

Electromagnetic Forces on Parallel Current-Carrying Wires

One ampere is defined as the constant current in two straight parallel conductors of infinite length and negligible circular cross section, placed one meter apart in vacuum, that produces a force of 2×10^{-7} newton per meter of length (N/m). This example shows a setup of two parallel wires in the spirit of this definition, but with the difference that the wires have finite cross sections.

For wires with circular cross section carrying a uniform current density as in this example, the mutual magnetic force is the same as for line currents. This can be understood by the following arguments: Start from a situation where both wires are line currents (I). Each line current is subject to a Lorentz force per meter $(\mathbf{I} \times \mathbf{B})$, where the magnetic flux density (\mathbf{B}) is the one produced by the other wire. Now, give one wire a finite radius. It follows directly from circular symmetry and Maxwell-Ampère's law that, outside this wire, the produced flux density is exactly the same as before so the force on the remaining line current is unaltered. Further, the net force on the wire with the distributed current density must be of exactly the same magnitude (but with opposite direction) as the force on the line current so that force did not change either. If the two wires exchange places, the forces must still be the same, and it follows from symmetry that the force is independent of wire radius as long as the wire cross sections do not intersect. The wires can even be cylindrical shells or any other shape with circular symmetry. For an experimental setup, negligible cross section is required as resistive voltage drop along the wires and Hall effect may cause electrostatic forces that increase with wire radius but such effects are not included in this example.

The force between the wires is computed using two different methods: first automatically by integrating the stress tensor on the boundaries, then by integrating the volume (Lorentz) force density over the wire cross section. The results converge to 2×10^{-7} N/ m for the 1 ampere definition, as expected.

Model Definition

The application is built using the 2D Magnetic Fields interface. The modeling plane is a cross section of the two wires and the surrounding air.

DOMAIN EQUATIONS

The equation formulation assumes that the only nonzero component of the magnetic vector potential is A_z . This corresponds to all currents being perpendicular to the modeling plane. The following equation is solved:

$$\nabla \times (\mu^{-1} \nabla \times A_z \mathbf{e}_z) = J_z^{e} \mathbf{e}_z$$

where μ is the permeability of the medium and J_z^e is the externally applied current density. J_z^e is set so that the applied current in the wires equals 1 A, but with different signs.

Surrounding the air is an infinite element domain. For details, see the *AC/DC Module User's Guide*.

Results and Discussion

The magnetic flux density of the two current carrying coils is shown in Figure 1 where the direction and magnitude of the magnetic flux density are illustrated with streamlines and color scale, respectively.

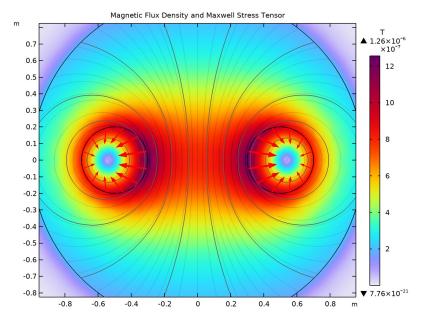


Figure 1: The magnetic flux density of the two current carrying coils.

As shown in Figure 1, the Maxwell surface stress tensor on the boundaries of conductors is represented by arrow plots. The expression for the surface stress reads

$$\mathbf{n}_1 \cdot T_2 = -\frac{1}{2} (\mathbf{H} \cdot \mathbf{B}) \mathbf{n}_1 + (\mathbf{n}_1 \cdot \mathbf{H}) \mathbf{B}^{\mathrm{T}}$$

where \mathbf{n}_1 is the boundary normal pointing out from the conductor wire and T_2 the stress tensor of air. The closed line integral of this expression around the circumference of either wire evaluates to -1.99×10^{-7} N/m. The minus sign indicates that the force between the wires is repulsive. The software automatically provides the coordinate components of the force on each wire.

The volume force density is given by

$$\mathbf{F} = \mathbf{J} \times \mathbf{B} = \begin{bmatrix} -J_z^{e} \cdot B_y, J_z^{e} \cdot B_x, 0 \end{bmatrix}$$

The surface integral of the x component of the volume force on the cross section of a wire gives the result -2.00×10^{-7} N/m.

By refining the mesh and re-solving the problem, you can verify that the solution with both methods converges to -2×10^{-7} (N/m), see Mesh Convergence. The volume force density integral is typically the most accurate one for reasons explained in the COMSOL Multiphysics Reference Manual.

Mesh Convergence

In order to investigate the accuracy of the model, it is recommended to perform a systematic mesh convergence analysis of the desired entity, here the force on the wire. In Figure 2 and Figure 3, the mesh convergence is shown for the absolute errors in the Maxwell surface stress method and the volumetric Lorentz force method, respectively. The Lorentz force is 2–3 orders of magnitude more accurate than the Maxwell stress tensor force for a given mesh density.

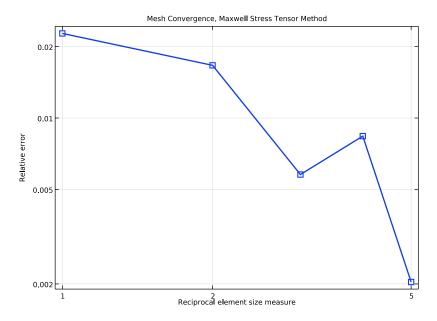


Figure 2: Mesh convergence is shown for the force computation using the Maxwell surface stress method.

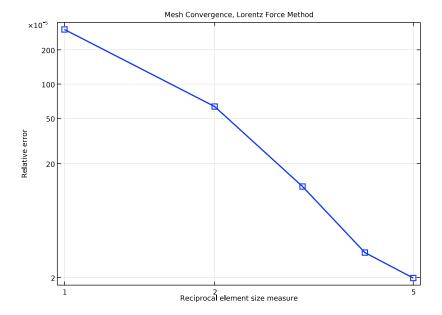


Figure 3: Mesh convergence is shown for the force computation using the volumetric Lorentz force method.

Application Library path: ACDC_Module/Introductory_Electromagnetic_Forces/parallel_wires

Modeling Instructions

From the File menu, choose New.

NEW

In the New window, click Model Wizard.

MODEL WIZARD

- I In the Model Wizard window, click **2** 2D.
- 2 In the Select Physics tree, select AC/DC>Electromagnetic Fields>Magnetic Fields (mf).
- 3 Click Add.

- 4 Click 🕞 Study.
- 5 In the Select Study tree, select General Studies>Stationary.
- 6 Click M Done.

GLOBAL DEFINITIONS

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description
r_wire	0.2[m]	0.2 m	Wire radius
10	1[A]	ΙA	Total current
J0	<pre>IO/(pi*r_wire^2)</pre>	7.9577 A/m ²	Current density
N	1	I	Mesh multiplier

GEOMETRY I

Add a circle for the main air domain. The outer layer will constitute an infinite element domain to approximate a region extending to infinity.

Circle I (c1)

- I In the Geometry toolbar, click Circle.
- 2 In the Settings window for Circle, locate the Size and Shape section.
- 3 In the Radius text field, type 1.5.
- **4** Click to expand the **Layers** section. In the table, enter the following settings:

Layer name	Thickness (m)
Layer 1	0.5

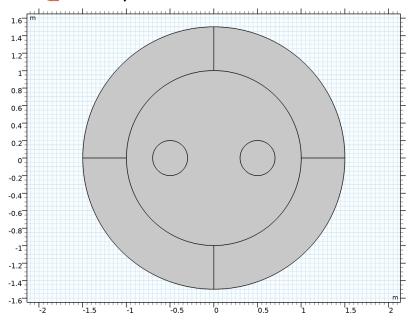
Circle 2 (c2)

- I In the Geometry toolbar, click Circle.
- 2 In the Settings window for Circle, locate the Size and Shape section.
- **3** In the **Radius** text field, type r_wire.
- 4 Locate the **Position** section. In the x text field, type 0.5.

Circle 3 (c3)

I In the Geometry toolbar, click • Circle.

- 2 In the Settings window for Circle, locate the Size and Shape section.
- 3 In the Radius text field, type r_wire.
- 4 Locate the **Position** section. In the **x** text field, type -0.5.
- 5 Click Build All Objects.



DEFINITIONS

Define an infinite element region in the outer domains.

Infinite Element Domain I (ie I)

- I In the Definitions toolbar, click on Infinite Element Domain.
- **2** Select Domains 1–4 only.
- 3 In the Settings window for Infinite Element Domain, locate the Geometry section.
- 4 From the Type list, choose Cylindrical.

ADD MATERIAL

- I In the Home toolbar, click **Add Material** to open the Add Material window.
- 2 Go to the Add Material window.
- 3 In the tree, select Built-in>Air.
- 4 Click Add to Component in the window toolbar.

5 In the Home toolbar, click 4 Add Material to close the Add Material window.

MAGNETIC FIELDS (MF)

By default, the first material you select will apply to your entire geometry. Air is defined with a zero conductivity, and relative permittivity and permeability both equal to 1. These properties are the same as those of vacuum, which is the assumed material in the definition of the ampere. Since the model assumes a given static and uniform current distribution, the electrical conductivity of the wires does not appear in the equations, so it is safe to use the same properties in the wires too.

External Current Density I

- I In the Model Builder window, under Component I (compl) right-click Magnetic Fields (mf) and choose External Current Density.
- **2** Select Domain 6 only(the wire on the left).
- 3 In the Settings window for External Current Density, locate the External Current Density section.
- **4** Specify the J_e vector as

0	x
0	у
J0	z

External Current Density 2

- I In the Physics toolbar, click **Domains** and choose **External Current Density**.
- **2** Select Domain 7 only(the wire on the right).
- 3 In the Settings window for External Current Density, locate the External Current Density section.
- **4** Specify the J_e vector as

0	×
0	у
-J0	z

The definition of the physics of the system is now complete. Add a Force Calculation feature to make Maxwell's stress tensor available as a variable.

Force Calculation I

I In the Physics toolbar, click **Domains** and choose Force Calculation.

- 2 Select Domain 6 only.
- 3 In the Settings window for Force Calculation, locate the Force Calculation section.
- 4 In the Force name text field, type wire1.

Force Calculation 2

- I In the Physics toolbar, click **Domains** and choose Force Calculation.
- 2 Select Domain 7 only.
- 3 In the Settings window for Force Calculation, locate the Force Calculation section.
- 4 In the Force name text field, type wire2.

The infinite element domain requires some attention when meshing. As it is steeply scaled in the radial direction to model a very large geometry (approximating a geometry extending to infinity), the mesh will effectively be stretched in that direction. A structured mesh is indicated in this case to prevent poor element quality. The Magnetic Fields interface can automatically create an appropriate mesh for this application.

MESH I

I In the Model Builder window, under Component I (compl) right-click Mesh I and choose **Build All.**

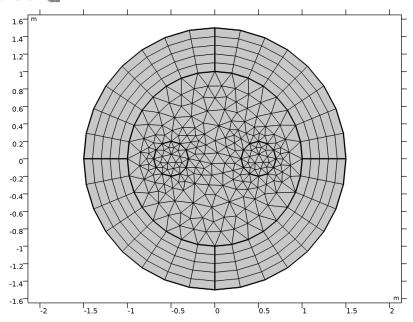
The automatically created mesh applies a Mapped operation on the finite element domain. Modify it according to the following instructions.

2 Right-click Component I (compl)>Mesh I and choose Edit Physics-Induced Sequence.

Size

- I In the Model Builder window, under Component I (compl)>Mesh I click Size.
- 2 In the Settings window for Size, locate the Element Size section.
- 3 From the Predefined list, choose Coarser.
- 4 Click to expand the Element Size Parameters section. In the Maximum element size text field, type 0.2.





The mesh should look like in the figure.

STUDY I

In the **Home** toolbar, click **Compute**.

RESULTS

Magnetic Flux Density and Maxwell Stress Tensor

- I In the Settings window for 2D Plot Group, click to expand the Title section.
- 2 From the Title type list, choose Label.
- 3 In the Label text field, type Magnetic Flux Density and Maxwell Stress Tensor.
- 4 Locate the Color Legend section. Select the Show units check box.
 The default plot shows the norm of the magnetic flux density. Note that the value inside the infinite element domain has no physical relevance.

Arrow Line 1

- I Right-click Magnetic Flux Density and Maxwell Stress Tensor and choose Arrow Line.
- 2 In the Settings window for Arrow Line, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Component 1 (comp1)>

Magnetic Fields>Mechanical>mf.nToutx_wirel,mf.nTouty_wirel -Maxwell surface stress tensor.

- 3 Locate the Coloring and Style section.
- **4** Select the **Scale factor** check box. In the associated text field, type **300000**.
- 6 Click the **Q** Zoom In button in the Graphics toolbar.

Arrow Line 2

- I Right-click Magnetic Flux Density and Maxwell Stress Tensor and choose Arrow Line.
- 2 In the Settings window for Arrow Line, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Component I (compl)> Magnetic Fields>Mechanical>mf.nToutx wire2,mf.nTouty wire2 -Maxwell surface stress tensor.
- 3 Locate the Coloring and Style section.
- 4 Select the Scale factor check box. In the associated text field, type 300000.
- The plot shows the Maxwell's stress tensor distribution on the surface of the wires. The total force on each wire is evaluated as the surface integral of the stress tensor and is available as a postprocessing variable.

Global Evaluation 1

- I In the Results toolbar, click (8.5) Global Evaluation.
- 2 In the Settings window for Global Evaluation, click Replace Expression in the upper-right corner of the Expressions section. From the menu, choose Component I (compl)> Magnetic Fields>Mechanical>Electromagnetic force - N>mf.Forcex_wire I -Electromagnetic force, x-component.
- 3 Click **= Evaluate**.

TABLE I

- I Go to the **Table I** window. The force in the x direction on the first wire evaluates to a value between -2.0×10^{-7} N/m and -1.9×10^{-7} N/m. Note that the force is given per meter, since the 2D model has an out-of-plane thickness of 1 m.
- 2 In the Settings window for Global Evaluation, click Replace Expression in the upper-right corner of the Expressions section. From the menu, choose Component I (compl)> Magnetic Fields>Mechanical>Electromagnetic force - N>mf.Forcex wire2 -Electromagnetic force, x-component.

- 3 Click **= Evaluate**.
- 4 Go to the Table I window.

As expected, the force on the second wire has a similar value but the opposite sign.

Proceed to compare the value with those from the Lorentz force distribution.

RESULTS

Surface Integration 1

- I In the Results toolbar, click 8.85 More Derived Values and choose Integration> Surface Integration.
- 2 Select Domain 6 only.
- 3 In the Settings window for Surface Integration, click Replace Expression in the upper-right corner of the Expressions section. From the menu, choose Component I (compl)> Magnetic Fields>Mechanical>Lorentz force contribution, instantaneous value - N/m3> mf.FLtzix - Lorentz force contribution, instantaneous value, x-component.
- 4 Click **= Evaluate**.

TABLE 2

I Go to the Table 2 window.

This time, the value is expected to be consistently closer to -2×10^{-7} N/m. When applicable, Lorentz force integrals usually give more accurate results than the Maxwell's stress tensor.

2 Select Domain 7 only.

RESULTS

- I In the Model Builder window, click Surface Integration 1.
- 2 In the Settings window for Surface Integration, click **Evaluate**. Once again, integration over the second wire gives a similar but positive result.

MESH I

Proceed with the mesh convergence analysis for the force. Create a parameterized mesh.

I In the Model Builder window, under Component I (compl) right-click Mesh I and choose Duplicate.

MESH 2

Size

- I In the Model Builder window, expand the Mesh 2 node, then click Size.
- 2 In the Settings window for Size, locate the Element Size Parameters section.
- 3 In the Maximum element size text field, type 0.2/N.

Distribution 1

- I In the Model Builder window, expand the Component I (compl)>Meshes>Mesh 2> Mapped I node, then click Component I (compl)>Meshes>Mesh 2>Distribution I.
- 2 In the Settings window for Distribution, locate the Distribution section.
- 3 In the Number of elements text field, type 4*N.

Perform the mesh convergence analysis in a new study.

ADD STUDY

- I In the Home toolbar, click Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select General Studies>Stationary.
- 4 Click Add Study in the window toolbar.
- 5 In the Home toolbar, click Add Study to close the Add Study window.

STUDY 2

Perform a sweep over the mesh multiplier parameter.

Parametric Sweep

- I In the Study toolbar, click Parametric Sweep.
- 2 In the Settings window for Parametric Sweep, locate the Study Settings section.
- 3 Click + Add.
- 4 From the list in the Parameter name column, choose N (Mesh multiplier).
- 5 Click Range.
- 6 In the Range dialog box, type 1 in the Start text field.
- 7 In the Step text field, type 1.
- **8** In the **Stop** text field, type 5.
- 9 Click Replace.

Define a nonlocal integration coupling to compute the total force from the Lorentz force contribution.

DEFINITIONS

Integration | (intob|)

- I In the **Definitions** toolbar, click Monlocal Couplings and choose Integration.
- **2** Select Domain 7 only.

STUDY 2

In the **Home** toolbar, click **Compute**.

RESULTS

Magnetic Flux Density (Mesh Convergence Study)

- I In the Settings window for 2D Plot Group, locate the Title section.
- 2 From the Title type list, choose Label.
- 3 In the Label text field, type Magnetic Flux Density (Mesh Convergence Study).
- 4 Locate the Color Legend section. Select the Show units check box.

ID Plot Group 3

- I In the Home toolbar, click **Add Plot Group** and choose ID Plot Group. Plot the absolute error versus the mesh multiplier parameter for the force computed using Maxwell's stress tensor.
- 2 In the Settings window for ID Plot Group, locate the Data section.
- 3 From the Dataset list, choose Study 2/Parametric Solutions 1 (sol3).

Global I

- I Right-click ID Plot Group 3 and choose Global.
- 2 In the Settings window for Global, locate the y-Axis Data section.
- **3** In the table, enter the following settings:

Expression	Unit	Description
abs(mf.Forcex_wire2-2e-7)/2e-7	N	

- 4 Click to expand the Coloring and Style section. From the Width list, choose 2.
- 5 Find the Line markers subsection. From the Marker list, choose Square.

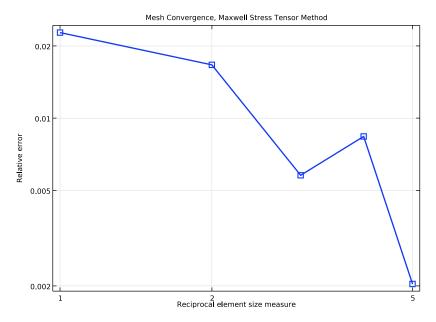
- 6 In the ID Plot Group 3 toolbar, click Plot. Switch to logarithmic scale and add suitable plot annotations.
- 7 Click the x-Axis Log Scale button in the Graphics toolbar.
- 8 Click the y-Axis Log Scale button in the Graphics toolbar.

Mesh Convergence, Maxwell Stress Tensor Method

- I In the Model Builder window, click ID Plot Group 3.
- 2 In the Settings window for ID Plot Group, click to expand the Title section.
- 3 From the Title type list, choose Label.
- 4 Locate the Plot Settings section.
- 5 Select the x-axis label check box. In the associated text field, type Reciprocal element size measure.
- 6 Select the y-axis label check box. In the associated text field, type Relative error.
- 7 In the Label text field, type Mesh Convergence, Maxwell Stress Tensor Method.

Global I

- I In the Model Builder window, click Global I.
- 2 In the Settings window for Global, click to expand the Legends section.
- **3** Clear the **Show legends** check box.



Mesh Convergence, Maxwell Stress Tensor Method

Plot the absolute error versus the mesh multiplier parameter for the force computed using the Lorentz force contribution.

I In the Model Builder window, right-click Mesh Convergence, Maxwell Stress Tensor Method and choose **Duplicate**.

Mesh Convergence, Lorentz Force Method

- I In the Model Builder window, under Results click Mesh Convergence, Maxwell Stress Tensor Method I.
- 2 In the Settings window for ID Plot Group, type Mesh Convergence, Lorentz Force Method in the Label text field.

Global I

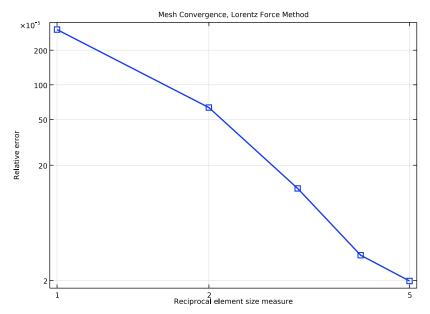
- I In the Model Builder window, expand the Mesh Convergence, Lorentz Force Method node, then click Global 1.
- 2 In the Settings window for Global, locate the y-Axis Data section.

3 In the table, enter the following settings:

Expression	Unit	Description
abs(intop1(mf.FLtzix)-2e-7)/2e-7	N/m	

Mesh Convergence, Lorentz Force Method

- I In the Model Builder window, click Mesh Convergence, Lorentz Force Method.



Using the Lorentz force method gives results that are 2-3 orders of magnitude more accurate than the ones obtained using Maxwell's stress tensor for a given mesh density.

In the following, it is presented another way to visualize Maxwell Stress tensor together with the Lorentz Force. The plot highlights clearly that the Maxwell Stress tensor is a boundary vector (whose main property is that its surface integral is the total force on the body) and that the Lorentz Force is a volumetric force. Differently from Maxwell Stress tensor, Lorentz Force is an actual volumetric force, and, for the present case where the objects are nonmagnetic conductor, Lorentz Force is the only contribution to the total force.

Lorentz Force and Maxwell Stress Tensor

I In the Home toolbar, click **Add Plot Group** and choose **2D Plot Group**.

2 In the Settings window for 2D Plot Group, type Lorentz Force and Maxwell Stress Tensor in the Label text field.

First, remove representation of all the edges, and reproduce only the coil edges twice, one above the other.

- **3** Locate the **Plot Settings** section. Clear the **Plot dataset edges** check box.
- 4 Click to expand the **Selection** section. From the **Geometric entity level** list, choose Domain.
- **5** Select Domains 6 and 7 only.

line 1

- I Right-click Lorentz Force and Maxwell Stress Tensor and choose Line.
- 2 In the Settings window for Line, locate the Expression section.
- **3** In the **Expression** text field, type 1.
- 4 Locate the Coloring and Style section. Clear the Color legend check box.
- 5 From the Coloring list, choose Uniform.
- 6 From the Color list, choose Black.
- 7 Right-click Line I and choose Duplicate.

Translation 1

- I In the Model Builder window, right-click Line 2 and choose Translation.
- 2 In the Settings window for Translation, locate the Translation section.
- 3 In the y text field, type 0.5.

Where the **Translation** feature is used to displace the two upper circles.

Add a title well representing what is going to be shown.

Lorentz Force and Maxwell Stress Tensor

- I In the Model Builder window, under Results click Lorentz Force and Maxwell Stress Tensor.
- 2 In the Settings window for 2D Plot Group, locate the Title section.
- 3 From the Title type list, choose Manual.
- 4 In the Title text area, type Maxwell Stress Tensor (Red) and Lorentz Force (Blue).

Now add Maxwell Stress Tensor representation on both conductors.

Arrow Line 1

I Right-click Lorentz Force and Maxwell Stress Tensor and choose Arrow Line.

- 2 In the Settings window for Arrow Line, locate the Expression section.
- 3 In the x-component text field, type try catch (mf.nToutx wire1, mf.nToutx wire2).
- 4 In the y-component text field, type try_catch(mf.nTouty_wire1, mf.nTouty wire2).
- 5 Locate the Coloring and Style section.
- **6** Select the **Scale factor** check box. In the associated text field, type 400000.
- 7 Locate the Arrow Positioning section. In the Number of arrows text field, type 20.

Finally add Lorentz Force representation, shifted up.

Arrow Surface 1

- I Right-click Lorentz Force and Maxwell Stress Tensor and choose Arrow Surface.
- 2 In the Settings window for Arrow Surface, locate the Coloring and Style section.
- **3** From the **Color** list, choose **Blue**.
- 4 Click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Component I (compl)>Magnetic Fields>Mechanical>mf.FLtzix,mf.FLtziy -Lorentz force contribution, instantaneous value.
- 5 Locate the Arrow Positioning section. Find the x grid points subsection. In the Points text field, type 21.
- 6 Find the y grid points subsection. In the Points text field, type 11.
- 7 Locate the Coloring and Style section.
- **8** Select the **Scale factor** check box. In the associated text field, type 10000.

Translation 1

- I Right-click Arrow Surface I and choose Translation.
- 2 In the Settings window for Translation, locate the Translation section.
- 3 In the y text field, type 0.5.

Execute the plot and rescale the view to get a representation of Maxwell Stress Tensor and Lorentz Force. The result will look like in the following figure.

Lorentz Force and Maxwell Stress Tensor

- I In the Model Builder window, under Results click Lorentz Force and Maxwell Stress Tensor.
- 2 In the Lorentz Force and Maxwell Stress Tensor toolbar, click Plot.

$\textbf{3} \ \ \text{Click the} \ \ \fbox{\textbf{Zoom Extents}} \ \text{button in the Graphics toolbar}.$

