1 变分法

$F[y]$ 泛函。

$$F(y(x)+\epsilon\eta(x)) = F(y(x)) + \eta\int\frac{\partial F}{\partial y(x)}\eta(x)dx + O(\epsilon^2)$$

$$\int\limits_x \frac{\partial F}{\partial y(x)}\eta(x)dx = 0$$

2 高斯分布的矩

一阶原点矩

$$E(x) = \frac{1}{(2\pi)^{\frac{D}{2}}}\frac{1}{|\Sigma|^{\frac{1}{2}}}\int\exp\left\{-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right\}xdx$$

做换元 $z = x-\mu$ 后，有：

$$E(x) = \frac{1}{(2\pi)^{\frac{D}{2}}}\frac{1}{|\Sigma|^{\frac{1}{2}}}\int\exp\left\{-\frac{1}{2}z^T\Sigma^{-1}z\right\}(z+\mu)dz$$

而由对称性：

$$\int\exp\left\{-\frac{1}{2}z^T\Sigma^{-1}z\right\}(z)dz = 0$$

故有：

$$E(x) = \frac{1}{(2\pi)^{\frac{D}{2}}}\frac{1}{|\Sigma|^{\frac{1}{2}}}\int\exp\left\{-\frac{1}{2}z^T\Sigma^{-1}z\right\}(z+\mu)dz = \mu$$

二阶原点矩 类似地，做换元 $z = x-\mu$ 后，有：

$$E(x) = \frac{1}{(2\pi)^{\frac{D}{2}}}\frac{1}{|\Sigma|^{\frac{1}{2}}}\int\exp\left\{-\frac{1}{2}z^T\Sigma^{-1}z\right\}(z+\mu)(z+\mu)^Tdz$$

将 $(z+\mu)(z+\mu)$ 展开，考虑对称性，有：

$$\begin{aligned} E(x) &= \mu\mu^T \\ &\quad + \frac{1}{(2\pi)^{\frac{D}{2}}}\frac{1}{|\Sigma|^{\frac{1}{2}}}\int\exp\left\{-\frac{1}{2}z^T\Sigma^{-1}z\right\}zz^Tdz \end{aligned}$$

考虑 $I = \frac{1}{(2\pi)^{\frac{D}{2}}}\frac{1}{|\Sigma|^{\frac{1}{2}}}\int\exp\left\{-\frac{1}{2}z^T\Sigma^{-1}z\right\}zz^Tdz$