

Self-Inductance and Mutual Inductance of a Single Conductor and a Helical Coil

The mutual inductance and induced currents between a single-turn primary and 20-turn secondary coil in a concentric coplanar arrangement is computed using a frequency domain model. Each turn of the secondary coil is modeled explicitly. Static and AC results are compared one against the other, and against analytic predictions.

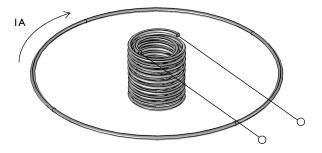


Figure 1: A 20-turn secondary coil inside of a single-turn primary coil (not to scale).

Model Definition

The physical situation being modeled is shown in Figure 1. A secondary coil composed of 20 turns, wound two deep, is concentric with the primary, and in the same plane. The radius of the centroid of the secondary coil is $R_2 = 10$ mm. The wire radii in both coils is $r_0 = 1$ mm. Although the coils are shown in 3D, they are modeled in the 2D axisymmetric space, assuming no physical variation around the centerline. First two DC analysis are solved in order to extract the inductance matrix of the system. Then a prescribed current of 1 A is flowing through a single-turn coil of radius $R_1 = 100$ mm, at a frequency of 1 kHz. The objective of this is computing the voltage difference at the secondary coil for the open-circuit case, and the induced currents for the closed-circuit case.

For the case of a secondary multiturn coil with N turns, and in the limit as $R_1 >> R_2 >>$ r_0 , the analytic expression for the mutual inductance between the two coils is:

$$M = N\mu_0 \frac{\pi R_2^2}{2R_1}$$

where μ_0 is the permeability of free space.

The two concentric coils are modeled in a 2D axisymmetric sense, as shown schematically in Figure 2. The modeling domain is surrounded by a region of Infinite Elements, which are a way to truncate a domain which stretches to infinity. Although the thickness of the Infinite Element Domain is finite, it can be thought of as a domain of infinite extent.

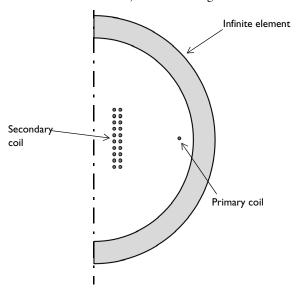


Figure 2: A schematic representation of the 2D axisymmetric model of the concentric coils.

The primary coil is modeled using the **Coil** feature, which can be thought of as introducing an infinitesimal slit in an otherwise continuous torus. Since the coil has a single turn and is made up of conductive material, the **Single conductor** model is used in the Coil feature. The feature is used to excite the coil by specifying a current of 1 A.

The secondary coil is modeled using the **Coil** feature with the **Coil group** setting that enforces that the same amount of current flow through each circular domain which represents one turn of the coil (the coil turns are connected in series). This feature can be used to model both the open circuit and the closed-circuit case. To model the open-circuit case, the current through the coil is specified to be 0 A, which specifies that there is no current flowing through the coil. To model a closed circuit it is enough to put 0 V. The Coil feature with the Coil group setting computes the total current and potential drop on the entire coil. Furthermore, if just one coil only is fed, also self-inductance and mutual inductance of the coil system are available in the output. For AC feeding, assuming the system to be purely reactive, mutual inductance can be computed via:

$$M = \frac{V_{coil}}{i\omega I_p} \tag{1}$$

Where ω is the angular frequency of excitation of the driving current in the primary, I_p . The inductance computed in this fashion will have a small imaginary component as, due to finite conductivity there are eddy current losses in the wires and the coil impedance, though mainly reactive, has a small resistive part. The deduced inductance is compared then also against the inductance predicted by taking the integral of the magnetic flux through the center of the coil.

For the closed-circuit case, the voltage drop across the coil is fixed at 0 V. Although this seems to imply a short circuit, the reactance of the copper coil is inherently included, so the case being modeled is analogous to a closed continuous loop of wire. The Coil feature with the Coil group setting enforces that the same current flows through each turn.

Via a simple circuital analogy, and for frequencies such that conductor size is significantly larger than skin depth, this current can be estimated from DC values via:

$$I_2 = -iw(L_{21}/Z_2) \tag{2}$$

where L_{21} is mutual inductance and $Z_2 = R_2 + iwL_2$ is the impedance of the inner coil. At the simulated 1 kHz deviations are really small. Increasing the frequency, this estimate will fail as the model is able to capture the autoinductance effects on the coil that are not included in the simplest circuital analogy.

Results and Discussion

From two initial static analyses — one feeding the single coil and the other feeding the 20turn coil group — it is possible to extract the following inductance matrix from built-in variables:

A similar estimate (which is however less accurate) is extracted computing the integral of the normal magnetic flux. In case of an AC feeding the magnetic flux lines are plotted in Figure 3 for the open-circuit case. The Coil feature with the Coil group setting computes the voltage across the secondary, which can be used to evaluate the mutual inductance, 39.9 - 0.6i nH. This agrees well with the mutual inductance predicted by static calculations and with the analytic mutual inductance estimate of 39.478 nH.

The induced currents of the secondary coil is plotted in Figure 4 for the closed-circuit case. The skin effect is apparent, the current is being driven to the boundaries of the domains. The induced current through the secondary coil is -10.94 - 3.5i mA, the imaginary component implies a reactive current. Both real and imaginary parts are correctly accounted for by Equation 2.

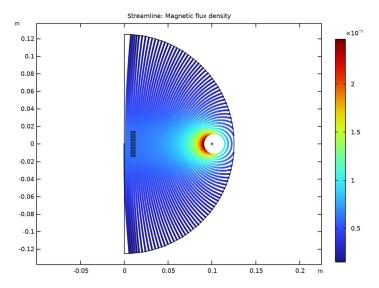


Figure 3: Magnetic flux lines for the open-circuit case.

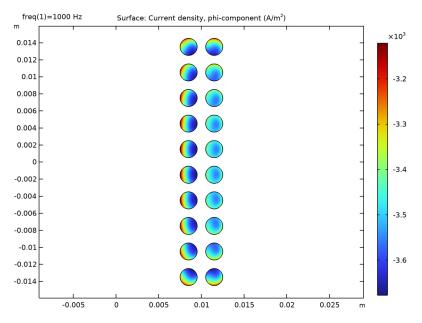


Figure 4: Induced currents in the coil for the closed-circuit case.

Application Library path: ACDC_Module/Tutorials,_Coils/
mutual_inductance_coil_group

Modeling Instructions

From the File menu, choose New.

NEW

In the New window, click Model Wizard.

MODEL WIZARD

- I In the Model Wizard window, click 2D Axisymmetric.
- 2 In the Select Physics tree, select AC/DC>Electromagnetic Fields>Magnetic Fields (mf).
- 3 Click Add.
- 4 Click Study.

- 5 In the Select Study tree, select General Studies>Stationary.
- 6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description
r_wire	1 [mm]	0.001 m	Radius, wire
R1	100[mm]	0.1 m	Radius, outer coil
R2	10[mm]	0.01 m	Radius, inner coil
M	20*(mu0_const*pi* R2^2)/(2*R1)	3.9478E-8 H	Analytic mutual inductance
I1	1[A]	I A	Current, outer coil
12	0[A]	0 A	Current, inner coil

Here, mu0_const is a predefined COMSOL constant for the permeability in vacuum.

GEOMETRY I

Create a circle for the simulation domain. Define a layer in the circle where you will assign the Infinite Element Domain.

Circle I (c1)

- I In the Geometry toolbar, click Circle.
- 2 In the Settings window for Circle, locate the Size and Shape section.
- 3 In the Sector angle text field, type 180.
- 4 In the Radius text field, type 1.75*R1.
- **5** Locate the **Rotation Angle** section. In the **Rotation** text field, type -90.
- **6** Click to expand the **Layers** section. In the table, enter the following settings:

Layer name	Thickness (m)
Layer 1	50[mm]

Create a circle for the outer coil.

Circle 2 (c2)

- I In the Geometry toolbar, click Circle.
- 2 In the Settings window for Circle, locate the Size and Shape section.
- 3 In the Radius text field, type r_wire.
- 4 Locate the **Position** section. In the r text field, type R1.

Then, create a circle for the inner coil.

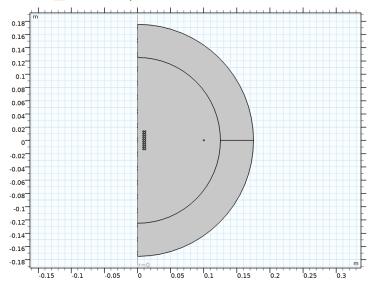
Circle 3 (c3)

- I In the Geometry toolbar, click Circle.
- 2 In the Settings window for Circle, locate the Size and Shape section.
- **3** In the **Radius** text field, type r_wire.
- 4 Locate the **Position** section. In the r text field, type R2-1.5*r wire.
- 5 In the z text field, type -13.5*r_wire.

Array I (arrI)

- I In the Geometry toolbar, click \(\sum_{\text{transforms}} \) Transforms and choose Array.
- 2 Select the object c3 only.
- 3 In the Settings window for Array, locate the Size section.
- 4 In the r size text field, type 2.
- 5 In the z size text field, type 10.
- 6 Locate the **Displacement** section. In the r text field, type 3*r_wire.
- 7 In the z text field, type 3*r_wire.

8 Click Build All Objects.



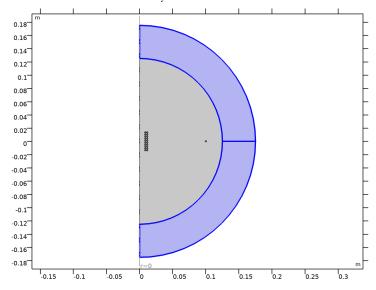
DEFINITIONS

Define the Infinite Element Domain to apply a coordinate transformation that mathematically stretches the layer to infinity. The Physics-controlled mesh creates a Swept Mesh inside the Infinite Elements domains.

Infinite Element Domain I (ie I)

I In the Definitions toolbar, click on Infinite Element Domain.

2 Select Domains 1 and 3 only.



MAGNETIC FIELDS (MF)

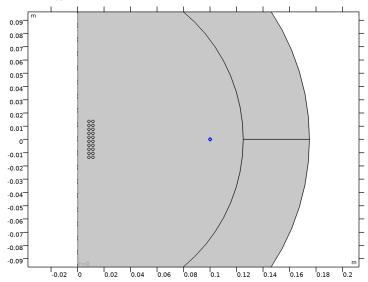
Now, set up the physics. Assign a Coil feature on the outer and the inner coil. The outer coil will be initially fed with a current of 1 A.

I Click the 2 Zoom In button in the Graphics toolbar.

Coil I

- I In the Model Builder window, under Component I (compl) right-click Magnetic Fields (mf) and choose the domain setting Coil.
- 2 Select Domain 24 only.
- 3 In the Settings window for Coil, locate the Coil section.

4 In the $I_{\rm coil}$ text field, type I1.

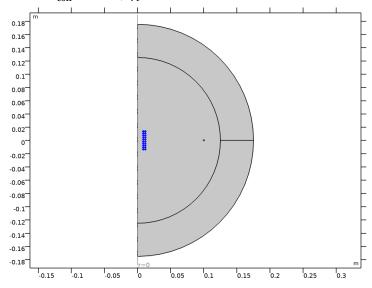


Specify 0~A current for the Coil feature assigned to the inner coil to model the open circuit case.

Coil 2

- I In the Physics toolbar, click **Domains** and choose **Coil**.
- 2 In the Settings window for Coil, locate the Coil section.
- **3** Select the **Coil group** check box.
- 4 Select Domains 4–23 only.

5 In the $I_{
m coil}$ text field, type I2.



MATERIALS

Next, assign material properties. Use Air for all domains.

ADD MATERIAL

- I In the Home toolbar, click 4 Add Material to open the Add Material window.
- 2 Go to the Add Material window.
- 3 In the tree, select Built-in>Air.
- 4 Click Add to Component in the window toolbar.

MATERIALS

Air (mat I)

Then, override the coil domains with copper.

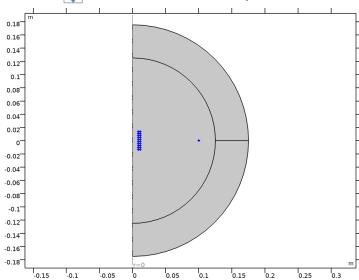
ADD MATERIAL

- I Go to the Add Material window.
- 2 In the tree, select AC/DC>Copper.
- 3 Click Add to Component in the window toolbar.
- 4 In the Home toolbar, click 4 Add Material to close the Add Material window.

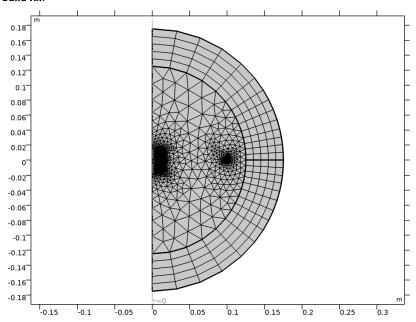
MATERIALS

Copper (mat2)

- I Select Domains 4–24 only.
- 2 Click the **Zoom Extents** button in the **Graphics** toolbar.



MESH I In the Model Builder window, under Component I (compl) right-click Mesh I and choose **Build All.**



Solve the first case in which the outer coil (named 1) is fed and the inner (named 2) is open.

STUDY I

- I In the Model Builder window, click Study I.
- 2 In the Settings window for Study, locate the Study Settings section.
- 3 Clear the Generate default plots check box.
- 4 In the Home toolbar, click **Compute**.

RESULTS

In the Model Builder window, expand the Results node.

Study I/Solution I (soll)

Select only the domains not part of the Infinite Element Domain to better visualize the magnetic flux density.

Selection

- I In the Model Builder window, expand the Results>Datasets node.
- 2 Right-click Study I/Solution I (soll) and choose Selection.
- 3 In the Settings window for Selection, locate the Geometric Entity Selection section.
- 4 From the Geometric entity level list, choose Domain.
- **5** Select Domains 2 and 4–24 only.

2D Plot Group 1

In the Home toolbar, click Add Plot Group and choose 2D Plot Group.

Streamline 1

- I Right-click 2D Plot Group I and choose Streamline.
- 2 In the Settings window for Streamline, locate the Streamline Positioning section.
- 3 From the Positioning list, choose Starting-point controlled.
- 4 From the Entry method list, choose Coordinates.
- 5 In the r text field, type range (0, 0.9*R1/49, 0.9*R1).
- **6** In the **z** text field, type 0.
- 7 Locate the Coloring and Style section. Find the Line style subsection. From the Type list, choose Tube.

Color Expression 1

- I Right-click Streamline I and choose Color Expression.
- 2 Click the **Zoom Extents** button in the **Graphics** toolbar.

This reproduces Figure 2.

Evaluate the self inductance of the external coil and the mutual inductance of the outer coil with respect to the inner. Some additional quantities are also computed to verify the results.

Global Evaluation 1

- I In the Results toolbar, click (8.5) Global Evaluation.
- 2 In the Settings window for Global Evaluation, locate the Expressions section.
- **3** In the table, enter the following settings:

Expression	Unit	Description	
mf.LCoil_1	nH	External coil inductance	
2*mf.intWm/1[A^2]	nH	Energy estimate for external coil inductance	

Expression	Unit	Description
mf.L_2_1	nH	Computed mutual inductance
М	nH	Analytical mutual inductance

4 Click **= Evaluate**.

Next, compute the self inductance of the inner coil and the mutual inductance of the inner coil with respect to the outer. Start by switching the currents in the coils.

GLOBAL DEFINITIONS

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description
I1	0[A]	0 A	Current, outer coil
12	1[A]	ΙA	Current, inner coil

Then add and solve a second study for this case. The solution previously computed will still be available in Study 1.

ADD STUDY

- I In the Home toolbar, click Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select General Studies>Stationary.
- 4 Click Add Study in the window toolbar.
- 5 In the Home toolbar, click Add Study to close the Add Study window.

STUDY 2

- I In the Model Builder window, click Study 2.
- 2 In the Settings window for Study, locate the Study Settings section.
- 3 Clear the Generate default plots check box.
- 4 In the Home toolbar, click **Compute**.

The quantities of interest are evaluated in the following steps.

RESULTS

Global Evaluation 2

- I In the Results toolbar, click (8.5) Global Evaluation.
- 2 In the Settings window for Global Evaluation, locate the Data section.
- 3 From the Dataset list, choose Study 2/Solution 2 (sol2).
- **4** Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
mf.LCoil_2	nH	Internal coil inductance
2*mf.intWm/1[A^2]	nH	Energy estimate for internal coil inductance
mf.L_1_2	nH	Computed mutual inductance
M	nH	Analytical mutual inductance

5 Click ▼ next to **= Evaluate**, then choose **New Table**.

TABLE 2

I Go to the Table 2 window.

Self and mutual inductance variables as computed above are derived via concatenated flux, which is defined as the line integral of the magnetic vector potential along the coil. This approach gives the best accuracy.

For simple geometries like the present one, concatenated flux can be also computed explicitly using its definition as the integral of magnetic flux through a surface, although this approach usually gives less accurate results.

RESULTS

Cut Line 2D I

- I In the Results toolbar, click Cut Line 2D.
- 2 In the Settings window for Cut Line 2D, locate the Line Data section.
- 3 In row Point 2, set r to R2.

Cut Line 2D 2

- I In the Results toolbar, click Cut Line 2D.
- 2 In the Settings window for Cut Line 2D, locate the Line Data section.
- 3 In row Point 2, set r to R1.

Line Integration I

- I In the Results toolbar, click 8.85 More Derived Values and choose Integration> Line Integration.
- 2 In the Settings window for Line Integration, locate the Data section.
- 3 From the Dataset list, choose Cut Line 2D 1.
- **4** Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
20*mf.Bz/I1	nH	

- 5 Click ▼ next to **= Evaluate**, then choose **New Table**.
- **6** Right-click **Line Integration I** and choose **Duplicate**.

Line Integration 2

- I In the Model Builder window, click Line Integration 2.
- 2 In the Settings window for Line Integration, locate the Data section.
- 3 From the Dataset list, choose Cut Line 2D 2.
- **4** Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
mf.Bz/I1	nH	

5 Click ▼ next to **= Evaluate**, then choose **Table 3 - Line Integration 1**.

Cut Line 2D L

- I In the Model Builder window, under Results>Datasets click Cut Line 2D 1.
- 2 In the Settings window for Cut Line 2D, locate the Data section.
- 3 From the Dataset list, choose Study 2/Solution 2 (sol2).

Cut Line 2D 2

- I In the Model Builder window, click Cut Line 2D 2.
- 2 In the Settings window for Cut Line 2D, locate the Data section.
- 3 From the Dataset list, choose Study 2/Solution 2 (sol2).

Line Integration I

- I In the Model Builder window, under Results>Derived Values click Line Integration I.
- 2 In the Settings window for Line Integration, locate the Expressions section.

3 In the table, enter the following settings:

Expression	Unit	Description
20*mf.Bz/I2	nH	

4 Click ▼ next to **= Evaluate**, then choose **Table 3 - Line Integration 1**.

Line Integration 2

- I In the Model Builder window, click Line Integration 2.
- 2 In the Settings window for Line Integration, locate the Expressions section.
- **3** In the table, enter the following settings:

Expression	Unit	Description
mf.Bz/I2	nH	

4 Click ▼ next to **= Evaluate**, then choose **Table 3 - Line Integration 1**.

Experimentally, mutual inductance is measured by feeding an AC signal in the primary coil and measuring the voltage induced in the open-circuit secondary coil. This procedure can be simulated by using a Frequency Domain study step. Start by setting the AC feed on Coil 1 and the open circuit (zero current) condition on Coil 2.

GLOBAL DEFINITIONS

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description
I1	1[A]	ΙA	Current, outer coil
12	0[A]	0 A	Current, inner coil

ADD STUDY

- I In the Home toolbar, click Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select General Studies> Frequency Domain.
- 4 Click Add Study in the window toolbar.

5 In the Home toolbar, click Add Study to close the Add Study window.

STUDY 3

Steb 1: Frequency Domain

- I In the Settings window for Frequency Domain, locate the Study Settings section.
- 2 In the Frequencies text field, type 1[kHz].
- 3 In the Model Builder window, click Study 3.
- 4 In the Settings window for Study, locate the Study Settings section.
- 5 Clear the Generate default plots check box.
- **6** In the **Home** toolbar, click **Compute**.

RESULTS

Study 3/Solution 3 (sol3)

Select the inner coil domains.

Selection

- I In the Model Builder window, right-click Study 3/Solution 3 (sol3) and choose Selection.
- 2 In the Settings window for Selection, locate the Geometric Entity Selection section.
- 3 From the Geometric entity level list, choose Domain.
- 4 Select Domains 4–23 only.

2D Plot Group 2

- I In the Home toolbar, click Add Plot Group and choose 2D Plot Group.
- 2 In the Settings window for 2D Plot Group, locate the Data section.
- 3 From the Dataset list, choose Study 3/Solution 3 (sol3).

Surface I

Right-click 2D Plot Group 2 and choose Surface.

Evaluate the mutual inductance using Equation 1.

Global Evaluation 3

- I In the Results toolbar, click (8.5) Global Evaluation.
- 2 In the Settings window for Global Evaluation, locate the Data section.
- 3 From the Dataset list, choose Study 3/Solution 3 (sol3).

4 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
mf.VCoil_2/1[A]/mf.iomega	nH	

5 Click **= Evaluate**.

Finally, simulate the system as if it were a transformer with a short-circuited secondary winding. Specify a voltage of 0 V for the Coil feature assigned to the inner coil to model the short-circuit condition.

MAGNETIC FIELDS (MF)

Coil 2

- I In the Model Builder window, under Component I (compl)>Magnetic Fields (mf) click Coil 2.
- 2 In the Settings window for Coil, locate the Coil section.
- 3 From the Coil excitation list, choose Voltage.
- **4** In the V_{coil} text field, type 0.

ADD STUDY

- I In the Home toolbar, click Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select General Studies> Frequency Domain.
- 4 Click Add Study in the window toolbar.
- 5 In the Home toolbar, click Add Study to close the Add Study window.

STUDY 4

Step 1: Frequency Domain

- I In the Settings window for Frequency Domain, locate the Study Settings section.
- 2 In the Frequencies text field, type 1 [kHz].
- 3 In the Model Builder window, click Study 4.
- 4 In the Settings window for Study, locate the Study Settings section.
- 5 Clear the Generate default plots check box.
- 6 In the Home toolbar, click **Compute**.

RESULTS

Study 4/Solution 4 (sol4)

Select the inner coil domains.

Selection

- I In the Model Builder window, right-click Study 4/Solution 4 (sol4) and choose Selection.
- 2 In the Settings window for Selection, locate the Geometric Entity Selection section.
- 3 From the Geometric entity level list, choose Domain.
- 4 Select Domains 4–23 only.

2D Plot Group 3

- I In the Home toolbar, click Add Plot Group and choose 2D Plot Group.
- 2 In the Settings window for 2D Plot Group, locate the Data section.
- 3 From the Dataset list, choose Study 4/Solution 4 (sol4).

Surface I

- I Right-click 2D Plot Group 3 and choose Surface.
- 2 In the Settings window for Surface, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Component I (compl)>Magnetic Fields> Currents and charge>Current density - A/m2>mf.Jphi - Current density, phi-component.
- 3 In the 2D Plot Group 3 toolbar, click Plot.
- 4 Click the **Zoom Extents** button in the **Graphics** toolbar.

Compare the plot with Figure 4 describing the induced currents in the coil for the closed circuit case. Skin effect and proximity effects are clearly visible.

Evaluate the total induced current on the inner (secondary) coil. This quantity is related to static quantities, being in the simplest approximation $i\omega M/(R_2 + i\omega L_2)$ times 1[A].

Global Evaluation 4

- I In the Results toolbar, click (8.5) Global Evaluation.
- 2 In the Settings window for Global Evaluation, locate the Data section.
- 3 From the Dataset list, choose Study 4/Solution 4 (sol4).

4 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
mf.ICoil_2	Α	Coil current
<pre>-mf.iomega*withsol('sol1',mf.L_2_1)/ (withsol('sol2',mf.RCoil_2)+withsol('sol2', mf.LCoil_2)*mf.iomega)*mf.ICoil_1</pre>	Α	

5 Click **= Evaluate**.