

Inkjet Nozzle — Level Set Method

Introduction

Inkjet printers are attractive tools for printing text and images because they combine low cost and high resolution with acceptable speed. The working principle behind inkjet technology is to eject small droplets of liquid from a nozzle onto a sheet of paper. Important properties of a printer are its speed and the resolution of the final images. Designers can vary several parameters to modify a printer's performance. For instance, they can vary the inkjet geometry and the type of ink to create droplets of different sizes. The size and speed of the ejected droplets are also strongly dependent on the speed at which ink is injected into the nozzle. Simulations can be useful to improve the understanding of the fluid flow and to predict the optimal design of an inkjet for a specific application.

Although initially invented to produce images on paper, the inkjet technique has since been adopted for other application areas. Instruments for the precise deposition of microdroplets often employ inkjets. These instruments are used within the life sciences for diagnosis, analysis, and drug discovery. Inkjets have also been used as 3D printers to synthesize tissue from cells and to manufacture microelectronics. For all of these applications it is important to be able to accurately control the inkjet performance.

This example demonstrates how to model the fluid flow within an inkjet using the Laminar Two-Phase Flow, Level Set interface.

Model Definition

Figure 1 shows the geometry of the inkjet studied in this example. Because of its symmetry you can use an axisymmetric 2D model. Initially, the space between the inlet and the nozzle is filled with ink. Additional ink is injected through the inlet during a period of 10 μs, and it consequently forces ink to flow out of the nozzle. When the injection stops, a droplet of ink snaps off and continues to travel until it hits the target.

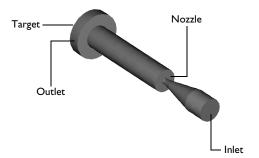


Figure 1: Geometry of the inkjet.

REPRESENTATION AND CONVECTION OF THE FLUID INTERFACE

Level Set Method

The Level Set interface uses a reinitialized, conservative level set method to describe and convect the fluid interface. The 0.5 contour of the level set function ϕ defines the interface, where ϕ equals 0 in air and 1 in ink. In a transition layer close to the interface, ϕ goes smoothly from 0 to 1. The interface moves with the fluid velocity, \mathbf{u} , at the interface. The following equation describes the convection of the reinitialized level set function:

$$\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi + \gamma \left[\left(\nabla \cdot \left(\phi (1 - \phi) \frac{\nabla \phi}{|\nabla \phi|} \right) \right) - \varepsilon \nabla \cdot \nabla \phi \right] = 0$$

The thickness of the transition layer is proportional to ε . For this model you can use $\varepsilon = h_c/2$, where h_c is the typical mesh size in the region passed by the droplet.

The parameter γ determines the amount of reinitialization. A suitable value for γ is the maximum magnitude occurring in the velocity field.

Beside defining the fluid interface, the level set function is used to smooth the density and viscosity jumps across the interface through the definitions

$$\rho = \rho_{air} + (\rho_{ink} - \rho_{air})\phi$$

$$\mu = \mu_{air} + (\mu_{ink} - \mu_{air})\phi$$

Here, ϕ acts as the volume fraction of the ink.

TRANSPORT OF MASS AND MOMENTUM

The incompressible Navier-Stokes equations, including surface tension, describe the transport of mass and momentum. Both ink and air can be considered incompressible as long as the fluid velocity is small compared to the speed of sound. The Navier-Stokes equations are

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) - \nabla \cdot \left(\mu (\nabla \mathbf{u} + \nabla \mathbf{u}^T) \right) + \nabla p = \mathbf{F}_{\text{st}}$$

$$(\nabla \cdot \mathbf{u}) = 0$$

Here, ρ denotes density (kg/m³), μ equals the dynamic viscosity (N·s/m²), \mathbf{u} represents the velocity (m/s), p denotes pressure (Pa), and \mathbf{F}_{st} is the surface tension force.

The surface tension force is computed as

$$\mathbf{F}_{st} = \sigma \delta \kappa \mathbf{n}$$

where **n** is the interface normal, σ is the surface tension coefficient (N/m), $\kappa = -\nabla \cdot \mathbf{n}$ is the curvature, and δ equals a Dirac delta function that is nonzero only at the fluid interface. The normal to the interface is

$$\mathbf{n} = \frac{\nabla \phi}{|\nabla \phi|}$$

while the delta function is approximated by

$$\delta = 6|\phi(1-\phi)||\nabla\phi||$$

INITIAL CONDITIONS

Figure 2 shows the initial distribution (t = 0) of ink and air. The velocity is initially 0.

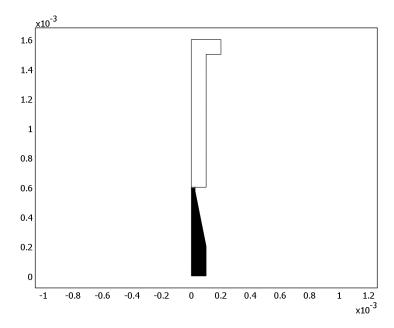


Figure 2: Initial distribution of ink. Black corresponds to ink and white corresponds to air.

BOUNDARY CONDITIONS

Inlet

The mean inlet velocity in the z direction increases from 0 to 0.56 m/s during the first 2 μ s. The velocity is then maintained for 10 μ s and finally decreases to 0 for another 2 μ s. The time-dependent mean velocity in the z direction can then be defined as

$$v_{av}(t) = \text{rect}(t) \cdot 0.56$$

where t is given in seconds and rect(t) is a smooth rectangular pulse function with the transition points at 1 μ s and 13 μ s with a 2 μ s transition period. (see Figure 3).

The velocity profiled is defined by the fully developed flow option in the inlet feature.

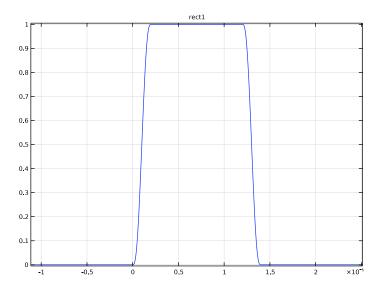


Figure 3: Smooth step function.

Use $\phi = 1$ as the inlet boundary condition for the level set variable.

Outlet

Set a constant pressure at the outlet. The value of the pressure given here is not important because the velocity depends only on the pressure gradient. You thus obtain the same velocity field regardless of whether the pressure is set to 1 atm or to 0.

Walls

On all other boundaries except the target, set No slip conditions. Use the Wetted wall condition on the target, with a contact angle of $\pi/2$ and a slip length of 10 μm .

Results and Discussion

Figure 4 shows the ink surface and the velocity field at different times.

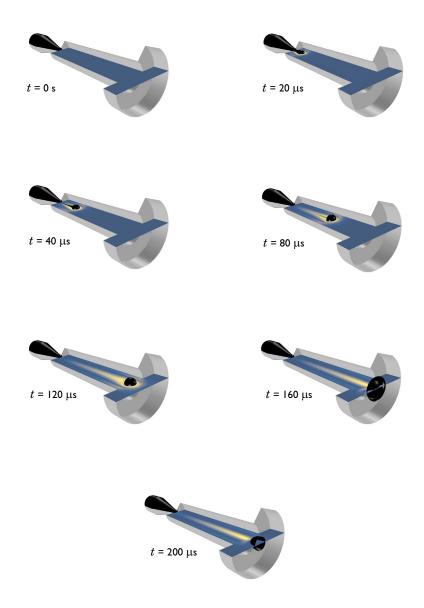
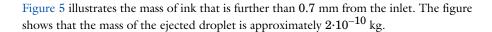


Figure 4: Position of air/ink interface and velocity field at various times.



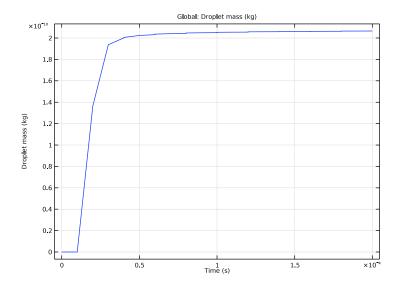


Figure 5: Amount of ink from just above the nozzle.

This example studies only one inkjet model, but it is easy to modify the model in several ways. You can, for example, change properties such as the geometry or the inlet velocity and study the influence on the size and the speed of the ejected droplets. You can also investigate how the inkjet would perform if the ink were replaced by a different fluid. It is also easy to add forces such as gravity to the model.

Notes About the COMSOL Implementation

You can readily set up the model using the Laminar Two-Phase Flow, Level Set interface. This interface adds the equations automatically, and you need only specify physical parameters of the fluids and the initial and boundary conditions.

In order to accurately resolve the interface between the air and ink, use the adaptive meshing. This means that as the interface moves during the simulation, the mesh is updated in order to keep the mesh refined in the interface region.

The simulation procedure involves two consecutive computations. First you calculate a smooth initial solution for the level set variable. Using this initial solution, you then start the time-dependent simulation of the fluid motion.

To calculate the droplet's mass, use a nonlocal integration coupling. To visualize the droplet in 3D, revolve the 2D axially symmetric solution to a 3D geometry.

References

- 1. J.-T. Yeh, "A VOF-FEM Coupled Inkjet Simulation," Proc. ASME FEDSM'01, New Orleans, Louisiana, 2001.
- 2. E. Olsson and G. Kreiss, "A Conservative Level Set Method for Two Phase Flow," J. Comput. Phys., vol. 210, pp. 225-246, 2005.
- 3. P. Yue, J. Feng, C. Liu, and J. Shen, "A Diffuse-Interface Method for Simulating Two-Phase Flows of Complex Fluids," *J. Fluid Mech.*, vol. 515, pp. 293–317, 2004.

Application Library path: Microfluidics Module/Two-Phase Flow/ inkjet nozzle ls

Modeling Instructions

From the File menu, choose New.

NEW

In the New window, click Model Wizard.

MODEL WIZARD

- I In the Model Wizard window, click 2D Axisymmetric.
- 2 In the Select Physics tree, select Fluid Flow>Multiphase Flow>Two-Phase Flow, Level Set> Laminar Flow.
- 3 Click Add.
- 4 Click 🔁 Study.
- 5 In the Select Study tree, select Preset Studies for Selected Multiphysics> Time Dependent with Phase Initialization.
- 6 Click M Done.

It is convenient to add some parameters for the geometry.

GLOBAL DEFINITIONS

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description	
InletR	O.1[mm]	IE-4 m	Nozzle inlet radius	
NozzleL	0.375[mm]	3.75E-4 m	Nozzle length	
NozzleR	0.025[mm]	2.5E-5 m	Nozzle radius	
ThroatL	0.025[mm]	2.5E-5 m	Throat length	
TargetL	1 [mm]	0.001 m	Distance to target	
AirW	O.1[mm]	IE-4 m	Air channel width	

GEOMETRY I

- I In the Model Builder window, under Component I (compl) click Geometry I.
- 2 In the Settings window for Geometry, locate the Units section.
- 3 From the Length unit list, choose mm.

Rectangle I (rI)

- I In the Geometry toolbar, click Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type InletR.
- 4 In the Height text field, type 2*InletR.

Polygon I (poll)

- I In the Geometry toolbar, click / Polygon.
- 2 In the Settings window for Polygon, locate the Coordinates section.
- **3** In the table, enter the following settings:

r (mm)	z (mm)
0	2*InletR
InletR	2*InletR
NozzleR	2*InletR+NozzleL
0	2*InletR+NozzleL

4 Click Build Selected.

Rectangle I (rI)

In the Model Builder window, right-click Rectangle I (rI) and choose Duplicate.

Rectangle 2 (r2)

- I In the Model Builder window, click Rectangle 2 (r2).
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type NozzleR.
- 4 In the Height text field, type ThroatL+TargetL.
- **5** Locate the **Position** section. In the **z** text field, type NozzleL+2*InletR.
- 6 Right-click Rectangle 2 (r2) and choose Duplicate.

Rectangle 3 (r3)

- I In the Model Builder window, click Rectangle 3 (r3).
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type AirW.
- 4 In the Height text field, type TargetL.
- **5** Locate the **Position** section. In the **z** text field, type ThroatL+NozzleL+2*InletR.
- 6 Right-click Rectangle 3 (r3) and choose Duplicate.

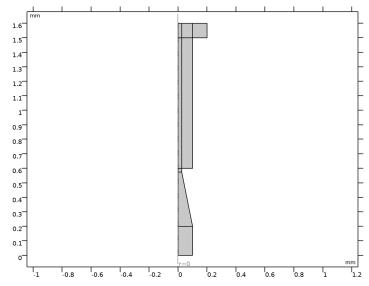
Rectangle 4 (r4)

- I In the Model Builder window, click Rectangle 4 (r4).
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type 2*AirW.
- 4 In the Height text field, type AirW.
- 5 Locate the Position section. In the z text field, type ThroatL+NozzleL+2*InletR+ TargetL-AirW.
- 6 Click | Build Selected.

Form Union (fin)

- I In the Model Builder window, click Form Union (fin).
- 2 In the Settings window for Form Union/Assembly, click 📔 Build Selected.
- 3 Click the **Zoom Extents** button in the **Graphics** toolbar.

This completes the geometry modeling stage.



Next, add a **Multiphase material** to get the effective material parameters from the volume fraction of each phase, and add the material parameters for each phase.

MULTIPHYSICS

Two-Phase Flow, Level Set 1 (tpf1)

- I In the Model Builder window, under Component I (compl)>Multiphysics click Two-Phase Flow, Level Set I (tpfl).
- 2 In the Settings window for Two-Phase Flow, Level Set, locate the Material Properties section.
- 3 Click Add Multiphase Material.

MATERIALS

Phase I (mpmat1.phase1)

- I In the Model Builder window, under Component I (compl)>Materials> Multiphase Material I (mpmatl) click Phase I (mpmatl.phasel).
- 2 In the Settings window for Phase, locate the Link Settings section.
- 3 Click Add Material from Library. This button is found when expanding the options next to the Material list.

ADD MATERIAL TO PHASE I (MPMATI.PHASEI)

- I Go to the Add Material to Phase I (mpmat1.phaseI) window.
- 2 In the tree, select Built-in>Air.
- 3 Click OK.

MATERIALS

Phase 2 (mpmat1.phase2)

- I In the Model Builder window, under Component I (compl)>Materials> Multiphase Material I (mpmatl) click Phase 2 (mpmatl.phase2).
- 2 In the Settings window for Phase, locate the Link Settings section.
- 3 Click Blank Material. This button is found when expanding the options next to the Material list.

GLOBAL DEFINITIONS

Ink

- I In the Model Builder window, under Global Definitions>Materials click Material 2 (mat2).
- 2 In the Settings window for Material, locate the Material Contents section.
- **3** In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Density	rho	1e3[kg/m^3]	kg/m³	Basic
Dynamic viscosity	mu	1e-2[Pa*s]	Pa·s	Basic

4 In the Label text field, type Ink.

Now, define a rectangular pulse function to use when defining the time dependence of the inlet velocity.

Rectangle I (rect1)

- I In the Home toolbar, click f(x) Functions and choose Global>Rectangle.
- 2 In the Settings window for Rectangle, click to expand the Smoothing section.
- 3 Locate the Parameters section. In the Lower limit text field, type 1e-6.
- 4 In the Upper limit text field, type 13e-6.
- 5 Locate the Smoothing section. In the Size of transition zone text field, type 2e-6.
- 6 Click Plot.

Next, define a nonlocal integration coupling that you will use when defining a variable for the droplet mass.

DEFINITIONS

Integration I (intobl)

- I In the **Definitions** toolbar, click Nonlocal Couplings and choose Integration.
- 2 In the Settings window for Integration, locate the Source Selection section.
- 3 From the Selection list, choose All domains.

After these preliminaries, you can define variables for the inlet velocity and the droplet mass.

Variables 1

- I In the **Definitions** toolbar, click **a= Local Variables**.
- 2 In the Settings window for Variables, locate the Variables section.
- **3** In the table, enter the following settings:

Name	Expression	Unit	Description
v_in	0.56[m/s]*rect1(t[1/s])	m/s	Inlet velocity
m_d	<pre>intop1(1e3[kg/m^3]*phils*(z> 0.7[mm]))</pre>	kg	Droplet mass

MULTIPHYSICS

Two-Phase Flow, Level Set I (tpfl)

- I In the Model Builder window, under Component I (compl)>Multiphysics click Two-Phase Flow, Level Set I (tpfl).
- 2 In the Settings window for Two-Phase Flow, Level Set, locate the Surface Tension section.
- 3 Select the Include surface tension force in momentum equation check box.
- 4 From the Surface tension coefficient list, choose User defined. In the σ text field, type 0.07.

LAMINAR FLOW (SPF)

Inlet I

- I In the Model Builder window, under Component I (compl) right-click Laminar Flow (spf) and choose Inlet.
- 2 Select Boundary 2 only.

- 3 In the Settings window for Inlet, locate the Boundary Condition section.
- 4 From the list, choose Fully developed flow.
- **5** Locate the **Fully Developed Flow** section. In the U_{av} text field, type v_in.

Outlet I

- I In the Physics toolbar, click Boundaries and choose Outlet.
- 2 Select Boundary 24 only.

LEVEL SET (LS)

Level Set Model 1

- I In the Model Builder window, under Component I (compl)>Level Set (Is) click Level Set Model I.
- 2 In the Settings window for Level Set Model, locate the Level Set Model section.
- 3 In the ε_{ls} text field, type 2.5e-6.
- 4 In the γ text field, type 10.

Initial Values, Fluid 2

- I In the Model Builder window, click Initial Values, Fluid 2.
- 2 Select Domains 1–3 only.

Inlet I

- I In the Physics toolbar, click Boundaries and choose Inlet.
- 2 Select Boundary 2 only.
- 3 In the Settings window for Inlet, locate the Level Set Condition section.
- 4 From the list, choose Fluid 2 ($\varphi = 1$).

Outlet I

- I In the Physics toolbar, click Boundaries and choose Outlet.
- 2 Select Boundary 24 only.

MULTIPHYSICS

Wetted Wall I (ww1)

- I In the Model Builder window, under Component I (compl)>Multiphysics click Wetted Wall I (wwl).
- 2 In the Settings window for Wetted Wall, locate the Boundary Selection section.
- 3 Click Paste Selection.

- 4 In the Paste Selection dialog box, type 11-13, 15, 18-20, 22, 23 in the Selection text field.
- 5 Click OK.
- 6 In the Settings window for Wetted Wall, locate the Wetted Wall section.
- 7 From the Slip length list, choose User defined. In the β text field, type 10 [um].

MESH I

In the Model Builder window, under Component I (compl) right-click Mesh I and choose **Build All.**

STUDY I

Step 2: Time Dependent

- I In the Model Builder window, under Study I click Step 2: Time Dependent.
- 2 In the Settings window for Time Dependent, locate the Study Settings section.
- 3 In the Output times text field, type range (0, 10e-6, 200e-6).
- **4** Click to expand the **Results While Solving** section. Select the **Plot** check box. This choice means that the **Graphics** window will show a contour line of the volume fraction of Fluid 1 and velocity field while solving, and this plot will be updated at each output time step.
- 5 Click to expand the Adaptation section. From the Adaptive mesh refinement list, choose Adaptive mesh refinement.

By adjusting the scaling of the fields manually, you can reduce the computation time.

Solution I (soll)

- I In the Study toolbar, click Show Default Solver.
- 2 In the Model Builder window, expand the Solution I (soll) node.
- 3 In the Model Builder window, expand the Study I>Solver Configurations> Solution I (soll)>Dependent Variables 2 node, then click Velocity field (compl.u).
- 4 In the Settings window for Field, locate the Scaling section.
- 5 From the Method list, choose Manual.
- 6 In the Scale text field, type 10.
- 7 In the Model Builder window, under Study 1>Solver Configurations>Solution I (sol1)> Dependent Variables 2 click Pressure (compl.p).
- 8 In the Settings window for Field, locate the Scaling section.

- 9 From the Method list, choose Manual.
- 10 In the Scale text field, type 1e4.
- II In the **Study** toolbar, click **Compute**.

RESULTS

Edge 2D I

- I In the Results toolbar, click More Datasets and choose Edge 2D.
- **2** Select Boundaries 12, 13, 15, 19, 20, 22, and 24 only.

Revolution 2D 3

- I In the Results toolbar, click More Datasets and choose Revolution 2D.
- 2 In the Settings window for Revolution 2D, locate the Data section.
- 3 From the Dataset list, choose Edge 2D 1.
- **4** Click to expand the **Revolution Layers** section. In the **Revolution angle** text field, type 230.

Edge 2D 2

- I In the Results toolbar, click More Datasets and choose Edge 2D.
- 2 Select Boundaries 12, 13, and 19 only.

Revolution 2D 4

- I In the Results toolbar, click More Datasets and choose Revolution 2D.
- 2 In the Settings window for Revolution 2D, locate the Data section.
- 3 From the Dataset list, choose Edge 2D 2.
- 4 Locate the Revolution Layers section. In the Start angle text field, type 230.
- 5 In the Revolution angle text field, type 130.

Isosurface I

- I In the Model Builder window, expand the Results>Volume Fraction of Fluid I (Is) I node, then click Isosurface I.
- 2 In the Settings window for Isosurface, locate the Coloring and Style section.
- 3 From the Color list, choose Black.

Slice 1

- I In the Model Builder window, right-click Volume Fraction of Fluid I (Is) I and choose Slice.
- 2 In the Settings window for Slice, locate the Plane Data section.

- 3 From the Plane list, choose zx-planes.
- 4 In the Planes text field, type 1.
- 5 Locate the Coloring and Style section. Click Change Color Table.
- 6 In the Color Table dialog box, select Linear>Cividis in the tree.
- 7 Click OK.

Surface I

- I Right-click Volume Fraction of Fluid I (Is) I and choose Surface.
- 2 In the Settings window for Surface, locate the Data section.
- 3 From the Dataset list, choose Revolution 2D 3.
- 4 Locate the Coloring and Style section. From the Coloring list, choose Uniform.
- 5 From the Color list, choose Gray.

Surface 2

- I Right-click Volume Fraction of Fluid I (Is) I and choose Surface.
- 2 In the Settings window for Surface, locate the Data section.
- 3 From the Dataset list, choose Revolution 2D 4.
- 4 Locate the Coloring and Style section. From the Coloring list, choose Uniform.
- **5** From the **Color** list, choose **Black**.

Volume Fraction of Fluid 1 (Is) 1

- I In the Model Builder window, click Volume Fraction of Fluid I (Is) I.
- 2 In the Settings window for 3D Plot Group, locate the Data section.
- **3** From the **Time (s)** list, choose **4E-5**.
- 4 Locate the Plot Settings section. Clear the Plot dataset edges check box.
- 5 In the Volume Fraction of Fluid I (Is) I toolbar, click Plot.
- 6 Click the **Zoom Extents** button in the **Graphics** toolbar.
- 7 Locate the Data section. From the Time (s) list, choose 0.
- 8 In the Volume Fraction of Fluid I (Is) I toolbar, click I Plot.

Compare the resulting plot with that in the upper panel of Figure 4.To create the remaining plots, plot the solution for the time values 2e-5, 4e-5, 8e-5, 1.2e-4, 1.6e-4, and 2e-4.

ID Plot Group 6

I In the Home toolbar, click **Add Plot Group** and choose ID Plot Group.

- 2 In the Settings window for ID Plot Group, locate the Data section.
- 3 From the Dataset list, choose Study I/Refined Mesh Solution I (sol3).
- 4 Locate the Legend section. Clear the Show legends check box.

Global I

- I Right-click ID Plot Group 6 and choose Global.
- 2 In the Settings window for Global, click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (compl)>Definitions> Variables>m_d - Droplet mass - kg.
- 3 In the ID Plot Group 6 toolbar, click Plot.