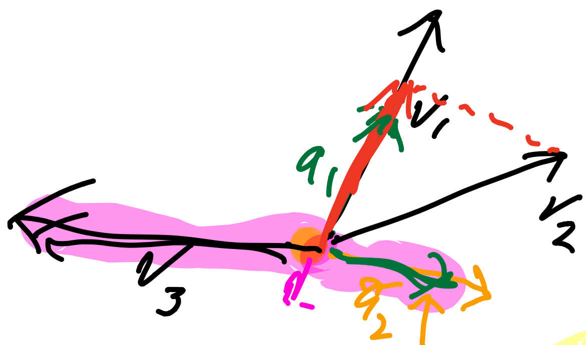


$$\tilde{q}_2 = v_2 - (q_1^T v_2)\, q_1$$

$$q_2 = \frac{\tilde{q}_2}{\|\tilde{q}_2\|}$$

$$\tilde{q}_3 = v_3 - (q_1^T v_3)\, q_1 - (q_2^T v_3)\, q_2$$

$$= 0$$

A with orthonormal cols

$$A^T A = I_{k \times k}$$

$$(AB)^T = B^T A^T$$

$$A_{n \times k} \qquad k \leq n$$

$$(Ax)^T (Ay) = x^T \underbrace{A^T A}_{I} y = x^T y$$

inner product of Ax and Ay

$$\angle (Ax, Ay) = \angle (x, y)$$

$$(Ax)^T (Ax) = x^T x = \|$$

$$||$$

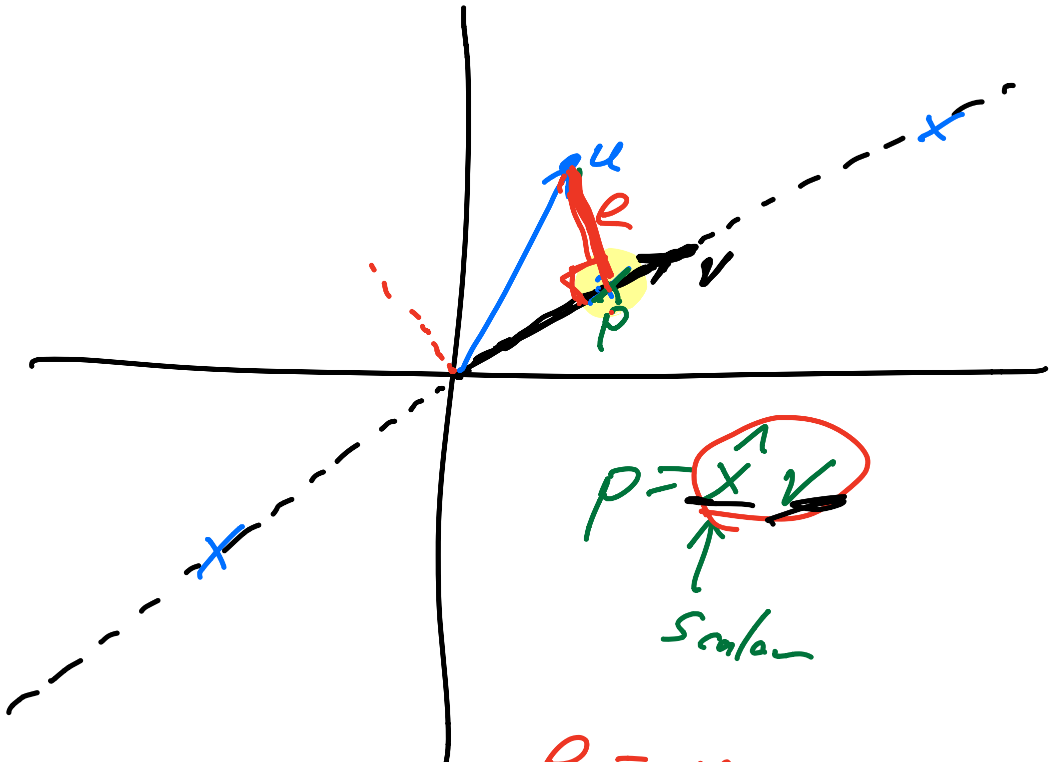
$$||Ax||^2 \qquad ||x||^2$$

$$||Ax|| = ||x||$$

$$A = \begin{bmatrix} | & | & | \\ v_1 & v_2 & v_3 \\ | & | & | \end{bmatrix} \qquad Q = \begin{bmatrix} | & | & | \\ q_1 & q_2 & q_3 \\ | & | & | \end{bmatrix}$$

$$\boxed{A = QR}$$

$$\begin{bmatrix} | & | & | \\ v_1 & v_2 & v_3 \\ | & | & | \end{bmatrix} = \begin{bmatrix} | & | & | \\ q_1 & q_2 & q_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ 0 & R_{22} & R_{23} \\ 0 & 0 & R_{33} \end{bmatrix}$$



$$p = \underbrace{\hat{x}}_{\text{scalar}} v$$

$$e = u - p$$

$$\underbrace{(u-p)}_{e}{}^{T} v = 0$$

$$u^T v - \underbrace{p^T}_{(\hat{x} v)^T} v = 0$$

$$u^T v - \hat{x}(v^T v) = 0$$

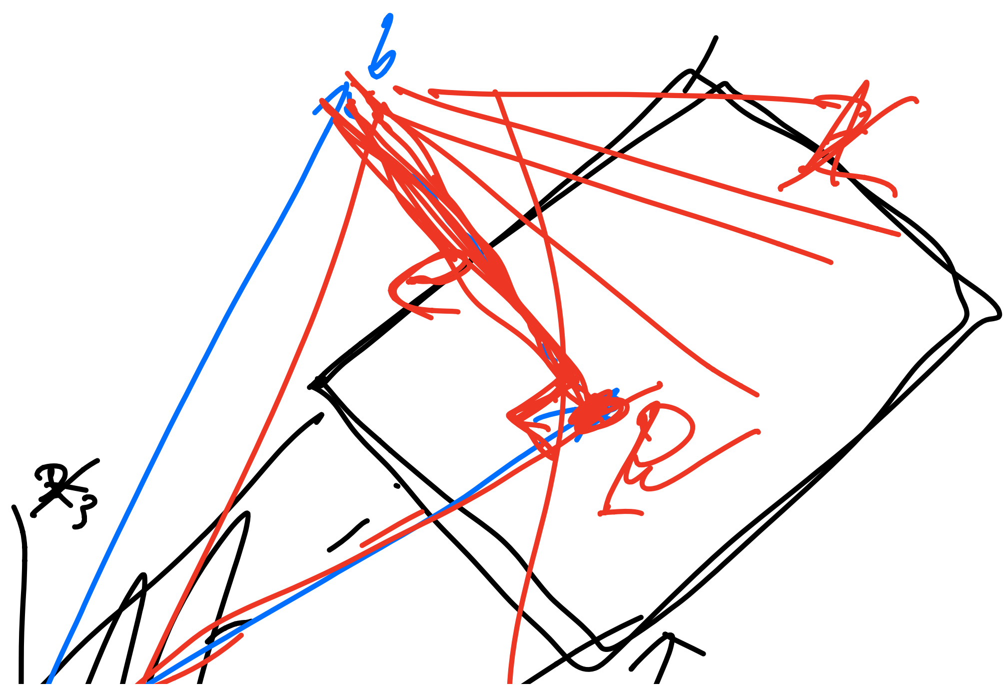
$$\underline{u^T v} = \hat{x}\, \underline{v^T v}$$

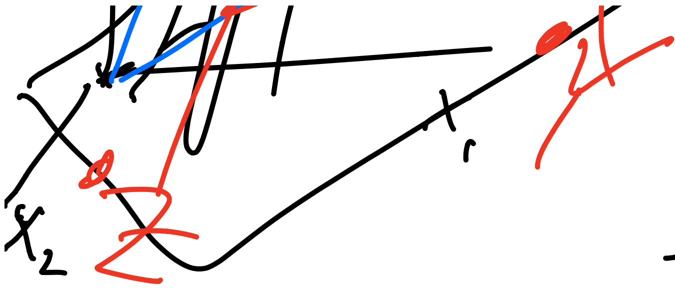
$$\hat{x} = \frac{u^T v}{v^T v} = \frac{u^T v}{\|v\|^2}$$

$$p = \frac{u^T v}{v^T v} v$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

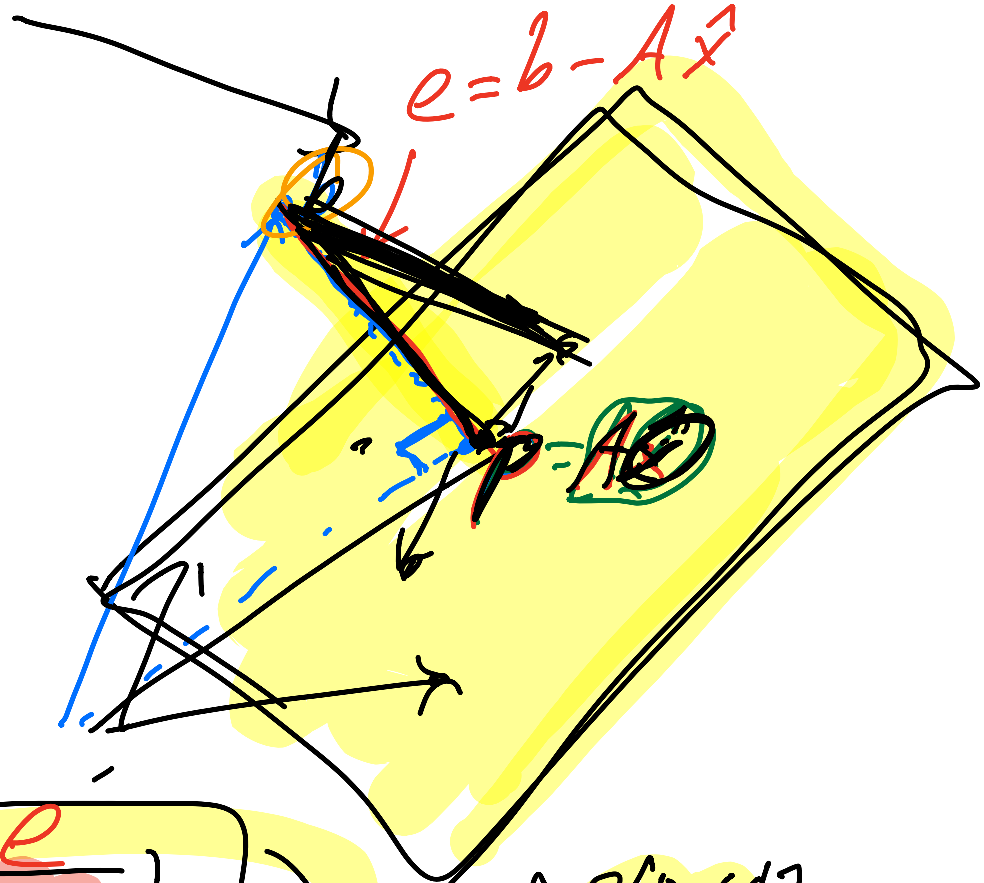
$$\boxed{Ax = b}$$





$Sp(\underline{col(A)})$

$R(A)$



$$\boxed{a_1^T(\overset{e}{\overline{b - A\hat{x}}}) = 0}$$

$$\underset{m \times n}{A} = \begin{bmatrix} | & | & & | \\ a_1 & a_2 & \cdots & a_n \\ | & | & & | \end{bmatrix}$$

$a_2^T(b - A\hat{x}) = 0$

$\vdots$

$a_n^T(b - A\hat{x}) = 0$

$$A^T = \begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ & \vdots & \\ - & a_n^T & - \end{bmatrix}_{n \times m}$$

$$\boxed{A^T(b - A\hat{x}) = 0}$$

$$A^T b - A^T A \hat{x} = 0$$

Normal equations

$$\boxed{A^T A \hat{x} = A^T b}$$

Assume A has LI cols'

A skinny full-rank

$\mathcal{N}(A) = 0$

$\det(A^T A) \neq 0$, $\operatorname{inv}(A^T A)$ exist.

$$\hat{x} = \underbrace{(A^T A)^{-1} A^T}_{A^+} b$$

pseudo-inverse

$$\boxed{(B^{-1})^T = (B^T)^{-1}}$$

$$p = A\hat{x}$$
$$= A(A^T A)^{-1} A^T b$$

$$P = AA^+$$

$$P^T = A^{TT} (A^T A)^{-T} A^T$$
$$= A (A^T A)^{-1} A^T$$

Sym ✓

$$P^2 = \underbrace{A(A^TA)^{-1}A^T}_{P}\,\underbrace{A(A^TA)^{-1}A^T}_{P} = P$$

$$P^{129} = P$$

Summary:

$$A^+ = (A^TA)^{-1}A^T$$

$$P = AA^+ = \boxed{A(A^TA)^{-1}A^T}$$

$$Ax = b$$

Approximate Sol

A^+b

$x = A b$ ...ation

$p = \hat{b} = A\hat{x} = Pb$

$\boxed{QR}$ A skinny full rank

$A^{+} = (A^T A)^{-1} A^T = A^{-1} \cancel{A^{-T}} \cancel{A^T}$

$A_{\boxed{n \times n}}$ full rank

$= A^{-1}$

A square

$A^{+} = ((QR)^T QR)^{-1} (QR)^T$

$= (R^T \underline{Q^T Q} R)^{-1} R^T Q^T$

$= (R^T R)^{-1} R^T Q^T$

$$= R^{-1} \underbrace{R^{-T} R^{T}} Q^{T}$$

$$= R^{-1} Q^{T}$$

$$P = AA^{+} = AR^{-1}Q^{T}$$

$$= Q\cancel{RR^{-1}}Q^{T}$$

$$= \boxed{QQ^{T}}$$