

# Flexible and Smooth Strip Footing on a Stratum of Clay

A typical verification example for geotechnical problems is a shallow stratum layer of clay, see Figure 1. In the example, a vertical load is applied on the clay stratum, and the static response as well as the collapse load are of interest. This example is adapted from Ref. 2.

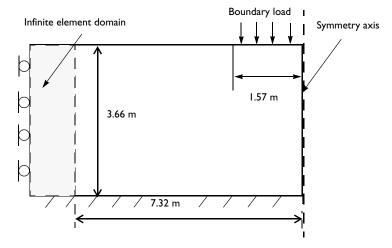


Figure 1: Dimensions, boundary conditions, and pressure load for the stratum of clay.

#### ANALYSIS TYPE

Yield Surface

Assume plane strain conditions, and model the clay with soil plasticity and the Drucker-Prager criterion.

The yield surface, F, for the Drucker-Prager criterion is given by

$$F = \sqrt{J_2} + \alpha I_1 - k = 0$$

where  $I_1$  is the first stress invariant and  $J_2$  is the second deviatoric stress invariant.

The first stress invariant is defined as the trace of Cauchy stress tensor:

$$I_1 = \operatorname{trace}(\sigma)$$

The second stress invariant is defined as

$$I_2 = \frac{1}{2}(I_1^2 - \text{trace}(\sigma^2))$$

The second deviatoric stress invariant can be expressed using the first and the second stress invariants:

$$J_2 = \frac{1}{3}I_1^2 - I_2$$

If 2D plane strain conditions prevail, the Drucker–Prager criterion matches the Mohr–Coulomb criterion. For this case the material parameters  $\alpha$  and k are given by the cohesion c and the angle of internal friction  $\phi$  (Ref. 1)

$$\alpha = \frac{\tan\phi}{\sqrt{(9 + 12\tan^2\phi)}}$$

$$k = \frac{3c}{\sqrt{(9 + 12\tan^2\phi)}}$$

The Drucker–Prager criterion is the default choice for the **Soil Plasticity** feature, and the check box **Match to Mohr–Coulomb criterion** applies the aforementioned matching of the material parameters.

Under Soil Plasticity, it is also possible to use Mohr-Coulomb criterion

$$F = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) + \frac{1}{2}(\sigma_{\max} + \sigma_{\min})\sin\phi - \cos\phi = 0 ,$$

where  $\sigma_{max}$  and  $\sigma_{min}$  are the biggest and smallest principal stresses, respectively. The Mohr–Coulomb criterion defines an irregular hexagon pyramid in the principal stress space. Since this yield function gives rise to singularities in the derivatives of the yield function, the use of a nonassociated flow rule with a Drucker–Prager plastic potential is chosen. This is done in the plastic potential list, with the option **Drucker–Prager matched at compressive meridian**.

Flow Rule

The flow rule defines the relation between the plastic strain increment in a given direction and the current level of stress in the same direction. The relation reads

$$\dot{\varepsilon}_{ij} = \lambda \frac{\partial Q}{\partial \sigma_{ij}}$$

where  $\lambda$  is the plastic multiplier and Q is the plastic potential. If the yield surface, F, and the plastic potential, Q, are identical, that is, if F = Q, then it is called an associated flow rule, otherwise it is called a nonassociated flow rule.

#### SOIL PROPERTIES

- Young's modulus, E = 207 MPa, and Poisson's ratio v = 0.3.
- Cohesion c = 69 MPa, and angle of internal friction  $\phi = 20^{\circ}$ .

#### CONSTRAINTS AND LOADS

- The clay layer is supported by a rigid and perfectly rough base. Therefore, apply a fixed constraint on the lower horizontal boundary.
- Model only the left half of the domain due to symmetry reasons. Use the symmetry boundary condition at the right vertical boundary.
- The stratum is subjected to a footing that is considered to be flexible and smooth. The width of the strip footing is 3.14 m, see Figure 2. Gradually increase the footing pressure until the clay layer reaches the collapse load.

#### INFINITE ELEMENT DOMAIN

- In order to mimic an infinite layer of soil, add an Infinite Element Domain. The scaling 1e3\*root.mod1.dGeomChar means that the spatial variables in this domain are scaled by thousand times the typical geometry length.
- The left vertical boundary is perfectly smooth and a can be assumed to be of the roller type.

#### Results and Discussion

The load-displacement curves for both the Mohr-Coulomb and the Drucker-Prager criteria are plotted in Figure 2. The relation between the applied footing pressure and the centerline displacement (directly beneath the center of the footing) in the y direction are shown. The lines show the load-displacement curves for the Mohr-Coulomb and Drucker-Prager criteria. The curves are identical up to 300 kPa because the whole domain is still within the elastic region. From that point when the pressure increases, the behavior diverges. Both curves reach the collapse load at approximately 1.1 MPa. The development of plastic strains in the soil at different stages of loading is shown in Figure 3.

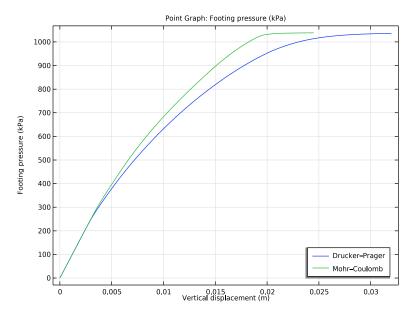


Figure 2: Footing pressure versus vertical displacement for the Mohr–Coulomb and Drucker–Prager material models.

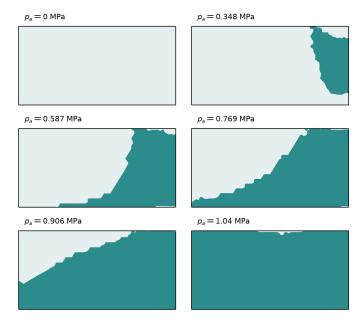


Figure 3: Evolution of the equivalent plastic strain on the clay layer during the parametric loading. Dark regions indicate the plastic region.

## Notes About the COMSOL Implementation

Both the Mohr-Coulomb or Drucker-Prager criterion are predefined in the Soil Plasticity subfeature. The default Mohr-Coulomb criterion uses a nonassociated flow rule, with a plastic potential implemented as Drucker-Prager matched at compressive meridian.

A suitable modeling technique in a case where the relation between the applied load and the displacement is highly nonlinear, is to use an algebraic equation that controls the applied pressure so that the model reaches the desired displacement increments. This is implemented using a Global Equation, and the parametric solver incrementally increases the displacement up to the desired vertical displacement.

# References

- 1. W.F. Chen and E. Mizuno, Nonlinear Analysis in Soil Mechanics, Elsevier, 1990.
- 2. A. Mar, How To Undertake Finite Element Based Geotechnical Analysis, NAFEMS, 2002.

## Application Library path: Geomechanics Module/Soil/flexible footing

# Modeling Instructions

From the File menu, choose New.

#### NEW

In the New window, click Model Wizard.

#### MODEL WIZARD

- I In the Model Wizard window, click **2** 2D.
- 2 In the Select Physics tree, select Structural Mechanics>Solid Mechanics (solid).
- 3 Click Add.
- 4 Click Study.
- 5 In the Select Study tree, select General Studies>Stationary.
- 6 Click **Done**.

#### GLOBAL DEFINITIONS

## Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description
v_prescr	O[m]	0 m	Prescribed displacement
W	7.32[m]	7.32 m	Width
Н	3.66[m]	3.66 m	Height

## GEOMETRY I

## Rectangle I (rI)

- I In the Geometry toolbar, click Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type W\*1.1.

- 4 In the Height text field, type H.
- **5** Locate the **Position** section. In the **x** text field, type -W\*0.1.
- **6** Click to expand the **Layers** section. In the table, enter the following settings:

Layer name	Thickness (m)		
Layer 1	W*0.1		

- 7 Clear the Layers on bottom check box.
- 8 Select the Layers to the left check box.

The left layer is used to model an infinite element domain.

Point I (ptl)

- I In the Geometry toolbar, click Point.
- 2 In the Settings window for Point, locate the Point section.
- 3 In the x text field, type W-1.57[m].
- 4 In the y text field, type H.

Form Union (fin)

- I In the Model Builder window, click Form Union (fin).
- 2 In the Settings window for Form Union/Assembly, click 📔 Build Selected.

### **DEFINITIONS**

Infinite Element Domain I (iel)

- I In the Definitions toolbar, click  $\[ \]$  Infinite Element Domain.
  - The infinite element domain is scaled by a factor of 1000.
- **2** Select Domain 1 only.

#### DEFINITIONS

Add a nonlocal integration coupling to evaluate the displacement at the center of the applied pressure (point 7).

Integration | (intob |)

- I In the **Definitions** toolbar, click Nonlocal Couplings and choose Integration.
- 2 In the Settings window for Integration, locate the Source Selection section.
- 3 From the Geometric entity level list, choose Point.
- **4** Select Point 7 only.

## SOLID MECHANICS (SOLID)

Linear Elastic Material I

In the Model Builder window, under Component I (compl)>Solid Mechanics (solid) click Linear Elastic Material I.

Soil Plasticity I

- I In the Physics toolbar, click Attributes and choose Soil Plasticity.
- 2 In the Settings window for Soil Plasticity, locate the Soil Plasticity section.
- 3 Select the Match to Mohr-Coulomb criterion check box.

Linear Elastic Material I

In the Model Builder window, click Linear Elastic Material 1.

Soil Plasticity 2

- I In the Physics toolbar, click Attributes and choose Soil Plasticity.
- 2 In the Settings window for Soil Plasticity, locate the Soil Plasticity section.
- 3 From the Material model list, choose Mohr-Coulomb.

Fixed Constraint I

- I In the Physics toolbar, click Boundaries and choose Fixed Constraint.
- **2** Select Boundaries 2 and 5 only.

Symmetry I

- I In the Physics toolbar, click Boundaries and choose Symmetry.
- 2 Select Boundary 8 only.

Roller I

- I In the Physics toolbar, click Boundaries and choose Roller.
- 2 Select Boundary 1 only.

Boundary Load 1

- I In the Physics toolbar, click Boundaries and choose Boundary Load.
- **2** Select Boundary 7 only.
- 3 In the Settings window for Boundary Load, locate the Force section.
- **4** Specify the  $\mathbf{F}_{A}$  vector as

0	x
footing_pressure	у

- 5 Click the Show More Options button in the Model Builder toolbar.
- 6 In the Show More Options dialog box, in the tree, select the check box for the node Physics>Equation-Based Contributions.
- 7 Click OK.

## Global Equations 1

- I In the Physics toolbar, click A Global and choose Global Equations.
- 2 In the Settings window for Global Equations, locate the Global Equations section.
- **3** In the table, enter the following settings:

Name	f(u,ut,utt,t) (I)	Initial value (u_0) (1)	Initial value (u_t0) (1/s)	Description
footing_pressure	intop1(v) - v_prescr	0	0	

- 4 Locate the Units section. Click Select Dependent Variable Quantity.
- 5 In the Physical Quantity dialog box, type pressure in the text field.
- 6 Click **Filter**.
- 7 In the tree, select General>Pressure (Pa).
- 8 Click OK.
- 9 In the Settings window for Global Equations, locate the Units section.
- 10 Click Select Source Term Quantity.
- II In the Physical Quantity dialog box, type displacement in the text field.
- 12 Click **Filter**.
- 13 In the tree, select General>Displacement (m).
- 14 Click OK.

## MATERIALS

Material I (mat1)

- I In the Model Builder window, under Component I (compl) right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, locate the Material Contents section.

**3** In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Young's modulus	Е	207[MPa]	Pa	Young's modulus and Poisson's ratio
Poisson's ratio	nu	0.3	I	Young's modulus and Poisson's ratio
Cohesion	cohesion	69[kPa]	Pa	Mohr-Coulomb
Angle of internal friction	internalphi	20[deg]	rad	Mohr-Coulomb

#### MESH I

- I In the Model Builder window, under Component I (compl) click Mesh I.
- 2 In the Settings window for Mesh, locate the Sequence Type section.
- **3** From the list, choose **User-controlled mesh**.

#### Size

- I In the Model Builder window, under Component I (compl)>Mesh I click Size.
- 2 In the Settings window for Size, locate the Element Size section.
- 3 From the Predefined list, choose Finer.

## Free Triangular 1

- I In the Model Builder window, click Free Triangular I.
- 2 In the Settings window for Free Triangular, locate the Domain Selection section.
- 3 From the Geometric entity level list, choose Domain.
- 4 Select Domain 2 only.

## Mapped I

- I In the Mesh toolbar, click Mapped.
  - Use a mapped mesh in the infinite element domain to improve convergence.
- 2 In the Settings window for Mapped, locate the Domain Selection section.
- 3 From the Geometric entity level list, choose Domain.
- 4 Select Domain 1 only.
- 5 Click Build All.

The first study is parametric and solves the model assuming the Drucker-Prager criterion. The stepping parameter v prescr represents the vertical displacement at the center of the applied pressure (point 7).

#### DRUCKER-PRAGER CRITERION

- I In the Model Builder window, click Study I.
- 2 In the Settings window for Study, type Drucker-Prager Criterion in the Label text field.

## Step 1: Stationary

- I In the Model Builder window, under Drucker-Prager Criterion click Step 1: Stationary.
- 2 In the Settings window for Stationary, locate the Physics and Variables Selection section.
- 3 Select the Modify model configuration for study step check box.
- 4 In the tree, select Component I (compl)>Solid Mechanics (solid)> Linear Elastic Material I>Soil Plasticity 2.
- 5 Click Disable.
- 6 Click to expand the Study Extensions section. Select the Auxiliary sweep check box.
- 7 Click + Add.
- **8** In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
v_prescr (Prescribed	range(0, -5e-4, -32e-3)	m
displacement)		

#### Solution I (soll)

- I In the Study toolbar, click Show Default Solver.
- 2 In the Model Builder window, expand the Solution I (soll) node.
- 3 In the Model Builder window, expand the Drucker-Prager Criterion> Solver Configurations>Solution I (soll)>Stationary Solver I node, then click Parametric I.
- 4 In the Settings window for Parametric, click to expand the Continuation section. To improve convergence, set the predictor of the parametric solver to constant.
- 5 From the Predictor list, choose Constant.
- 6 In the Study toolbar, click **Compute**.

Add a predefined plot to easily visualize the plasticized region.

#### ADD PREDEFINED PLOT

- I In the Home toolbar, click Add Predefined Plot to open the Add Predefined Plot
- 2 Go to the Add Predefined Plot window.
- 3 In the tree, select Drucker-Prager Criterion/Solution I (soll)>Solid Mechanics> Equivalent Plastic Strain (solid).
- 4 Click Add Plot in the window toolbar.

#### ROOT

The second study is also parametric and solves the model assuming a Mohr-Coulomb criterion. Again, the stepping parameter v prescr represents the vertical displacement at the center of the applied pressure (point 7).

#### ADD STUDY

- I In the Home toolbar, click Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select General Studies>Stationary.
- 4 Click Add Study in the window toolbar.
- 5 In the Home toolbar, click Add Study to close the Add Study window.

#### MOHR-COULOMB CRITERION

- I In the Model Builder window, click Study 2.
- 2 In the Settings window for Study, type Mohr-Coulomb Criterion in the Label text field.

#### Step 1: Stationary

- I In the Model Builder window, under Mohr-Coulomb Criterion click Step 1: Stationary.
- 2 In the Settings window for Stationary, locate the Physics and Variables Selection section.
- 3 Select the Modify model configuration for study step check box.
- 4 In the tree, select Component I (compl)>Solid Mechanics (solid)> Linear Elastic Material I>Soil Plasticity I.
- 5 Click ODisable.
- 6 Locate the Study Extensions section. Select the Auxiliary sweep check box.
- 7 Click + Add.

**8** In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
v_prescr (Prescribed displacement)	range(0, -5e-4, -24.5e- 3)	m

## Solution 2 (sol2)

I In the Study toolbar, click Show Default Solver.

To improve convergence, change the relative tolerance from 1e-3 to 1e-4 and set the predictor of the continuation solver to constant.

- 2 In the Model Builder window, expand the Solution 2 (sol2) node, then click Stationary Solver 1.
- 3 In the Settings window for Stationary Solver, locate the General section.
- 4 In the Relative tolerance text field, type 1e-4.
- 5 In the Model Builder window, expand the Mohr-Coulomb Criterion>Solver Configurations> Solution 2 (sol2)>Stationary Solver I node, then click Parametric I.
- 6 In the Settings window for Parametric, locate the Continuation section.
- 7 From the Predictor list, choose Constant.
- 8 In the Study toolbar, click **Compute**.

#### ADD PREDEFINED PLOT

- I Go to the Add Predefined Plot window.
- 2 In the tree, select Mohr-Coulomb Criterion/Solution 2 (sol2)>Solid Mechanics> Equivalent Plastic Strain (solid).
- 3 Click Add Plot in the window toolbar.
- 4 In the Home toolbar, click Add Predefined Plot to close the Add Predefined Plot window.

#### RESULTS

Remove the infinite element domain from the dataset for plotting.

#### Selection

- I In the Model Builder window, expand the Results>Datasets node.
- 2 Right-click Drucker-Prager Criterion/Solution I (soll) and choose Selection.
- 3 In the Settings window for Selection, locate the Geometric Entity Selection section.
- 4 From the Geometric entity level list, choose Domain.

**5** Select Domain 2 only.

#### Selection

- I In the Model Builder window, right-click Mohr-Coulomb Criterion/Solution 2 (sol2) and choose Selection.
- 2 In the Settings window for Selection, locate the Geometric Entity Selection section.
- 3 From the Geometric entity level list, choose Domain.
- **4** Select Domain 2 only.

Add mirror datasets to improve the result visualization.

## Mirror 2D I

- I In the Results toolbar, click More Datasets and choose Mirror 2D.
- 2 In the Settings window for Mirror 2D, locate the Axis Data section.
- 3 In row Point I, set X to W.
- 4 In row Point 2, set X to W.

#### Mirror 2D 2

- I Right-click Mirror 2D I and choose Duplicate.
- 2 In the Settings window for Mirror 2D, locate the Data section.
- 3 From the Dataset list, choose Mohr-Coulomb Criterion/Solution 2 (sol2).

#### Stress, Drucker-Prager Criterion

- I In the Model Builder window, under Results click Stress (solid).
- 2 In the Settings window for 2D Plot Group, type Stress, Drucker-Prager Criterion in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Mirror 2D 1.
- 4 Locate the Plot Settings section. Clear the Plot dataset edges check box.

#### Surface 1

- I In the Model Builder window, expand the Stress, Drucker-Prager Criterion node, then click Surface 1.
- 2 In the Settings window for Surface, locate the Expression section.
- 3 From the Unit list, choose kPa.

## Deformation

- I In the Model Builder window, expand the Surface I node, then click Deformation.
- 2 In the Settings window for Deformation, locate the Scale section.

3 Select the Scale factor check box. In the associated text field, type 10.

Arrow Line 1

- I In the Model Builder window, right-click Stress, Drucker-Prager Criterion and choose Arrow Line.
- 2 In the Settings window for Arrow Line, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Component I (compl)> Solid Mechanics>Load>solid.F\_Ax,solid.F\_Ay - Load (spatial frame).
- 3 Click to expand the **Title** section. From the **Title type** list, choose **None**.
- 4 Locate the Coloring and Style section. From the Arrow base list, choose Head.
- **5** Select the **Scale factor** check box. In the associated text field, type 5E-7.
- 6 Click to expand the Inherit Style section. From the Plot list, choose Surface 1.

#### Deformation I

Right-click Arrow Line I and choose Deformation.

Stress, Drucker-Prager Criterion

In the Stress, Drucker-Prager Criterion toolbar, click  **Plot**.

Stress, Mohr-Coulomb Criterion

- I In the Model Builder window, under Results click Stress (solid) I.
- 2 In the Settings window for 2D Plot Group, type Stress, Mohr-Coulomb Criterion in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Mirror 2D 2.
- **4** Locate the **Plot Settings** section. Clear the **Plot dataset edges** check box.

Plastic Region, Mohr-Coulomb Criterion

- I In the Model Builder window, under Results click Equivalent Plastic Strain (solid) I.
- 2 In the Settings window for 2D Plot Group, type Plastic Region, Mohr-Coulomb Criterion in the Label text field.
- **3** Locate the **Plot Settings** section. Clear the **Plot dataset edges** check box.
- 4 Locate the Data section. From the Dataset list, choose Mirror 2D 2.

Surface I

- I In the Model Builder window, expand the Plastic Region, Mohr-Coulomb Criterion node, then click Surface 1.
- 2 In the Settings window for Surface, locate the Expression section.
- 3 In the Expression text field, type solid.epeGp>0.

## Deformation I

- I Right-click Surface I and choose Deformation.
- 2 In the Settings window for Deformation, locate the Scale section.
- 3 Select the Scale factor check box. In the associated text field, type 10.
- 4 In the Plastic Region, Mohr-Coulomb Criterion toolbar, click on Plot.

#### Plastic Region, Drucker-Prager Criterion

- I In the Model Builder window, under Results click Equivalent Plastic Strain (solid).
- 2 In the Settings window for 2D Plot Group, type Plastic Region, Drucker-Prager Criterion in the Label text field.

#### Surface 1

- I In the Model Builder window, expand the Plastic Region, Drucker-Prager Criterion node, then click Surface 1.
- 2 In the Settings window for Surface, locate the Expression section.
- 3 In the Expression text field, type solid.epeGp>0.

## Deformation I

- I Right-click Surface I and choose Deformation.
- 2 In the Settings window for Deformation, locate the Scale section.
- 3 Select the Scale factor check box. In the associated text field, type 10.
- 4 In the Plastic Region, Drucker-Prager Criterion toolbar, click **Plot**.
- 5 Click the **Zoom Extents** button in the **Graphics** toolbar.

## Footing Pressure vs. Displacement

- I In the Home toolbar, click **Add Plot Group** and choose ID Plot Group.
- 2 Drag and drop below Plastic Region, Mohr-Coulomb Criterion.
- 3 In the Settings window for ID Plot Group, type Footing Pressure vs. Displacement in the Label text field.
- **4** Locate the **Legend** section. From the **Position** list, choose **Lower right**.

## Point Graph 1

- I Right-click Footing Pressure vs. Displacement and choose Point Graph.
- 2 In the Settings window for Point Graph, locate the Data section.
- 3 From the Dataset list, choose Drucker-Prager Criterion/Solution I (soll).
- **4** Select Point 7 only.

- 5 Locate the y-Axis Data section. In the Expression text field, type abs(footing pressure).
- 6 From the Unit list, choose kPa.
- **7** Select the **Description** check box. In the associated text field, type Footing pressure.
- 8 Locate the x-Axis Data section. From the Parameter list, choose Expression.
- 9 In the Expression text field, type abs(v).
- 10 Select the **Description** check box. In the associated text field, type Vertical displacement.
- II Click to expand the **Legends** section. Select the **Show legends** check box.
- 12 From the Legends list, choose Manual.
- **I3** In the table, enter the following settings:

# Legends Drucker-Prager

## Point Graph 2

- I Right-click Point Graph I and choose Duplicate.
- 2 In the Settings window for Point Graph, locate the Data section.
- 3 From the Dataset list, choose Mohr-Coulomb Criterion/Solution 2 (sol2).
- 4 Click to expand the **Title** section. From the **Title type** list, choose **None**.
- **5** Locate the **Legends** section. In the table, enter the following settings:

# Legends Mohr-Coulomb

Footing Pressure vs. Displacement

- I In the Model Builder window, click Footing Pressure vs. Displacement.
- 2 In the Footing Pressure vs. Displacement toolbar, click Plot.