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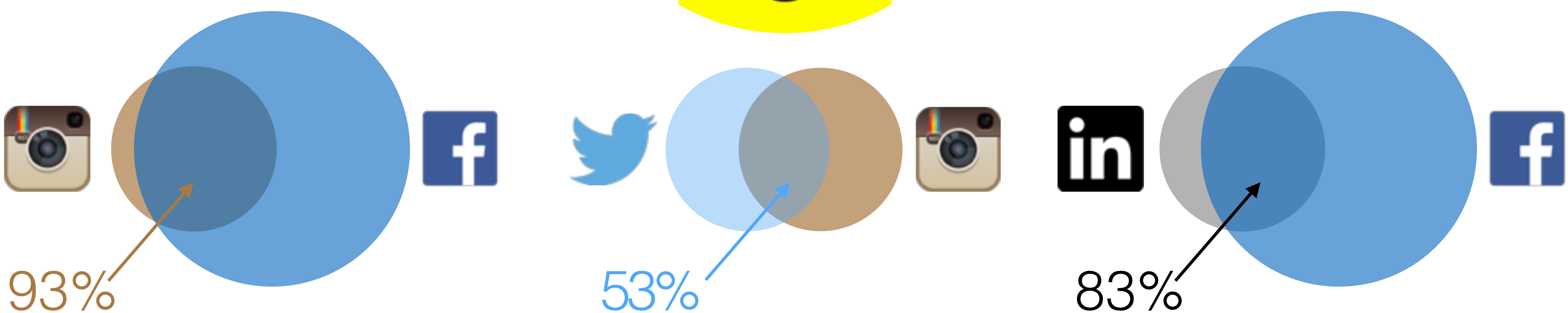
Multiple Anonymized Social Networks Alignment

Jiawei Zhang, Philip S. Yu

University of Illinois at Chicago, Chicago, IL

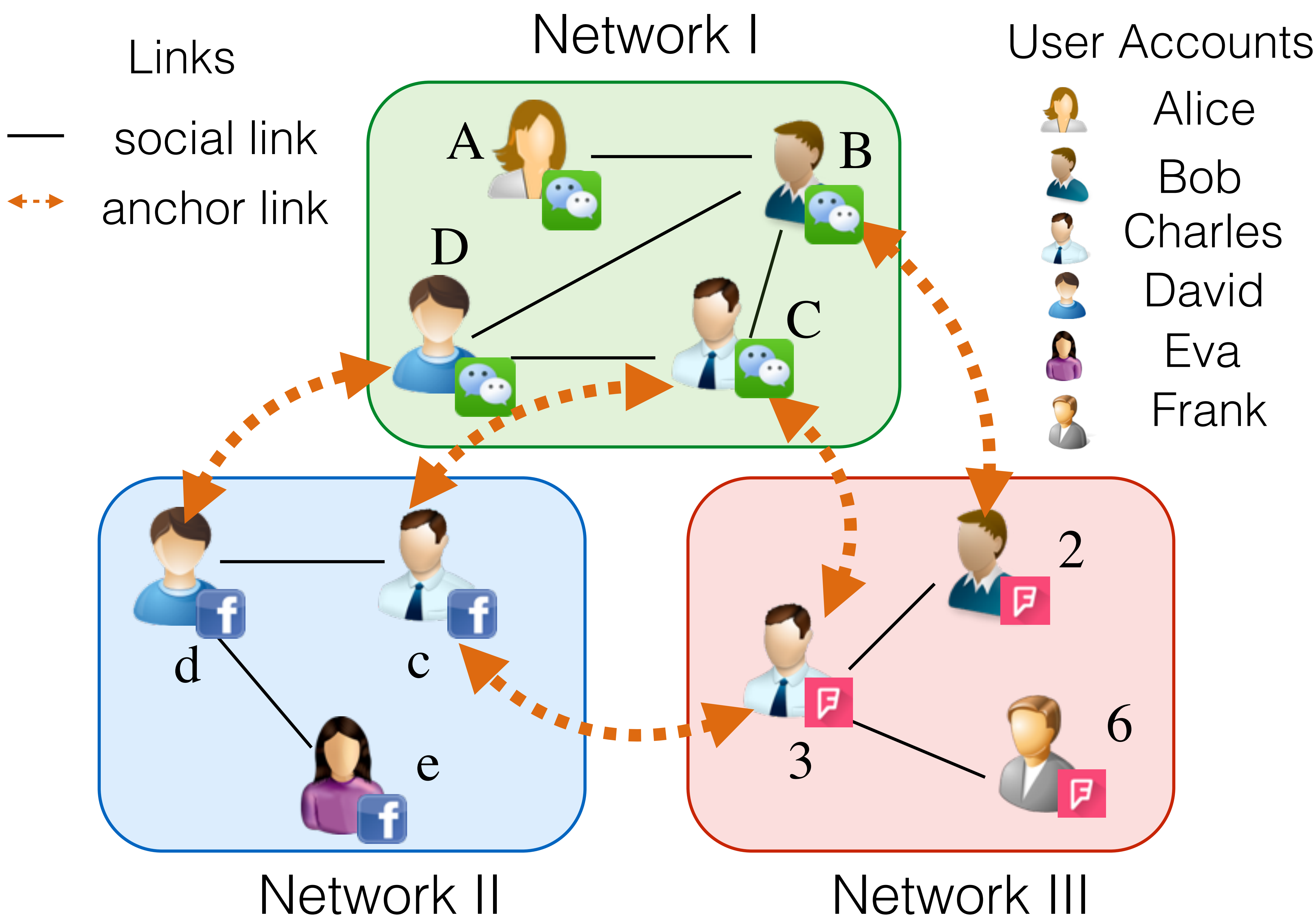
UIC UNIVERSITY OF ILLINOIS AT CHICAGO

People are using multiple social networks simultaneously



[1] Zhang et al. PNA: Partial Network Alignment with Generic Stable Matching, 2015 IEEE IRI.
[2] Duggan et al. Social media update 2013.

Align multiple anonymized social networks via the common users

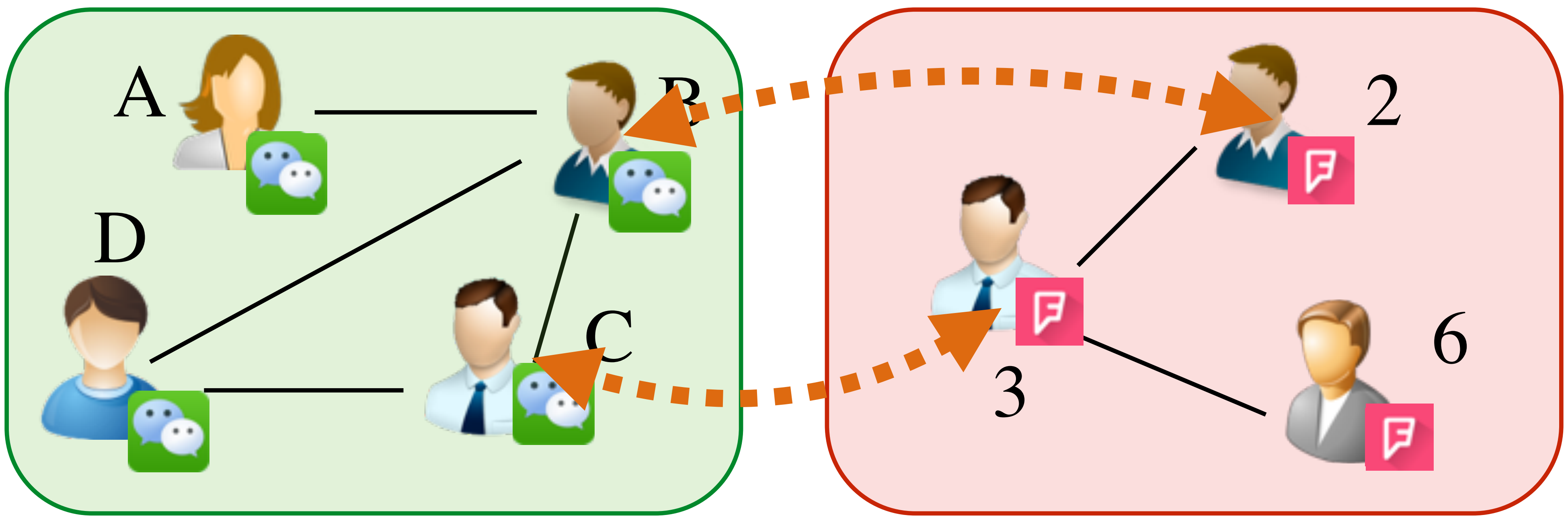


Motivation: fuse multiple information sources

- gain a more comprehensive understanding about users' social activities in all social networks
  - provide better recommendation services [Zhang ICDM 2013, Zhang WSDM 2014, Zhang CIKM 2015, Zhang IJCAI 2015]
  - detect more accurate community structures [Jin IEEE BigData 2014, Zhang IEEE IRI 2015, Zhang SDM 2015]
- cross-network information exchange and diffusion
  - resolve the information sparsity problem [Zhang ICDM 2013, Zhang WSDM 2014, Zhang IJCAI 2015, Zhang SDM 2015]
  - overcome the cold-start problem [Zhang ICDM 2013, Zhang SDM 2015]
  - broader influence in viral marketing [Zhan PAKDD 2015]

Challenge 1: networks are pre-anonymized

Solution: use the structure information only



	A	B	C	D
A	0	1	0	0
B	1	0	1	1
C	0	1	0	1
D	0	1	1	0

Adjacency Matrix $\mathbf{S}^{(i)}$

	2	3	6
A	0	0	0
B	1	0	0
C	0	1	0
D	0	0	0

Transition Matrix $\mathbf{T}^{(i,j)}$

	2	3	6
2	0	1	0
3	1	0	1
6	0	1	0

Adjacency Matrix $\mathbf{S}^{(j)}$

Challenge 1: networks are pre-anonymized

Assumption: shared users have similar social structures in different networks

	A	B	C	D
A	0	1	0	0
B	1	0	1	1
C	0	1	0	1
D	0	1	1	0

Adjacency Matrix $\mathbf{S}^{(i)}$

	2	3	6
A	0	0	0
B	1	0	0
C	0	1	0
D	0	0	0

Transition Matrix $\mathbf{T}^{(i,j)}$

	2	3	6
2	0	1	0
3	1	0	1
6	0	1	0

Adjacency Matrix $\mathbf{S}^{(j)}$

Via transition matrix $\mathbf{T}^{(i,j)}$(i.e., anchor links), we can map the social connections among shared users from network I to network II:

$$(\mathbf{T}^{(i,j)})^\top \mathbf{S}^{(i)} \mathbf{T}^{(i,j)}$$

The optimal transition matrix $\mathbf{T}^{(i,j)}$(i.e., anchor links) should minimize the mapping cost

$$\min \left\| (\mathbf{T}^{(i,j)})^\top \mathbf{S}^{(i)} \mathbf{T}^{(i,j)} - \mathbf{S}^{(j)} \right\|_F^2$$

Challenge 2: one-to-one constraint on anchor links

- Add one-to-one constraint on the transitional matrix $\mathbf{T}^{(i,j)}$
  - $\mathbf{T}^{(i,j)}$ Should be binary matrix
  - each row and each column of $\mathbf{T}^{(i,j)}$ should contain at most one entry being filled with value 1
- The complete objective function should be

$$
\begin{aligned}
\bar{\mathbf{T}}^{(i,j)} &= \arg\min_{\mathbf{T}^{(i,j)}} \left\| (\mathbf{T}^{(i,j)})^\top \mathbf{S}^{(i)} \mathbf{T}^{(i,j)} - \mathbf{S}^{(j)} \right\|_F^2 \\
s.t. \quad & \mathbf{T}^{(i,j)} \in \{0,1\}^{|\mathcal{U}^{(i)}| \times |\mathcal{U}^{(j)}|}, \\
& \mathbf{T}^{(i,j)} \mathbf{1}^{|\mathcal{U}^{(j)}| \times 1} \preccurlyeq \mathbf{1}^{|\mathcal{U}^{(i)}| \times 1}, \\
& (\mathbf{T}^{(i,j)})^\top \mathbf{1}^{|\mathcal{U}^{(i)}| \times 1} \preccurlyeq \mathbf{1}^{|\mathcal{U}^{(j)}| \times 1},
\end{aligned}
$$

Challenge 3: multiple network alignment

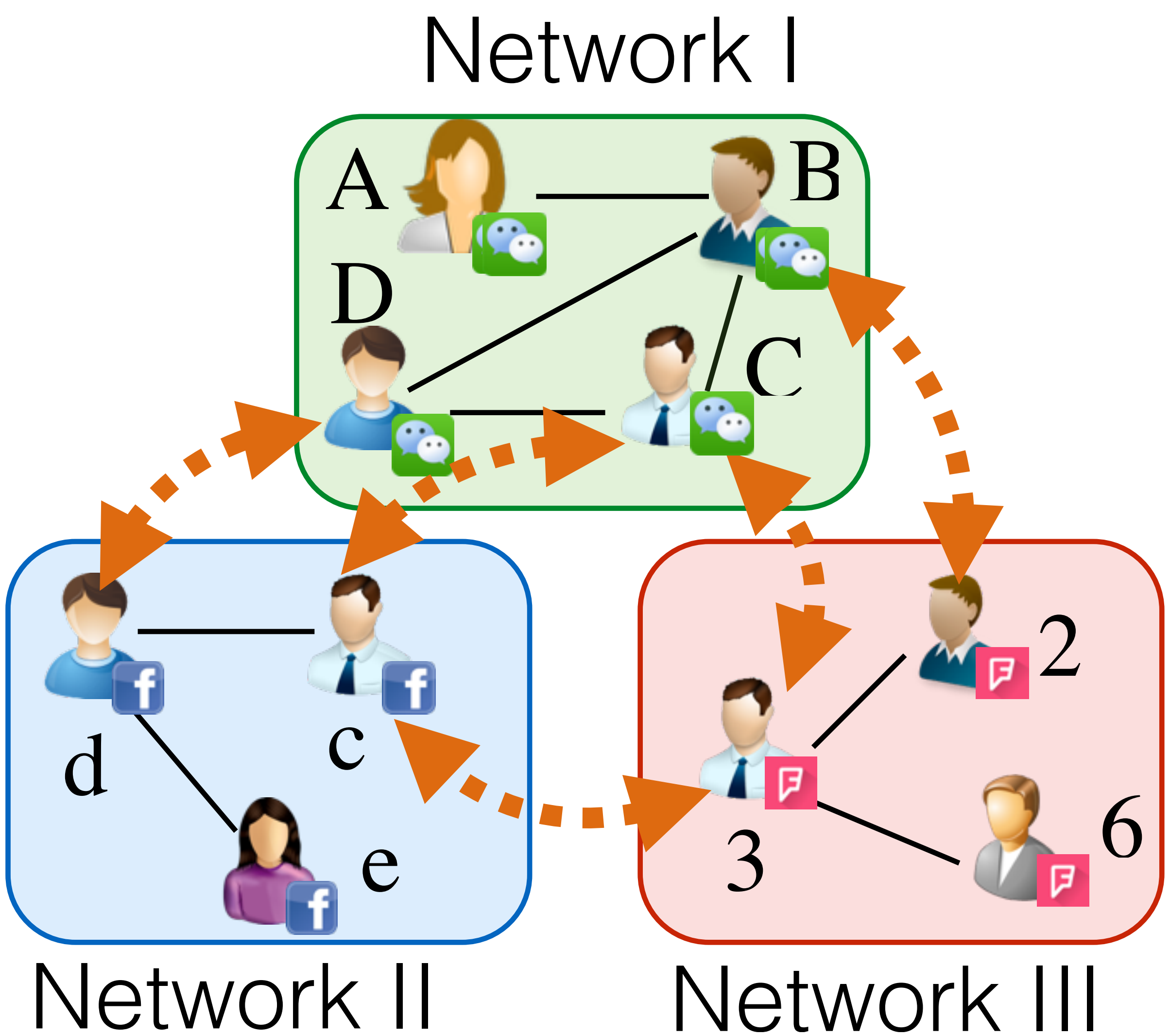
- Anchor links meet the "Transitivity Law"

$$\forall a, b, c \in \mathcal{X}, (a, b) \in \mathcal{R} \wedge (b, c) \in \mathcal{R} \rightarrow (a, c) \in \mathcal{R}.$$

- if (u, v) are connected by anchor links between network 1 and network 2;
- and (v, w) are connected by anchor links between network 2 and network 3;
- then (u, w) should be also connected by anchor links between network 1 and network 3

Challenge 3: multiple network alignment

- Anchor links meet the "Transitivity Law"



- The alignment results of any 3 networks should be consistent
- In other words, the mapping from network 1 to network 3 via paths
  - network 1 -> network 2 -> network 3
  - network 1 -> network 3

  should be similar
- we introduce the "Alignment Transitivity Penalty" cost function to be

$$C(\{G^{(i)}, G^{(j)}, G^{(k)}\})$$
$$= \left\| (\mathbf{T}^{(j,k)})^\top (\mathbf{T}^{(i,j)})^\top \mathbf{S}^{(i)} \mathbf{T}^{(i,j)} \mathbf{T}^{(j,k)} - (\mathbf{T}^{(i,k)})^\top \mathbf{S}^{(i)} \mathbf{T}^{(i,k)} \right\|_F^2 .$$

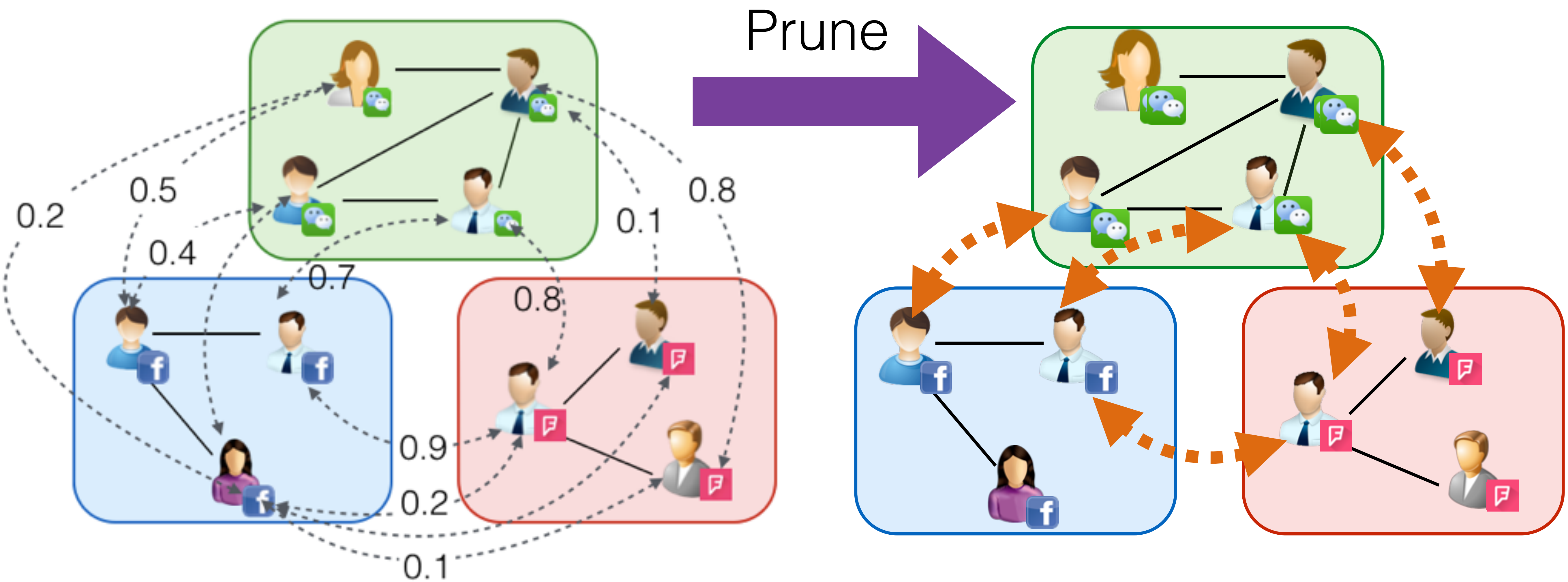
Joint Objective Function

$$
\begin{aligned}
&\bar{\mathbf{T}}^{(i,j)}, \bar{\mathbf{T}}^{(j,k)}, \bar{\mathbf{T}}^{(k,i)} \\
&= \arg\min_{\mathbf{T}^{(i,j)}, \mathbf{T}^{(j,k)}, \mathbf{T}^{(k,i)}} \left\| (\mathbf{T}^{(i,j)})^\top \mathbf{S}^{(i)} \mathbf{T}^{(i,j)} - \mathbf{S}^{(j)} \right\|_F^2 \\
&+ \left\| (\mathbf{T}^{(j,k)})^\top \mathbf{S}^{(j)} \mathbf{T}^{(j,k)} - \mathbf{S}^{(k)} \right\|_F^2 + \left\| (\mathbf{T}^{(k,i)})^\top \mathbf{S}^{(k)} \mathbf{T}^{(k,i)} - \mathbf{S}^{(i)} \right\|_F^2 \\
&+ \alpha \cdot \left\| (\mathbf{T}^{(j,k)})^\top (\mathbf{T}^{(i,j)})^\top \mathbf{S}^{(i)} \mathbf{T}^{(i,j)} \mathbf{T}^{(j,k)} - \mathbf{T}^{(k,i)} \mathbf{S}^{(i)} (\mathbf{T}^{(k,i)})^\top \right\|_F^2 \\
&s.t.\ \mathbf{T}^{(i,j)} \in \{0,1\}^{|\mathcal{U}^{(i)}| \times |\mathcal{U}^{(j)}|}, \mathbf{T}^{(j,k)} \in \{0,1\}^{|\mathcal{U}^{(j)}| \times |\mathcal{U}^{(k)}|} \\
&\qquad \mathbf{T}^{(k,i)} \in \{0,1\}^{|\mathcal{U}^{(k)}| \times |\mathcal{U}^{(i)}|} \\
&\qquad \mathbf{T}^{(i,j)} \mathbf{1}^{|\mathcal{U}^{(j)}| \times 1} \preccurlyeq \mathbf{1}^{|\mathcal{U}^{(i)}| \times 1}, (\mathbf{T}^{(i,j)})^\top \mathbf{1}^{|\mathcal{U}^{(i)}| \times 1} \preccurlyeq \mathbf{1}^{|\mathcal{U}^{(j)}| \times 1}, \\
&\qquad \mathbf{T}^{(j,k)} \mathbf{1}^{|\mathcal{U}^{(k)}| \times 1} \preccurlyeq \mathbf{1}^{|\mathcal{U}^{(j)}| \times 1}, (\mathbf{T}^{(j,k)})^\top \mathbf{1}^{|\mathcal{U}^{(j)}| \times 1} \preccurlyeq \mathbf{1}^{|\mathcal{U}^{(k)}| \times 1}, \\
&\qquad \mathbf{T}^{(k,i)} \mathbf{1}^{|\mathcal{U}^{(i)}| \times 1} \preccurlyeq \mathbf{1}^{|\mathcal{U}^{(k)}| \times 1}, (\mathbf{T}^{(k,i)})^\top \mathbf{1}^{|\mathcal{U}^{(k)}| \times 1} \preccurlyeq \mathbf{1}^{|\mathcal{U}^{(i)}| \times 1},
\end{aligned}
$$

b to solve the objective function, ts

the constraints are relaxed

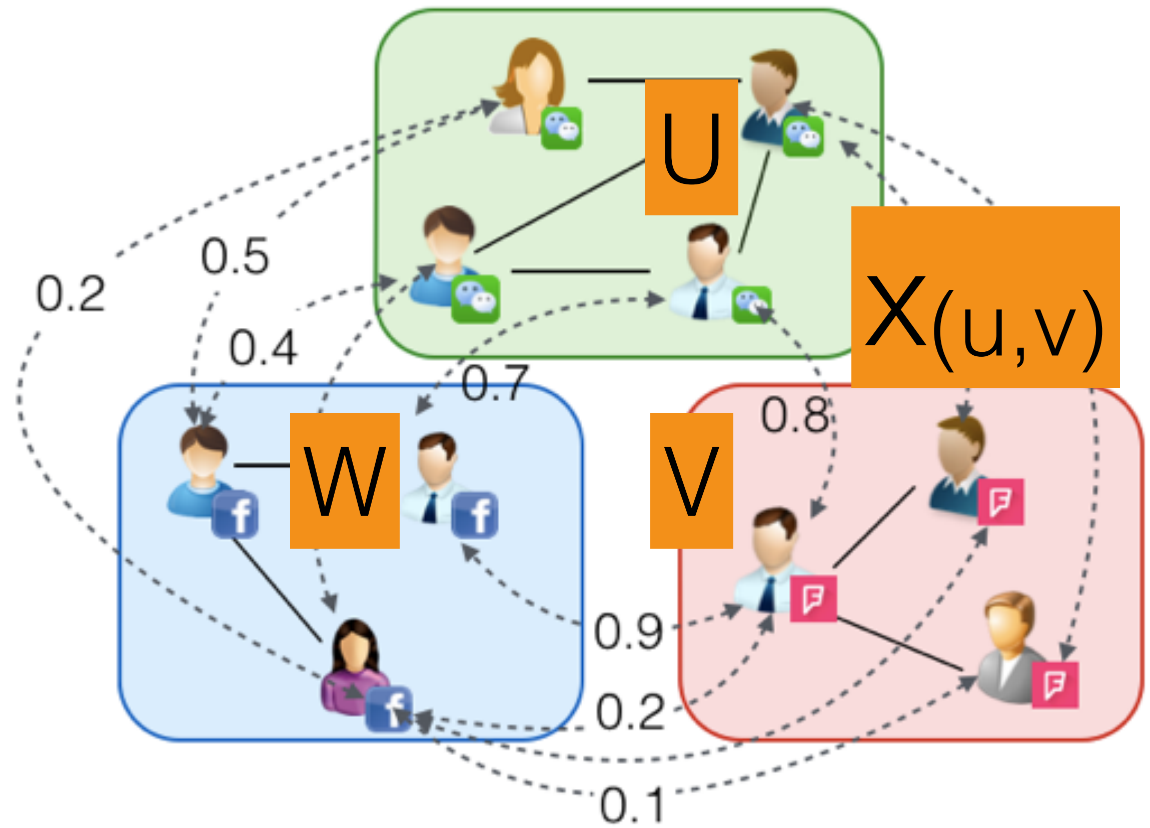
Challenge 4: Transitive Network Mapping



- Prune the non-existing anchor links introduced by constraint relaxation
- We introduce the **transitive multiple network matching** method, with considerations about the "one-to-one" constraint and "Transitivity Law" property of anchor links

Transitive multiple network matching

- Network matching:
  - pick the high confidence anchor links from the prediction results; prune the remaining ones of low confidence scores
- Constraints on network matching
  - one-to-one constraint, for each use
  - "Transitivity Law" constraint



- define $x_{(u,v)}$ to denote potential anchor link (u,v) is selected or not (1: selected, 0 otherwise).
- in the alignment of two networks, for each user u, at most one variable $x_{(u,v)}$ is assigned with value 1 (one-to-one constraint)
- in the alignment of any 3 networks, for any 3 users u, v, w
  - if $x_{(u,v)} = 1$, $x_{(v,w)} = 1$, then $x_{(u,w)} = 1$ ("Transitivity Law" constraint)
  - in other words $x_{(u,v)} + x_{(v,w)} + x_{(u,w)} \,!= 2$ for any user u, v, w in 3 networks

Transitive multiple network matching

$$\max_{\mathbf{x}^{(i,j)},\mathbf{x}^{(j,k)},\mathbf{x}^{(k,i)}} \sum_{l,m} x_{l,m}^{(i,j)} \mathbf{T}^{(i,j)}(l,m) + \sum_{l,m} x_{l,m}^{(i,j)} \mathbf{T}^{(i,j)}(l,m) + \sum_{l,m} x_{l,m}^{(i,j)} \mathbf{T}^{(i,j)}(l,m),$$

select anchor links of high confidences

$$s.t. \sum_{u_m^{(j)} \in \mathcal{U}^{(j)}} x_{l,m}^{(i,j)} \leq 1, \sum_{u_o^{(k)} \in \mathcal{U}^{(k)}} x_{l,o}^{(i,k)} \leq 1, \forall u_l^{(i)} \in \mathcal{U}^{(i)},$$

$$\sum_{u_l^{(i)} \in \mathcal{U}^{(i)}} x_{m,l}^{(j,i)} \leq 1, \sum_{u_o^{(k)} \in \mathcal{U}^{(k)}} x_{m,o}^{(j,k)} \leq 1, \forall u_m^{(j)} \in \mathcal{U}^{(j)},$$

$$\sum_{u_l^{(i)} \in \mathcal{U}^{(i)}} x_{o,l}^{(k,i)} \leq 1, \sum_{u_m^{(j)} \in \mathcal{U}^{(j)}} x_{o,m}^{(k,j)} \leq 1, \forall u_o^{(k)} \in \mathcal{U}^{(k)},$$

one-to-one constraint

$$x_{l,m}^{(i,j)} + x_{m,o}^{(j,k)} + x_{o,l}^{(k,i)} \neq 2, \forall l \in \{1, 2, \cdots, |\mathcal{U}^{(i)}|\},$$
$$\forall m \in \{1, 2, \cdots, |\mathcal{U}^{(j)}|\}, \forall o \in \{1, 2, \cdots, \mathcal{U}^{(k)}|,$$

transitivity law constraint

$$x_{l,m}^{(i,j)} \in \{0,1\}, \forall u_l^{(i)} \in \mathcal{U}^{(i)}, u_m^{(j)} \in \mathcal{U}^{(j)}.$$
$$x_{m,o}^{(j,k)} \in \{0,1\}, \forall u_m^{(j)} \in \mathcal{U}^{(j)}, u_o^{(k)} \in \mathcal{U}^{(k)}.$$
$$x_{o,l}^{(k,i)} \in \{0,1\}, \forall u_o^{(k)} \in \mathcal{U}^{(k)}, u_l^{(i)} \in \mathcal{U}^{(i)}.$$

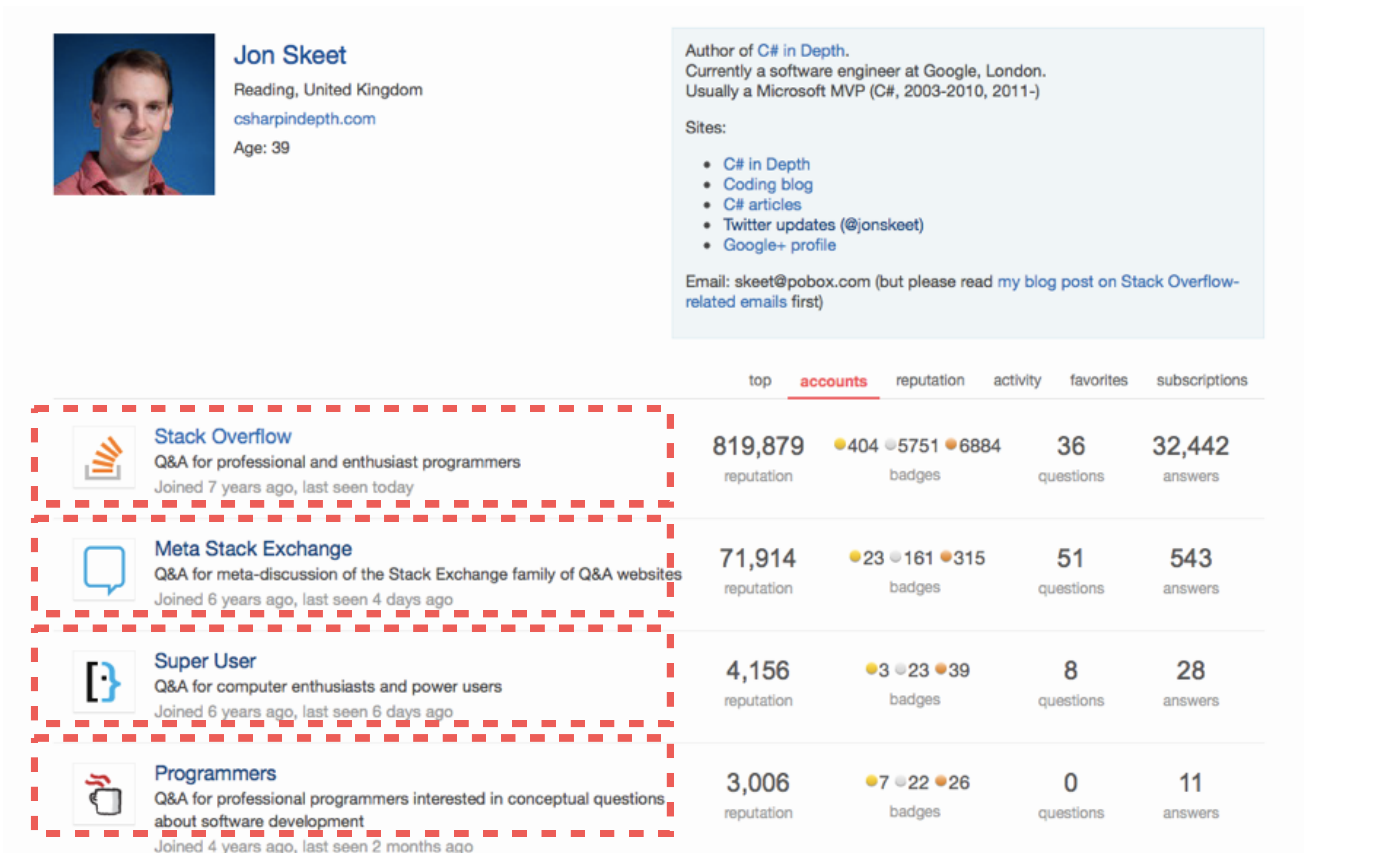
binary value of variables

Experiments

- Dataset - 3 Q&A sites
    - Stack Overflow (10,000 users)
    - Super User (10,000 users)
    - Programmers (10,000 users)
- Anchor links among these 3 networks
    - Stack Overflow - Super User: 3,677
    - Stack Overflow - Programmers: 2,626
    - Super User - Programmers: 1,953
- Experiment Setting: unsupervised learning settings
    - the anchor links are used as the ground truth for evaluating the prediction results
    - known anchor links are not involved in the model building

Experiments

- Ground Truth Anchor Links



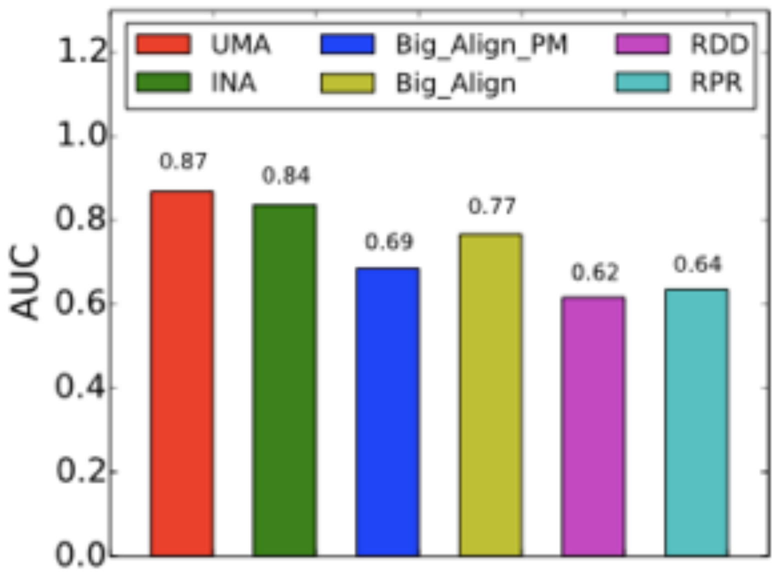
Experiments

	UMA	INA	Big-A	Big-A-PM	RDD	RPR
prediction	√	√	√ (pairwise)	√ (pairwise)		
matching	√			√ (pairwise)		
score					√	√

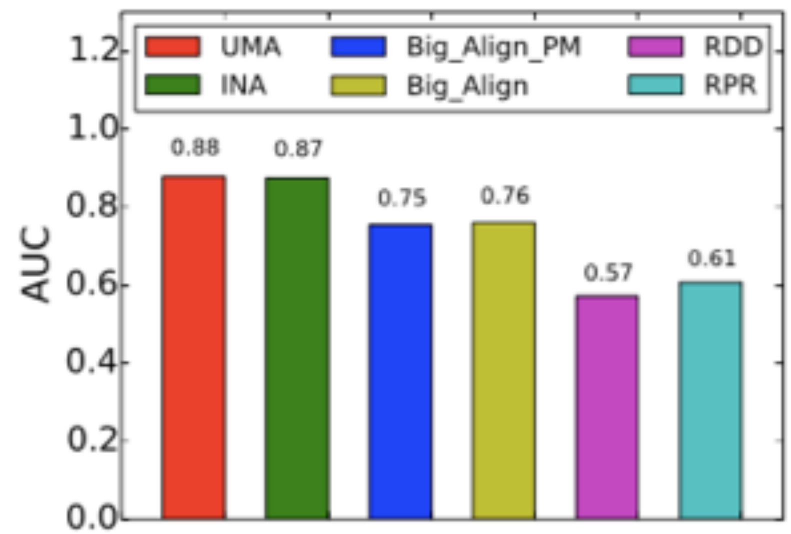
- Comparison Methods
  - UMA: multiple network alignment + matching
  - INA: multiple network alignment (the 1st step of UMA)
  - Big-Align: pairwise network alignment [13]
  - Big-Align-PM: Big-Align + pairwise network matching
  - RDD: calculate the Relative Degree Distance as the confidence scores of potential anchor links [13]
  - RPR: calculate the Relative PageRank score as the confidence scores of potential anchor links
- Evaluation Metrics
  - AUC and Precision@100
  - Accuracy, Precision, Recall, F1 (for methods UMA and Big-Align-PM with the matching step)

[13] D. Koutra, H. Tong, and D. Lubensky. Big-align: Fast bipartite graph alignment. In *ICDM*, 2013

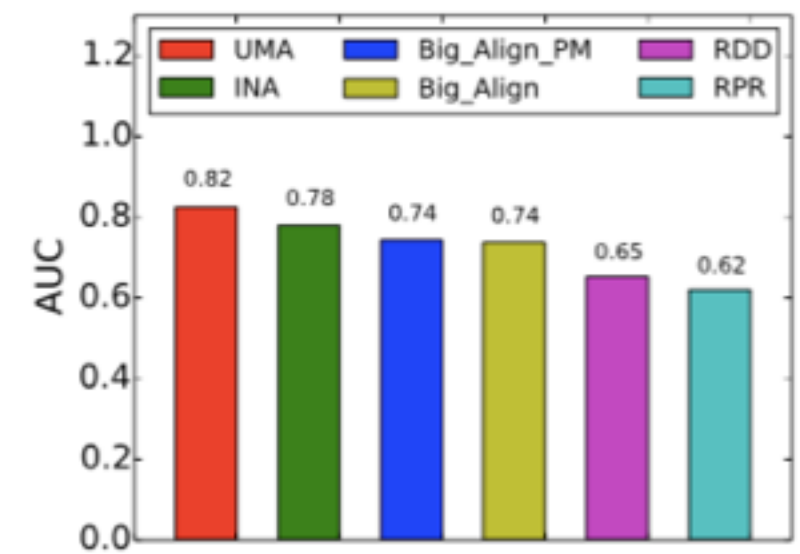
Experiment results



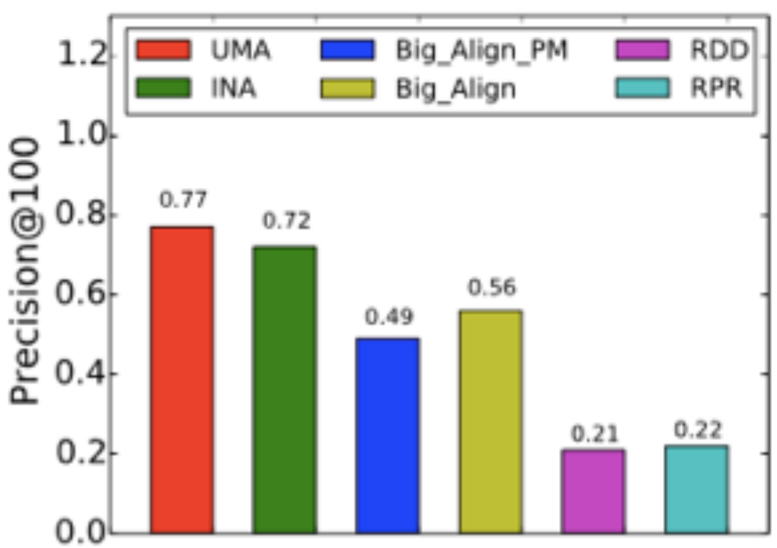
(a) AUC ($G^{(i)}$, $G^{(j)}$)



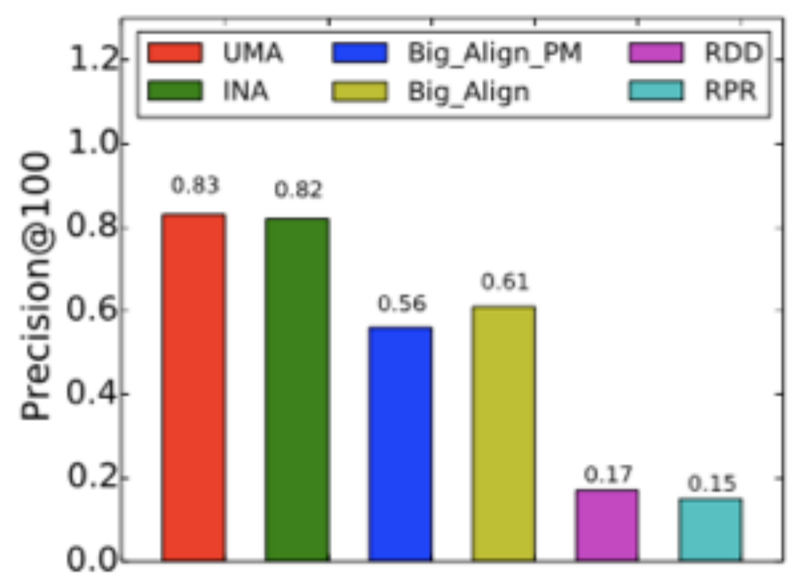
(b) AUC ($G^{(j)}$, $G^{(k)}$)



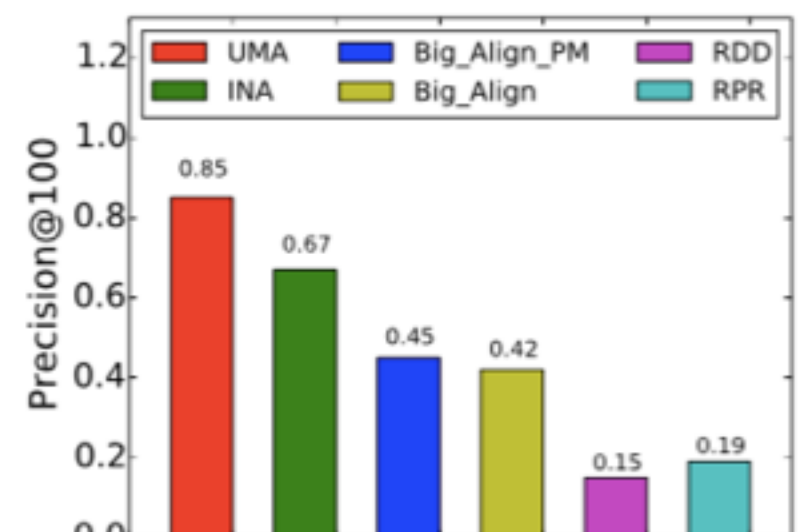
(c) AUC ($G^{(k)}$, $G^{(i)}$)



(d) Precision@100 ($G^{(i)}$, $G^{(j)}$)

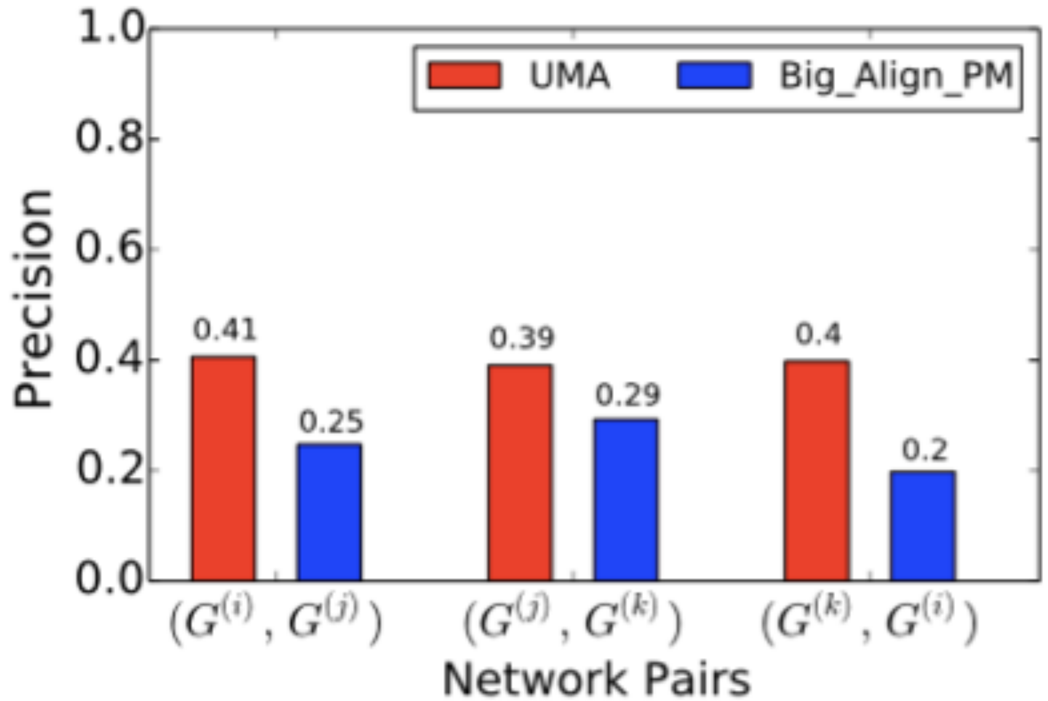


(e) Precision@100 ($G^{(j)}$, $G^{(k)}$)

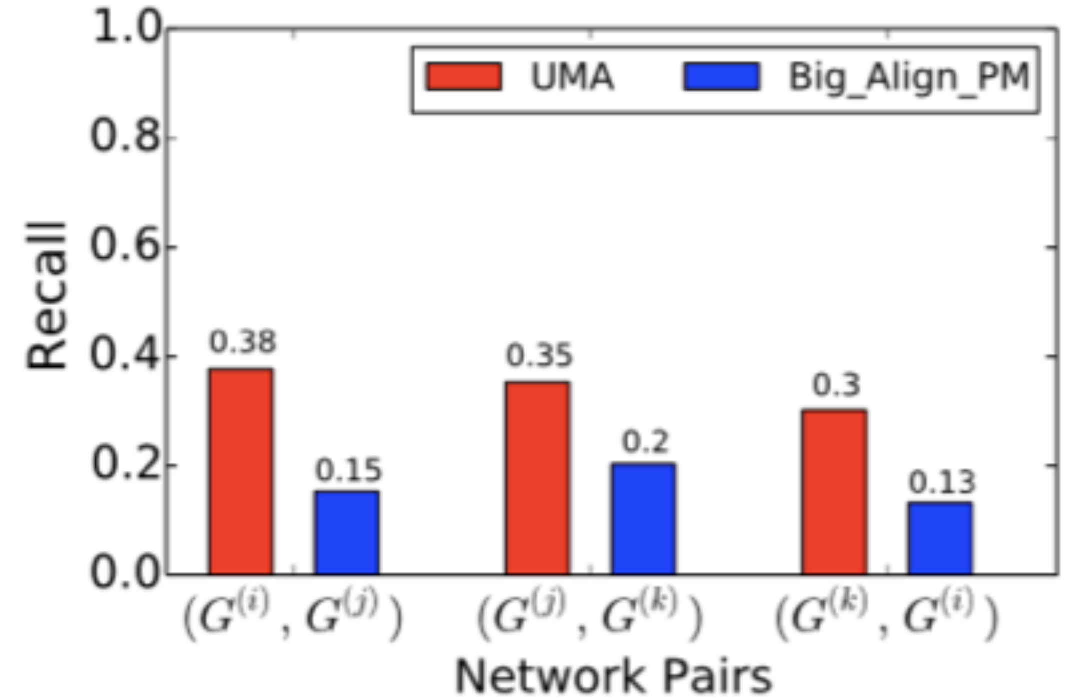


(f) Precision@100 ($G^{(k)}$, $G^{(i)}$)

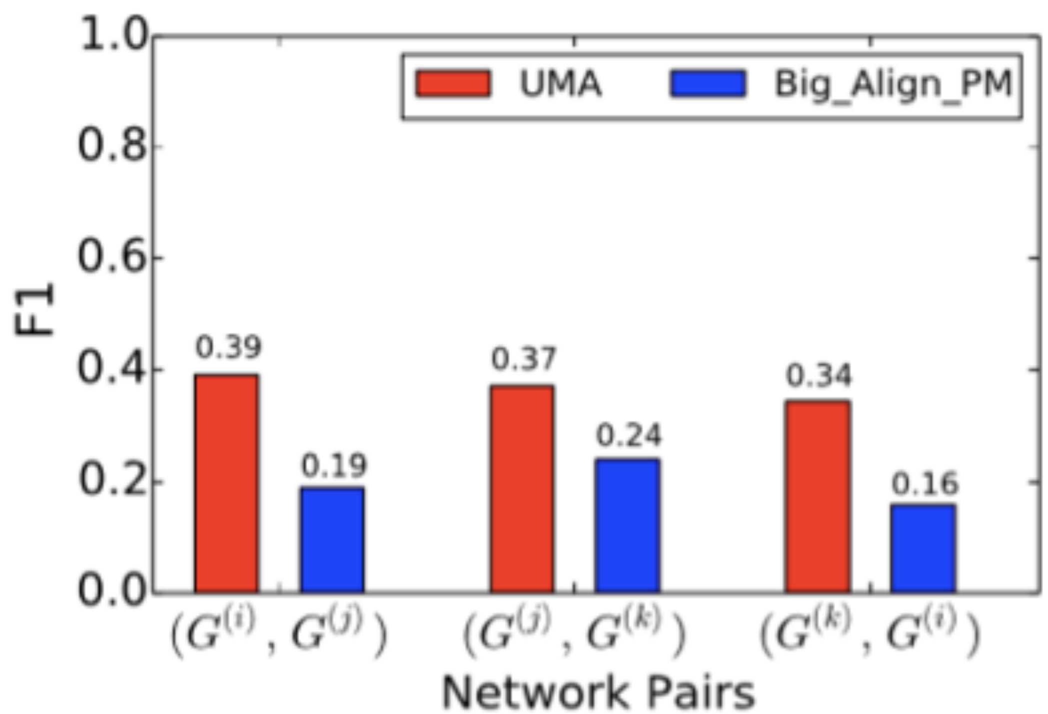
Experiment results



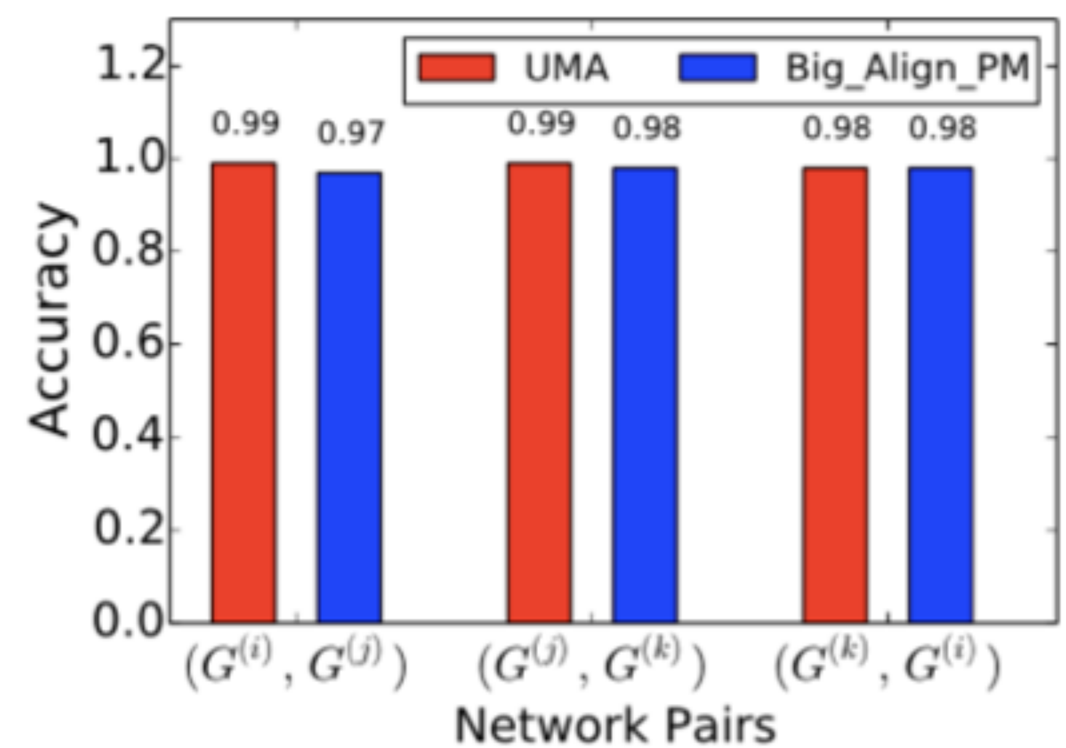
(a) Precision



(b) Recall



(c) F1



(d) Accuracy

Summary

- Problem studied
  - simultaneous alignment of multiple anonymized social networks
- Propose method:
  - an joint optimization function to minimize mapping cost and “transitivity penalty’’ cost with one-to-one constraint
  - a transitive multiple network matching algorithm to prune the non-existing anchor links with one-to-one constraint and “transitivity law” constraint

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Q&A

Jiawei Zhang, Philip S. Yu
jzhan9@uic.edu, psyu@cs.uic.edu

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