Multipactor Saturation

Introduction

The multipactor effect is a sometimes deleterious phenomenon in which the number of electrons in a microwave cavity can grow exponentially. Free electrons within the cavity are accelerated by an RF signal, causing them to strike the cavity walls. If the electrons hit the walls with sufficient energy, the walls may then release secondary electrons. Under the right conditions, a single electron impact can cause the release of multiple secondary electrons, thus causing the total number of electrons in the cavity to increase.

If a certain relationship exists between the size of the cavity and the frequency and amplitude of the RF signal, then the electron emission occurs in resonance with the driving signal; the emitted secondary electrons are accelerated to the other end of the cavity, where they cause an even greater number of secondary electrons to be released, and so on.

When the number of electrons in the cavity becomes sufficiently large, then the mutual electrostatic repulsion between the electrons can disrupt their resonance with the driving RF signal, so that the total population of electrons does not grow without bound. Instead, the population of electrons eventually reaches a dynamic equilibrium in which the effects of space charge and the RF signal counterbalance each other. This verification model is a quasi-1D, two-way coupled particle field interaction model that shows how growth of space charge in a microwave cavity leads to multipactor saturation.

Model Definition

This example uses the Electrostatics interface to solve for the electric potential in a cavity and the Charged Particle Tracing interface to track electrons within the cavity. The RF voltage is applied by using the **Ground** boundary condition at one end of the geometry and the **Electric Potential** boundary condition at the other end, where the specified potential is a sinusoidal function of time,

$$V = V_0 \sin(2\pi f t)$$

where V_0 (SI unit: V) is the amplitude of the signal and f (SI unit: 1/s) is the frequency.

The direction from the **Ground** boundary condition to the **Electric Potential** boundary condition is taken to be the positive x direction. Let D (SI unit: m) represent the gap thickness, the perpendicular distance between these two surfaces. The dimensions of the cavity in the y and z directions are assumed to be much larger than in the x direction. In addition, a constant, uniform magnetic field is applied within the cavity,

$$\mathbf{B} = \langle 0, 0, B_0 \rangle$$

Multipaction can occur for specific combinations of the values of D, V_0, f, and B_0. This can include situations in which $B_0 = 0$. A stability analysis of such a one-dimensional cavity geometry is demonstrated in Ref. 1. The model parameters used in this example are also based on Ref. 1: $V_0 = 1078$ V, $B_0 = 360$ G, $f = 2.5$ GHz, and $D = 0.16$ cm.

When an electron hits the wall at either end of the microwave cavity, the incident electron is removed from the simulation using a **Wall** node with the **Disappear** wall condition. Depending on the kinetic energy of the incident particle, it may release one or more secondary particles into the domain using the **Secondary Emission** subnode. The relationship between the kinetic energy of the incident particle and the number of emitted secondary electrons to release is called the secondary electron yield or SEY, an energy-dependent expression also from Ref. 1 and plotted in Figure 1,

$$\text{SEY} = \begin{cases} 2.8\left[\dfrac{\text{KE}}{300 \text{ eV}}\exp\left(1 - \dfrac{\text{KE}}{300 \text{ eV}}\right)\right]^{0.6} & \text{KE} < 300 \text{ eV} \\[2em] 2.8\left[\dfrac{\text{KE}}{300 \text{ eV}}\exp\left(1 - \dfrac{\text{KE}}{300 \text{ eV}}\right)\right]^{0.2} & \text{KE} > 300 \text{ eV} \end{cases}$$
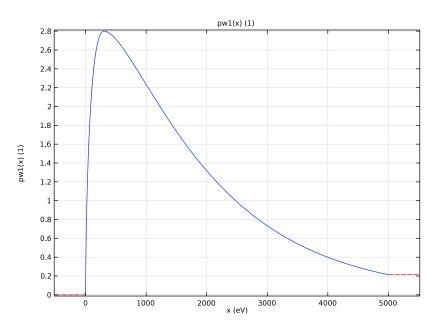

Figure 1: Secondary electron yield (SEY) as a function of incident electron kinetic energy.

Although the model geometry in this example is three-dimensional, the model is essentially 1D-3V, meaning that only one component of particle position and all three components of the particle velocity are relevant. The spatial density of particles and the electric potential are assumed to be completely uniform in the y and z directions. To enforce this uniformity without introducing spurious y- and z-components of the electric field, a highly structured mesh was used with only a single element in these directions and a large number of very thin elements in the x direction, as shown in Figure 2.
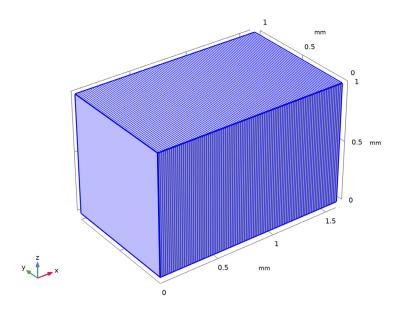

Figure 2: Structured mesh to simulate the multipactor as a 1D-3V model.

To account for space-charge-induced debunching, in which the mutual electrostatic repulsion of electrons causes some of the model particles to slow down before hitting the wall (and thus release fewer secondary electrons), the particles were bidirectionally coupled to the electric potential in the cavity via the **Electric Particle Field Interaction** multiphysics coupling. At every time step, each model particle contributes to a volumetric space charge density term within the domain mesh element that it occupies, and then the electric potential is updated based on this space charge density term. Because the mesh only has a single element in the y and z directions, and the space charge density of particles is considered uniform within each mesh element, this method of coupling the particles and field does not introduce any y- or z-components of the electric field.

The actual number of electrons in the multipactor can be very large, so to reduce the computational cost a macroparticle-based approach was implemented. Each model particle represents n electrons for the purpose of computing the volumetric space charge density, where in this example $n = 50,000$. This value is entered into the **Charge Multiplication Factor** text field in the settings for the **Electric Particle Field Interaction** multiphysics coupling node. To check that the model is statistically converged, consider recomputing with a lower value of n, such as 10,000. This might require an increase in the **Maximum number of secondary particles** (see below), but the total charge of particles hitting the cavity walls should be unaffected if the model is statistically converged (see Figure 3).

The secondary electron yield is a continuous function of energy and is usually not integer-valued. For instance, consider a value of 2.4. In this scenario the **Wall** boundary condition has been configured to always release two secondary electrons, and to release a third electron with a 40% probability. For this reason, each **Wall** node has two **Secondary Emission** attributes, one for a whole number of guaranteed secondary electrons, and one for an additional secondary electron with an emission probability between 0 and 1.

It is important to allocate a sufficiently large number of secondary particles so that all of the secondary electron emission in the model can be simulated. The **Maximum number of secondary particles** can be controlled in the physics interface **Particle Release and Propagation** section. Because electrons are constantly added to and removed from the simulation domain, it is highly beneficial to recycle the degrees of freedom associated with the removed model particles. In the physics interface **Advanced Settings** section, the option **All disappeared particles** was selected from the **Reuse particle degrees of freedom list**. The default behavior is not to recycle degrees of freedom so that every emitted secondary particle has a unique index, but at the cost of significantly more DOFs overall.

The constant, spatially uniform magnetic field was applied using the **Magnetic Force** node. Because the particles in this example are nonrelativistic, the contribution of the moving electrons to the background magnetic field was deemed negligible.

The length of the simulation domain in the y and z directions should not affect the solution, so any length can be selected that allows the solution to be viewed easily in the Graphics window. When a particle hits one of the surfaces parallel to the xy- or zx-planes, the particle is immediately moved to the opposite face and then carries on with its precollision velocity. Particles can be mapped from a source boundary to a destination boundary using the **Periodic Condition** feature. This example uses a pair of **Periodic Condition** nodes, one for the two surfaces parallel to the xy-plane and another for the two surfaces parallel to the zx-plane.

Results and Discussion

The total charge of impacting particles on the cavity walls per half period is plotted over 20 RF cycles in Figure 3. Compare this figure to Figure 6a in Ref. 1.

After saturation has been reached, two distinct populations of electron can be observed by carefully inspecting every frame of the default **Particle Trajectories** plot. For many electrons, after being released from one wall, they are accelerated enough so that they hit the opposite wall about half of one RF cycle later. For some other particles, the trajectory is sufficiently curved by the magnetic field that they do not reach the opposite wall before the RF signal reverses direction, so they are instead accelerated back toward the wall that released them.

Another way to observe the behavior after saturation is to plot and animate the distribution of space charge density in a **Slice** plot through the model geometry. Figure 4 shows one such plot at a time after saturation has been reached. The dark band is the main band containing most of the newly released secondary electrons. The lighter band includes some recently released secondary particles that lead or trail the main band due to space charge effects, as well as electrons that did not quite reach the opposite wall during the last RF cycle. The space charge density uses constant shape functions within each mesh element, so the sharp changes in color just indicate where one element ends and the next element begins. To make plot look smoother, consider refining the mesh in the x direction (which might also require a larger number of model particles for statistical convergence) or smoothing the solution during results processing.

You can also try animating either the **Particle Trajectories** plot or the **Slice** plot shown in Figure 4 to visualize how the band of maximum charge density moves in phase with the applied RF voltage.
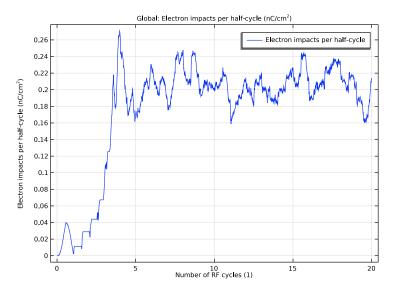
Figure 3: Total charge of impacting electrons per unit area, per RF half-cycle.
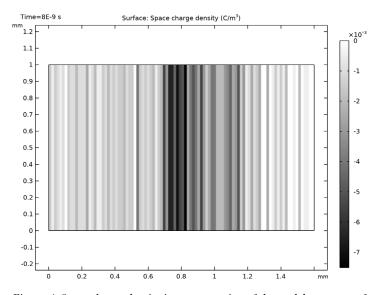

Figure 4: Space charge density in a cross section of the model geometry after saturation.

Reference

1. S. Riyopoulos, "Multipactor saturation due to space-charge-induced debunching," *Phys. Plasmas*, vol. 4, no. 5, pp. 1448–1462, 1997.

Application Library path: `Particle_Tracing_Module/`
`Charged_Particle_Tracing/multipactor_saturation`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.

2 In the **Select Physics** tree, select **AC/DC>Particle Tracing>Particle-Field Interaction, Nonrelativistic**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **Preset Studies for Selected Multiphysics>Time Dependent**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1
Load the model parameters from a file.

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 Click **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file `multipactor_saturation_parameters.txt`.

Variables 1

1 In the **Home** toolbar, click a= **Variables** and choose **Global Variables**.

Load the global variables from a file.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 Click 📁 **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file
`multipactor_saturation_variables_global.txt`.

These global variables will be used to suppress the release of secondary electrons when the RF field exerts a force in the outward normal direction.

Piecewise 1 (pw1)

1 In the **Home** toolbar, click f(x) **Functions** and choose **Global>Piecewise**.

2 In the **Settings** window for **Piecewise**, locate the **Definition** section.

3 Find the **Intervals** subsection. In the table, enter the following settings:

Start	End	Function
0	300	`2.8*((x/300)*exp(1-x/300))^0.6`
300	5000	`2.8*((x/300)*exp(1-x/300))^0.2`

4 Locate the **Units** section. In the **Arguments** text field, type eV.

5 In the **Function** text field, type 1.

6 Click ▣ **Plot**. Compare the resulting plot to Figure 1. This function will be used to define the secondary electron yield (SEY) as a function of the kinetic energy of the incident particle.

GEOMETRY 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.

2 In the **Settings** window for **Geometry**, locate the **Units** section.

3 From the **Length unit** list, choose **mm**.

Block 1 (blk1)

1 In the **Geometry** toolbar, click 📦 **Block**.

2 In the **Settings** window for **Block**, locate the **Size and Shape** section.

3 In the **Width** text field, type D.

4 In the **Depth** text field, type L.

5 In the **Height** text field, type L.

6 Click ▦ **Build All Objects**.

Any value of L should work, since the geometry extends infinitely far in the y and z directions, so a value was chosen that allows the geometry to be viewed easily in the Graphics window.

DEFINITIONS

Variables 2

1 In the **Home** toolbar, click $a=$ **Variables** and choose **Local Variables**.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 Click 📂 **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file multipactor_saturation_variables_local.txt.

Some of these variables will be used to define the number of released secondary electrons at the boundaries. Others are postprocessing variables that will be used to plot the number of electron impacts at the cavity walls over time. Note how the at operator is used to evaluate an expression at a previous solution time.

ADD MATERIAL

1 In the **Home** toolbar, click ⬛ **Add Material** to open the **Add Material** window.

2 Go to the **Add Material** window.

3 In the tree, select **Built-in>Perfect vacuum**.

4 Click **Add to Component** in the window toolbar.

5 In the **Home** toolbar, click ⬛ **Add Material** to close the **Add Material** window.

ELECTROSTATICS (ES)

Ground 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Electrostatics (es)** and choose **Ground**.

2 Select Boundary 1 only.

Electric Potential 1

1 In the **Physics** toolbar, click ▭ **Boundaries** and choose **Electric Potential**.

2 Select Boundary 6 only.

3 In the **Settings** window for **Electric Potential**, locate the **Electric Potential** section.

4 In the V_0 text field, type V0*sin(2*pi*f0*t).

Next, allow **Advanced Physics Options** to be shown if they are not already shown. This is required to access the settings that govern recycling of degrees of freedom for the Charged Particle Tracing interface.

ROOT

1 Click the 👁 **Show More Options** button in the **Model Builder** toolbar.

2 In the **Show More Options** dialog box, in the tree, select the check box for the node **Physics>Advanced Physics Options**.

3 Click **OK**.

CHARGED PARTICLE TRACING (CPT)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Charged Particle Tracing (cpt)**.

2 In the **Settings** window for **Charged Particle Tracing**, locate the **Particle Release and Propagation** section.

3 From the **Particle release specification** list, choose **Specify release times**.

4 In the **Maximum number of secondary particles** text field, type 1000.

5 Click to expand the **Advanced Settings** section. From the **Reuse particle degrees of freedom** list, choose **All disappeared particles**.

Electric Force 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Charged Particle Tracing (cpt)** click **Electric Force 1**.

2 In the **Settings** window for **Electric Force**, locate the **Electric Force** section.

3 From the **E** list, choose **Electric field (es/ccn1)**.

Magnetic Force 1

1 In the **Physics** toolbar, click ▬ **Domains** and choose **Magnetic Force**.

2 Select Domain 1 only.

3 In the **Settings** window for **Magnetic Force**, locate the **Magnetic Force** section.

4 Specify the **B** vector as

0	x
0	y
B0	z

Release from Grid 1

1 In the **Physics** toolbar, click ⚛ **Global** and choose **Release from Grid**.

2 In the **Settings** window for **Release from Grid**, locate the **Initial Coordinates** section.

3 In the $q_{x,0}$ text field, type range(0.01,0.02,0.99)*D.

4 In the $q_{y,0}$ text field, type L/2.

5 In the $q_{z,0}$ text field, type L/2.

The initial particle positions are distributed across the width of the cavity so that at least some of these seed charges will hit the walls at sufficiently high energy to start the electron avalanche.

Next, two **Secondary Emission** nodes will be added to **Wall** nodes on opposite sides of the cavity. The first releases a guaranteed number of secondary electrons equal to the largest integer less than the secondary electron yield. The second releases one more electron with a probability based on the remainder of the secondary electron yield. Two **Wall** nodes have been used because each has an additional requirement that the force from the RF signal should point into the simulation domain, or else no particles can be released.

Wall 2

1 In the **Physics** toolbar, click ▭ **Boundaries** and choose **Wall**.

2 Select Boundary 1 only.

3 In the **Settings** window for **Wall**, locate the **Wall Condition** section.

4 From the **Wall condition** list, choose **Disappear**.

Secondary Emission 1

1 In the **Physics** toolbar, click ▭ **Attributes** and choose **Secondary Emission**.

2 In the **Settings** window for **Secondary Emission**, locate the **Secondary Particles** section.

3 In the N_s text field, type SEY_int_left.

4 From the **Initial velocity** list, choose **Thermal**.

5 In the T text field, type T0.

Secondary Emission 2

1 Right-click **Secondary Emission 1** and choose **Duplicate**.

2 In the **Settings** window for **Secondary Emission**, locate the **Secondary Emission Condition** section.

3 From the **Secondary emission condition** list, choose **Probability**.

4 In the γ text field, type SEY_frac_left.

5 Locate the **Secondary Particles** section. In the N_s text field, type 1.

Wall 2

In the **Model Builder** window, click **Wall 2**.

Accumulator 1

1 In the **Physics** toolbar, click ⚙ **Attributes** and choose **Accumulator**.

2 In the **Settings** window for **Accumulator**, locate the **Units** section.

3 Click ▦ **Select Quantity**.

4 In the **Physical Quantity** dialog box, type surfacecharge in the text field.

5 Click ▼ **Filter**.

6 In the tree, select **Electromagnetics>Surface charge density (C/m^2)**.

7 Click **OK**.

8 In the **Settings** window for **Accumulator**, locate the **Accumulator Settings** section.

9 In the R text field, type e_const*n.

   These accumulated variables will be used in results processing to track the total number of electron impacts on each surface over time.

Now duplicate this **Wall** boundary condition and apply the copy to the opposite wall, with the appropriate modifications.

Wall 3

1 Right-click **Wall 2** and choose **Duplicate**.

2 Select Boundary 6 only.

Secondary Emission 1

1 In the **Model Builder** window, expand the **Wall 3** node, then click **Secondary Emission 1**.

2 In the **Settings** window for **Secondary Emission**, locate the **Secondary Particles** section.

3 In the N_s text field, type SEY_int_right.

Secondary Emission 2

1 In the **Model Builder** window, click **Secondary Emission 2**.

2 In the **Settings** window for **Secondary Emission**, locate the **Secondary Emission Condition** section.

3 In the γ text field, type SEY_frac_right.

Periodic boundary conditions will be used on the remaining surfaces to represent the infinite extent of the cavity in the y and z directions. Each **Periodic Condition** node is applied to one pair of boundaries on opposite sides.

Periodic Condition 1

1 In the **Physics** toolbar, click 🟦 **Boundaries** and choose **Periodic Condition**.

2 Select Boundaries 3 and 4 only.

Periodic Condition 2

1 In the **Physics** toolbar, click 🟦 **Boundaries** and choose **Periodic Condition**.

2 Select Boundaries 2 and 5 only.

MULTIPHYSICS

Electric Particle-Field Interaction 1 (epfi1)

1 In the **Model Builder** window, under **Component 1 (comp1)>Multiphysics** click **Electric Particle-Field Interaction 1 (epfi1)**.

2 In the **Settings** window for **Electric Particle-Field Interaction**, click to expand the **Charge Multiplication Factor** section.

3 In the n text field, type n.

This multiplication factor is the number of real electrons that are represented by each model particle for the purpose of computing the space charge density. This prevents an inordinate number of particle degrees of freedom from being required.

MESH 1

A structured mesh will be designed with a single element in the y and z directions, but very fine resolution in the x direction in which the space charge density may vary greatly.

Mapped 1

1 In the **Mesh** toolbar, click △ **Boundary** and choose **Mapped**.

2 Select Boundary 1 only.

Distribution 1

1 In the **Mesh** toolbar, click ▦ **Distribution**.

2 Select Edges 1, 2, 4, and 6 only.

3 In the **Settings** window for **Distribution**, locate the **Distribution** section.

4 In the **Number of elements** text field, type 1.

Swept 1

In the **Mesh** toolbar, click 🔺 **Swept**.

Distribution 1

1 In the **Mesh** toolbar, click ▦ **Distribution**.

2 In the **Settings** window for **Distribution**, locate the **Distribution** section.

3 In the **Number of elements** text field, type 100.

4 Click ▦ **Build All**. Compare the resulting plot to Figure 2.

STUDY 1

Now adjust the study settings to trace the particles for up to 20 RF cycles. A rather small time step is used in order to create higher-quality animations in postprocessing, but a larger time step could also be used to reduce file size.

Step 1: Time Dependent

1 In the **Model Builder** window, under **Study 1** click **Step 1: Time Dependent**.

2 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.

3 In the **Output times** text field, type range(0,1/(100*f0),20/f0).

Solution 1 (sol1)

1 In the **Study** toolbar, click ▐┲ **Show Default Solver**.

2 In the **Model Builder** window, expand the **Solution 1 (sol1)** node.

3 In the **Model Builder** window, expand the **Study 1>Solver Configurations> Solution 1 (sol1)>Time-Dependent Solver 1** node.

4 Right-click **Study 1>Solver Configurations>Solution 1 (sol1)>Time-Dependent Solver 1** and choose **Fully Coupled**.

5 In the **Settings** window for **Fully Coupled**, locate the **General** section.

6 From the **Linear solver** list, choose **Direct**.

7 In the **Study** toolbar, click ═ **Compute**.

If the **Maximum number of secondary particles** in the settings for the Charged Particle Tracing interface is too low, a warning might appear, saying that some secondary particles were not released. For the parameter values in these instructions, 1000 secondary particles is expected to suffice and this warning should not appear. If you later reduce the value of the model parameter n, the charge multiplication factor, then an increase in the preallocated secondary particles may be required.

RESULTS

Some default plots, as well as a default **Particle** dataset, are created. These can be viewed at any time. To see the quantitative information that indicates whether saturation has been reached, create a **1D Plot Group** to track the number of electron impacts over time.

1D Plot Group 5

In the **Home** toolbar, click ⊞ **Add Plot Group** and choose **1D Plot Group**.

Global 1

1 In the **1D Plot Group 5** toolbar, click ⊟ **Global**.

2 In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)>Definitions> Variables>impacts_change - Electron impacts per half-cycle - C/m²**.

3 Locate the **y-Axis Data** section. In the table, enter the following settings:

Expression	Unit	Description
impacts_change	nC/cm^2	Electron impacts per half-cycle

4 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

5 Click **Replace Expression** in the upper-right corner of the **x-Axis Data** section. From the menu, choose **Global definitions>Variables>tau - Number of RF cycles - 1**.

6 In the **1D Plot Group 5** toolbar, click ▶ **Plot**. Compare the resulting plot to Figure 3.

Cut Plane 1

1 In the **Results** toolbar, click ▱ **Cut Plane**.

2 In the **Settings** window for **Cut Plane**, locate the **Plane Data** section.

3 From the **Plane** list, choose **xy-planes**.

4 In the **z-coordinate** text field, type L/2.

5 Click ▶ **Plot**, to verify that the cut plane passes through the middle of the domain.

2D Plot Group 6

1 In the **Results** toolbar, click ▇ **2D Plot Group**.

2 In the **Settings** window for **2D Plot Group**, locate the **Data** section.

3 From the **Dataset** list, choose **Cut Plane 1**.

Surface 1

1 In the **2D Plot Group 6** toolbar, click ▇ **Surface**.

2 In the **Settings** window for **Surface**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)> Currents and charge>epfil.rhos - Space charge density - C/m³**.

3 Locate the **Coloring and Style** section. Click ▮▮ **Change Color Table**.

4 In the **Color Table** dialog box, select **Linear>GrayScale** in the tree.

5 Click **OK**.

6 In the **2D Plot Group 6** toolbar, click [image] **Plot**. Compare the resulting plot to Figure 4.

Optionally, you may animate the **Slice** plot or the **Particle Trajectories** plot.

Animation 1

1 In the **2D Plot Group 6** toolbar, click [image] **Animation** and choose **Player**.

2 In the **Settings** window for **Animation**, locate the **Frames** section.

3 In the **Number of frames** text field, type 200.

4 Click the [image] **Play** button in the **Graphics** toolbar.