Created in COMSOL Multiphysics 6.2

Self-Lubricating Journal Bearing

Introduction

A self-lubricating journal bearing typically has a porous bush which is press-fitted to the bearing, such that, the bush's inner surface and the rotating journal maintain a small, lubricant-filled clearance. The porous bush, usually made of sintered metal powder, is impregnated with the lubricant. During operation, the bush acts as a reservoir and redistributes the lubricant in the clearance.

Inside the porous bush, the lubricant is permitted to flow along axial, radial and circumferential directions. When the journal is loaded, it is no longer concentric with the bush. This eccentricity creates a zone of high lubricant pressure in the region of reducing clearance and a zone of low lubricant pressure in the region of increasing clearance. The lubricant is supplied into the clearance from the pores in the low pressure zone and simultaneously squeezed out of the high pressure zone into the porous bush, thus facilitating the act of self-lubrication.

These bearings are generally employed in applications operating under high rotational speed and low load-carrying requirements. They are most useful in situations where frequent maintenance and lubrication of the bearing is not feasible.

Model Definition

This model simulates a journal bearing operating in the hydrodynamic lubrication regime. In this regime, the lubricant is assumed to fill the entire clearance between the rotating journal and the bush, thereby generating high- and low-pressure zones, of which the former helps to support the external load. The phenomenon of cavitation, which can typically occur in the low-pressure zone, is ignored in the model.

The migration of lubricant in the porous bush is modeled using Darcy's law. The flow of the thin film of lubricant in the clearance is modeled using the Reynolds equation. At the interface between the porous and thin-film flow regions, the boundary layer developed is modeled using the Beavers–Joseph condition, which proposes an interface slip velocity for the clear fluid (thin-film fluid in the present example) that is proportional to the normal exterior velocity gradient. For details, see Ref. 1.

The dimensionless quantities of interest, reported in Ref. 1, are

$$s = \frac{\sqrt{\Phi}}{\alpha C}, \quad \Psi = \frac{H\Phi}{C^3} \tag{1}$$

Here, s is the dimensionless slip parameter, Φ is the permeability of the porous bush, α is the Beavers–Joseph slip coefficient, C is the concentric clearance between the journal and the bush, Ψ is the dimensionless permeability parameter, and H is the radial thickness of the bush. The thickness of the thin film of fluid in the clearance, h_t, is determined by the eccentricity, ε, as $h_t = C(1+\varepsilon\cos\theta)$, where, $\theta = \mathrm{atan}(y/x)$ is the angular position in the xy-plane.

The model solves for the stationary solution of the flow field in the porous domain and the clearance for the parameter values $s = [0.25, 0.5]$, $\Psi = [1, 0.1, 0.01, 0.001]$, and $\varepsilon = [0.1, 0.3, 0.5, 0.7, 0.9]$. It compares the numerical solution with those obtained from the analytic expressions in Ref. 1 for the short-bearing approximation, where the radius is much larger than the bearing length. Further, the flow field is visualized for a typical bearing without the short-bearing assumption.

GEOMETRY

The geometry represents the porous bush as a cylindrical annulus, wherein the clearance is represented by the bush's inner surface, as shown in Figure 1.

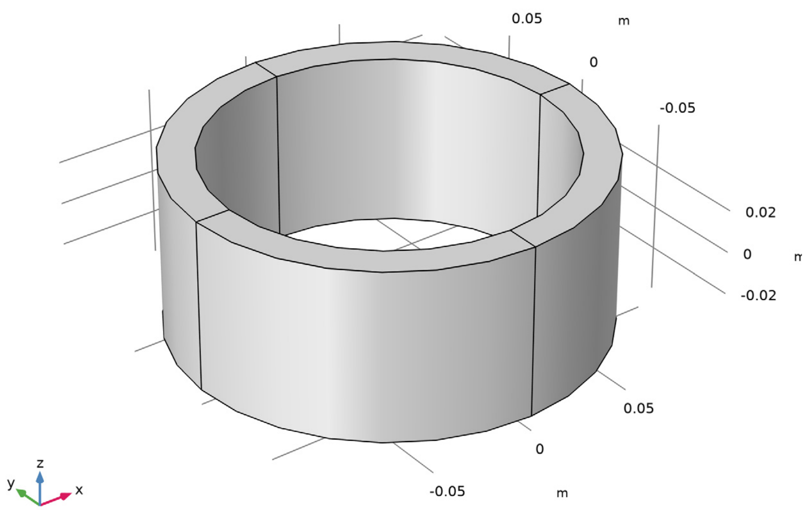

Figure 1: Computational domain of the porous bush. The clearance is represented by the inner surface of the cylinder.

The journal and the bearing are not represented in the geometry of the model, but their influence is taken into consideration in the form of boundary conditions. The bush has an inner radius, R, and length along the axis, $L = 5H$. For the short-bearing assumption, $R = 20L$; otherwise, $R = L$. For all simulations, the concentric clearance, $C = 10^{-5}$ m/Ψ.

PHYSICS INTERFACE SETTINGS

The lubricant density is $\rho = 1$ kg·m^{-3} and the dynamic viscosity $\mu = 1$ Pa·s. The journal's tangential velocity is $U = 1$ m/s. The Thin-Film Flow interface solves the Reynolds equation. The fluid-film thickness in the clearance is specified as the height of the wall, while the tangential velocity is specified at the base. The two ends of the bearing are assumed to be exposed to ambient conditions. Hence, a zero-pressure boundary condition is applicable. Meanwhile, a no-penetration condition is suitable at the bush surface that is in contact with the nonporous bearing to model the press-fit. For both interfaces, the reference pressure, $p_{\mathrm{ref}} = 0$. Also, a zero-pressure initial condition is applied. The Thin-Film and Porous Media Flow multiphysics coupling with a Beavers–Joseph slip condition couples the interfaces.

MESHING

A structured mesh composed of hexahedral elements is constructed with 200, 5, and 15 elements in the circumferential, radial, and axial directions, respectively, as shown in Figure 2.

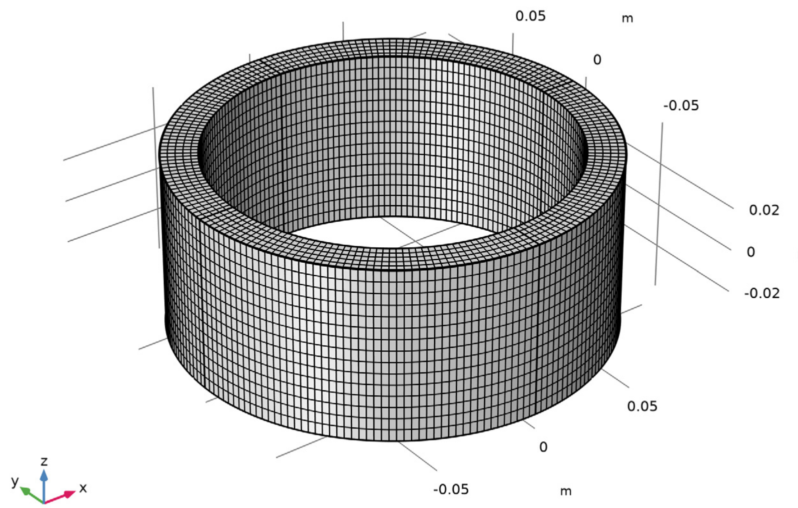

Figure 2: A structured, hexahedral mesh for the cylindrical geometry.

Results and Discussion

The load-carrying capacity is the main quantity of interest for postprocessing the solution. A dimensionless form of the load-carrying capacity, W_{nd}, can be calculated from the computed pressure as

$$W_{nd} = \frac{C^2}{\mu U L^3}\sqrt{\left(\int_{z=-L/2}^{z=L/2}\int_{y=0}^{y=R}\int_{x=-R}^{x=R} p_{film}\cos\theta\right)^2 + \left(\int_{z=-L/2}^{z=L/2}\int_{y=0}^{y=R}\int_{x=-R}^{x=R} p_{film}\sin\theta\right)^2} \qquad (2)$$

Here, p_{film} is the thin-film pressure. Negative pressures are ignored according to Ocvirk's approximations; see Ref. 1. The analytic expression for the nondimensional load-carrying capacity, $W_{nd,ana}$, obtained from Ref. 1, is given by

$$W_{nd,\,ana} = \sqrt{\left(\int_{z=-L/2}^{z=L/2}\int_{y=0}^{y=R}\int_{x=-R}^{x=R} p_{nd}\cos\theta\right)^2 + \left(\int_{z=-L/2}^{z=L/2}\int_{y=0}^{y=R}\int_{x=-R}^{x=R} p_{nd}\sin\theta\right)^2} \qquad (3)$$

where the dimensionless pressure, p_{nd}, is given by

$$p_{\mathrm{nd}} = \frac{24}{\pi^3}\sum_{n=1}^{10}\frac{(-1)^{n+1}\varepsilon\, g_n(\theta)\cos(\pi\beta_n z/L)}{(2n-1)^3} \qquad (4)$$

Here,

$$g_n(\theta) = \frac{\sin\theta(2s^2 + 2s(1+\varepsilon\cos\theta) + (1+\varepsilon\cos\theta)^2)}{(s+1+\varepsilon\cos\theta)(A+B)} \qquad (5)$$

$$A = (1+\varepsilon\cos\theta)^2(6\alpha^2 s^2 + 4s(1+\varepsilon\cos\theta) + (1+\varepsilon\cos\theta)^2) \qquad (6)$$

$$B = \frac{12\Psi}{\pi\beta_n(H/L)}(s+1+\varepsilon\cos\theta)\tanh(\pi\beta_n(H/L)) \qquad (7)$$

$$\beta_n = 2n-1 \qquad (8)$$

Figure 3 shows a plot of the nondimensional load-carrying capacity as a function of ε for various values of Ψ. The numeric values match well with the analytic curves.

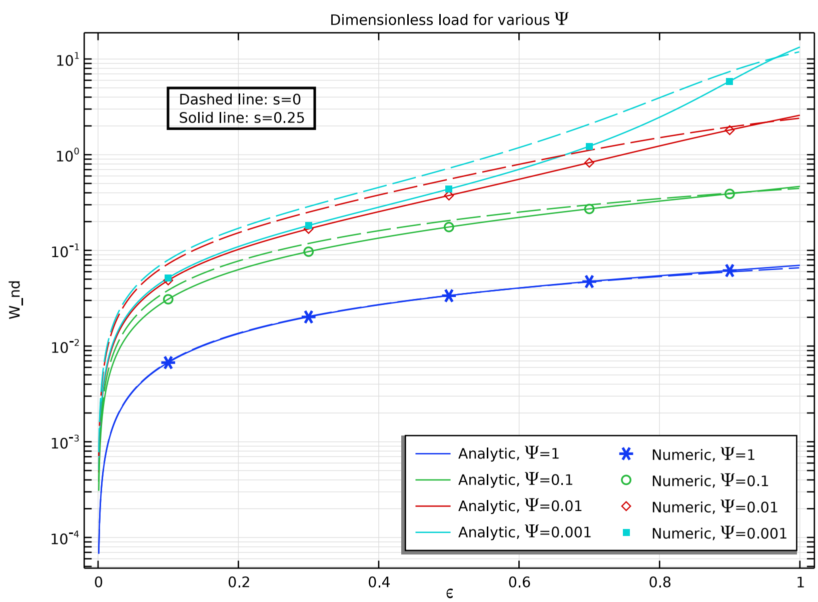

Figure 3: The dimensionless load-carrying capacity as a function of eccentricity for different values of the permeability parameter. Here, s = 0.25.

Dotted lines show the load capacity with no slip at the interface. Load capacity generally increases with increased eccentricity for all values of Ψ. Accounting for the slip results in a reduction in the load capacity for low values of eccentricity. This is true up to a certain critical value of transition of ε that depends on Ψ. Thereafter, the trend reverses for larger values of eccentricity. Also, for small values of Ψ, the load capacity is large, indicating that the load capacity deteriorates with increases in the permeability of the porous bush.

Figure 4 shows the plot of the nondimensional load-carrying capacity as a function of Ψ for various values of ε. The numeric values match well with the analytic curves.

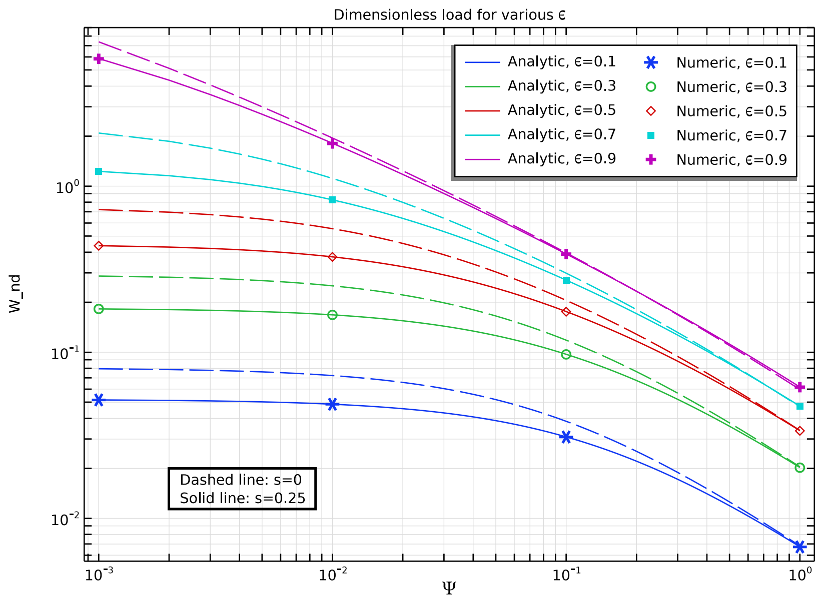

Figure 4: The dimensionless load-carrying capacity as a function of the permeability parameter for different values of eccentricity. Here, s = 0.25.

In general, the load capacity decreases with increases in the permeability parameter, where higher values of eccentricity support higher load values. Here, the dotted lines show the load capacity with no slip at the interface. Accounting for the slip velocity, the load capacities are smaller than those with no-slip condition up to a certain critical value of Ψ, after which the trend is observed to reverse. Moreover, the critical value of transition for Ψ decreases with increased eccentricity.

Figure 5 and Figure 6 show a similar behavior to that in Figure 3 and Figure 4, respectively, thus indicating that the effect of an increase in the slip coefficient value is not significant.

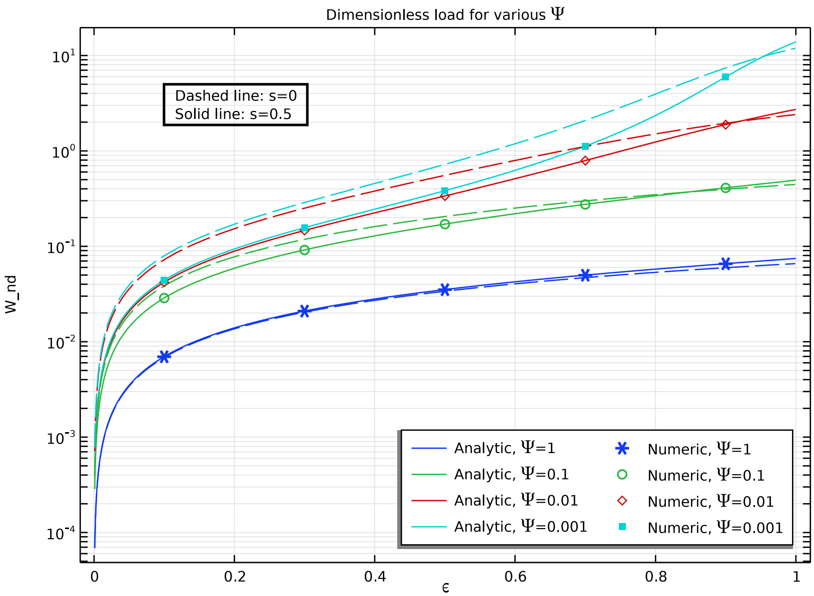

Figure 5: The dimensionless load carrying capacity as a function of eccentricity for different values of the permeability parameter. Here, s = 0.5.

Figure 7 shows a polar plot of the attitude angle, ϕ versus the eccentricity for various values of the permeability parameter. Again, the dotted lines show the load capacity with no slip at the interface. The attitude angle is the angle formed by the line joining the centers of the journal and the bearing with the load line. It is computed from the numerical solution as

$$\phi = \mathrm{atan}\left(-\left(\int_{z=-L/2}^{z=L/2}\int_{y=0}^{y=R}\int_{x=-R}^{x=R} p_{\mathrm{film}}\sin\theta\right)\Big/\left(\int_{z=-L/2}^{z=L/2}\int_{y=0}^{y=R}\int_{x=-R}^{x=R} p_{\mathrm{film}}\cos\theta\right)\right) \tag{9}$$

The analytic expression for the attitude angle, obtained from Ref. 1, is given by

$$\phi_{\mathrm{ana}} = \mathrm{atan}\left(-\left(\int_{z=-L/2}^{z=L/2}\int_{y=0}^{y=R}\int_{x=-R}^{x=R} p_{\mathrm{nd}}\sin\theta\right)\Big/\left(\int_{z=-L/2}^{z=L/2}\int_{y=0}^{y=R}\int_{x=-R}^{x=R} p_{\mathrm{nd}}\cos\theta\right)\right) \tag{10}$$

The attitude angle generally decreases with increases in the eccentricity and decreases in the permeability parameter. Compared to the case of no slip at the interface, a slip condition generally produces a lower attitude angle. The exceptional case is for the low value of $\Psi = 0.001$, where for lower values of eccentricity the attitude angle is larger for the slip condition as compared to that for the no-slip condition.

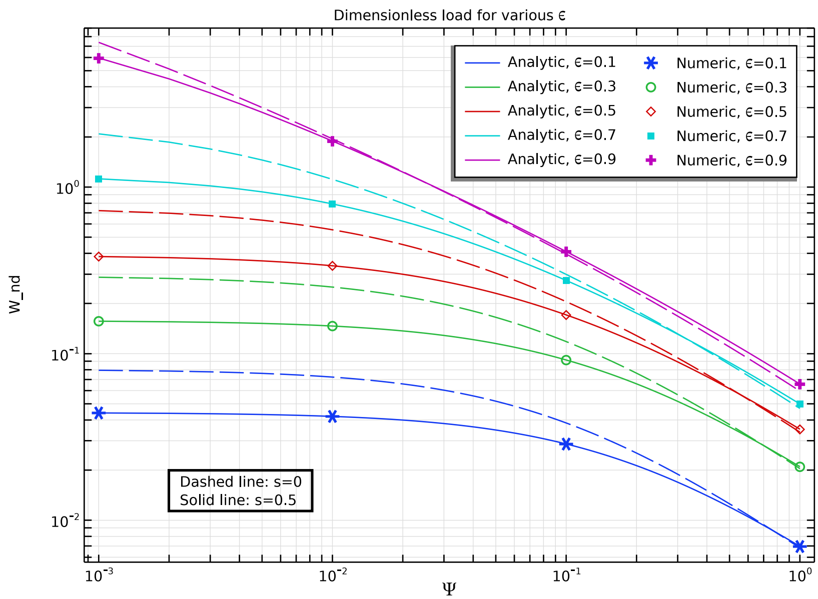

Figure 6: The dimensionless load carrying capacity as a function of the permeability parameter for different values of eccentricity. Here, s = 0.5.

Figure 8 shows the steady-state flow patterns in a typical self-lubricating bearing. The pressure contours show the development of differential pressure zones. The streamlines show the flow of the lubricant from high pressure zone to the low pressure zone and also the loss of lubricant at the ends of the bearing which are exposed to ambient conditions.

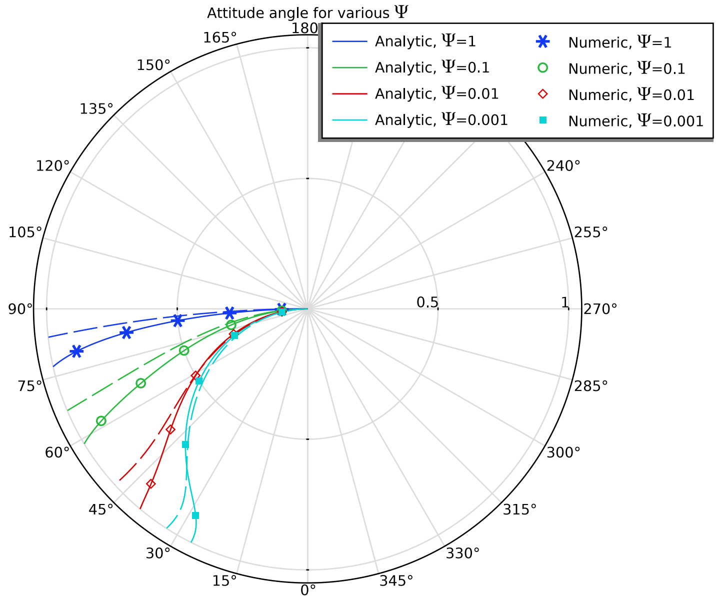

Figure 7: Polar plot of the attitude angle vs. eccentricity for different values of the permeability parameter.

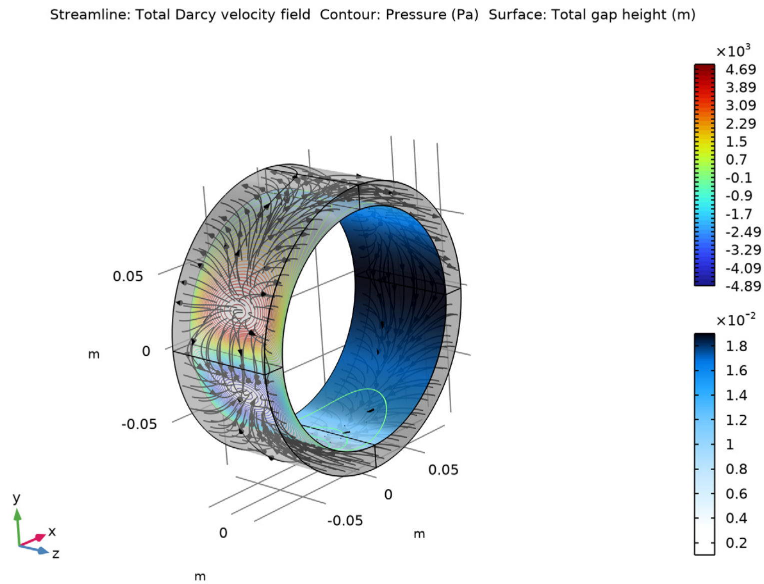

Figure 8: Streamlines showing the flow of lubricant in the porous bush of a self-lubricating journal bearing. Also shown are the contours of pressure and the variation of thickness of the thin film of fluid in the gap between the bush and the journal.

Reference

1. J. Prakash and S.K. Vij, “Analysis of Narrow Porous Journal Bearing Using Beavers-Joseph Criterion of Velocity Slip,” *J. Appl. Mech.*, vol. 41, no. 2, pp. 348–354, 1974.

Application Library path: `CFD_Module/Verification_Examples/self_lubricating_bearing`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.

2 In the **Select Physics** tree, select **Fluid Flow>Porous Media and Subsurface Flow>Thin-Film and Porous Media Flow**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Stationary**.

6 Click **Done**.

GLOBAL DEFINITIONS

Load parameters and variables from file.

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 Click **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file `self_lubricating_bearing_parameters.txt`.

DEFINITIONS

Variables 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Definitions** and choose **Variables**.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 Click **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file `self_lubricating_bearing_variables.txt`.

Analytic 1 (an1)

Create help functions for the analytic solution.

1 In the **Home** toolbar, click f(X) **Functions** and choose **Local>Analytic**.

2 In the **Settings** window for **Analytic**, type `beta_n` in the **Function name** text field.

3 Locate the **Definition** section. In the **Expression** text field, type `2*n-1`.

4 In the **Arguments** text field, type `n`.

5 Locate the **Plot Parameters** section. In the table, enter the following settings:

Plot	Argument	Lower limit	Upper limit	Fixed value	Unit
√	n	0	n_upper_lim	0	

Analytic 2 (an2)

1 In the **Home** toolbar, click f(X) **Functions** and choose **Local>Analytic**.

2 In the **Settings** window for **Analytic**, type `g_n` in the **Function name** text field.

3 Locate the **Definition** section. In the **Expression** text field, type `sin(theta)*(2*s^2+2*s*(1+ep*cos(theta))+(1+ep*cos(theta))^2)/((s+1+ep*cos(theta))*(((6*(alpha^2)*(s^2)+4*s*(1+ep*cos(theta))+(1+ep*cos(theta))^2)*(1+ep*cos(theta))^2)+(12*Psi*(s+1+ep*cos(theta))*tanh(pi*beta_n(n)*H/L)/(pi*beta_n(n)*H/L))))`.

4 In the **Arguments** text field, type `ep, n, Psi, s, theta`.

5 Locate the **Plot Parameters** section. In the table, enter the following settings:

Plot	Argument	Lower limit	Upper limit	Fixed value	Unit
	n	0	1	0	
√	s	0	1	0	

Analytic 3 (an3)

1 In the **Home** toolbar, click f(X) **Functions** and choose **Local>Analytic**.

2 In the **Settings** window for **Analytic**, type `pbar_summand` in the **Function name** text field.

3 Locate the **Definition** section. In the **Expression** text field, type `(-1)^(n+1)*ep*g_n(ep,n,Psi,s,theta)*cos(pi*beta_n(n)*z/L)/(2*n-1)^3`.

4 In the **Arguments** text field, type `ep, n, Psi, s, theta, z`.

5 Locate the **Plot Parameters** section. In the table, enter the following settings:

Plot	Argument	Lower limit	Upper limit	Fixed value	Unit
	n	0	1	0	
√	s	0	1	0	

GEOMETRY 1

Cylinder 1 (cyl1)

1 In the **Geometry** toolbar, click **Cylinder**.

2 In the **Settings** window for **Cylinder**, locate the **Object Type** section.

3 From the **Type** list, choose **Surface**.

4 Locate the **Size and Shape** section. In the **Radius** text field, type `R+H`.

5 In the **Height** text field, type `L`.

6 Locate the **Position** section. In the **z** text field, type `-L/2`.

7 Click to expand the **Layers** section. In the table, enter the following settings:

Layer name	Thickness (m)
Layer 1	H

Form Union (fin)

In the **Geometry** toolbar, click **Build All**.

THIN-FILM FLOW (TFF)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Thin-Film Flow (tff)**.

2 In the **Settings** window for **Thin-Film Flow**, locate the **Boundary Selection** section.

3 Click **Clear Selection**.

4 Click **Paste Selection**.

5 In the **Paste Selection** dialog box, type `8, 9, 14, 16` in the **Selection** text field.

6 Click **OK**.

7 In the **Settings** window for **Thin-Film Flow**, locate the **Reference Pressure** section.

8 In the p_{ref} text field, type `0`.

Fluid-Film Properties 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Thin-Film Flow (tff)** click **Fluid-Film Properties 1**.

2 In the **Settings** window for **Fluid-Film Properties**, locate the **Wall Properties** section.

3 In the h_{w1} text field, type `h_f`.

4 Locate the **Base Properties** section. From the $\mathbf{v}_b$ list, choose **User defined**. Specify the vector as

u_b	x
v_b	y

5 Locate the **Fluid Properties** section. From the μ list, choose **User defined**. In the associated text field, type `mu`.

6 From the ρ list, choose **User defined**. In the associated text field, type `rho`.

DARCY'S LAW (DL)

1. In the **Model Builder** window, under **Component 1 (comp1)** click **Darcy's Law (dl)**.
2. In the **Settings** window for **Darcy's Law**, locate the **Physical Model** section.
3. In the p_{ref} text field, type `0`.

Fluid 1

1. In the **Model Builder** window, under **Component 1 (comp1)>Darcy's Law (dl)>Porous Medium 1** click **Fluid 1**.
2. In the **Settings** window for **Fluid**, locate the **Fluid Properties** section.
3. From the ρ list, choose **User defined**. In the associated text field, type `rho`.
4. From the μ list, choose **User defined**. In the associated text field, type `mu`.

Porous Matrix 1

1. In the **Model Builder** window, click **Porous Matrix 1**.
2. In the **Settings** window for **Porous Matrix**, locate the **Matrix Properties** section.
3. From the κ list, choose **User defined**. In the associated text field, type `Phi`.
4. From the ε_{p} list, choose **User defined**.

Pressure 1

1. In the **Physics** toolbar, click **Boundaries** and choose **Pressure**.
2. In the **Settings** window for **Pressure**, locate the **Boundary Selection** section.
3. Click **Paste Selection**.
4. In the **Paste Selection** dialog box, type `4-7, 12, 13, 17, 18` in the **Selection** text field.
5. Click **OK**.

MULTIPHYSICS

Thin-Film and Porous Media Flow 1 (tfpf1)

1. In the **Model Builder** window, under **Component 1 (comp1)>Multiphysics** click **Thin-Film and Porous Media Flow 1 (tfpf1)**.
2. In the **Settings** window for **Thin-Film and Porous Media Flow**, locate the **Coupling** section.
3. From the **Coupling type** list, choose **Beavers-Joseph slip condition**.
4. In the $\alpha_{\mathrm{BJ,w}}$ text field, type `alpha`.
5. In the $\alpha_{\mathrm{BJ,b}}$ text field, type `alpha`.

MESH 1

Create a structured mesh.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.
2 In the **Settings** window for **Mesh**, locate the **Sequence Type** section.
3 From the list, choose **User-controlled mesh**.

Edge 1

1 In the **Mesh** toolbar, click **More Generators** and choose **Edge**.
2 In the **Settings** window for **Edge**, locate the **Edge Selection** section.
3 Click **Paste Selection**.
4 In the **Paste Selection** dialog box, type `3, 4, 9, 10, 15, 19, 23, 27` in the **Selection** text field.
5 Click **OK**.

Distribution 1

1 Right-click **Edge 1** and choose **Distribution**.
2 In the **Settings** window for **Distribution**, locate the **Distribution** section.
3 In the **Number of elements** text field, type `50`.
4 Select the **Equidistant** check box.

Mapped 1

1 In the **Mesh** toolbar, click **More Generators** and choose **Mapped**.
2 In the **Settings** window for **Mapped**, locate the **Boundary Selection** section.
3 Click **Paste Selection**.
4 In the **Paste Selection** dialog box, type `4, 5, 12, 17` in the **Selection** text field.
5 Click **OK**.

Distribution 1

1 Right-click **Mapped 1** and choose **Distribution**.
2 In the **Settings** window for **Distribution**, locate the **Edge Selection** section.
3 Click **Paste Selection**.
4 In the **Paste Selection** dialog box, type `2, 14, 22, 30` in the **Selection** text field.
5 Click **OK**.

Swept 1

In the **Mesh** toolbar, click **Swept**.

Distribution 1

1 Right-click **Swept 1** and choose **Distribution**.

2 In the **Settings** window for **Distribution**, locate the **Distribution** section.

3 In the **Number of elements** text field, type 15.

Free Tetrahedral 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Mesh 1** right-click **Free Tetrahedral 1** and choose **Disable**.

2 Right-click **Mesh 1** and choose **Build All**.

DEFINITIONS

Add integration operators required later for solution post-processing.

Integration 1 (intop1)

1 In the **Definitions** toolbar, click **Nonlocal Couplings** and choose **Integration**.

2 In the **Settings** window for **Integration**, locate the **Source Selection** section.

3 From the **Geometric entity level** list, choose **Boundary**.

4 Click **Paste Selection**.

5 In the **Paste Selection** dialog box, type `9, 16` in the **Selection** text field.

6 Click **OK**.

Integration 2 (intop2)

1 In the **Definitions** toolbar, click **Nonlocal Couplings** and choose **Integration**.

2 In the **Settings** window for **Integration**, locate the **Source Selection** section.

3 From the **Geometric entity level** list, choose **Boundary**.

4 Click **Paste Selection**.

5 In the **Paste Selection** dialog box, type `8, 9, 14, 16` in the **Selection** text field.

6 Click **OK**.

GLOBAL DEFINITIONS

Solve the Stationary study step for different values of parameters s, Phi and ep. The short-bearing assumption is applicable.

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
R	20*L	1.6 m	Journal radius

STUDY 1

Parametric Sweep

1 In the **Study** toolbar, click **Parametric Sweep**.

2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

3 Click **Add**.

4 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
s (Slip parameter)	0.25 0.5	

5 Click **Add**.

6 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
Psi (Permeability parameter)	1 0.1 0.01 0.001	

7 Click **Add**.

8 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
ep (Journal eccentricity)	0.1 0.3 0.5 0.7 0.9	

9 From the **Sweep type** list, choose **All combinations**.

10 In the **Model Builder** window, click **Study 1**.

11 In the **Settings** window for **Study**, locate the **Study Settings** section.

12 Clear the **Generate default plots** check box.

13 In the **Study** toolbar, click **Compute**.

GLOBAL DEFINITIONS

Solve the Stationary study step without the short-bearing assumption

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
R	L	0.08 m	Journal radius

ADD STUDY

1 In the **Study** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies>Stationary**.

4 Click **Add Study** in the window toolbar.

5 In the **Study** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 2

1 In the **Model Builder** window, click **Study 2**.

2 In the **Settings** window for **Study**, locate the **Study Settings** section.

3 Clear the **Generate default plots** check box.

4 In the **Study** toolbar, click **Compute**.

RESULTS

1 In the **Model Builder** window, click **Results**.

2 In the **Settings** window for **Results**, locate the **Update of Results** section.

3 Select the **Only plot when requested** check box.

Grid 1D 1

Create datasets to plot the analytic expressions of the solution.

1 In the **Model Builder** window, expand the **Results** node.

2 Right-click **Results>Datasets** and choose **More Datasets>Grid 1D**.

3 In the **Settings** window for **Grid 1D**, locate the **Data** section.

4 From the **Source** list, choose **Function**.

5 From the **Function** list, choose **All**.

6 Locate the **Parameter Bounds** section. In the **Name** text field, type `ep`.

7 Right-click **Grid 1D 1** and choose **Duplicate**.

Grid 1D 2

1 In the **Model Builder** window, click **Grid 1D 2**.

2 In the **Settings** window for **Grid 1D**, locate the **Parameter Bounds** section.

3 In the **Name** text field, type `Psi`.

4 In the **Minimum** text field, type `0.001`.

Global Evaluation 1

Derive numeric values of the dimensionless load and attitude angle for different parameter combinations.

1 In the **Results** toolbar, click **8.5** **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1/Parametric Solutions 1 (sol2)**.

4 From the **Parameter selection (s)** list, choose **From list**.

5 In the **Parameter values (s)** list, select **0.25**.

6 From the **Parameter selection (Psi)** list, choose **From list**.

7 In the **Parameter values (Psi)** list, select **1**.

8 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
`(C^2/(mu*U*L^3))*sqrt((intop1(pfilm*cos(theta)))^2+(intop1(pfilm*sin(theta)))^2)`	1	
`atan(-(intop1(pfilm*sin(theta)))/(intop1(pfilm*cos(theta))))`	rad	

9 Right-click **Global Evaluation 1** and choose **Duplicate**.

Global Evaluation 2

1 In the **Model Builder** window, click **Global Evaluation 2**.

2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.

3 In the **Parameter values (Psi)** list, select **0.1**.

4 Right-click **Global Evaluation 2** and choose **Duplicate**.

Global Evaluation 3

1 In the **Model Builder** window, click **Global Evaluation 3**.

2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.

3 In the **Parameter values (Psi)** list, select **0.01**.

4 Right-click **Global Evaluation 3** and choose **Duplicate**.

Global Evaluation 4

1 In the **Model Builder** window, click **Global Evaluation 4**.
2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.
3 In the **Parameter values (Psi)** list, select **0.001**.

Global Evaluation 5

1 In the **Results** toolbar, click 8.5 **Global Evaluation**.
2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.
3 From the **Dataset** list, choose **Study 1/Parametric Solutions 1 (sol2)**.
4 From the **Parameter selection (s)** list, choose **From list**.
5 In the **Parameter values (s)** list, select **0.25**.
6 From the **Parameter selection (ep)** list, choose **From list**.
7 In the **Parameter values (ep)** list, select **0.1**.
8 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
`(C^2/(mu*U*L^3))*sqrt((intop1(pfilm*cos(theta)))^2+(intop1(pfilm*sin(theta)))^2)`	1	

9 Right-click **Global Evaluation 5** and choose **Duplicate**.

Global Evaluation 6

1 In the **Model Builder** window, click **Global Evaluation 6**.
2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.
3 In the **Parameter values (ep)** list, select **0.3**.
4 Right-click **Global Evaluation 6** and choose **Duplicate**.

Global Evaluation 7

1 In the **Model Builder** window, click **Global Evaluation 7**.
2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.
3 In the **Parameter values (ep)** list, select **0.5**.
4 Right-click **Global Evaluation 7** and choose **Duplicate**.

Global Evaluation 8

1 In the **Model Builder** window, click **Global Evaluation 8**.
2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.
3 In the **Parameter values (ep)** list, select **0.7**.

4 Right-click **Global Evaluation 8** and choose **Duplicate**.

Global Evaluation 9

1 In the **Model Builder** window, click **Global Evaluation 9**.

2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.

3 In the **Parameter values (ep)** list, select **0.9**.

Global Evaluation 10

1 In the **Results** toolbar, click **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1/Parametric Solutions 1 (sol2)**.

4 From the **Parameter selection (s)** list, choose **From list**.

5 In the **Parameter values (s)** list, select **0.5**.

6 From the **Parameter selection (Psi)** list, choose **From list**.

7 In the **Parameter values (Psi)** list, select **1**.

8 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
`(C^2/(mu*U*L^3))*sqrt((intop1(pfilm*cos(theta)))^2+(intop1(pfilm*sin(theta)))^2)`	1	

9 Right-click **Global Evaluation 10** and choose **Duplicate**.

Global Evaluation 11

1 In the **Model Builder** window, click **Global Evaluation 11**.

2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.

3 In the **Parameter values (Psi)** list, select **0.1**.

4 Right-click **Global Evaluation 11** and choose **Duplicate**.

Global Evaluation 12

1 In the **Model Builder** window, click **Global Evaluation 12**.

2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.

3 In the **Parameter values (Psi)** list, select **0.01**.

4 Right-click **Global Evaluation 12** and choose **Duplicate**.

Global Evaluation 13

1 In the **Model Builder** window, click **Global Evaluation 13**.

2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.

3 In the **Parameter values (Psi)** list, select **0.001**.

Global Evaluation 14

1 In the **Results** toolbar, click (8.5) **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1/Parametric Solutions 1 (sol2)**.

4 From the **Parameter selection (s)** list, choose **From list**.

5 In the **Parameter values (s)** list, select **0.5**.

6 From the **Parameter selection (ep)** list, choose **From list**.

7 In the **Parameter values (ep)** list, select **0.1**.

8 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
`(C^2/(mu*U*L^3))*sqrt((intop1(pfilm*cos(theta)))^2+(intop1(pfilm*sin(theta)))^2)`	1	

9 Right-click **Global Evaluation 14** and choose **Duplicate**.

Global Evaluation 15

1 In the **Model Builder** window, click **Global Evaluation 15**.

2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.

3 In the **Parameter values (ep)** list, select **0.3**.

4 Right-click **Global Evaluation 15** and choose **Duplicate**.

Global Evaluation 16

1 In the **Model Builder** window, click **Global Evaluation 16**.

2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.

3 In the **Parameter values (ep)** list, select **0.5**.

4 Right-click **Global Evaluation 16** and choose **Duplicate**.

Global Evaluation 17

1 In the **Model Builder** window, click **Global Evaluation 17**.

2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.

3 In the **Parameter values (ep)** list, select **0.7**.

4 Right-click **Global Evaluation 17** and choose **Duplicate**.

Global Evaluation 18

1 In the **Model Builder** window, click **Global Evaluation 18**.

2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.

3 In the **Parameter values (ep)** list, select **0.9**.

4 In the **Results** toolbar, click **Evaluate** and choose **Evaluate All**.

Dimensionless Load vs. ep: s=0.25

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Dimensionless Load vs. ep: s=0.25` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **None**.

4 Click to expand the **Title** section. From the **Title type** list, choose **Manual**.

5 In the **Title** text area, type `Dimensionless load for various \Psi`.

6 Locate the **Plot Settings** section.

7 Select the **x-axis label** check box. In the associated text field, type `\epsilon`.

8 Select the **y-axis label** check box. In the associated text field, type `W_nd`.

9 Locate the **Axis** section. Select the **y-axis log scale** check box.

10 Locate the **Legend** section. From the **Position** list, choose **Lower right**.

11 In the **Number of columns** text field, type `2`.

12 In the **Maximum relative width** text field, type `1`.

Function 1

1 In the **Dimensionless Load vs. ep: s=0.25** toolbar, click **More Plots** and choose **Function**.

2 In the **Settings** window for **Function**, locate the **Data** section.

3 From the **Dataset** list, choose **Grid 1D 1**.

4 Locate the **y-Axis Data** section. In the **Expression** text field, type

```
sqrt((integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,1,0.25,
theta,z),n,1,n_upper_lim)*cos(theta),theta,0,pi),z,-L/2,L/2))^2+
(integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,1,0.25,theta,
z),n,1,n_upper_lim)*sin(theta),theta,0,pi),z,-L/2,L/2))^2)/L.
```

5 Locate the **x-Axis Data** section. In the **Expression** text field, type `ep`.

6 Click to expand the **Coloring and Style** section. From the **Color** list, choose **Cycle (reset)**.

7 Click to expand the **Legends** section. Select the **Show legends** check box.

8 From the **Legends** list, choose **Manual**.

9 In the table, enter the following settings:

Legends
`Analytic, \Psi=1`

10 Right-click **Function 1** and choose **Duplicate**.

Function 2

1 In the **Model Builder** window, click **Function 2**.

2 In the **Settings** window for **Function**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `sqrt((integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,0.1,0.25,theta,z),n,1,n_upper_lim)*cos(theta),theta,0,pi),z,-L/2,L/2))^2+(integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,0.1,0.25,theta,z),n,1,n_upper_lim)*sin(theta),theta,0,pi),z,-L/2,L/2))^2)/L`.

4 Locate the **Coloring and Style** section. From the **Color** list, choose **Cycle**.

5 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Analytic, \Psi=0.1`

6 Right-click **Function 2** and choose **Duplicate**.

Function 3

1 In the **Model Builder** window, click **Function 3**.

2 In the **Settings** window for **Function**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `sqrt((integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,0.01,0.25,theta,z),n,1,n_upper_lim)*cos(theta),theta,0,pi),z,-L/2,L/2))^2+(integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,0.01,0.25,theta,z),n,1,n_upper_lim)*sin(theta),theta,0,pi),z,-L/2,L/2))^2)/L`.

4 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Analytic, \Psi=0.01`

5 Right-click **Function 3** and choose **Duplicate**.

Function 4

1 In the **Model Builder** window, click **Function 4**.

2 In the **Settings** window for **Function**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `sqrt((integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,0.001,0.25,theta,z),n,1,n_upper_lim)*cos(theta),theta,0,pi),z,-L/2,L/2))^2+(integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,0.001,0.25,theta,z),n,1,n_upper_lim)*sin(theta),theta,0,pi),z,-L/2,L/2))^2)/L`.

4 Locate the **Legends** section. In the table, enter the following settings:

Legends
Analytic, \Psi=0.001

5 Right-click **Function 4** and choose **Duplicate**.

Function 5

1 In the **Model Builder** window, click **Function 5**.

2 In the **Settings** window for **Function**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `sqrt((integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,1,0,theta,z),n,1,n_upper_lim)*cos(theta),theta,0,pi),z,-L/2,L/2))^2+(integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,1,0,theta,z),n,1,n_upper_lim)*sin(theta),theta,0,pi),z,-L/2,L/2))^2)/L`.

4 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dashed**.

5 From the **Color** list, choose **Cycle (reset)**.

6 Locate the **Legends** section. Clear the **Show legends** check box.

7 Right-click **Function 5** and choose **Duplicate**.

Function 6

1 In the **Model Builder** window, click **Function 6**.

2 In the **Settings** window for **Function**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `sqrt((integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,0.1,0,theta,z),n,1,n_upper_lim)*cos(theta),theta,0,pi),z,-L/2,L/2))^2+(integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,0.1,0,theta,z),n,1,n_upper_lim)*sin(theta),theta,0,pi),z,-L/2,L/2))^2)/L`.

4 Locate the **Coloring and Style** section. From the **Color** list, choose **Cycle**.

5 Right-click **Function 6** and choose **Duplicate**.

Function 7

1 In the **Model Builder** window, click **Function 7**.

2 In the **Settings** window for **Function**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `sqrt((integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,0.01,0,theta,z),n,1,n_upper_lim)*cos(theta),theta,0,pi),z,-L/2,L/2))^2+(integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,0.01,0,theta,z),n,1,n_upper_lim)*sin(theta),theta,0,pi),z,-L/2,L/2))^2)/L`.

4 Right-click **Function 7** and choose **Duplicate**.

Function 8

1 In the **Model Builder** window, click **Function 8**.

2 In the **Settings** window for **Function**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `sqrt((integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,0.001,0,theta,z),n,1,n_upper_lim)*cos(theta),theta,0,pi),z,-L/2,L/2))^2+(integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,0.001,0,theta,z),n,1,n_upper_lim)*sin(theta),theta,0,pi),z,-L/2,L/2))^2)/L`.

Table Graph 1

1 In the **Model Builder** window, right-click **Dimensionless Load vs. ep: s=0.25** and choose **Table Graph**.

2 In the **Settings** window for **Table Graph**, locate the **Data** section.

3 From the **x-axis data** list, choose **ep**.

4 From the **Plot columns** list, choose **Manual**.

5 In the **Columns** list, select **(C^2/(mu*U*L^3))*sqrt((intop1(pfilm*cos(theta)))^2+(intop1(pfilm*sin(theta)))^2) (1)**.

6 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

7 From the **Color** list, choose **Cycle (reset)**.

8 Find the **Line markers** subsection. From the **Marker** list, choose **Cycle (reset)**.

9 Click to expand the **Legends** section. Select the **Show legends** check box.

10 From the **Legends** list, choose **Manual**.

11 In the table, enter the following settings:

Legends
`Numeric, \Psi=1`

12 Right-click **Table Graph 1** and choose **Duplicate**.

Table Graph 2

1 In the **Model Builder** window, click **Table Graph 2**.

2 In the **Settings** window for **Table Graph**, locate the **Data** section.

3 From the **Table** list, choose **Table 2**.

4 Locate the **Coloring and Style** section. From the **Color** list, choose **Cycle**.

5 Find the **Line markers** subsection. From the **Marker** list, choose **Cycle**.

6 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Numeric, \Psi=0.1`

7 Right-click **Table Graph 2** and choose **Duplicate**.

Table Graph 3

1 In the **Model Builder** window, click **Table Graph 3**.

2 In the **Settings** window for **Table Graph**, locate the **Data** section.

3 From the **Table** list, choose **Table 3**.

4 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Numeric, \Psi=0.01`

5 Right-click **Table Graph 3** and choose **Duplicate**.

Table Graph 4

1 In the **Model Builder** window, click **Table Graph 4**.

2 In the **Settings** window for **Table Graph**, locate the **Data** section.

3 From the **Table** list, choose **Table 4**.

4 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Numeric, \Psi=0.001`

Annotation 1

1 In the **Model Builder** window, right-click **Dimensionless Load vs. ep: s=0.25** and choose **Annotation**.

2 In the **Settings** window for **Annotation**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1/Solution 1 (sol1)**.

4 Locate the **Annotation** section. In the **Text** text field, type `Dashed line: s=0\\Solid line: s=0.25`.

5 Select the **LaTeX markup** check box.

6 Locate the **Position** section. In the **X** text field, type `0.1`.

7 In the **Y** text field, type `5`.

8 Locate the **Coloring and Style** section. Clear the **Show point** check box.

9 Select the **Show frame** check box.

Dimensionless Load vs. ep: s=0.25

1 In the **Model Builder** window, click **Dimensionless Load vs. ep: s=0.25**.

2 In the **Dimensionless Load vs. ep: s=0.25** toolbar, click **Plot**.

Dimensionless Load vs. Psi: s=0.25

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Dimensionless Load vs. Psi: s=0.25` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **None**.

4 Locate the **Title** section. From the **Title type** list, choose **Manual**.

5 In the **Title** text area, type `Dimensionless load for various \epsilon`.

6 Locate the **Plot Settings** section.

7 Select the **x-axis label** check box. In the associated text field, type `\Psi`.

8 Select the **y-axis label** check box. In the associated text field, type `W_nd`.

9 Locate the **Axis** section. Select the **x-axis log scale** check box.

10 Select the **y-axis log scale** check box.

11 Locate the **Legend** section. In the **Number of columns** text field, type `2`.

12 In the **Maximum relative width** text field, type `1`.

Function 1

1 In the **Dimensionless Load vs. Psi: s=0.25** toolbar, click **More Plots** and choose **Function**.

2 In the **Settings** window for **Function**, locate the **Data** section.

3 From the **Dataset** list, choose **Grid 1D 2**.

4 Locate the **y-Axis Data** section. In the **Expression** text field, type `sqrt((integrate(integrate((24/pi^3)*sum(pbar_summand(0.1,n,Psi,0.25, theta,z),n,1,n_upper_lim)*cos(theta),theta,0,pi),z,-L/2,L/2))^2+ (integrate(integrate((24/pi^3)*sum(pbar_summand(0.1,n,Psi,0.25, theta,z),n,1,n_upper_lim)*sin(theta),theta,0,pi),z,-L/2,L/2))^2)/L`.

5 Locate the **x-Axis Data** section. In the **Expression** text field, type `Psi`.

6 In the **Lower bound** text field, type `0.001`.

7 Locate the **Coloring and Style** section. From the **Color** list, choose **Cycle (reset)**.

8 Locate the **Legends** section. Select the **Show legends** check box.

9 From the **Legends** list, choose **Manual**.

10 In the table, enter the following settings:

Legends
`Analytic, \epsilon=0.1`

11 Right-click **Function 1** and choose **Duplicate**.

Function 2

1 In the **Model Builder** window, click **Function 2**.

2 In the **Settings** window for **Function**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `sqrt((integrate(integrate((24/pi^3)* sum(pbar_summand(0.3,n,Psi,0.25,theta,z),n,1,n_upper_lim)* cos(theta),theta,0,pi),z,-L/2,L/2))^2+(integrate(integrate((24/ pi^3)*sum(pbar_summand(0.3,n,Psi,0.25,theta,z),n,1,n_upper_lim)* sin(theta),theta,0,pi),z,-L/2,L/2))^2)/L`.

4 Locate the **Coloring and Style** section. From the **Color** list, choose **Cycle**.

5 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Analytic, \epsilon=0.3`

6 Right-click **Function 2** and choose **Duplicate**.

Function 3

1 In the **Model Builder** window, click **Function 3**.

2 In the **Settings** window for **Function**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `sqrt((integrate(integrate((24/pi^3)*sum(pbar_summand(0.5,n,Psi,0.25,theta,z),n,1,n_upper_lim)*cos(theta),theta,0,pi),z,-L/2,L/2))^2+(integrate(integrate((24/pi^3)*sum(pbar_summand(0.5,n,Psi,0.25,theta,z),n,1,n_upper_lim)*sin(theta),theta,0,pi),z,-L/2,L/2))^2)/L`.

4 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Analytic, \epsilon=0.5`

5 Right-click **Function 3** and choose **Duplicate**.

Function 4

1 In the **Model Builder** window, click **Function 4**.

2 In the **Settings** window for **Function**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `sqrt((integrate(integrate((24/pi^3)*sum(pbar_summand(0.7,n,Psi,0.25,theta,z),n,1,n_upper_lim)*cos(theta),theta,0,pi),z,-L/2,L/2))^2+(integrate(integrate((24/pi^3)*sum(pbar_summand(0.7,n,Psi,0.25,theta,z),n,1,n_upper_lim)*sin(theta),theta,0,pi),z,-L/2,L/2))^2)/L`.

4 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Analytic, \epsilon=0.7`

5 Right-click **Function 4** and choose **Duplicate**.

Function 5

1 In the **Model Builder** window, click **Function 5**.

2 In the **Settings** window for **Function**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `sqrt((integrate(integrate((24/pi^3)*sum(pbar_summand(0.9,n,Psi,0.25,theta,z),n,1,n_upper_lim)*cos(theta),theta,0,pi),z,-L/2,L/2))^2+(integrate(integrate((24/pi^3)*sum(pbar_summand(0.9,n,Psi,0.25,theta,z),n,1,n_upper_lim)*sin(theta),theta,0,pi),z,-L/2,L/2))^2)/L`.

4 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Analytic, \epsilon=0.9`

Dimensionless Load vs. Psi: s=0.25

In the **Model Builder** window, click **Dimensionless Load vs. Psi: s=0.25**.

Function 6

1 In the **Dimensionless Load vs. Psi: s=0.25** toolbar, click **More Plots** and choose **Function**.

2 In the **Settings** window for **Function**, locate the **Data** section.

3 From the **Dataset** list, choose **Grid 1D 2**.

4 Locate the **y-Axis Data** section. In the **Expression** text field, type `sqrt((integrate(integrate((24/pi^3)*sum(pbar_summand(0.1,n,Psi,0,theta,z),n,1,n_upper_lim)*cos(theta),theta,0,pi),z,-L/2,L/2))^2+(integrate(integrate((24/pi^3)*sum(pbar_summand(0.1,n,Psi,0,theta,z),n,1,n_upper_lim)*sin(theta),theta,0,pi),z,-L/2,L/2))^2)/L`.

5 Locate the **x-Axis Data** section. In the **Expression** text field, type `Psi`.

6 In the **Lower bound** text field, type `0.001`.

7 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dashed**.

8 From the **Color** list, choose **Cycle (reset)**.

9 Right-click **Function 6** and choose **Duplicate**.

Function 7

1 In the **Model Builder** window, click **Function 7**.

2 In the **Settings** window for **Function**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `sqrt((integrate(integrate((24/pi^3)*sum(pbar_summand(0.3,n,Psi,0,theta,z),n,1,n_upper_lim)*cos(theta),theta,0,pi),z,-L/2,L/2))^2+(integrate(integrate((24/pi^3)*sum(pbar_summand(0.3,n,Psi,0,theta,z),n,1,n_upper_lim)*sin(theta),theta,0,pi),z,-L/2,L/2))^2)/L`.

4 Locate the **Coloring and Style** section. From the **Color** list, choose **Cycle**.

5 Right-click **Function 7** and choose **Duplicate**.

Function 8

1 In the **Model Builder** window, click **Function 8**.

2 In the **Settings** window for **Function**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `sqrt((integrate(integrate((24/pi^3)* sum(pbar_summand(0.5,n,Psi,0,theta,z),n,1,n_upper_lim)*cos(theta), theta,0,pi),z,-L/2,L/2))^2+(integrate(integrate((24/pi^3)* sum(pbar_summand(0.5,n,Psi,0,theta,z),n,1,n_upper_lim)*sin(theta), theta,0,pi),z,-L/2,L/2))^2)/L`.

4 Right-click **Function 8** and choose **Duplicate**.

Function 9

1 In the **Model Builder** window, click **Function 9**.

2 In the **Settings** window for **Function**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `sqrt((integrate(integrate((24/pi^3)* sum(pbar_summand(0.7,n,Psi,0,theta,z),n,1,n_upper_lim)*cos(theta), theta,0,pi),z,-L/2,L/2))^2+(integrate(integrate((24/pi^3)* sum(pbar_summand(0.7,n,Psi,0,theta,z),n,1,n_upper_lim)*sin(theta), theta,0,pi),z,-L/2,L/2))^2)/L`.

4 Right-click **Function 9** and choose **Duplicate**.

Function 10

1 In the **Model Builder** window, click **Function 10**.

2 In the **Settings** window for **Function**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `sqrt((integrate(integrate((24/pi^3)* sum(pbar_summand(0.9,n,Psi,0,theta,z),n,1,n_upper_lim)*cos(theta), theta,0,pi),z,-L/2,L/2))^2+(integrate(integrate((24/pi^3)* sum(pbar_summand(0.9,n,Psi,0,theta,z),n,1,n_upper_lim)*sin(theta), theta,0,pi),z,-L/2,L/2))^2)/L`.

Table Graph 1

1 In the **Model Builder** window, right-click **Dimensionless Load vs. Psi: s=0.25** and choose **Table Graph**.

2 In the **Settings** window for **Table Graph**, locate the **Data** section.

3 From the **Table** list, choose **Table 5**.

4 From the **x-axis data** list, choose **Psi**.

5 From the **Plot columns** list, choose **Manual**.

6 In the **Columns** list, select **(C^2/(mu*U*L^3))*sqrt((intop1(pfilm*cos(theta)))^2+(intop1(pfilm*sin(theta)))^2) (1)**.
7 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.
8 From the **Color** list, choose **Cycle (reset)**.
9 Find the **Line markers** subsection. From the **Marker** list, choose **Cycle (reset)**.
10 Locate the **Legends** section. Select the **Show legends** check box.
11 From the **Legends** list, choose **Manual**.
12 In the table, enter the following settings:

Legends
`Numeric, \epsilon=0.1`

13 Right-click **Table Graph 1** and choose **Duplicate**.

Table Graph 2

1 In the **Model Builder** window, click **Table Graph 2**.
2 In the **Settings** window for **Table Graph**, locate the **Data** section.
3 From the **Table** list, choose **Table 6**.
4 Locate the **Coloring and Style** section. From the **Color** list, choose **Cycle**.
5 Find the **Line markers** subsection. From the **Marker** list, choose **Cycle**.
6 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Numeric, \epsilon=0.3`

7 Right-click **Table Graph 2** and choose **Duplicate**.

Table Graph 3

1 In the **Model Builder** window, click **Table Graph 3**.
2 In the **Settings** window for **Table Graph**, locate the **Data** section.
3 From the **Table** list, choose **Table 7**.
4 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Numeric, \epsilon=0.5`

5 Right-click **Table Graph 3** and choose **Duplicate**.

Table Graph 4

1 In the **Model Builder** window, click **Table Graph 4**.

2 In the **Settings** window for **Table Graph**, locate the **Data** section.

3 From the **Table** list, choose **Table 8**.

4 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Numeric, \epsilon=0.7`

5 Right-click **Table Graph 4** and choose **Duplicate**.

Table Graph 5

1 In the **Model Builder** window, click **Table Graph 5**.

2 In the **Settings** window for **Table Graph**, locate the **Data** section.

3 From the **Table** list, choose **Table 9**.

4 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Numeric, \epsilon=0.9`

Annotation 1

1 In the **Model Builder** window, right-click **Dimensionless Load vs. Psi: s=0.25** and choose **Annotation**.

2 In the **Settings** window for **Annotation**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1/Solution 1 (sol1)**.

4 Locate the **Annotation** section. In the **Text** text field, type `Dashed line: s=0\\Solid line: s=0.25`.

5 Select the **LaTeX markup** check box.

6 Locate the **Position** section. In the **X** text field, type `0.002`.

7 In the **Y** text field, type `0.02`.

8 Locate the **Coloring and Style** section. Clear the **Show point** check box.

9 Select the **Show frame** check box.

Dimensionless Load vs. Psi: s=0.25

1 In the **Model Builder** window, click **Dimensionless Load vs. Psi: s=0.25**.

2 In the **Dimensionless Load vs. Psi: s=0.25** toolbar, click **Plot**.

Dimensionless Load vs. ep: s=0.5

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.
2 In the **Settings** window for **1D Plot Group**, type `Dimensionless Load vs. ep: s=0.5` in the **Label** text field.
3 Locate the **Data** section. From the **Dataset** list, choose **None**.
4 Locate the **Title** section. From the **Title type** list, choose **Manual**.
5 In the **Title** text area, type `Dimensionless load for various \Psi`.
6 Locate the **Plot Settings** section.
7 Select the **x-axis label** check box. In the associated text field, type `\epsilon`.
8 Select the **y-axis label** check box. In the associated text field, type `W_nd`.
9 Locate the **Axis** section. Select the **y-axis log scale** check box.
10 Locate the **Legend** section. From the **Position** list, choose **Lower right**.
11 In the **Number of columns** text field, type `2`.
12 In the **Maximum relative width** text field, type `1`.

Function 1

1 In the **Dimensionless Load vs. ep: s=0.5** toolbar, click **More Plots** and choose **Function**.
2 In the **Settings** window for **Function**, locate the **Data** section.
3 From the **Dataset** list, choose **Grid 1D 1**.
4 Locate the **y-Axis Data** section. In the **Expression** text field, type
```
(C^2/(mu*U*L^3))*
sqrt((integrate(integrate(((mu*U*L^2)/(R*C^2))*(24/pi^3)*sum((-
1)^(n+1)*ep*g_n(ep,n,1,0.5,theta)*cos(pi*beta_n(n)*z/L)/(2*n-1)^3,n,
1,n_upper_lim)*cos(theta)*R,theta,0,pi),z,-L/2,L/2))^2+
(integrate(integrate(((mu*U*L^2)/(R*C^2))*(24/pi^3)*sum((-1)^(n+1)*
ep*g_n(ep,n,1,0.5,theta)*cos(pi*beta_n(n)*z/L)/(2*n-1)^3,n,1,
n_upper_lim)*sin(theta)*R,theta,0,pi),z,-L/2,L/2))^2).
```
5 Locate the **x-Axis Data** section. In the **Expression** text field, type `ep`.
6 Locate the **Coloring and Style** section. From the **Color** list, choose **Cycle (reset)**.
7 Locate the **Legends** section. Select the **Show legends** check box.
8 From the **Legends** list, choose **Manual**.

9 In the table, enter the following settings:

Legends
Analytic, \Psi=1

10 Right-click **Function 1** and choose **Duplicate**.

Function 2

1 In the **Model Builder** window, click **Function 2**.

2 In the **Settings** window for **Function**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `(C^2/(mu*U*L^3))*sqrt((integrate(integrate(((mu*U*L^2)/(R*C^2))*(24/pi^3)*sum((-1)^(n+1)*ep*g_n(ep,n,0.1,0.5,theta)*cos(pi*beta_n(n)*z/L)/(2*n-1)^3,n,1,n_upper_lim)*cos(theta)*R,theta,0,pi),z,-L/2,L/2))^2+(integrate(integrate(((mu*U*L^2)/(R*C^2))*(24/pi^3)*sum((-1)^(n+1)*ep*g_n(ep,n,0.1,0.5,theta)*cos(pi*beta_n(n)*z/L)/(2*n-1)^3,n,1,n_upper_lim)*sin(theta)*R,theta,0,pi),z,-L/2,L/2))^2)`.

4 Locate the **Coloring and Style** section. From the **Color** list, choose **Cycle**.

5 Locate the **Legends** section. In the table, enter the following settings:

Legends
Analytic, \Psi=0.1

6 Right-click **Function 2** and choose **Duplicate**.

Function 3

1 In the **Model Builder** window, click **Function 3**.

2 In the **Settings** window for **Function**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `(C^2/(mu*U*L^3))*sqrt((integrate(integrate(((mu*U*L^2)/(R*C^2))*(24/pi^3)*sum((-1)^(n+1)*ep*g_n(ep,n,0.01,0.5,theta)*cos(pi*beta_n(n)*z/L)/(2*n-1)^3,n,1,n_upper_lim)*cos(theta)*R,theta,0,pi),z,-L/2,L/2))^2+(integrate(integrate(((mu*U*L^2)/(R*C^2))*(24/pi^3)*sum((-1)^(n+1)*ep*g_n(ep,n,0.01,0.5,theta)*cos(pi*beta_n(n)*z/L)/(2*n-1)^3,n,1,n_upper_lim)*sin(theta)*R,theta,0,pi),z,-L/2,L/2))^2)`.

4 Locate the **Legends** section. In the table, enter the following settings:

Legends
Analytic, \Psi=0.01

5 Right-click **Function 3** and choose **Duplicate**.

Function 4

1 In the **Model Builder** window, click **Function 4**.

2 In the **Settings** window for **Function**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `(C^2/(mu*U*L^3))*sqrt((integrate(integrate(((mu*U*L^2)/(R*C^2))*(24/pi^3)*sum((-1)^(n+1)*ep*g_n(ep,n,0.001,0.5,theta)*cos(pi*beta_n(n)*z/L)/(2*n-1)^3,n,1,n_upper_lim)*cos(theta)*R,theta,0,pi),z,-L/2,L/2))^2+(integrate(integrate(((mu*U*L^2)/(R*C^2))*(24/pi^3)*sum((-1)^(n+1)*ep*g_n(ep,n,0.001,0.5,theta)*cos(pi*beta_n(n)*z/L)/(2*n-1)^3,n,1,n_upper_lim)*sin(theta)*R,theta,0,pi),z,-L/2,L/2))^2)`.

4 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Analytic, \Psi=0.001`

Annotation 1, Function 5, Function 6, Function 7, Function 8, Table Graph 1, Table Graph 2, Table Graph 3, Table Graph 4

1 In the **Model Builder** window, under **Results>Dimensionless Load vs. ep: s=0.25**, Ctrl-click to select **Function 5**, **Function 6**, **Function 7**, **Function 8**, **Table Graph 1**, **Table Graph 2**, **Table Graph 3**, **Table Graph 4**, and **Annotation 1**.

2 Right-click and choose **Copy**.

Dimensionless Load vs. ep: s=0.5

In the **Model Builder** window, under **Results** right-click **Dimensionless Load vs. ep: s=0.5** and choose **Paste Multiple Items**.

Table Graph 1

1 In the **Model Builder** window, click **Table Graph 1**.

2 In the **Settings** window for **Table Graph**, locate the **Data** section.

3 From the **Table** list, choose **Table 10**.

Table Graph 2

1 In the **Model Builder** window, click **Table Graph 2**.

2 In the **Settings** window for **Table Graph**, locate the **Data** section.

3 From the **Table** list, choose **Table 11**.

Table Graph 3

1 In the **Model Builder** window, click **Table Graph 3**.
2 In the **Settings** window for **Table Graph**, locate the **Data** section.
3 From the **Table** list, choose **Table 12**.

Table Graph 4

1 In the **Model Builder** window, click **Table Graph 4**.
2 In the **Settings** window for **Table Graph**, locate the **Data** section.
3 From the **Table** list, choose **Table 13**.

Annotation 1

1 In the **Model Builder** window, click **Annotation 1**.
2 In the **Settings** window for **Annotation**, locate the **Annotation** section.
3 In the **Text** text field, type `Dashed line: s=0\\Solid line: s=0.5`.

Dimensionless Load vs. ep: s=0.5

1 In the **Model Builder** window, click **Dimensionless Load vs. ep: s=0.5**.
2 In the **Dimensionless Load vs. ep: s=0.5** toolbar, click **Plot**.

Dimensionless Load vs. Psi: s=0.5

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.
2 In the **Settings** window for **1D Plot Group**, type `Dimensionless Load vs. Psi: s=0.5` in the **Label** text field.
3 Locate the **Data** section. From the **Dataset** list, choose **None**.
4 Locate the **Title** section. From the **Title type** list, choose **Manual**.
5 In the **Title** text area, type `Dimensionless load for various \epsilon`.
6 Locate the **Plot Settings** section.
7 Select the **x-axis label** check box. In the associated text field, type `\Psi`.
8 Select the **y-axis label** check box. In the associated text field, type `W_nd`.
9 Locate the **Axis** section. Select the **y-axis log scale** check box.
10 Select the **x-axis log scale** check box.
11 Locate the **Legend** section. In the **Number of columns** text field, type `2`.
12 In the **Maximum relative width** text field, type `1`.

Function 1

1 In the **Dimensionless Load vs. Psi: s=0.5** toolbar, click **More Plots** and choose **Function**.

2 In the **Settings** window for **Function**, locate the **Data** section.

3 From the **Dataset** list, choose **Grid 1D 2**.

4 Locate the **y-Axis Data** section. In the **Expression** text field, type `(C^2/(mu*U*L^3))*sqrt((integrate(integrate(((mu*U*L^2)/(R*C^2))*(24/pi^3)*sum((-1)^(n+1)*0.1*g_n(0.1,n,Psi,0.5,theta)*cos(pi*beta_n(n)*z/L)/(2*n-1)^3,n,1,n_upper_lim)*cos(theta)*R,theta,0,pi),z,-L/2,L/2))^2+(integrate(integrate(((mu*U*L^2)/(R*C^2))*(24/pi^3)*sum((-1)^(n+1)*0.1*g_n(0.1,n,Psi,0.5,theta)*cos(pi*beta_n(n)*z/L)/(2*n-1)^3,n,1,n_upper_lim)*sin(theta)*R,theta,0,pi),z,-L/2,L/2))^2)`.

5 Locate the **x-Axis Data** section. In the **Expression** text field, type `Psi`.

6 In the **Lower bound** text field, type `0.001`.

7 Locate the **Coloring and Style** section. From the **Color** list, choose **Cycle (reset)**.

8 Locate the **Legends** section. Select the **Show legends** check box.

9 From the **Legends** list, choose **Manual**.

10 In the table, enter the following settings:

Legends
`Analytic, \epsilon=0.1`

11 Right-click **Function 1** and choose **Duplicate**.

Function 2

1 In the **Model Builder** window, click **Function 2**.

2 In the **Settings** window for **Function**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `(C^2/(mu*U*L^3))*sqrt((integrate(integrate(((mu*U*L^2)/(R*C^2))*(24/pi^3)*sum((-1)^(n+1)*0.3*g_n(0.3,n,Psi,0.5,theta)*cos(pi*beta_n(n)*z/L)/(2*n-1)^3,n,1,n_upper_lim)*cos(theta)*R,theta,0,pi),z,-L/2,L/2))^2+(integrate(integrate(((mu*U*L^2)/(R*C^2))*(24/pi^3)*sum((-1)^(n+1)*0.3*g_n(0.3,n,Psi,0.5,theta)*cos(pi*beta_n(n)*z/L)/(2*n-1)^3,n,1,n_upper_lim)*sin(theta)*R,theta,0,pi),z,-L/2,L/2))^2)`.

4 Locate the **Coloring and Style** section. From the **Color** list, choose **Cycle**.

5 Locate the **Legends** section. In the table, enter the following settings:

Legends
Analytic, \epsilon=0.3

6 Right-click **Function 2** and choose **Duplicate**.

Function 3

1 In the **Model Builder** window, click **Function 3**.

2 In the **Settings** window for **Function**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `(C^2/(mu*U*L^3))*sqrt((integrate(integrate(((mu*U*L^2)/(R*C^2))*(24/pi^3)*sum((-1)^(n+1)*0.5*g_n(0.5,n,Psi,0.5,theta)*cos(pi*beta_n(n)*z/L)/(2*n-1)^3,n,1,n_upper_lim)*cos(theta)*R,theta,0,pi),z,-L/2,L/2))^2+(integrate(integrate(((mu*U*L^2)/(R*C^2))*(24/pi^3)*sum((-1)^(n+1)*0.5*g_n(0.5,n,Psi,0.5,theta)*cos(pi*beta_n(n)*z/L)/(2*n-1)^3,n,1,n_upper_lim)*sin(theta)*R,theta,0,pi),z,-L/2,L/2))^2)`.

4 Locate the **Legends** section. In the table, enter the following settings:

Legends
Analytic, \epsilon=0.5

5 Right-click **Function 3** and choose **Duplicate**.

Function 4

1 In the **Model Builder** window, click **Function 4**.

2 In the **Settings** window for **Function**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `(C^2/(mu*U*L^3))*sqrt((integrate(integrate(((mu*U*L^2)/(R*C^2))*(24/pi^3)*sum((-1)^(n+1)*0.7*g_n(0.7,n,Psi,0.5,theta)*cos(pi*beta_n(n)*z/L)/(2*n-1)^3,n,1,n_upper_lim)*cos(theta)*R,theta,0,pi),z,-L/2,L/2))^2+(integrate(integrate(((mu*U*L^2)/(R*C^2))*(24/pi^3)*sum((-1)^(n+1)*0.7*g_n(0.7,n,Psi,0.5,theta)*cos(pi*beta_n(n)*z/L)/(2*n-1)^3,n,1,n_upper_lim)*sin(theta)*R,theta,0,pi),z,-L/2,L/2))^2)`.

4 Locate the **Legends** section. In the table, enter the following settings:

Legends
Analytic, \epsilon=0.7

5 Right-click **Function 4** and choose **Duplicate**.

Function 5

1 In the **Model Builder** window, click **Function 5**.

2 In the **Settings** window for **Function**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `(C^2/(mu*U*L^3))*sqrt((integrate(integrate(((mu*U*L^2)/(R*C^2))*(24/pi^3)*sum((-1)^(n+1)*0.9*g_n(0.9,n,Psi,0.5,theta)*cos(pi*beta_n(n)*z/L)/(2*n-1)^3,n,1,n_upper_lim)*cos(theta)*R,theta,0,pi),z,-L/2,L/2))^2+(integrate(integrate(((mu*U*L^2)/(R*C^2))*(24/pi^3)*sum((-1)^(n+1)*0.9*g_n(0.9,n,Psi,0.5,theta)*cos(pi*beta_n(n)*z/L)/(2*n-1)^3,n,1,n_upper_lim)*sin(theta)*R,theta,0,pi),z,-L/2,L/2))^2)`.

4 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Analytic, \epsilon=0.9`

Annotation 1, Function 10, Function 6, Function 7, Function 8, Function 9, Table Graph 1, Table Graph 2, Table Graph 3, Table Graph 4, Table Graph 5

1 In the **Model Builder** window, under **Results>Dimensionless Load vs. Psi: s=0.25**, Ctrl-click to select **Function 6**, **Function 7**, **Function 8**, **Function 9**, **Function 10**, **Table Graph 1**, **Table Graph 2**, **Table Graph 3**, **Table Graph 4**, **Table Graph 5**, and **Annotation 1**.

2 Right-click and choose **Copy**.

Dimensionless Load vs. Psi: s=0.5

In the **Model Builder** window, under **Results** right-click **Dimensionless Load vs. Psi: s=0.5** and choose **Paste Multiple Items**.

Table Graph 1

1 In the **Model Builder** window, click **Table Graph 1**.

2 In the **Settings** window for **Table Graph**, locate the **Data** section.

3 From the **Table** list, choose **Table 14**.

Table Graph 2

1 In the **Model Builder** window, click **Table Graph 2**.

2 In the **Settings** window for **Table Graph**, locate the **Data** section.

3 From the **Table** list, choose **Table 15**.

Table Graph 3

1 In the **Model Builder** window, click **Table Graph 3**.

2 In the **Settings** window for **Table Graph**, locate the **Data** section.

3 From the **Table** list, choose **Table 16**.

Table Graph 4

1 In the **Model Builder** window, click **Table Graph 4**.

2 In the **Settings** window for **Table Graph**, locate the **Data** section.

3 From the **Table** list, choose **Table 17**.

Table Graph 5

1 In the **Model Builder** window, click **Table Graph 5**.

2 In the **Settings** window for **Table Graph**, locate the **Data** section.

3 From the **Table** list, choose **Table 18**.

Annotation 1

1 In the **Model Builder** window, click **Annotation 1**.

2 In the **Settings** window for **Annotation**, locate the **Annotation** section.

3 In the **Text** text field, type `Dashed line: s=0\\Solid line: s=0.5`.

Dimensionless Load vs. Psi: s=0.5

1 In the **Model Builder** window, click **Dimensionless Load vs. Psi: s=0.5**.

2 In the **Dimensionless Load vs. Psi: s=0.5** toolbar, click **Plot**.

Attitude Angle vs. ep: s=0.25

1 In the **Home** toolbar, click **Add Plot Group** and choose **Polar Plot Group**.

2 In the **Settings** window for **Polar Plot Group**, type `Attitude Angle vs. Psi` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **None**.

4 Click to expand the **Title** section. From the **Title type** list, choose **Manual**.

5 In the **Title** text area, type `Attitude angle for various \Psi`.

6 Locate the **Axis** section. From the **Zero angle** list, choose **Down**.

7 From the **Rotation direction** list, choose **Clockwise**.

8 Locate the **Legend** section. In the **Number of columns** text field, type `2`.

9 In the **Maximum relative width** text field, type `1`.

10 Click to expand the **Window Settings** section. In the **Label** text field, type `Attitude Angle vs. ep: s=0.25`.

Line Graph 1

1 Right-click **Attitude Angle vs. ep: s=0.25** and choose **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Grid 1D 1**.

4 Locate the **r-Axis Data** section. In the **Expression** text field, type `ep`.

5 Locate the θ **Angle Data** section. From the **Parameter** list, choose **Expression**.

6 In the **Expression** text field, type `atan(-(integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,1,0.25,theta,z),n,1,n_upper_lim)*sin(theta),theta,0,pi),z,-L/2,L/2))/(integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,1,0.25,theta,z),n,1,n_upper_lim)*cos(theta),theta,0,pi),z,-L/2,L/2)))`.

7 Click to expand the **Coloring and Style** section. From the **Color** list, choose **Cycle (reset)**.

8 Click to expand the **Legends** section. Select the **Show legends** check box.

9 From the **Legends** list, choose **Manual**.

10 In the table, enter the following settings:

Legends
`Analytic, \Psi=1`

11 Right-click **Line Graph 1** and choose **Duplicate**.

Line Graph 2

1 In the **Model Builder** window, click **Line Graph 2**.

2 In the **Settings** window for **Line Graph**, locate the θ **Angle Data** section.

3 In the **Expression** text field, type `atan(-(integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,0.1,0.25,theta,z),n,1,n_upper_lim)*sin(theta),theta,0,pi),z,-L/2,L/2))/(integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,0.1,0.25,theta,z),n,1,n_upper_lim)*cos(theta),theta,0,pi),z,-L/2,L/2)))`.

4 Locate the **Coloring and Style** section. From the **Color** list, choose **Cycle**.

5 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Analytic, \Psi=0.1`

6 Right-click **Line Graph 2** and choose **Duplicate**.

Line Graph 3

1 In the **Model Builder** window, click **Line Graph 3**.

2 In the **Settings** window for **Line Graph**, locate the θ **Angle Data** section.

3 In the **Expression** text field, type `atan(-(integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,0.01,0.25,theta,z),n,1,n_upper_lim)*sin(theta),theta,0,pi),z,-L/2,L/2))/(integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,0.01,0.25,theta,z),n,1,n_upper_lim)*cos(theta),theta,0,pi),z,-L/2,L/2)))`.

4 Locate the **Legends** section. In the table, enter the following settings:

Legends
Analytic, \Psi=0.01

5 Right-click **Line Graph 3** and choose **Duplicate**.

Line Graph 4

1 In the **Model Builder** window, click **Line Graph 4**.

2 In the **Settings** window for **Line Graph**, locate the θ **Angle Data** section.

3 In the **Expression** text field, type `atan(-(integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,0.001,0.25,theta,z),n,1,n_upper_lim)*sin(theta),theta,0,pi),z,-L/2,L/2))/(integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,0.001,0.25,theta,z),n,1,n_upper_lim)*cos(theta),theta,0,pi),z,-L/2,L/2)))`.

4 Locate the **Legends** section. In the table, enter the following settings:

Legends
Analytic, \Psi=0.001

Line Graph 5

1 In the **Model Builder** window, right-click **Attitude Angle vs. ep: s=0.25** and choose **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Grid 1D 1**.

4 Locate the **r-Axis Data** section. In the **Expression** text field, type `ep`.

5 Locate the θ **Angle Data** section. From the **Parameter** list, choose **Expression**.

6 In the **Expression** text field, type `atan(-(integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,1,0,theta,z),n,1,n_upper_lim)*sin(theta),theta,0,pi),z,-L/2,L/2))/(integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,1,0,theta,z),n,1,n_upper_lim)*cos(theta),theta,0,pi),z,-L/2,L/2)))`.

7 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dashed**.

8 From the **Color** list, choose **Cycle (reset)**.

9 Right-click **Line Graph 5** and choose **Duplicate**.

Line Graph 6

1 In the **Model Builder** window, click **Line Graph 6**.

2 In the **Settings** window for **Line Graph**, locate the **Coloring and Style** section.

3 From the **Color** list, choose **Cycle**.

4 Locate the θ **Angle Data** section. In the **Expression** text field, type `atan(-(integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,0.1,0,theta,z),n,1,n_upper_lim)*sin(theta),theta,0,pi),z,-L/2,L/2))/(integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,0.1,0,theta,z),n,1,n_upper_lim)*cos(theta),theta,0,pi),z,-L/2,L/2)))`.

5 Right-click **Line Graph 6** and choose **Duplicate**.

Line Graph 7

1 In the **Model Builder** window, click **Line Graph 7**.

2 In the **Settings** window for **Line Graph**, locate the θ **Angle Data** section.

3 In the **Expression** text field, type `atan(-(integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,0.01,0,theta,z),n,1,n_upper_lim)*sin(theta),theta,0,pi),z,-L/2,L/2))/(integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,0.01,0,theta,z),n,1,n_upper_lim)*cos(theta),theta,0,pi),z,-L/2,L/2)))`.

4 Right-click **Line Graph 7** and choose **Duplicate**.

Line Graph 8

1 In the **Model Builder** window, click **Line Graph 8**.

2 In the **Settings** window for **Line Graph**, locate the θ **Angle Data** section.

3 In the **Expression** text field, type `atan(-(integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,0.001,0,theta,z),n,1,n_upper_lim)*sin(theta),theta,0,pi),z,-L/2,L/2))/(integrate(integrate((24/pi^3)*sum(pbar_summand(ep,n,0.001,0,theta,z),n,1,n_upper_lim)*cos(theta),theta,0,pi),z,-L/2,L/2)))`.

Table Graph 1

1 In the **Model Builder** window, right-click **Attitude Angle vs. ep: s=0.25** and choose **Table Graph**.

2 In the **Settings** window for **Table Graph**, locate the **Data** section.

3 From the θ **angle data** list, choose **atan(-(intop1(pfilm*sin(theta)))/(intop1(pfilm*cos(theta)))) (rad)**.

4 From the **Plot columns** list, choose **Manual**.

5 In the **Columns** list, select **ep**.

6 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

7 From the **Color** list, choose **Cycle (reset)**.

8 Find the **Line markers** subsection. From the **Marker** list, choose **Cycle (reset)**.

9 Click to expand the **Legends** section. Select the **Show legends** check box.

10 From the **Legends** list, choose **Manual**.

11 In the table, enter the following settings:

Legends
`Numeric, \Psi=1`

12 Right-click **Table Graph 1** and choose **Duplicate**.

Table Graph 2

1 In the **Model Builder** window, click **Table Graph 2**.

2 In the **Settings** window for **Table Graph**, locate the **Data** section.

3 From the **Table** list, choose **Table 2**.

4 Locate the **Coloring and Style** section. From the **Color** list, choose **Cycle**.

5 Find the **Line markers** subsection. From the **Marker** list, choose **Cycle**.

6 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Numeric, \Psi=0.1`

7 Right-click **Table Graph 2** and choose **Duplicate**.

Table Graph 3

1 In the **Model Builder** window, click **Table Graph 3**.

2 In the **Settings** window for **Table Graph**, locate the **Data** section.

3 From the **Table** list, choose **Table 3**.

4 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Numeric, \Psi=0.01`

5 Right-click **Table Graph 3** and choose **Duplicate**.

Table Graph 4

1 In the **Model Builder** window, click **Table Graph 4**.

2 In the **Settings** window for **Table Graph**, locate the **Data** section.

3 From the **Table** list, choose **Table 4**.

4 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Numeric, \Psi=0.001`

Attitude Angle vs. ep: s=0.25

1 In the **Model Builder** window, click **Attitude Angle vs. ep: s=0.25**.

2 In the **Attitude Angle vs. ep: s=0.25** toolbar, click **Plot**.

Darcy's Velocity Streamlines, Film Pressure Contour and Film Thickness

1 In the **Home** toolbar, click **Add Plot Group** and choose **3D Plot Group**.

2 In the **Settings** window for **3D Plot Group**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 2/Solution 43 (sol43)**.

4 In the **Label** text field, type `Darcy's Velocity Streamlines, Film Pressure Contour and Film Thickness`.

5 Locate the **Color Legend** section. From the **Position** list, choose **Right double**.

Streamline 1

1 Right-click **Darcy's Velocity Streamlines, Film Pressure Contour and Film Thickness** and choose **Streamline**.

2 In the **Settings** window for **Streamline**, locate the **Expression** section.

3 In the **X-component** text field, type `dl.u`.

4 In the **Y-component** text field, type `dl.v`.

5 In the **Z-component** text field, type `dl.w`.

6 Click to expand the **Title** section. Locate the **Streamline Positioning** section. From the **Positioning** list, choose **Uniform density**.

7 Locate the **Coloring and Style** section. Find the **Point style** subsection. From the **Type** list, choose **Arrow**.

8 From the **Color** list, choose **Black**.

Contour 1

1 In the **Model Builder** window, right-click **Darcy's Velocity Streamlines, Film Pressure Contour and Film Thickness** and choose **Contour**.

2 In the **Settings** window for **Contour**, click to expand the **Title** section.

3 Locate the **Levels** section. In the **Total levels** text field, type `50`.

4 Locate the **Coloring and Style** section. From the **Legend type** list, choose **Filled**.

Selection 1

1 Right-click **Contour 1** and choose **Selection**.

2 In the **Settings** window for **Selection**, locate the **Selection** section.

3 Click **Paste Selection**.

4 In the **Paste Selection** dialog box, type `8, 9, 14, 16` in the **Selection** text field.

5 Click **OK**.

Surface 1

1 In the **Model Builder** window, right-click **Darcy's Velocity Streamlines, Film Pressure Contour and Film Thickness** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `tff.h`.

4 Click to expand the **Title** section. Locate the **Coloring and Style** section. Click **Change Color Table**.

5 In the **Color Table** dialog box, select **Aurora>JupiterAuroraBorealis** in the tree.

6 Click **OK**.

7 In the **Settings** window for **Surface**, locate the **Coloring and Style** section.

8 From the **Color table transformation** list, choose **Reverse**.

Selection 1

1 Right-click **Surface 1** and choose **Selection**.

2 In the **Settings** window for **Selection**, locate the **Selection** section.

3 Click **Paste Selection**.

4 In the **Paste Selection** dialog box, type `8, 9, 14, 16` in the **Selection** text field.

5 Click **OK**.

Transparency 1

1 In the **Model Builder** window, right-click **Surface 1** and choose **Transparency**.

2 In the **Settings** window for **Transparency**, locate the **Transparency** section.

3 In the **Transparency** text field, type `0.1`.

Surface 2

1 In the **Model Builder** window, right-click **Darcy's Velocity Streamlines, Film Pressure Contour and Film Thickness** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `1`.

4 Locate the **Title** section. From the **Title type** list, choose **None**.

5 Locate the **Coloring and Style** section. From the **Coloring** list, choose **Uniform**.

6 From the **Color** list, choose **Gray**.

Selection 1

1 Right-click **Surface 2** and choose **Selection**.

2 In the **Settings** window for **Selection**, locate the **Selection** section.

3 Click **Paste Selection**.

4 In the **Paste Selection** dialog box, type `2-7, 11-13, 17-19` in the **Selection** text field.

5 Click **OK**.

Transparency 1

In the **Model Builder** window, right-click **Surface 2** and choose **Transparency**.

Darcy's Velocity Streamlines, Film Pressure Contour and Film Thickness

1 In the **Settings** window for **3D Plot Group**, locate the **Plot Settings** section.

2 From the **View** list, choose **New view**.

3 In the **Darcy's Velocity Streamlines, Film Pressure Contour and Film Thickness** toolbar, click **Plot**.

4 Click **Go to Source**.

View 3D 2

Adjust the view to obtain Figure 8.