Created in COMSOL Multiphysics 6.2

Active Flame Validation

Introduction

In jet engines and gas turbines, an interaction between the heat release from the flame and the acoustic resonances in the engines can lead to unstable modes that can be damaging to the engine. This occurs because oscillations in the fuel supply results in oscillations in the heat release. The oscillations in the heat release can interfere positively or negatively with the acoustic resonances in the engine such that the oscillations are either dampened or amplified.

To model this effect, it is necessary to have a model of the heat release and how it depends on the acoustic field. This model demonstrates the usage of the domain feature **Flame Model** in the **Pressure Acoustics, Frequency Domain** interface with a simple model that can be compared to an analytical solution.

Model Definition

The model consists of a simple 2D geometry, a rectangle with length L, representing a pipe with one closed end and one open end. At the closed end and the two sides, a hard wall boundary condition is used, while at the open end the acoustic pressure is set to zero. In the middle of the rectangle, there is a small domain with an active compact flame. To the left of the flame, the temperature is 300 K and to the right the flame has heated up the air to 1200 K.

The active flame is modeled with the domain feature **Flame Model**, which adds a heat source in the domain that depends on the acoustic velocity at a reference point. The reference point is set to be on the left boundary of the **Flame Model** domain. The heat $q(x)$ released by the flame is given by

$$q(x) = \frac{n_u(x)q_\mathrm{s}}{U_\mathrm{s}}\exp(-i\omega\tau(x)) \tag{1}$$

where $\tau(x)$ is the time-lag, $n_u(x)$ is the interaction index, U_s is the steady velocity field, and q_s is the steady heat release. To ease the analytical modeling of the eigenfrequencies, define the interaction index $n_u(x)$ by the parameter n as

$$n_u(x) = \frac{nU_s}{q_s} \times \frac{\gamma p_0}{\gamma - 1} \tag{2}$$

where the specific heat capacity ratio γ and the background pressure p_0 are used. Both τ and n are chosen to be independent of the spatial coordinates. The eigenfrequencies can be found analytically as functions of τ and n (see Ref. 1):

$$0 = \cos\left(\frac{L\omega}{4c_1}\right)\left[\cos\left(2\frac{L\omega}{4c}\right)^2 - \frac{\Gamma(1+n\exp(i\omega\tau))-1}{4\Gamma(1+n\exp(i\omega\tau))+1} - \frac{3}{4}\right] \quad (3)$$

Here, Γ is given as $\Gamma = \rho_2 c_2/\rho_1 c_1$, where the subscript 1 refers to the material parameters to the left of the flame and subscript 2 to those to the right of the flame. The first cosine gives solutions where the active flame does not interfere with the eigenfrequency, while the second parenthesis gives the modes where the active flame impacts the eigenfrequency. The active flame will both shift the resonance frequency either up or down in frequency but it will also dampen or amplify the resonance mode. The damping or amplification is represented by the imaginary part of the eigenfrequency. With the used time convention of $\exp(i\omega t)$ a positive imaginary part represents damping of the mode while a mode with a negative imaginary part is amplified by the active flame. Note the time convention is opposite to the convention used in Ref. 1. A mode that is amplified by the active flame results in an unstable mode. Unstable modes in an engine can be damaging to the engine and they are therefore important to avoid during the design.

Results and Discussion

With an eigenfrequency study step, the first four eigenmodes of the system are found, corresponding to modes with 1/4, 3/4, 5/4, and 7/4 wavelengths in the pipe. The four modes are shown in Figure 1 with the closed end to the left and the open end to the right. Because of the difference in temperature on opposite sides of the active flame, the sound speed and therefore the wavelength is different on each side.

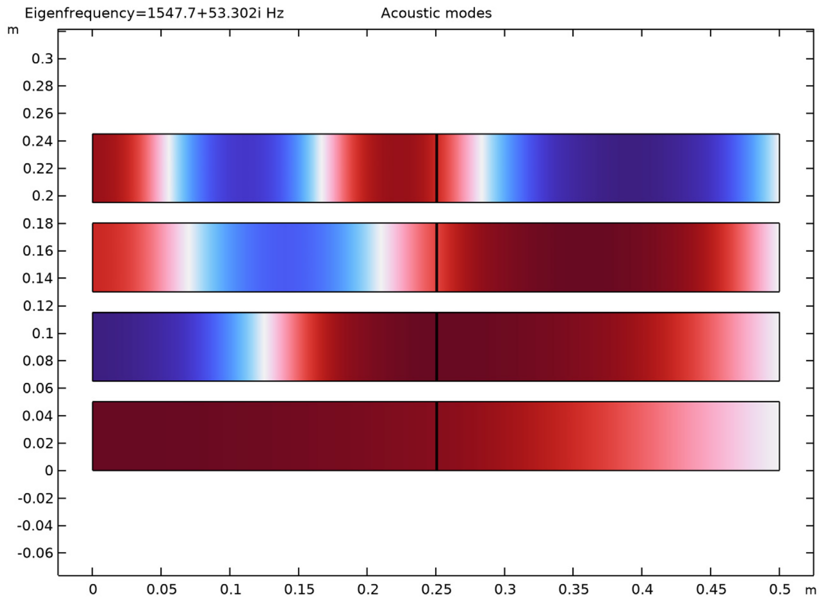


Figure 1: The four lowest eigenmodes of the pipe and active flame.

In Figure 2, the eigenfrequencies are plotted with the real part of the eigenfrequency on the x-axis and the imaginary part on the y-axis. The eigenfrequencies are plotted for both $n = 0$ (inactive flame) with blue plus signs and $n = 5$ (active flame) with green plus signs. The theoretical predictions are plotted with blue circles for both $n = 0$ and $n = 5$. It can be seen that all the eigenfrequencies for the inactive flame has zero imaginary part, this is because there is no damping in the system when the flame is inactive. For the second eigenfrequency (at around 700 Hz) the eigenfrequency of the active mode is identical to the eigenfrequency of the passive mode. This mode is not influenced by the active flame and is described by the first cosine in Equation 2.

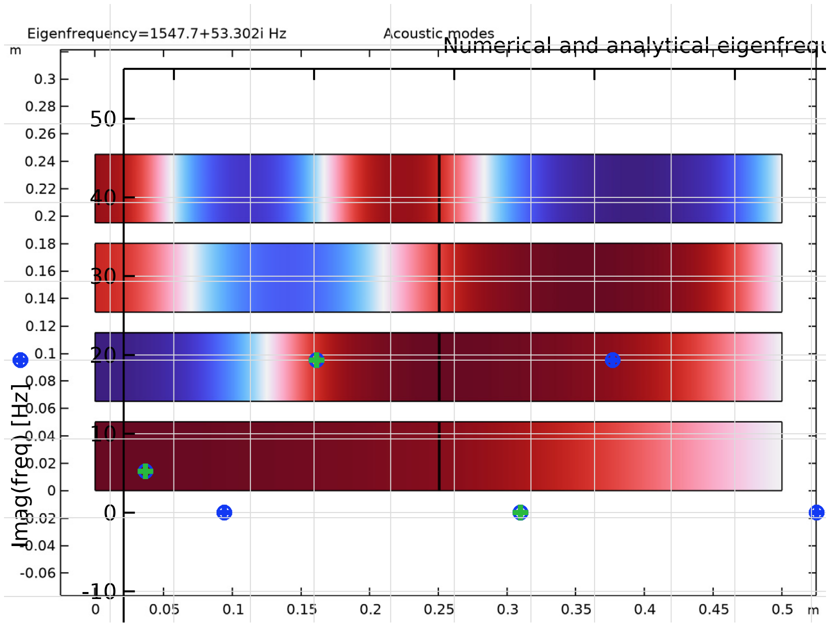


Figure 2: The real and imaginary part of the eigenfrequencies plotted for the lowest four eigenmodes. The analytical solution plotted with blue circles, the simulations with a passive flame with blue plus signs and the active flame with the green plus signs.

For the first and fourth modes, the imaginary part of the eigenfrequency is positive with the active flame. This means that the modes are dampened by the active flame. This interaction between the acoustic field and the heat released from the flame also reduces the real part of the eigenfrequency. For the third mode, the imaginary part of the eigenfrequency is negative and the mode is amplified by the active flame. The real part of the frequency is shifted upward due to the active flame.

Notes About the COMSOL Implementation

The contribution from the flame model depends nonlinearly on the frequency through the exponential function in Equation 1. Therefore, the system presents some challenges when solving an eigenfrequency study step. In the eigenfrequency study step the effect of the frequency is linearized around a linearization point, `freq_lin`. However, when the equations depend nonlinearly on the frequency, this is not sufficient; thus a parametric sweep over the linearization frequency is used. Thereby, different linearization points are used for the different eigenfrequencies, ensuring that the linearization points are located relatively close to the eigenfrequencies.

Reference

1. F. Nicoud, L. Benoit, C. Sensiau, and T. Poinsot, "Acoustic modes in combustors with complex impedances and multidimensional active flames," *AIAA J.*, vol. 45, no. 2, pp. 426–441, 2007.

Application Library path: `Acoustics_Module/Tutorials,_Pressure_Acoustics/active_flame_validation`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.

2 In the **Select Physics** tree, select **Acoustics>Pressure Acoustics>Pressure Acoustics, Frequency Domain (acpr)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Eigenfrequency**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 Click **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file `active_flame_validation_parameters.txt`.

In the parameter file the analytical eigenfrequencies are calculated. The are calculated iteratively due to the form of the differential equation determining the eigenfrequencies.

DEFINITIONS

In the **Model Builder** window, expand the **Component 1 (comp1)>Definitions** node.

GEOMETRY 1

Rectangle 1 (r1)

1 In the **Model Builder** window, expand the **Component 1 (comp1)>Geometry 1** node.

2 Right-click **Geometry 1** and choose **Rectangle**.

3 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

4 In the **Width** text field, type `L`.

5 In the **Height** text field, type `L/10`.

6 Click to expand the **Layers** section. In the table, enter the following settings:

Layer name	Thickness (m)
Layer 1	L/2
Layer 2	delta_f

7 Select the **Layers to the left** check box.

8 Clear the **Layers on bottom** check box.

9 Click **Build All Objects**.

ADD MATERIAL FROM LIBRARY

In the **Home** toolbar, click **Windows** and choose **Add Material from Library**.

ADD MATERIAL

1 Go to the **Add Material** window.

2 In the tree, select **Built-in>Air**.

3 Click **Add to Component** in the window toolbar.

4 In the **Home** toolbar, click **Add Material** to close the **Add Material** window.

DEFINITIONS

Variables 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Definitions** and choose **Variables**.
2 In the **Settings** window for **Variables**, locate the **Geometric Entity Selection** section.
3 From the **Geometric entity level** list, choose **Domain**.
4 Select Domain 1 only.
5 Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
T_s	300[K]	K	
n_u	n/delta_f*u_s/Q_s*(acpr.rho*acpr.flm1.Cp)/acpr.flm1.alpha_p		

Variables 2

1 In the **Model Builder** window, right-click **Definitions** and choose **Variables**.
2 In the **Settings** window for **Variables**, locate the **Geometric Entity Selection** section.
3 From the **Geometric entity level** list, choose **Domain**.
4 Select Domains 2 and 3 only.
5 Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
T_s	1200[K]	K	
n_u	n/delta_f*u_s/Q_s*(acpr.rho*acpr.flm1.Cp)/acpr.flm1.alpha_p		

PRESSURE ACOUSTICS, FREQUENCY DOMAIN (ACPR)

Pressure Acoustics 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Pressure Acoustics, Frequency Domain (acpr)** click **Pressure Acoustics 1**.
2 In the **Settings** window for **Pressure Acoustics**, locate the **Model Input** section.
3 In the T text field, type T_s.
4 In the p_A text field, type P_s.

Pressure 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Pressure**.

2 Select Boundary 10 only.

Flame Model 1

1 In the **Physics** toolbar, click **Domains** and choose **Flame Model**.

2 Select Domain 2 only.

3 In the **Settings** window for **Flame Model**, locate the **Flame Model** section.

4 In the n_u text field, type `n_u`.

5 In the τ_u text field, type `tau_u`.

6 In the Q_s text field, type `Q_s`.

7 In the U_s text field, type `u_s`.

8 Locate the **Acoustic Reference** section. From the **Acoustic reference** list, choose **Reference boundary**.

9 Locate the **Reference Boundary** section. Click to select the **Activate Selection** toggle button.

10 Select Boundary 4 only.

11 Locate the **Acoustic Reference** section. Select the **Reverse normal direction** check box.

MESH 1

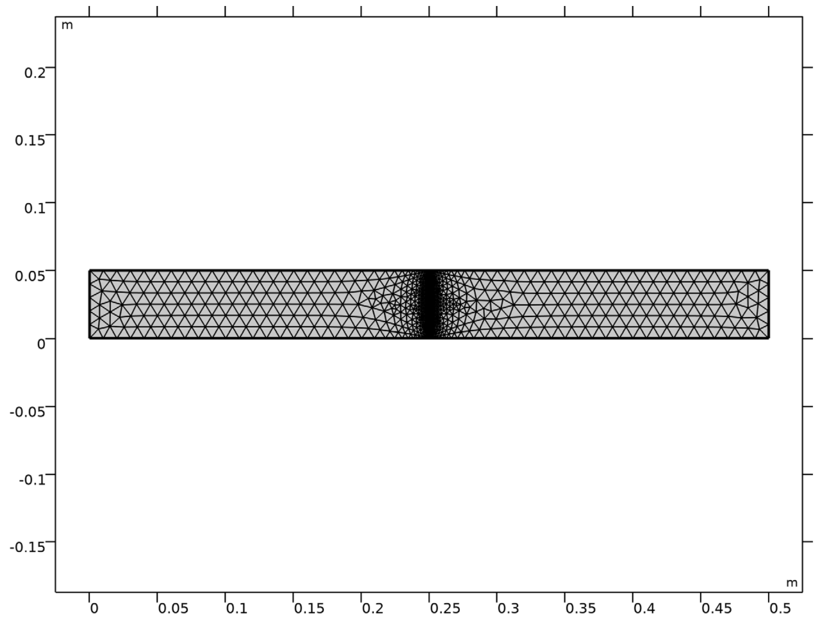
1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2 In the **Settings** window for **Mesh**, locate the **Physics-Controlled Mesh** section.

3 In the table, clear the **Use** check box for **Pressure Acoustics, Frequency Domain (acpr)**.

4 From the **Element size** list, choose **Extra fine**.

5 Click **Build All**.



STUDY 1

Step 1: Eigenfrequency

1 In the **Model Builder** window, under **Study 1** click **Step 1: Eigenfrequency**.

2 In the **Settings** window for **Eigenfrequency**, locate the **Study Settings** section.

3 Select the **Desired number of eigenfrequencies** check box. In the associated text field, type `1`.

4 From the **Search method around shift** list, choose **Larger real part**.

5 In the **Search for eigenfrequencies around shift** text field, type `freq_lin-100[Hz]`.

Solution 1 (sol1)

1 In the **Study** toolbar, click **Show Default Solver**.

2 In the **Model Builder** window, expand the **Solution 1 (sol1)** node, then click **Eigenvalue Solver 1**.

3 In the **Settings** window for **Eigenvalue Solver**, locate the **General** section.

4 Find the **Eigenvalue linearization point** subsection. In the **Value of eigenvalue linearization point** text field, type `freq_lin`.

Parametric Sweep

1 In the **Study** toolbar, click **Parametric Sweep**.

Add a parametric sweep over the linearization point.

2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

3 Click **Add**.

4 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
freq_lin (Linearization frequency for the solver)	`range(200,450,1550)`	Hz

Parametric Sweep 2

1 In the **Study** toolbar, click **Parametric Sweep**.

Solve for an inactive flame n=0 and active flame n=5.

2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

3 Click **Add**.

4 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
n (Scale parameter)	`{0,5}`	

5 In the **Study** toolbar, click **Compute**.

RESULTS

Numerical and analytical eigenfrequencies

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Numerical and analytical eigenfrequencies` in the **Label** text field.

3 Click to expand the **Title** section. From the **Title type** list, choose **Label**.

4 Locate the **Data** section. From the **Dataset** list, choose **Study 1/ Parametric Solutions 1 (sol2)**.

5 From the **Parameter selection (freq_lin)** list, choose **First**.

6 From the **Eigenfrequency selection** list, choose **First**.

7 Locate the **Plot Settings** section. Select the **x-axis label** check box.

8 Select the **y-axis label** check box.

9 In the **x-axis label** text field, type `Frequency [Hz]`.

10 In the **y-axis label** text field, type `Imag(freq) [Hz]`.

11 Locate the **Legend** section. Clear the **Show legends** check box.

Table Graph 1

1 Right-click **Numerical and analytical eigenfrequencies** and choose **Table Graph**.

2 In the **Settings** window for **Table Graph**, locate the **Data** section.

3 From the **Source** list, choose **Evaluation group**.

4 From the **x-axis data** list, choose **Eigenfrequency (Hz)**.

5 From the **Plot columns** list, choose **Manual**.

6 In the **Columns** list, select **Eigenfrequency (Hz)**.

7 Select the **Plot imaginary part** check box.

8 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

9 Find the **Line markers** subsection. From the **Marker** list, choose **Circle**.

Numerical and analytical eigenfrequencies

Right-click **Table Graph 1** and choose **Global**.

Global 1

1 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

2 In the table, enter the following settings:

Expression	Unit	Description
imag(omega1Res3)/(2*pi)	rad/s	

3 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

4 In the **Expression** text field, type `real(omega1Res3)/(2*pi)`.

5 Click to expand the **Coloring and Style** section. Find the **Line markers** subsection. From the **Marker** list, choose **Plus sign**.

6 Find the **Line style** subsection. From the **Line** list, choose **None**.

7 From the **Color** list, choose **Cycle (reset)**.

8 Right-click **Global 1** and choose **Duplicate**.

Global 2

1 In the **Model Builder** window, click **Global 2**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
0	1	

4 Locate the **x-Axis Data** section. In the **Expression** text field, type `c1/L`.

5 Right-click **Global 2** and choose **Duplicate**.

Global 3

1 In the **Model Builder** window, click **Global 3**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
imag(omega3Res3)/(2*pi)	rad/s	

4 Locate the **x-Axis Data** section. In the **Expression** text field, type `real(omega3Res3)/(2*pi)`.

5 Right-click **Global 3** and choose **Duplicate**.

Global 4

1 In the **Model Builder** window, click **Global 4**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
imag(omega4Res3)/(2*pi)	rad/s	

4 Locate the **x-Axis Data** section. In the **Expression** text field, type `real(omega4Res3)/(2*pi)`.

5 In the **Numerical and analytical eigenfrequencies** toolbar, click **Plot**.

Rayleigh Criterion

1 In the **Results** toolbar, click 8.85 e-12 **More Derived Values** and choose **Average>Surface Average**.

2 In the **Settings** window for **Surface Average**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1/Parametric Solutions 1 (sol2)**.

4 Select Domain 2 only.

5 Locate the **Expressions** section. In the table, enter the following settings:

Expression	**Unit**	**Description**
freq	Hz	Frequency
realdot(p,acpr.flm1.Q_heat)	kg^2/(m^2*s^5)	

6 In the **Label** text field, type `Rayleigh Criterion`.

7 Click **Evaluate**.

Acoustic modes

1 In the **Results** toolbar, click **2D Plot Group**.

2 In the **Settings** window for **2D Plot Group**, click to expand the **Title** section.

3 From the **Title type** list, choose **Label**.

4 Click to expand the **Plot Array** section. Select the **Enable** check box.

5 From the **Array axis** list, choose **y**.

6 In the **Label** text field, type `Acoustic modes`.

7 Locate the **Color Legend** section. Clear the **Show legends** check box.

Surface 1

1 Right-click **Acoustic modes** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1/Parametric Solutions 1 (sol2)**.

4 From the **Parameter value (freq_lin (Hz))** list, choose **200**.

5 Right-click **Surface 1** and choose **Duplicate**.

Surface 2

1 In the **Model Builder** window, click **Surface 2**.

2 In the **Settings** window for **Surface**, locate the **Data** section.

3 From the **Parameter value (freq_lin (Hz))** list, choose **650**.

4 Right-click **Surface 2** and choose **Duplicate**.

Surface 3

1 In the **Model Builder** window, click **Surface 3**.

2 In the **Settings** window for **Surface**, locate the **Data** section.

3 From the **Parameter value (freq_lin (Hz))** list, choose **1100**.

4 Right-click **Surface 3** and choose **Duplicate**.

Surface 4

1 In the **Model Builder** window, click **Surface 4**.
2 In the **Settings** window for **Surface**, locate the **Data** section.
3 From the **Parameter value (freq_lin (Hz))** list, choose **1550**.
4 In the **Acoustic modes** toolbar, click **Plot**.