Linköpings tekniska högskola
IEI
Produktionsekonomi

Finansiell optimering
Laboration 1
November 3, 2014

Computer exercise 1 - Maximum Likelihood

Aim: You will be introduced to some of the tools that are used on the financial markets. Given historical financial data the parameters of a stochastic process will be determined by Maximum Likelihood estimation.
Background: You will be working in Matlab and the Excel environment of Thomson Reuters Eikon. In the Excel document historical prices are loaded from ThomsonReuters.

Download ml.zip from www.iei.liu.se/prodek/blomvall/finopt/lab, which includes a Matlab program (runMLGARCH.m) for reading historical data from the file labML.xls and for calling the optimization algorithm which use the objective function value computed in likelihoodModGARCH.m.

The GARCH(1,1) process models a long run variance level and can also describe volatility clustering. To improve the model to also consider the aspect that volatility usually increase more when asset prices decrease, and an increase in asset prices usually lead to decreasing volatility, a modification is proposed. The stochastic process for the share price, S_i, $i = 0, \ldots, T$, is given by

$$\begin{aligned} \ln \tfrac{S_{i+1}}{S_i} &= \nu \Delta t + \sigma_i \xi_i \sqrt{\Delta t} \\ \sigma_{i+1}^2 &= \beta_0 + \beta_1 \sigma_i^2 + \beta_2 \tfrac{1}{\Delta t}\left(\ln \tfrac{S_{i+1}}{S_i} - \alpha_0 \Delta t\right)^2 \quad \text{if } \ln \tfrac{S_{i+1}}{S_i} < 0 \\ \sigma_{i+1}^2 &= \beta_0 + \beta_1 \sigma_i^2 + \beta_2 \tfrac{1}{\Delta t}\left(\ln \tfrac{S_{i+1}}{S_i} - \alpha_1 \Delta t\right)^2 \quad \text{if } \ln \tfrac{S_{i+1}}{S_i} \geq 0 \end{aligned} \tag{1}$$

where

$$\xi_i \sim N(0,1). \tag{2}$$

By selecting appropriate values for α_0 and α_1 a larger increase in volatility when asset prices decrease, and a smaller increase in volatility when asset prices increase can be obtained.
To simplify the calculations $v_i = \sigma_i^2$ can be used. Maximum Likelihood estimation of the parameters gives the following optimization problem

$$\begin{aligned} \max_{\nu,\lambda} \quad & l = \sum_{i=0}^{T-1} -\frac{1}{2}\ln v_i - \frac{1}{2}\frac{\left(\ln \frac{S_{i+1}}{S_i} - \nu\Delta t\right)^2}{v_i \Delta t} \\ \text{s.t.} \quad & v_{i+1} = \beta_0 + \beta_1 v_i + \beta_2 \tfrac{1}{\Delta t}\left(\ln \tfrac{S_{i+1}}{S_i} - \alpha_0 \Delta t\right)^2 \quad \text{if } \ln \tfrac{S_{i+1}}{S_i} < 0 \\ & v_{i+1} = \beta_0 + \beta_1 v_i + \beta_2 \tfrac{1}{\Delta t}\left(\ln \tfrac{S_{i+1}}{S_i} - \alpha_1 \Delta t\right)^2 \quad \text{if } \ln \tfrac{S_{i+1}}{S_i} \geq 0 \end{aligned} \tag{3}$$

Note that in this particular instance the if-clause does not cause any problem for the optimization algorithm, since it is determined by $\ln \frac{S_{i+1}}{S_i}$ which is independent of the variables in the ML estimation.

The function runMLGARCH.m loads the input data and calls the optimization algorithm fmincon, with the function likelihoodModGARCH(x,r,dt) as an argument.

Exercise: Compute the likelihood value in the function likelihoodModGARCH and determine the optimal parameters using the interior point solver in fmincon!

Preparation: Determine the expression for ξ_i from historical prices using (1)!

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Question: Are the values for ν, β_0, β_1, β_2, α_0 and α_1 reasonable?

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Exercise: Determine the qq-plot for ξ!
Question: What are the the improvements?

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Exercise: Retrieve data for another asset using ThomsonReuters Eikon and rerun the estimation!
Question: Which is the asset, and what is the optimal parameter values? How well does the modified GARCH model describe the asset prices?

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