Created in COMSOL Multiphysics 6.2

Thin Layer Interfaces

Introduction

This model demonstrates alternative implementations for describing a thin layer, and the impact of the choice on the continuity of the displacement and stress fields. It is shown how a perfect interface can be obtained by asymptotically changing the material parameters.

Model Definition

Figure 1 shows the undeformed geometry composed by two domains forming a square of side L and the contacting surface where the thin layer interface will be placed. The full domain is a square of side 1 m and the interface is built by joining two circular arcs of radius 0.707.

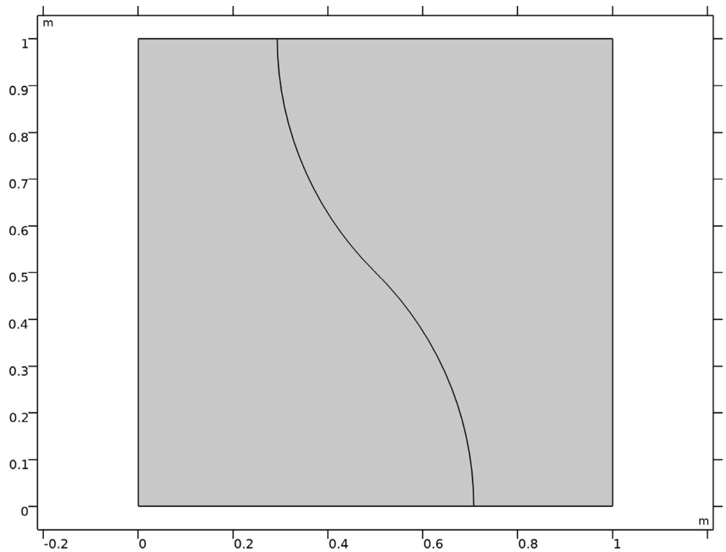

Figure 1: Model geometry.

A nearly incompressible Saint Venant–Kirchhoff hyperelastic material is used for both domains. The thin layer of thickness d << L contains a compressible Saint Venant–Kirchhoff hyperelastic material. The domain material properties are shown in Table 1.

TABLE 1: DOMAIN MATERIAL PROPERTIES.

VARIABLE	VALUE
Bulk modulus	50 N/mm^2
Shear modulus	10 N/mm^2

THIN LAYER APPROXIMATIONS

This study uses three approximations:

- Solid
- Membrane
- Spring

In the solid approximation, a slit is introduced and consequently a discontinuity of the displacement field is allowed. The slit is filled with a 3D thin material whose deformation gradient, F, is approximated as

$$F \approx I + \nabla_t \mathbf{u}_a + \frac{1}{d}\mathbf{u}_e \otimes \mathbf{N} \tag{1}$$

Here, $\mathbf{u}_\mathrm{e}$ is the extension of the layer, $\mathbf{u}_\mathrm{a}$ is the average displacement of the layer mid-plane, and $\mathbf{N}$ is the normal.

In the membrane approximation, no slit is introduced; the continuity of the displacement is assured and only a jump in the stress is permitted. The deformation gradient is approximated as follows:

$$F \approx I - \mathbf{N} \otimes \mathbf{N} + \nabla_t \mathbf{u} + \lambda_n \mathbf{n} \otimes \mathbf{N} \tag{2}$$

The material properties for both the solid and membrane approximations are given in terms of the bulk modulus k_b and the shear modulus μ_b.

If a spring material is used, a slit is introduced as in the solid case and the two sides of the interface are connected by springs. The deformation gradient is approximated as

$$F \approx I + \frac{1}{d}\mathbf{u}_e \otimes \mathbf{N} \tag{3}$$

and the resulting geometric nonlinear spring force $\mathbf{f}_\mathrm{s}$ per unit area is defined as

$$\mathbf{f}_s = -\alpha\left(I + \frac{1}{2d}\mathbf{N} \otimes \mathbf{u}_e\right)\mathbf{u}_e \tag{4}$$

where α is the spring stiffness constant.

BOUNDARY CONDITIONS

A prescribed displacement boundary condition is applied on the lateral faces in the normal direction up to a stretch of 50%. The upper and lower faces are constrained with a roller.

Results and Discussion

In the case of a perfect interface, the displacement and stresses are continuous, as shown in Figure 2.

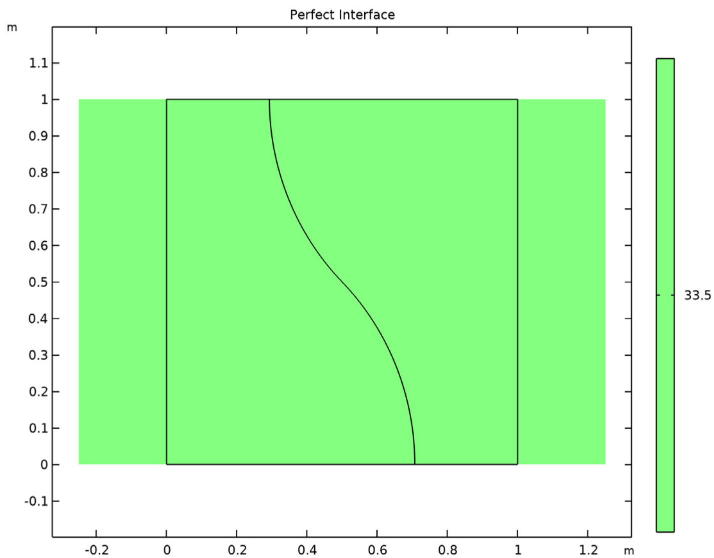

Figure 2: First Piola–Kirchhoff stress (xX component) for the perfect interface case after a 50% stretch. Continuity of displacements and stresses along the interface are enforced.

If a thin layer of material is inserted between the domains, the perfect continuity of the stress and displacements is no longer ensured. For example, when using a solid approximation, both the stress and displacement fields are discontinuous, as shown in Figure 3.

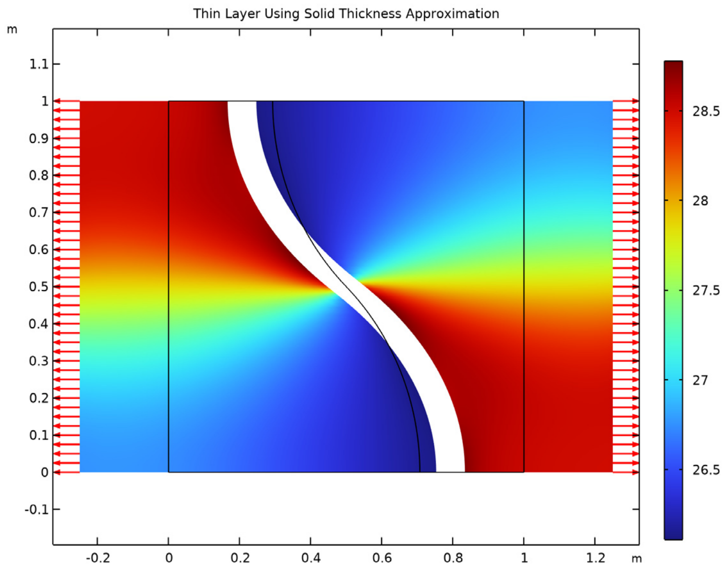

Figure 3: First Piola–Kirchhoff stress distribution with a thin layer using a solid approximation.

The displacement can be enforced to be continuous using the membrane approximation. Moreover, if the ratio between the shear modulus of the thin layer and the bulk material goes to zero, the continuity of the stress is also restored.

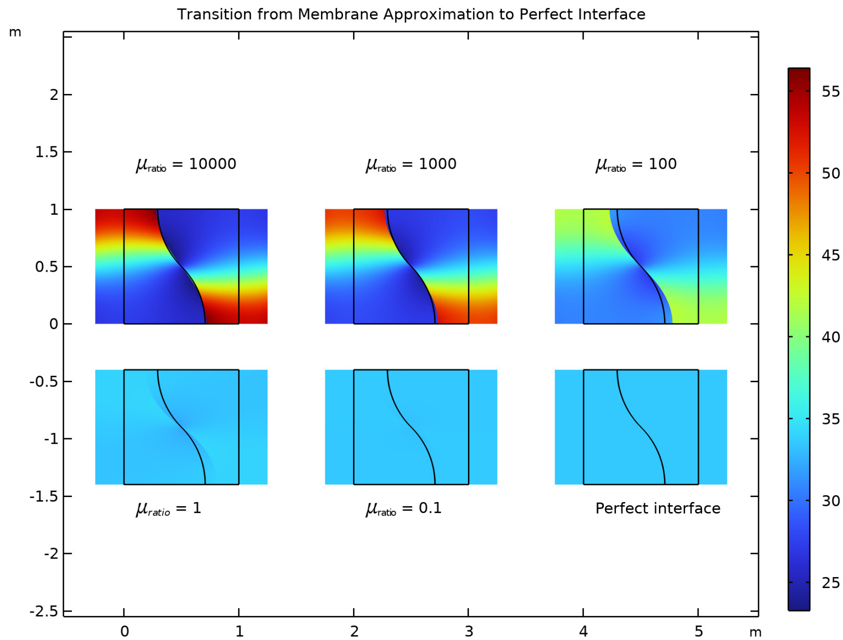

Figure 4: First Piola–Kirchhoff stress distribution with a thin layer using the membrane approximation.

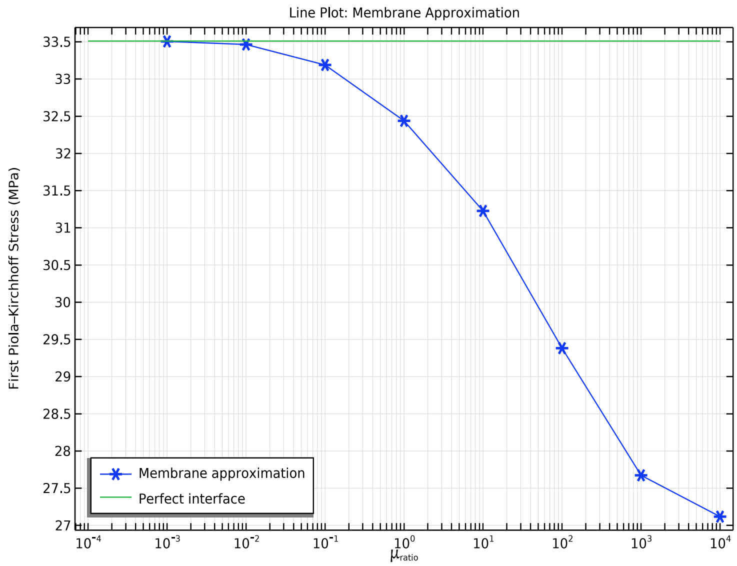

Figure 5: Stress at the center point. Comparison between thin layer (membrane approximation) and perfect interface.

If a spring material is used, the displacements are no more continuous but the stresses are. If the stiffness of the spring increases, the displacement jump disappears resulting in a perfect interface.

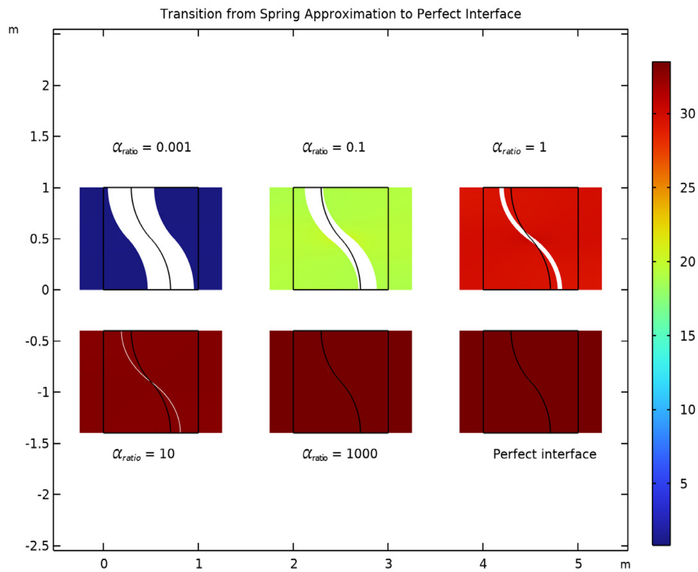

Figure 6: First Piola–Kirchhoff stress distribution with a thin layer using a spring material.

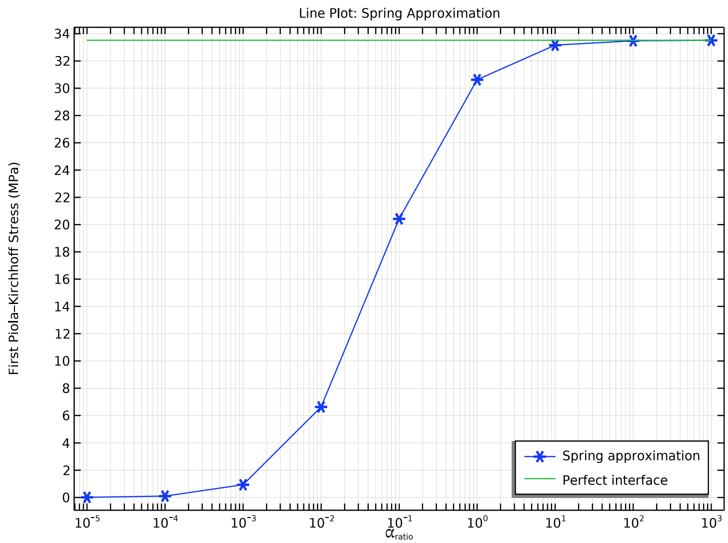

Figure 7: Stress at the center point. Comparison between thin layer (spring material) and perfect interface.

Reference

1. A. Javili, "Variational formulation of generalized interfaces for finite deformation elasticity," *Math. Mech. Solids*, vol. 23, 2018.

Application Library path: `Nonlinear_Structural_Materials_Module/Hyperelasticity/thin_layer_interfaces`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.

2 In the **Select Physics** tree, select **Structural Mechanics>Solid Mechanics (solid)**.

3 Click **Add**.

4 Click **Study**.

5 Click **Done**.

GLOBAL DEFINITIONS

Parameters: Geometry

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, type `Parameters: Geometry` in the **Label** text field.

3 Locate the **Parameters** section. In the table, enter the following settings:

Name	Expression	Value	Description
L	1[m]	1 m	Length
R	1[m]/sqrt(2)	0.70711 m	Radius
d	1[m]	1 m	Out-of-Plane dimension

Parameters: Bulk Material

1 In the **Home** toolbar, click Pi **Parameters** and choose **Add>Parameters**.

2 In the **Settings** window for **Parameters**, type `Parameters: Bulk Material` in the **Label** text field.

3 Locate the **Parameters** section. In the table, enter the following settings:

Name	Expression	Value	Description
RhoBulk	1[kg/m^3]	1 kg/m³	Density
KBulk	50 [N/mm^2]	5E7 N/m²	Bulk Modulus
MuBulk	10 [N/mm^2]	1E7 N/m²	Lame Parameter

Parameters: Thin Layer

1 In the **Home** toolbar, click Pi **Parameters** and choose **Add>Parameters**.

2 In the **Settings** window for **Parameters**, type `Parameters: Thin Layer` in the **Label** text field.

3 Locate the **Parameters** section. In the table, enter the following settings:

Name	Expression	Value	Description
th	L/10	0.1 m	Thin layer thickness
RhoBnd	1[kg/m^3]	1 kg/m³	Density
KRatio	0.2	0.2	Scale factor for bulk modulus
KBnd	KRatio*KBulk	1E7 N/m²	Bulk modulus
MuRatio	100	100	Scale factor for Lame parameter
MuBnd	MuRatio*MuBulk	1E9 N/m²	Lame parameter
AlphaRatio	1	1	Scale factor for stiffness
AlphaBnd	AlphaRatio*(KBulk/th)	5E8 N/m³	Spring stiffness

Parameters: Boundary Conditions

1 In the **Home** toolbar, click Pi **Parameters** and choose **Add>Parameters**.

2 In the **Settings** window for **Parameters**, type `Parameters: Boundary Conditions` in the **Label** text field.

3 Locate the **Parameters** section. In the table, enter the following settings:

Name	Expression	Value	Description
stretch	0	0	Stretch

Parameters: Mesh

1 In the **Home** toolbar, click Pi **Parameters** and choose **Add>Parameters**.

2 In the **Settings** window for **Parameters**, type `Parameters: Mesh` in the **Label** text field.

3 Locate the **Parameters** section. In the table, enter the following settings:

Name	Expression	Value	Description
nel	20	20	Number of elements

GEOMETRY 1

Square 1 (sq1)

1 In the **Model Builder** window, expand the **Component 1 (comp1)>Geometry 1** node.

2 Right-click **Geometry 1** and choose **Square**.

3 In the **Settings** window for **Square**, locate the **Size** section.

4 In the **Side length** text field, type L.

Circular Arc 1 (ca1)

1 In the **Geometry** toolbar, click **More Primitives** and choose **Circular Arc**.

2 In the **Settings** window for **Circular Arc**, locate the **Radius** section.

3 In the **Radius** text field, type R.

4 Locate the **Angles** section. In the **End angle** text field, type 45.

5 Right-click **Circular Arc 1 (ca1)** and choose **Duplicate**.

Circular Arc 2 (ca2)

1 In the **Model Builder** window, click **Circular Arc 2 (ca2)**.

2 In the **Settings** window for **Circular Arc**, locate the **Center** section.

3 In the **x** text field, type L.

4 In the **y** text field, type L.

5 Locate the **Angles** section. In the **Start angle** text field, type 180.

6 In the **End angle** text field, type 225.

Partition Objects 1 (par1)

1 In the **Geometry** toolbar, click **Booleans and Partitions** and choose **Partition Objects**.

2 Select the object **sq1** only.

3 In the **Settings** window for **Partition Objects**, locate the **Partition Objects** section.

4 Click to select the **Activate Selection** toggle button for **Tool objects**.

5 Select the objects **ca1** and **ca2** only.

Form Union (fin)

1 In the **Geometry** toolbar, click **Build All**.

2 Click the **Zoom Extents** button in the **Graphics** toolbar.

Add an average operator for plotting purposes.

DEFINITIONS

Average 1 (aveop1)

1 In the **Definitions** toolbar, click **Nonlocal Couplings** and choose **Average**.

2 Click in the **Graphics** window and then press Ctrl+A to select both domains.

SOLID MECHANICS (SOLID)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Solid Mechanics (solid)**.

2 In the **Settings** window for **Solid Mechanics**, locate the **Thickness** section.

3 In the *d* text field, type `d`.

Hyperelastic Material 1

1 In the **Physics** toolbar, click **Domains** and choose **Hyperelastic Material**.

2 Click in the **Graphics** window and then press Ctrl+A to select both domains.

3 In the **Settings** window for **Hyperelastic Material**, locate the **Hyperelastic Material** section.

4 From the **Material model** list, choose **St Venant–Kirchhoff**.

5 From the **Specify** list, choose **Bulk modulus and shear modulus**.

6 From the **Compressibility** list, choose **Nearly incompressible**.

MATERIALS

Bulk

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type `Bulk` in the **Label** text field.

3 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Bulk modulus	K	`KBulk`	N/m²	Bulk modulus and shear modulus
Shear modulus	G	`MuBulk`	N/m²	Bulk modulus and shear modulus
Density	rho	`RhoBulk`	kg/m³	Basic

SOLID MECHANICS (SOLID)

Roller 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Roller**.

2 Select Boundaries 2–5 only.

Prescribed Displacement 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Prescribed Displacement**.

2 Select Boundaries 1 and 6 only.

3 In the **Settings** window for **Prescribed Displacement**, locate the **Coordinate System Selection** section.

4 From the **Coordinate system** list, choose **Boundary System 1 (sys1)**.

5 Locate the **Prescribed Displacement** section. From the **Displacement in t1 direction** list, choose **Prescribed**.

6 From the **Displacement in n direction** list, choose **Prescribed**.

7 In the u_{0n} text field, type `L*stretch/2`.

MESH 1

Mapped 1

In the **Mesh** toolbar, click **Mapped**.

Distribution 1

1 Right-click **Mapped 1** and choose **Distribution**.

2 Select Boundaries 3, 5, 7, and 8 only.

3 In the **Settings** window for **Distribution**, locate the **Distribution** section.

4 In the **Number of elements** text field, type `nel`.

Distribution 2

1 In the **Model Builder** window, right-click **Mapped 1** and choose **Distribution**.

2 In the **Settings** window for **Distribution**, locate the **Distribution** section.

3 From the **Distribution type** list, choose **Predefined**.

4 Select Boundary 2 only.

5 In the **Number of elements** text field, type `nel`.

6 In the **Element ratio** text field, type `3`.

7 Right-click **Distribution 2** and choose **Duplicate**.

Distribution 3

1 In the **Model Builder** window, click **Distribution 3**.

2 Select Boundary 4 only.

Distribution 4

1 In the **Model Builder** window, right-click **Mapped 1** and choose **Distribution**.

2 Select Boundaries 1 and 6 only.

3 In the **Settings** window for **Distribution**, locate the **Distribution** section.

4 In the **Number of elements** text field, type `nel*2`.

5 Click **Build All**.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies>Stationary**.

4 Click **Add Study** in the window toolbar.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

PERFECT INTERFACE

1 In the **Model Builder** window, click **Study 1**.

2 In the **Settings** window for **Study**, type `Perfect Interface` in the **Label** text field.

3 Locate the **Study Settings** section. Clear the **Generate default plots** check box.

Step 1: Stationary

1 In the **Model Builder** window, under **Perfect Interface** click **Step 1: Stationary**.

2 In the **Settings** window for **Stationary**, click to expand the **Study Extensions** section.

3 Select the **Auxiliary sweep** check box.

4 Click **Add**.

5 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
stretch (Stretch)		

6 Click **Range**.

7 In the **Range** dialog box, type `0.1` in the **Start** text field.

8 In the **Step** text field, type `0.1`.

9 In the **Stop** text field, type `0.5`.

10 Click **Replace**.

11 In the **Home** toolbar, click **Compute**.

RESULTS

Perfect Interface

1 In the **Model Builder** window, expand the **Results** node.

2 Right-click **Results** and choose **2D Plot Group**.

3 In the **Settings** window for **2D Plot Group**, type `Perfect Interface` in the **Label** text field.

4 Click to expand the **Title** section. From the **Title type** list, choose **Manual**.

5 In the **Title** text area, type `Perfect Interface`.

6 Clear the **Parameter indicator** text field.

Surface 1

1 Right-click **Perfect Interface** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `solid.PxX`.

4 From the **Unit** list, choose **MPa**.

Deformation 1

1 Right-click **Surface 1** and choose **Deformation**.

2 In the **Settings** window for **Deformation**, locate the **Scale** section.

3 Select the **Scale factor** check box. In the associated text field, type `1`.

4 In the **Perfect Interface** toolbar, click **Plot**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar.

Add a thin layer between the domains using a solid approximation.

SOLID MECHANICS (SOLID)

Solid Approximation

1 In the **Physics** toolbar, click **Boundaries** and choose **Thin Layer**.

2 Select Boundaries 7 and 8 only.

3 In the **Settings** window for **Thin Layer**, locate the **Boundary Properties** section.

4 In the L_{th} text field, type `th`.

5 In the **Label** text field, type `Solid Approximation`.

Hyperelastic Material 1

1 In the **Physics** toolbar, click **Attributes** and choose **Hyperelastic Material**.

2 Select Boundaries 7 and 8 only.

3 In the **Settings** window for **Hyperelastic Material**, locate the **Hyperelastic Material** section.

4 From the **Material model** list, choose **St Venant–Kirchhoff**.

5 From the **Specify** list, choose **Bulk modulus and shear modulus**.

MATERIALS

Thin Layer

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.
2 In the **Settings** window for **Material**, type `Thin Layer` in the **Label** text field.
3 Locate the **Geometric Entity Selection** section. From the **Geometric entity level** list, choose **Boundary**.
4 Select Boundaries 7 and 8 only.
5 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Bulk modulus	K	KBnd	N/m²	Bulk modulus and shear modulus
Shear modulus	G	MuBnd	N/m²	Bulk modulus and shear modulus
Density	rho	RhoBnd	kg/m³	Basic

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.
2 Go to the **Add Study** window.
3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies>Stationary**.
4 Click **Add Study** in the window toolbar.
5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 2

Step 1: Stationary

1 In the **Settings** window for **Stationary**, locate the **Study Extensions** section.
2 Select the **Auxiliary sweep** check box.
3 Click **Add**.
4 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
stretch (Stretch)	`range(0.1,0.1,0.5)`	

5 Click **Add**.

6 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
MuRatio (Scale factor for Lame parameter)	`range(1,-0.25,0.25) 0.01`	

7 From the **Sweep type** list, choose **All combinations**.

8 From the **Run continuation for** list, choose **Manual**.

9 From the **Continuation parameter** list, choose **stretch**.

10 In the **Model Builder** window, click **Study 2**.

11 In the **Settings** window for **Study**, locate the **Study Settings** section.

12 Clear the **Generate default plots** check box.

13 In the **Label** text field, type `Solid Approximation`.

14 In the **Home** toolbar, click **Compute**.

RESULTS

Solid Approximation

1 In the **Home** toolbar, click **Add Plot Group** and choose **2D Plot Group**.

2 In the **Settings** window for **2D Plot Group**, type `Solid Approximation` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Solid Approximation/ Solution 2 (sol2)**.

4 Locate the **Title** section. From the **Title type** list, choose **Manual**.

5 In the **Title** text area, type `Thin Layer Using Solid Thickness Approximation`.

6 Clear the **Parameter indicator** text field.

Surface 1

1 Right-click **Solid Approximation** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Data** section.

3 From the **Dataset** list, choose **Solid Approximation/Solution 2 (sol2)**.

4 From the **Parameter value (MuRatio)** list, choose **0.25**.

5 Locate the **Expression** section. In the **Expression** text field, type `solid.PxX`.

6 From the **Unit** list, choose **MPa**.

7 In the **Solid Approximation** toolbar, click **Plot**.

Deformation 1

1 Right-click **Surface 1** and choose **Deformation**.

2 In the **Settings** window for **Deformation**, locate the **Scale** section.

3 Select the **Scale factor** check box. In the associated text field, type `1`.

4 In the **Solid Approximation** toolbar, click **Plot**.

Arrow Line 1

In the **Model Builder** window, right-click **Solid Approximation** and choose **Arrow Line**.

Deformation 1

1 In the **Model Builder** window, right-click **Arrow Line 1** and choose **Deformation**.

2 In the **Settings** window for **Deformation**, locate the **Scale** section.

3 Select the **Scale factor** check box. In the associated text field, type `1`.

4 In the **Solid Approximation** toolbar, click **Plot**.

Arrow Line 1

1 In the **Model Builder** window, click **Arrow Line 1**.

2 In the **Settings** window for **Arrow Line**, locate the **Arrow Positioning** section.

3 From the **Placement** list, choose **Mesh nodes**.

4 Click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)>Solid Mechanics>Displacement>u,v - Displacement field**.

Selection 1

1 Right-click **Arrow Line 1** and choose **Selection**.

2 Select Boundaries 1 and 6 only.

3 In the **Solid Approximation** toolbar, click **Plot**.

4 Click the **Zoom Extents** button in the **Graphics** toolbar.

Add a thin layer using a membrane approximation to get an elastic interface model.

SOLID MECHANICS (SOLID)

Membrane Approximation

1 In the **Physics** toolbar, click **Boundaries** and choose **Thin Layer**.

2 In the **Settings** window for **Thin Layer**, type `Membrane Approximation` in the **Label** text field.

3 Select Boundaries 7 and 8 only.

4 Locate the **Boundary Properties** section. In the L_{th} text field, type `th`.

5 Locate the **Thin Layer** section. From the **Approximation** list, choose **Membrane**.

Hyperelastic Material 1

1 In the **Physics** toolbar, click **Attributes** and choose **Hyperelastic Material**.

2 Select Boundaries 7 and 8 only.

3 In the **Settings** window for **Hyperelastic Material**, locate the **Hyperelastic Material** section.

4 From the **Material model** list, choose **St Venant–Kirchhoff**.

5 From the **Specify** list, choose **Bulk modulus and shear modulus**.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies>Stationary**.

4 Click **Add Study** in the window toolbar.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

MEMBRANE APPROXIMATION

1 In the **Model Builder** window, click **Study 3**.

2 In the **Settings** window for **Study**, type `Membrane Approximation` in the **Label** text field.

3 Locate the **Study Settings** section. Clear the **Generate default plots** check box.

Step 1: Stationary

1 In the **Model Builder** window, under **Membrane Approximation** click **Step 1: Stationary**.

2 In the **Settings** window for **Stationary**, locate the **Physics and Variables Selection** section.

3 Select the **Modify model configuration for study step** check box.

4 In the tree, select **Component 1 (comp1)>Solid Mechanics (solid), Controls spatial frame>Solid Approximation**.

5 Click **Disable**.

6 Locate the **Study Extensions** section. Select the **Auxiliary sweep** check box.

7 Click **Add**.

8 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
stretch (Stretch)		

9 Click **Range**.

10 In the **Range** dialog box, type `0.1` in the **Start** text field.

11 In the **Step** text field, type `0.1`.

12 In the **Stop** text field, type `0.5`.

13 Click **Add**.

14 In the **Settings** window for **Stationary**, locate the **Study Extensions** section.

15 Click **Add**.

16 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
MuRatio (Scale factor for Lame parameter)	`10^{range(-3,1,4)}`	

17 From the **Sweep type** list, choose **All combinations**.

18 From the **Run continuation for** list, choose **Manual**.

19 From the **Continuation parameter** list, choose **stretch**.

20 In the **Home** toolbar, click **Compute**.

RESULTS

Membrane Approximation

1 In the **Home** toolbar, click **Add Plot Group** and choose **2D Plot Group**.

2 In the **Settings** window for **2D Plot Group**, type `Membrane Approximation` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Membrane Approximation/ Solution 3 (sol3)**.

4 Locate the **Title** section. From the **Title type** list, choose **Manual**.

5 In the **Title** text area, type `Transition from Membrane Approximation to Perfect Interface`.

6 Clear the **Parameter indicator** text field.

7 Click to expand the **Plot Array** section. Select the **Enable** check box.

8 From the **Array shape** list, choose **Square**.

9 From the **Padding** list, choose **Absolute**.

10 In the **Column padding length** text field, type `L`.

11 In the **Row padding length** text field, type `-2.4*L`.

Surface 1

1. Right-click **Membrane Approximation** and choose **Surface**.
2. In the **Settings** window for **Surface**, locate the **Data** section.
3. From the **Dataset** list, choose **Membrane Approximation/Solution 3 (sol3)**.
4. Locate the **Expression** section. In the **Expression** text field, type `solid.PxX`.
5. From the **Unit** list, choose **MPa**.

Deformation 1

1. Right-click **Surface 1** and choose **Deformation**.
2. In the **Settings** window for **Deformation**, locate the **Scale** section.
3. Select the **Scale factor** check box. In the associated text field, type `1`.
4. In the **Membrane Approximation** toolbar, click **Plot**.

Surface 1

In the **Model Builder** window, right-click **Surface 1** and choose **Duplicate**.

Surface 2

1. In the **Model Builder** window, click **Surface 2**.
2. In the **Settings** window for **Surface**, locate the **Data** section.
3. From the **Parameter value (MuRatio)** list, choose **1000**.
4. Click to expand the **Inherit Style** section. From the **Plot** list, choose **Surface 1**.
5. Right-click **Surface 2** and choose **Duplicate**.

Surface 3

1. In the **Model Builder** window, click **Surface 3**.
2. In the **Settings** window for **Surface**, locate the **Data** section.
3. From the **Parameter value (MuRatio)** list, choose **100**.
4. Right-click **Surface 3** and choose **Duplicate**.

Surface 4

1. In the **Model Builder** window, click **Surface 4**.
2. In the **Settings** window for **Surface**, locate the **Data** section.
3. From the **Parameter value (MuRatio)** list, choose **1**.
4. Right-click **Surface 4** and choose **Duplicate**.

Surface 5

1. In the **Model Builder** window, click **Surface 5**.

2 In the **Settings** window for **Surface**, locate the **Data** section.

3 From the **Parameter value (MuRatio)** list, choose **0.1**.

4 Right-click **Surface 5** and choose **Duplicate**.

Surface 6

1 In the **Model Builder** window, click **Surface 6**.

2 In the **Settings** window for **Surface**, locate the **Data** section.

3 From the **Dataset** list, choose **Perfect Interface/Solution 1 (sol1)**.

Membrane Approximation

In the **Model Builder** window, click **Membrane Approximation**.

Table Annotation 1

1 In the **Membrane Approximation** toolbar, click **More Plots** and choose **Table Annotation**.

2 In the **Settings** window for **Table Annotation**, locate the **Data** section.

3 From the **Source** list, choose **Local table**.

4 In the table, enter the following settings:

x-coordinate	y-coordinate	Annotation
0	3/2*L	μ_ratio = 10000
2*L	3/2*L	μ_ratio = 1000
4*L	3/2*L	μ_ratio = 100
0	-3/2*L	μ_{ratio} = 1
2*L	-3/2*L	μ_ratio = 0.1
4*L	-3/2*L	Perfect interface

5 Locate the **Coloring and Style** section. Clear the **Show point** check box.

6 Locate the **Data** section. Select the **LaTeX markup** check box.

7 In the **Membrane Approximation** toolbar, click **Plot**.

Line Plot: Membrane Approximation

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Line Plot: Membrane Approximation` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Membrane Approximation/ Solution 3 (sol3)**.

4 Locate the **Axis** section. Select the **x-axis log scale** check box.

5 Locate the **Plot Settings** section.

6 Select the **x-axis label** check box. In the associated text field, type \mu_{ratio}.

7 Select the **y-axis label** check box. In the associated text field, type First Piola-Kirchhoff Stress (MPa).

8 Click to expand the **Title** section. From the **Title type** list, choose **Label**.

9 Locate the **Legend** section. From the **Position** list, choose **Lower left**.

Point Graph 1

1 Right-click **Line Plot: Membrane Approximation** and choose **Point Graph**.

2 Select Point 4 only.

3 In the **Settings** window for **Point Graph**, locate the **y-Axis Data** section.

4 In the **Expression** text field, type mean(side(1,solid.PxX)).

5 From the **Unit** list, choose **MPa**.

6 Locate the **x-Axis Data** section. From the **Axis source data** list, choose **MuRatio**.

7 Locate the **Data** section. From the **Dataset** list, choose **Membrane Approximation/Solution 3 (sol3)**.

8 From the **Parameter selection (stretch)** list, choose **Last**.

9 Click to expand the **Legends** section. Select the **Show legends** check box.

10 From the **Legends** list, choose **Manual**.

11 In the table, enter the following settings:

Legends
Membrane approximation

12 Click to expand the **Coloring and Style** section. Find the **Line markers** subsection. From the **Marker** list, choose **Cycle**.

13 In the **Line Plot: Membrane Approximation** toolbar, click **Plot**.

Line Segments 1

1 In the **Model Builder** window, right-click **Line Plot: Membrane Approximation** and choose **Line Segments**.

2 In the **Settings** window for **Line Segments**, locate the **Data** section.

3 From the **Dataset** list, choose **Perfect Interface/Solution 1 (sol1)**.

4 From the **Parameter selection (stretch)** list, choose **Last**.

5 Locate the **x-Coordinates** section. In the table, enter the following settings:

Expression	Unit	Description
`0`	`1`	
`1e-4`	`1`	
`1e4`	`1`	

6 Locate the **y-Coordinates** section. In the table, enter the following settings:

Expression	Unit	Description
`aveop1(solid.PxX)`	`MPa`	`Average 1`
`aveop1(solid.PxX)`	`MPa`	`Average 1`
`aveop1(solid.PxX)`	`MPa`	`Average 1`

7 Click to expand the **Legends** section. Select the **Show legends** check box.

8 From the **Legends** list, choose **Manual**.

9 In the table, enter the following settings:

Legends
`Perfect interface`

10 Click the **Zoom Extents** button in the **Graphics** toolbar.

Add a thin layer using a spring approximation to mimic a cohesive interface model.

SOLID MECHANICS (SOLID)

Spring Approximation

1 In the **Physics** toolbar, click **Boundaries** and choose **Thin Layer**.

2 Select Boundaries 7 and 8 only.

3 In the **Settings** window for **Thin Layer**, locate the **Boundary Properties** section.

4 In the L_{th} text field, type `th`.

5 Locate the **Thin Layer** section. From the **Approximation** list, choose **Spring**.

6 In the **Label** text field, type `Spring Approximation`.

Spring Material 1

1 In the **Physics** toolbar, click **Attributes** and choose **Spring Material**.

2 In the **Settings** window for **Spring Material**, locate the **Spring** section.

3 In the $\mathbf{k}_{\mathrm{A}}$ text field, type `AlphaBnd`.

4 In the ρ_V text field, type `RhoBnd`.

5 Select Boundaries 7 and 8 only.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies>Stationary**.

4 Click **Add Study** in the window toolbar.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

SPRING APPROXIMATION

1 In the **Model Builder** window, click **Study 4**.

2 In the **Settings** window for **Study**, type `Spring Approximation` in the **Label** text field.

3 Locate the **Study Settings** section. Clear the **Generate default plots** check box.

Step 1: Stationary

1 In the **Model Builder** window, under **Spring Approximation** click **Step 1: Stationary**.

2 In the **Settings** window for **Stationary**, locate the **Physics and Variables Selection** section.

3 Select the **Modify model configuration for study step** check box.

4 In the tree, select **Component 1 (comp1)>Solid Mechanics (solid), Controls spatial frame>Solid Approximation** and **Component 1 (comp1)>Solid Mechanics (solid), Controls spatial frame>Membrane Approximation**.

5 Click **Disable**.

6 Locate the **Study Extensions** section. Select the **Auxiliary sweep** check box.

7 Click **Add**.

8 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
stretch (Stretch)	`range(0.1,0.1,0.5)`	

9 Click **Add**.

10 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
AlphaRatio (Scale factor for stiffness)	`10^{range(-5,1,3)}`	

11 From the **Sweep type** list, choose **All combinations**.

12 From the **Run continuation for** list, choose **Manual**.

13 From the **Continuation parameter** list, choose **stretch**.

14 In the **Home** toolbar, click **Compute**.

RESULTS

Spring Approximation

1 In the **Home** toolbar, click **Add Plot Group** and choose **2D Plot Group**.

2 In the **Settings** window for **2D Plot Group**, type `Spring Approximation` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Spring Approximation/ Solution 4 (sol4)**.

4 Locate the **Title** section. From the **Title type** list, choose **Manual**.

5 In the **Title** text area, type `Transition from Spring Approximation to Perfect Interface`.

6 Clear the **Parameter indicator** text field.

7 Locate the **Plot Array** section. Select the **Enable** check box.

8 From the **Array shape** list, choose **Square**.

9 From the **Padding** list, choose **Absolute**.

10 In the **Column padding length** text field, type `L`.

11 In the **Row padding length** text field, type `-2.4*L`.

Surface 1

1 Right-click **Spring Approximation** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Data** section.

3 From the **Dataset** list, choose **Spring Approximation/Solution 4 (sol4)**.

4 From the **Parameter value (AlphaRatio)** list, choose **0.001**.

5 Locate the **Expression** section. In the **Expression** text field, type `solid.PxX`.

6 From the **Unit** list, choose **MPa**.

Deformation 1

1 Right-click **Surface 1** and choose **Deformation**.

2 In the **Settings** window for **Deformation**, locate the **Scale** section.

3 Select the **Scale factor** check box. In the associated text field, type `1`.

4 In the **Spring Approximation** toolbar, click **Plot**.

Surface 1

In the **Model Builder** window, right-click **Surface 1** and choose **Duplicate**.

Surface 2

1 In the **Model Builder** window, click **Surface 2**.

2 In the **Settings** window for **Surface**, locate the **Data** section.

3 From the **Parameter value (AlphaRatio)** list, choose **0.1**.

4 Locate the **Inherit Style** section. From the **Plot** list, choose **Surface 1**.

5 Right-click **Surface 2** and choose **Duplicate**.

Surface 3

1 In the **Model Builder** window, click **Surface 3**.

2 In the **Settings** window for **Surface**, locate the **Data** section.

3 From the **Parameter value (AlphaRatio)** list, choose **1**.

4 Right-click **Surface 3** and choose **Duplicate**.

Surface 4

1 In the **Model Builder** window, click **Surface 4**.

2 In the **Settings** window for **Surface**, locate the **Data** section.

3 From the **Parameter value (AlphaRatio)** list, choose **10**.

4 Right-click **Surface 4** and choose **Duplicate**.

Surface 5

1 In the **Model Builder** window, click **Surface 5**.

2 In the **Settings** window for **Surface**, locate the **Data** section.

3 From the **Parameter value (AlphaRatio)** list, choose **1000**.

4 Right-click **Surface 5** and choose **Duplicate**.

Surface 6

1 In the **Model Builder** window, click **Surface 6**.

2 In the **Settings** window for **Surface**, locate the **Data** section.

3 From the **Dataset** list, choose **Perfect Interface/Solution 1 (sol1)**.

Spring Approximation

In the **Model Builder** window, click **Spring Approximation**.

Table Annotation 1

1. In the **Spring Approximation** toolbar, click **More Plots** and choose **Table Annotation**.
2. In the **Settings** window for **Table Annotation**, locate the **Data** section.
3. From the **Source** list, choose **Local table**.
4. In the table, enter the following settings:

x-coordinate	y-coordinate	Annotation
0	3/2*L	α_ratio = 0.001
2*L	3/2*L	α_ratio = 0.1
4*L	3/2*L	α_{ratio} = 1
0	-3/2*L	α_{ratio} = 10
2*L	-3/2*L	α_ratio = 1000
4*L	-3/2*L	Perfect interface

5. Locate the **Coloring and Style** section. Clear the **Show point** check box.
6. Locate the **Data** section. Select the **LaTeX markup** check box.
7. In the **Spring Approximation** toolbar, click **Plot**.
8. Click the **Zoom Extents** button in the **Graphics** toolbar.

Line Plot: Membrane Approximation

In the **Model Builder** window, under **Results** right-click **Line Plot: Membrane Approximation** and choose **Duplicate**.

Line Plot: Spring Approximation

1. In the **Model Builder** window, click **Line Plot: Membrane Approximation 1**.
2. In the **Settings** window for **1D Plot Group**, locate the **Data** section.
3. From the **Dataset** list, choose **Spring Approximation/Solution 4 (sol4)**.
4. Locate the **Plot Settings** section. In the **x-axis label** text field, type \alpha_{ratio}.
5. In the **Label** text field, type Line Plot: Spring Approximation.
6. Locate the **Legend** section. From the **Position** list, choose **Lower right**.

Point Graph 1

1. In the **Model Builder** window, expand the **Line Plot: Spring Approximation** node, then click **Point Graph 1**.
2. In the **Settings** window for **Point Graph**, locate the **Data** section.
3. From the **Dataset** list, choose **Spring Approximation/Solution 4 (sol4)**.

4 Locate the **Legends** section. In the table, enter the following settings:

Legends
Spring approximation

5 In the **Line Plot: Spring Approximation** toolbar, click **Plot**.

Line Segments 1

1 In the **Model Builder** window, click **Line Segments 1**.

2 In the **Settings** window for **Line Segments**, locate the **x-Coordinates** section.

3 In the table, enter the following settings:

Expression	Unit	Description
0	1	
1e-5	1	
1e3	1	

4 In the **Line Plot: Spring Approximation** toolbar, click **Plot**.

Disable some features in the studies to be able to rerun them.

PERFECT INTERFACE

Step 1: Stationary

1 In the **Model Builder** window, under **Perfect Interface** click **Step 1: Stationary**.

2 In the **Settings** window for **Stationary**, locate the **Physics and Variables Selection** section.

3 Select the **Modify model configuration for study step** check box.

4 In the tree, select **Component 1 (comp1)>Solid Mechanics (solid), Controls spatial frame>Solid Approximation**, **Component 1 (comp1)>Solid Mechanics (solid), Controls spatial frame>Membrane Approximation**, and **Component 1 (comp1)>Solid Mechanics (solid), Controls spatial frame>Spring Approximation**.

5 Click **Disable**.

SOLID APPROXIMATION

1 In the **Model Builder** window, under **Solid Approximation** click **Step 1: Stationary**.

2 In the **Settings** window for **Stationary**, locate the **Physics and Variables Selection** section.

3 Select the **Modify model configuration for study step** check box.

4 In the tree, select **Component 1 (comp1)>Solid Mechanics (solid), Controls spatial frame>Membrane Approximation** and **Component 1 (comp1)>Solid Mechanics (solid), Controls spatial frame>Spring Approximation**.

5 Click **Disable**.

MEMBRANE APPROXIMATION

1 In the **Model Builder** window, under **Membrane Approximation** click **Step 1: Stationary**.

2 In the **Settings** window for **Stationary**, locate the **Physics and Variables Selection** section.

3 In the tree, select **Component 1 (comp1)>Solid Mechanics (solid), Controls spatial frame>Spring Approximation**.

4 Click **Disable**.