Created in COMSOL Multiphysics 6.2

Implementing a Point Source

Introduction

Consider Poisson's equation on the unit circle with a point source at the origin. Its formal expression is:

$$\begin{cases} -\nabla \cdot (\nabla u) = \delta & \Omega \\ u = 0 & \partial\Omega \end{cases}$$

where δ is the Dirac δ distribution located at the origin. The exact solution to this boundary value problem is $-(1/2\pi)\log(r)$, which has a singularity at the origin. You can model the point source by adding a **Point Source** node to your COMSOL Multiphysics model.

Application Library path: `COMSOL_Multiphysics/Equation_Based/point_source`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.

2 In the **Select Physics** tree, select **Mathematics>Classical PDEs>Laplace's Equation (lpeq)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Stationary**.

6 Click **Done**.

GEOMETRY

Circle 1 (c1)

In the **Geometry** toolbar, click **Circle**.

Point 1 (pt1)

1 In the **Geometry** toolbar, click **Point**.

2 In the **Settings** window for **Point**, click **Build All Objects**.

LAPLACE'S EQUATION (LPEQ)

Dirichlet Boundary Condition 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Laplace's Equation (lpeq)** and choose **Dirichlet Boundary Condition**.

2 In the **Settings** window for **Dirichlet Boundary Condition**, locate the **Boundary Selection** section.

3 From the **Selection** list, choose **All boundaries**.

Point Source 1

1 In the **Physics** toolbar, click **Points** and choose **Point Source**.

2 Select Point 3 only.

3 In the **Settings** window for **Point Source**, locate the **Source Term** section.

4 In the *f* text field, type 1.

STUDY 1

Step 1: Stationary

1 In the **Model Builder** window, under **Study 1** click **Step 1: Stationary**.

2 In the **Settings** window for **Stationary**, click to expand the **Adaptation and Error Estimates** section.

3 From the **Adaptation and error estimates** list, choose **Adaptation and error estimates**.

4 In the **Home** toolbar, click **Compute**.

RESULTS

1 In the **Settings** window for **2D Plot Group**, locate the **Color Legend** section.

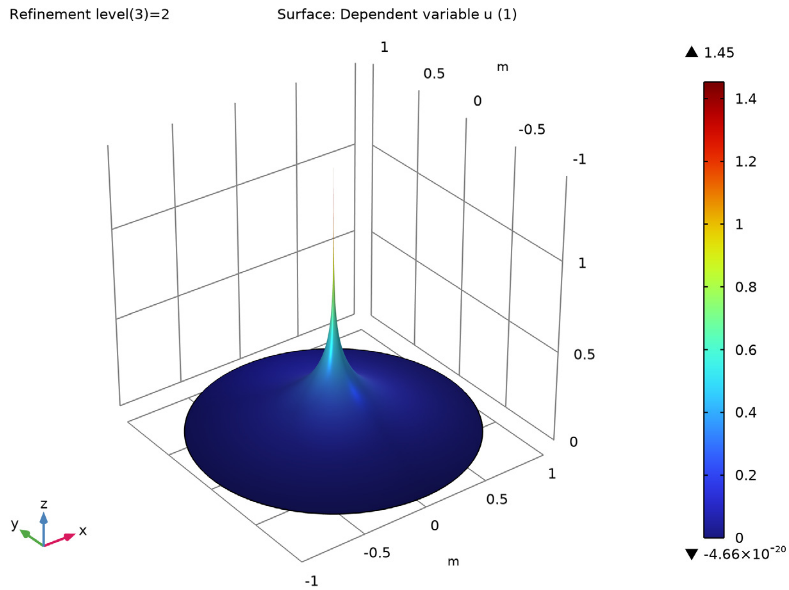
2 Select the **Show maximum and minimum values** check box.

Height Expression 1

1 In the **Model Builder** window, expand the **Laplace's Equation** node.

2 Right-click **Surface 1** and choose **Height Expression**.

The height plot appears directly.



Cut Line 2D 1

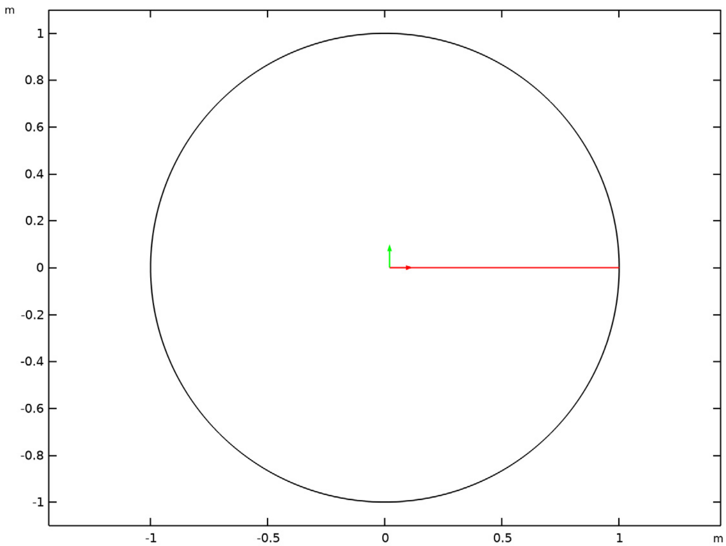
1 In the **Results** toolbar, click **Cut Line 2D**.

2 In the **Settings** window for **Cut Line 2D**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1/Adaptive Mesh Refinement Solutions 1 (sol2)**.

4 Locate the **Line Data** section. In row **Point 1**, set **X** to `0.02`.

5 Click **Plot**.



1D Plot Group 2

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, locate the **Data** section.

3 From the **Dataset** list, choose **Cut Line 2D 1**.

4 From the **Parameter selection (Refinement level)** list, choose **Last**.

Line Graph 1

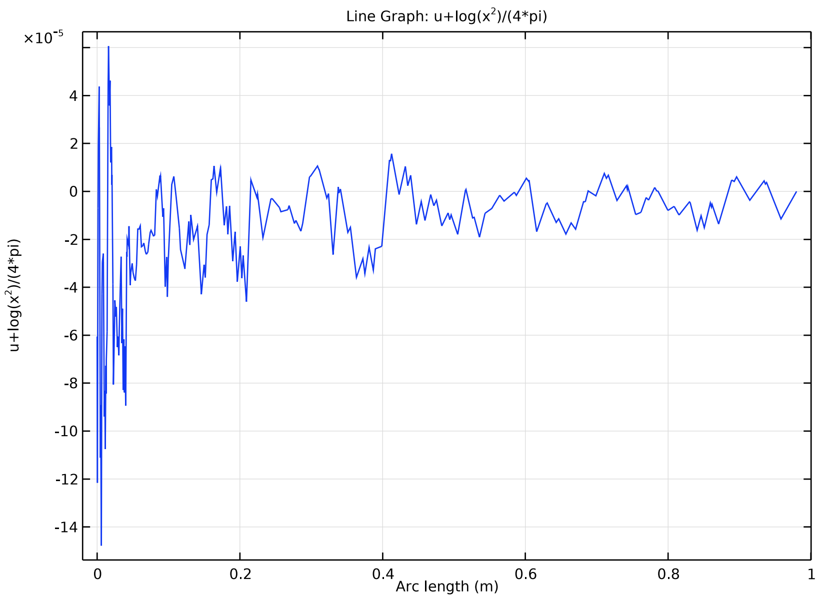
1 Right-click **1D Plot Group 2** and choose **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `u+log(x^2)/(4*pi)`.

4 In the **1D Plot Group 2** toolbar, click **Plot**.

The resulting plot shows the error in the solution.



Surface Integration 1

1 In the **Results** toolbar, click **More Derived Values** and choose **Integration>Surface Integration**.

2 In the **Settings** window for **Surface Integration**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1/Adaptive Mesh Refinement Solutions 1 (sol2)**.

4 From the **Parameter selection (Refinement level)** list, choose **Last**.

5 Select Domain 1 only.

6 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
abs(u+log(sqrt(x^2+y^2))/(2*pi))		

7 Click **Evaluate**.

TABLE 1

1 Go to the **Table 1** window.

The result of this integration shows that the error is small.