

Fatigue Analysis of a Nonproportionally Loaded Shaft with a Fillet

Introduction

This benchmark model is based on the example found in section 5.4.3 of Ref. 1. It shows how to perform a high-cycle fatigue analysis for nonproportional loading using critical plane methods.

Model Definition

The geometry is a circular shaft with two different diameters, 10 mm and 16 mm. At the transition between the two diameters there is a fillet with a radius of 2 mm.

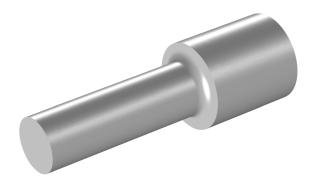


Figure 1: The notched shaft.

Two time-dependent loads are applied at the small end of the shaft: a transverse force and a twisting moment. The force varies between 0 and 1.94 kN and the torque varies between

-28.7 and +28.7 Nm. Figure 2 shows the history of one loading cycle.

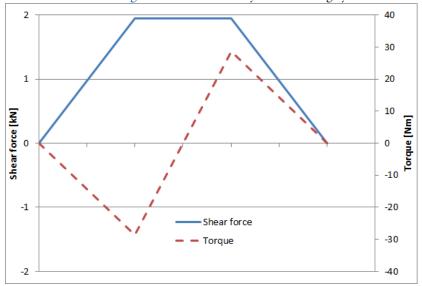


Figure 2: Load history.

The big end of the shaft is fixed. The material is Elastic with E = 100 GPa and v = 0.

In Ref. 1 it is stated that the fatigue limit for completely reversed axial tension is 700 MPa, while the fatigue limit for pure tension is 560 MPa. These values are the stress amplitudes. In uniaxial loading, the Findley criterion can be written as

$$\sqrt{(\sigma_{\rm a})^2 + (k \cdot \sigma_{\rm max})^2} + k \cdot \sigma_{\rm max} = 2f$$

where σ_a is the stress amplitude and σ_{max} is the maximum stress experienced in a fatigue cycle. This means that you have to solve the simultaneous equations

$$\sqrt{700^{2} + (k \cdot 700)^{2}} + k \cdot 700 = 2f$$
$$\sqrt{560^{2} + (k \cdot 1120)^{2}} + k \cdot 1120 = 2f$$

to get the Findley parameters f and k. The result is f = 440 MPa and k = 0.23.

The Matake criterion is similar to the Findley criterion, with the difference that the critical plane is defined solely by the maximum shear stress. For a uniaxial case, the Matake expression is

$$\frac{\sigma_{\rm a}}{2} \left(1 + \frac{k \sigma_{\rm max}}{\sigma_{\rm a}} \right) = f$$

which gives the corresponding system of equations as

$$350 \cdot (1+k) = f \\ 280 \cdot (1+2k) = f$$

The solution is f = 467 MPa and k = 0.33 as parameters for the Matake case.

The Dang Van criterion utilizes the stress history on a plane that has undergone an elastic shakedown. It evaluates fatigue using the expression

$$\tau_{\text{max}} + a\sigma_{\text{H}} = b$$

where τ_{max} is the maximum shear stress and σ_{H} is the hydrostatic stress, while a and b are material parameters. For the fatigue tests above, the Dang Van relation transforms to

$$\frac{\sigma_a}{2} + a \frac{\sigma_{\text{max}}}{3} = b$$

where σ_a is the stress amplitude and σ_{max} is the maximum stress. The material constants are obtained from the following set of equations

$$350 + a \cdot 233 = b$$

 $280 + a \cdot 373 = b$

The material parameters of the Dang Van model are b = 467 MPa and a = 0.5.

Results and Discussion

Figure 3 and Figure 4 show stress distributions from the two basic load cases. The location for the maximum equivalent stress is at the surface of the fillet, at a radius slightly larger than the minimum radius of the shaft.

In Figure 5 the equivalent stress from the combined load case with transverse force and positive torque is shown. It is symmetric with respect to the XY-plane and is identical also for the case when the torque is reversed.

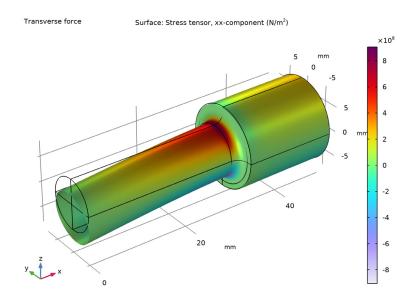


Figure 3: Axial stress from transverse force.

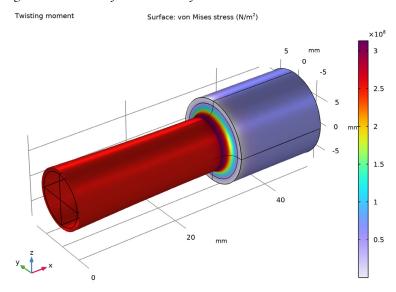


Figure 4: Equivalent stress from torque.

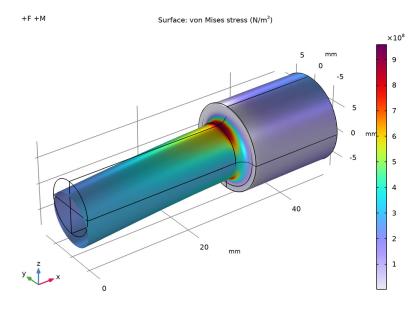


Figure 5: Equivalent stress distribution for one of the combined load cases.

The results from the fatigue evaluation is shown in Figure 6, Figure 7, and Figure 8. With the Findley criterion, the fatigue usage factor is computed to 0.98, in perfect agreement with Ref. 1.

Using the Matake criterion, the fatigue usage factor decreases to 0.88, which shows that there can be significant differences between results from seemingly similar models. The critical plane computed in the Matake model differs from the one used in the Findley model. As a consequence, the maximum normal stress on the critical plane can be significantly lower in the Matake case than in the Findley case.

The Dang Van model predicts the fatigue usage factor to 0.94. This is slightly larger than 0.92 as predicted in Ref. 1. The difference arises from the fact that in the reference the shear stress arising from the constant bending force is approximated to zero. In reality, this shear force is 16% as compared with the shear force arising from the twisting moment and has an impact on the results. Moreover, when a comparison between different models is made, attention must be made to the discretization of the geometry. The critical point of each model is found in different locations. Thus, a perfect agreement in one model does not mean that there will be a perfect agreement in another model. Based on the discretization, the evaluation for certain models is made closer to its critical point and thus results are in better agreement with the theoretical value. A stress state that is only a few percent from its theoretical value can have a larger impact on the fatigue prediction, since the fatigue model requires further data manipulation.

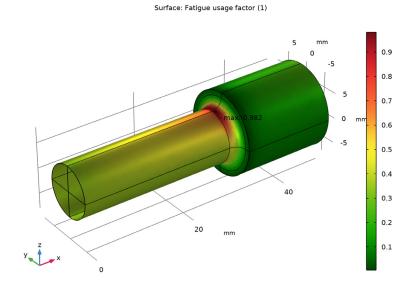


Figure 6: Fatigue usage factor using the Findley criterion.

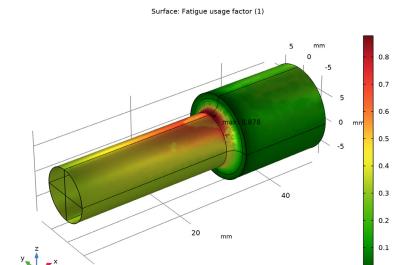


Figure 7: Fatigue usage factor using the Matake criterion.



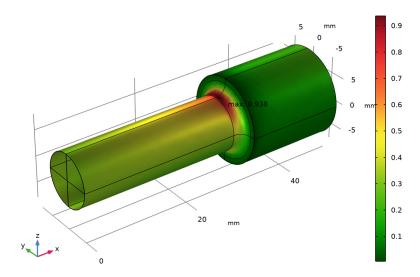


Figure 8: Fatigue usage factor using the Dang Van criterion.

There is a large difference in the fatigue usage factor between the top (tension) and bottom (compression) side of the bar (see Figure 9), even though the equivalent stress is the same at both locations. This shows how the criterion captures the difference between the predominantly tensile stress states at the critical spot and the compressive stress states on the other side. Moreover, there is a clear difference between the models. In tension, the Dang Van model predicts higher fatigue usage factor than the Matake model, while in compression, this is reversed. Therefore, the computed values should not be taken as an exact prediction of the fatigue but rather as a probable outcome. Each model has a

preferred area of use and application in a slightly different domain can result in less accurate fatigue prediction.

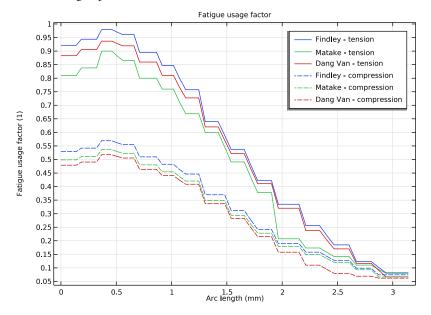


Figure 9: Fatigue usage factor prediction in tension and compression.

Notes About the COMSOL Implementation

In this model, you use the load case functionality in COMSOL Multiphysics to produce the load cycle. In the first study, the two basic load cases are analyzed. This study is not essential for the analysis, but it allows you to inspect the results of the individual basic load cases.

In order to resolve stresses on the surface of the shaft more accurately, it is cladded with a thin layer of the same material. The computed surface stresses are then used in the fatigue evaluation.

Reference

1. D.F. Socie and G.B. Marquis, Multiaxial Fatigue, SAE, 1999.

Application Library path: Fatigue Module/Stress Based/shaft with fillet

Modeling Instructions

From the File menu, choose New.

NEW

In the New window, click Model Wizard.

MODEL WIZARD

- I In the Model Wizard window, click 📋 3D.
- 2 In the Select Physics tree, select Structural Mechanics>Solid Mechanics (solid).
- 3 Click Add.
- 4 Click 🗪 Study.
- 5 In the Select Study tree, select General Studies>Stationary.
- 6 Click M Done.

GEOMETRY I

- I In the Model Builder window, under Component I (compl) click Geometry I.
- 2 In the Settings window for Geometry, locate the Units section.
- 3 From the Length unit list, choose mm.

Work Plane I (wpl)

- I In the Geometry toolbar, click Work Plane.
- 2 In the Settings window for Work Plane, click A Go to Plane Geometry.

Work Plane I (wbl)>Plane Geometry

In the Model Builder window, click Plane Geometry.

Work Plane I (wbl)>Polygon I (boll)

- I In the Work Plane toolbar, click / Polygon.
- 2 In the Settings window for Polygon, locate the Coordinates section.
- **3** From the **Data source** list, choose **Vectors**.
- 4 In the xw text field, type 0 0 32 32 50 50 0.
- **5** In the **yw** text field, type 0 5 5 8 8 0 0.

Work Plane I (wpl)>Fillet I (fill)

- I In the Work Plane toolbar, click Fillet.
- 2 On the object **poll**, select Point 3 only.
- 3 In the Settings window for Fillet, locate the Radius section.
- 4 In the Radius text field, type 2.
- 5 Click | Build Selected.

Revolve I (rev I)

- I In the Model Builder window, right-click Geometry I and choose Revolve.
- 2 In the Settings window for Revolve, locate the Revolution Angles section.
- 3 Clear the Keep original faces check box.
- 4 Locate the Revolution Axis section. Find the Direction of revolution axis subsection. In the xw text field, type 1.
- **5** In the **yw** text field, type 0.
- 6 Click | Build Selected.

DEFINITIONS

Create the boundary selection that will be used for fatigue evaluation.

Selection for fatigue evaluation

- I In the Model Builder window, expand the Component I (compl)>Definitions node.
- 2 Right-click Definitions and choose Selections>Explicit.
- 3 In the Settings window for Explicit, type Selection for fatigue evaluation in the Label text field.
- 4 Locate the Input Entities section. From the Geometric entity level list, choose Boundary.
- 5 Select the Group by continuous tangent check box.
- 6 Select Boundaries 4 and 16 only.

SOLID MECHANICS (SOLID)

Thin Layer I

- I In the Model Builder window, under Component I (compl) right-click Solid Mechanics (solid) and choose Thin Layer.
- 2 In the Settings window for Thin Layer, locate the Boundary Selection section.
- 3 From the Selection list, choose Selection for fatigue evaluation.
- **4** Locate the **Boundary Properties** section. In the $L_{\rm th}$ text field, type 1e-6[m].

5 Locate the **Thin Layer** section. From the **Approximation** list, choose **Membrane**.

Fixed Constraint I

- I In the Physics toolbar, click **Boundaries** and choose Fixed Constraint.
- **2** Select Boundaries 21–24 only.

Rigid Connector I

- I In the Physics toolbar, click **Boundaries** and choose **Rigid Connector**.
- **2** Select Boundaries 1, 3, 5, and 7 only.

Applied Force 1

- I In the Physics toolbar, click 🕞 Attributes and choose Applied Force.
- 2 In the Settings window for Applied Force, locate the Applied Force section.
- **3** Specify the \mathbf{F} vector as

0	х
0	у
-1.94[kN]	z

4 In the Physics toolbar, click Load Group and choose New Load Group.

Rigid Connector 1

In the Model Builder window, click Rigid Connector 1.

Applied Moment 1

- I In the Physics toolbar, click 🕞 Attributes and choose Applied Moment.
- 2 In the Settings window for Applied Moment, locate the Applied Moment section.
- **3** Specify the **M** vector as

-	
28.7[N*m]	x
0	у
0	z

4 In the Physics toolbar, click Load Group and choose New Load Group.

GLOBAL DEFINITIONS

Transverse force

I In the Model Builder window, under Global Definitions>Load and Constraint Groups click Load Group 1.

- 2 In the Settings window for Load Group, type Transverse force in the Label text field.
- 3 In the Parameter name text field, type 1gF.

Twisting moment

- I In the Model Builder window, under Global Definitions>Load and Constraint Groups click Load Group 2.
- 2 In the Settings window for Load Group, type Twisting moment in the Label text field.
- 3 In the Parameter name text field, type 1gM.

Material I (mat I)

In the Model Builder window, under Global Definitions right-click Materials and choose Blank Material.

MATERIALS

Material Link I (matlnk I)

In the Model Builder window, under Component I (compl) right-click Materials and choose More Materials>Material Link.

GLOBAL DEFINITIONS

Material I (mat I)

- I In the Settings window for Material, locate the Material Contents section.
- 2 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Young's modulus	E	100[GPa]	Pa	Young's modulus and Poisson's ratio
Poisson's ratio	nu	0	I	Young's modulus and Poisson's ratio
Density	rho	0	kg/m³	Basic

Because fatigue usage is to be evaluated on the surface, you need to define a material on the boundaries.

MATERIALS

Material Link 2 (matlnk2)

- I Right-click Materials and choose More Materials>Material Link.
- 2 In the Settings window for Material Link, locate the Geometric Entity Selection section.

- 3 From the Geometric entity level list, choose Boundary.
- 4 From the Selection list, choose Selection for fatigue evaluation.

MESH I

- I In the Model Builder window, under Component I (compl) click Mesh I.
- 2 In the Settings window for Mesh, locate the Physics-Controlled Mesh section.
- **3** From the **Element size** list, choose **Fine**.
- 4 Click III Build All.

A finer mesh is needed in the fillet in order to resolve the stress concentration.

5 Locate the **Sequence Type** section. From the list, choose **User-controlled mesh**.

Size 1

- I In the Model Builder window, right-click Free Tetrahedral I and choose Size.
- 2 In the Settings window for Size, locate the Element Size section.
- 3 From the Predefined list, choose Finer.

Size 2

- I Right-click Free Tetrahedral I and choose Size.
- 2 In the Settings window for Size, locate the Geometric Entity Selection section.
- 3 From the Geometric entity level list, choose Edge.
- 4 Select Edges 13, 14, 16, and 18 only.
- **5** Locate the **Element Size** section. Click the **Custom** button.
- 6 Locate the Element Size Parameters section.
- 7 Select the Maximum element size check box. In the associated text field, type 0.5.
- 8 Select the Maximum element growth rate check box. In the associated text field, type 1.2.
- 9 Click Build All.

STUDY

Step 1: Stationary

- I In the Model Builder window, under Study I click Step I: Stationary.
- 2 In the Settings window for Stationary, click to expand the Study Extensions section.
- 3 Select the **Define load cases** check box.
- 4 Click Add two times.

5 In the table, enter the following settings:

Load case	lgF	Weight	lgM	Weight
Transverse force	\checkmark	1.0		1.0
Twisting moment		1.0	V	1.0

- 6 In the Model Builder window, click Study 1.
- 7 In the Settings window for Study, type Study 1 (Basic load cases) in the Label text field.
- 8 In the Home toolbar, click **Compute**.

RESULTS

Stress, Thin Layer (solid)

Visualize the difference between the tension and compression on the opposite sides of the shaft.

- I In the Model Builder window, under Results click Stress, Thin Layer (solid).
- 2 In the Stress, Thin Layer (solid) toolbar, click Plot.
- 3 In the Settings window for 3D Plot Group, locate the Data section.
- 4 From the Load case list, choose Transverse force.

Surface I

- I In the Model Builder window, expand the Stress, Thin Layer (solid) node, then click Surface 1.
- 2 In the Settings window for Surface, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Component I (compl)>Solid Mechanics> Stress>Stress tensor (spatial frame) - N/m2>solid.sGpxx - Stress tensor, xx-component.
- 3 In the Stress, Thin Layer (solid) toolbar, click Plot.

ADD STUDY

- I In the Home toolbar, click Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select General Studies>Stationary.
- 4 Click Add Study in the window toolbar.
- 5 In the Home toolbar, click Add Study to close the Add Study window.

STUDY 2

Step 1: Stationary

- I In the Settings window for Stationary, locate the Study Extensions section.
- 2 Select the Define load cases check box.
- 3 Click Add three times.
- **4** In the table, enter the following settings:

Load case	lgF	Weight	IgM	Weight
No load		1.0		1.0
+F -M	\checkmark	1.0	V	-1.0
+F +M	V	1.0	V	1.0

- 5 In the Model Builder window, click Study 2.
- 6 In the Settings window for Study, type Study 2 (Combined load cases) in the Label text field.
- 7 In the Home toolbar, click **Compute**.

RESULTS

Stress, Thin Layer (solid) I

- I In the Model Builder window, expand the Results>Stress, Thin Layer (solid) I node, then click Stress, Thin Layer (solid) 1.
- 2 In the Stress, Thin Layer (solid) I toolbar, click Plot.

As a last step, perform a fatigue analysis on the load cycle.

ADD PHYSICS

- I In the Home toolbar, click open the Add Physics window.
- 2 Go to the Add Physics window.
- 3 In the tree, select Structural Mechanics>Fatigue (ftg).
- 4 Find the Physics interfaces in study subsection. In the table, clear the Solve check boxes for Study I (Basic load cases) and Study 2 (Combined load cases).
- 5 Click Add to Component I in the window toolbar.
- 6 In the Home toolbar, click Add Physics to close the Add Physics window.

FATIGUE (FTG)

Findley

- I Right-click Component I (compl)>Fatigue (ftg) and choose the boundary evaluation Stress-Based.
- 2 In the Settings window for Stress-Based, type Findley in the Label text field.
- 3 Locate the Boundary Selection section. From the Selection list, choose Selection for fatigue evaluation.
- **4** Locate the **Solution Field** section. From the **Physics interface** list, choose Solid Mechanics (solid).
- **5** Locate the **Evaluation Settings** section. In the Q text field, type 16.

Matake

- I In the Physics toolbar, click **Boundaries** and choose **Stress-Based**.
- 2 In the Settings window for Stress-Based, type Matake in the Label text field.
- 3 Locate the Boundary Selection section. From the Selection list, choose Selection for fatigue evaluation.
- 4 Locate the Fatigue Model Selection section. From the Criterion list, choose Matake.
- **5** Locate the **Solution Field** section. From the **Physics interface** list, choose Solid Mechanics (solid).
- **6** Locate the **Evaluation Settings** section. In the Q text field, type 16.

Dang Van

- I In the Physics toolbar, click **Boundaries** and choose **Stress-Based**.
- 2 In the Settings window for Stress-Based, type Dang Van in the Label text field.
- 3 Locate the Boundary Selection section. From the Selection list, choose Selection for fatigue evaluation.
- 4 Locate the Fatigue Model Selection section. From the Criterion list, choose Dang Van.
- 5 Locate the Solution Field section. From the Physics interface list, choose Solid Mechanics (solid).

GLOBAL DEFINITIONS

Material I (mat I)

- I In the Model Builder window, under Global Definitions>Materials click Material I (matl).
- 2 In the Settings window for Material, locate the Material Contents section.

3 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group Findley	
Normal stress sensitivity coefficient	k_Findley	0.23	1		
Limit factor	f_Findley	440[MPa]	Pa	Findley	
Normal stress sensitivity coefficient	k_Matake	0.33	1	Matake	
Limit factor	f_Matake	467[MPa]	Pa	Matake	
Hydrostatic stress sensitivity coefficient	a_DangVan	0.5	I	Dang Van	
Limit factor	b_DangVan	467[MPa]	Pa	Dang Van	

ADD STUDY

- I In the Home toolbar, click Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- **3** Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** check box for Solid Mechanics (solid).
- 4 Find the Studies subsection. In the Select Study tree, select Preset Studies for Selected Physics Interfaces>Fatigue.
- **5** Click **Add Study** in the window toolbar.
- 6 In the Home toolbar, click Add Study to close the Add Study window.

STUDY 3

Steb 1: Fatigue

- I In the Settings window for Fatigue, locate the Values of Dependent Variables section.
- 2 Find the Values of variables not solved for subsection. From the Settings list, choose User controlled.
- 3 From the Method list, choose Solution.
- 4 From the Study list, choose Study 2 (Combined load cases), Stationary.
- 5 In the Model Builder window, click Study 3.
- 6 In the Settings window for Study, type Study 3 (Fatigue) in the Label text field.
- 7 In the Home toolbar, click **Compute**.

RESULTS

Fatigue Usage Factor (Findley)

In the Settings window for 3D Plot Group, type Fatigue Usage Factor (Findley) in the Label text field.

Surface 1

- I In the Model Builder window, expand the Fatigue Usage Factor (Findley) node, then click Surface I.
- 2 In the Settings window for Surface, locate the Expression section.
- 3 In the Expression text field, type ftg.stre1.fus.

Fatigue Usage Factor (Findley)

In the Model Builder window, right-click Fatigue Usage Factor (Findley) and choose Duplicate.

Fatigue Usage Factor (Matake)

- I In the Model Builder window, under Results click Fatigue Usage Factor (Findley) I.
- 2 In the Settings window for 3D Plot Group, type Fatigue Usage Factor (Matake) in the Label text field.

Surface I

- I In the Model Builder window, expand the Fatigue Usage Factor (Matake) node, then click Surface I.
- 2 In the Settings window for Surface, locate the Expression section.
- 3 In the Expression text field, type ftg.stre2.fus.

Fatigue Usage Factor (Matake)

In the Model Builder window, right-click Fatigue Usage Factor (Matake) and choose Duplicate.

Fatigue Usage Factor (Dang Van)

- I In the Model Builder window, under Results click Fatigue Usage Factor (Matake) I.
- 2 In the Settings window for 3D Plot Group, type Fatigue Usage Factor (Dang Van) in the Label text field.

Surface I

- I In the Model Builder window, expand the Fatigue Usage Factor (Dang Van) node, then click Surface 1.
- 2 In the Settings window for Surface, locate the Expression section.

3 In the Expression text field, type ftg.stre3.fus.

Compare fatigue results in tension and compression.

Fatigue Usage Factor, ID

- I In the Home toolbar, click Add Plot Group and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Fatigue Usage Factor, 1D in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Study 3 (Fatigue)/Solution 3 (sol3).
- 4 Click to expand the Title section. From the Title type list, choose Manual.
- 5 In the **Title** text area, type Fatigue usage factor.

Line Graph 1

- I Right-click Fatigue Usage Factor, ID and choose Line Graph.
- **2** Select Edge 19 only.
- 3 In the Settings window for Line Graph, click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (compl)>Fatigue> ftg.strel.fus - Fatigue usage factor - 1.
- 4 Click to expand the **Legends** section. Select the **Show legends** check box.
- 5 From the Legends list, choose Manual.
- **6** In the table, enter the following settings:

Legends Findley - tension

7 Right-click Line Graph I and choose Duplicate.

Line Graph 2

- I In the Model Builder window, click Line Graph 2.
- 2 In the Settings window for Line Graph, locate the y-Axis Data section.
- 3 In the Expression text field, type ftg.stre2.fus.
- **4** Locate the **Legends** section. In the table, enter the following settings:

Legends Matake - tension

5 Right-click Line Graph 2 and choose Duplicate.

Line Graph 3

- I In the Model Builder window, click Line Graph 3.
- 2 In the Settings window for Line Graph, locate the y-Axis Data section.
- 3 In the Expression text field, type ftg.stre3.fus.
- **4** Locate the **Legends** section. In the table, enter the following settings:

Legends Dang Van - tension

Line Graph 4

- I In the Model Builder window, right-click Fatigue Usage Factor, ID and choose Line Graph.
- **2** Select Edge 17 only.
- 3 In the Settings window for Line Graph, click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose ftg.strel.fus - Fatigue usage factor.
- 4 Click to expand the Coloring and Style section. Find the Line style subsection. From the Line list, choose Dashed.
- 5 From the Color list, choose Cycle (reset).
- **6** Locate the **Legends** section. Select the **Show legends** check box.
- 7 From the Legends list, choose Manual.
- **8** In the table, enter the following settings:

Legends Findley - compression

9 Right-click Line Graph 4 and choose Duplicate.

Line Graph 5

- I In the Model Builder window, click Line Graph 5.
- 2 In the Settings window for Line Graph, locate the y-Axis Data section.
- 3 In the Expression text field, type ftg.stre2.fus.
- 4 Locate the Coloring and Style section. From the Color list, choose Cycle.
- **5** Locate the **Legends** section. In the table, enter the following settings:

Legends Matake - compression

6 Right-click Line Graph 5 and choose Duplicate.

Line Graph 6

- I In the Model Builder window, click Line Graph 6.
- 2 In the Settings window for Line Graph, locate the y-Axis Data section.
- 3 In the Expression text field, type ftg.stre3.fus.
- **4** Locate the **Legends** section. In the table, enter the following settings:

Leger	ıds		
Dang	Van	-	compression

