

# Squeeze Film Between Rectangular Porous and Nonporous Plates

## Introduction

This example describes how to simulate the flow of a thin film of fluid in the gap between two rectangular plates, one of them with a porous facing, when the fluid is squeezed as a consequence of the relative motion between the plates. The model accounts for the ingress and egress of fluid between the porous material and the thin-film region. The simple geometry of the model allows for the comparison of the computed solution with analytic expressions in Ref. 1.

## Model Definition

The model is set up with parameterized quantities in a unitless base system. It uses a thinfilm flow approximation, described by the Reynolds equation, for the fluid motion in the gap between the plates. The fluid flow in the porous region is described by Darcy's law. The coupling between the Thin-Film Flow interface and the Darcy's Law interface is handled by the Thin-Film and Porous Media multiphysics coupling.

#### GEOMETRY

The length of the rectangular plates,  $\alpha$ , is taken to be of unit value. The width, b is controlled by the aspect ratio parameter, k = a/b. Using symmetry planes orthogonal to the x- and y-axes, only one quarter of the plate is modeled, as shown in Figure 1. The height of the porous facing, H, is suitably scaled with respect to the plate length using the parameter,  $\overline{H} = H/a$ . A rigid body motion is considered for the nonporous plate and it is therefore not included in the model geometry. The relative motion between the porous and nonporous plates can be accounted for by the motion of the "wall" or "base" defined in the Thin-Film Flow interface.

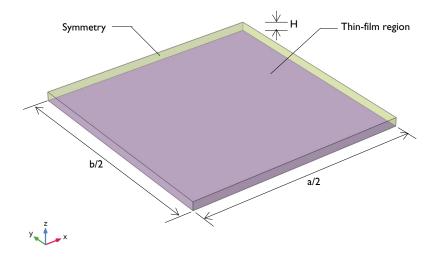


Figure 1: Computational domain with symmetry boundaries (in yellow) and the thin-film region (in purple).

#### PHYSICS INTERFACE SETTINGS

The Thin-Film Flow interface is set up to solve the Reynolds equation. The reference pressure,  $p_{ref}$ , is set to zero. The parameterized fluid density,  $\rho$ , and viscosity,  $\eta$ , are both set to unit value. The thin-film thickness is chosen to be,  $h_t = 0.001$ , and set as the height of the wall above the reference plane. The squeeze action is induced by applying a wall velocity in the positive z direction of unit magnitude controlled by the parameter,  $v_{sq}$ . Symmetry condition is applied to edges corresponding to the symmetry boundaries in the geometry. Border condition with zero pressure is applied to other edges as default.

The Darcy's Law interface is set up with zero reference pressure and similar fluid properties to that in the Thin-Film Flow interface. The porosity,  $\varepsilon_p$ , is set to zero and the permeability,  $\boldsymbol{\phi}$  , is chosen to be isotropic in nature, with a value controlled by the permeability parameter,  $\psi = \phi H/h_t^3$ . The boundary conditions include a No Flow condition on the top face, a Symmetry condition on the symmetry boundaries, and a Pressure condition with zero prescribed value elsewhere.

The multiphysics coupling, added automatically, acts at the intersection of the two interfaces. It accounts for the fluid motion and the pressure balance in the region.

#### MESHING

A structured mesh with hexahedral elements is constructed using the Mapped and Swept mesh features. The porous domain thickness is meshed with four elements. The resulting mesh is shown in Figure 2.

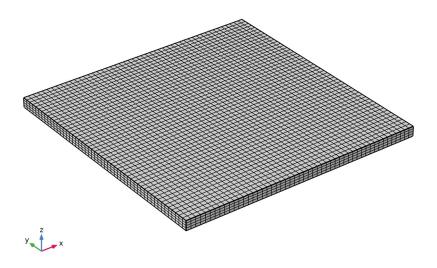


Figure 2: Computational domain with structured, hexahedral mesh.

# Results and Discussion

The analysis presented in Ref. 1 provides the expression for the dimensionless pressure at the intersection of the physics interfaces. It is given in the form of infinite series as

$$\frac{h_t^3 p(x,y)}{\eta a^2 v_{\text{sq}}} = \frac{192}{\pi^3} \sum_{m, n \text{ odd}}^{\infty} \left( \frac{\sin(\alpha_m x) \sin(\beta_n y)}{m n \sqrt{m^2 + k^2 n^2} (\pi \sqrt{m^2 + k^2 n^2} + \psi G_{\text{mn}})} \right), \tag{1}$$

where

$$\alpha_m = (m\pi)/a \,, \tag{2}$$

$$\beta_n = (n\pi)/b \,, \tag{3}$$

$$G_{\rm mn} = \frac{12(e^{2\Upsilon H} - 1)}{\overline{H}(e^{2\Upsilon H} + 1)},\tag{4}$$

$$\Upsilon = \frac{\pi}{a} \sqrt{m^2 + k^2 n^2} \,. \tag{5}$$

The dimensionless load carrying capacity of the squeeze film is given as

$$\frac{h_t^3 W}{\eta b a^2 v_{\text{sq}}} = \frac{768}{\pi^5} \sum_{m, n \text{ odd}}^{\infty} \left( \frac{1}{m^2 n^2 \sqrt{m^2 + k^2 n^2} (\pi \sqrt{m^2 + k^2 n^2} + \psi G_{\text{mn}})} \right). \tag{6}$$

In the comparison plots presented in the following paragraphs, the values of m and n are taken to be equal to 29, as suggested in Ref. 1. This ensures accurate pressure and load capacity values up to five significant digits.

Figure 3 shows pressure distribution for different values of  $\psi$  on the intersecting boundary of the physics interfaces at the location y = b/2, for values k = 1 and  $\overline{H} = 0.02$ . The

maximum pressure rapidly decreases with an increase in the permeability parameter. For values of  $\psi < 0.001$ , the behavior is close to the nonporous limit as reported in Ref. 1.

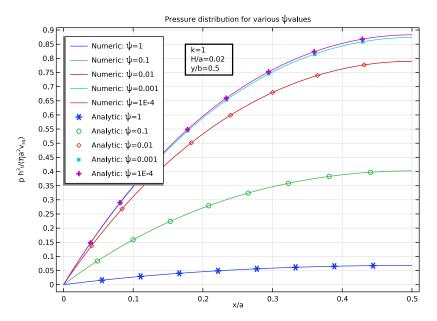


Figure 3: Pressure distribution for various values of  $\psi$ .

Figure 4 shows the load capacity for different values of k at the intersecting boundary of the physics interfaces for the value of  $\overline{H} = 0.02$ . The load-carrying capacity generally decreases with an increase in the aspect ratio, as such plates cannot contain the pressure generated due to increased side leakage. In fact, square plates provide the highest loadbearing capacity for a given quadrilateral area of coverage.

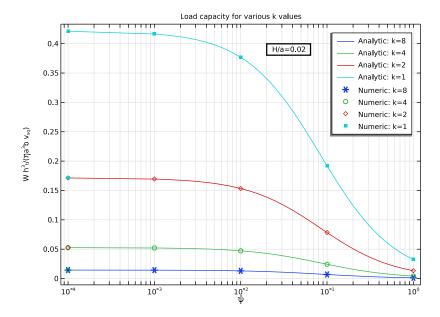


Figure 4: Load capacity for various values of k.

Figure 5 shows the loss of load capacity with increasing aspect ratio and permeability parameter.

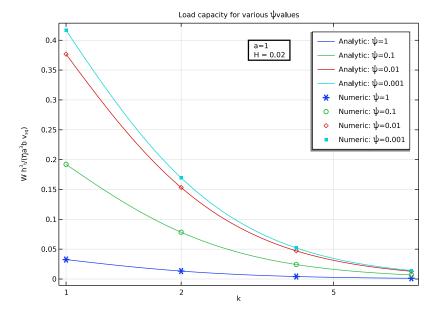


Figure 5: Load capacity for various values of  $\psi$ .

Figure 6 shows load capacity for different values of  $\phi$  at the intersection of the physics interfaces for the values of k = 1 and  $h_t = 0.001$ . The load capacity is large and shows little variation with changes in the porous domain thickness for low permeabilities. With an increase in the permeability and porous domain thickness, however, the load capacity rapidly deteriorates.

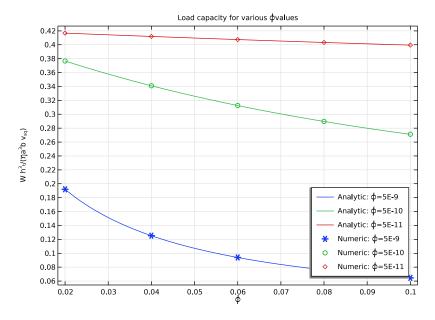


Figure 6: Load capacity for various values of  $\phi$ .

All comparison plots show excellent agreement between computed and analytic solutions.

Figure 7 shows the pressure developed at the intersection of the physics interfaces and the subsequent percolation flow in the porous domain as a consequence of the squeezing action of the plates on the fluid in the thin-film region for the case k = 1,  $\overline{H} = 0.02$ , and  $\psi = 0.01$ .

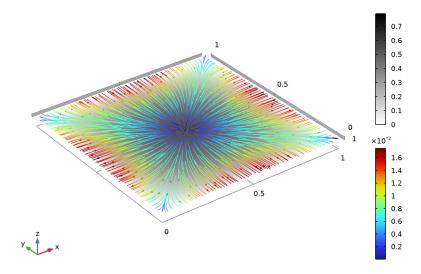


Figure 7: Darcy pressure and velocity streamlines.

# Reference

1. H. Wu, "An Analysis of the Squeeze Film Between Porous Rectangular Plates," J. Lubr. Technol., vol. 94, no. 1, pp. 64-68, 1972.

Application Library path: CFD\_Module/Verification\_Examples/ squeeze\_film\_porous

# Modeling Instructions

From the File menu, choose New.

## NEW

In the New window, click Model Wizard.

#### MODEL WIZARD

- I In the Model Wizard window, click **3D**.
- 2 In the Select Physics tree, select Fluid Flow>Porous Media and Subsurface Flow>Thin-Film and Porous Media Flow.
- 3 Click Add.
- 4 Click 🔵 Study.
- 5 In the Select Study tree, select General Studies>Stationary.
- 6 Click M Done.

Choose a unitless base system.

#### ROOT

- I In the Model Builder window, click the root node.
- 2 In the root node's **Settings** window, locate the **Unit System** section.
- 3 From the Unit system list, choose None.

#### **GLOBAL DEFINITIONS**

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- 3 Click Load from File.
- **4** Browse to the model's Application Libraries folder and double-click the file squeeze film porous parameters.txt.

Define analytic expressions of the solution. The expressions are available in Equation 1-Equation 6.

#### DEFINITIONS

Analytic I (an I)

- I In the Home toolbar, click f(X) Functions and choose Local>Analytic.
- 2 In the Settings window for Analytic, type a\_m in the Function name text field.
- 3 Locate the **Definition** section. In the **Expression** text field, type m\*pi/a.
- 4 In the Arguments text field, type m, a.

**5** Locate the **Plot Parameters** section. In the table, enter the following settings:

Plot	Argument	Lower limit	Upper limit	Fixed value	Unit
$\sqrt{}$	m	0	29	0	

## Analytic 2 (an2)

- I In the Home toolbar, click f(X) Functions and choose Local>Analytic.
- 2 In the Settings window for Analytic, type b n in the Function name text field.
- 3 Locate the **Definition** section. In the **Expression** text field, type n\*pi/b.
- 4 In the Arguments text field, type n, b.
- **5** Locate the **Plot Parameters** section. In the table, enter the following settings:

Plot	Argument	Lower limit	Upper limit	Fixed value	Unit
	n	0	29	0	

### Analytic 3 (an3)

- I In the Home toolbar, click f(x) Functions and choose Local>Analytic.
- 2 In the Settings window for Analytic, type gamma mn in the Function name text field.
- 3 Locate the **Definition** section. In the **Expression** text field, type pi\*sqrt(m^2+(k\* n)^2)/a.
- 4 In the Arguments text field, type m, n, k, a.
- **5** Locate the **Plot Parameters** section. In the table, enter the following settings:

Plot	Argument	Lower limit	Upper limit	Fixed value	Unit
<b>V</b>	m	0	29	0	
<b>V</b>	n	0	29	0	
<b>V</b>	k	0	8	0	

## Analytic 4 (an4)

- I In the Home toolbar, click f(x) Functions and choose Local>Analytic.
- 2 In the Settings window for Analytic, type G mn in the Function name text field.
- 3 Locate the **Definition** section. In the **Expression** text field, type 12\*(exp(2\*gamma mn\*  $H)-1)/(H*(exp(2*gamma_mn*H)+1)/a).$
- 4 In the Arguments text field, type gamma\_mn, H, a.

**5** Locate the **Plot Parameters** section. In the table, enter the following settings:

Plot	Argument	Lower limit	Upper limit	Fixed value	Unit
$\sqrt{}$	gamma_mn	0	1e6	0	

### Analytic 5 (an5)

- I In the Home toolbar, click f(x) Functions and choose Local>Analytic.
- 2 In the Settings window for Analytic, type p\_summand in the Function name text field.
- 3 Locate the **Definition** section. In the **Expression** text field, type (sin(a m((2\*m-1),a)\* x)\*sin(b n((2\*n-1),b)\*y))/((2\*m-1)\*(2\*n-1)\*sqrt((2\*m-1)^2+(k\*(2\*n-1)\*)\* 1)) $^2$ )\*(pi\*sqrt((2\*m-1) $^2$ +(k\*(2\*n-1)) $^2$ )+(psi\*G\_mn(gamma\_mn((2\*m-1), (2\*n-1),k,a),H,a)))).
- 4 In the Arguments text field, type m, n, x, y, a, b, psi, k, H.
- **5** Locate the **Plot Parameters** section. In the table, enter the following settings:

Plot	Argument	Lower limit	Upper limit	Fixed value	Unit
1	m	0	29	0	
$\sqrt{}$	n	0	29	0	
<b>V</b>	×	0	a	0	

## Analytic 6 (an6)

- I In the Home toolbar, click f(X) Functions and choose Local>Analytic.
- 2 In the Settings window for Analytic, type W summand psi in the Function name text field.
- 3 Locate the **Definition** section. In the **Expression** text field, type 1/(((2\*m-1)\*(2\*n-1))^2\*sqrt((2\*m-1)^2+(k\*(2\*n-1))^2)\*((pi\*sqrt((2\*m-1)^2+(k\*(2\*n-1)) $^2$ ))+(psi\*G mn(gamma mn((2\*m-1),(2\*n-1),k,a),H,a)))).
- 4 In the Arguments text field, type m, n, a, psi, k, H.
- **5** Locate the **Plot Parameters** section. In the table, enter the following settings:

Plot	Argument	Lower limit	Upper limit	Fixed value	Unit
$\sqrt{}$	m	0	29	0	
$\checkmark$	n	0	29	0	

#### Analytic 7 (an7)

- I In the Home toolbar, click f(x) Functions and choose Local>Analytic.
- 2 In the Settings window for Analytic, type W summand phi in the Function name text field.

- 3 Locate the **Definition** section. In the **Expression** text field, type 1/(((2\*m-1)\*(2\*n-1)))
  - 1))^2\*sqrt((2\*m-1)^2+(k\*(2\*n-1))^2)\*((pi\*sqrt((2\*m-1)^2+(k\*(2\*n-1))^2+(k
  - 1)) $^2$ ))+(phi\*H/ht $^3$ \*G\_mn(gamma\_mn((2\*m-1),(2\*n-1),k,a),H,a)))).
- 4 In the Arguments text field, type m, n, a, phi, k, H.
- **5** Locate the **Plot Parameters** section. In the table, enter the following settings:

Plot	Argument	Lower limit	Upper limit	Fixed value	Unit
<b>√</b>	m	0	29	0	
	n	0	29	0	

#### **GEOMETRY I**

Block I (blk I)

- I In the Geometry toolbar, click Block.
- 2 In the Settings window for Block, locate the Size and Shape section.
- 3 In the Width text field, type a/2.
- 4 In the **Depth** text field, type b/2.
- 5 In the Height text field, type H.

Form Union (fin)

In the Geometry toolbar, click **Build All**.

Create selections for the thin-film domain and symmetry boundaries.

## **DEFINITIONS**

Thin-Film Boundary

- 2 In the Settings window for Explicit, type Thin-Film Boundary in the Label text field.
- 3 Locate the Input Entities section. From the Geometric entity level list, choose Boundary.
- 4 Click Paste Selection.
- 5 In the Paste Selection dialog box, type 3 in the Selection text field.
- 6 Click OK.
- 7 In the Settings window for Explicit, locate the Color section.
- 8 From the Color list, choose Color 1.

Symmetry Boundaries

I In the **Definitions** toolbar, click **\( \bigcap\_{\bigcap} \) Explicit**.

- 2 In the Settings window for Explicit, type Symmetry Boundaries in the Label text field.
- 3 Locate the Input Entities section. From the Geometric entity level list, choose Boundary.
- 4 Click Paste Selection.
- 5 In the Paste Selection dialog box, type 5-6 in the Selection text field.
- 6 Click OK.
- 7 In the Settings window for Explicit, locate the Color section.
- 8 From the Color list, choose Color 4.
- **9** Click the **Zoom Extents** button in the **Graphics** toolbar.

Next, specify parameters for the Thin-Film Flow interface: Set the reference pressure to zero and reverse the reference normal to point in the positive *z* direction.

## THIN-FILM FLOW (TFF)

- I In the Model Builder window, under Component I (compl) click Thin-Film Flow (tff).
- 2 In the Settings window for Thin-Film Flow, locate the Boundary Selection section.
- 3 From the Selection list, choose Thin-Film Boundary.
- **4** Locate the **Reference Pressure** section. In the  $p_{ref}$  text field, type **0**.

#### Fluid-Film Properties 1

- I In the Model Builder window, under Component I (comp1)>Thin-Film Flow (tff) click Fluid-Film Properties I.
- 2 In the Settings window for Fluid-Film Properties, locate the Model Input section.
- **3** From the T list, choose Common model input.
- 4 Click to expand the Reference Surface Properties section. From the Reference normal orientation list, choose Opposite direction to geometry normal.
- **5** Locate the **Wall Properties** section. In the  $h_{w1}$  text field, type ht.
- **6** From the  $\mathbf{v}_w$  list, choose **User defined**. Specify the vector as

# sqvel z

- 7 Locate the Fluid Properties section. From the  $\mu$  list, choose User defined. In the associated text field, type eta.
- **8** From the  $\rho$  list, choose **User defined**. In the associated text field, type rho.

#### Symmetry I

- I In the Physics toolbar, click Edges and choose Symmetry.
- 2 In the Settings window for Symmetry, locate the Edge Selection section.

- 3 Click Paste Selection.
- 4 In the Paste Selection dialog box, type 7, 10 in the Selection text field.
- 5 Click OK.

Now specify parameters for the Darcy's Law interface, including a zero reference pressure.

## DARCY'S LAW (DL)

- I In the Model Builder window, under Component I (compl) click Darcy's Law (dl).
- 2 In the Settings window for Darcy's Law, locate the Physical Model section.
- 3 In the  $p_{ref}$  text field, type 0.

#### Fluid 1

- I In the Model Builder window, under Component I (compl)>Darcy's Law (dl)> Porous Medium I click Fluid I.
- 2 In the Settings window for Fluid, locate the Fluid Properties section.
- **3** From the p list, choose **User defined**. In the associated text field, type rho.
- **4** From the  $\mu$  list, choose **User defined**. In the associated text field, type eta.

#### Porous Matrix I

- I In the Model Builder window, click Porous Matrix I.
- 2 In the Settings window for Porous Matrix, locate the Matrix Properties section.
- 3 From the  $\epsilon_p$  list, choose User defined. From the  $\kappa$  list, choose User defined. In the associated text field, type phi.

#### Symmetry 1

- I In the Physics toolbar, click **Boundaries** and choose Symmetry.
- 2 In the Settings window for Symmetry, locate the Boundary Selection section.
- 3 From the Selection list, choose Symmetry Boundaries.

#### Pressure 1

- I In the Physics toolbar, click **Boundaries** and choose **Pressure**.
- 2 In the Settings window for Pressure, locate the Boundary Selection section.
- 3 Click Paste Selection.
- 4 In the Paste Selection dialog box, type 1, 2 in the Selection text field.
- 5 Click OK.

Create a structured hexahedral mesh.

#### MESH I

- I In the Model Builder window, under Component I (compl) click Mesh I.
- 2 In the Settings window for Mesh, locate the Physics-Controlled Mesh section.
- 3 From the Element size list, choose Extremely fine.
- 4 Locate the Sequence Type section. From the list, choose User-controlled mesh.

- I In the Mesh toolbar, click More Generators and choose Mapped.
- 2 In the Settings window for Mapped, locate the Boundary Selection section.
- 3 From the Selection list, choose Thin-Film Boundary.

## Swebt I

In the Mesh toolbar, click A Swept.

#### Distribution I

- I Right-click Swept I and choose Distribution.
- 2 In the Settings window for Distribution, locate the Distribution section.
- 3 In the Number of elements text field, type 4.

#### Free Tetrahedral I

- I In the Model Builder window, under Component I (compl)>Mesh I right-click Free Tetrahedral I and choose Disable.
- 2 Right-click Mesh I and choose Build All.
- 3 Click the **Zoom Extents** button in the **Graphics** toolbar.

#### STUDY I

- I In the Model Builder window, click Study I.
- 2 In the Settings window for Study, locate the Study Settings section.
- 3 Clear the Generate default plots check box.

Add a Parametric Sweep step for computing the Stationary solution for specified parameter combinations.

#### Parametric Sweep

- I In the Study toolbar, click Parametric Sweep.
- 2 In the Settings window for Parametric Sweep, locate the Study Settings section.
- 3 Click + Add.

**4** In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
k (Ratio of plate length to width)	8*1^range(1,5) 4* 1^range(1,5) 2* 1^range(1,5) 1^range(1, 5) 1^range(1,15)	

- 5 Click + Add.
- **6** In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
psi (Permeability parameter)	10^{range(0,-1,-4)} 10^{range(0,-1,-4)} 10^{range(0,-1,-4)} 10^{range(0,-1,-4)} range(0.1,0.1,0.5) range(0.01,0.01,0.05) range(0.001,0.001,	

- 7 Click + Add.
- **8** In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
Hbar (Ratio of porous domain thickness to plate length)	0.02*1^range(1,20) range(0.02,0.02,0.1) range(0.02,0.02,0.1) range(0.02,0.02,0.1)	

9 In the Study toolbar, click **Compute**.

#### RESULTS

- I In the Model Builder window, click Results.
- 2 In the Settings window for Results, locate the Update of Results section.
- 3 Select the Only plot when requested check box.

Prepare datasets for processing the results and analytic functions.

#### Mirror 3D I

- I In the **Model Builder** window, expand the **Results** node.
- 2 Right-click Results>Datasets and choose More 3D Datasets>Mirror 3D.
- 3 In the Settings window for Mirror 3D, locate the Data section.

- 4 From the Dataset list, choose Study I/Parametric Solutions I (sol2).
- 5 Locate the Plane Data section. From the Plane list, choose XZ-planes.
- 6 In the Y-coordinate text field, type b/2.

## Mirror 3D 2

- I In the Results toolbar, click More Datasets and choose Mirror 3D.
- 2 In the Settings window for Mirror 3D, locate the Plane Data section.
- 3 In the X-coordinate text field, type a/2.
- 4 Locate the Data section. From the Dataset list, choose Mirror 3D 1.

## Cut Line 3D I

- I In the Results toolbar, click Cut Line 3D.
- 2 In the Settings window for Cut Line 3D, locate the Data section.
- 3 From the Dataset list, choose Mirror 3D 2.
- 4 Locate the Line Data section. In row Point 1, set y to b/2.
- 5 In row Point 2, set x to a.
- 6 In row Point 2, set y to b/2.

## Grid ID I

- I In the Results toolbar, click More Datasets and choose Grid>Grid ID.
- 2 In the Settings window for Grid ID, locate the Data section.
- **3** From the **Source** list, choose **Function**.
- **4** From the **Function** list, choose **All**.
- 5 Locate the Parameter Bounds section. In the Name text field, type psi.
- 6 In the Minimum text field, type 1e-4.

#### Grid ID 2

- I In the Results toolbar, click More Datasets and choose Grid>Grid ID.
- 2 In the Settings window for Grid ID, locate the Data section.
- **3** From the **Source** list, choose **Function**.
- **4** From the **Function** list, choose **All**.
- 5 Locate the Parameter Bounds section. In the Name text field, type k.
- 6 In the Minimum text field, type 1.
- 7 In the Maximum text field, type 8.

#### Grid ID 3

- I In the Results toolbar, click More Datasets and choose Grid>Grid ID.
- 2 In the Settings window for Grid ID, locate the Data section.
- **3** From the **Source** list, choose **Function**.
- **4** From the **Function** list, choose **All**.
- 5 Locate the Parameter Bounds section. In the Name text field, type H.
- **6** In the **Minimum** text field, type **0.02**.
- 7 In the Maximum text field, type 0.1.

Compute the load capacity for different parameter values.

#### Surface Integration 1

- I In the Results toolbar, click 8.85 More Derived Values and choose Integration> Surface Integration.
- 2 In the Settings window for Surface Integration, locate the Data section.
- 3 From the Dataset list, choose Mirror 3D 2.
- 4 From the Parameter selection (k, psi, Hbar) list, choose Manual.
- 5 In the Parameter indices (1-35) text field, type range (1,1,5).
- 6 Locate the Selection section. From the Selection list, choose Thin-Film Boundary.
- 7 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
p*ht^3/(eta*a^3*b*sqvel)		

8 Right-click Surface Integration I and choose Duplicate.

## Surface Integration 2

- I In the Model Builder window, click Surface Integration 2.
- 2 In the Settings window for Surface Integration, locate the Data section.
- 3 In the Parameter indices (1-35) text field, type range (5, 1, 10).
- 4 Right-click Surface Integration 2 and choose Duplicate.

#### Surface Integration 3

- I In the Model Builder window, click Surface Integration 3.
- 2 In the Settings window for Surface Integration, locate the Data section.
- 3 In the Parameter indices (1-35) text field, type range (10, 1, 15).
- 4 Right-click Surface Integration 3 and choose Duplicate.

## Surface Integration 4

- I In the Model Builder window, click Surface Integration 4.
- 2 In the Settings window for Surface Integration, locate the Data section.
- 3 In the Parameter indices (1-35) text field, type range (15, 1, 20).
- 4 Right-click Surface Integration 4 and choose Duplicate.

## Surface Integration 5

- I In the Model Builder window, click Surface Integration 5.
- 2 In the Settings window for Surface Integration, locate the Data section.
- 3 In the Parameter indices (1-35) text field, type range (1,5,20).
- 4 Right-click Surface Integration 5 and choose Duplicate.

#### Surface Integration 6

- I In the Model Builder window, click Surface Integration 6.
- 2 In the Settings window for Surface Integration, locate the Data section.
- 3 In the Parameter indices (1-35) text field, type range (2,5,20).
- 4 Right-click Surface Integration 6 and choose Duplicate.

### Surface Integration 7

- I In the Model Builder window, click Surface Integration 7.
- 2 In the Settings window for Surface Integration, locate the Data section.
- 3 In the Parameter indices (1-35) text field, type range (3,5,20).
- 4 Right-click Surface Integration 7 and choose Duplicate.

## Surface Integration 8

- I In the Model Builder window, click Surface Integration 8.
- 2 In the Settings window for Surface Integration, locate the Data section.
- 3 In the Parameter indices (1-35) text field, type range (4,5,20).
- 4 Right-click Surface Integration 8 and choose Duplicate.

## Surface Integration 9

- I In the Model Builder window, click Surface Integration 9.
- 2 In the Settings window for Surface Integration, locate the Data section.
- 3 In the Parameter indices (1-35) text field, type range (21, 1, 25).
- 4 Right-click Surface Integration 9 and choose Duplicate.

#### Surface Integration 10

- I In the Model Builder window, click Surface Integration 10.
- 2 In the Settings window for Surface Integration, locate the Data section.
- 3 In the Parameter indices (1-35) text field, type range (26, 1, 30).
- 4 Right-click Surface Integration 10 and choose Duplicate.

#### Surface Integration 11

- I In the Model Builder window, click Surface Integration II.
- 2 In the Settings window for Surface Integration, locate the Data section.
- 3 In the Parameter indices (1-35) text field, type range (31, 1, 35).
- 4 In the Results toolbar, click **= Evaluate** and choose Clear and Evaluate All.

#### Pressure Distribution for Various psi Values

- I In the Results toolbar, click \( \subseteq \text{ID Plot Group.} \)
- 2 In the Settings window for ID Plot Group, type Pressure Distribution for Various psi Values in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Cut Line 3D 1.
- 4 From the Parameter selection (k, psi, Hbar) list, choose From list.
- 5 In the Parameter values (k,psi,Hbar) list, choose 16: k=1, psi=1, Hbar=0.02, 17: k=1, psi=0.1, Hbar=0.02, 18: k=1, psi=0.01, Hbar=0.02, 19: k=1, psi=0.001, Hbar=0.02, and 20: k=1, psi=1E-4, Hbar=0.02.
- 6 Click to expand the Title section. From the Title type list, choose Manual.
- 7 In the Title text area, type Pressure distribution for various \psi values.
- 8 Locate the **Plot Settings** section.
- **9** Select the **x-axis label** check box. In the associated text field, type x/a.
- 10 Select the y-axis label check box. In the associated text field, type p h<sup>3</sup> <sub>t</sub>/(\eta a<sup>2</sup>v<sub>sq</sub>).
- II Locate the Legend section. From the Position list, choose Upper left.

#### Line Grabh I

- I Right-click Pressure Distribution for Various psi Values and choose Line Graph.
- 2 In the Settings window for Line Graph, locate the y-Axis Data section.
- 3 In the Expression text field, type p\*ht^3/(eta\*a^2\*sqvel).
- 4 Locate the x-Axis Data section. From the Parameter list, choose Expression.
- **5** In the **Expression** text field, type x/a.

- **6** Click to expand the **Coloring and Style** section. Click to expand the **Legends** section. Select the **Show legends** check box.
- 7 From the Legends list, choose Manual.
- **8** In the table, enter the following settings:

# Legends Numeric: \psi=1 Numeric: \psi=0.1 Numeric: \psi=0.01 Numeric: \psi=0.001 Numeric: \psi=1E-4

## Line Graph 2

- I In the Model Builder window, right-click Pressure Distribution for Various psi Values and choose Line Graph.
- 2 In the Settings window for Line Graph, locate the y-Axis Data section.
- 3 In the Expression text field, type  $192*sum(sum(p_summand(m, n, x, y, a, b, psi,$ 1, 0.02), n, 1, 15), m, 1, 15)/(pi<sup>3</sup>).
- 4 Locate the x-Axis Data section. From the Parameter list, choose Expression.
- **5** In the **Expression** text field, type x/a.
- 6 Locate the Coloring and Style section. From the Color list, choose Cycle (reset).
- 7 Find the Line markers subsection. From the Marker list, choose Cycle (reset).
- **8** Find the **Line style** subsection. From the **Line** list, choose **None**.
- 9 Find the Line markers subsection. From the Positioning list, choose Interpolated.
- **10** Locate the **Legends** section. Select the **Show legends** check box.
- II From the Legends list, choose Manual.
- 12 In the table, enter the following settings:

Legends	
Analytic:	\psi=1
Analytic:	\psi=0.1
Analytic:	\psi=0.01
Analytic:	\psi=0.001
Analytic:	\psi=1E-4

#### Annotation I

- I Right-click Pressure Distribution for Various psi Values and choose Annotation.
- 2 In the Settings window for Annotation, locate the Annotation section.
- 3 In the **Text** text field, type k=eval(k) \\ H/a=eval(Hbar) \\ y/b=0.5.
- 4 Select the LaTeX markup check box.
- **5** Locate the **Position** section. In the **x** text field, type **0.175**.
- 6 In the y text field, type 0.85.
- 7 Click to expand the Advanced section. In the Expression precision text field, type 1.
- 8 Locate the Coloring and Style section. Clear the Show point check box.
- 9 Select the Show frame check box.

#### Pressure Distribution for Various psi Values

- I In the Model Builder window, click Pressure Distribution for Various psi Values.
- 2 In the Pressure Distribution for Various psi Values toolbar, click **Tool** Plot.

## Load Capacity for Various k Values

- I In the Home toolbar, click **Add Plot Group** and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Load Capacity for Various k Values in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose None.
- 4 Locate the Title section. From the Title type list, choose Manual.
- 5 In the **Title** text area, type Load capacity for various k values.
- 6 Locate the Plot Settings section.
- 7 Select the x-axis label check box. In the associated text field, type \psi.
- 8 Select the y-axis label check box. In the associated text field, type W h<sup>3</sup> <sub>t</sub>/(\eta a<sup>3</sup>b v<sub>sq</sub>).
- 9 Locate the Axis section. Select the x-axis log scale check box.

#### Function I

- I In the Load Capacity for Various k Values toolbar, click \( \subseteq \text{More Plots} and choose \)
- 2 In the Settings window for Function, locate the Data section.
- 3 From the Dataset list, choose Grid ID 1.

- 4 Locate the y-Axis Data section. In the Expression text field, type 768\* sum(sum(W\_summand\_psi(m, n, a, psi, 8, 0.02), n, 1, 15), m, 1, 15)/ (pi<sup>5</sup>).
- 5 Locate the x-Axis Data section. In the Expression text field, type psi.
- 6 In the Lower bound text field, type 1e-4.
- 7 Click to expand the Coloring and Style section. From the Color list, choose Cycle (reset).
- 8 Click to expand the **Legends** section. Select the **Show legends** check box.
- 9 From the Legends list, choose Manual.
- **10** In the table, enter the following settings:

# Legends Analytic: k=8

II Right-click Function I and choose Duplicate.

#### Function 2

- I In the Model Builder window, click Function 2.
- 2 In the Settings window for Function, locate the y-Axis Data section.
- 3 In the Expression text field, type 768\*sum(sum(W\_summand\_psi(m, n, a, psi, 4, 0.02), n, 1, 15), m, 1, 15)/(pi<sup>5</sup>).
- 4 Locate the Coloring and Style section. From the Color list, choose Cycle.
- **5** Locate the **Legends** section. In the table, enter the following settings:

# Legends Analytic: k=4

6 Right-click Function 2 and choose Duplicate.

#### Function 3

- I In the Model Builder window, click Function 3.
- 2 In the Settings window for Function, locate the y-Axis Data section.
- 3 In the Expression text field, type 768\*sum(sum(W summand psi(m, n, a, psi, 2, 0.02), n, 1, 15), m, 1, 15)/(pi<sup>5</sup>).
- **4** Locate the **Legends** section. In the table, enter the following settings:

Legends	
Analytic:	k=2

5 Right-click Function 3 and choose Duplicate.

#### Function 4

- I In the Model Builder window, click Function 4.
- 2 In the Settings window for Function, locate the y-Axis Data section.
- 3 In the Expression text field, type 768\*sum(sum(W summand psi(m, n, a, psi, 1, 0.02), n, 1, 15), m, 1, 15)/(pi<sup>5</sup>).
- **4** Locate the **Legends** section. In the table, enter the following settings:

Legends	
Analytic:	k=1

#### Table Graph 1

- I In the Model Builder window, right-click Load Capacity for Various k Values and choose Table Graph.
- 2 In the Settings window for Table Graph, locate the Data section.
- 3 From the x-axis data list, choose psi.
- 4 From the Plot columns list, choose Manual.
- 5 In the Columns list, select p\*ht^3/(eta\*a^3\*b\*sqvel).
- 6 Locate the Coloring and Style section. Find the Line style subsection. From the Line list, choose None.
- 7 From the Color list, choose Cycle (reset).
- 8 Find the Line markers subsection. From the Marker list, choose Cycle (reset).
- **9** Click to expand the **Legends** section. Select the **Show legends** check box.
- 10 From the Legends list, choose Manual.
- II In the table, enter the following settings:

# Legends Numeric: k=8

12 Right-click Table Graph I and choose Duplicate.

## Table Graph 2

- I In the Model Builder window, click Table Graph 2.
- 2 In the Settings window for Table Graph, locate the Data section.
- 3 From the Table list, choose Table 2.
- 4 Locate the Coloring and Style section. From the Color list, choose Cycle.

- 5 Find the Line markers subsection. From the Marker list, choose Cycle.
- **6** Locate the **Legends** section. In the table, enter the following settings:

# Legends Numeric: k=4

7 Right-click Table Graph 2 and choose Duplicate.

#### Table Graph 3

- I In the Model Builder window, click Table Graph 3.
- 2 In the Settings window for Table Graph, locate the Data section.
- 3 From the Table list, choose Table 3.
- **4** Locate the **Legends** section. In the table, enter the following settings:

# Legends Numeric: k=2

5 Right-click Table Graph 3 and choose Duplicate.

## Table Graph 4

- I In the Model Builder window, click Table Graph 4.
- 2 In the Settings window for Table Graph, locate the Data section.
- **3** From the **Table** list, choose **Table 4**.
- **4** Locate the **Legends** section. In the table, enter the following settings:

# Legends Numeric: k=1

## Annotation I

- I In the Model Builder window, right-click Load Capacity for Various k Values and choose Annotation
- 2 In the Settings window for Annotation, locate the Data section.
- 3 From the Dataset list, choose Study I/Parametric Solutions I (sol2).
- 4 From the Parameter value (k,psi,Hbar) list, choose 1: k=8, psi=1, Hbar=0.02.
- **5** Locate the **Annotation** section. In the **Text** text field, type H/a=0.02.
- 6 Select the LaTeX markup check box.
- 7 Locate the **Position** section. In the **X** text field, type 0.02.
- 8 In the Y text field, type 0.4.

- **9** Locate the **Advanced** section. In the **Expression precision** text field, type 1.
- 10 Locate the Coloring and Style section. Clear the Show point check box.
- II Select the **Show frame** check box.

## Load Capacity for Various k Values

- I In the Model Builder window, click Load Capacity for Various k Values.
- 2 In the Load Capacity for Various k Values toolbar, click **1** Plot.

## Load Cabacity for Various bsi Values

- I In the Home toolbar, click **Add Plot Group** and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Load Capacity for Various psi Values in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose None.
- 4 Locate the Title section. From the Title type list, choose Manual.
- 5 In the Title text area, type Load capacity for various \psi values.
- 6 Locate the Plot Settings section.
- 7 Select the x-axis label check box. In the associated text field, type k.
- 8 Select the y-axis label check box. In the associated text field, type W h<sup>3</sup> <sub>t</sub>/(\eta a<sup>3</sup>b v<sub>sq</sub>).
- **9** Locate the **Axis** section. Select the **x-axis** log scale check box.

#### Function I

- I In the Load Capacity for Various psi Values toolbar, click \to More Plots and choose Function.
- 2 In the Settings window for Function, locate the Data section.
- 3 From the Dataset list, choose Grid ID 2.
- 4 Locate the y-Axis Data section. In the Expression text field, type 768\* sum(sum(W\_summand\_psi(m, n, a, 1, k, 0.02), n, 1, 15), m, 1, 15)/ (pi^5).
- 5 Locate the x-Axis Data section. In the Expression text field, type k.
- 6 In the Lower bound text field, type 1.
- 7 In the **Upper bound** text field, type 8.
- 8 Locate the Coloring and Style section. From the Color list, choose Cycle (reset).
- **9** Locate the **Legends** section. Select the **Show legends** check box.
- 10 From the Legends list, choose Manual.

II In the table, enter the following settings:

```
Legends
Analytic: \psi=1
```

**12** Right-click **Function I** and choose **Duplicate**.

## Function 2

- I In the Model Builder window, click Function 2.
- 2 In the Settings window for Function, locate the y-Axis Data section.
- 3 In the Expression text field, type 768\*sum(sum(W summand psi(m, n, a, 0.1, k, 0.02), n, 1, 15), m, 1, 15)/(pi<sup>5</sup>).
- 4 Locate the Coloring and Style section. From the Color list, choose Cycle.
- **5** Locate the **Legends** section. In the table, enter the following settings:

Legends	
Analytic:	\psi=0.1

6 Right-click Function 2 and choose Duplicate.

#### Function 3

- I In the Model Builder window, click Function 3.
- 2 In the Settings window for Function, locate the y-Axis Data section.
- 3 In the Expression text field, type 768\*sum(sum(W summand psi(m, n, a, 0.01, k, 0.02), n, 1, 15), m, 1, 15)/(pi<sup>5</sup>).
- **4** Locate the **Legends** section. In the table, enter the following settings:

```
Legends
Analytic: \psi=0.01
```

**5** Right-click **Function 3** and choose **Duplicate**.

#### Function 4

- I In the Model Builder window, click Function 4.
- 2 In the Settings window for Function, locate the y-Axis Data section.
- 3 In the Expression text field, type 768\*sum(sum(W summand psi(m, n, a, 0.001, k, 0.02), n, 1, 15), m, 1, 15)/(pi<sup>5</sup>).

**4** Locate the **Legends** section. In the table, enter the following settings:

Legends	
Analytic:	\psi=0.001

#### Table Graph 1

- I In the Model Builder window, right-click Load Capacity for Various psi Values and choose Table Graph.
- 2 In the Settings window for Table Graph, locate the Data section.
- 3 From the Table list, choose Table 5.
- 4 From the x-axis data list, choose k.
- 5 From the Plot columns list, choose Manual.
- 6 In the Columns list, select p\*ht^3/(eta\*a^3\*b\*sqvel).
- 7 Locate the Coloring and Style section. Find the Line style subsection. From the Line list, choose None.
- 8 From the Color list, choose Cycle (reset).
- 9 Find the Line markers subsection. From the Marker list, choose Cycle (reset).
- 10 Locate the Legends section. Select the Show legends check box.
- II From the Legends list, choose Manual.
- 12 In the table, enter the following settings:

Legends	
Numeric:	\psi=1

**13** Right-click **Table Graph I** and choose **Duplicate**.

## Table Graph 2

- I In the Model Builder window, click Table Graph 2.
- 2 In the Settings window for Table Graph, locate the Data section.
- 3 From the Table list, choose Table 6.
- 4 Locate the Coloring and Style section. From the Color list, choose Cycle.
- 5 Find the Line markers subsection. From the Marker list, choose Cycle.
- **6** Locate the **Legends** section. In the table, enter the following settings:

Legends	
Numeric:	\psi=0.1

7 Right-click Table Graph 2 and choose Duplicate.

## Table Graph 3

- I In the Model Builder window, click Table Graph 3.
- 2 In the Settings window for Table Graph, locate the Data section.
- **3** From the **Table** list, choose **Table 7**.
- **4** Locate the **Legends** section. In the table, enter the following settings:

# Legends Numeric: \psi=0.01

5 Right-click **Table Graph 3** and choose **Duplicate**.

## Table Graph 4

- I In the Model Builder window, click Table Graph 4.
- 2 In the Settings window for Table Graph, locate the Data section.
- 3 From the Table list, choose Table 8.
- **4** Locate the **Legends** section. In the table, enter the following settings:

# Legends Numeric: \psi=0.001

#### Annotation I

- I In the Model Builder window, right-click Load Capacity for Various psi Values and choose Annotation.
- 2 In the Settings window for Annotation, locate the Data section.
- 3 From the Dataset list, choose Mirror 3D 2.
- 4 From the Parameter value (k,psi,Hbar) list, choose 1: k=8, psi=1, Hbar=0.02.
- 5 Locate the Annotation section. In the Text text field, type a=1 \\ H = 0.02.
- **6** Select the **LaTeX markup** check box.
- 7 Locate the **Position** section. In the **x** text field, type 3.
- **8** In the **y** text field, type 0.4.
- **9** Locate the Coloring and Style section. Clear the Show point check box.
- 10 Select the Show frame check box.

## Load Capacity for Various psi Values

I In the Model Builder window, click Load Capacity for Various psi Values.

2 In the Load Capacity for Various psi Values toolbar, click Plot.

Load Capacity for Various phi Values

- I In the Home toolbar, click **Add Plot Group** and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Load Capacity for Various phi Values in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose None.
- 4 Locate the Title section. From the Title type list, choose Manual.
- 5 In the Title text area, type Load capacity for various \phi values.
- 6 Locate the Plot Settings section.
- 7 Select the x-axis label check box. In the associated text field, type \phi.
- 8 Select the y-axis label check box. In the associated text field, type W h<sup>3</sup> <sub>t</sub>/(\eta a<sup>3</sup>b v<sub>sq</sub>).
- 9 Locate the Legend section. From the Position list, choose Lower right.

#### Function I

- I In the Load Capacity for Various phi Values toolbar, click More Plots and choose Function.
- 2 In the Settings window for Function, locate the Data section.
- 3 From the Dataset list, choose Grid ID 3.
- 4 Locate the y-Axis Data section. In the Expression text field, type 768\* sum(sum(W\_summand\_phi(m, n, a, 5e-9, 1, H), n, 1, 15), m, 1, 15)/ (pi<sup>5</sup>).
- **5** Locate the **x-Axis Data** section. In the **Expression** text field, type H.
- **6** In the **Lower bound** text field, type 0.02.
- 7 In the **Upper bound** text field, type 0.1.
- 8 Locate the Coloring and Style section. From the Color list, choose Cycle (reset).
- **9** Locate the **Legends** section. Select the **Show legends** check box.
- 10 From the Legends list, choose Manual.
- II In the table, enter the following settings:

Legends	
Analytic:	\phi=5E-9

12 Right-click Function I and choose Duplicate.

#### Function 2

- I In the Model Builder window, click Function 2.
- 2 In the Settings window for Function, locate the y-Axis Data section.
- 3 In the Expression text field, type 768\*sum(sum(W\_summand\_phi(m, n, a, 5e-10, 1, H), n, 1, 15), m, 1, 15)/(pi<sup>5</sup>).
- 4 Locate the Coloring and Style section. From the Color list, choose Cycle.
- **5** Locate the **Legends** section. In the table, enter the following settings:

# Legends Analytic: \phi=5E-10

6 Right-click Function 2 and choose Duplicate.

#### Function 3

- I In the Model Builder window, click Function 3.
- 2 In the Settings window for Function, locate the y-Axis Data section.
- 3 In the Expression text field, type 768\*sum(sum(W summand phi(m, n, a, 5e-11, 1, H), n, 1, 15), m, 1, 15)/( $pi^5$ ).
- **4** Locate the **Legends** section. In the table, enter the following settings:

# Legends Analytic: \phi=5E-11

#### Table Graph 1

- I In the Model Builder window, right-click Load Capacity for Various phi Values and choose Table Graph.
- 2 In the Settings window for Table Graph, locate the Data section.
- 3 From the Table list, choose Table 9.
- 4 From the x-axis data list, choose Hbar.
- 5 From the Plot columns list, choose Manual.
- 6 In the Columns list, select p\*ht^3/(eta\*a^3\*b\*sqvel).
- 7 Locate the Coloring and Style section. Find the Line style subsection. From the Line list, choose None.
- 8 From the Color list, choose Cycle (reset).
- 9 Find the Line markers subsection. From the Marker list, choose Cycle (reset).
- 10 Locate the Legends section. Select the Show legends check box.

II From the Legends list, choose Manual.

**12** In the table, enter the following settings:

#### Legends

Numeric: \phi=5E-9

**13** Right-click **Table Graph I** and choose **Duplicate**.

## Table Graph 2

- I In the Model Builder window, click Table Graph 2.
- 2 In the Settings window for Table Graph, locate the Data section.
- 3 From the Table list, choose Table 10.
- 4 Locate the Coloring and Style section. From the Color list, choose Cycle.
- 5 Find the Line markers subsection. From the Marker list, choose Cycle.
- **6** Locate the **Legends** section. In the table, enter the following settings:

## Legends

Numeric: \phi=5E-10

7 Right-click Table Graph 2 and choose Duplicate.

## Table Graph 3

- I In the Model Builder window, click Table Graph 3.
- 2 In the Settings window for Table Graph, locate the Data section.
- 3 From the Table list, choose Table 11.
- **4** Locate the **Legends** section. In the table, enter the following settings:

## Legends

Numeric: \phi=5E-11

Load Capacity for Various phi Values

- I In the Model Builder window, click Load Capacity for Various phi Values.
- 2 In the Load Capacity for Various phi Values toolbar, click Plot.

Darcy Pressure and Velocity Fields

- I In the Home toolbar, click Add Plot Group and choose 3D Plot Group.
- 2 In the Settings window for 3D Plot Group, type Darcy Pressure and Velocity Fields in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Mirror 3D 2.

- 4 From the Parameter value (k,psi,Hbar) list, choose 18: k=1, psi=0.01, Hbar=0.02.
- **5** Click to expand the **Selection** section. Click to expand the **Title** section. From the Title type list, choose Manual.
- 6 In the Title text area, type Surface: Darcy pressure, Streamline: Darcy velocity fields.
- 7 Locate the Plot Settings section. Clear the Plot dataset edges check box.
- 8 Locate the Color Legend section. From the Position list, choose Right double.

## Surface I

- I Right-click Darcy Pressure and Velocity Fields and choose Surface.
- 2 In the Settings window for Surface, locate the Expression section.
- 3 In the Expression text field, type p\*ht^3/(eta\*a^2\*sqvel).
- 4 Locate the Coloring and Style section. Click Change Color Table.
- 5 In the Color Table dialog box, select Linear>GrayScale in the tree.
- 6 Click OK.
- 7 In the Settings window for Surface, locate the Coloring and Style section.
- 8 From the Color table transformation list, choose Reverse.

#### Selection 1

- I Right-click Surface I and choose Selection.
- 2 In the Settings window for Selection, locate the Selection section.
- 3 From the Selection list, choose Thin-Film Boundary.

#### Streamline 1

- I In the Model Builder window, right-click Darcy Pressure and Velocity Fields and choose Streamline.
- 2 In the Settings window for Streamline, locate the Expression section.
- 3 In the x-component text field, type dl.u\*dl.rho\*a/eta.
- 4 In the y-component text field, type dl.v\*dl.rho\*a/eta.
- 5 In the **z-component** text field, type dl.w\*dl.rho\*a/eta.
- 6 Locate the Streamline Positioning section. From the Positioning list, choose Uniform density.
- 7 In the Separating distance text field, type 0.05.
- 8 Locate the Coloring and Style section. Find the Point style subsection. From the Type list, choose Arrow.

## Color Expression 1

- I Right-click Streamline I and choose Color Expression.
- 2 In the Settings window for Color Expression, locate the Expression section.
- 3 In the Expression text field, type dl.U\*dl.rho\*a/eta.

## Darcy Pressure and Velocity Fields

- I In the Model Builder window, under Results click Darcy Pressure and Velocity Fields.
- 3 Click the **Zoom Extents** button in the **Graphics** toolbar.