Created in COMSOL Multiphysics 6.2

Iron Sphere in a Magnetic Field — 60 Hz

Introduction

This tutorial is part of a series on modeling an iron sphere in a background magnetic field within the Introduction to Electromagnetics tutorial group. This tutorial focuses the case of an magnetically permeable iron sphere in a spatially uniform magnetic field where the magnetic field sinusoidally varies in time at a frequency of 60 Hz. At this frequency, the skin depth of the iron used in the model is ~ 0.3 mm, which is larger than the sphere itself.

A factor that requires consideration in this tutorial scenario is the conductivity of air, σ_{air}. In practice σ_{air} is negligible but setting it to zero leads to numerical difficulties. This tutorial covers two methods of approaching this by using a stabilization conductivity in the **Free Space** feature or using **Gauge Fixing**. To assess the performance of these methods, we will look at the magnetic dissipation. Using a stabilization conductivity in **Free Space** will produce artificial magnetic dissipation in that domain. This will be calculated for the different methods and compared to the larger dissipation in the iron sphere itself.

Model Definition

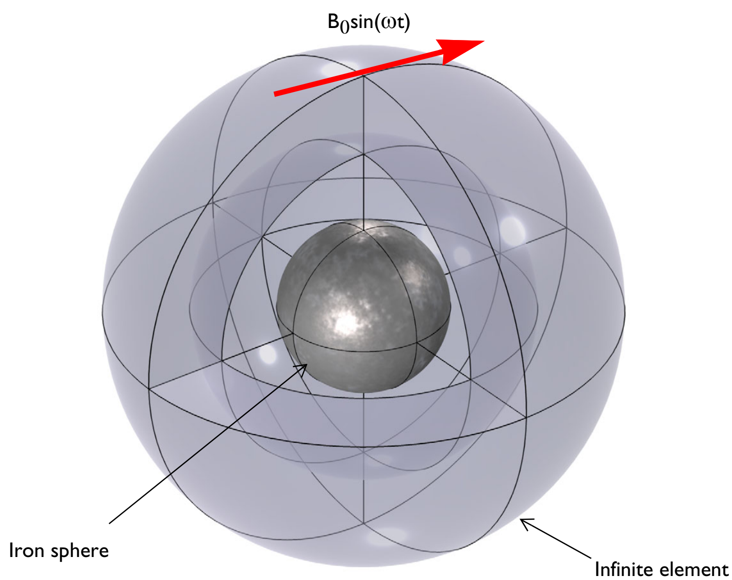


Figure 1: A magnetically permeable iron sphere in a spatially uniform background magnetic field that sinusoidally varies in time with angular frequency, w. The sphere at the center is surrounded by air and enclosed in a region of Infinite Elements.

Each model in this tutorial series uses the same basic structure illustrated in Figure 1. It consists of a 0.25 mm diameter iron sphere, with a relative permeability of, $\mu_r = 4000$, placed in a spatially uniform background magnetic field of strength $B_0 = 1$ mT. In this case, that magnetic field oscillates at a frequency of 60 Hz.

As discussed in the introduction, the surrounding air normally has an infinite skin depth, which leads to numerical difficulties when solving. We can overcome this in two ways:

1. 1) Use a stabilization conductivity corresponding to a ratio of 1000:1 between the largest and smallest skin depths in the model. In this case, this means using a value of $\sigma_{stab} = 5000$ S/m. This will add artificial magnetic dissipation in the air domains in the model.
2. Use gauge fixing. This adds an additional equation to the system of equations being solved, and as a consequence significantly increases the computational effort needed to solve the model. However, this approach does not require a stabilization conductivity, resulting in negligible artificial magnetic dissipation in the air domain.

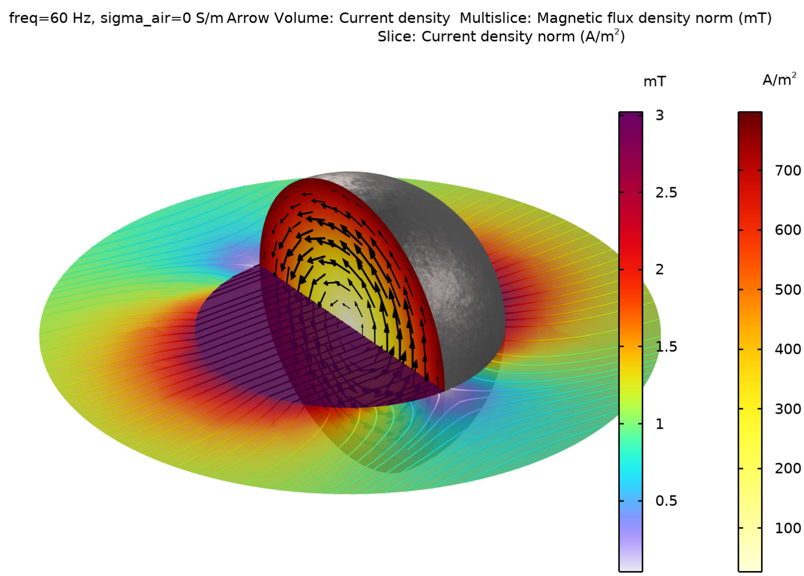


Figure 2: Cross-section of the iron sphere showing the induced current in the sphere and the magnetic flux calculated using gauge fixing.

Figure 2 plots the magnetic field and the induced current density for the model with gauge fixing. Without gauge fixing, a stabilization conductivity of 5000 S/m is used, leading to a skin depth in the air of ~ 0.9 m and a total dissipation in the air of 3.8×10^{-16} W. When using gauge fixing, the artificial conductivity is not needed and set to 0 S/m. This corresponds to an infinite skin depth in the surrounding air and negligible dissipation. In both cases, the total dissipation in the iron sphere is 9.1×10^{-14} W. This is the dominating value in this model.

Using gauge fixing increases the solution time and memory needed to solve the problem, and generally only slightly improves the solution. Therefore, it should be used sparingly. In any case, it is always recommended to carefully study the effects of artificial conductivity on the relative skin depths in the model and to keep in mind that this is a function of the operating frequency.

Application Library path: `ACDC_Module/Introductory_Electromagnetics/iron_sphere_bfield_02_60hz`

Modeling Instructions

This tutorial will demonstrate the physics of an iron sphere in a spatially uniform magnetic field sinusoidally varying at 60 Hz. The instructions on the following pages will help you to build, configure, solve, and analyze the model. If anything seems out of order, please retrace your steps. The finalized model — available in the model's Application Libraries folder — can help you out. You can compare it directly to your current model by means of the **Compare** option in the **Developer** toolbar.

ROOT

The geometry, materials, and selections have been prepared in the *Introduction* tutorial (chapter 1). They have been saved in the file `iron_sphere_bfield_00_introduction.mph`. You can start by opening this file and saving it under a new name.

Hint: if you are new to COMSOL Multiphysics, it is worthwhile to check out the Introduction tutorial first.

1 From the **File** menu, choose **Open**.

2 Browse to the model's Application Libraries folder and double-click the file `iron_sphere_bfield_00_introduction.mph`.

3 From the **File** menu, choose **Save As**.

4 Browse to a suitable folder and type the filename `iron_sphere_bfield_02_60hz.mph`.

MAGNETIC FIELDS (MF)

Free Space 1

1 In the **Model Builder** window, expand the **Component 1 (comp1)>Magnetic Fields (mf)** node, then click **Free Space 1**.

2 In the **Settings** window for **Free Space**, locate the **Domain Selection** section.

3 In the list, select **9 (overridden)**.

Next, set up the stabilization conductivity of **Free Space** to the use the parameter sigma_air. This allows you to set the desired value in the studies later in this tutorial.

4 Locate the **Stabilization** section. Find the **Typical frequency** subsection. From the f_{typ} list, choose **User defined**.

5 From the σ_{stab} list, choose **User defined**. In the associated text field, type `sigma_air`.

This tutorial starts by investigating the case without using Gauge Fixing. The first steps initiate this study.

MESH 1

For these simulations, the default Physics controlled mesh with a fine element size is sufficient.

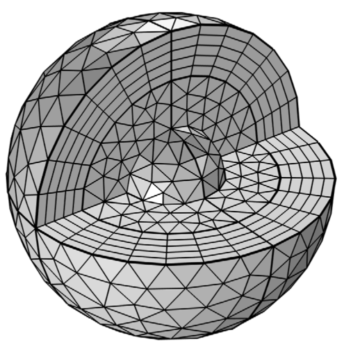
1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2 In the **Settings** window for **Mesh**, locate the **Physics-Controlled Mesh** section.

3 From the **Element size** list, choose **Fine**.

4 Click **Build All**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar.



Note: In the introduction modeling steps, the nearest upper quarter sphere was hidden to improve visibility in the result plots. This allows the visibility of the mesh layers of the Infinite Element Domain and the Analysis Domain.

STUDY 1 - WITHOUT GAUGE FIXING

1 In the **Model Builder** window, click **Study 1**.

2 In the **Settings** window for **Study**, type `Study 1 - Without Gauge Fixing` in the **Label** text field.

Step 1: Frequency Domain

1 In the **Model Builder** window, expand the **Study 1 - Without Gauge Fixing** node, then click **Step 1: Frequency Domain**.

2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

3 In the **Frequencies** text field, type `60`.

For the first study without using Gauge Fixing, a relatively high value of stabilization conductivity needed. In this case it is 5000 S/m.

4 Click to expand the **Study Extensions** section. Select the **Auxiliary sweep** check box.

5 Click **Add**.

6 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
sigma_air (Stabilization electrical conductivity of air)	5000	S/m

7 In the **Home** toolbar, click **Compute**.

RESULTS

Magnetic Flux Density Norm (mf)

In the **Model Builder** window, expand the **Magnetic Flux Density Norm (mf)** node.

Study 1 - Without Gauge Fixing

1 In the **Model Builder** window, expand the **Results>Datasets** node, then click **Study 1 - Without Gauge Fixing/Solution 1 (sol1)**.

2 In the **Settings** window for **Solution**, type `Study 1 - Without Gauge Fixing` in the **Label** text field.

Selection

1 Right-click **Study 1 - Without Gauge Fixing** and choose **Selection**.

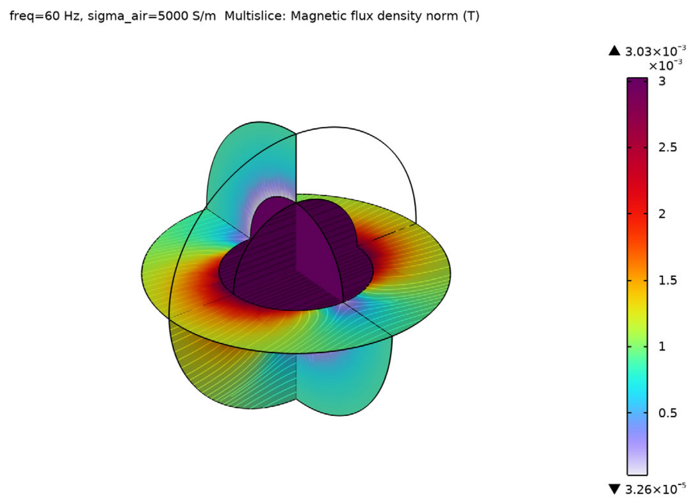
2 In the **Settings** window for **Selection**, locate the **Geometric Entity Selection** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 From the **Selection** list, choose **Analysis domain**.

Magnetic Flux Density Norm (mf)

1. In the **Model Builder** window, under **Results** click **Magnetic Flux Density Norm (mf)**.
2. In the **Magnetic Flux Density Norm (mf)** toolbar, click **Plot**.
3. Click the **Zoom Extents** button in the **Graphics** toolbar.



Next, plot the current induced in the sphere.

Current Density - Without Gauge Fixing

1. In the **Home** toolbar, click **Add Plot Group** and choose **3D Plot Group**.
2. In the **Settings** window for **3D Plot Group**, type `Current Density - Without Gauge Fixing` in the **Label** text field.
3. Click to expand the **Selection** section. From the **Geometric entity level** list, choose **Domain**.
4. From the **Selection** list, choose **Iron Sphere**.

Slice 1

1. Right-click **Current Density - Without Gauge Fixing** and choose **Slice**.
2. In the **Settings** window for **Slice**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)>Magnetic Fields>Currents and charge>mf.normJ - Current density norm - A/m²**.
3. Locate the **Plane Data** section. In the **Planes** text field, type `1`.
4. Locate the **Coloring and Style** section. Click **Change Color Table**.

5 In the **Color Table** dialog box, select **Thermal>Thermal** in the tree.

6 Click **OK**.

7 In the **Settings** window for **Slice**, locate the **Coloring and Style** section.

8 From the **Color table transformation** list, choose **Reverse**.

9 In the **Current Density - Without Gauge Fixing** toolbar, click **Plot**.

Arrow Volume 1

1 In the **Model Builder** window, right-click **Current Density - Without Gauge Fixing** and choose **Arrow Volume**.

2 In the **Settings** window for **Arrow Volume**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)> Magnetic Fields>Currents and charge>mf.Jx,mf.Jy,mf.Jz - Current density**.

3 Locate the **Arrow Positioning** section. Find the **x grid points** subsection. In the **Points** text field, type 1.

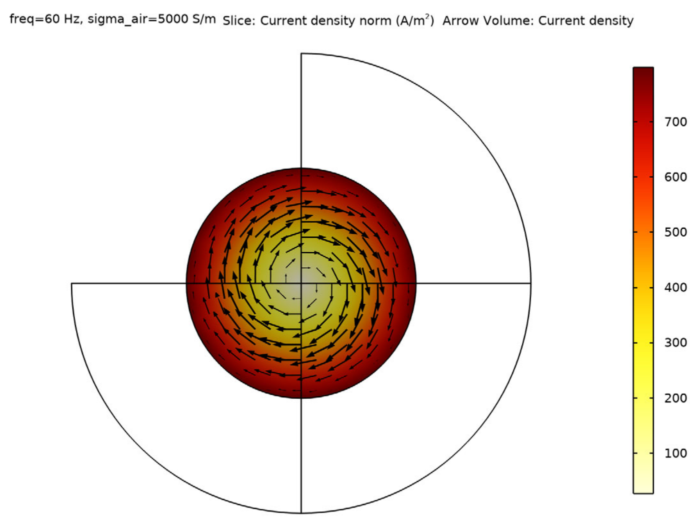
4 Find the **y grid points** subsection. In the **Points** text field, type 15.

5 Find the **z grid points** subsection. In the **Points** text field, type 15.

6 Locate the **Coloring and Style** section. From the **Color** list, choose **Black**.

7 In the **Current Density - Without Gauge Fixing** toolbar, click **Plot**.

8 Click the **Go to YZ View** button in the **Graphics** toolbar.



9 Click the **Go to Default View** button in the **Graphics** toolbar.

Skin Depth - Iron Sphere

1 In the **Results** toolbar, click 8.85 e-12 **More Derived Values** and choose **Maximum>Volume Maximum**.

The skin depth of each material at this frequency is defined in the introduction. This can be confirmed with the values used in the different domains in the model. The value is uniform throughout the domain so this can be easily evaluated by using the maximum value in the domain.

2 In the **Settings** window for **Volume Maximum**, type `Skin Depth - Iron Sphere` in the **Label** text field.

3 Locate the **Selection** section. From the **Selection** list, choose **Iron Sphere**.

4 Click **Replace Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (comp1)>Magnetic Fields>Material properties>mf.deltaS - Skin depth - m**.

5 Click **Evaluate**.

The skin depth of the iron sphere should be about 0.3 mm.

Skin Depth - Air (Without Gauge Fixing)

1 In the **Results** toolbar, click 8.85 e-12 **More Derived Values** and choose **Maximum>Volume Maximum**.

2 In the **Settings** window for **Volume Maximum**, type `Skin Depth - Air (Without Gauge Fixing)` in the **Label** text field.

3 Locate the **Selection** section. From the **Selection** list, choose **Analysis domain**.

4 Click **Replace Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (comp1)>Magnetic Fields>Material properties>mf.deltaS - Skin depth - m**.

5 Click **Evaluate**.

The skin depth of air with a stabilization conductivity of 5000 S/m is around 919 mm.

To assess the dissipation in the different domains, integrate over the respective volumes.

Magnetic Dissipation - Iron Sphere

1 In the **Results** toolbar, click 8.85 e-12 **More Derived Values** and choose **Integration>Volume Integration**.

2 In the **Settings** window for **Volume Integration**, type `Magnetic Dissipation - Iron Sphere` in the **Label** text field.

3 Locate the **Selection** section. From the **Selection** list, choose **Iron Sphere**.

4 Click **Replace Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (comp1)>Magnetic Fields>Heating and losses>mf.Qrh - Volumetric loss density, electric - W/m³**.

5 Click **Evaluate**.

The dissipation in the iron sphere is largest in the simulation, having a value of 9.1e-14 W.

Dissipation - Air (Without Gauge Fixing)

1 In the **Results** toolbar, click 8.85 e-12 **More Derived Values** and choose **Integration>Volume Integration**.

2 In the **Settings** window for **Volume Integration**, type `Dissipation - Air (Without Gauge Fixing)` in the **Label** text field.

This integral should be over the air volume in the analysis domain, so we remove the iron sphere domain from the integral volume.

3 Locate the **Selection** section. From the **Selection** list, choose **Analysis domain**.

4 In the list, select **9**.

5 Click **Remove from Selection**.

6 Select Domains 5–8, 12, 13, 15, and 16 only.

7 Click **Replace Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (comp1)>Magnetic Fields>Heating and losses>mf.Qrh - Volumetric loss density, electric - W/m³**.

8 Click **Evaluate**.

Without using Gauge fixing and using an artificial conductivity of 5000 S/m, the dissipation in the air is 3.8e-16 W.

MAGNETIC FIELDS (MF)

That was the results given using the artificially high stabilization conductivity of air. Next, we will investigate the effect of using Gauge Fixing in the simulation. In this case, the stabilization conductivity is not needed so it is set to 0 S/m.

Gauge Fixing for A-field 1

1 In the **Model Builder** window, expand the **Component 1 (comp1)>Magnetic Fields (mf)** node.

2 Right-click **Magnetic Fields (mf)** and choose **Gauge Fixing for A-field**.

STUDY 1 - WITHOUT GAUGE FIXING

The first study did not use Gauge fixing, this can be disabled in the study setting windows in case the user wishes to rerun the first study.

Step 1: Frequency Domain

1 In the **Model Builder** window, under **Study 1 - Without Gauge Fixing** click **Step 1: Frequency Domain**.

2 In the **Settings** window for **Frequency Domain**, locate the **Physics and Variables Selection** section.

3 Select the **Modify model configuration for study step** check box.

4 In the tree, select **Component 1 (comp1)>Magnetic Fields (mf)>Gauge Fixing for A-field 1**.

5 Right-click and choose **Disable**.

Next, create a second study to investigate the case with Gauge Fixing.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies> Frequency Domain**.

4 Click **Add Study** in the window toolbar.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 2 - WITH GAUGE FIXING

1 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

2 In the **Frequencies** text field, type `60`.

3 Click to expand the **Study Extensions** section. Select the **Auxiliary sweep** check box.

4 Click **Add**.

5 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
sigma_air (Stabilization electrical conductivity of air)	0	S/m

6 In the **Model Builder** window, click **Study 2**.

7 In the **Settings** window for **Study**, type `Study 2 - With Gauge Fixing` in the **Label** text field.

8 In the **Home** toolbar, click **Compute**.

RESULTS

Study 2 - With Gauge Fixing/Solution 2 (sol2)

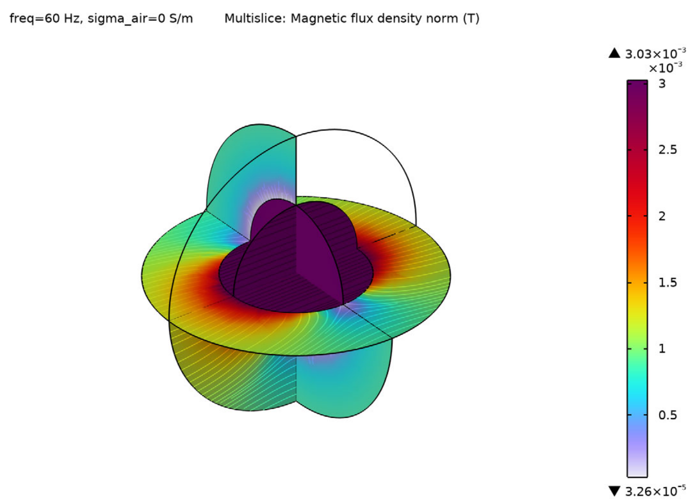
The new results are now plotted in the same manner as the previous study.

Selection

1. In the **Model Builder** window, right-click **Study 2 - With Gauge Fixing/Solution 2 (sol2)** and choose **Selection**.
2. In the **Settings** window for **Selection**, locate the **Geometric Entity Selection** section.
3. From the **Geometric entity level** list, choose **Domain**.
4. From the **Selection** list, choose **Analysis domain**.
5. Right-click **Selection** and choose **Plot**.

Magnetic Flux Density Norm (mf) 1

1. In the **Model Builder** window, under **Results** click **Magnetic Flux Density Norm (mf) 1**.
2. In the **Magnetic Flux Density Norm (mf) 1** toolbar, click **Plot**.



Current Density - Without Gauge Fixing

In the **Model Builder** window, right-click **Current Density - Without Gauge Fixing** and choose **Duplicate**.

Current Density - With Gauge Fixing

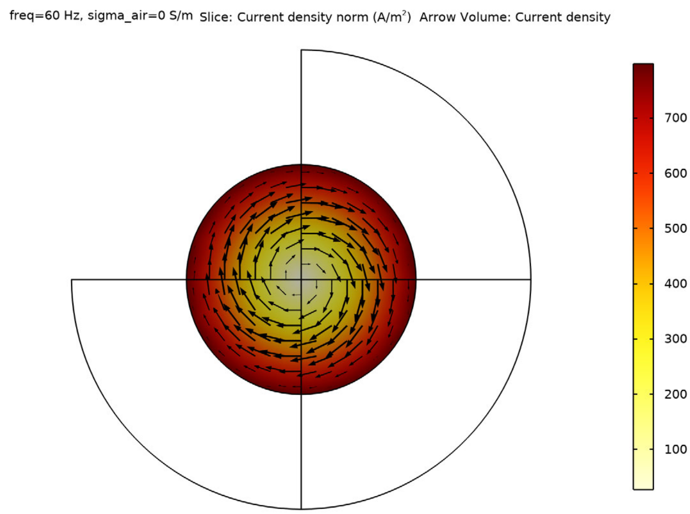
1 In the **Model Builder** window, under **Results** click **Current Density - Without Gauge Fixing 1**.

2 In the **Settings** window for **3D Plot Group**, type `Current Density - With Gauge Fixing` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Study 2 - With Gauge Fixing/Solution 2 (sol2)**.

4 Click the **Go to YZ View** button in the **Graphics** toolbar.

5 In the **Current Density - With Gauge Fixing** toolbar, click **Plot**.



The resultant current induced within the iron is comparable to the results produced without using the gauge fixing, thus demonstrating the applicability of both techniques for modeling the current behavior in the iron sphere.

6 Click the **Go to Default View** button in the **Graphics** toolbar.

For a quantitative comparison, evaluate the skin depth and dissipation in the free space domain.

Skin Depth - Air (With Gauge Fixing)

1 In the **Results** toolbar, click 8.85 e-12 **More Derived Values** and choose **Maximum>Volume Maximum**.

2 In the **Settings** window for **Volume Maximum**, type `Skin Depth - Air (With Gauge Fixing)` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Study 2 - With Gauge Fixing/ Solution 2 (sol2)**.

4 Locate the **Selection** section. From the **Selection** list, choose **Analysis domain**.

5 Click **Replace Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (comp1)>Magnetic Fields>Material properties>mf.deltaS - Skin depth - m**.

6 Click **Evaluate**.

This new skin depth should now be infinite.

Dissipation - Air (With Gauge Fixing)

1 In the **Results** toolbar, click 8.85 e-12 **More Derived Values** and choose **Integration> Volume Integration**.

Again, evaluate over the analysis domain but with the iron sphere removed from the volume integral.

2 In the **Settings** window for **Volume Integration**, type `Dissipation - Air (With Gauge Fixing)` in the **Label** text field.

3 Locate the **Selection** section. From the **Selection** list, choose **Analysis domain**.

4 In the list, select **9**.

5 Click **Remove from Selection**.

6 Select Domains 5–8, 12, 13, 15, and 16 only.

7 Click **Replace Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (comp1)>Magnetic Fields>Heating and losses>mf.Qrh - Volumetric loss density, electric - W/m³**.

8 Locate the **Data** section. From the **Dataset** list, choose **Study 2 - With Gauge Fixing/ Solution 2 (sol2)**.

9 Click **Evaluate**.

The dissipation using the gauge fixing and zero stabilization conductivity should be negligible.

Both of the modeling techniques demonstrated in this tutorial produce a comparable result for the current and magnetic flux within the iron sphere. The difference lies in the computational time as the gauge fixing takes longer to compute but requires less adjustment to the stabilization conductivity. The gauge fixing provides a more accurate result and results in three order of magnitude less artificial dissipation in the surrounding free space. The default settings of the **Free Space** feature estimates an appropriate stabilization conductivity using the model geometry scale and the solver frequency.

However, as demonstrated in this tutorial, the stabilization conductivity can be explicitly defined. This allows the user to optimize the stabilization conductivity according to their modeling requirements.