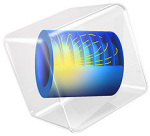

Atmospheric Pressure Corona Discharge in Air

Introduction

This example presents a study of a coaxial DC corona discharge in dry air at atmospheric pressure. The dimensions and operation conditions are similar to the ones found in electrostatic precipitators with wire-to-plate configuration. The inner electrode has 100 μm radius and the gap between electrodes is 10 cm. The simulations presented are for steady-state regimes with the discharge sustained with 10 kV applied to the inner electrode while the exterior electrode is grounded. Emphasis is placed on the charged particles creation and transport and how that translates into the current-voltage characteristic of the discharge.

Model Definition

Figure 1 shows a cross section of the model geometry. The discharge is assumed to be diffuse and uniform in the radial direction. The model is one dimensional in the radial direction between the electrodes and describes the behavior of charged species using fluid-type equations. We assume that the gas temperature and air number density are constant. The gas temperature is kept at 600 K.

The model solves the electron and ions continuity and momentum equations, in the drift-diffusion approximation, self-consistently coupled with Poisson's equation. The local field approximation is used, which means that transport and source coefficients are assumed to be well parameterized through the reduced electric field ($\boldsymbol{E}/\boldsymbol{N}$). In the local field approximation the fluid equation for the mean electron energy is not solved, which reduces significantly the complexity of the numerical problem.

The local field approximation is valid in a situation where the rate of electron energy gain from the electric field is locally balanced by the energy loss rate. When this condition is met the electrons are said to be in local equilibrium with the electric field and the electron mean properties can be expressed as a function the reduced electric field.

The model presented in the following section is used to simulate the ionization of the neutral gas as well as the flux of charged particles when the negative electric potential is applied at the inner conductor (cathode). The high electric field generated by the combination of high potential and small conductor curvature radius (inner conductor, r_i) causes electron drift and ionization of the neutral gas surrounding the cathode. The resulting positive ions generate more electrons through secondary emission at the cathode surface. These electrons are accelerated through a small region away from the cathode, where they can acquire significant energy. This can lead to ionization which creates new

electron-ion pairs. The secondary ions migrate toward the cathode where they eject more secondary electrons. This process is responsible for sustaining the discharge.

The model uses a Scharfetter–Gummel upwind scheme to eliminate numerical instabilities in the number density of the charged particles associated with the finite element method. This is needed, in particular close to the cathode, where the ion flux is particularly high.

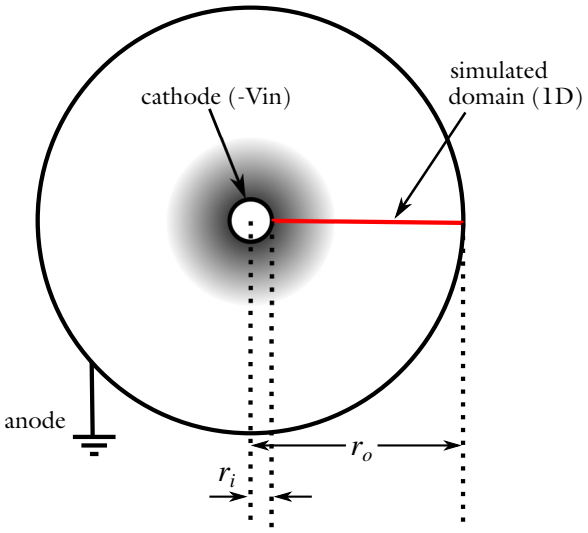

Figure 1: Not-to-scale cross section of the coaxial configuration. The negative potential ($-V_{in}$) is applied at the inner conductor (cathode) and the outer electrode is grounded (anode). The shaded area represents the ionization region created by the positive space charge distribution generated in the vicinity of the cathode.

DOMAIN EQUATIONS

The electron density is computed by solving the drift-diffusion equation for the electron density.

$$\frac{\partial}{\partial t}(n_e) + \nabla \cdot [-n_e(\mu_e \bullet \mathbf{E}) - \mathbf{D}_e \bullet \nabla n_e] = R_e \qquad (1)$$

Convection of electrons due to fluid motion is neglected. For more detailed information on electron transport see the section *Theory for the Drift Diffusion Interface* in the *Plasma Module User's Guide*.

When using the local field approximation the electron density equation is not solved and the transport and source coefficients are mapped by the reduced electric field. In practice,

when using the local field approximation or the local energy approximation, the transport and source coefficients are still given as a function of the mean electron energy. When using the local field approximation however, it must be provided a function that relates the mean electron energy and the reduced electric field

$$\varepsilon = F(E/N)\,. \tag{2}$$

The electron source R_e is defined later. The electron diffusivity is computed from the electron mobility using the relation

$$\mathbf{D}_e = \mu_e T_e \tag{3}$$

The source coefficients in the above equation is determined by the plasma chemistry using rate coefficients. Suppose that there are M reactions that contribute to the growth or decay of electron density and P inelastic electron-neutral collisions. In general, $P \gg M$. In the case of rate coefficients, the electron source term is given by

$$R_e = \sum_{j=1}^{M} x_j k_j N_n n_e \tag{4}$$

where x_j is the mole fraction of the target species for reaction j, k_j is the rate coefficient for reaction j (SI unit: m^3/s), and N_n is the total neutral number density (SI unit: $1/\mathrm{m}^3$). For DC discharges it is better practice to use Townsend coefficients instead of rate coefficients to define reaction rates. Townsend coefficients provide a better description of what happens in the cathode fall region Ref. 1. When Townsend coefficients are used, the electron source term is given by:

$$R_e = \sum_{j=1}^{M} x_j \alpha_j N_n |\Gamma_e| \tag{5}$$

where α_j is the Townsend coefficient for reaction j (SI unit: m^2), and Γ_e is the electron flux as defined above (SI unit: $1/(\mathrm{m}^2 \cdot \mathrm{s})$). Townsend coefficients can increase the stability of the numerical scheme when the electron flux is field driven as is the case with DC discharges.

For nonelectron species, the following equation is solved for the mass fraction of each species:

$$\rho \frac{\partial}{\partial t}(w_k) + \rho(\mathbf{u} \cdot \nabla) w_k = \nabla \cdot \mathbf{j}_k + R_k \tag{6}$$

For detailed information on the transport of the nonelectron species see the section *Theory for the Heavy Species Transport Interface* in the *Plasma Module User's Guide*.

The electrostatic field is computed using the equation

$$-\nabla \cdot \varepsilon_0 \varepsilon_r \nabla V = \rho \tag{7}$$

The space charge density ρ is automatically computed based on the plasma chemistry specified in the model using the formula

$$\rho = q\left(\sum_{k=1}^{N} Z_k n_k - n_e\right) \tag{8}$$

For detailed information about electrostatics see *Theory for the Electrostatics Interface* in the *Plasma Module User's Guide*.

Boundary Conditions

Electrons are lost to the wall due to random motion within a few mean free paths of the wall and gained due to secondary emission effects, resulting in the boundary condition for the electron flux

$$\mathbf{n} \cdot \Gamma_e = \left(\frac{1}{2} \nu_{e,\mathrm{th}} n_e\right) - \sum_p \gamma_p (\Gamma_p \cdot \mathbf{n}) \tag{9}$$

The second term on the right-hand side of Equation 9 is the gain of electrons due to secondary emission effects, γ_p being the secondary emission coefficient. For the heavy species, ions are lost to the wall due to surface reactions and the fact that the electric field is directed toward the wall:

$$\mathbf{n} \cdot \mathbf{j}_k = M_w R_k + M_w c_k Z \mu_k (\mathbf{E} \cdot \mathbf{n})[Z_k \mu_k (\mathbf{E} \cdot \mathbf{n}) > 0] \tag{10}$$

The discharge is driven by a DC electric potential (V_0) applied to the inner conductor of the coaxial geometry (at coordinate $r = r_i$). The other boundary (at coordinate $r = r_o$) is grounded. To facilitate the beginning of the numerical simulation a step function is used to modulate V_0 with the transient applied potential assuming the form

$$V = V_0 \tanh\left(\frac{t}{\tau}\right). \tag{11}$$

This numeric technique does not interfere with the results at steady state, which are the ones of interest in this work.

PLASMA CHEMISTRY

The chemistry of a plasma sustained in air can be very complex and a detailed study of the main excited states could easily have hundreds of reactions. The main goal of this model is to study the charge particle density profiles and currents. With that in mind it is used a simplified set of reactions that describes correctly the creation and destruction of charged species in a background of dry air.

Table 1 lists the chemical reactions considered Ref. 2. In the fluid equations nitrogen and oxygen are not treated separately as in a detailed chemistry. Instead a general species A is used for the background gas. A can be ionized forming positive ions p, and A can attach electrons forming negative ions n.

The creation and destruction of electrons in the volume is described by ionization and attachment Townsend coefficients, and by rate constants for a three-body attachment and electron-ion recombination. The Townsend coefficients are obtained as a function of the mean electron energy by suitably averaging over the electron energy distribution computed using a Boltzmann solver with a consistent set of electron scattering collisions cross sections of nitrogen and oxygen Ref. 3. It is used a mixture of 80% nitrogen and 20% oxygen. The relation between the mean electron energy and the reduced electric field is also obtain from the Boltzmann solver and is given in Figure 2.

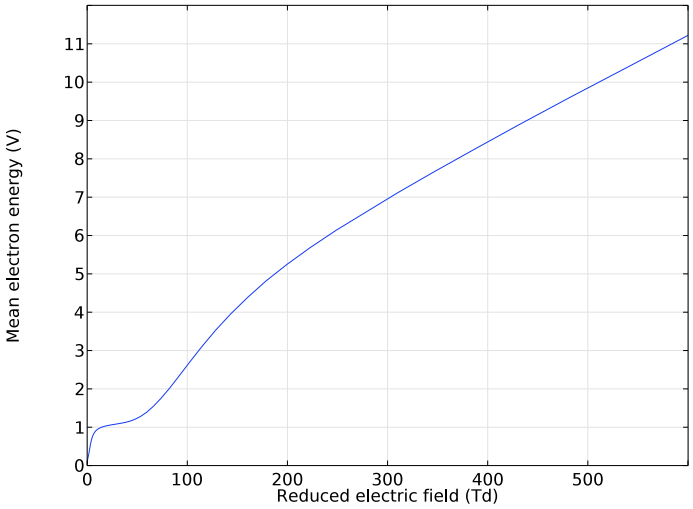

Figure 2: Mean electron energy as a function of the reduced electric field for a mixture of 80% nitrogen and 20% oxygen.

For detailed information on how to compute source coefficients from a Boltzmann solver see the section *The Boltzmann Equation, Two-Term Approximation Interface* in the *Plasma Module User's Guide*.

TABLE 1: TABLE OF COLLISIONS AND REACTIONS MODELED.

Reaction	**Formula**	**Type**	$\Delta\varepsilon$ **(eV)**	k_f **(m^3/s)**
1	e+A=>p+2e	Ionization	15	-
2	e+A=>n	Attachment	-	-
3	e+2A=>n+A	Attachment	-	-
4	e+p=>A	Reaction	-	$5 \cdot 10^{-14}$
5	n+p=>2A	Reaction	-	$2 \cdot 10^{-12}$

At steady state, the plasma main charged species are ions. For this reason, the initial conditions have an equal density of positive and negative ions and a small density of electrons. These initial conditions preserve charge neutrality as it is important for numerical reasons.

In addition to the volumetric reactions, the following surface reactions are implemented:

TABLE 2: TABLE OF SURFACE REACTIONS.

Reaction	Formula	Sticking coefficient
1	p=>A	1
2	n=>A	1

When the ions reach the wall, they are assumed to change back to neutral atoms. Note that the secondary emission coefficient for positive ions is set to 0.05 on the cathode boundary (at coordinate $r = r_i$) and to 0 at the outer electrode (at coordinate $r = r_o$). The mean electron energy of the secondary electron is set to 4 eV. When using the local field approximation the mean energy of the secondary electron is only used in postprocessing.

Results and Discussion

Throughout most of this section, results are presented and discussed for a DC negative corona sustained with –45 kV applied to the inner electrode. At the end, the current-voltage characteristic of the discharge are presented. The background gas is kept at a constant density as obtained by the ideal gas law at atmospheric pressure and at a temperature of 600 K. All results presented and discussed correspond to a steady state operation.

When comparing with corona discharges sustained in noble gases, air corona discharges need much higher voltage to breakdown the background gas and to sustain a discharge. There are two main reasons for this: (i) the electron collision frequency in air is higher (in part due to rotational and vibrational interactions) making it more difficult to accelerate electrons; and (ii) oxygen is electronegative.

Figure 3 presents the spatial profiles of the charged species. The discharge can be separated in two regions: (a) corresponding to a region of less than 1 mm near the cathode where most of the ionization occurs; and (b) the rest of the volume that reaches to the ground electrode.

The strongly negative potential at the inner electrode accelerates positive ions toward it and repeals negative charged particles. The result is a region of positive charge separation (region (a)) where strong electric fields exist and the electrons are accelerated to energies capable of ionizing the background gas. The electron temperature, the electric potential, and the reduced electric field are represented in Figure 4, Figure 5, and Figure 6. As can be seen, region (b) has weak electric field and consequent electron temperatures. In this region, electrons have not enough energy to ionize and are efficiently attached forming negative ions. The result is a long spatial portion of the discharge dominated by negative

ions that drift toward the ground electrode. Note also that in region (b) the charge separation barely deforms the applied potential.

Figure 7, Figure 8, and Figure 9 are 2D representations (obtained by a revolution of the 1D solution) of the charged species number density. A representation of the charge distribution with distances in linear scale helps to build a more realistic image of this type of discharges. Notice how small the inner electrode and the ionization region are, and how most of the volume is filled with negative ions drifting in the directions of the anode. This observation does not mean that region (a) should be neglected. In fact, it is in region (a) that the mechanisms that sustain the discharge occur.

Comparing with corona discharges in noble gases, this type of corona discharges have a much higher resistivity because the charged particle number density is much lower, and ions, much less mobile than electrons, are the main charge carriers.

Figure 10 shows the absolute value of the total ion current at the ground electrode as a function of the absolute value of the applied voltage at the inner electrode. The current-voltage characteristic follows a quadratic law as expected. The obtained values of the current density are also coherent with the ones found in this type of discharges.

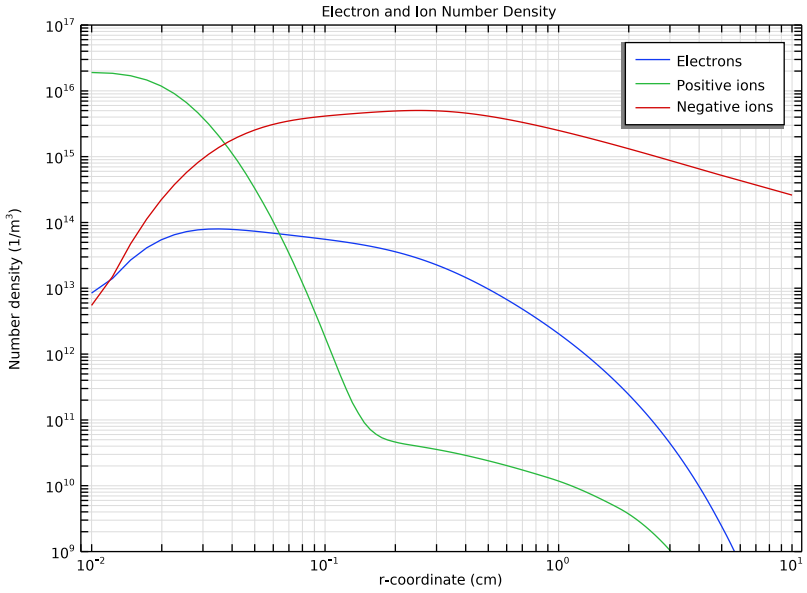

Figure 3: Spatial profiles of the charged species number density at steady state: electrons (blue), positive ions (green), and negative ions (red).

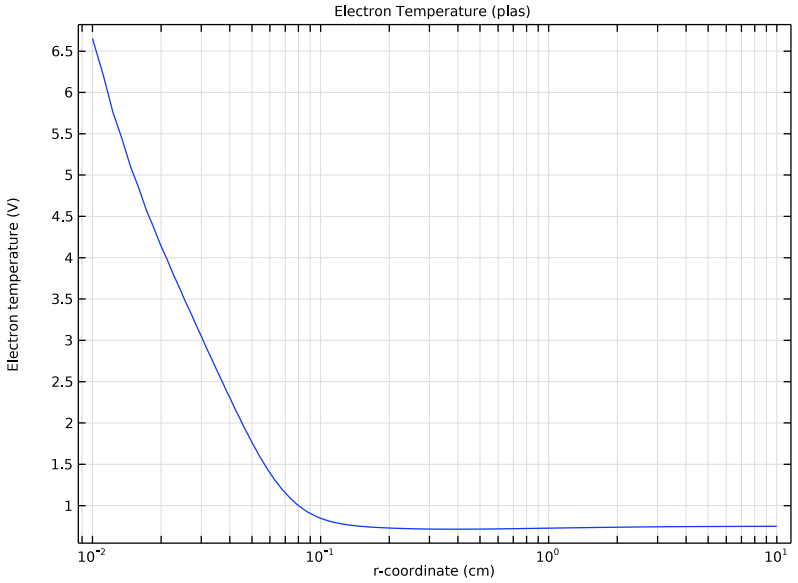

Figure 4: Spatial profile of the electron temperature.

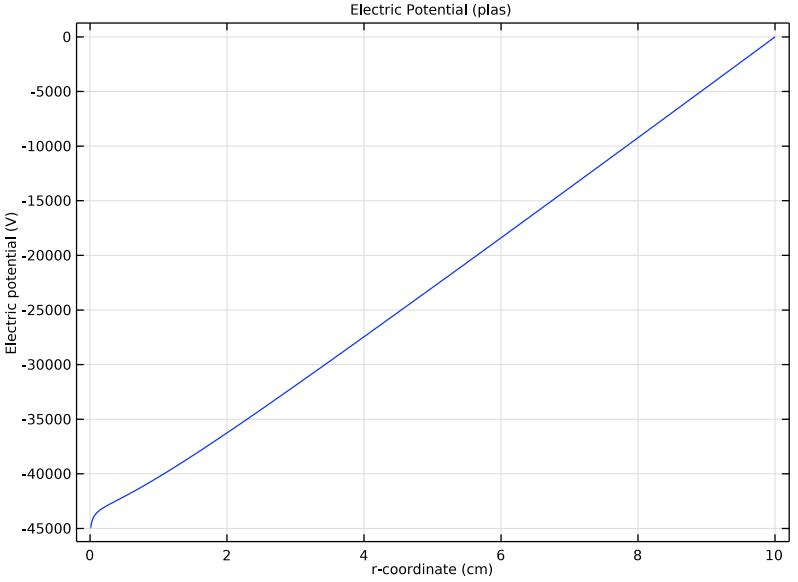

Figure 5: Spatial profile of the electric potential.

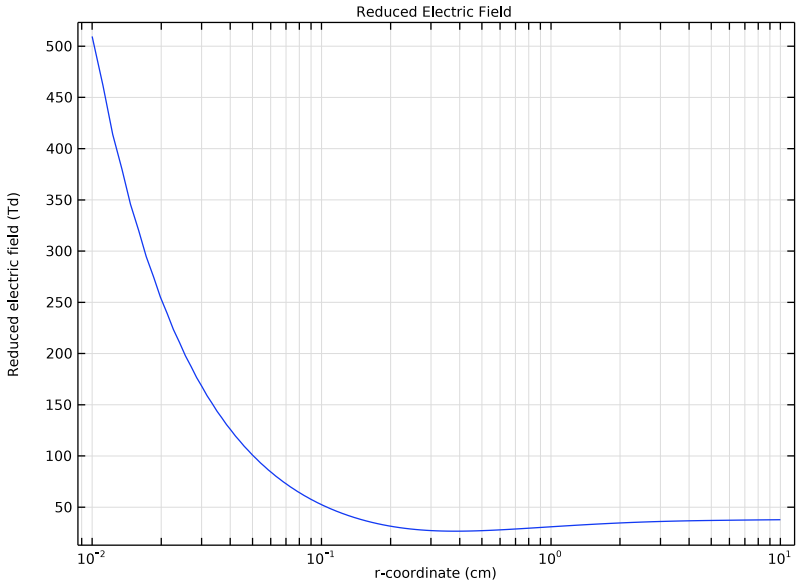

Figure 6: Spatial profile of the reduced electric field.

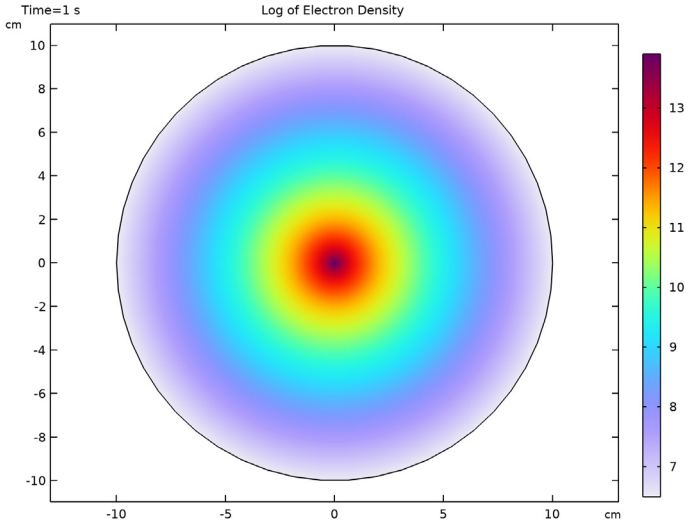

Figure 7: 2D representation of the electron density (the scale is in log base 10).

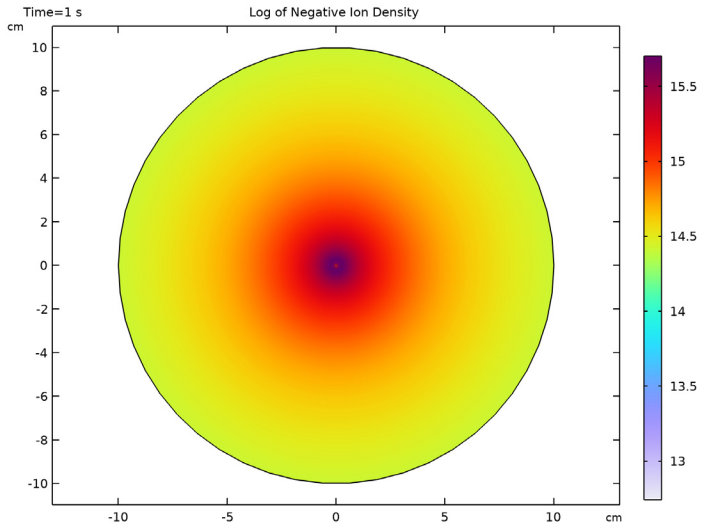

Figure 8: 2D representation of the negative ion density (the scale is in log base 10).

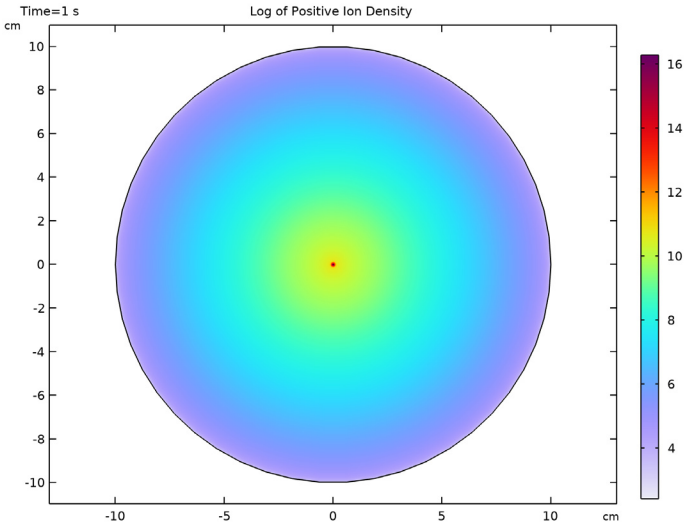

Figure 9: 2D representation of the positive ion density (the scale is in log base 10).

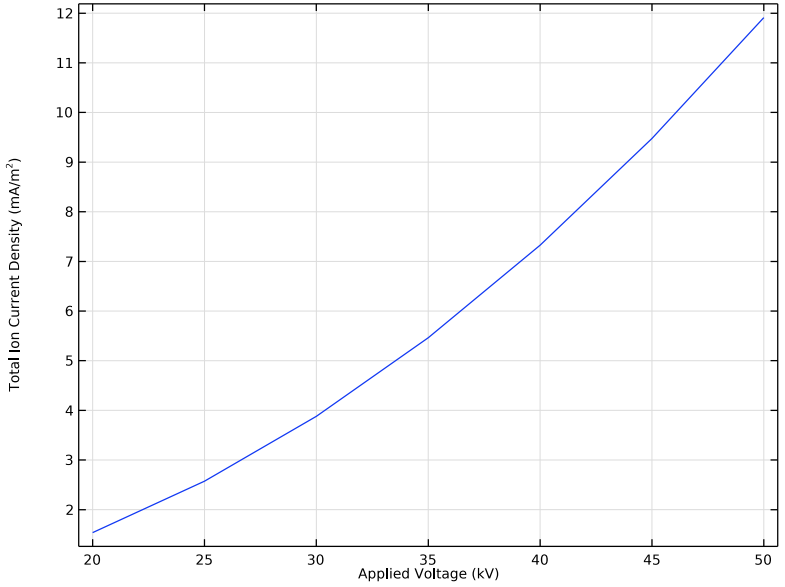

Figure 10: Total ion current density (absolute value) at the ground electrode as a function of the applied voltage (absolute value) at the inner electrode.

References

1. M.A. Lieberman and A.J. Lichtenberg, *Principles of Plasma Discharges and Materials Processing*, John Wiley & Sons, 2005.

2. A.A. Kulikovsky, "Positive streamer between parallel plate electrode in atmospheric pressure air", *J. Phys. D: Appl. Phys.*, vol. 30, pp. 441–450, 1997.

3. Phelps database, www.lxcat.net, retrieved 2016.

Application Library path: `Plasma_Module/Corona_Discharges/corona_discharge_air_1d`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **1D Axisymmetric**.

2 In the **Select Physics** tree, select **Plasma>Plasma (plas)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Time Dependent**.

6 Click **Done**.

GEOMETRY 1

Follow the steps below to create the model geometry: a simple 1D geometry consisting of a single domain bounded by the cathode (left, inner conductor) and the anode (right, outer conductor).

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.

2 In the **Settings** window for **Geometry**, locate the **Units** section.

3 From the **Length unit** list, choose **cm**.

Interval 1 (i1)

1 Right-click **Component 1 (comp1)>Geometry 1** and choose **Interval**.

2 In the **Settings** window for **Interval**, locate the **Interval** section.

3 In the table, enter the following settings:

Coordinates (cm)
10
0.01

4 Click **Build All Objects**.

DEFINITIONS

Variables 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Definitions** and choose **Variables**.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 In the table, enter the following settings:

Name	Expression	Unit	Description
mueN	3.74e24*(plas.Erd*1e21)^-0.22[1/(V*m*s)]		Reduced electron mobility
muiN	6e21[1/(V*s*m)]	1/(V·m·s)	Reduced ion mobility
rnp	2e-6[cm^3/s]	m^3/s	ion-ion recombination
rei	5e-8[cm^3/s]	m^3/s	electron-ion recombination
Vapp	-V0*ramp		Applied Voltage
ramp	tanh(1e5*t)		
p0	760[torr]	Pa	Gas pressure
t0	600[K]	K	Gas temperature
ni0	1e17[m^-3]	$1/m^3$	Initial ion number density
ne0	1e10[m^-3]	$1/m^3$	Initial electron number density

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
V0	45[kV]	45000 V	

PLASMA (PLAS)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Plasma (plas)**.

2 In the **Settings** window for **Plasma**, locate the **Vertical Height** section.

3 In the d_z text field, type `1[m]`.

4 Locate the **Plasma Properties** section. Select the **Use reduced electron transport properties** check box.

This model uses the local field approximation to parameterize in space source and transport coefficients.

5 From the **Mean electron energy** list, choose **Local field approximation**.

Solve this model using the finite volume method and a Scharfetter-Gummel scheme. You can switch from the finite element method to the finite volume method by suitable choices in the **Discretization** section.

6 Click to expand the **Discretization** section. From the **Formulation** list, choose **Finite volume (constant shape function)**.

Electron Impact Reaction 1

1 In the **Physics** toolbar, click — **Domains** and choose **Electron Impact Reaction**.

Add an ionization reaction.

2 In the **Settings** window for **Electron Impact Reaction**, locate the **Reaction Formula** section.

3 In the **Formula** text field, type `A+e=>p+2e`.

4 Locate the **Collision Type** section. From the **Collision type** list, choose **Ionization**.

5 In the Δε text field, type `15`.

Import the ionization Townsend coefficient.

6 Locate the **Collision** section. From the **Specify reaction using** list, choose **Use lookup table**.

7 Locate the **Reaction Parameters** section. From the **Rate constant form** list, choose **Townsend coefficient**.

8 Find the **Townsend coefficient data** subsection. Click **Load from File**.

9 Browse to the model's Application Libraries folder and double-click the file `alpha.txt`.

Electron Impact Reaction 2

1 In the **Physics** toolbar, click **Domains** and choose **Electron Impact Reaction**.

Add an attachment reaction.

2 In the **Settings** window for **Electron Impact Reaction**, locate the **Reaction Formula** section.

3 In the **Formula** text field, type `A+e=>n`.

4 Locate the **Collision Type** section. From the **Collision type** list, choose **Attachment**.

Import the attachment Townsend coefficient.

5 Locate the **Collision** section. From the **Specify reaction using** list, choose **Use lookup table**.

6 Locate the **Reaction Parameters** section. From the **Rate constant form** list, choose **Townsend coefficient**.

7 Find the **Townsend coefficient data** subsection. Click **Load from File**.

8 Browse to the model's Application Libraries folder and double-click the file `eta.txt`.

Electron Impact Reaction 3

1 In the **Physics** toolbar, click **Domains** and choose **Electron Impact Reaction**.

Add a 3-body attachment reaction.

2 In the **Settings** window for **Electron Impact Reaction**, locate the **Reaction Formula** section.

3 In the **Formula** text field, type `A + A + e => n + A`.

4 Locate the **Collision Type** section. From the **Collision type** list, choose **Attachment**.

5 Locate the **Reaction Parameters** section. In the k^{f} text field, type `1.4e-41*(0.026/plas.Te)*exp(100/t0-0.061/plas.Te)*N_A_const^2*0.1`.

Reaction 1

1 In the **Physics** toolbar, click **Domains** and choose **Reaction**.

Add electron-ion recombination.

2 In the **Settings** window for **Reaction**, locate the **Reaction Formula** section.

3 In the **Formula** text field, type `e+p=>A`.

4 Locate the **Reaction Parameters** section. In the k^{f} text field, type `rei*N_A_const`.

Reaction 2

1 In the **Physics** toolbar, click — **Domains** and choose **Reaction**.

Add ion-ion recombination.

2 In the **Settings** window for **Reaction**, locate the **Reaction Formula** section.

3 In the **Formula** text field, type `n+p=>A+A`.

4 Locate the **Reaction Parameters** section. In the k^{f} text field, type `rnp*N_A_const`.

Species: A

When solving any type of reacting flow problem one species must always be chosen to fulfill the mass constraint. This should be taken as the species with the largest mass fraction.

1 In the **Model Builder** window, click **Species: A**.

2 In the **Settings** window for **Species**, locate the **Species Formula** section.

3 Select the **From mass constraint** check box.

4 Locate the **General Parameters** section. From the **Preset species data** list, choose **N2**.

Species: p

1 In the **Model Builder** window, click **Species: p**.

2 In the **Settings** window for **Species**, locate the **Species Formula** section.

3 From the **Species type** list, choose **Ion**.

4 Locate the **General Parameters** section. From the **Preset species data** list, choose **N2**.

5 In the z text field, type `1`.

6 In the n_0 text field, type `ni0`.

7 Locate the **Mobility and Diffusivity Expressions** section. From the **Specification** list, choose **Specify mobility, compute diffusivity**.

8 Locate the **Mobility Specification** section. In the u_{m} text field, type `muiN/plas.Nn`.

Species: n

1 In the **Model Builder** window, click **Species: n**.

2 In the **Settings** window for **Species**, locate the **Species Formula** section.

3 From the **Species type** list, choose **Ion**.

4 Locate the **General Parameters** section. From the **Preset species data** list, choose **N2**.

5 In the z text field, type `-1`.

6 In the n_0 text field, type `ni0`.

7 Locate the **Mobility and Diffusivity Expressions** section. From the **Specification** list, choose **Specify mobility, compute diffusivity**.

8 Locate the **Mobility Specification** section. In the u_m text field, type `muiN/plas.Nn`.

Surface Reaction 1

1 In the **Physics** toolbar, click → **Boundaries** and choose **Surface Reaction**.

Surface reactions must always be included in a plasma model because they describe how ions, excited, and radical species interact with the wall.Secondary emission of electrons is important to sustain a DC discharge. In this example, add a secondary emission coefficient on the left wall (cathode).

2 In the **Settings** window for **Surface Reaction**, locate the **Reaction Formula** section.

3 In the **Formula** text field, type `p=>A`.

4 Select Boundary 1 only.

5 Locate the **Secondary Emission Parameters** section. In the γ_i text field, type `0.05`.

6 In the ε_i text field, type `4`.

Surface Reaction 2

1 In the **Physics** toolbar, click → **Boundaries** and choose **Surface Reaction**.

2 In the **Settings** window for **Surface Reaction**, locate the **Reaction Formula** section.

3 In the **Formula** text field, type `p=>A`.

4 Select Boundary 2 only.

Surface Reaction 3

1 In the **Physics** toolbar, click → **Boundaries** and choose **Surface Reaction**.

2 In the **Settings** window for **Surface Reaction**, locate the **Reaction Formula** section.

3 In the **Formula** text field, type `n=>A`.

4 Locate the **Boundary Selection** section. From the **Selection** list, choose **All boundaries**.

Plasma Model 1

Set up the background gas information, the electron mobility and import the table that gives the relation between the mean electron energy and the reduced electric field.

1 In the **Model Builder** window, click **Plasma Model 1**.

2 In the **Settings** window for **Plasma Model**, locate the **Model Inputs** section.

3 In the T text field, type `t0`.

4 In the p_A text field, type `p0`.

5 Locate the **Electron Density and Energy** section. In the $\mu_e N_n$ text field, type `mueN`.

6 Locate the **Mean Electron Energy Specification** section. From the **Specify using** list, choose **Use lookup table**.

7 Find the **Mean electron energy** subsection. Click **Load from File**.

8 Browse to the model's Application Libraries folder and double-click the file `EN_to_Nrg.txt`.

Initial Values 1

1 In the **Model Builder** window, click **Initial Values 1**.

2 In the **Settings** window for **Initial Values**, locate the **Initial Values** section.

3 In the $n_{e,0}$ text field, type `ne0`.

Ground 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Ground**.

2 Select Boundary 2 only.

Metal Contact 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Metal Contact**.

2 Select Boundary 1 only.

3 In the **Settings** window for **Metal Contact**, locate the **Terminal** section.

4 In the V_0 text field, type `Vapp`.

Wall 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Wall**.

2 In the **Settings** window for **Wall**, locate the **Boundary Selection** section.

3 From the **Selection** list, choose **All boundaries**.

MESH 1

Meshing is a critical step in any plasma model. A fine mesh is needed close to the electrodes to capture the separation of space charge between the electrons and ions close to the wall.

Edge 1

In the **Mesh** toolbar, click **Edge**.

Distribution 1

1 Right-click **Edge 1** and choose **Distribution**.

2 In the **Settings** window for **Distribution**, locate the **Distribution** section.

3 From the **Distribution type** list, choose **Predefined**.

4 In the **Number of elements** text field, type `200`.

5 In the **Element ratio** text field, type `100`.

6 From the **Growth rate** list, choose **Exponential**.

7 Select the **Symmetric distribution** check box.

8 Click **Build All**.

STUDY 1

Step 1: Time Dependent

1 In the **Model Builder** window, under **Study 1** click **Step 1: Time Dependent**.

2 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.

3 In the **Output times** text field, type `0 10^{range(-8,8/49,0)}`.

4 Click to expand the **Results While Solving** section. Select the **Plot** check box.

Solution 1 (sol1)

1 In the **Study** toolbar, click **Show Default Solver**.

2 In the **Model Builder** window, expand the **Solution 1 (sol1)** node.

Set the Jacobian update to minimal to decrease the computational time.

3 In the **Model Builder** window, expand the **Study 1>Solver Configurations>Solution 1 (sol1)>Time-Dependent Solver 1** node, then click **Fully Coupled 1**.

4 In the **Settings** window for **Fully Coupled**, click to expand the **Method and Termination** section.

5 From the **Jacobian update** list, choose **Minimal**.

Get the initial values to generate the default plots and then set them up to show the electron and ion densities while the solver runs.

6 In the **Study** toolbar, click **Get Initial Value**.

RESULTS

Electron and Ion Number Density

1 In the **Settings** window for **1D Plot Group**, type `Electron and Ion Number Density` in the **Label** text field.

2 Click to expand the **Title** section. From the **Title type** list, choose **Label**.

3 Locate the **Data** section. From the **Time selection** list, choose **Last**.

4 Locate the **Plot Settings** section. Select the **x-axis label** check box.

5 Select the **y-axis label** check box. In the associated text field, type `Number density (1/m<sup>3</sup>)`.

6 Locate the **Axis** section. Select the **Manual axis limits** check box.

7 In the **x minimum** text field, type `0.009`.

8 In the **x maximum** text field, type `11`.

9 In the **y minimum** text field, type `1e9`.

10 In the **y maximum** text field, type `1e17`.

11 Select the **x-axis log scale** check box.

12 Select the **y-axis log scale** check box.

Electrons

1 In the **Model Builder** window, expand the **Electron and Ion Number Density** node, then click **Line Graph 1**.

2 In the **Settings** window for **Line Graph**, type `Electrons` in the **Label** text field.

3 Click to expand the **Legends** section. Select the **Show legends** check box.

4 Find the **Include** subsection. Clear the **Solution** check box.

5 Select the **Label** check box.

6 Click to expand the **Quality** section. From the **Resolution** list, choose **No refinement**.

Positive ions

1 Right-click **Electrons** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, type `Positive ions` in the **Label** text field.

3 Locate the **y-Axis Data** section. In the **Expression** text field, type `plas.n_wp`.

Negative ions

1 Right-click **Positive ions** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, type `Negative ions` in the **Label** text field.

3 Locate the **y-Axis Data** section. In the **Expression** text field, type `plas.n_wn`.

Electron Temperature (plas)

1 In the **Model Builder** window, under **Results** click **Electron Temperature (plas)**.

2 In the **Settings** window for **1D Plot Group**, locate the **Data** section.

3 From the **Time selection** list, choose **Last**.

4 Locate the **Title** section. From the **Title type** list, choose **Label**.

5 Locate the **Axis** section. Select the **x-axis log scale** check box.

Electric Potential (plas)

1 In the **Model Builder** window, click **Electric Potential (plas)**.

2 In the **Settings** window for **1D Plot Group**, locate the **Data** section.

3 From the **Time selection** list, choose **Last**.

4 Locate the **Title** section. From the **Title type** list, choose **Label**.

Electron and Ion Number Density

In the **Study** toolbar, click **Compute**.

Reduced Electric Field

1 Right-click **Electric Potential (plas)** and choose **Duplicate**.

2 In the **Settings** window for **1D Plot Group**, type `Reduced Electric Field` in the **Label** text field.

3 Locate the **Axis** section. Select the **x-axis log scale** check box.

Line Graph 1

1 In the **Model Builder** window, expand the **Reduced Electric Field** node, then click **Line Graph 1**.

2 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `plas.Erd`.

4 From the **Unit** list, choose **Td**.

5 In the **Reduced Electric Field** toolbar, click **Plot**.

Revolution 1D 1

In the **Results** toolbar, click **More Datasets** and choose **Revolution 1D**.

Log of Electron Density

1 In the **Results** toolbar, click **2D Plot Group**.

2 In the **Settings** window for **2D Plot Group**, type `Log of Electron Density` in the **Label** text field.

3 Click to expand the **Title** section. From the **Title type** list, choose **Label**.

Surface 1

1 Right-click **Log of Electron Density** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `log10(plas.ne)`.

4 In the **Log of Electron Density** toolbar, click **Plot**.

Log of Negative Ion Density

1 In the **Model Builder** window, right-click **Log of Electron Density** and choose **Duplicate**.

2 In the **Settings** window for **2D Plot Group**, type `Log of Negative Ion Density` in the **Label** text field.

3 Click to expand the **Title** section. From the **Title type** list, choose **Label**.

Surface 1

1 In the **Model Builder** window, expand the **Log of Negative Ion Density** node, then click **Surface 1**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `log10(plas.n_wn)`.

4 In the **Log of Negative Ion Density** toolbar, click **Plot**.

Log of Positive Ion Density

1 In the **Model Builder** window, right-click **Log of Negative Ion Density** and choose **Duplicate**.

2 In the **Settings** window for **2D Plot Group**, type `Log of Positive Ion Density` in the **Label** text field.

3 Click to expand the **Title** section. From the **Title type** list, choose **Label**.

Surface 1

1 In the **Model Builder** window, expand the **Log of Positive Ion Density** node, then click **Surface 1**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `log10(plas.n_wp)`.

4 In the **Log of Positive Ion Density** toolbar, click **Plot**.

STUDY 1

Prepare a parameterization of the applied voltage.

1 In the **Model Builder** window, click **Study 1**.

2 In the **Settings** window for **Study**, locate the **Study Settings** section.

3 Clear the **Generate default plots** check box.

4 Clear the **Generate convergence plots** check box.

Parametric Sweep

1 In the **Study** toolbar, click **Parametric Sweep**.

2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

3 Click **Add**.

4 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
V0	20 25 30 35 40 45 50	kV

Step 1: Time Dependent

1 In the **Model Builder** window, click **Step 1: Time Dependent**.

2 In the **Settings** window for **Time Dependent**, locate the **Results While Solving** section.

3 Clear the **Plot** check box.

Parametric Sweep

In the **Study** toolbar, click **Compute**.

RESULTS

Current Vs. Voltage

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1/Parametric Solutions 1 (sol2)**.

4 From the **Time selection** list, choose **Last**.

5 In the **Label** text field, type `Current Vs. Voltage`.

6 Locate the **Title** section. From the **Title type** list, choose **None**.

Point Graph 1

1 Right-click **Current Vs. Voltage** and choose **Point Graph**.

2 Select Boundary 2 only.

3 In the **Settings** window for **Point Graph**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)> Plasma>Current>plas.nJt - Total ion current density on wall - A/m²**.

4 Locate the **y-Axis Data** section. In the **Expression** text field, type `abs(plas.nJt)`.

5 In the **Unit** field, type `mA/m^2`.

6 Select the **Description** check box.

7 Locate the **x-Axis Data** section. From the **Axis source data** list, choose **V0**.

8 From the **Parameter** list, choose **Expression**.

9 In the **Expression** text field, type `V0`.

10 From the **Unit** list, choose **kV**.

11 Select the **Description** check box. In the associated text field, type `Applied Voltage`.

Current Vs. Voltage

1 In the **Model Builder** window, click **Current Vs. Voltage**.

2 In the **Settings** window for **1D Plot Group**, locate the **Plot Settings** section.

3 Select the **y-axis label** check box. In the associated text field, type `Total Ion Current Density (mA/m<sup>2</sup>)`.

4 In the **Current Vs. Voltage** toolbar, click  **Plot**.