$$Ax = b$$

$$\begin{bmatrix} \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \end{bmatrix} \to \begin{bmatrix} \times & \times & \times & \times \\ 0 & \tilde{\times} & \tilde{\times} & \tilde{\times} \\ 0 & \tilde{\times} & \tilde{\times} & \tilde{\times} \\ 0 & \tilde{\times} & \tilde{\times} & \tilde{\times} \end{bmatrix} \to \begin{bmatrix} \times & \times & \times & \times \\ 0 & \tilde{\times} & \tilde{\times} & \tilde{\times} \\ 0 & 0 & \approx & \approx \\ 0 & 0 & \approx & \approx \end{bmatrix}$$

$$A \qquad l_1 A \qquad l_2 l_1 A$$

$$\to \begin{bmatrix} \times & \times & \times & \times \\ 0 & \tilde{\times} & \tilde{\times} & \tilde{\times} \\ 0 & 0 & \approx & \approx \\ 0 & 0 & 0 & \approx \end{bmatrix} = \underline{U}$$

$$l_3 l_2 l_1 A$$

$$\cancel{l_1^{-1}} \cancel{l_2^{-1}} \cancel{l_3^{-1}} \cancel{l_3} \cancel{l_2} \cancel{l_1} A = l_1^{-1} l_2^{-1} l_3^{-1} U$$

$$L = l_1^{-1} l_2^{-1} l_3^{-1}$$

$$A = \boxed{L}\, U$$

$$\boxed{Uw = c} \quad \underline{\text{easy}}$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 \\ l_{31} & l_{32} & 1 & 0 \\ l_{41} & l_{42} & l_{43} & 1 \end{bmatrix}$$

Finding A^{-1}

$$A^{-1}A = I$$

$$AA^{-1} = I$$

$Ax = b$

$$\underbrace{\begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}}_{A^{-1}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & I \end{bmatrix} \rightarrow \begin{bmatrix} \tilde{A} & \tilde{I} \end{bmatrix}$$

$$\to \left[\tilde{A}\ \tilde{I}\right] \to \cdots$$

$$\left[I\ \tilde{\tilde{\tilde{I}}}\right]$$

$$Ax = b$$

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Unit Vector

$$e_7 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$PA = LU$

$LUx_1 = e_1$

$$LUx_2 = e_2$$

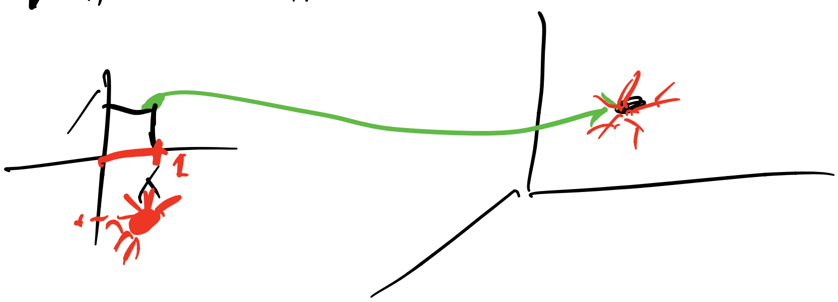
$$LUx_3 = e_3$$

$$\vdots$$

$$LUx_n = e_n$$

$$A^{-1} = \begin{bmatrix} | & | & & | \\ x_1 & x_2 & \cdots & x_n \\ | & | & & | \end{bmatrix}$$

$f: \mathbb{R}^2 \to \mathbb{R}^3$



$$f([x,y]^T) = \begin{bmatrix} f_1(x,y) = x^2 y \\ f_2(x,y) = e^{x+y} \\ f_3(x,y) = xe^y \end{bmatrix}$$

$f(x,y)$

a) $\|f(1,-1)\| = \sqrt{(1^2(-1))^2 + (e^{1+-1})^2 + (1e^{-1})^2}$

b) $\mathrm{Tr}(\underbrace{f(x,y)\, f(x,y)^T})$

$$= (x^2 y)^2 + (e^{x+y})^2 + (xe^y)^2$$

$x=1 \quad y=1$

c) $$Df = \begin{bmatrix} -\nabla f_1^T- \\ -\nabla f_2^T- \\ -\nabla f_3^T- \end{bmatrix}_{3\times 2}$$

$$Df(x,y) = \begin{bmatrix} 2xy & x^2 \\ e^{x+y} & e^{x+y} \\ e^y & xe^y \end{bmatrix}$$

Linear approx around $x=1$, $y=-1$

$$\underbrace{\hat{f}(x, (1,-1))}_{z} = f(1,-1) + D$$

$$= \underbrace{\begin{bmatrix} 1(-1) \\ e^0 = 1 \\ e^{-1} \end{bmatrix}}_{b} + \underbrace{\begin{bmatrix} 2(1)(-1) & 1^2 \\ e^{1+-1} & e^{1+-1} \\ e^{-1} & 1e^{-1} \end{bmatrix}}_{A} \begin{pmatrix} x_1 - 1 \\ x_2 - (-1) \end{pmatrix}$$

$\hat{}$

d) Rank (D) answer no.

"

Rank ≤ 2

e) $g(x) = 2x^2y + 3e^{x+y} - xe^y$

$= u^T f(x,y)$

$u = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$

$f : \mathbb{R}^n \to \mathbb{R}^m$ and linear

i.e. $f(\alpha_1 x + \alpha_2 y) = \alpha_1 f(x) + \alpha_2 f(y)$

Then $\exists A$

$$f(x) = Ax$$

$$x = x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \dots + x_n \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

$x \in \mathbb{R}^n$

$$= x_1 e_1 + x_2 e_2 + \dots + x_n e_n$$

$$f(x) = f(x_1 e_1 + x_2 e_2 + \dots + x_n e_n)$$

$$= x_1 f(e_1) + x_2 f(e_2) + \dots + x_n f(e_n)$$

$$= \underbrace{\begin{bmatrix} | & | & & | \\ f(e_1) & f(e_2) & \dots & f(e_n) \\ | & | & & | \end{bmatrix}}_{A_{m \times n}} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$