$$X \longrightarrow Y$$

Features   Labels

Supervised Learning

Regression   classification

perceptron

Height

= blue
lord

= green
Lord

Assumption
if
Linearly Separable

Weight

$$X \quad \text{Feature} = \begin{bmatrix} \text{Weight} \\ \text{Height} \end{bmatrix}$$

$$Y \quad \text{Label} \quad \begin{cases} \nearrow & \bullet \\ \searrow & \bullet \end{cases}$$

Model

IR_f

Not
Lin
Seperable

F_3

F_1

Γ_2

Hyper-plane

$\mathbb{R}^n \quad \mathbb{R}^2$

$w \in \mathbb{R}^n$

x

x

$$HP = \left\{ x : \boxed{w^T x + b = 0} \right\}$$
$\in \mathbb{R}^2$

$$\boxed{w^T x + b} \Big) \begin{array}{c} > 0 \\ ? \\ < 0 \end{array}$$

weights   bias

$$\boxed{w_1 \quad w_2} \quad \boxed{\begin{array}{c} x_1 \\ x_2 \end{array}} \quad \left(+ \quad \boxed{b} \right) \begin{array}{c} > 0 \\ ? \\ < 0 \end{array}$$

$$\boxed{\tilde{w}_1 \quad \tilde{w}_2 \quad \tilde{w}_3} \quad \boxed{\begin{array}{c} 1 \\ x_1 \\ x_2 \end{array}} \begin{array}{c} > 0 \\ ? \\ < 0 \end{array}$$
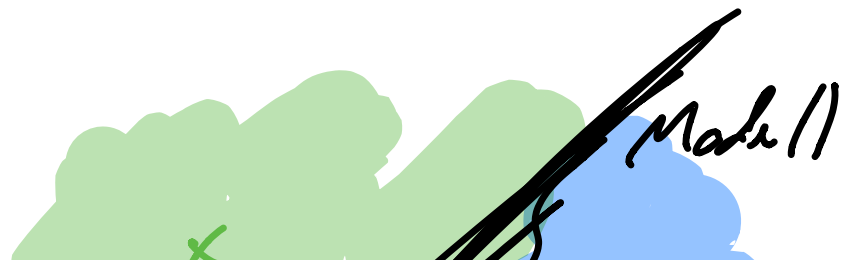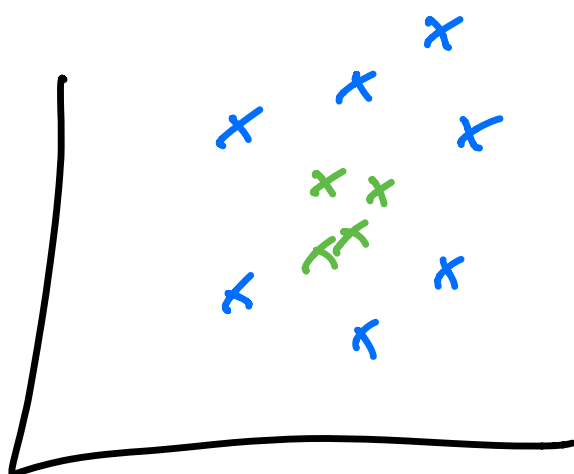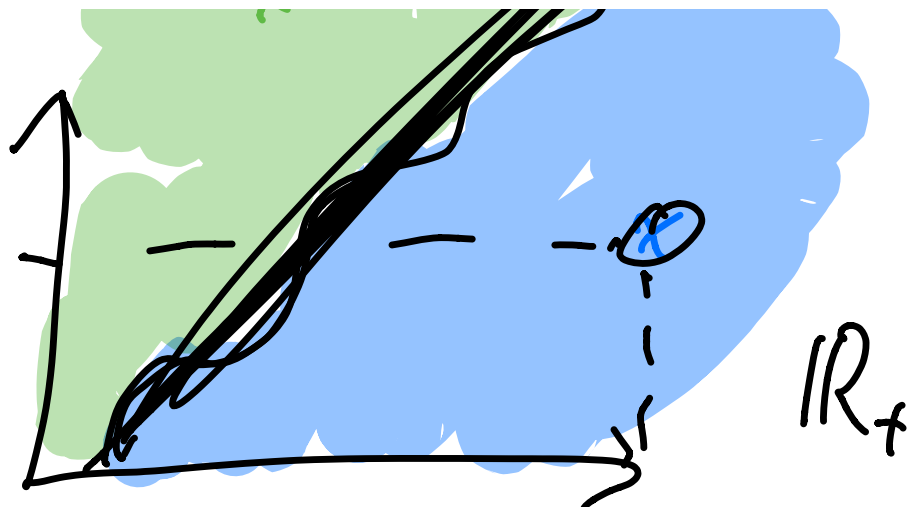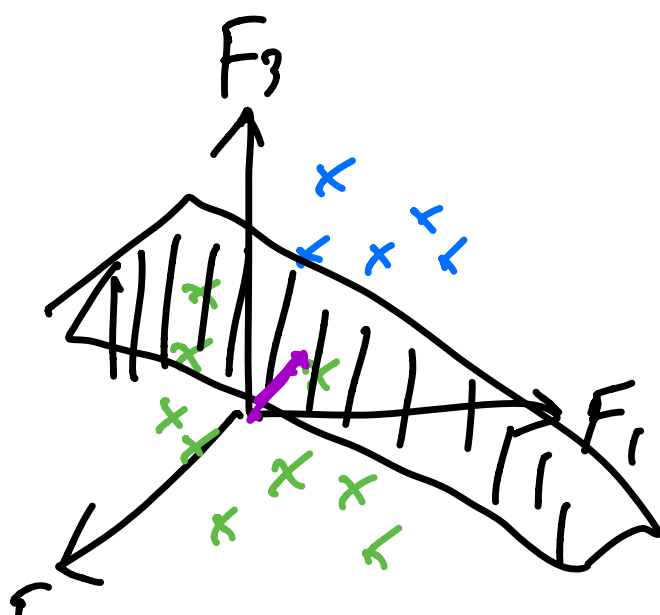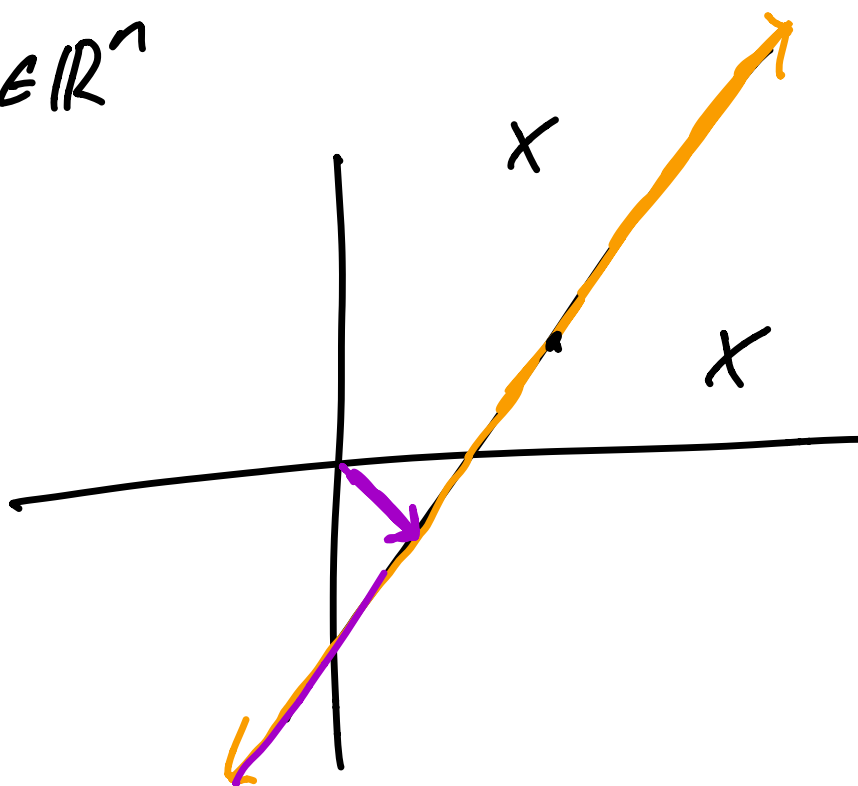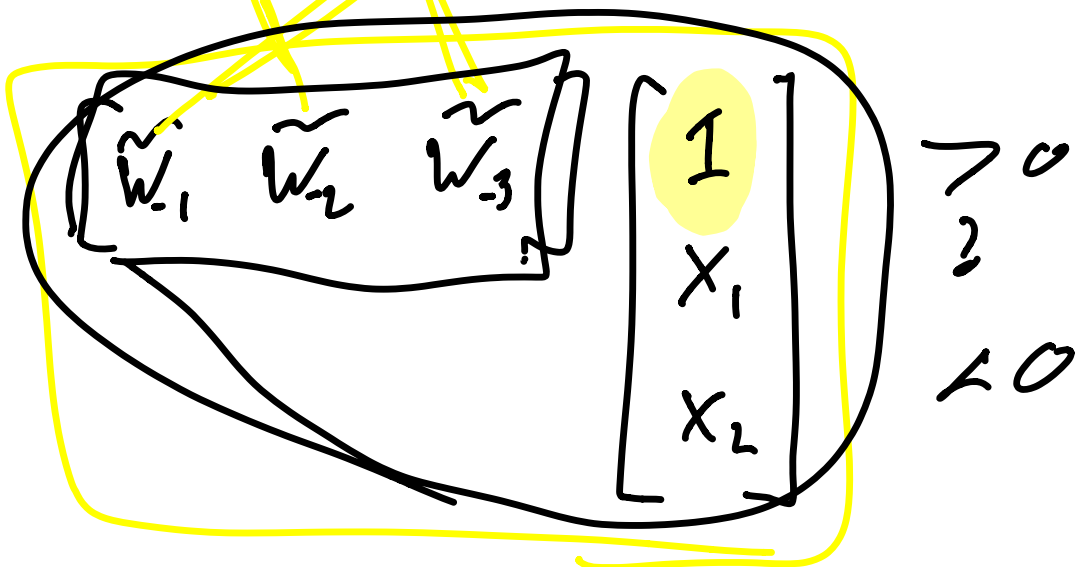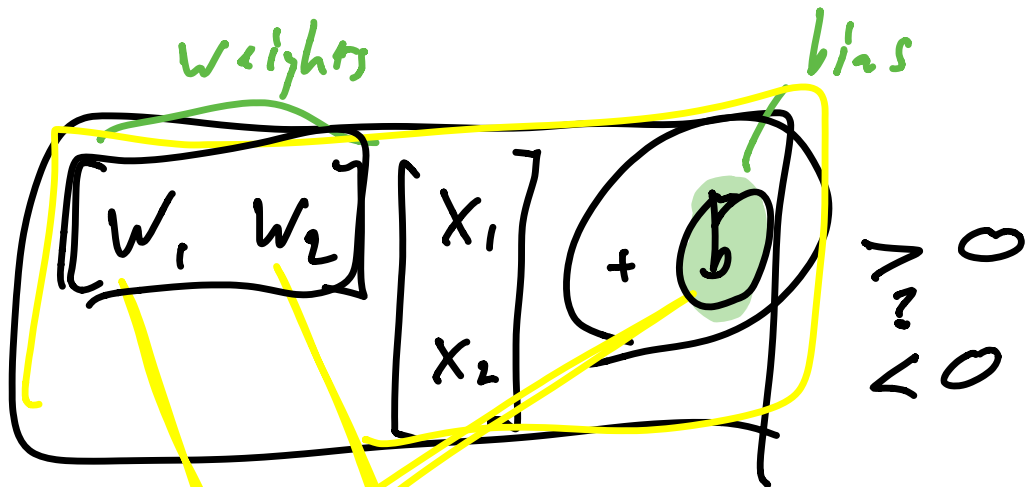
$$\begin{bmatrix} w_1 & w_2 & \left(w_3 \right) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Linear Transformations

$$f(\alpha_1 x + \alpha_2 y) = \alpha_1 f(x_1) + \alpha_2 f(x_2)$$

↑

~~transform~~

$$f(x) = Ax$$

$$f(\alpha_1 x + \alpha_2 y) = A\alpha_1 x + A\alpha_2 y$$
$$= \alpha_1 Ax + \alpha_2 Ay$$
$$= \alpha_1 f(x) + \alpha_2 f(y)$$

$$X = x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \cdots + x_n \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$
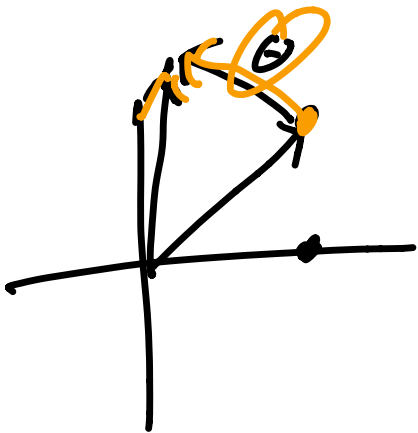
$$\underbrace{}$$

$$e_1 \qquad\qquad e_2 \qquad\qquad\qquad e_n$$

$$f(x) = f\left(\sum_{i=1}^{n} x_i \, e_i \right) \Big\} LT$$

$$= \sum_{i=1}^{n} x_i \, f(e_i)$$

$$= \begin{bmatrix} | & | & & | \\ f(e_1) & f(e_2) & \cdots & f(e_n) \\ | & | & & | \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ \vdots \\ x_n \end{bmatrix}$$

$$= A X$$

The Matrix

or a LT

$$f: \mathbb{R}^2 \to \mathbb{R}^2$$

↑ Rotated by θ

$$f\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} | & | \\ f(\vec{e_1}) & f(\vec{e_2}) \\ | & | \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

A

↓

Got the rotation matrix

$\vec{e}, \vec{j}, \vec{k}$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

e_2