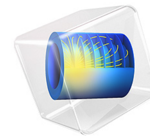
Created in COMSOL Multiphysics 6.2



Axisymmetric Cavity Resonator

Introduction

In this example, the resonant frequencies and fields of an axisymmetric cavity are obtained using the 2D axisymmetric formulation that is available with either the RF Module or the Wave Optics Module. The cross section of the cavity is rectangular and the walls are perfect electric conductors (PEC). For this geometry, applying separation of variables results in transcendental equations for the eigenfrequencies, and so this example serves as a benchmark for the 2D axisymmetric formulation. The resonant frequencies obtained with COMSOL Multiphysics agree with the solutions of the transcendental equations to 1 part in 10^6, which is the default tolerance of the eigenvalue solver. The model also demonstrates that the solutions obtained using the 2D axisymmetric formulation can be rotated rotate about the z-axis to obtain a 3D solution.

Model Definition

The cavity is axisymmetric with a rectangular cross section as shown in Figure 1. The cavity walls are PEC and the region of interest is the interior of the cavity, which is vacuum.

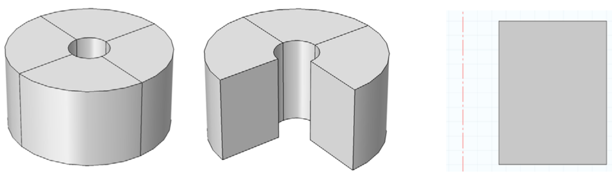


Figure 1: Geometry of the resonant cavity. From left to right: full view, 3/4 cut-away view, and cross section in the rz-plane.

The problem is solved in the frequency domain. The assumed time dependence is $e^{j\omega t}$, where ω is the angular frequency and is related to the frequency f by $\omega = 2\pi f$. The time-harmonic form of the curl-curl equation is a homogeneous eigenvalue equation in the electric field $\mathbf{E}$ and the unknown eigenvalue ω^2, shown below.

$$\nabla\times\frac{1}{\mu}(\nabla\times\mathbf{E})-\omega^2\varepsilon\mathbf{E} = 0 \tag{1}$$

Since the material properties and the geometry are axisymmetric, the model can be solved with the 2D axisymmetric formulation. The computational domain is the rz-plane and the dependent variables are the cylindrical components of the electric field $E_r(r, z)$, $E_\phi(r, z)$, and $E_z(r, z)$, which are, in general, complex.

The angular dependence of the field is assumed to be of the form $e^{-jm\phi}$, where m is an integer called the azimuthal mode number. The azimuthal mode number is specified by the user. The eigenfrequencies for the resonant cavity will be solved for $m = 0$, 1, and 2 using a parametric sweep, which is similar to a `for` loop.

The combined temporal and angular dependence of the field components is $e^{j(\omega t - m\phi)}$. The physical quantities are obtained from the dependent variables using the expressions below, where $\mathrm{Re}\{\ldots\}$ indicates the real part of the complex quantity.

$$E_r(r, \phi, z, t) = \mathrm{Re}\{E_r(r, z)e^{j(\omega t - m\phi)}\} \tag{2}$$

$$E_\phi(r, \phi, z, t) = \mathrm{Re}\{E_\phi(r, z)e^{j(\omega t - m\phi)}\} \tag{3}$$

$$E_z(r, \phi, z, t) = \mathrm{Re}\{E_z(r, z)e^{j(\omega t - m\phi)}\} \tag{4}$$

Suppose that $E_z(r, z)$ is purely real, in which case the preceding equation reduces to

$$E_z(r, \phi, z, t) = E_z(r, z)\cos(\omega t - m\phi) \quad . \tag{5}$$

For $m > 0$, the field rotates in the $+\phi$ direction and for $m < 0$, the field rotates in the $-\phi$ direction. This is also true if $E_z(r, z)$ is complex and it also holds for the other field components. This is shown explicitly in the animation at the end of the example.

Results and Discussion

The default plot is the norm of the electric field. This plot can be used as a diagnostic tool to determine whether a particular mode is physical or spurious. The lowest-frequency physical modes have spatial variation that is comparable to the size of the cavity, while the spurious modes have a spatial variation that is comparable to the mesh size. The norm of the electric field is plotted for four of the lowest-frequency (physical) modes below in Figure 2 to Figure 5.

An arrow plot of the electric field for the $m = 1$ mode at 2.122059 GHz is shown in Figure 6. This plot is animated at the end of the example.

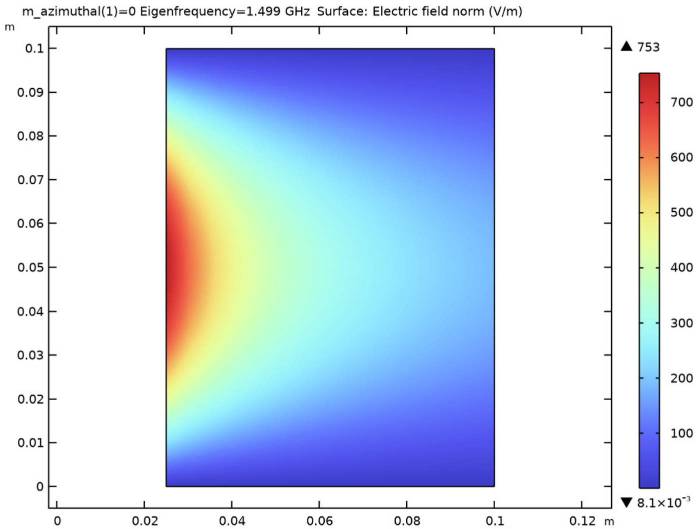


Figure 2: Norm of the electric field for the m=0 mode at 1.499 GHz.

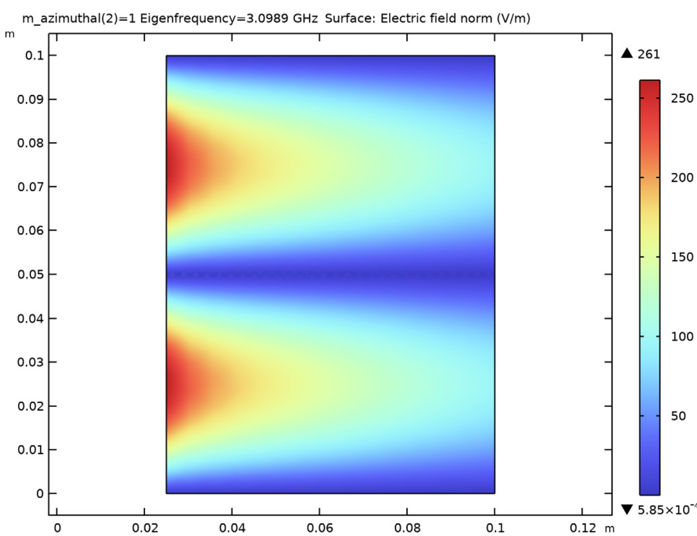


Figure 3: Norm of the electric field for the m=1 mode at 1.692 GHz.

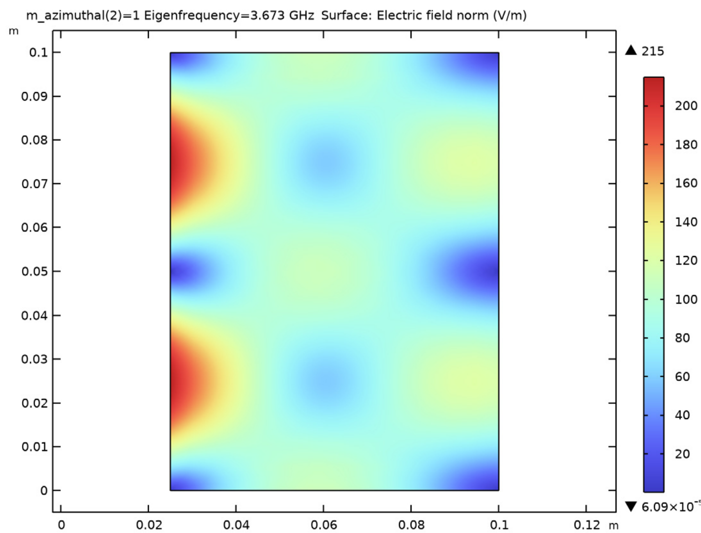


Figure 4: Norm of the electric field for the m=1 mode at 2.122 GHz.

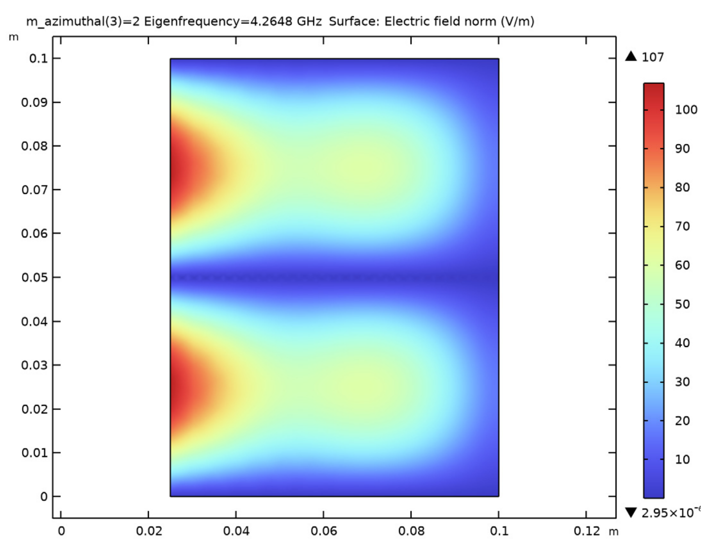


Figure 5: Norm of the electric field for the m=2 mode at 2.076 GHz.

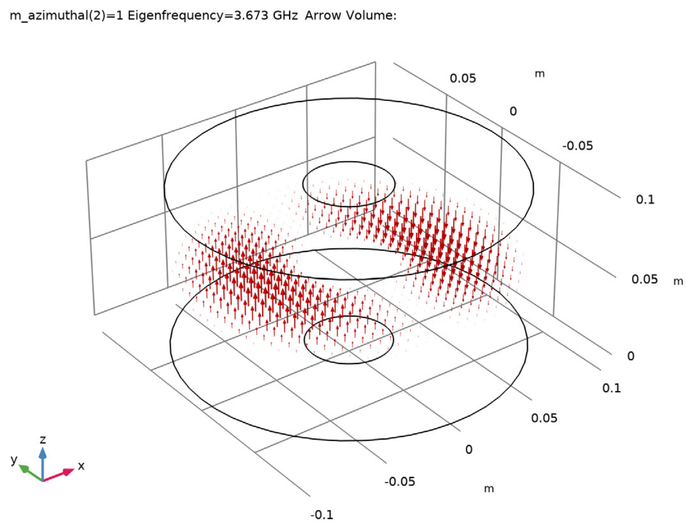


Figure 6: An arrow plot of the electric field for the m=1 mode at 2.122 GHz.

An analytical solution to this problem can be obtained using separation of variables. Details of the derivation can be found in Ref. 1. There is a transcendental equation involving Bessel functions for the eigenvalue $k^2 = \omega^2/c^2$. The values for 8 modes are contained in the left column of the table below. The right column contains the values obtained using COMSOL Multiphysics. The results agree to 1 part in 106, which is the default tolerance of the eigenvalue solver.

TABLE 1: EIGENVALUE COMPARISON OF ANALYTICAL AND COMSOL MULTIPHYSICS SOLUTIONS.

TRANSCENDENTAL EQUATION	**COMSOL MULTIPHYSICS**
986.96	986.96
1257.40	1257.42
1679.11	1679.10
1892.55	1892.57
1978.03	1978.03
2666.07	2666.07
2830.13	2830.10

References

1. C.M. Pinciuc, *Basis Functions With Divergence Constraints For The Finite Element Method*, PhD thesis, Dept. Electrical and Computer Eng., Univ. of Toronto, 2012, Appendix C, pp. 174–181.

2. J.D. Jackson, *Classical Electrodynamics*, 3rd Edition, John Wiley & Sons, 1999, p. 399 (Problem 8.7).

Application Library path: `RF_Module/Verification_Examples/axisymmetric_cavity_resonator`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D Axisymmetric**.

2 In the **Select Physics** tree, select **Radio Frequency>Electromagnetic Waves, Frequency Domain (emw)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Eigenfrequency**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
a	2.5[cm]	0.025 m	inner radius
b	10[cm]	0.1 m	outer radius
height	10[cm]	0.1 m	height
m_azimuthal	0	0	azimuthal mode number

GEOMETRY 1

Create a rectangle for the 2D axisymmetric geometry.

Rectangle 1 (r1)

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

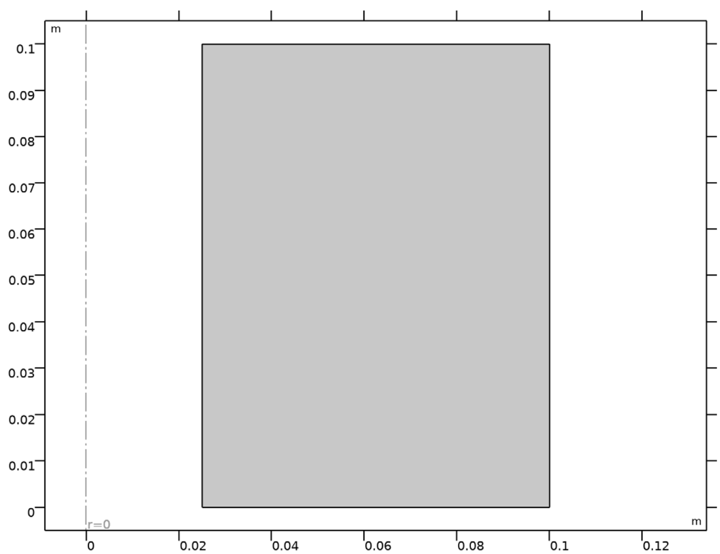
3 In the **Width** text field, type `b-a`.

4 In the **Height** text field, type `height`.

5 Locate the **Position** section. In the **r** text field, type `a`.

6 In the **Geometry** toolbar, click **Build All**.

The completed geometry should look the same as in the figure that follows.



MATERIALS

Assign material properties of the vacuum to the interior of the cavity.

Vacuum

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type `Vacuum` in the **Label** text field.

3 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	1	1	Basic
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	1	1	Basic
Electrical conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	0	S/m	Basic

ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN (EMW)

Change the azimuthal mode number to the parameter defined in the table above.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electromagnetic Waves, Frequency Domain (emw)**.

2 In the **Settings** window for **Electromagnetic Waves, Frequency Domain**, locate the **Out-of-Plane Wave Number** section.

3 In the m text field, type `m_azimuthal`.

MESH 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2 In the **Settings** window for **Mesh**, locate the **Physics-Controlled Mesh** section.

3 In the table, clear the **Use** check box for **Electromagnetic Waves, Frequency Domain (emw)**.

For electromagnetic wave problems, the maximum mesh size should be smaller than 1/5 of the wavelength. The spatial variation of the lowest frequency resonant modes is comparable to the cavity size, so the default mesh is a reasonable starting point. This will turn out to be very accurate for this simple model. However, in a general case where the exact solutions are not known, a mesh refinement analysis is useful.

4 In the **Home** toolbar, click **Build Mesh**.

STUDY 1

Solve for 12 cavity modes near 2 GHz, an estimate for the lowest resonant frequency based on the size of the cavity. A reasonable value is necessary so that not all of the solutions are spurious modes.

Step 1: Eigenfrequency

1 In the **Model Builder** window, under **Study 1** click **Step 1: Eigenfrequency**.

2 In the **Settings** window for **Eigenfrequency**, locate the **Study Settings** section.

3 Select the **Desired number of eigenfrequencies** check box. In the associated text field, type `12`.

4 In the **Search for eigenfrequencies around shift** text field, type `2[GHz]`.

Perform a parametric sweep over the azimuthal mode number, which is similar to a **for** loop.

Parametric Sweep

1 In the **Study** toolbar, click **Parametric Sweep**.

2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

3 Click **Add**.

4 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
m_azimuthal (azimuthal mode number)	0 1 2	

5 In the **Study** toolbar, click **Compute**.

RESULTS

Look at plots of the norm of the electric field. The smoothly varying solutions are physical modes. The spatial variation of the field is comparable to the cavity size. The spurious modes have spatial variation comparable to the mesh size. In fact, the resonant frequency and fields of the spurious modes are extremely mesh-dependent. From the plots, it is possible to determine that the lowest frequency resonant mode is approximately 1.49896 GHz.

Electric Field (emw)

1 In the **Model Builder** window, under **Results** click **Electric Field (emw)**.

2 In the **Settings** window for **2D Plot Group**, locate the **Data** section.

3 From the **Parameter value (m_azimuthal)** list, choose **0**.

4 From the **Eigenfrequency (GHz)** list, choose **1.499**.

5 In the **Electric Field (emw)** toolbar, click **Plot**.

This plot should look like Figure 2 above. The other 2D plots can be produced following the instructions below.

6 From the **Parameter value (m_azimuthal)** list, choose **1**.

7 From the **Eigenfrequency (GHz)** list, choose **3.0989**.

8 In the **Electric Field (emw)** toolbar, click **Plot**.

9 From the **Eigenfrequency (GHz)** list, choose **3.673**.

10 In the **Electric Field (emw)** toolbar, click **Plot**.

11 From the **Parameter value (m_azimuthal)** list, choose **2**.

12 From the **Eigenfrequency (GHz)** list, choose **4.2648**.

13 In the **Electric Field (emw)** toolbar, click **Plot**.

Evaluate k^2 to compare with the solutions obtained via separation of variables. The conditional statement that appears in the expression ensures that there are nonzero values for the physical modes only, that is, the expression evaluates to zero for the spurious modes.

Global Evaluation 2

1 In the **Results** toolbar, click **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1/Parametric Solutions 1 (sol2)**.

4 From the **Table columns** list, choose **Outer solutions**.

5 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
`(2*pi*freq/c_const)^2 * (freq > 1.4[GHz])`	1/m^2	

6 Click **Evaluate**.

Revolution 2D

After computing the solution to the 2D axisymmetric problem, a revolution of the 2D dataset is produced. The revolved dataset can be used in 3D plots. Set the start angle to 0 and the revolution angle to 360.

1 In the **Model Builder** window, expand the **Results>Datasets** node, then click **Revolution 2D**.

2 In the **Settings** window for **Revolution 2D**, click to expand the **Revolution Layers** section.

3 In the **Start angle** text field, type `0`.

4 In the **Revolution angle** text field, type `360`.

5 Click to expand the **Advanced** section. Select the **Define variables** check box.

The last step defines a variable for the angle, `rev1phi`, which can be used in a 3D arrow plot of the electric field.

3D Plot Group 2

In the **Results** toolbar, click **3D Plot Group**.

Arrow Volume 1

1 In the **3D Plot Group 2** toolbar, click **Arrow Volume**.

2 In the **Settings** window for **Arrow Volume**, locate the **Expression** section.

3 In the **R-component** text field, type `Er*exp(-i*m_azimuthal*rev1phi)`.

4 In the **PHI-component** text field, type `Ephi*exp(-i*m_azimuthal*rev1phi)`.

5 In the **Z-component** text field, type `Ez*exp(-i*m_azimuthal*rev1phi)`.

6 Locate the **Arrow Positioning** section. Find the **X grid points** subsection. In the **Points** text field, type `30`.

7 Find the **Y grid points** subsection. In the **Points** text field, type `30`.

8 Find the **Z grid points** subsection. In the **Points** text field, type `1`.

9 In the **3D Plot Group 2** toolbar, click **Plot**.

3D Electric Field

1 In the **Model Builder** window, under **Results** click **3D Plot Group 2**.

2 In the **Settings** window for **3D Plot Group**, type `3D Electric Field` in the **Label** text field.

3 Locate the **Data** section. From the **Parameter value (m_azimuthal)** list, choose **1**.

4 From the **Eigenfrequency (GHz)** list, choose **3.673**.

5 In the **3D Electric Field** toolbar, click **Plot**.

This plot should look like Figure 6 above. Now convert the plot into a movie. Note that to make a frequency domain dataset oscillate in time, the sequence type is set to dynamic data extension.

Animation 1

1. In the **Results** toolbar, click **Animation** and choose **File**.
2. In the **Settings** window for **Animation**, locate the **Target** section.
3. From the **Target** list, choose **Player**.
4. Locate the **Scene** section. From the **Subject** list, choose **3D Electric Field**.
5. Locate the **Animation Editing** section. From the **Sequence type** list, choose **Dynamic data extension**.
6. Click the **Play** button in the **Graphics** toolbar.