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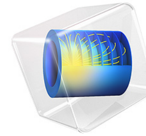

Electric Field Between Concentric Cylinders

Introduction

This introductory model treats the electrostatics problem of two concentric cylinders of infinite length, which is commonly found in textbooks. Since the problem can be solved analytically, the model can be used to compare theory with numerical simulation results. Two cases are considered:

- A fixed potential on each cylinder
- A surface charge density on one cylinder and a fixed potential on the other

Model Definition

First, consider the analytical solution to this problem to use for comparison. Let r_i be the radius of the inner cylinder and r_o the radius of the outer cylinder. In the first case, where we have a fixed potential V_0 on the inner boundary and ground on the outer boundary, the electric potential satisfies

$$V = \frac{V_0}{\ln(r_\mathrm{i}/r_\mathrm{o})}\ln(r/r_\mathrm{o})$$

which results in the electric field

$$\mathbf{E} = -\nabla V = \frac{-V_0}{r\ln(r_\mathrm{i}/r_\mathrm{o})}\hat{r}$$

where $\hat{r}$ is a unit vector in the radial direction.

In the case where there is a surface charge q_0 on the inner cylinder, the electric potential instead reads

$$V = \frac{-q_0}{2\pi\varepsilon_0}\ln(r/r_\mathrm{o})$$

which gives the electric field

$$\mathbf{E} = \frac{q_0}{2\pi\varepsilon_0 r}\hat{r}$$

where ε_0 is the permittivity of vacuum.

To simplify the geometry of the model, take advantage of its twofold symmetry. First, since the cylinders are infinitely long, a perpendicular cross section is sufficient to represent the whole length. Furthermore, such a cross section is rotationally symmetric and it — and by extension, the full geometry — can thus be represented by a 1D component.

In the case with a fixed surface charge density on the inner boundary, an interesting note can be made about why it might be preferable to keep ground on the outer boundary instead of another surface charge q_0 with the opposite sign. Since a surface charge is added on the inner cylinder, the latter could be considered to be more intuitive. However, there is one important difference: Having ground on a boundary sets the zero level of the electric potential, and therefore acts as a gauge fix. With only the surface charges specified, there is an infinite number of viable solutions for the electric potential. In order to solve such a problem numerically, one would have to change the default direct solver to an iterative one. Iterative solvers can solve even ungauged problems, finding one solution out of the many possible ones. It is of course important to remember that the electric field will be the same no matter which potential is chosen.

Results

The electric potential in the two cases is plotted in Figure 1 and Figure 2, where the numerical solution is compared with its analytical counterpart.

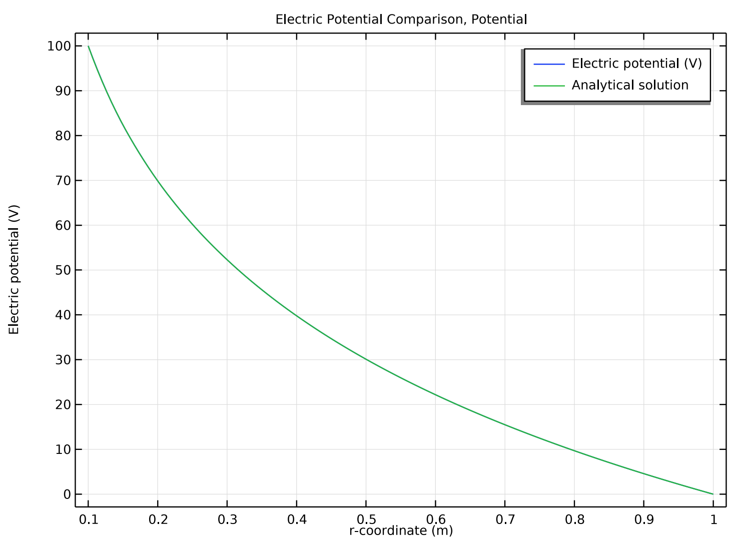

Figure 1: The electric potential from the numerical and analytical solutions for the case with a specified potential.

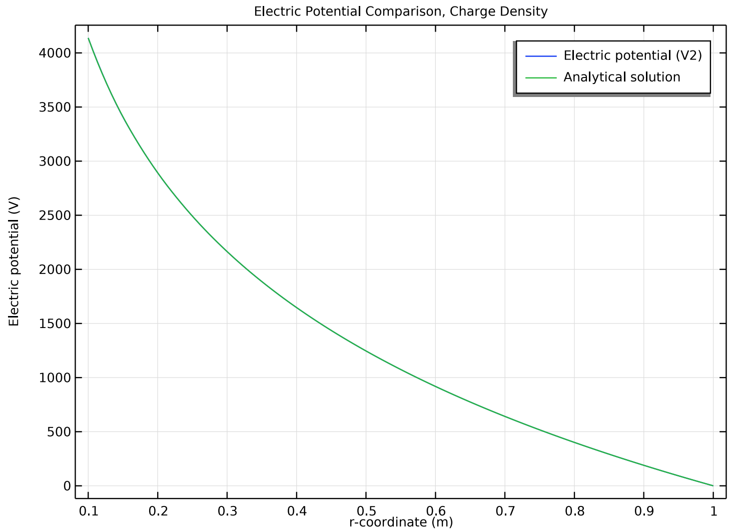

Figure 2: The electric potential from the numerical and analytical solutions for the case with a specified surface charge density.

Similarly, the solutions for the radial component of the electric field are plotted and compared in Figure 3 and Figure 4. In these plots, it can be seen how well the numerical solutions for the two different approaches agree with the analytical solutions.

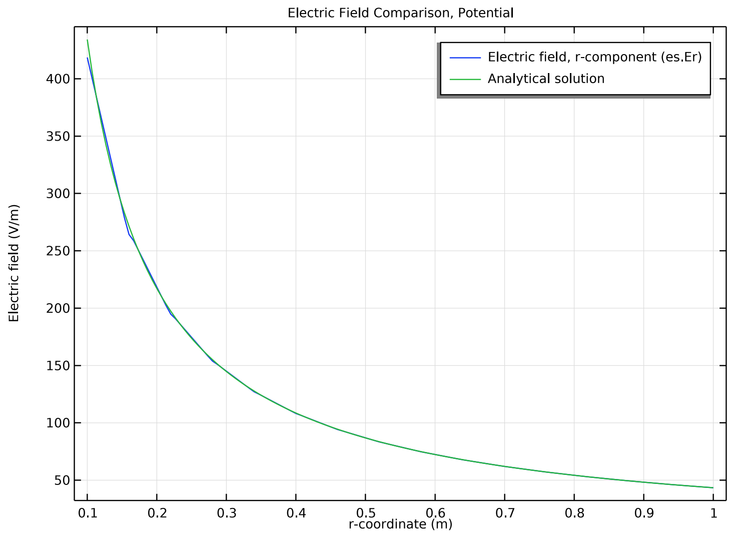

Figure 3: The radial component of the electric field from the numerical and analytical solutions for the case with a specified potential.

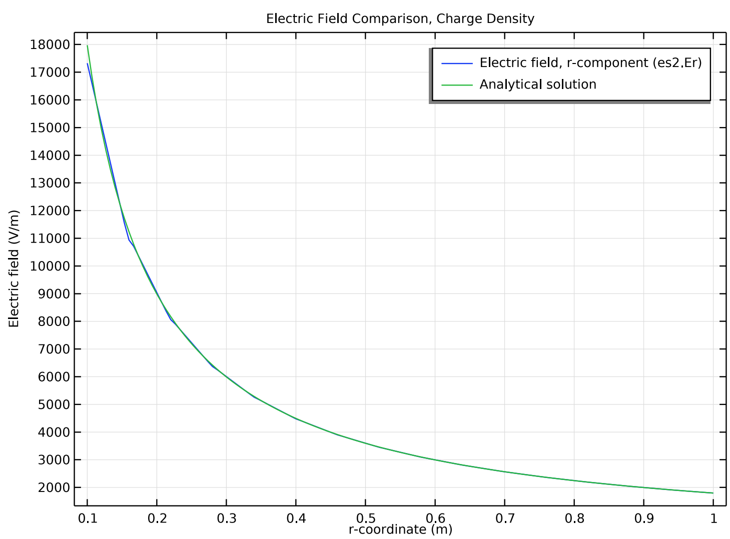

Figure 4: The radial component of the electric field from the numerical and analytical solutions for the case with a specified surface charge density.

Application Library path: `ACDC_Module/Introductory_Electrostatics/electric_field_concentric_cylinders`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **1D Axisymmetric**.

2 In the **Select Physics** tree, select **AC/DC>Electric Fields and Currents>Electrostatics (es)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Stationary**.

6 Click **Done**.

First, define some parameters that will be used when building the model.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
ri	0.1[m]	0.1 m	Radius of inner cylinder
ro	1[m]	1 m	Radius of outer cylinder
V0	100[V]	100 V	Potential at inner cylinder
q0	1e-7[C/m]	1E-7 C/m	Charge at inner cylinder

Building the geometry is simplified by using 1D axisymmetric, since only an interval is needed to create two concentric cylinders of infinite length.

GEOMETRY 1

Interval 1 (i1)

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Geometry 1** and choose **Interval**.

2 In the **Settings** window for **Interval**, locate the **Interval** section.

3 In the table, enter the following settings:

Coordinates (m)
`ri`
`ro`

4 Click **Build All Objects**.

MATERIALS

Material 1 (mat1)

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, locate the **Material Contents** section.

3 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	1	1	Basic

In the first physics interface, add ground at the outer cylinder and a fixed potential at the inner cylinder.

ELECTROSTATICS (ES)

Ground 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Electrostatics (es)** and choose **Ground**.

2 Select Boundary 2 only.

Electric Potential 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Electric Potential**.

2 Select Boundary 1 only.

3 In the **Settings** window for **Electric Potential**, locate the **Electric Potential** section.

4 In the V_0 text field, type `V0`.

Now, add a second Electrostatics physics interface. Here, a surface charge density will be added on the inner cylinder, while keeping ground on the outer cylinder.

ADD PHYSICS

1 In the **Physics** toolbar, click **Add Physics** to open the **Add Physics** window.

2 Go to the **Add Physics** window.

3 In the tree, select **Recently Used>Electrostatics (es)**.

4 Click **Add to Component 1** in the window toolbar.

5 In the **Physics** toolbar, click **Add Physics** to close the **Add Physics** window.

ELECTROSTATICS 2 (ES2)

Ground 1

1 Right-click **Component 1 (comp1)>Electrostatics 2 (es2)** and choose **Ground**.

2 Select Boundary 2 only.

Surface Charge Density 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Surface Charge Density**.

2 Select Boundary 1 only.

3 In the **Settings** window for **Surface Charge Density**, locate the **Surface Charge Density** section.

4 In the ρ_s text field, type `q0/(2*pi*ri)`.

STUDY 1

In the **Home** toolbar, click **Compute**.

RESULTS

Electric Potential Comparison, Potential

Now, it is time to compare the results with the known analytical solutions. Plots of the electric potential are added automatically, so for those only the comparison needs to be added. Then, add plots of the computed electric field and compare those with analytical solutions as well.

1 In the **Settings** window for **1D Plot Group**, type `Electric Potential Comparison, Potential` in the **Label** text field.

2 Locate the **Plot Settings** section. Select the **y-axis label** check box.

3 Click to expand the **Title** section. From the **Title type** list, choose **Label**.

Line Graph 2

1 Right-click **Electric Potential Comparison, Potential** and choose **Line Graph**.

2 Select Domain 1 only.

3 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.

4 In the **Expression** text field, type `V0*log(r/ro)/log(ri/ro)`.

5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

6 In the **Expression** text field, type `r`.

7 Click to expand the **Legends** section. Select the **Show legends** check box.

8 From the **Legends** list, choose **Manual**.

9 In the table, enter the following settings:

Legends
`Analytical solution`

Line Graph 1

1 In the **Model Builder** window, click **Line Graph 1**.

2 In the **Settings** window for **Line Graph**, locate the **Legends** section.

3 Select the **Show legends** check box.

4 Find the **Include** subsection. Select the **Expression** check box.

5 Clear the **Solution** check box.

6 Select the **Description** check box.

7 In the **Electric Potential Comparison, Potential** toolbar, click **Plot**.

Electric Potential Comparison, Charge Density

1 In the **Model Builder** window, under **Results** click **Electric Potential (es2)**.

2 In the **Settings** window for **1D Plot Group**, type `Electric Potential Comparison, Charge Density` in the **Label** text field.

3 Locate the **Plot Settings** section. Select the **y-axis label** check box.

4 Locate the **Title** section. From the **Title type** list, choose **Label**.

Line Graph 2

1 Right-click **Electric Potential Comparison, Charge Density** and choose **Line Graph**.

2 Select Domain 1 only.

3 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.

4 In the **Expression** text field, type `-q0*log(r/ro)/(2*pi*epsilon0_const)`.

5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

6 In the **Expression** text field, type `r`.

7 Locate the **Legends** section. Select the **Show legends** check box.

8 From the **Legends** list, choose **Manual**.

9 In the table, enter the following settings:

Legends
`Analytical solution`

Line Graph 1

1 In the **Model Builder** window, click **Line Graph 1**.

2 In the **Settings** window for **Line Graph**, locate the **Legends** section.

3 Select the **Show legends** check box.

4 Find the **Include** subsection. Select the **Expression** check box.

5 Clear the **Solution** check box.

6 Select the **Description** check box.

7 In the **Electric Potential Comparison, Charge Density** toolbar, click **Plot**.

Electric Field Comparison, Potential

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Electric Field Comparison, Potential` in the **Label** text field.

3 Locate the **Plot Settings** section.

4 Select the **y-axis label** check box. In the associated text field, type `Electric field (V/m)`.

5 Locate the **Title** section. From the **Title type** list, choose **Label**.

Line Graph 1

1 Right-click **Electric Field Comparison, Potential** and choose **Line Graph**.

2 Select Domain 1 only.

3 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.

4 In the **Expression** text field, type `es.Er`.

5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

6 In the **Expression** text field, type `r`.

7 Locate the **Legends** section. Select the **Show legends** check box.

8 Find the **Include** subsection. Select the **Expression** check box.

9 Clear the **Solution** check box.

10 Select the **Description** check box.

Line Graph 2

1 In the **Model Builder** window, right-click **Electric Field Comparison, Potential** and choose **Line Graph**.

2 Select Domain 1 only.

3 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.

4 In the **Expression** text field, type `-V0/(r*log(ri/ro))`.

5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

6 In the **Expression** text field, type `r`.

7 Locate the **Legends** section. Select the **Show legends** check box.

8 From the **Legends** list, choose **Manual**.

9 In the table, enter the following settings:

Legends
`Analytical solution`

10 In the **Electric Field Comparison, Potential** toolbar, click **Plot**.

Electric Field Comparison, Charge Density

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Electric Field Comparison, Charge Density` in the **Label** text field.

3 Locate the **Plot Settings** section.

4 Select the **y-axis label** check box. In the associated text field, type `Electric field (V/m)`.

5 Locate the **Title** section. From the **Title type** list, choose **Label**.

Line Graph 1

1. Right-click **Electric Field Comparison, Charge Density** and choose **Line Graph**.
2. Select Domain 1 only.
3. In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.
4. In the **Expression** text field, type `es2.Er`.
5. Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.
6. In the **Expression** text field, type `r`.
7. Locate the **Legends** section. Select the **Show legends** check box.
8. Find the **Include** subsection. Select the **Expression** check box.
9. Clear the **Solution** check box.
10. Select the **Description** check box.

Line Graph 2

1. In the **Model Builder** window, right-click **Electric Field Comparison, Charge Density** and choose **Line Graph**.
2. Select Domain 1 only.
3. In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.
4. In the **Expression** text field, type `q0/(2*pi*epsilon0_const*r)`.
5. Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.
6. In the **Expression** text field, type `r`.
7. Locate the **Legends** section. Select the **Show legends** check box.
8. From the **Legends** list, choose **Manual**.
9. In the table, enter the following settings:

Legends
`Analytical solution`

10. In the **Electric Field Comparison, Charge Density** toolbar, click **Plot**.