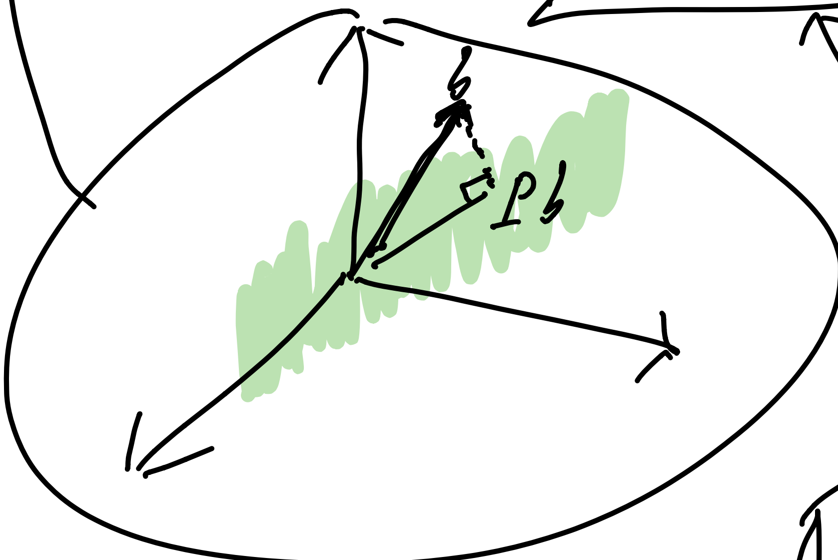
$$\boxed{A_{3\times 2}\, x_{[2\times 1]} = b_{3\times 1}}$$

$$\hat{x} = A^{+} b$$

$$= \underbrace{(A^T A)^{-1} A^T}_{A^{+}}\, b$$

~~SVD~~

$$P = A A^{+} = \boxed{A (A^T A)^{-1} A^T}$$



$$A = QR$$

$$f(x) = \beta^T [1 \ x]$$

$\beta_0 + \beta_1 x$

$$A = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}$$

$$\hat{\beta} = A^{\dagger} y$$

$$\boxed{Ax = \lambda x} \qquad x \neq 0$$

$$\boxed{Ax - \lambda I x = 0}$$

$$\boxed{(A - \lambda I)x = 0}$$

B_λ

$B_\lambda x = 0$

Want B_λ singular

$\mathrm{Det}(B_\lambda) = 0$

$$p(\lambda) = \boxed{\mathrm{Det}(\underbrace{A - \lambda I}_{B_\lambda}) = 0}$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\mathrm{Det}\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) = 0$$

$$\mathrm{Det}\begin{pmatrix} 1-\lambda & 2 \\ 3 & 4-\lambda \end{pmatrix} = 0$$

$$(1-\lambda)(4-\lambda)-6=0$$

$$\underbrace{1}_{a}\lambda^2 \underbrace{-5}_{b}\lambda \underbrace{-2}_{c}=0$$

$$\lambda_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \qquad \sqrt{-1}=i$$

$\lambda_1 \approx 5.37$

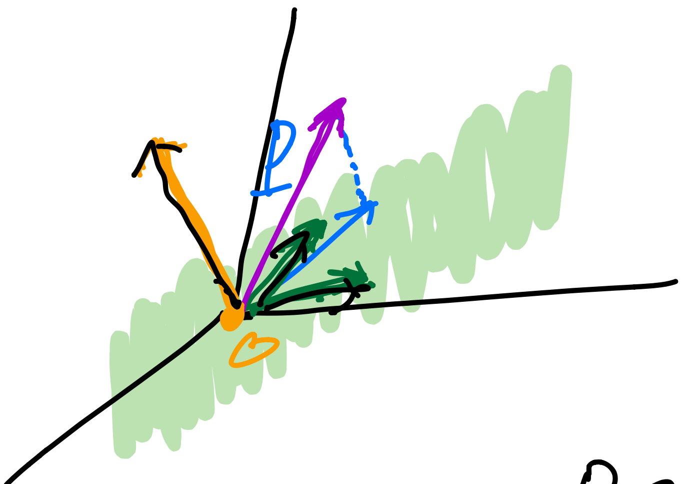
$\lambda_2 \approx -0.37$

$$(A-\lambda_2 I)\boxed{x}=0$$

Any x in nullspace of $A-\lambda_2 I$ is an eigenvector ass' with λ_2

$A_{n\times m}$ $\qquad m\leq n$

$$\boxed{P_{n\times n} = A(A^TA)^{-1}A^T}$$



$$\boxed{PA = A}$$

$$\left[\,|\;|\;|\cdots|\,\right] = \left[\,|\;|\;|\;|\,\right]$$

$P(x) = 1x$

$$\underbrace{A(A^TA)^{-1}A^T}_{P}A = A \quad \left(\begin{array}{l} \lambda = 1 \\ \text{for} \\ \text{cols of} \\ A \end{array} \right)$$

$$P\,| = \boxed{0}\,| = 0$$

$\lambda = 0$

for

nullspace of P.

$$Ax = \lambda x \qquad \det(A) \neq 0$$

$$\Downarrow$$

$$A^{-1}Ax = \lambda A^{-1}x \qquad \lambda_i \neq 0$$

$$\boxed{x = \lambda A^{-1}x}$$

$$\frac{1}{\lambda}x = A^{-1}x$$

$$\boxed{A^{-1}x = \frac{1}{\lambda}x}$$

$$AB \qquad BA$$

$$x \neq 0$$

$$BABx = \lambda Bx \qquad \lambda \neq 0$$

$$BA(Bx) = \lambda(Bx) \qquad Bx \neq 0$$

Hence Bx is an eigenvector of BA with eigenvalue λ

$A^T A$ $\qquad$ $A A^T$

same $\lambda \neq 0$

$$A M = M \tilde{A} \qquad \det(M) \neq 0$$

$$A = M \tilde{A} M^{-1}$$

$\det(BC) = \det(B)\det(C)$

$$\boxed{\det(A - \lambda I)}$$

$$= \det(M\tilde{A}M^{-1} - \lambda I)$$

$$= \det(M\tilde{A}M^{-1} - \lambda M M^{-1})$$

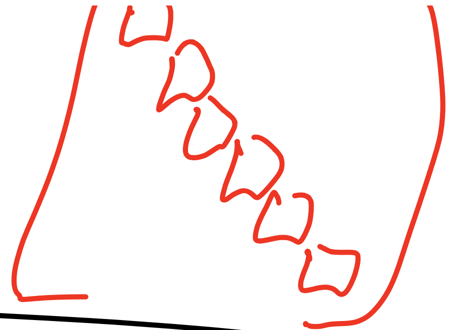
$$= \det(M(\tilde{A} - \lambda I)M^{-1})$$

$$= \cancel{\det(M)}\,\det(\tilde{A} - \lambda I)\,\cancel{\det(M^{-1})}$$

$$= \boxed{\det(\tilde{A} - \lambda I)}$$

$$\boxed{A = \boxed{M}\,\Lambda\,M^{-1}}$$

$$\begin{bmatrix} | & & | \\ v_1 & \cdots & v_n \\ | & & | \end{bmatrix} \qquad \tilde{A} \qquad \Lambda = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 \ddots & \\ 0 & & \lambda_n \end{bmatrix}$$



$$A^2 = M\Lambda \cancel{M^{-1} M} \Lambda M^{-1}$$

$$= M\Lambda^2 M^{-1}$$

$$= M \begin{bmatrix} \lambda_1^2 & & & \\ & \lambda_2^2 & & \\ & & \ddots & \\ & & & \lambda_n^2 \end{bmatrix} M^{-1}$$

$$\boxed{A^k = M\Lambda^k M^{-1}}$$

Fibbonaci

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

$$\boxed{y(n) = y(n-1) + y(n-2)}$$

$$X(n) = \begin{bmatrix} \square \\ \square \end{bmatrix}$$

$y(n)$ current

$y(n-1)$ previous

$$\boxed{X(n) = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} X(n-1)}$$

$$\underbrace{\begin{bmatrix} y(n) \\ y(n-1) \end{bmatrix}}_{X(n)} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} y(n-1) \\ y(n-2) \end{bmatrix}}_{X(n-1)}$$

$$X(n) = A\,X(n)$$

$$X(1) = (1, 0)$$

$$X(2) = AX(1)$$

$$X(3) = AX(2) = AAX(1) = A^2 X(1)$$

$$\vdots$$

$$X(n) = A^{n-1} X(1)$$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= M \Lambda M^{-1}$$

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$$\boxed{S} = Q \Lambda Q^T = \lambda_1 q_1 q_1^T$$

10^6

10×10

10^{12}

eigenvectors chosen orthonormal

$$Q^T = Q^{-1}$$

$$Q = \begin{bmatrix} | & | & & | \\ q_1 & q_2 & \cdots & q_n \\ | & | & & | \end{bmatrix}$$

$$+ \lambda_2 q_2 q_2^T + \lambda_3 q_3 q_3^T + \cdots + \lambda_n q_n q_n^T$$

$$\sum_{i=1}^{n} \lambda_i q_i q_i^T$$

Spectral Decomp of Sym' Matrix.