Extending the Use and Prediction Precision of Subnational Public Opinion Estimation*

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Abstract

The comparative study of subnational units is on the rise. To estimate public opinion on the subnational level, multilevel regression and poststratification (MRP) has become the standard method. Unfortunately, MRP comes with stringent data requirements, as it requires data for the joint distribution. As a consequence, scholars cannot apply MRP in countries without a census, and when census data is available, the modeling is restricted to a few variables. This article introduces multilevel regression with *marginal* poststratification (MR*m*P), which relies on marginal distributions only. The relaxed data requirement increases the prediction precision of the method and extends the use of MRP to countries without census data. Monte Carlo analyses as well as U.S. and Swiss applications show that, using the same predictors, MR*m*P performs as well as the current standard approach, and it is superior when additional predictors are modeled. These improvements promise that MR*m*P will further stimulate subnational research.

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1 Introduction

The comparative study of subnational units attracts growing interest in the literature. There are a number of reasons for this: subnational units are potentially better suited for comparative analysis than countries because they are less heterogeneous, more accurate data is available, country-specific factors are constant, and controlled comparisons allow to develop interesting identification strategies for causal inference (e.g., Snyder, 2001; Ziblatt, 2008; Tausanovitch and Warshaw, 2014). A critical but challenging element of subnational comparative research is the estimation of public opinion. The recent introduction of hierarchical modeling and poststratification, so-called MRP, generates reliable public opinion estimates for subnational units. Unfortunately, the use of MRP, as currently applied in the literature, comes with stringent data requirements. We build on the recent methodological advances and develop a new approach that extends the use of MRP beyond a few (post-)industrialized countries and that increases the prediction precision of the method substantially.

Early attempts in the estimation of public opinion disaggregated national surveys into subnational subsamples (Miller and Stokes, 1963). One solution for overcoming the small-n problem of the disaggregation approach was to combine multiple surveys with the same questions into one mega-poll (Erikson et al., 1993). Gelman and Little (1997), then, laid the methodological ground work for MRP, which replaced the older methods and can be applied by using standard national survey data (Lax and Phillips, 2009*b*; Warshaw and Rodden, 2012). The quick spread of MRP is quite remarkable—not least when we consider that the method has, to the best of our knowledge, only been applied in the U.S., U.K., and Switzerland so far.

The narrow spatial scope is mainly because of the stringent data requirement of the current standard application of the method. The precondition for using MRP is that detailed census data in the form of joint distributions is available for poststratification. Researchers need to know, for example, how many 18–35-year-old women with a university degree live in each subnational unit. This data requirement makes it impossible to apply MRP in countries where such data is not available—whether this is because of data protection laws (e.g., in India) or because the data is not gathered by a single agency (e.g., in Afghanistan). Furthermore, if census data is available, researchers can only use three or four demographic variables that are provided in the restrictive format of joint distributions as individual-level predictors of political preferences. Strong predictors such as party identification and income cannot be modeled.

We develop an alternative application of MRP, which we call multilevel regression with *marginal* poststratification (MRmP). MRmP relies on marginal distributions. For applying MRmP, researchers only need to know, for example, the shares (marginals) of women, of university graduates, and of 18–35-year-old citizens in each subnational unit. Due to this relaxed data requirement, MRmP can be applied in countries where it has not been possible (e.g., in India and Afghanistan). A further important advantage of MRmP is that it increases the prediction precision of subnational public opinion estimation for countries with a census (e.g., the U.S. and Switzerland). The findings of U.S. and Swiss data analyses show that the use of additional strong predictors such as income and party identification beyond the standard demographic variables improves the prediction precision of MRP substantially. Finally, we provide pre-modeling guidance to help researchers specify models that derive precise subnational public opinion estimates with MRP. In short, MRmP, the approach developed in this article, goes beyond the data limitations of the current standard application, extends the use of MRP, and substantially improves the prediction precision of the still young method. We thus believe that MRmP will further stimulate comparative research using public opinion estimates for empirical analysis.

This article first summarizes the standard application of MRP in the literature, which we call "classic MRP", before introducing MRmP. The theoretical part then shows that potential deviations in predictions between MRmP and classic MRP are solely due to the non-constant marginal effects in the response model. We derive the conditions under which this can make a difference and further investigate MRmP with Monte Carlo and real-world data analyses.

The findings document that MRmP substantially improves upon classic MRP. Finally, we provide MRP pre-modeling guidance and conclude.

2 "Classic MRP" and Its Limits

The application of MRP has its origins in the Gelman and Little (1997) study, which combined hierarchical modeling and poststratification. Park et al. (2004) subsequently introduced the method to political science with a remarkable impact on the discipline, as the substantial number of recent studies using the approach documents (Lax and Phillips, 2009*a*,*b*, 2012; Kastellec, Lax and Phillips, 2010; Warshaw and Rodden, 2012; Pacheco, 2012; Tausanovitch and Warshaw, 2014). In the last years, MRP has been established as the state-of-the-art method for comparative subnational research studying public opinion. Accordingly, Selb and Munzert (2011, 456) conclude that MRP is the "gold standard" in estimating political preferences on the subnational level.

MRP estimates public opinion on the subnational level in four steps. The first is to conduct a survey that identifies personal characteristics and asks a number of political preference questions; second, a hierarchical model is fitted to the data to make predictions for specific voter types; in the third step, predictions for all predefined voter types are calculated using the estimates of the hierarchical model; and, finally, researchers calculate, based on fine-grained census data, public opinion estimates in the subnational units by weighting the predictions of each voter type according to the number of voters living in the subnational units with the same characteristics.

Let us further illustrate the data requirements and the method. Researchers start with the collection of national survey data of usually somewhere between 500 and 1,500 respondents and model the responses on the support for a specific policy. For illustrative purposes, we explore an MRP response model with two individual-level variables – gender (men/women) and education (no high school, high school, college, post-graduate) – and with random effects for the subnational unit (α_c) and the region (α_r). In addition, we include a number of contextual factors explaining variation between the subnational units ($\mathbf{X}_c$). Accordingly, we write the following hierarchical probit, as a standard response model, to estimate the support for a specific policy:

$$
\begin{aligned}
Pr(y_i = 1) &= \Phi\left(\beta_0 + \alpha^{gender}_{j[i]} + \alpha^{education}_{m[i]} + \alpha^{subnational\ unit}_{c}\right) \\
\alpha^{gender}_{j} &\sim N(0, \sigma^2_{gender}),\ \text{for } j = 1, ..., J \\
\alpha^{education}_{m} &\sim N(0, \sigma^2_{education}),\ \text{for } m = 1, ..., M \\
\alpha^{subnational\ unit}_{c} &\sim N(\alpha^{region}_{r[c]} + \boldsymbol{\beta}\mathbf{X}_c, \sigma^2_{subnational\ unit}),\ \text{for } c = 1, ..., C \\
\alpha^{region}_{r} &\sim N(0, \sigma^2_{region}),\ \text{for } r = 1, ..., R
\end{aligned}
$$

One of the distinguishing features of MRP is the partial pooling and the shrinkage to the mean induced by modeling random effects (Steenbergen and Jones, 2002). This partly accounts for the good predictive performance of MRP. In this example, there are 8 voter types in each subnational unit (4 education categories and gender: $J \times M = 8$). The response model estimates are used to calculate predictions $\hat{\pi}_{cjm}$ for all possible combinations of j and m in each subnational unit c. For poststratification, the researcher needs to know the joint distributions in each subnational unit, that is, the frequency of each voter type ($N_{11}, N_{12}, ..., N_{22}$). We call MRP that relies on joint distributions "classic MRP" as it is the current standard method in the literature. Census officials provide, if available, the information on the joint distributions. Table 1 illustrates the data requirement of classic MRP.

Table 1: Census Data Requirement Example of Classic MRP

`education` / `gender`	*no high school*	*high school*	*college*	*post-graduate*	*Total*
men	$\mathbf{N_{11}}$	$\mathbf{N_{12}}$	$\mathbf{N_{13}}$	$\mathbf{N_{14}}$	$N_{1\cdot}$
women	$\mathbf{N_{21}}$	$\mathbf{N_{22}}$	$\mathbf{N_{23}}$	$\mathbf{N_{24}}$	$N_{2\cdot}$
Total	$N_{\cdot 1}$	$N_{\cdot 2}$	$N_{\cdot 3}$	$N_{\cdot 4}$	N

Finally, each prediction is weighted by the joint distribution data and the total sum divided by the number of all residents:

$$\hat{\pi}_c = \frac{\sum_j \sum_m \hat{\pi}_{jm\in c} N_{jm\in c}}{N_{n\in c}} = \frac{\sum_j \sum_m \Phi\left(\hat{\beta}_0 + \hat{\alpha}_m + \hat{\alpha}_j + \hat{\alpha}_c\right) N_{jm\in c}}{N_{n\in c}}$$

In this example, we rely on eight voter types. Real-world applications include more individual-level random effects. Lax and Phillips (2009*b*), for example, work with gender, three races, and four education and age categories (96 types) and thus need, among others, data on the exact number of 18–29-year-old black women with a high school degree in each state. Such fine-grained information is only available if a detailed census has been carried out. If joint distribution data is available, the specification of the response model is pre-determined by the data of the census bureau (not the modeling decisions of researchers) and usually restricted to standard demographic variables. The demanding data requirements of classic MRP are problematic for the following reasons:

- In developing countries, fine-grained census data is not available. In Afghanistan, for example, no census is carried out, while in the more developed India, census data on the village level is not available as a joint distribution.
- The joint distributions of the census in (post-)industrialized countries do not include variables that are potentially important predictors of political preferences. Party identification, for example, is neither available in the Swiss nor in the U.S. census as joint distributions.

Before political scientists started working with MRP, the standard method for deriving preference measures on the subnational level was the disaggregation of national surveys into subnational samples. While this method is free from data availability restrictions, the estimates for small constituencies are unreliable, as they stem from very few observations (Levendusky et al., 2008). Several studies have shown that MRP estimates are more precise than disaggregation estimates (e.g., Lax and Phillips, 2009*b*). Following the MRP literature, we use disaggregation estimates as a baseline measure to gauge the extent to which MRP improves predictive precision. Our main goal in this analysis is to build on the strength of MRP by relaxing the strict data requirements of the current standard application in the literature.

3 MRP with Synthetic Joint Distributions (MR*m*P)

Even some of the most sophisticated MRP contributions are constrained by the data requirements discussed above. Warshaw and Rodden (2012, 208), for example, study district-level public opinion in the U.S. without using age as a predictor in their response model because the "census factfinder does not include age breakdowns for each race/gender/education subgroup." This example highlights the data availability problem of classic MRP: while U.S. district data on the age structure is available as a marginal distribution, data on the exact number of elderly people with a given gender, age, and education is not. More generally, marginal distributions are available for many interesting variables in most countries. In the Afghan case, for example, the Asia Foundation collects data on the ethnic and linguistic structures of subnational populations.

The key deviation of MR*m*P, the approach developed in this article, is that it relies on synthetic joint distributions that are created with data on the marginal distributions (while classic MRP relies on the "true" joint distributions). Our point of departure is that researchers can collect data on the population structures of subnational units as marginal distributions for various variables that are potentially important predictors of political preferences. Instead of relying exclusively on the demographic variables of the census, MR*m*P allows the modeling of any political, social, economic, or demographic variable asked in the survey. Researchers only need the marginal distributions of these variables in the subnational units, which are more widely available.

MRmP calculates the synthetic joint distributions for the poststratification step with data on the marginal distributions. The synthetic joints can be computed in two different ways: either by extracting the data from multiple surveys (Kastellec et al., 2014), or, as we propose, by estimating the product of the marginal distributions as a rough approximation of the true joint distribution. The estimated synthetic joint will be correct, when the used variables are independent of one another. If they are not—which is in all likelihood the case—the synthetic joint will deviate from the true joint distributions. However, as we will show, the deviation only affects the MRP predictions through the non-constant marginal effects of the probit link function in the response model. In terms of prediction precision, the differences are essentially irrelevant, as the findings of the Monte Carlo simulations and the real-world data applications show. We will come back to this shortly.

The following theoretical discussion illustrates the difference between classic MRP and MRmP by emphasizing the scenario under which the two estimation procedures are most distinct. Table 2 shows an example of two binary individual-level variables, `v1` and `v2`, that are completely separated and thus totally dependent. The synthetic joint distribution, estimated as the product of the marginal distributions (60% and 40%), deviates quite strongly from the true joint distribution in this most extreme case (compare Table 2.1 and 2.2).

Table 2: True and Synthetic Joint Distributions for the Most Extreme Case

v1 \ v2	i=1	i=2	
j=1	60%	0%	60%
j=2	0%	40%	40%
	60%	40%	100%

2.1: True Joint Distribution

v1 \ v2	i=1	i=2	
j=1	36%	24%	60%
j=2	24%	16%	40%
	60%	40%	100%

2.2: Synthetic Joint Distribution

v1 \ v2	i=1	i=2
j=1	7%	50%
j=2	69%	98%

2.3: Predicted Support

Notes: Example of a synthetic joint distribution for the most extreme case where the two individual-level variables are fully dependent. The predicted support in Table 2.3 is based on a model that includes two random effects (α^{v1} and α^{v2}) with the following estimates: $\hat{\alpha}_1^{v1} = -1$, $\hat{\alpha}_2^{v1} = +1$, $\hat{\alpha}_1^{v2} = -0.5$, and $\hat{\alpha}_2^{v2} = +1$.

What matters for applied scholars is how much the predictions differ with the use of the synthetic joint distribution (MRmP) compared to the true joint distribution (classic MRP). To estimate the difference in predictions, we assume a response model, which is identical in both procedures, with two random effects on the individual level (age and education) and an estimated constant with the value 0. As estimates for the random effects, we pick fairly large numbers (from -1 to $+1$) to magnify the difference between classic MRP and MRmP. In applied work, random effects are clearly smaller (see Appendix A.3). The predicted probability for an individual of a specific cell is estimated using the random effects (e.g., for an individual from the upper left cell: $\hat{p}_{11} = \Phi(\hat{\alpha}_1^{v1} + \hat{\alpha}_1^{v2})$). Table 2.3 reports the predicted support for each cell based on the response model estimates. The support in the subnational unit is estimated by weighting the predictions for each type by its frequency in the population. With the true joint distribution (classic MRP, see Table 2.1), the support in the subnational unit, $\hat{p}_{true}$, is estimated as follows:

$$
\begin{aligned}
\hat{p}_{true} &= 0.6 \cdot \Phi(\hat{\alpha}_1^{v1} + \hat{\alpha}_1^{v2}) + 0.4 \cdot \Phi(\hat{\alpha}_2^{v1} + \hat{\alpha}_2^{v2}) \\
&= 0.6 \cdot \underbrace{\Phi(-1.5)}_{0.07} + 0.4 \cdot \underbrace{\Phi(+2)}_{0.98} = 0.431
\end{aligned}
$$

Using the synthetic joint distribution (MRmP, see Table 2.2), the support in the subnational unit, $\hat{p}_{syn}$, is estimated as follows:

$$
\begin{aligned}
\hat{p}_{syn} &= \underbrace{0.24 \cdot \Phi(\hat{\alpha}_1^{v1} + \hat{\alpha}_2^{v2})}_{\text{should be 0\% of pop}} + \underbrace{0.36 \cdot \Phi(\hat{\alpha}_1^{v1} + \hat{\alpha}_1^{v2})}_{\text{should be 60\% of pop}} + \underbrace{0.24 \cdot \Phi(\hat{\alpha}_2^{v1} + \hat{\alpha}_1^{v2})}_{\text{should be 0\% of pop}} + \underbrace{0.16 \cdot \Phi(\hat{\alpha}_2^{v1} + \hat{\alpha}_2^{v1})}_{\text{should be 40\% of pop}} \\
&= 0.24 \cdot \underbrace{\Phi(0)}_{0.5} + 0.36 \cdot \underbrace{\Phi(-1.5)}_{0.07} + 0.24 \cdot \underbrace{\Phi(+0.5)}_{0.69} + 0.16 \cdot \underbrace{\Phi(+2)}_{0.98} \\
&= 0.466
\end{aligned}
$$

In this most extreme example, the deviation in predicted support is only 3.5%. The prediction deviation is surprisingly small, considering that we choose variables that are perfectly correlated and high values for the estimated random effects. To understand the source of the deviation, it is important to recall that the synthetic joint distribution is computed using the correct marginal distributions: it accounts for 60% of the population with the characteristic $\texttt{v1} = 1$, for 60% with $\texttt{v2} = 1$, for 40% with $\texttt{v1} = 2$, and for 40% with $\texttt{v2} = 2$; wrong are only the joint distribution values. However, the prediction deviation of 3.5% is not so much a product of the wrong synthetic joint distribution values but rather because of the non-constant marginal effects of the probit link function in the response model. In a probit model, adding α_1^{v1} to a hypothetical person with `v2 = 0` has a different marginal effect than it has on a person with `v2 = 1`.

Let us illustrate that point by looking at how the support in the subnational unit is estimated in both procedures. In this illustrative case, all individuals of the first row of the matrix ($\texttt{v1} = 1$) also have the characteristic $\texttt{v2} = 1$ (see Table 2.1). For example, all women are also university graduates (and no man has a university degree). Accordingly, women's predicted support of 7% for the policy is weighted by 0.6 (see the first part of the equation using the true joint distribution). In the estimation with the synthetic joint distribution, women's predicted support is overestimated, as the 7% support is only weighted by 0.36 (see the second part of the equation using the synthetic joint distribution), while the additional 0.24 of women (with the characteristic $\texttt{v1} = 1$, i.e., no university degree) is multiplied with a higher predicted probability of 50% (see the first part of the equation using the synthetic joint distribution). As there are no women with no university degree in that illustrative example, women's predicted support for the policy is overestimated.

For men ($\texttt{v1} = 2$), the equation using the true joint distribution weights the very high 98% probability of men's support for the policy by 0.40. The prediction using the synthetic joint distribution, however, underestimates men's support for the policy, as it weighs the 98% support only for 0.16 and adds 0.24 with 69% support. Overall, MRmP overestimates women's and underestimates men's support for the policy in this example. In a linear model, this over- and underestimation of predicted support between men and women would cancel each other out—but not in a probit model: the non-constant marginal effects in a probit model explain the small prediction deviation between classic MRP and MRmP of 3.5%.[1] Based on this analysis, we derive two important implications. First, the deviation in predictions between classic MRP and MRmP is only because of the non-linear link function of the probit model. Second, the non-constant marginal effects of the probit model only cause deviations in predictions to the extent to which the individual-level variables are correlated.

MRmP is related to *raking*, a procedure in survey methodology, which is an alternative to poststratification when only marginal distributions are available. Raking works as follows: weights are assigned based on one of the marginal distributions and then, conditional on the derived weights, new weights are calculated based on the marginal distribution of the second variable, and so on. This estimation process, known as iterative proportional fitting, is repeated until the marginals of the weighted sample converge to the marginals of the population and is applied when the sample is not representative (Deming and Stephan, 1940; Fienberg, 1970). Unlike raking, MRmP does not create weights for every single observation, but rather a weight for each ideal type as specified in the response model. What is similar, however, is that both methods rely on marginal distributions instead of joint distributions.

Table 3 summarizes the distinction between classic MRP and MRmP. The most important advantage of MRmP is that it allows the modeling of individual-level variables, of which only the marginal distributions are available, while having data on the true joint distribution is a *conditio sine qua non* for classic MRP. MRmP thus extends the use of MRP to countries without a census and makes the application of MRP more flexible for countries where census data is available.

[1] Note that in the linear case, the two equations predicting public opinion are identical. We have explored MRP with a linear model, as there is something like a folk theorem (at least among economists) that a linear model performs as well as a probit model for binary outcomes (Angrist and Pischke, 2008; Beck, 2011). However, MRP with a linear model does not perform optimally. The main problem is that it tends to produce estimates that are less than 0 or greater than 1. This is one of the criticisms of using linear models for binary outcome variables (Maddala, 1983, 16).

Table 3: Date Requirement and Model Flexibility of Classic MRP and MRmP

	MRmP	Classic MRP
True joint distribution needed	✗	✓
Marginal distributions are sufficient	✓	✗
Flexible modeling of response model	✓	✗

The potential downside of MRmP is that the correlation between individual-level variables induces a small deviation in prediction because of the non-constant marginal effects of the probit model.[2] The discussed illustrative example explored the most extreme (and unrealistic) case with a perfect correlation between the two variables. If they were totally independent—which is also unrealistic—classic MRP and MRmP would provide the same results. In general, the deviation between the predictions becomes larger as the correlation increases.[3] The deviation in prediction between the two MRP approaches also increases with higher variances of the random effects. The next sections analyze whether MRmP performs as well as MRP in applied settings, and whether MRmP outperforms classic MRP when additional, powerful, individual-level predictors are modeled.

[2] A second limitation is that deep interactions can only be modeled if the two constituting variables are available in the census data (Ghitza and Gelman, 2013).

[3] See Appendix A.1 for correlations in Swiss and U.S. data.

4 MRmP and Classic MRP with the Same Data

Should we expect that predictions differ between MRP and MRmP in applied work? To answer that question, we execute Monte Carlo analyses with two manipulated parameters: first, we change the sample sizes from a small sample of 500 respondents to a medium sample of 1,000 respondents and a large sample of 2,000 respondents. Second, we use four different correlations (ρ) among the individual-level variables (0, 0.2, 0.4, and 0.6). In the case of no correlation, MRmP and classic MRP are equivalent (with perfect independence, the product of the marginals equals the true joint distribution). Correlations of 0.2 and partly also of 0.4 are realistic for standard demographic values, while 0.6 is exceptionally high (see Appendix A.1). As discussed in the previous section, the prediction deviation between the two procedures should increase as the correlation becomes larger.

The data-generating-process (DGP) assumes three variables on the individual level and a variable on the subnational level. We analyze 25 subnational units; 10 of them are large (each covering 7% of the total population) and 15 are small (each covering 3% of the total population). The following equations describe the DGP:

$$\begin{aligned}
y_i^* &= \beta_0 + \gamma_{1[i]} + \gamma_{2[i]} + \gamma_{3[i]} + \alpha_c^{subnational\ unit} + \varepsilon_{ic} \\
Pr(y_i = 1) &= \Phi(y_i^*),\ \varepsilon_{ic} \sim N(0,4) \\
\bar{\gamma}_{1i} &\sim N(0,1)\ \&\ \bar{\gamma}_{2i} \sim N(0,1)\ \&\ \bar{\gamma}_{3i} \sim N(0,1) \\
\alpha_c^{subnational\ unit} &\sim N(X_c, 4),\ \text{for } c = 1, ..., 25
\end{aligned}$$

The individual-level variables ($\gamma_1, \gamma_2, \gamma_3$) are based on draws from a multivariate normal distribution and transformed to discrete variables with four categories each:

$$\gamma_{ki} = \begin{cases} 1 & \text{if } \bar{\gamma}_{ki} < -1, \\ 2 & \text{if } -1 < \bar{\gamma}_{ki} < 0, \\ 3 & \text{if } 0 < \bar{\gamma}_{ki} < 1, \\ 4 & \text{if } \bar{\gamma}_{ki} > 1 \end{cases} \quad \text{and Var}(\bar{\boldsymbol{\gamma}}) = \begin{pmatrix} 1 & \rho & \rho \\ \rho & 1 & \rho \\ \rho & \rho & 1 \end{pmatrix} \text{for } k = 1,\ 2,\ 3.$$

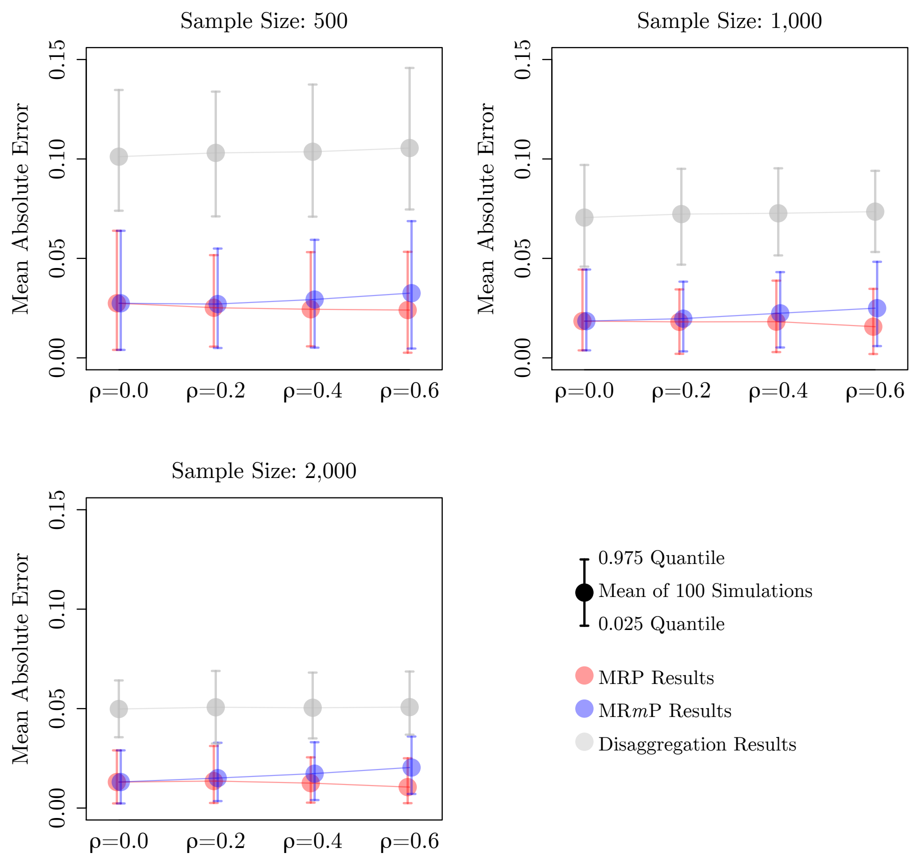
The selected random effects (γ_{ki}) are quite large in size as they are based on the following normal distributions ($\gamma_k \sim N(0,1)$, $\forall\ k$). In real-world examples, the random effects are smaller (the conservative setup of the analysis tends to overestimate the prediction deviation between classic MRP and MRmP).[4] The subnational level variable, X_c, is evenly spread between -2 and $+2$. For the Monte Carlo analyses, we create a "true" population of one million citizens, draw for every simulation a new sample, and estimate disaggregation, classic MRP, and MRmP predictions.

Figure 1 shows the prediction precision for 12 different Monte Carlo analyses (with three different sample sizes and four different correlations) for disaggregation, classic MRP, and MRmP. Each of the three plots reports the simulation results for the four different correlations (0, 0.2, 0.4, 0.6) of the individual-level variables and for one of the three sample sizes (500, 1,000, 2,000), respectively. Concretely, the dots show the mean absolute error (MAE) for disaggregation, classic MRP, and MRmP of 25 subnational unit predictions that were estimated with 100 simulations each. The intervals document the range of the 100 MAEs for the three estimation approaches.

The findings confirm our expectations. First, as other studies have already shown, MRP systematically outperforms disaggregation (e.g., Lax and Phillips, 2009*b*). Second, in case the correlation between the individual-level variables is 0, classic MRP and MRmP lead to exactly the same predictions. Third, increasing the sample size improves disaggregation but does not change the relative performance of MRP versus MRmP. Finally, and most importantly, the deviations in predictions between classic MRP and MRmP grows as the correlation between the individual-level variables increases. Yet the predictions between classic MRP and MRmP are only distinguishable if the correlation is at a high level of 0.6. Such a high correlation of individual-level variables is very unusual in applied work (see Appendix A.1), which suggests that the deviations in prediction between classic MRP and MRmP are most likely negligible for real-world data applications.

[4]For example, in the MRP analysis of state public opinion by Kastellec, Lax and Philipps (2010), the largest random effect has a variance of 0.3. In this study, the largest random effect has a variance of 0.45 (see Appendix A.3).

Figure 1: Monte Carlo Analyses for Three Sample Sizes and Four Correlation Levels



Notes: Each plot shows the simulation results for a specific sample size ($N \in \{500, 1000, 2000\}$). The y-axis shows the mean absolute error (MAE) and the x-axis the different correlations among the individual-level variables.

To further investigate the claim that there are most likely no deviations in prediction between classic MRP and MRmP in applied work, we analyze data on 186 direct democratic votes in Switzerland between 1990 and 2010.[5] We estimate the cantonal support with the true joint distributions (classic MRP) and the synthetic joint distribution (MRmP) by using data from the national VOX surveys (n $\approx 500 - 1,000$) and compare the predictions to the actual vote outcomes. We rely on a standard response model, including the demographic variables

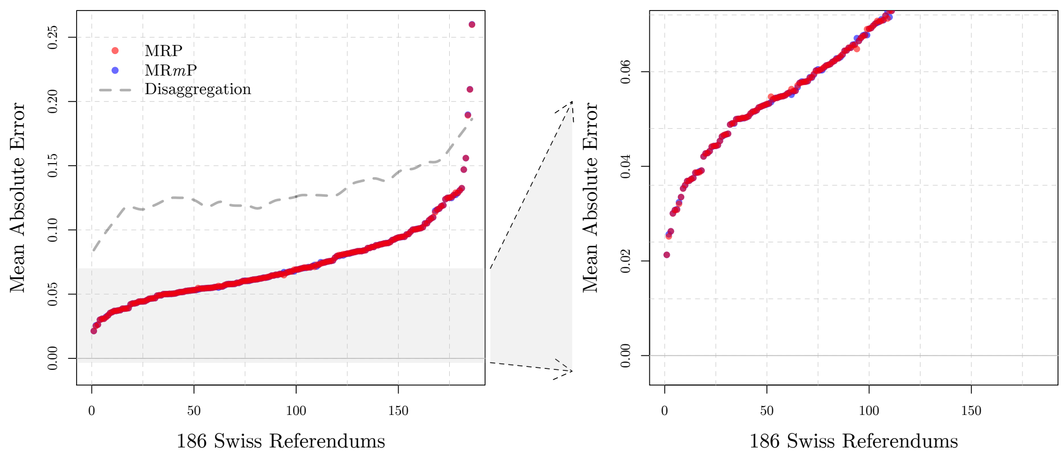
[5]For the analysis, we use three different data sources: the Federal Statistical Office (BfS) collects vote outcome data for the cantons for all 186 direct democratic votes; the joint distributions for each canton are from the 2000 census; and the survey data is from the VOX research (Kriesi, 2005).

available as joint distributions from the census (gender, education, and age), the shares of German speakers and of Catholics as predictors on the subnational (i.e., cantonal) level, and random effects for regions and cantons. For all 186 votes, we estimate the following response model:

$$
\begin{aligned}
Pr(y_i = 1) &= \Phi\left(\alpha_0 + \boldsymbol{\beta}\mathbf{X}_c + \alpha^{gender}_{j[i]} + \alpha^{education}_{k[i]} + \alpha^{age}_{m[i]} + \alpha^{canton}_{c[i]} + \alpha^{region}_{r[i]}\right) \\
\alpha^{gender}_{j} &\sim N(0, \sigma^2_{gender}), \text{ for } j = 1, ..., J \\
\alpha^{education}_{k} &\sim N(0, \sigma^2_{education}), \text{ for } j = 1, ..., K \\
\alpha^{age}_{m} &\sim N(0, \sigma^2_{age}), \text{ for } m = 1, ..., M \\
\alpha^{canton}_{c} &\sim N(0, \sigma^2_{canton}), \text{ for } c = 1, ..., C \\
\alpha^{region}_{r} &\sim N(0, \sigma^2_{region}), \text{ for } r = 1, ..., R
\end{aligned}
$$

Figure 2 reports the MAEs for disaggregation, classic MRP, and MRmP over all 186 direct democratic votes. The classic MRP and MRmP performance similarities are striking. The estimates are so close that we can only identify differences once we zoom in (see right plot). The findings show that there is no difference in prediction precision between the two methods. While the Monte Carlo analysis already suggested that we will not find prediction deviations between classic MRP and MRmP in real-world data applications, the findings of the Swiss analysis support that claim. The results are virtually identical because both factors that theoretically drive the estimates of the two methods apart are small. The random effects in the response model are lower than in the Monte Carlos analyses (the variances of the random effects are less than 0.1), and the correlations among the individual-level variables are small (with a maximum of $\rho = -0.2$, see Appendix A.1).

Figure 2: Prediction Precision of Disaggregation, Classic MRP, and MRmP Estimates with the Same Data for 186 Swiss Votes



Notes: MAEs for 186 public votes. The right plot zooms in on the lower region of the left plot. The gray line reports the MAEs for disaggregation, the red dots show for classic MRP, and the blue dots for MRmP.

5 How MRmP Outperforms Classic MRP

The advantage of MRmP—namely, that it allows the modeling of additional powerful individual-level predictors—promises to improve prediction precision. The specification of the response model on the individual level is in classic MRP pre-determined by the census data (and restricted to three or four demographic variables). This limitation constrains even the most sophisticated research in the literature. For example, in the works of Lax and Phillips (2009*a*) and Warshaw and Rodden (2012), religion and age are important predictors of the political preferences they are investigating. Yet they could not include these variables on the individual level in their studies because of data availability reasons. Following Lax and Phillips (2009*a*), the standard procedure in the literature is to model variables that are not available in the joint distribution format as predictors on the subnational level of the response model (instead of the individual level). This is a reasonable strategy when these variables better explain variation among the units rather than within.

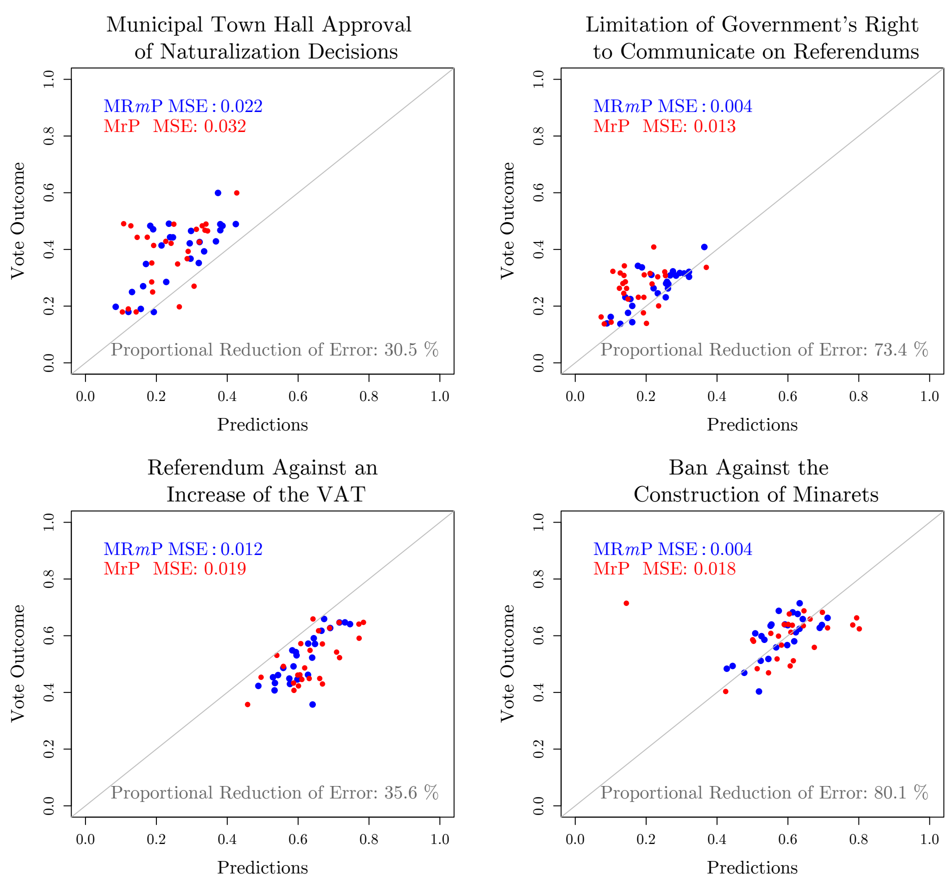
In case of MRmP, however, researchers only need marginal distributions (which is obviously unproblematic for age and religion in subnational units of the U.S.). Accordingly, the set of variables that can be modeled on the individual level is greatly enhanced with MRmP. Potentially interesting predictors are party identification, income, and employment status—just to name a few. The marginal distributions of these variables are typically available for subnational units. Which of these (or other) variables are potentially powerful predictors depends on the political preferences of interest. The key question is whether we can improve the prediction precision, when interesting predictors of political preferences are modeled as random effects on the individual level (MRmP) as compared to classic MRP, where such variables are included on the subnational level.

We first investigate that question with the Swiss data introduced above. The most important recent development in Swiss politics is the rise of the Swiss People's Party (SVP, see Kriesi et al., 2005). Particularly after 2007, when the leader of the SVP was not re-elected in the federal government, the party relied strongly on direct democratic campaigns to reinforce the narrative that they are in opposition against the "classe politique". Accordingly, in the legislative period from 2007–2011, identification with the SVP was a strong predictor of whether voters support SVP referendums and initiatives, as several exit poll analyses show. We analyze the following four public votes of that legislative period, where the SVP was starkly engaged against the unified coalition of all other relevant Swiss parties and for which VOX survey data is available:

- Initiative for municipal town hall approval of naturalization decisions.
- Initiative to limit the government's right to communicate in referendum campaigns.
- Referendum against an increase of the VAT for disability insurance.
- Initiative to ban the construction of minarets.

For the estimation of subnational public opinion, we specify the same baseline response model as discussed above with gender, education, and age as individual-level random effects, the shares of German speakers and of Catholics as cantonal variables, and random effects for regions and cantons. The baseline specification is extended for MRmP by adding party identification as an additional random effect on the individual level, while party identification is modeled for classic MRP as a subnational (i.e., cantonal) variable (like the shares of German speakers and of Catholics). We again predict the cantonal support for the initiatives and referendums with MRmP and classic MRP and compare the predictions to the actual results. Figure 3 plots the MRmP and classic MRP predictions against the true vote outcomes. In all four public votes, MRmP clearly outperforms classic MRP. The improvements in prediction precision are substantial, going up to an 80% reduction of prediction error in the case of the ban against the minarets (i.e., a more than 4 times smaller mean squared error (MSE)). The significant improvements show that modeling party identification as a random effect on the individual level for SVP public votes leads to more accurate predictions than introducing that variable on Level 2.[6]

Figure 3: Public Vote Outcomes and Classic MRP and MRmP Estimates for SVP Initiatives and Referendums with Party Identification as Additional Predictor



Notes: The $x-$axis reports the estimated share of yes votes and the $y-$axis the true cantonal vote outcomes. MRmP includes party identification as a random effect on the individual level, while classic MRP includes party identification as a cantonal variable on Level-2. The sample sizes (N) vary between 525 and 680.

[6] Appendix A.2 reports a second Swiss example with income as an additional predictor of tax policy preferences. The findings are substantively the same.

We additionally investigate a U.S. example, building on the Warshaw and Rodden (2012) analysis of the estimation of public opinion in state legislative districts. To cross validate the findings, they compare the MRP predictions to the actual vote outcomes for direct democratic votes on same-sex marriage in Arizona, California, Michigan, Ohio, and Wisconsin. The response model includes race, gender, and education as individual-level predictors and, as district-level predictors, the median income, the shares of the urban population, of veterans, and of same-sex couples. The authors explicitly state that age is a critical predictor that they cannot model because the census has no data breakdown for race/gender/education *and* age (Warshaw and Rodden, 2012, 208). This is a case where MRmP goes beyond the data limitation of current MRP applications as the marginal distributions of age are available for U.S. state legislative districts.

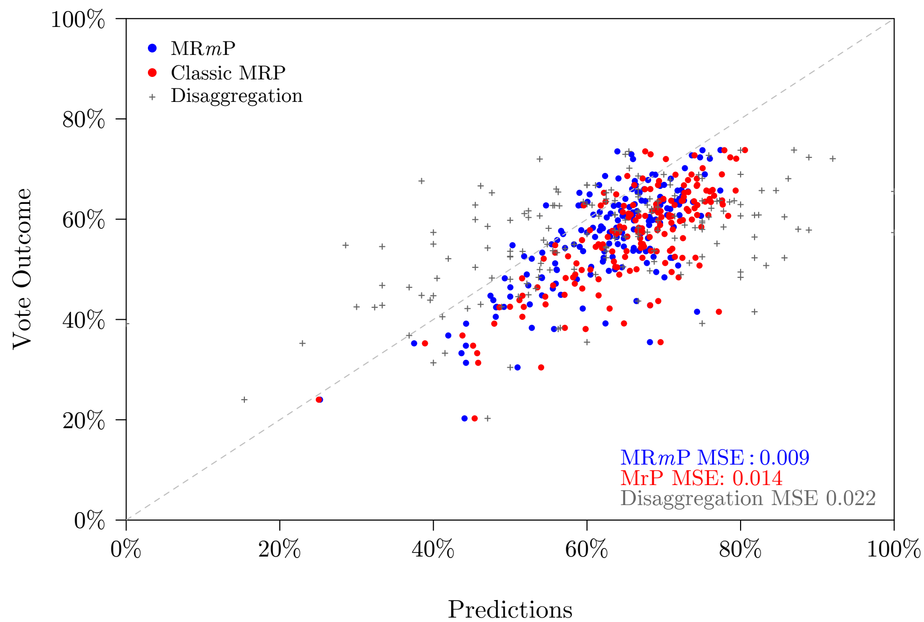
For the analysis, we replicate the Warshaw and Rodden (2012) public opinion estimates[7] before executing the MRmP analysis using synthetic joint distributions for race, gender, education, and age. In the case of MRmP, we model age, which cannot be modeled in classic MRP, as an individual-level predictor of same-sex marriage preferences. Figure 4 plots the disaggregation, classic MRP, and MRmP estimates against the true vote outcome. The disaggregation MSE is 0.022, that of classic MRP 0.014, and that of MRmP 0.009. Relying on that measure, MRmP improves upon classic MRP as much as classic MRP improves upon disaggregation.

The U.S. analysis highlights the conditions under which MRmP provides substantially better predictions than classic MRP. The improvement in prediction is because age is not strongly correlated with gender, race, and education,[8] and because age is an important individual-level predictor of political preferences in same-sex marriage questions: no other estimated random effect is that large in the response model (education is the second largest with a variance of almost half the size, see Appendix A.3). Both questions, how strong the predictive power of the individual-level variables are and the extent to which they correlate, can be analyzed before

[7] We are indebted to the authors for providing us with detailed replication files and their dataset.

[8] The correlations vary from -0.17 (race) to -0.01 (education), and 0.05 (gender). See Appendix A.1.

Figure 4: Public Vote Outcomes and Disaggregation, Classic MRP, and MRmP Estimates for the Warshaw and Rodden (2012) Analysis on Same?Sex Marriage Referendums in Arizona, California, Michigan, Ohio, and Wisconsin with Age as Additional Predictor



Notes: The $x-$axis reports the estimated share of yes votes and the $y-$axis the true vote outcome for state senate districts. MRmP includes age as a additional individual-level random effect, while age cannot be modeled in classic MRP.

making model specification decisions. Based on the presented Monte Carlo simulations and the Swiss and U.S. analyses, applied researchers should proceed as follows, when considering using MRP:

1) Analyze the survey data and explore which individual-level variables are strong predictors of the studied political preferences (see Appendix A.3).

2) Check which variables are available as joint distributions from the census.

   2.1) Use classic MRP if all important individual-level predictors are available as joint distribution: poststratify with the joint distribution data.

   2.2) Use MRmP if joint distributions are not available; collect the marginal distributions of the individual-level predictors, multiply the marginals to compute the synthetic joint distribution, and poststratify with the synthetic joint distribution.

   2.3) Consider using MRmP if joint distributions are available, but not for a strong individual-level predictor (e.g., income or party identification in the U.S. and Swiss cases).

      2.3.1) Check the correlations of the individual-level variables.

2.3.2) Use MRmP if the correlations are small to moderate; create the synthetic joint distribution by multiplying the marginal distribution of the strong individual-level predictor with the joint distribution data and poststratify with the synthetic joint distribution.

To sum up, while for some applications classic MRP provides good measures of subnational public opinion, there are many other cases where MRmP improves upon the standard application of the method. MRmP extends the use and prediction precision of MRP—depending on the data availability and the strength of the individual-level predictors.

6 Conclusion

The comparative study of subnational units has attracted growing interest, and the estimation of reliable public preference measures for subnational units is a critical element in empirical research in that literature. MRP generates reliable subnational public opinion estimates with standard national polling data. The numerous MRP studies published in the last years show that the method stimulates interesting research – for example, on the responsiveness of subnational politicians and administrations to voters' preferences (Lax and Phillips, 2012; Tausanovitch and Warshaw, 2014). However, the application of MRP has been restricted to a few countries because of the stringent data requirement of the current standard approach, which requires detailed census data in the form of joint distributions (researchers need to know, for example, how many 18–35-year-old women with a university degree live in each subnational unit).

The presented alternative application of MRP, MRmP, relies on the marginal distribution of individual-level variables (e.g., the shares of women, of university graduates, and of 18–35-year-old citizens in each subnational unit), which extends the use of the method to countries without joint distribution census data. This extension of subnational public opinion estimation with MRP is important to stimulate comparative subnational research in less developed countries with more restricted data availability. This article compared MRmP and the current standard MRP approach theoretically using Monte Carlo analyses and Swiss and U.S. data examples. The findings show that using the same predictors, MRmP performs as well as the current standard approach, and that MRmP increases the prediction precision when additional strong predictors beyond the standard demographic variables are added to the response model.

The improvements of MRmP should also further stimulate subnational comparative research in (post-)industrialized countries with census data. So far, scholars have relied on rather generic response models with three or four demographic individual-level variables as predictors of various policy preferences. With MRmP, the individual-level predictors can be selected depending on the political preferences of interest. Public opinion polls show, for example, that churchgoing is associated with policy views on abortion, and views on free trade policies are correlated with the trade exposure of an individual's job (Mayda and Rodrik, 2005). The presented findings suggest that modeling such strong predictors increases the prediction precision of MRP substantially. MRmP thus takes MRP to new countries and improves the method by allowing more model flexibility. The guidance provided in the article helps scholars to develop MRP applications that take full advantage of the data that is available to estimate the subnational public policy preferences of interest.

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A Appendix

A.1 Correlation of Individual-Level Variables

The Monte Carlo analyses suggests that correlations below $\approx |0.4|$ should be unproblematic for MRmP applications. In the U.S. and Swiss data, the correlations are much lower, as the Table 4 and 5 show.

Table 4: Average Correlation Matrix over 186 Swiss Exit Polls

	Education	Age	Gender
Education	1.00	-0.12	-0.20
Age	-0.12	1.00	0.02
Gender	-0.20	0.02	1.00

Table 5: Correlation Matrix from U.S. Data (Warshaw and Rodden (2012))

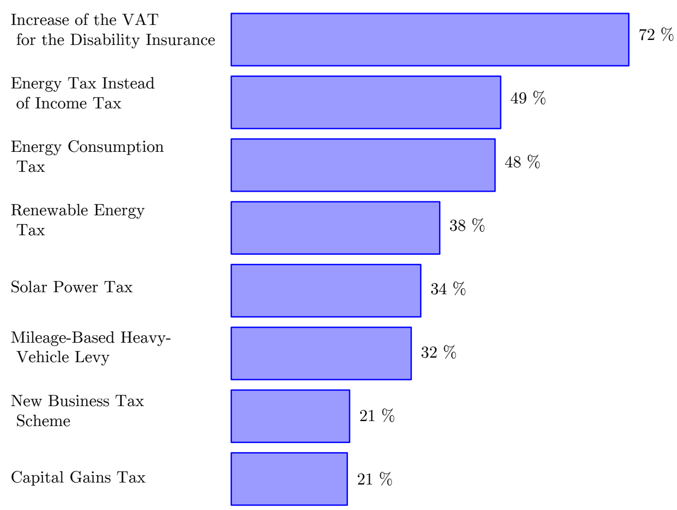
	Age	Education	Race	Gender
Age	1.00	-0.01	-0.17	0.05
Education	-0.01	1.00	-0.09	-0.04
Race	-0.17	-0.09	1.00	-0.01
Gender	0.05	-0.04	-0.01	1.00

A.2 Additional Swiss Analysis

The article shows that MRmP increases the prediction precision compared to classic MRP with the example of introducing party identification as an additional individual-level predictor for SVP initiatives and referendums for the 2007 − 2011 legislative period. Another interesting individual-level predictor of political preferences is income. Earned income has been identified in the literature as an important determinant of tax policy preferences that is typically politicized by the left (Corneo and Grüner, 2002; Bartels, 2008).

We find 8 public votes on taxation with a distinct left-right campaign dynamic in the Swiss survey data (VOX). Like in the SVP example, we rely on the baseline specification of the response model. For assessing the gains in prediction precision, we introduce income as an additional random effect for MRmP and compare the MRmP predictions to classic MRP, where we model income as a variable on the subnational (i.e., cantonal) level.

Figure 5: Reduction in Mean Squared Errors between Classic MRP and MRmP Estimates



Notes: Swiss votes on taxation with income as additional predictor; sample sizes vary between 445 and 819.

The proportional mean squared error reductions reported in in the figure above corroborate that MRmP outperforms classic MRP, when a powerful predictor of the investigated political preferences is introduced on the individual level. The mean squared error reductions between 21% and 72% are again substantial improvements in prediction precision.

A.3 Replication of Warshaw and Rodden (2012)

Table 6 presents the response model estimates of the Warshaw and Rodden (2012) replication analysis. The MRmP model includes age as an individual-level predictor. The random effect for age is large (and the other random effects are also larger in the MRmP model), which shows, together with the AIC and BIC values, that introducing age increases the predictive power of the model substantially.

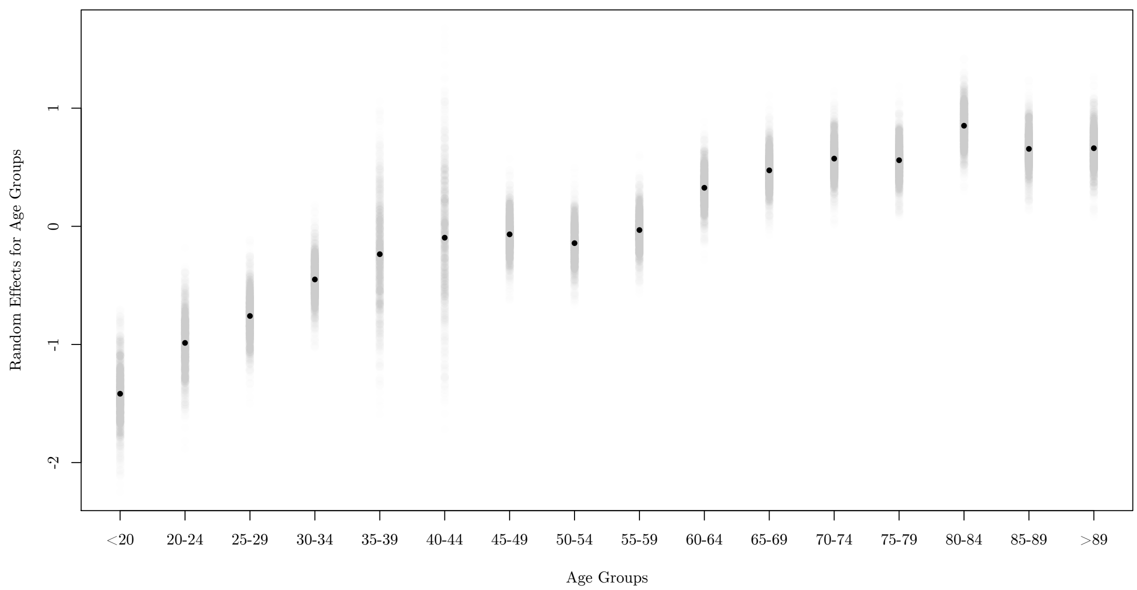
Table 6: MRP and MRmP Response Model Estimates

	MRmP Model	MRP Model
Gender	-0.49^{***}	-0.43^{***}
	(0.04)	(0.03)
Income (district)	-0.01^{***}	-0.01^{***}
	(0.00)	(0.00)
Urban (district)	-0.65^{***}	-0.60^{***}
	(0.10)	(0.10)
Veteran (district)	-0.59	-0.42
	(0.56)	(0.55)
Religion (state)	1.70^{***}	1.56^{***}
	(0.28)	(0.27)
Union members (state)	-0.91	-0.82
	(0.63)	(0.61)
Same sex couples (district)	-34.17^{***}	-34.35^{***}
	(3.41)	(3.13)
Constant	2.48^{***}	2.15^{***}
	(0.36)	(0.29)
Variance: `district`	0.08	0.07
Variance: `state`	0.01	0.00
Variance: `age.group`	0.45	not included
Variance: `education.group`	0.24	0.21
Variance: `region`	0.01	0.01
Variance: `race`	0.07	0.04
AIC	20415.85	21103.44
BIC	20524.72	21204.53
Num. obs.	17611	17611
Num. of districts	1779	1779
Num. of states	48	48
Num. groups: age.group	16	
Num. groups: education5	5	5
Num. groups: region	4	4
Num. groups: race2	4	4

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

To further illustrate how strongly age is correlated with attitudes towards same-sex marriage, Figure 6 plots the 16 different random effects for age from the MRmP response model. The older a respondent is, the larger is the estimated random effect ($y = 1$ is equivalent to being opposed to same-sex marriages).

Figure 6: Random Effects for Age from the MRmP Response Model



Notes: Estimates of the random effects and 1,000 simulations drawn from the posterior vector for illustrating estimation uncertainty.