Created in COMSOL Multiphysics 6.2

Lossy Circular Waveguide

Introduction

In mode analysis it is usually the primary goal to find a *propagation constant*. This quantity is often, but not always, real valued; if the analysis involves some lossy part, such as a nonzero conductivity or an open boundary, the eigenvalue is complex. In such situations, the real and imaginary parts have separate interpretations:

- The real part is the propagation constant
- The imaginary part is the *attenuation constant*, measuring the damping in space

Model Definition

The mode analysis study for electromagnetic waves solves the eigenvalue problem

$$\nabla \times (\mu^{-1} \nabla \times \mathbf{E}) - \lambda \mathbf{E} = 0$$

where

$$\lambda = k_0^2 \left(\varepsilon_\mathrm{r} - \frac{j\sigma}{\omega} \right)$$

is the eigenvalue. For time-harmonic problems, the electric field for out-of-plane propagation can be written as

$$\mathbf{E}(\mathbf{r}, t) = \mathrm{Re}(\tilde{\mathbf{E}}(\mathbf{r}) e^{j\omega t - \alpha z})$$

where z is the known out-of-plane direction.

The spatial parameter, $\alpha = \delta_z + j\beta = -\lambda$, can have a real part and an imaginary part. The propagation constant is equal to the imaginary part, and the real part, δ_z, represents the damping along the propagation direction.

VARIABLES INFLUENCED BY MODE ANALYSIS

The following table lists the variables that are influenced by the mode analysis in terms of the eigenvalue `lambda`:

NAME	EXPRESSION	CAN BE COMPLEX	DESCRIPTION
`beta`	`imag(-lambda)`	No	Propagation constant
`dampz`	`real(-lambda)`	No	Attenuation constant

NAME	EXPRESSION	CAN BE COMPLEX	DESCRIPTION
dampzdB	20*log10(exp(1))*dampz	No	Attenuation constant per meter, dB
neff	j*lambda/k0	Yes	Effective mode index

This two-dimensional model finds the modes of a circular waveguide with walls made of a nonperfect conductor, which is copper in this case. The losses in the walls lead to attenuation of the propagating wave. The propagation constant β is obtained as the imaginary part of $\alpha = -\lambda$ and the damping δ_z is obtained as the real part. Since the wave in the waveguide is attenuated in the z direction as $e^{-\delta_z z}$, the attenuation in dB scale is calculated using the formula

$$\Delta_{\mathrm{dB}} = 20\delta_z \log e$$

Results and Discussion

The eigenvalue solver returns six eigenvalues. Table 1 shows the six effective mode indices, n_{eff}, closest to 1, where

$$n_{\mathrm{eff}} = j\frac{\lambda}{k_0}$$

and k_0 is the wave number in vacuum. The table also lists the propagation constant and damping in dB/m for each eigenmode.

TABLE 1: EFFECTIVE MODE INDICES, PROPAGATION CONSTANTS, AND ATTENUATION.

Effective mode index	Propagation constant (1/m)	Attenuation constant per meter (dB/m)
$0.9308 - 2.2082\cdot10^{-6}i$	19.5071	$4.0199\cdot10^{-4}$
$0.9733 - 2.1116\cdot10^{-6}i$	20.3992	$3.844\cdot10^{-4}$
$0.9566 - 1.7954\cdot10^{-6}i$	20.0486	$3.2684\cdot10^{-4}$
$0.9566 - 1.7954\cdot10^{-6}i$	20.0486	$3.2684\cdot10^{-4}$
$0.9844 - 9.38\cdot10^{-7}i$	20.6324	$1.7076\cdot10^{-4}$
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The default surface plot shows the norm of the electric field for the effective mode index $0.9308 - 2.208 \cdot 10^{-6} j$. This plot is shown in Figure 1.

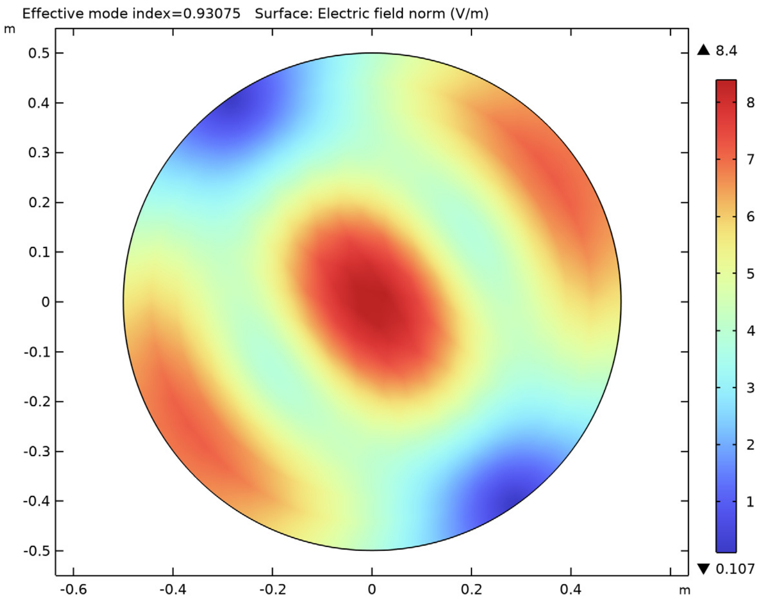

Figure 1: The surface plot visualizes the norm of the electric field for the effective mode index $0.9308 - 2.208 \cdot 10^{-6} j$.

Application Library path: `RF_Module/Transmission_Lines_and_Waveguides/lossy_circular_waveguide`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.

2 In the **Select Physics** tree, select **Radio Frequency>Electromagnetic Waves, Frequency Domain (emw)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces> Mode Analysis**.

6 Click **Done**.

GEOMETRY 1

Circle 1 (c1)

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `0.5`.

4 Click **Build All Objects**.

ADD MATERIAL

1 In the **Home** toolbar, click **Add Material** to open the **Add Material** window.

2 Go to the **Add Material** window.

3 In the tree, select **Built-in>Air**.

4 Click **Add to Component** in the window toolbar.

MATERIALS

Air (mat1)

By default the first material you add apply for all domains.

Next, specify copper as the material on the boundaries.

ADD MATERIAL

1 Go to the **Add Material** window.

2 In the tree, select **Built-in>Copper**.

3 Click **Add to Component** in the window toolbar.

4 In the **Home** toolbar, click **Add Material** to close the **Add Material** window.

MATERIALS

Copper (mat2)

1. In the **Settings** window for **Material**, locate the **Geometric Entity Selection** section.
2. From the **Geometric entity level** list, choose **Boundary**.
3. From the **Selection** list, choose **All boundaries**.

ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN (EMW)

Impedance Boundary Condition 1

1. In the **Model Builder** window, under **Component 1 (comp1)** right-click **Electromagnetic Waves, Frequency Domain (emw)** and choose the boundary condition **Impedance Boundary Condition**.
2. In the **Settings** window for **Impedance Boundary Condition**, locate the **Boundary Selection** section.
3. From the **Selection** list, choose **All boundaries**.

MESH 1

1. In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.
2. In the **Settings** window for **Mesh**, locate the **Physics-Controlled Mesh** section.
3. In the table, clear the **Use** check box for **Electromagnetic Waves, Frequency Domain (emw)**.

Solve for the 6 effective mode indices closest to 1.

STUDY 1

Step 1: Mode Analysis

1. In the **Model Builder** window, under **Study 1** click **Step 1: Mode Analysis**.
2. In the **Settings** window for **Mode Analysis**, locate the **Study Settings** section.
3. Select the **Desired number of modes** check box.
4. In the **Home** toolbar, click = **Compute**.

RESULTS

Electric Field (emw)

The default plot shows the electric field norm for the lowest mode found; compare with Figure 1.

Calculate the propagation constant and the attenuation constant (in dB) for each effective mode index.

Global Evaluation 1

1 In the **Results** toolbar, click 8.5 **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, click **Replace Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (comp1)>Electromagnetic Waves, Frequency Domain>Global>emw.beta - Propagation constant - rad/m**.

3 Click **Evaluate**.

Compare the results with those in the second column of Table 1.

4 Click **Replace Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (comp1)>Electromagnetic Waves, Frequency Domain>Global>emw.dampzdB - Attenuation constant per meter, dB - dB/m**.

5 Click **Evaluate**.

Compare with the third column of Table 1.