Claim: If $S = S^T$ and $S \in \mathbb{R}^{n \times n}$

λ is eigval of S, then

λ is real.

$$\lambda = \bar{\lambda}$$

$$\Downarrow$$

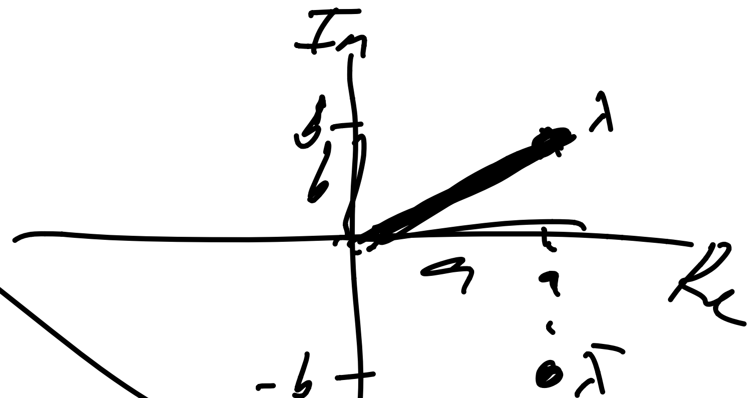
$$b = 0$$

$$\lambda = a + bi$$

$$a = Re(\lambda) \qquad i = \sqrt{-1}$$

$$b = Im(\lambda)$$

$$\bar{\lambda} = a - bi \qquad \overline{wz} = \bar{w}\,\bar{z}$$



$$Sx = \lambda x \qquad x \neq 0$$

$$S\bar{x} = \bar{\lambda}\bar{x}$$

$$\bar{x}^T S^T = \bar{x}^T \bar{\lambda}$$

$$\overline{\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}} = \begin{bmatrix} \bar{u}_1 \\ \bar{u}_2 \\ \bar{u}_3 \end{bmatrix}$$

$$\bar{x}^T S = \bar{x}^T \bar{\lambda}$$

$$Sx = \lambda x$$

$$\boxed{\bar{x}^T S x} = \bar{x}^T \bar{\lambda} x$$

$$= \bar{\lambda} \bar{x}^T x$$

$$\boxed{\bar{x}^T S x} = \lambda \bar{x}^T x$$

$$\boxed{\bar{\lambda} \bar{x}^T x = \lambda \bar{x}^T x}$$

$$\bar{x}^T x = \sum_{i=1}^{n} \bar{x}_i x_i = \sum_{i=1}^{n} |x_i|^2 \neq 0$$

$$> 0$$

$$\boxed{\bar{\lambda} = \lambda}$$

$$|a+bi| = \sqrt{a^2+b^2}$$

Hena λ is real.

Claim: $S = S^T$

Assum $\lambda_1 \neq \lambda_2$

$Sx = \lambda_1 x$ $\qquad x, y \neq 0$

$Sy = \lambda_2 y$

$$\boxed{y^T S x} = \boxed{\lambda_1 y^T x}$$

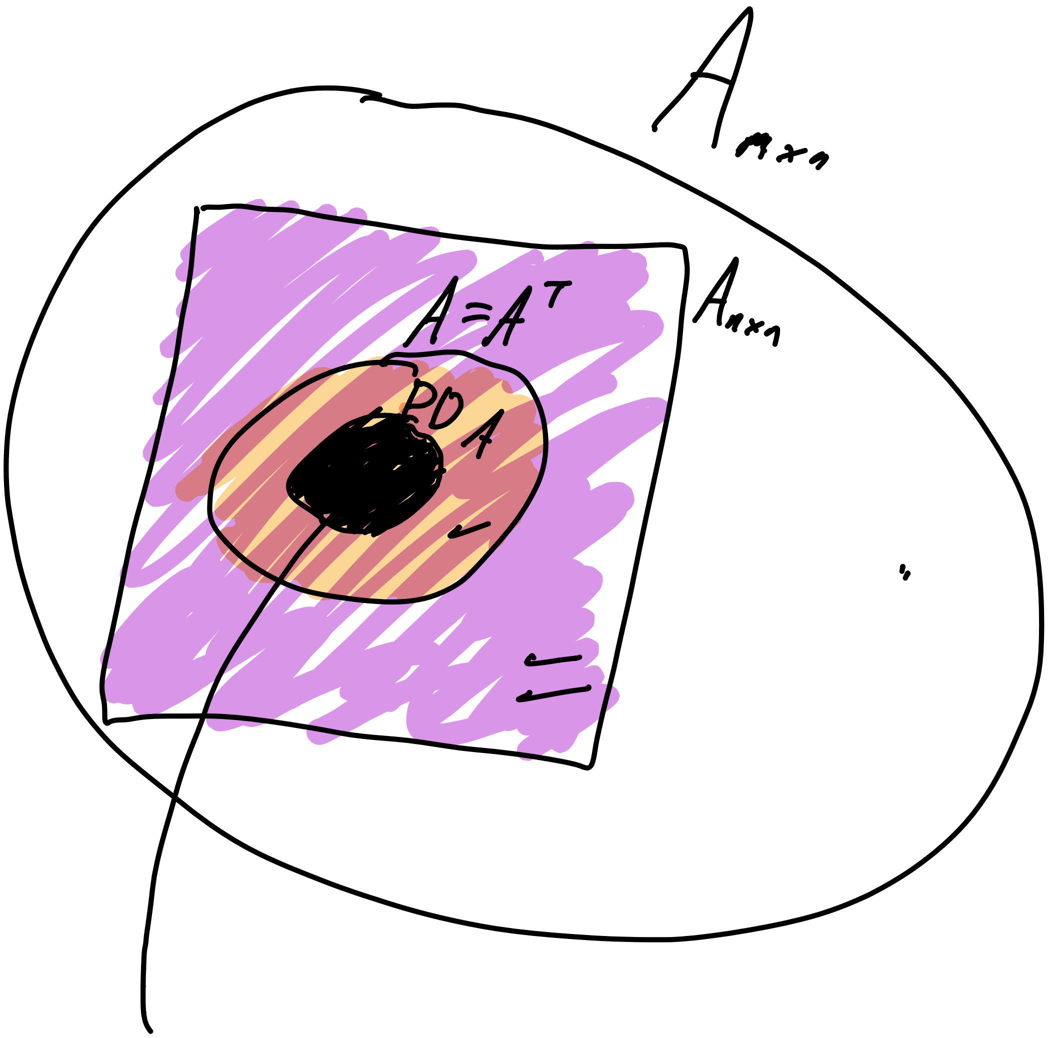
$$x^T S y = \lambda_2 x^T y$$

$$\boxed{y^T S x} = \boxed{\lambda_2 y^T x}$$

$$\lambda_1 y^T x = \lambda_2 y^T x$$

$$y^T x = 0$$

Hence $x \perp y$.



$S \succ 0$ PSD

1) Eigvals > 0

2) $\boxed{x^T S x > 0}$ $\forall x \neq 0$

PSD

$$x^T S x \geq 0 \quad \forall x$$

$Sx = \lambda x$ $\quad$ $x \neq 0$

$\lambda > 0$ $\quad$ λ real

$$x^T S x = \lambda x^T x$$
$$= \lambda \|x\|^2 > 0$$

Claim:

If A PD

and B PD

$C = A + B$ is PD

$$x^T C x = x^T (A+B) x$$
$$= (x^T A + x^T B) x$$

$$= \underbrace{x^T A x}_{>0} + \underbrace{x^T B x}_{>0} > 0$$

C is PD

Criterion 3: $\boxed{S = A^T A}$

for A with indep cols!

$$S = A^T A$$

$$\boxed{S = Q \Lambda Q^T}$$

$$= Q \sqrt{\Lambda} \sqrt{\Lambda} Q^T$$

$$\begin{bmatrix}\sqrt{\lambda_1} & & \\ & \ddots & \\ & & \sqrt{\lambda_n}\end{bmatrix}$$

$$A = Q\sqrt{\Lambda}Q^T$$

$$A^TA = \underbrace{Q\sqrt{\Lambda}\cancel{Q^T}}_{A^T}\cancel{Q}\sqrt{\Lambda}Q^T$$

$$= Q\Lambda Q^T$$

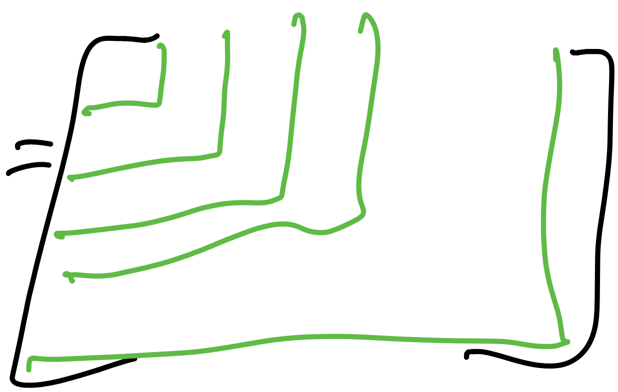
Cholekskiy decomposition

$$S = A^TA$$

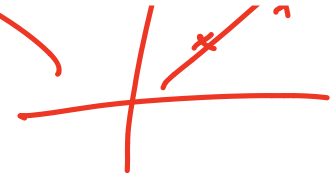
It is lower triangular

(using LU decomp')

Sylvester Criterion

$S =$ 

Least Squares



$$\boxed{Ax = b} \qquad A_{m\times n} \quad m>n$$

$$\text{null}(A)=\{0\}$$

$$A^\dagger = (A^TA)^{-1}A^T$$

$$A^\dagger A x = A^\dagger b$$

$$\boxed{\hat{x} = A^\dagger b}$$

$$\arg\min_{x\in\mathbb{R}^n} \|Ax-b\|^2 = \arg\min_{x\in\mathbb{R}^n} \|Ax-b\|$$

$$L(x) = \|Ax-b\|^2 = (Ax-b)^T(Ax-b)$$

$$= (x^T A^T - b^T)(Ax - b)$$

$$= x^T A^T A x - b^T A x - x^T A^T b + b^T b$$

$$= x^T A^T A x - b^T A x - b^T A x + b^T b$$

$$= x^T \boxed{A^T A} x - 2 b^T A x + b^T b$$

$$- 2 (A^T b^T)^T x + b^T b$$

$$= \boxed{x^T S x + c^T x + d}$$

$$a x^2 + b x + c$$

$HL = S = A^T A$ is PD

Quiz 3

1) Projections $\quad \overline{Ax = b}$

$$\underline{P} = A\underbrace{(A^TA)^{-1}A^T}$$

$A = \underline{QR}$

2) Least Squares with $A^+ = $

3) Eigen stuff

4) Diagonalization

$A = M \Lambda M^{-1}$

$A =$ [illegible]

5) Sym.

6) $x^T(A+B)x \geq 0$

7) Hessian Matrix

$f: \mathbb{R}^2 \to \mathbb{R}$

$$f(u,v) = \begin{bmatrix} u + e^{v} \\ uv^2 \end{bmatrix}$$

$\boxed{\text{SVD}}$

11

NOQ

$$\boxed{A^{+} = U\Sigma^{-1*}V^{T}}$$

$$\hat{x} = A^{+}b$$

$$A\cancel{x} = b$$

$$\begin{bmatrix} \frac{1}{\sigma_1} & & & \\ & \frac{1}{\sigma_2} & & \\ & & \ddots & \\ & & & \frac{1}{\sigma_r} \end{bmatrix}$$

$$A^{+} = (A^{T}A)^{-1}A^{T}$$