Created in COMSOL Multiphysics 6.2

Variably Saturated Flow

Introduction

This example uses the Richards' Equation interface to assess how well geophysical irrigation sensors detect the true level of fluid saturation in variably saturated soils. Andrew Hinnell, Alex Furman, and Ty Ferre from the Department of Hydrology and Water Resources at the University of Arizona brought the example to us. They originally worked out the problem in COMSOL Multiphysics' PDE interfaces, but this discussion shares their elegant model reformulated in the Richards' Equation interface.

A major challenge when characterizing fluid movement in variably saturated porous media lies primarily in the need to describe how the capacity to transmit and store fluids changes as fluids enter and fill the pore spaces. Experimental data for these properties are difficult to obtain. Moreover, the properties that change value as the soil saturates happen to be equation coefficients, which makes the mathematics notoriously nonlinear. The Richards' Equation interface provides interfaces that automate the van Genuchten (Ref. 1) as well as the Brooks and Corey (Ref. 2) relationships for fluid retention and material properties that vary with the solution.

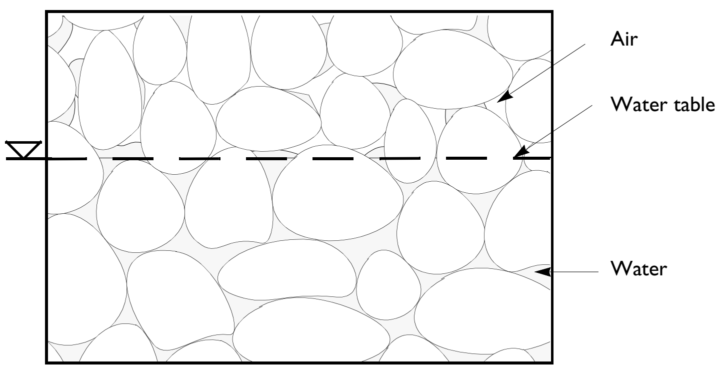


Figure 1: A variably saturated porous medium.

This example uses the model of Hinnell, Furman, and Ferre to characterize how the distribution of water changes around three impermeable sensors inserted into two different blocks of uniform soil partially saturated with water. The question for the model to address is this: Does the saturation localized around the sensor give a valid picture of the saturation within the total block?

This example demonstrates how to use the Richards' Equation interface including the van Genuchten (Ref. 1) as well as the Brooks and Corey (Ref. 2) retention models.

Model Definition

The problem setup is as follows. Two homogeneous columns of soil, each 2 m-by-2 m on a face, are partially saturated with water. A plot of the hydraulic properties of the first soil (Soil Type 1) fits the van Genuchten retention and permeability formulas. The other soil (Soil Type 2) has material properties that suit the Brooks and Corey formulas. Within each soil column are three impermeable rods, each with a 0.1 m radius. The rods are spaced at 0.5 m increments so they run horizontally down the centerline of each block; see Figure 2. Just after the rods are emplaced, the pressure head is still uniform, but water begins to move vertically downward in steady drainage. Because all vertical slices down a block are identical, you can model a 2D cross section and observe the changes in the flow field for 900 s or 15 minutes.

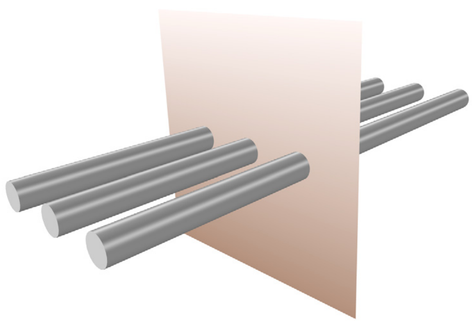


Figure 2: Soil block with three rods. The shaded plane represents a vertical cross section.

GOVERNING EQUATION

Richards' equation describes the unsaturated-saturated flow of water in the soils. In this problem you can only use Richards' equation for the water since the air in the soil is supposed to be at atmospheric pressure. The governing equation for the model is

$$[C + \mathrm{Se}S]\frac{\partial H_p}{\partial t} + \nabla \cdot [-K\nabla(H_p + D)] = 0$$

Pressure head, H_p (m), is the dependent variable. C denotes specific moisture capacity (m^{-1}), Se is the effective saturation, S is a storage coefficient (m^{-1}), t is time, K denotes the hydraulic conductivity (m/s), and D is the coordinate (for example x, y, or z) for the vertical elevation (m). The equation does not show the volumetric fraction of water, θ, which is a constitutive relation that depends on H_p. Nonlinearities appear because C, Se, and K change with H_p and θ.

The first term in the equation explains that fluid storage can change with time during both unsaturated and saturated conditions. When the soil is unsaturated, the pores fill with (or drain) water. After the pore spaces completely fill, there is slight compression of the fluid and the pore space. The specific moisture capacity $C = \partial\theta/\partial H_p$ describes the change in fluid volume fraction θ with pressure head. The storage coefficient addresses storage changes due to compression and expansion of the pore spaces and the water when the soil is fully wet. To model the storage coefficient, this example uses the specific storage option, which sets $S = \rho_\mathrm{f} g(\chi_\mathrm{p} + \theta\chi_\mathrm{f})$. Here, ρ_f is the fluid density ($\mathrm{kg/m^3}$), g is the acceleration of gravity, while χ_p and χ_f are the compressibilities of the solid particles and fluid, respectively ($\mathrm{m{\cdot}s^2/kg}$).

The van Genuchten and the Brooks and Corey formulas that describe the change in C, Se, K, and θ with H_p require data for the saturated and liquid volume fractions, θ_s and θ_r, as well as for other constants α, n, m, and l, which specify a particular type of medium. With the van Genuchten equations that follow, you consider the soil as being saturated when fluid pressure is atmospheric (that is, $H_p = 0$). With the Brooks and Corey approach, an air-entry pressure distinguishes saturated ($H_p > -1/\alpha$) and unsaturated ($H_p < -1/\alpha$) soil. For the detailed formulas, see the section *Retention and Permeability Relationships* in the *Subsurface Flow Module User's Guide*.

To find a unique solution to this problem, you must specify initial and boundary conditions. Initially, the column has uniform pressure head of H_{p0}. No flow conditions are applied to the rings and to the sides. At the surface and bottom of the geometry the pressure head $H_{\mathrm{p}0}$ is specified.

MODEL DATA

The following table gives the data needed to complete the two example problems:

VARIABLE	UNIT	DESCRIPTION	VAN GENUCHTEN	BROOKS & COREY
g	$\mathrm{m/s^2}$	Gravity	9.82	9.82
ρ_f	$\mathrm{kg/m^3}$	Fluid density	1000	1000
χ_p	$\mathrm{m{\cdot}s^2/kg}$	Compressibility solid particles	10^{-8}	10^{-8}
χ_f	$\mathrm{m{\cdot}s^2/kg}$	Compressibility of fluid	$4.4{\cdot}10^{-10}$	$4.4{\cdot}10^{-10}$

VARIABLE	UNIT	DESCRIPTION	VAN GENUCHTEN	BROOKS & COREY
K_s	m/s	Saturated hydraulic conductivity	$8.25 \cdot 10^{-5}$	$5.83 \cdot 10^{-5}$
θ_s		Porosity/void fraction	0.43	0.417
θ_r		Residual saturation	0.045	0.02
α	m^{-1}	alpha parameter	14.5	13.8
n		n parameter	2.68	0.592
m		m parameter	1-1/n	n/a
l		Pore connectivity parameter	0.5	1
H_{p0}	m	Specified pressure	-0.06	-0.2
H_{p0}	m	Initial pressure	-0.06	-0.2

Results and Discussion

Figure 3 and Figure 4 are solutions to the Richards' equation problem of Prof. Ty Ferre, Andrew Hinnell, and Alex Furman from the University of Arizona's Department of Hydrology and Water Resources. Each figure gives results for similar variably saturated flow problem posed for different soil types. Each snapshot shows effective fluid saturation (surface plot), pressure head (contours), and fluid velocities (arrows). The flow field varies around the rods but remains largely uniform over the remainder of each block.

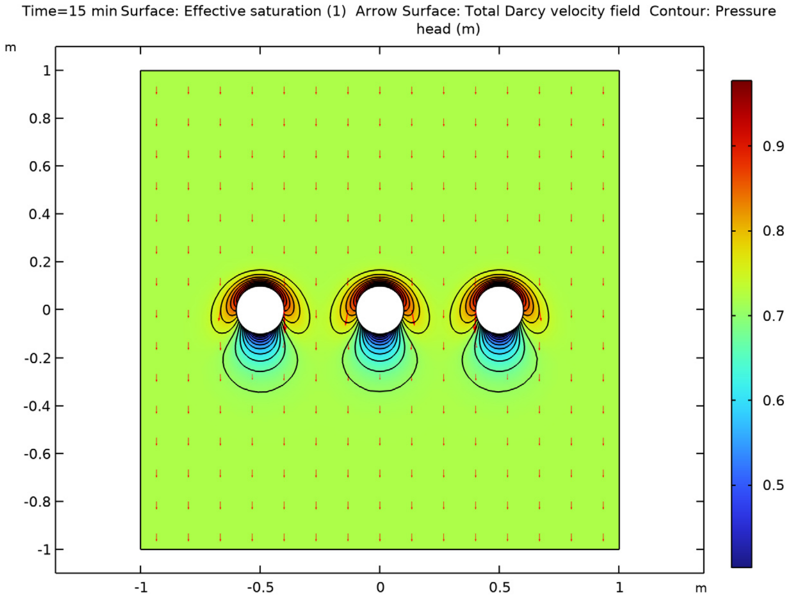


Figure 3: Solution for effective saturation (surface plot), pressure head (contours), and velocity (arrows) at 15 minutes for Soil Type 1 (van Genuchten).

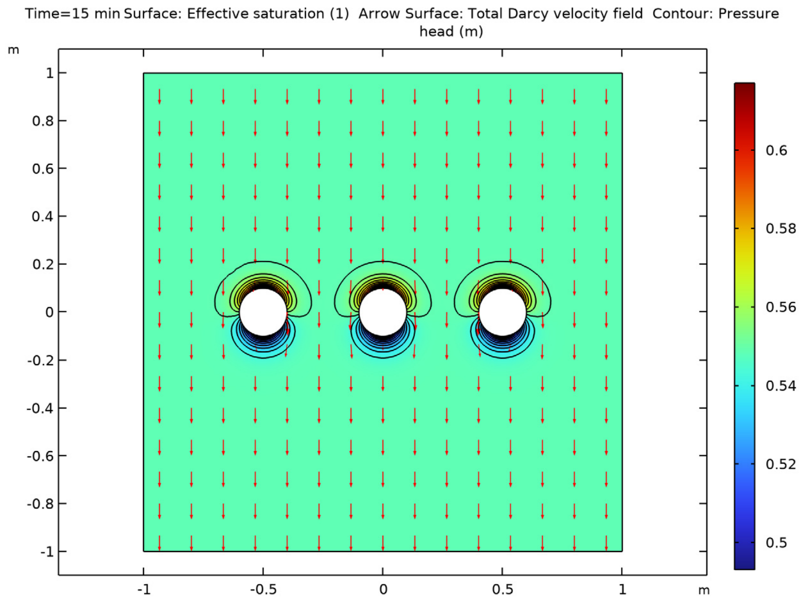


Figure 4: Solution for effective saturation (surface plot), pressure head (contours), and velocity (arrows) at 15 minutes for Soil Type 2 (Brooks and Corey).

Figure 5 shows the effective saturation evolving over time at the rod-soil boundary. The intervals (–180°, 0°) and (0°, 180°) for the angular coordinate along the horizontal axis correspond to the boundary's lower and upper halves, respectively. In the figure, the solid lines denote the solution for Soil Type 1, and the dashed lines correspond to Soil Type 2. The soil is wetter just above the rods than below them.

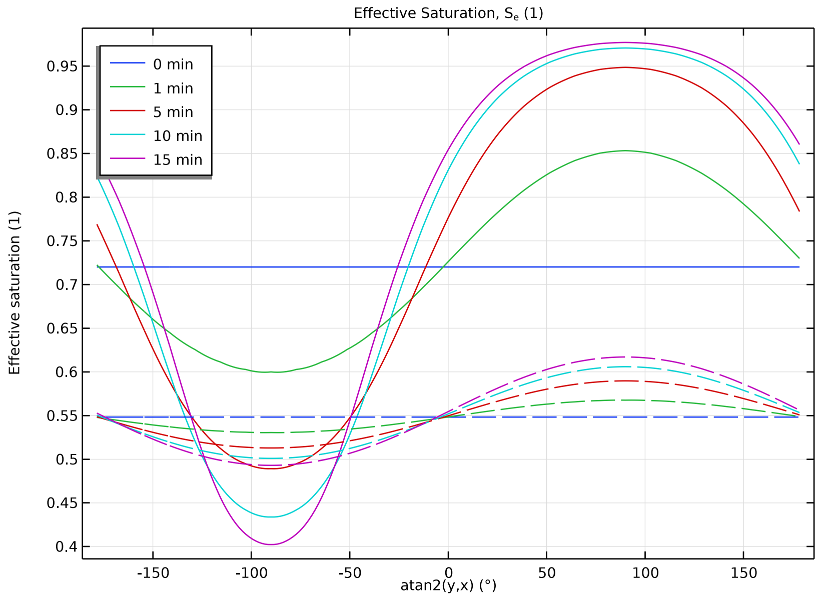


Figure 5: Effective saturation around the center rod in Soil Type 1 (solid lines) and Soil Type 2 (dashed lines).

Figure 6 compares the average fluid saturations at the rod boundary with the average within the two blocks of soil. The range of effective saturation estimates at the rod boundary appears as a scatter plot for different time steps. The solid line is the average of the effective saturation at the rod boundary. The dashed line is the average of effective saturation for the soil. Clearly the average effective saturation at the rods increases with time, but the average for the soil does not change. While the effects shown here are more pronounced in Soil Type 1 than Soil Type 2, the results call to question whether the sensors can accurately assess soil moisture if kept in situ for long times.

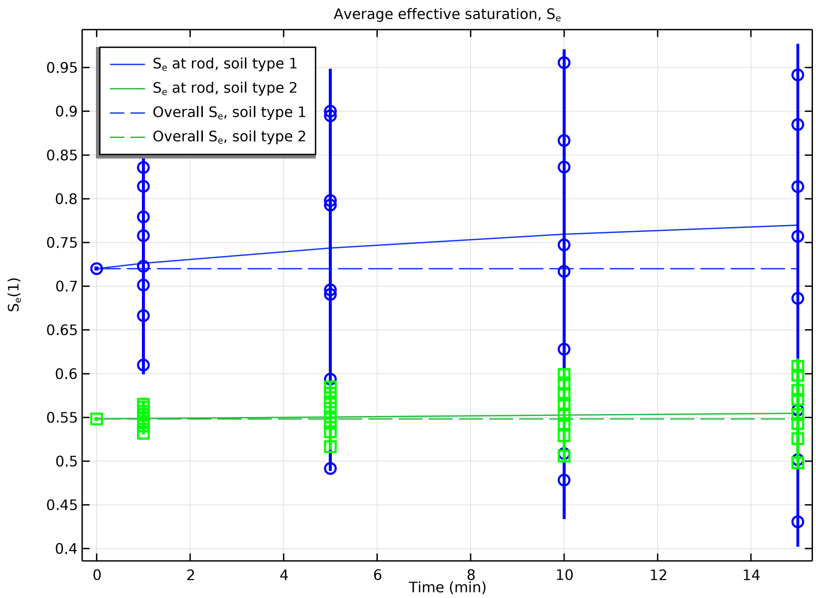


Figure 6: Average effective saturation at sensor circumference and overall soil block for Soil Type 1 (blue) and Soil Type 2 (green). Also shown is the range of effective saturation at the rod circumference for Soil Type 1 (circles) and Soil Type 2 (squares).

The results can also be displayed in 3D using an extrusion dataset. As an example, the effective saturation is plotted in slices within a 3D domain in Figure 7 and Figure 8.

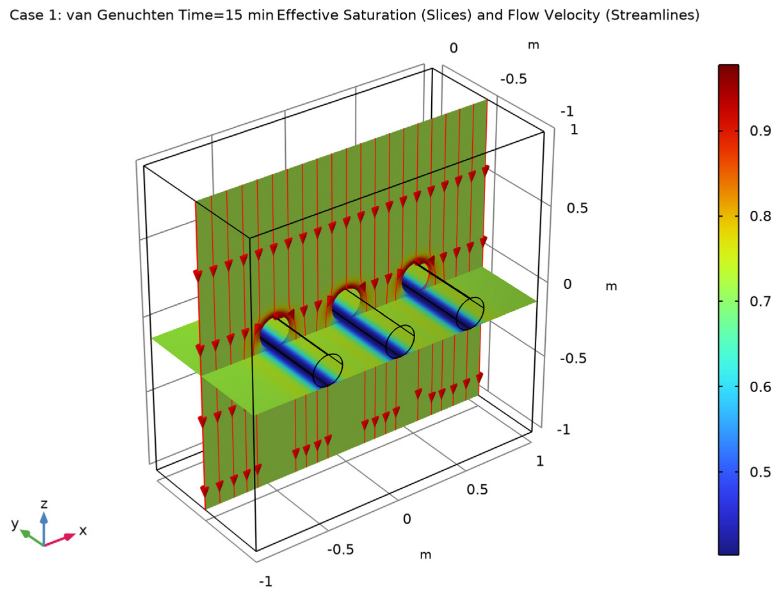


Figure 7: Slice plot of the effective saturation and streamlines of the velocity at 15 minutes for Soil Type 1 (van Genuchten), displayed within a 3D domain.

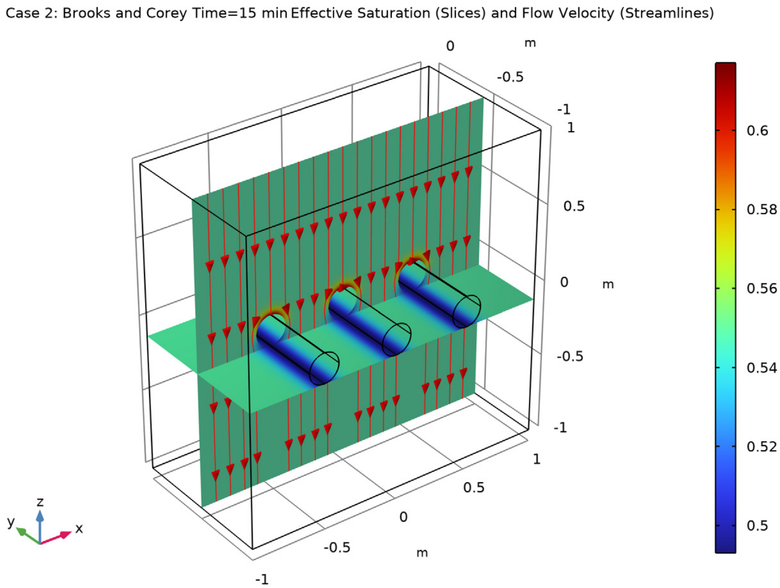


Figure 8: Slice plot of the effective saturation and streamlines of the velocity at 15 minutes for Soil Type 2(Brooks and Corey), displayed within a 3D domain.

References

1. M.Th. van Genuchten, “A Closed-form Equation for Predicting the Hydraulic of Conductivity of Unsaturated Soils,” *Soil Sci. Soc. Am. J.*, vol. 44, pp. 892–898, 1980.

2. R.H. Brooks and A.T. Corey, “Properties of Porous Media Affecting Fluid Flow,” *J. Irrig. Drainage Div., ASCE Proc*, vol. 72 (IR2), pp. 61–88, 1966.

Application Library path: `Subsurface_Flow_Module/Fluid_Flow/variably_saturated_flow`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.

2 In the **Select Physics** tree, select **Fluid Flow>Porous Media and Subsurface Flow>Richards' Equation (dl)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Time Dependent**.

6 Click **Done**.

GEOMETRY 1

Square 1 (sq1)

1 In the **Geometry** toolbar, click **Square**.

2 In the **Settings** window for **Square**, locate the **Size** section.

3 In the **Side length** text field, type `2`.

4 Locate the **Position** section. From the **Base** list, choose **Center**.

Circle 1 (c1)

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `0.1`.

4 Locate the **Position** section. In the **x** text field, type `-0.5`.

Array 1 (arr1)

1 In the **Geometry** toolbar, click **Transforms** and choose **Array**.

2 Select the object **c1** only.

3 In the **Settings** window for **Array**, locate the **Size** section.

4 In the **x size** text field, type `3`.

5 Locate the **Displacement** section. In the **x** text field, type `0.5`.

Difference 1 (dif1)

1 In the **Geometry** toolbar, click **Booleans and Partitions** and choose **Difference**.

2 Select the object **sq1** only, to add it to the **Objects to add** list.

3 In the **Settings** window for **Difference**, locate the **Difference** section.

4 Click to select the **Activate Selection** toggle button for **Objects to subtract**.

5 Select the objects **arr1(1,1)**, **arr1(2,1)**, and **arr1(3,1)** only.

6 Click **Build All Objects**.

DEFINITIONS

Next, create the selection to simplify the evaluation of the results.

Rod

1 In the **Definitions** toolbar, click **Explicit**.

2 In the **Settings** window for **Explicit**, type `Rod` in the **Label** text field.

3 Locate the **Input Entities** section. From the **Geometric entity level** list, choose **Boundary**.

4 Select Boundary 9 only.

5 Select the **Group by continuous tangent** check box.

To investigate the two different retention models, it is useful to work with parameter cases. First, load a set of parameters and create two different parameter sets.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 Click **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file `variably_saturated_flow_parameters.txt`.

5 In the **Home** toolbar, click **Parameter Case**.

6 In the **Settings** window for **Case**, type `Case 1: van Genuchten` in the **Label** text field.

7 In the **Home** toolbar, click **Parameter Case**.

8 In the **Settings** window for **Case**, type `Case 2: Brooks and Corey` in the **Label** text field.

9 Locate the **Parameters** section. In the table, enter the following settings:

Name	**Expression**	**Description**
poro	`0.417`	Porosity
theta_r	`0.02`	Residual liquid volume fraction
K	`5.8333e-5[m/s]`	Hydraulic conductivity
alpha	`13.8[1/m]`	Constitutive relation constant
n	`0.592`	Constitutive relation constant
l	`1`	Constitutive relation constant
Hp0	`-0.2[m]`	Pressure head

Continue with setting up the physics using the parameters.

RICHARDS' EQUATION (DL)

Fluid 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Richards' Equation (dl)>Unsaturated Porous Medium 1** click **Fluid 1**.

2 In the **Settings** window for **Fluid**, locate the **Fluid Properties** section.

3 From the **Fluid type** list, choose **Compressible, linearized**.

4 From the ρ_{ref} list, choose **User defined**. In the associated text field, type `rho`.

5 From the χ_f list, choose **User defined**. In the associated text field, type `chi_f`.

Porous Matrix 1

1 In the **Model Builder** window, click **Porous Matrix 1**.

2 In the **Settings** window for **Porous Matrix**, locate the **Matrix Properties** section.

3 From the ε_p list, choose **User defined**. In the associated text field, type `poro`.

4 In the χ_p text field, type `chi_p`.

5 From the **Permeability model** list, choose **Hydraulic conductivity**.

6 In the K_s text field, type `K`.

7 Locate the **Retention Model** section. In the α text field, type `alpha`.

8 In the l text field, type `l`.

9 In the n text field, type `n`.

10 In the θ_r text field, type `theta_r`.

Unsaturated Porous Medium (van Genuchten)

1 In the **Model Builder** window, under **Component 1 (comp1)>Richards' Equation (dl)** click **Unsaturated Porous Medium 1**.
2 In the **Settings** window for **Unsaturated Porous Medium**, type `Unsaturated Porous Medium (van Genuchten)` in the **Label** text field.
3 Right-click **Unsaturated Porous Medium (van Genuchten)** and choose **Duplicate**.

Unsaturated Porous Medium (Brooks and Corey)

1 In the **Model Builder** window, under **Component 1 (comp1)>Richards' Equation (dl)** click **Unsaturated Porous Medium (van Genuchten) 1**.
2 In the **Settings** window for **Unsaturated Porous Medium**, type `Unsaturated Porous Medium (Brooks and Corey)` in the **Label** text field.

Porous Matrix 1

1 In the **Model Builder** window, expand the **Unsaturated Porous Medium (Brooks and Corey)** node, then click **Porous Matrix 1**.
2 In the **Settings** window for **Porous Matrix**, locate the **Retention Model** section.
3 From the **Retention model** list, choose **Brooks and Corey**.

Initial Values 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Richards' Equation (dl)** click **Initial Values 1**.
2 In the **Settings** window for **Initial Values**, locate the **Initial Values** section.
3 Click the **Pressure head** button.
4 In the H_p text field, type `Hp0`.

Pressure Head 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Pressure Head**.
2 Select Boundaries 2 and 3 only.
3 In the **Settings** window for **Pressure Head**, locate the **Pressure Head** section.
4 In the H_{p0} text field, type `Hp0`.

MESH 1

Free Triangular 1

In the **Mesh** toolbar, click **Free Triangular**.

Size 1

1 Right-click **Free Triangular 1** and choose **Size**.

2 In the **Settings** window for **Size**, locate the **Geometric Entity Selection** section.

3 From the **Geometric entity level** list, choose **Boundary**.

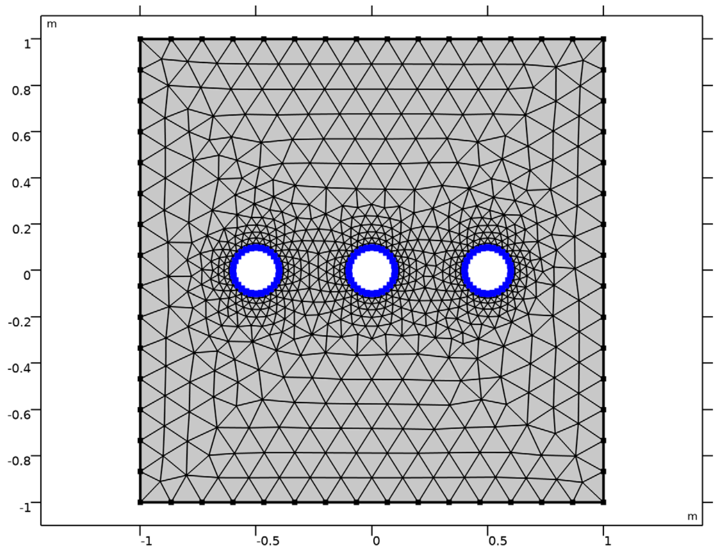
4 Select Boundaries 5–16 only.

5 Locate the **Element Size** section. Click the **Custom** button.

6 Locate the **Element Size Parameters** section.

7 Select the **Maximum element size** check box. In the associated text field, type `0.025`.

8 Click **Build All**.



STUDY 1

1 In the **Model Builder** window, click **Study 1**.

2 In the **Settings** window for **Study**, locate the **Study Settings** section.

3 Clear the **Generate default plots** check box.

Step 1: Time Dependent

1 In the **Model Builder** window, under **Study 1** click **Step 1: Time Dependent**.

2 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.

3 From the **Time unit** list, choose **min**.

4 In the **Output times** text field, type `0 1 5 10 15`.

Disable the **Unsaturated Porous Medium** feature that uses the Brooks and Corey retention model for this study.

5 Locate the **Physics and Variables Selection** section. Select the **Modify model configuration for study step** check box.

6 In the tree, select **Component 1 (comp1)>Richards' Equation (dl)> Unsaturated Porous Medium (Brooks and Corey)**.

7 Click **Disable**.

Add a **Parametric Sweep** to run this study for the set of the van Genuchten parameters.

Parametric Sweep

1 In the **Study** toolbar, click **Parametric Sweep**.

2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

3 From the **Sweep type** list, choose **Parameter switch**.

4 Click **Add**.

5 In the table, enter the following settings:

Switch	Cases	Case numbers
Parameters 1	User defined	1

6 In the **Model Builder** window, click **Study 1**.

7 In the **Settings** window for **Study**, type `Study 1: van Genuchten` in the **Label** text field.

8 In the **Study** toolbar, click **Compute**.

RESULTS

Follow the steps below to reproduce Figure 3 and Figure 4.

Soil Type 1

1 In the **Home** toolbar, click **Add Plot Group** and choose **2D Plot Group**.

2 In the **Settings** window for **2D Plot Group**, type `Soil Type 1` in the **Label** text field.

Surface 1

1 Right-click **Soil Type 1** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `dl.Se`.

Arrow Surface 1

In the **Model Builder** window, right-click **Soil Type 1** and choose **Arrow Surface**.

Contour 1

1 Right-click **Soil Type 1** and choose **Contour**.

2 In the **Settings** window for **Contour**, locate the **Expression** section.

3 In the **Expression** text field, type `dl.Hp`.

4 Locate the **Coloring and Style** section. From the **Coloring** list, choose **Uniform**.

5 From the **Color** list, choose **Black**.

6 Clear the **Color legend** check box.

7 In the **Soil Type 1** toolbar, click **Plot**.

Soil Type 1

Set up a new study for the Brooks and Corey parameter set.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies> Time Dependent**.

4 Click **Add Study** in the window toolbar.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 2

Step 1: Time Dependent

1 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.

2 From the **Time unit** list, choose **min**.

3 In the **Output times** text field, type `0 1 5 10 15`.

Parametric Sweep

1 In the **Study** toolbar, click **Parametric Sweep**.

2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

3 From the **Sweep type** list, choose **Parameter switch**.

4 Click **Add**.

5 In the table, enter the following settings:

Switch	Cases	Case numbers
Parameters 1	User defined	2

6 In the **Model Builder** window, click **Study 2**.

7 In the **Settings** window for **Study**, type `Study 2: Brooks and Corey` in the **Label** text field.

8 Locate the **Study Settings** section. Clear the **Generate default plots** check box.

9 In the **Study** toolbar, click **Compute**.

RESULTS

Soil Type 1

In the **Model Builder** window, under **Results** right-click **Soil Type 1** and choose **Duplicate**.

Soil Type 2

1 In the **Model Builder** window, under **Results** click **Soil Type 1.1**.

2 In the **Settings** window for **2D Plot Group**, type `Soil Type 2` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Study 2: Brooks and Corey/Solution 4 (sol4)**.

To generate Figure 5, continue with the steps below.

Effective Saturation

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Effective Saturation` in the **Label** text field.

Line Graph 1

1 Right-click **Effective Saturation** and choose **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **Selection** section.

3 From the **Selection** list, choose **Rod**.

4 Locate the **y-Axis Data** section. In the **Expression** text field, type `dl.Se`.

5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

6 In the **Expression** text field, type `atan2(y,x)`.

7 From the **Unit** list, choose **°**.

The function `atan2(y,x)` along the circle representing the rod produces unphysical values at one point, because there is 180°=-180°. To avoid this, the data are filtered.

Filter 1

1 Right-click **Line Graph 1** and choose **Filter**.

2 In the **Settings** window for **Filter**, locate the **Element Selection** section.

3 In the **Logical expression for inclusion** text field, type `abs(atan2(y,x))<pi`.

4 In the **Effective Saturation** toolbar, click **Plot**.

Line Graph 1

In the **Model Builder** window, right-click **Line Graph 1** and choose **Duplicate**.

Line Graph 2

1 In the **Model Builder** window, click **Line Graph 2**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 2: Brooks and Corey/Solution 4 (sol4)**.

4 Click to expand the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dashed**.

5 From the **Color** list, choose **Cycle (reset)**.

6 Click to expand the **Legends** section.

Line Graph 1

1 In the **Model Builder** window, click **Line Graph 1**.

2 In the **Settings** window for **Line Graph**, locate the **Legends** section.

3 Select the **Show legends** check box.

Effective Saturation

1 In the **Model Builder** window, click **Effective Saturation**.

2 In the **Settings** window for **1D Plot Group**, click to expand the **Title** section.

3 From the **Title type** list, choose **Manual**.

4 In the **Title** text area, type `Effective Saturation, S<sub>e</sub> (1)`.

5 Locate the **Legend** section. From the **Position** list, choose **Upper left**.

6 In the **Effective Saturation** toolbar, click **Plot**.

To reproduce Figure 6, start by evaluating the average fluid saturation at the rod boundary and the average within the block.

Surface Average 1

1 In the **Results** toolbar, click **More Derived Values** and choose **Average> Surface Average**.

2 Select Domain 1 only.

3 In the **Settings** window for **Surface Average**, locate the **Expressions** section.

4 In the table, enter the following settings:

Expression	Unit	Description
`dl.Se`	1	`Effective saturation`

5 Click **Evaluate**.

Access the solution for the Brooks and Corey retention model an evaluate again. An alternative way is to create a second **Surface Average node**.

6 Locate the **Data** section. From the **Dataset** list, choose **Study 2: Brooks and Corey/ Solution 4 (sol4)**.

7 Click **Evaluate**.

Line Average 2

1 In the **Results** toolbar, click **More Derived Values** and choose **Average>Line Average**.

2 In the **Settings** window for **Line Average**, locate the **Selection** section.

3 From the **Selection** list, choose **Rod**.

4 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
`dl.Se`	1	`Effective saturation`

5 Click next to **Evaluate**, then choose **New Table**.

6 Locate the **Data** section. From the **Dataset** list, choose **Study 2: Brooks and Corey/ Solution 4 (sol4)**.

7 Click **Evaluate**.

TABLE 2

1 Go to the **Table 2** window.

2 Click **Table Graph** in the window toolbar.

RESULTS

Table Graph 1

1 In the **Model Builder** window, under **Results>1D Plot Group 4** click **Table Graph 1**.

2 In the **Settings** window for **Table Graph**, click to expand the **Legends** section.

3 Select the **Show legends** check box.

4 From the **Legends** list, choose **Manual**.

5 In the table, enter the following settings:

Legends
`S<sub>e</sub> at rod, soil type 1`
`S<sub>e</sub> at rod, soil type 2`

6 Right-click **Results>1D Plot Group 4>Table Graph 1** and choose **Duplicate**.

Table Graph 2

1 In the **Model Builder** window, click **Table Graph 2**.

2 In the **Settings** window for **Table Graph**, locate the **Data** section.

3 From the **Table** list, choose **Table 1**.

4 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dashed**.

5 From the **Color** list, choose **Cycle (reset)**.

6 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Overall S<sub>e</sub>, soil type 1`
`Overall S<sub>e</sub>, soil type 2`

Continue by adding the distribution of effective saturation estimates at each output time.

1D Plot Group 4

In the **Model Builder** window, click **1D Plot Group 4**.

Line Graph 1

1 In the **1D Plot Group 4** toolbar, click **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1: van Genuchten/Solution 1 (sol1)**.

4 Locate the **Selection** section. From the **Selection** list, choose **Rod**.

5 Locate the **y-Axis Data** section. In the **Expression** text field, type `dl.Se`.

6 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

7 In the **Expression** text field, type `t`.

8 From the **Unit** list, choose **min**.

9 Locate the **Coloring and Style** section. From the **Color** list, choose **Blue**.

10 From the **Width** list, choose **2**.

11 Find the **Line markers** subsection. From the **Marker** list, choose **Circle**.

12 From the **Positioning** list, choose **Interpolated**.

13 Right-click **Line Graph 1** and choose **Duplicate**.

Line Graph 2

1 In the **Model Builder** window, click **Line Graph 2**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 2: Brooks and Corey/Solution 4 (sol4)**.

4 Locate the **Coloring and Style** section. From the **Color** list, choose **Green**.

5 Find the **Line markers** subsection. From the **Marker** list, choose **Square**.

Average effective saturation

1 In the **Model Builder** window, under **Results** click **1D Plot Group 4**.

2 In the **Settings** window for **1D Plot Group**, type Average effective saturation in the **Label** text field.

3 Locate the **Title** section. From the **Title type** list, choose **Manual**.

4 In the **Title** text area, type Average effective saturation, S_e.

5 Locate the **Plot Settings** section.

6 Select the **y-axis label** check box. In the associated text field, type S_e(1).

7 Locate the **Legend** section. From the **Position** list, choose **Upper left**.

Extrusion 2D 1

Finally, to reproduce the 3D Plots Figure 7 and Figure 8, follow the steps below.

1 In the **Model Builder** window, expand the **Results>Datasets** node.

2 Right-click **Results>Datasets** and choose **More 2D Datasets>Extrusion 2D**.

3 In the **Settings** window for **Extrusion 2D**, locate the **Data** section.

4 From the **Dataset** list, choose **Study 1: van Genuchten/Parametric Solutions 1 (sol2)**.

5 Locate the **Extrusion** section. Find the **Embedding** subsection. From the **Map plane to** list, choose **xz-plane**.

6 Click **Plot**.

7 Right-click **Extrusion 2D 1** and choose **Duplicate**.

Extrusion 2D 2

1 In the **Model Builder** window, click **Extrusion 2D 2**.

2 In the **Settings** window for **Extrusion 2D**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 2: Brooks and Corey/Parametric Solutions 2 (sol5)**.

3D Plot: Soil Type 1

1 In the **Results** toolbar, click **3D Plot Group**.

2 In the **Settings** window for **3D Plot Group**, type `3D Plot: Soil Type 1` in the **Label** text field.

Slice 1

1 Right-click **3D Plot: Soil Type 1** and choose **Slice**.

2 In the **Settings** window for **Slice**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)>Richards' Equation>Retention model>dl.Se - Effective saturation - 1**.

3 Locate the **Plane Data** section. From the **Plane** list, choose **zx-planes**.

4 In the **Planes** text field, type `1`.

5 Click to expand the **Title** section. From the **Title type** list, choose **None**.

6 In the **3D Plot: Soil Type 1** toolbar, click **Plot**.

3D Plot: Soil Type 1

1 In the **Model Builder** window, click **3D Plot: Soil Type 1**.

2 In the **Settings** window for **3D Plot Group**, click to expand the **Title** section.

3 From the **Title type** list, choose **Custom**.

4 Find the **User** subsection. In the **Suffix** text field, type `Effective Saturation (Slices) and Flow Velocity (Streamlines)`.

5 In the **3D Plot: Soil Type 1** toolbar, click **Plot**.

Slice 1

In the **Model Builder** window, right-click **Slice 1** and choose **Duplicate**.

Slice 2

1 In the **Model Builder** window, click **Slice 2**.

2 In the **Settings** window for **Slice**, locate the **Plane Data** section.

3 From the **Plane** list, choose **xy-planes**.

4 In the **Planes** text field, type `1`.

5 Select the **Interactive** check box.

Now use the slider to move the slice to match Figure 7

6 Click to expand the **Inherit Style** section. From the **Plot** list, choose **Slice 1**.

7 In the **3D Plot: Soil Type 1** toolbar, click **Plot**.

Streamline 1

1 In the **Model Builder** window, right-click **3D Plot: Soil Type 1** and choose **Streamline**.

2 In the **Settings** window for **Streamline**, locate the **Expression** section.

3 In the **y-component** text field, type `0`.

4 In the **z-component** text field, type `dl.v`. This has to be done to make sure that the vertical velocity is displayed correctly.

5 Click to expand the **Title** section. From the **Title type** list, choose **None**.

6 Locate the **Streamline Positioning** section. From the **Entry method** list, choose **Coordinates**.

7 In the **x** text field, type `range(-1,0.1,1)`.

8 In the **y** text field, type `-0.5`.

9 In the **z** text field, type `1`.

10 Locate the **Coloring and Style** section. Find the **Point style** subsection. From the **Type** list, choose **Arrow**.

11 In the **3D Plot: Soil Type 1** toolbar, click **Plot**.

12 Click the **Go to Default View** button in the **Graphics** toolbar.

13 Click the **Zoom Extents** button in the **Graphics** toolbar.

3D Plot: Soil Type 1

Right-click **3D Plot: Soil Type 1** and choose **Duplicate**.

3D Plot: Soil Type 2

1 In the **Model Builder** window, expand the **Results>3D Plot: Soil Type 1.1** node, then click **3D Plot: Soil Type 1.1**.

2 In the **Settings** window for **3D Plot Group**, type `3D Plot: Soil Type 2` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Extrusion 2D 2**.

4 In the **3D Plot: Soil Type 2** toolbar, click **Plot**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar.