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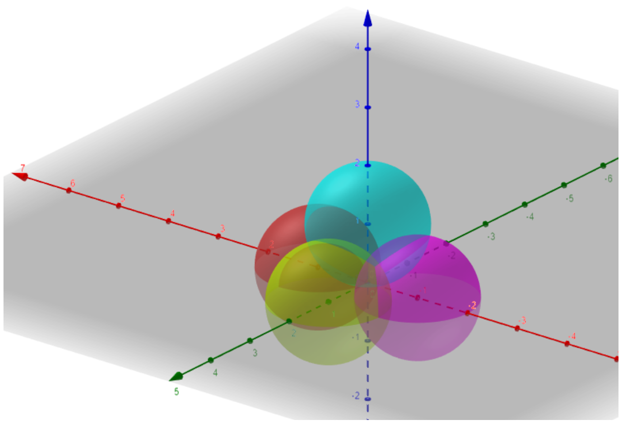
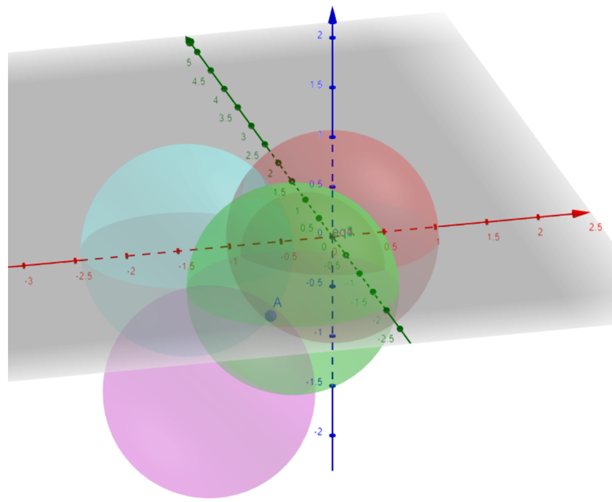
MP 1 – 3D Multilateration

CS 460
October 9, 2019





The idea behind this implementation is to apply some transformations to the landmarks so that they remain relative to each other, but are positioned in a favorable way in a new frame of reference that makes calculating the point of intersection easier. Then we can just apply all the transformations inversely and get the location of the point in its original frame.

In more detail, we first take the center of one landmark and translate it to the origin, moving the other landmarks by the same amount. Then we rotate along the z-axis and y-axis to fix a second landmark onto the x-axis. Finally we rotate along the x-axis to fix a third landmark in the xy-plane. After all these transformations the position of last landmark is fully determined.

The overall effect in this setup is that we have the following equations for the spheres in 3D:

$$x^2 + y^2 + z^2 = a^2 \tag{1}$$
$$(x-e)^2 + y^2 + z^2 = b^2 \tag{2}$$
$$(x-f)^2 + (y-g)^2 + z^2 = c^2 \tag{3}$$
$$(x-h)^2 + (y-i)^2 + (z-j)^2 = d^2 \tag{4}$$

Then the solution to the intersection of these spheres is:

$$x = \frac{e^2 + a^2 - b^2}{2e}$$
$$y = \frac{g^2 + f^2 - 2xf - a^2 - e^2}{2g}$$
$$z = \pm\sqrt{a^2 - x^2 - y^2}$$

Pick positive square root if the point (x, y, z) is within the boundary of the last landmark's radius. Choose the negative otherwise.