select **Traffic>TrafficLight** in the tree.

6 Click **OK**.

Point Trajectories: Point Mass

In the **Model Builder** window, under **Results>Velocity (mbd)** right-click **Point Trajectories: Point Mass** and choose **Duplicate**.

Point Trajectories: Contact Force

1 In the **Model Builder** window, under **Results>Velocity (mbd)** click **Point Trajectories: Point Mass 1**.

2 In the **Settings** window for **Point Trajectories**, type `Point Trajectories: Contact Force` in the **Label** text field.

3 Locate the **Trajectory Data** section. From the **Plot data** list, choose **Global**.

4 In the **X-expression** text field, type `aveop2(x)`.

5 In the **Y-expression** text field, type `aveop2(y)-Rc`.

6 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Type** list, choose **None**.

7 Find the **Point style** subsection. From the **Type** list, choose **Arrow**.

8 In the **Arrow, Y-component** text field, type `-mbd.rbc1.Fny`.

9 Select the **Scale factor** check box. In the associated text field, type `0.08`.

10 From the **Color** list, choose **Red**.

11 Right-click **Point Trajectories: Contact Force** and choose **Duplicate**.

Point Trajectories: Friction Force

1 In the **Model Builder** window, under **Results>Velocity (mbd)** click **Point Trajectories: Contact Force 1**.

2 In the **Settings** window for **Point Trajectories**, type `Point Trajectories: Friction Force` in the **Label** text field.

3 Locate the **Coloring and Style** section. Find the **Point style** subsection. In the **Arrow, X-component** text field, type `mbd.rbc1.Ffx`.

4 In the **Arrow, Y-component** text field, type `0`.

5 Click to expand the **Inherit Style** section. From the **Plot** list, choose **Point Trajectories: Contact Force**.

Similarly loop through the different initial velocities to reproduce the plots in Figure 2.

Follow the instructions below to see the dynamics of the system in 3D.

Extrusion 2D 1

1 In the **Results** toolbar, click **More Datasets** and choose **Extrusion 2D**.

2 In the **Settings** window for **Extrusion 2D**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1/Parametric Solutions 1 (sol2)**.

4 Locate the **Extrusion** section. Find the **Embedding** subsection. From the **Map plane to** list, choose **xz-plane**.

Velocity (3D)

1 In the **Results** toolbar, click **3D Plot Group**.

2 In the **Settings** window for **3D Plot Group**, type `Velocity (3D)` in the **Label** text field.

3 Locate the **Data** section. From the **Parameter value (v0 (m/s))** list, choose **3.1**.

4 Locate the **Plot Settings** section. From the **Frame** list, choose **Spatial (x, y, z)**.

5 Locate the **Color Legend** section. Clear the **Show legends** check box.

Volume 1

1 Right-click **Velocity (3D)** and choose **Volume**.

2 In the **Settings** window for **Volume**, locate the **Expression** section.

3 In the **Expression** text field, type `mbd.vel`.

4 Locate the **Coloring and Style** section. Click **Change Color Table**.

5 In the **Color Table** dialog box, select **Rainbow>SpectrumLight** in the tree.

6 Click **OK**.

Deformation 1

1 Right-click **Volume 1** and choose **Deformation**.

2 In the **Settings** window for **Deformation**, locate the **Expression** section.

3 In the **y-component** text field, type `0`.

4 In the **z-component** text field, type `v`.

5 Locate the **Scale** section.

6 Select the **Scale factor** check box. In the associated text field, type `1`.

Velocity (3D)

In the **Model Builder** window, under **Results** click **Velocity (3D)**.

Point Trajectories 1

1 In the **Velocity (3D)** toolbar, click **More Plots** and choose **Point Trajectories**.

2 In the **Settings** window for **Point Trajectories**, locate the **Trajectory Data** section.

3 From the **Plot data** list, choose **Global**.

4 In the **x-expression** text field, type `aveop1(x)`.

5 In the **z-expression** text field, type `aveop1(y)`.

6 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Type** list, choose **Tube**.

Color Expression 1

1 Right-click **Point Trajectories 1** and choose **Color Expression**.

2 In the **Settings** window for **Color Expression**, locate the **Expression** section.

3 In the **Expression** text field, type `aveop2(y)>1e-3`.

4 Locate the **Coloring and Style** section. Click **Change Color Table**.

5 In the **Color Table** dialog box, select **Traffic>TrafficLight** in the tree.

6 Click **OK**.

Velocity (3D)

1 In the **Model Builder** window, under **Results** click **Velocity (3D)**.

2 In the **Settings** window for **3D Plot Group**, locate the **Plot Settings** section.

3 From the **View** list, choose **New view**.

4 In the **Velocity (3D)** toolbar, click **Plot**.

Follow the instructions below to plot translational and normalized angular velocities as a function of rotation angle for $v_0 = 0.1$ m/s as shown in Figure 3.

Velocity vs. Rotation Angle

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Velocity vs. Rotation Angle` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Study 1/ Parametric Solutions 1 (sol2)**.

4 Click to expand the **Title** section. From the **Title type** list, choose **None**.

5 Locate the **Data** section. From the **Parameter selection (v0)** list, choose **From list**.

6 In the **Parameter values (v0 (m/s))** list, choose **0.1** and **2**.

7 Locate the **Plot Settings** section.

8 Select the **x-axis label** check box. In the associated text field, type `Rotation Angle (°)`.

9 Select the **y-axis label** check box. In the associated text field, type `Velocity (m/s)`.

10 Locate the **Axis** section. Select the **Manual axis limits** check box.

11 In the **x minimum** text field, type `0`.

12 In the **x maximum** text field, type `1080`.

13 In the **y minimum** text field, type `-0.1`.

14 In the **y maximum** text field, type `14`.

15 Locate the **Grid** section. Select the **Manual spacing** check box.

16 In the **x spacing** text field, type `180`.

17 Locate the **Legend** section. In the **Maximum relative width** text field, type `1`.

Global 1

1 Right-click **Velocity vs. Rotation Angle** and choose **Global**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
`aveop2(mbd.u_tX)`	`m/s`	`Translational velocity`
`-mbd.rd2.th_tz*Rc`	`m/s`	`Angular velocity (normalized)`

4 Locate the **x-Axis Data** section. From the **Axis source data** list, choose **Inner solutions**.

5 From the **Parameter** list, choose **Expression**.

6 In the **Expression** text field, type `-mbd.rd2.thz`.

7 From the **Unit** list, choose **°**.

8 Click to expand the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Cycle**.

9 From the **Width** list, choose **4**.

10 In the **Velocity vs. Rotation Angle** toolbar, click **Plot**.

Follow the instructions below to plot translational and normalized angular velocities as a function of time.

Velocity vs. Rotation Angle

In the **Model Builder** window, right-click **Velocity vs. Rotation Angle** and choose **Duplicate**.

Velocity vs. Time

1. In the **Model Builder** window, under **Results** click **Velocity vs. Rotation Angle 1**.
2. In the **Settings** window for **1D Plot Group**, type `Velocity vs. Time` in the **Label** text field.
3. Locate the **Plot Settings** section. In the **x-axis label** text field, type `Time (s)`.
4. Locate the **Data** section. In the **Parameter values (v0 (m/s))** list, select **3.1**.
5. Locate the **Axis** section. Clear the **Manual axis limits** check box.
6. Locate the **Grid** section. Clear the **Manual spacing** check box.

Global 1

1. In the **Model Builder** window, expand the **Velocity vs. Time** node, then click **Global 1**.
2. In the **Settings** window for **Global**, locate the **x-Axis Data** section.
3. From the **Parameter** list, choose **Time**.
4. In the **Velocity vs. Time** toolbar, click **Plot**.

Follow the instructions below to plot energy as a function of rotation angle for v_0 = 0.1 m/s as shown in Figure 4.

Velocity vs. Rotation Angle

In the **Model Builder** window, under **Results** right-click **Velocity vs. Rotation Angle** and choose **Duplicate**.

Energy vs. Rotation Angle

1. In the **Model Builder** window, under **Results** click **Velocity vs. Rotation Angle 1**.
2. In the **Settings** window for **1D Plot Group**, type `Energy vs. Rotation Angle` in the **Label** text field.
3. Locate the **Data** section. In the **Parameter values (v0 (m/s))** list, select **0.1**.
4. Locate the **Plot Settings** section. In the **y-axis label** text field, type `Energy (J)`.
5. Locate the **Grid** section. In the **y spacing** text field, type `5`.
6. Locate the **Axis** section. In the **y minimum** text field, type `-16`.

7 In the **y maximum** text field, type 30.

Global 1

1 In the **Model Builder** window, expand the **Energy vs. Rotation Angle** node, then click **Global 1**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
mbd.Wk_tot	J	Total kinetic energy
Wg	J	Current potential energy
mbd.Wk_tot+Wg	J	Total energy

4 Click **Add Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)>Multibody Dynamics>Rigid body contacts> Rigid Body Contact 1>Friction>mbd.rbc1.Wf - Frictional energy dissipation - J**.

5 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Solid**.

6 In the **Energy vs. Rotation Angle** toolbar, click **Plot**.

Follow the instructions below to plot energy as a function of time.

Energy vs. Rotation Angle

In the **Model Builder** window, right-click **Energy vs. Rotation Angle** and choose **Duplicate**.

Energy vs. Time

1 In the **Model Builder** window, under **Results** click **Energy vs. Rotation Angle 1**.

2 In the **Settings** window for **1D Plot Group**, type Energy vs. Time in the **Label** text field.

3 Locate the **Data** section. In the **Parameter values (v0 (m/s))** list, select **3.1**.

4 Locate the **Plot Settings** section. In the **x-axis label** text field, type Time (s).

5 Locate the **Axis** section. Clear the **Manual axis limits** check box.

6 Locate the **Grid** section. Clear the **Manual spacing** check box.

Global 1

1 In the **Model Builder** window, expand the **Energy vs. Time** node, then click **Global 1**.

2 In the **Settings** window for **Global**, locate the **x-Axis Data** section.

3 From the **Parameter** list, choose **Time**.

4 In the **Energy vs. Time** toolbar, click **Plot**.

Follow the instructions below to plot contact and friction forces for $v_0 = 0.1$ m/s as shown in Figure 5.

Velocity vs. Rotation Angle

In the **Model Builder** window, under **Results** right-click **Velocity vs. Rotation Angle** and choose **Duplicate**.

Contact Forces

1 In the **Model Builder** window, under **Results** click **Velocity vs. Rotation Angle 1**.

2 In the **Settings** window for **1D Plot Group**, type `Contact Forces` in the **Label** text field.

3 Locate the **Plot Settings** section. In the **y-axis label** text field, type `Force (N)`.

4 Locate the **Grid** section. In the **y spacing** text field, type `10`.

5 Locate the **Axis** section. In the **y maximum** text field, type `115`.

6 In the **y minimum** text field, type `-12`.

Global 1

1 In the **Model Builder** window, expand the **Contact Forces** node, then click **Global 1**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 Click **Clear Table**.

4 In the table, enter the following settings:

Expression	Unit	Description
`-mbd.rbc1.Fny`	N	`Contact force, vertical`
`mbd.rbc1.Ffx`	N	`Friction force, horizontal`

5 In the **Contact Forces** toolbar, click **Plot**.

Follow the instructions below to plot friction utilization factor for $v_0 = 0.1$ m/s as shown in Figure 6.

Contact Forces

In the **Model Builder** window, right-click **Contact Forces** and choose **Duplicate**.

Friction Utilization Factor

1 In the **Model Builder** window, click **Contact Forces 1**.

2 In the **Settings** window for **1D Plot Group**, locate the **Title** section.

3 From the **Title type** list, choose **Label**.

4 In the **Label** text field, type `Friction Utilization Factor`.

5 Locate the **Plot Settings** section. Clear the **y-axis label** check box.

6 Locate the **Grid** section. In the **y spacing** text field, type `0.2`.

7 Locate the **Axis** section. In the **y maximum** text field, type `1.6`.

8 In the **y minimum** text field, type `-1.1`.

Global 1

1 In the **Model Builder** window, expand the **Friction Utilization Factor** node, then click **Global 1**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 Click **Clear Table**.

4 In the table, enter the following settings:

Expression	Unit	Description
`mbd.rbc1.Ffx/(mbd.rbc1.Fny+0.001)/ mbd.rbc1.fric1.mu`	1	

5 In the **Friction Utilization Factor** toolbar, click **Plot**.

Follow the instructions below to plot velocity as a function of rotation angle for $v_0 = 2.8$ m/s as shown in Figure 7.

Velocity vs. Rotation Angle

1 In the **Model Builder** window, under **Results** click **Velocity vs. Rotation Angle**.

2 In the **Settings** window for **1D Plot Group**, locate the **Data** section.

3 In the **Parameter values (v0 (m/s))** list, select **2.8**.

4 Locate the **Axis** section. In the **x maximum** text field, type `1740`.

5 In the **y maximum** text field, type `13`.

6 In the **y minimum** text field, type `2`.

7 In the **Velocity vs. Rotation Angle** toolbar, click **Plot**.

Follow the instructions below to plot energy as a function of rotation angle for $v_0 = 2.8$ m/s as shown in Figure 8.

Energy vs. Rotation Angle

1 In the **Model Builder** window, click **Energy vs. Rotation Angle**.

2 In the **Settings** window for **1D Plot Group**, locate the **Data** section.

3 In the **Parameter values (v0 (m/s))** list, select **2.8**.

4 Locate the **Axis** section. In the **x maximum** text field, type `1740`.

5 In the **y maximum** text field, type `47`.

6 In the **Energy vs. Rotation Angle** toolbar, click **Plot**.

Follow the instructions below to plot contact and friction forces for v_0 = 2.8 m/s as shown in Figure 9.

Contact Forces

1 In the **Model Builder** window, click **Contact Forces**.

2 In the **Settings** window for **1D Plot Group**, locate the **Data** section.

3 In the **Parameter values (v0 (m/s))** list, select **2.8**.

4 Locate the **Axis** section. In the **y maximum** text field, type 118.

5 In the **y minimum** text field, type -15.

6 In the **x maximum** text field, type 1740.

7 In the **Contact Forces** toolbar, click **Plot**.

Follow the instructions below to plot friction utilization factor for v_0 = 2.8 m/s as shown in Figure 10.

Friction Utilization Factor

1 In the **Model Builder** window, click **Friction Utilization Factor**.

2 In the **Settings** window for **1D Plot Group**, locate the **Data** section.

3 In the **Parameter values (v0 (m/s))** list, select **2.8**.

4 Locate the **Axis** section. In the **x maximum** text field, type 1740.

5 In the **Friction Utilization Factor** toolbar, click **Plot**.

Follow the instructions below to plot velocity as a function of rotation angle for v_0 = 3.1 m/s as shown in Figure 11.

Velocity vs. Rotation Angle

1 In the **Model Builder** window, click **Velocity vs. Rotation Angle**.

2 In the **Settings** window for **1D Plot Group**, locate the **Data** section.

3 In the **Parameter values (v0 (m/s))** list, select **3.1**.

4 Locate the **Axis** section. In the **x maximum** text field, type 1440.

5 In the **y minimum** text field, type 1.

6 In the **Velocity vs. Rotation Angle** toolbar, click **Plot**.

Follow the instructions below to plot energy as a function of rotation angle for v_0 = 2.8 m/s as shown in Figure 12.

Energy vs. Rotation Angle

1 In the **Model Builder** window, click **Energy vs. Rotation Angle**.

2 In the **Settings** window for **1D Plot Group**, locate the **Data** section.

3 In the **Parameter values (v0 (m/s))** list, select **3.1**.

4 Locate the **Axis** section. In the **x maximum** text field, type `1440`.

5 In the **y maximum** text field, type `46`.

6 In the **Energy vs. Rotation Angle** toolbar, click **Plot**.

Follow the instructions below to plot contact and friction forces for v_0 = 3.1 m/s as shown in Figure 13.

Contact Forces

1 In the **Model Builder** window, click **Contact Forces**.

2 In the **Settings** window for **1D Plot Group**, locate the **Data** section.

3 In the **Parameter values (v0 (m/s))** list, select **3.1**.

4 Locate the **Axis** section. In the **x maximum** text field, type `1440`.

5 In the **y maximum** text field, type `105`.

6 In the **Contact Forces** toolbar, click **Plot**.

Follow the instructions below to plot friction utilization factor for v_0 = 3.1 m/s as shown in Figure 14.

Friction Utilization Factor

1 In the **Model Builder** window, click **Friction Utilization Factor**.

2 In the **Settings** window for **1D Plot Group**, locate the **Data** section.

3 In the **Parameter values (v0 (m/s))** list, select **3.1**.

4 Locate the **Axis** section. In the **x maximum** text field, type `1440`.

5 In the **Friction Utilization Factor** toolbar, click **Plot**.

Velocity (3D), Velocity (mbd), Velocity vs. Rotation Angle, Velocity vs. Time

1 In the **Model Builder** window, under **Results**, Ctrl-click to select **Velocity (mbd)**, **Velocity (3D)**, **Velocity vs. Rotation Angle**, and **Velocity vs. Time**.

2 Right-click and choose **Group**.

Velocity

In the **Settings** window for **Group**, type `Velocity` in the **Label** text field.

Energy vs. Rotation Angle, Energy vs. Time

1 In the **Model Builder** window, under **Results**, Ctrl-click to select **Energy vs. Rotation Angle** and **Energy vs. Time**.
2 Right-click and choose **Group**.

Energy

In the **Settings** window for **Group**, type `Energy` in the **Label** text field.

Contact Forces, Friction Utilization Factor

1 In the **Model Builder** window, under **Results**, Ctrl-click to select **Contact Forces** and **Friction Utilization Factor**.
2 Right-click and choose **Group**.

Contact Forces

In the **Settings** window for **Group**, type `Contact Forces` in the **Label** text field.

Velocity

1 In the **Results** toolbar, click **Animation** and choose **Player**.
2 In the **Settings** window for **Animation**, type `Velocity` in the **Label** text field.
3 Locate the **Scene** section. From the **Subject** list, choose **Velocity (mbd)**.
4 Locate the **Animation Editing** section. From the **Parameter value (v0 (m/s))** list, choose **3.1**.
5 Locate the **Frames** section. In the **Number of frames** text field, type `100`.
6 Right-click **Velocity** and choose **Duplicate**.

Velocity (3D)

1 In the **Model Builder** window, under **Results>Export** click **Velocity 1**.
2 In the **Settings** window for **Animation**, type `Velocity (3D)` in the **Label** text field.
3 Locate the **Scene** section. From the **Subject** list, choose **Velocity (3D)**.