COA 620 Introduction to Bayesian Statistics in Ecology

Lecture 4 – Probability distributions

Wei Wu

January 29, 2026

THE UNIVERSITY OF SOUTHERN MISSISSIPPI
GULF COAST RESEARCH LABORATORY

1

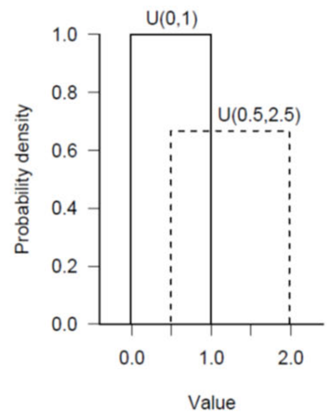
Binomial distribution

Question: Suppose that we are sampling fruits for infestation by a pest and know that the chance that a fruit is infested is p. Use a program to predict the distribution of infested fruit if we sample 10 fruits, and p is 0.1, 0.2, or 0.3.

2

Continuous models – Uniform distribution

range	$a \le x \le b$
distribution	$1/(b-a)$
R	`dunif`, `punif`, `qunif`, `runif`
parameters	minimum (a) and maximum (b) limits (real) [`min`, `max`]
mean	$(a+b)/2$
variance	$(b-a)^2/12$
CV	$(b-a)/((a+b)\sqrt{3})$



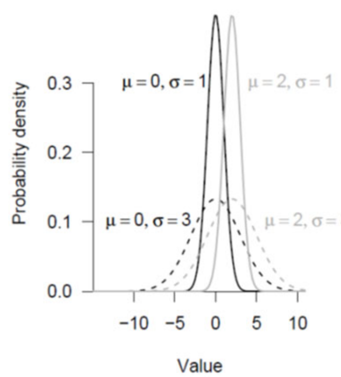
U(a,b) has a constant probability density of 1/(b-a) for a<=x<=b and 0 elsewhere.

$$[X \mid a,b] = \frac{1}{b-a}$$

3

Continuous models – Normal distribution

$$[X \mid \mu,\sigma] = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$



range	all real values
distribution	$\frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$
R	`dnorm`, `pnorm`, `qnorm`, `rnorm`
parameters	μ (real), mean [`mean`]
	σ (real, positive), standard deviation [`sd`]
mean	μ
variance	σ^2
CV	σ/μ

How about cumulative probability distribution?

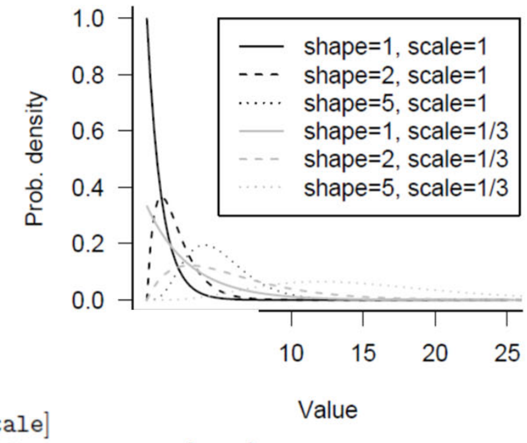
4

Continuous model – Gamma distribution

Distribution of waiting times until a certain number of events take place.

Gamma(shape=3, scale=2)

$$[X \mid a,s] = \frac{1}{s^a \Gamma(a)} X^{a-1} e^{-\frac{X}{s}}$$

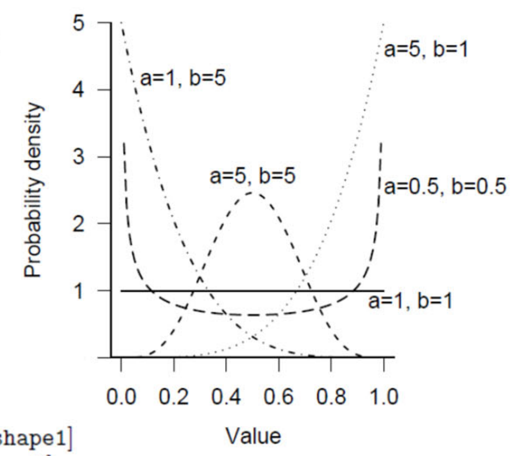


range	Nonnegative real
R	dgamma, pgamma, qgamma, rgamma
distribution	$\frac{1}{s^a \Gamma(a)} x^{a-1} e^{-x/s}$
parameters	s (real, positive), scale: length per event [scale]
	or r (real, positive), rate $= 1/s$; rate at which events occur [rate]
	a (real, positive), shape: number of events [shape]
mean	as or a/r
variance	as^2 or a/r^2
CV	$1/\sqrt{a}$

5

Continuous models – Beta distribution

$$[X \mid a,b] = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} X^{a-1}(1-X)^{b-1}$$



range	real, 0 to 1
R	dbeta, pbeta, qbeta, rbeta
density	$\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1}$
parameters	a (real, positive), shape 1: number of successes +1 [shape1]
	b (real, positive), shape 2: number of failures +1 [shape2]
mean	$a/(a+b)$
mode	$(a-1)/(a+b-2)$
variance	$ab/((a+b)^2)(a+b+1)$
CV	$\sqrt{(b/a)/(a+b+1)}$

$$variance = \frac{ab}{(a+b)^2(a+b+1)}$$

6

Simulate data

We will apply simulated data to test the accuracy of Central Limit Theorem.

Central Limit Theorem

When one takes random samples from any distribution with true mean μ and standard deviation σ, the distribution of the sample means will follow a normal distribution with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$

7