Created in COMSOL Multiphysics 6.2

Quarter-Wave Transformer

Introduction

Transmission lines are used when the frequency of the electromagnetic signals is so high that the wave nature of the signals must be taken into account. A consequence of the wave nature is that the signals are reflected if there are abrupt changes of the characteristic impedance along the transmission line. Similarly, the load impedance, Z_L, at the end of the transmission line must match its characteristic impedance, Z_0. Otherwise there are reflections from the transmission line's end.

A quarter-wave transformer (see Figure 1) is a component that can be inserted between the transmission line and the load to match the load impedance to the transmission line's characteristic impedance. To get this functionality, the transformer must be a quarter of a wavelength long and the relation between the impedances involved must be

$$\frac{Z_{in}}{Z} = \frac{Z}{Z_L} \tag{1}$$

If the length and the impedance requirements are fulfilled, the load impedance does not give rise to any reflections.

Typically, the characteristic impedance of transmission lines, Z_0, is 50 Ω. Thus, Z_{in} in Equation 1 should be set to

$$Z_{in} = Z_0 = 50\,\Omega$$

when solving for the characteristic impedance of the quarter-wave transformer, Z.

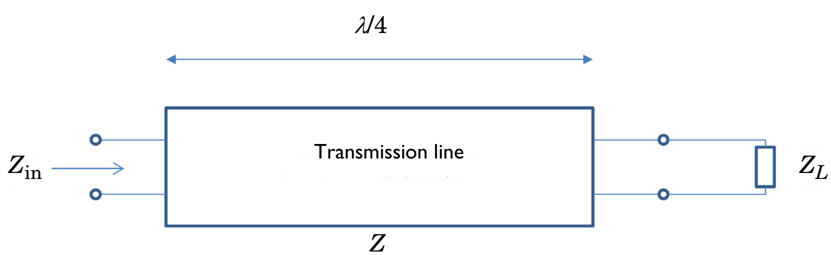


Figure 1: Schematic of a quarter-wave transformer. The input impedance is Z_{in}, the impedance of the transformer transmission line is Z, and the load impedance is Z_L.

This example exemplifies some of the characteristics of a quarter-wave transformer. In particular, the model simulation shows that the transformer only provides matching for one particular frequency, namely that for which the transformer is a quarter of a wavelength long.

Model Definition

The 1D geometry of the example consists of two line intervals. Each line interval represents a separate transmission line, with different electrical parameters (distributed capacitance and inductance) and lengths.

To excite and terminate the transmission lines, use lumped ports. This also makes it easy to obtain the reflection (S_{11}) and transmission (S_{21}) coefficients for the system.

Results and Discussion

As an example of the output from the model, Figure 2 shows the voltage amplitude distribution along the transmission lines for a frequency where the quarter-wave transformer matches the load impedance to the characteristic impedance of the incoming transmission line. The figure shows that the amplitude is constant, indicating that there is no reflection and therefore no standing waves. Figure 3 shows the frequency spectrum for the same transformer. As is evident from the graph, the quarter-wave transformer only operates without reflection in a certain wavelength range.

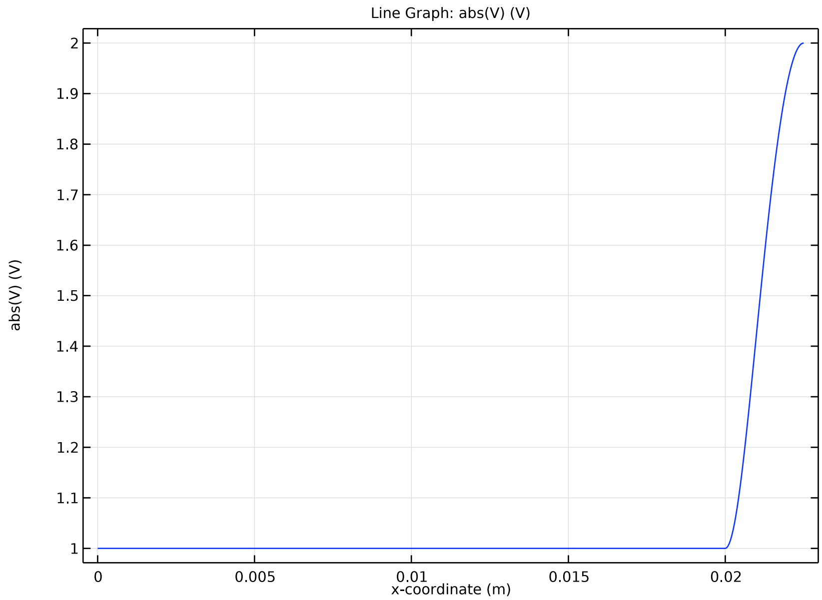


Figure 2: Absolute value of the voltage versus the x-coordinate. The quarter-wave transformer starts at x-coordinate 0.02 m.

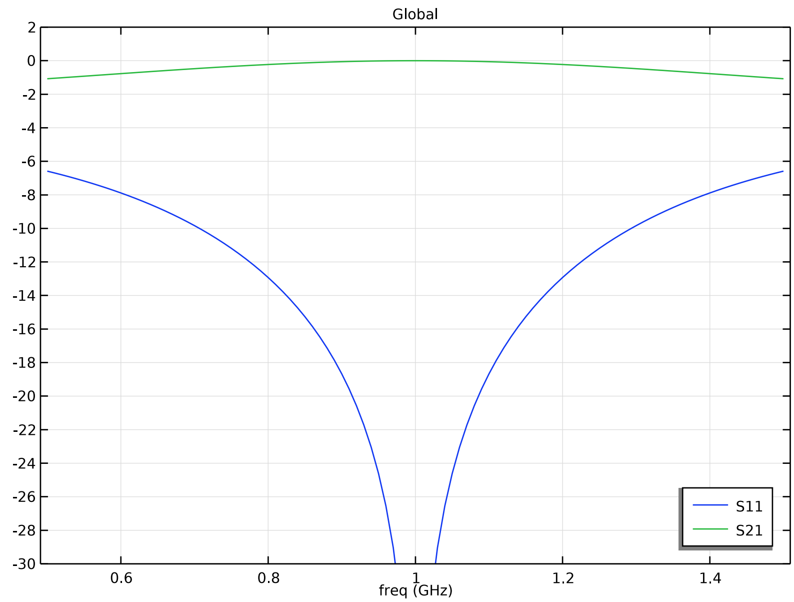


Figure 3: Spectral response for the transmission line. Notice that the transmission coefficient (S_{21}) peaks at the frequency (1 GHz) for which the transformer is a quarter-wave long. At that frequency the reflection coefficient (S_{11}) is zero (approaches negative infinity which the dB scale used in the graph).

Application Library path: `RF_Module/Transmission_Lines_and_Waveguides/quarter_wave_transformer`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **1D**.

2 In the **Select Physics** tree, select **Radio Frequency>Transmission Line (tl)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Frequency Domain**.

6 Click **Done**.

GLOBAL DEFINITIONS

First add some parameters that defines the electrical and geometrical properties of the transmission lines.

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
L1	2.5e-6[H/m]	2.5E-6 H/m	Distributed inductance, first transmission line
C1	1e-9[F/m]	1E-9 F/m	Distributed capacitance, first transmission line
f	1[GHz]	1E9 Hz	Frequency
wl1	1/(f*sqrt(L1*C1))	0.02 m	Wavelength, first transmission line
d1	wl1	0.02 m	Length, first transmission line
Z1	sqrt(L1/C1)	50 Ω	Characteristic impedance, first transmission line
ZL	4*Z1	200 Ω	Terminating impedance
Z2	sqrt(Z1*ZL)	100 Ω	Characteristic impedance, second transmission line
C2	C1	1E-9 F/m	Distributed capacitance, second transmission line
L2	C2*Z2^2	1E-5 H/m	Distributed inductance, second transmission line
wl2	1/(f*sqrt(L2*C2))	0.01 m	Wavelength, second transmission line
d2	wl2/4	0.0025 m	Length, second transmission line
hmax	d2/10	2.5E-4 m	Maximum discretization step

GEOMETRY 1

Set up the geometry as two intervals.

Interval 1 (i1)

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Geometry 1** and choose **Interval**.

2 In the **Settings** window for **Interval**, locate the **Interval** section.

3 From the **Specify** list, choose **Interval lengths**.

4 In the table, enter the following settings:

Lengths (m)
d1
d2

5 Click **Build All Objects**.

TRANSMISSION LINE (TL)

Assign the first transmission line the distributed capacitance and inductance `C1` and `L1`, respectively.

Transmission Line Equation 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Transmission Line (tl)** click **Transmission Line Equation 1**.

2 In the **Settings** window for **Transmission Line Equation**, locate the **Transmission Line Equation** section.

3 In the L text field, type `L1`.

4 In the C text field, type `C1`.

Now define the second transmission line by adding a transmission line equation feature to the second interval.

Transmission Line Equation 2

1 In the **Physics** toolbar, click **Domains** and choose **Transmission Line Equation**.

2 Click the **Zoom Extents** button in the **Graphics** toolbar to make the size of the transmission line suitable for selecting the second interval.

3 Select Domain 2 only.

Add the electrical parameters for the second transmission line.

4 In the **Settings** window for **Transmission Line Equation**, locate the **Transmission Line Equation** section.

5 In the L text field, type `L2`.

6 In the C text field, type `C2`.

Replace the default absorbing boundary condition with lumped ports. With the lumped ports, it is easy to excite the transmission line and also to plot the S-parameters, that is, the reflection and transmission coefficient, for the transmission line.

Lumped Port 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Lumped Port**.

2 Select Boundary 1 only.

Select that this port shall be excited. You can use the default voltage for the port.

3 In the **Settings** window for **Lumped Port**, locate the **Port Properties** section.

4 From the **Wave excitation at this port** list, choose **On**.

Lumped Port 2

1 In the **Physics** toolbar, click **Boundaries** and choose **Lumped Port**.

2 Select Boundary 3 only.

This lumped port should have a different characteristic impedance than the first lumped port and the two transmission lines.

3 In the **Settings** window for **Lumped Port**, locate the **Settings** section.

4 In the Z_{ref} text field, type `ZL`.

MESH 1

Let the mesh have a maximum subinterval that is one tenth of the quarter-wave part of the transmission line.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2 In the **Settings** window for **Mesh**, locate the **Sequence Type** section.

3 From the list, choose **User-controlled mesh**.

Size

1 In the **Model Builder** window, under **Component 1 (comp1)>Mesh 1** click **Size**.

2 In the **Settings** window for **Size**, locate the **Element Size** section.

3 Click the **Custom** button.

4 Locate the **Element Size Parameters** section. In the **Maximum element size** text field, type `hmax`.

STUDY 1

Step 1: Frequency Domain

Set the frequency for the **Frequency Domain** study and create a first default plot.

1 In the **Model Builder** window, under **Study 1** click **Step 1: Frequency Domain**.

2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

3 In the **Frequencies** text field, type `f`.

4 In the **Home** toolbar, click **Compute**.

RESULTS

To clearly demonstrate that the quarter-wave transformer works, replace the plot expression with the absolute value of the voltage.

Line Graph

1 In the **Model Builder** window, expand the **Electric Potential (tl)** node, then click **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `abs(V)`.

4 Click **Replace Expression** in the upper-right corner of the **x-Axis Data** section. From the menu, choose **Component 1 (comp1)>Geometry>Coordinate>x - x-coordinate**.

5 In the **Electric Potential (tl)** toolbar, click **Plot**.

You should now have a graph as in Figure 2. Notice that the left part of the curve is flat, with a unit amplitude, indicating that there are no standing waves, despite the fact that the second lumped port has a load impedance that normally would not be matched with the transmission line.

STUDY 1

Step 1: Frequency Domain

Now modify the study settings to create a frequency sweep around 1 GHz, but first define the frequency sweep parameters.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
`df`	`500[MHz]`	5E8 Hz	`Half of frequency sweep`
`fstep`	`10[MHz]`	1E7 Hz	`Frequency step`

STUDY 1

Step 1: Frequency Domain

1 In the **Model Builder** window, under **Study 1** click **Step 1: Frequency Domain**.

2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

3 Click **Range**.

4 In the **Range** dialog box, type `f-df` in the **Start** text field.

5 In the **Step** text field, type `fstep`.

6 In the **Stop** text field, type `f+df`.

7 Click **Replace**.

8 In the **Home** toolbar, click **Compute**.

RESULTS

Create a new plot group for a global plot of the S_{11}(reflection) and S_{21}(transmission) coefficients.

1D Plot Group 2

In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

Global 1

1 Right-click **1D Plot Group 2** and choose **Global**.

2 In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)>Transmission Line>Ports>S-parameter, dB>tl.S11dB - S11**.

3 Click **Add Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)>Transmission Line>Ports>S-parameter, dB>tl.S21dB - S21**.

4 In the **1D Plot Group 2** toolbar, click **Plot**.

Modify the *y*-axis limits to show that S_{21} actually has its maximum value for the frequency where S_{11} is at its minimum value.

1D Plot Group 2

1. In the **Model Builder** window, click **1D Plot Group 2**.
2. In the **Settings** window for **1D Plot Group**, locate the **Axis** section.
3. Select the **Manual axis limits** check box.
4. In the **y minimum** text field, type `-30`.
5. In the **y maximum** text field, type `2`.

   Move the legend panel, so it does not cover the curves.
6. Locate the **Legend** section. From the **Position** list, choose **Lower right**.
7. In the **1D Plot Group 2** toolbar, click **Plot**.

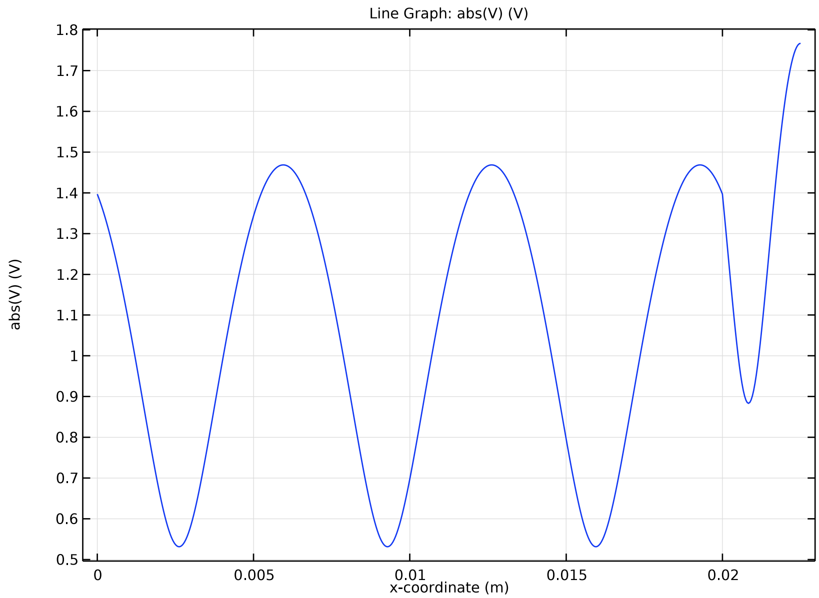
   You should now have a plot of the spectrum for S_{11} and S_{21}, similar to the one in Figure 3.

Electric Potential (tl)

To demonstrate that the quarter-wave transformer only eliminates the reflection at one frequency, plot the last frequency in the first plot group.

1. In the **Model Builder** window, click **Electric Potential (tl)**.
2. In the **Settings** window for **1D Plot Group**, locate the **Data** section.
3. From the **Parameter selection (freq)** list, choose **Last**.
4. In the **Electric Potential (tl)** toolbar, click **Plot**.

   Notice that the curve in the left part of the plot is not flat. The sinusoidal oscillation in the absolute value of the voltage is a signature of the standing wave that appears when there is a reflection point along the transmission line. For the selected frequency the quarter-wave transformer is not a quarter-wave long and, thus, there are now reflections.



To demonstrate that the quarter-wave transformer not only should have a matched length, but also a matched characteristic impedance, set the characteristic impedance of the second lumped port to 50Ω.

TRANSMISSION LINE (TL)

Lumped Port 2

1 In the **Model Builder** window, under **Component 1 (comp1)>Transmission Line (tl)** click **Lumped Port 2**.
2 In the **Settings** window for **Lumped Port**, locate the **Settings** section.
3 In the Z_{ref} text field, type 50.

Compute the spectral plot again.

STUDY 1

In the **Home** toolbar, click **Compute**.

RESULTS

Electric Potential (tl)

Select the central frequency (1 GHz) in the **Electric Potential (tl)** plot group.

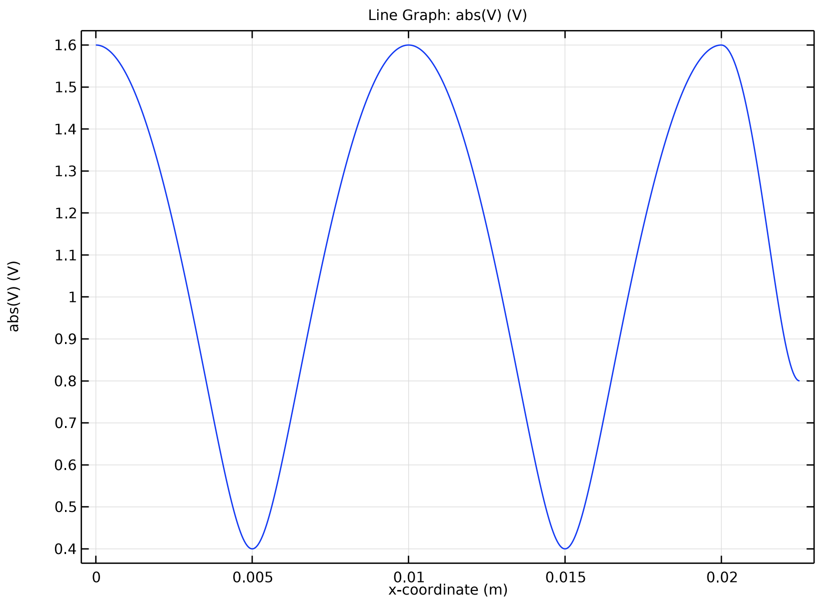
1 In the **Settings** window for **1D Plot Group**, locate the **Data** section.

2 From the **Parameter selection (freq)** list, choose **From list**.

3 In the **Parameter values (freq (GHz))** list, select **1**.

4 In the **Electric Potential (tl)** toolbar, click **Plot**.

Notice that there is now a standing wave also at the center frequency.



Select the second plot group to see the spectral response.

1D Plot Group 2

Notice that there is still a resonance at the center frequency. However, as was already indicated by the spatial plot, there is considerable reflection also at the resonance frequency.

1 In the **Model Builder** window, click **1D Plot Group 2**.