UM-SJTU Joint Institute Probabilistic Methods in Engineering (Ve401)

TERM PROJECT 2

POLICE SHOOTINGS IN THE U.S.

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Group 39

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Abstract

In this project, the analysis of fatal police shooting is performed. The goodness-of-fit test is performed to find that the data follows a poisson distribution in 2015-2018. Moreover, using the chi-squared test for independence, we find that the average number of fatal police shootings depends on weekdays. Next, a confidence interval for parameter k of the poisson distribution is derived and calculated using the data from 2015 to 2018. We also use the data from Jan. 1, 2019 to Feb. 14, 2019 to investigate whether it follows a poisson distribution. Since we cannot reject the hypothesis that it follows a poisson distribution, we say that it follows distribution. Finally, a prediction interval for the number of mass shootings is obtained using the Nelson's formula and a plot for the estimated data and prediction interval is given.

Key words: Statistics \cdot Hypothesis Test \cdot fatal police shooting in the US

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1 Source of Data

The data of fatal police shooting in the United States are collected by The Washington Post since Jan.1, 2015. Julie Tate, along with her collegues, explained in an article published in 2016, "How The Washington Post is examining police shootings in the United States", that in 2015, The Post gathered various details about killing by "culling local news reports, law enforcement websites and social media" (Tate). They also utilized "independent databases such as Killed by Police and Fatal Encounters" and "conducted additional reporting in many cases" (Tate). In 2016, the Post improved their collection by adding more information, such as the officers' names, and requiring more open-records from the departments for each fatal shooting. In the article, it can be seen that The Post has been seeking assistence from the public as well. The database is recognized as a more complete database, compared with the one FBI has established.

It is also worth noticing that the term "fatal police shooting" here only refers to "those shootings in which a police officer, in the line of duty, shoots and kills a civilian" (Tate). The database excludes the situations such as deaths when being held in custody by police and "fatal shootings by off-duty officers or non-shooting deaths" (Tate).

2 Police Homicides in the US Each Day

Figure (2.1) shows the number of homicides each day between January 1^{st} 2015 and December 31^{st} 2018 in the US. There have been 3943 homicides in total during these 1461 days. There are only 108 days having no homicide at all. And surprisingly, there is one day with 10 homicides and three days with 9 homicides.

It is noteworthy that 2016 is a leap year, which means there are 29 days in February 2016. We include the extra day to make our data more complete and also pay attention when we are assigning weekdays to those three years.

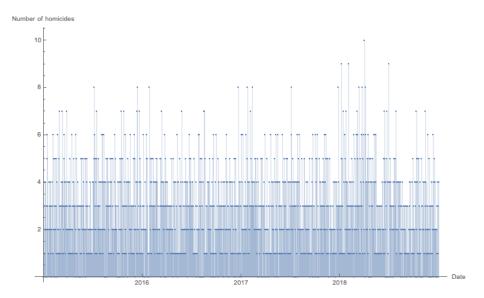


Figure 2.1: Number of homicides recorded each day in US between January 1^{st} 2015 and December 31^{st} 2018

3 Goodness of Fit Test for the Poisson Distribution

In order to investigate whether the data follows a Poisson distribution over the three years, we use the goodness of fit test for the poisson distribution. For the sample, the following number of deaths are observed:

| Number of deaths | Observed days |
|------------------|---------------|
| 0 | 108 |
| 1 | 287 |
| 2 | 324 |
| 3 | 310 |
| 4 | 227 |
| 5 | 116 |
| 6 | 53 |
| 7 | 21 |
| 8 | 11 |
| 9 | 3 |
| 10 | 1 |

The parameter k for the Poisson distribution is estimated using the sample

$$\hat{k} = \bar{X} = \frac{1}{1461} (1 \times 287 + 2 \times 324 + 3 \times 310 + 4 \times 227 + 5 \times 116 + 6 \times 53 + 7 \times 21 + 8 \times 11 + 9 \times 3 + 10 \times 1).$$
(3.1)

Then we have

$$\hat{k} = 2.69884. (3.2)$$

Next, We calculate

$$P[X=0] = \frac{e^{-\hat{k}}\hat{k}^0}{0!} = 0.0673;$$

$$P[X=1] = \frac{e^{-\hat{k}}\hat{k}^1}{1!} = 0.1816;$$

$$P[X=2] = \frac{e^{-\hat{k}}\hat{k}^2}{2!} = 0.2450;$$

$$P[X=3] = \frac{e^{-\hat{k}}\hat{k}^3}{3!} = 0.2204;$$

$$P[X=4] = \frac{e^{-\hat{k}}\hat{k}^4}{4!} = 0.1487;$$

$$P[X=5] = \frac{e^{-\hat{k}}\hat{k}^5}{5!} = 0.0803;$$

$$P[X=6] = \frac{e^{-\hat{k}}\hat{k}^6}{6!} = 0.0361;$$

$$P[X=7] = \frac{e^{-\hat{k}}\hat{k}^7}{7!} = 0.0139;$$

$$P[X=8] = \frac{e^{-\hat{k}}\hat{k}^8}{8!} = 0.0047;$$

$$P[X=9] = \frac{e^{-\hat{k}}\hat{k}^9}{9!} = 0.0014;$$

$$P[X=10] = \frac{e^{-\hat{k}}\hat{k}^{10}}{10!} = 0.0004.$$

Thus we have

| Number of deaths | Observed days | Expected days |
|------------------|---------------|---------------|
| 0 | 108 | 98.3 |
| 1 | 287 | 265.3 |
| 2 | 324 | 358.0 |
| 3 | 310 | 322.0 |
| 4 | 227 | 217.3 |
| 5 | 116 | 117.3 |
| 6 | 53 | 52.8 |
| 7 | 21 | 20.3 |
| 8 | 11 | 6.9 |
| 9 | 3 | 2.1 |
| 10 | 1 | 0.7 |

However, we see that $E_9, E_{10} < 5$. Althought this meets the requirement of the Pearson's test: $Ei \ge 5$ for 80% of the data, but we would like to make the result more accurate. We thus combine the last two categories:

| Number of deaths | Observed days | Expected days |
|------------------|---------------|---------------|
| 0 | 108 | 98.3 |
| 1 | 287 | 265.3 |
| 2 | 324 | 358.0 |
| 3 | 310 | 322.0 |
| 4 | 227 | 217.3 |
| 5 | 116 | 117.3 |
| 6 | 53 | 52.8 |
| 7 | 21 | 20.3 |
| 8 | 11 | 6.9 |
| 9 or more | 4 | 2.8 |

We plot the obtained data in Figure (3.1) and (3.2).

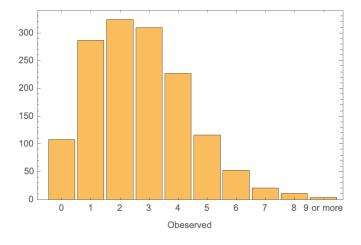


Figure 3.1: Frequency of occurrence of days in the US(observed)

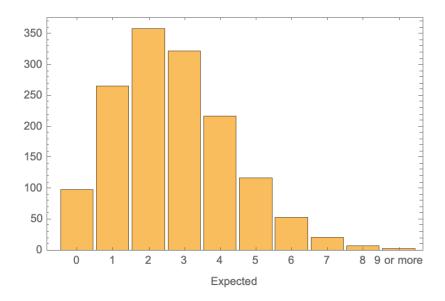


Figure 3.2: Frequency of occurrence of days in the US(expected)

We test in the three years 2015-2018,

$$H_0$$
: The occurrence of police shootings in the US follows a Poisson distribution, (3.4)

which is equivalent to the test whether the number of police shootings follows a categorical distribution with parameters (98.3, 265.3, 358, 322, 217.3, 117.3, 52.8, 20.3, 6.9, 2.8). We have the following category:

| Category | Number of deaths | Observed days | Expected days |
|----------|------------------|---------------|---------------|
| 1 | 0 | 108 | 98.3 |
| 2 | 1 | 287 | 265.3 |
| 3 | 2 | 324 | 358.0 |
| 4 | 3 | 310 | 322.0 |
| 5 | 4 | 227 | 217.3 |
| 6 | 5 | 116 | 117.3 |
| 7 | 6 | 53 | 52.8 |
| 8 | 7 | 21 | 20.3 |
| 9 | 8 | 11 | 6.9 |
| 10 | 9 or more | 4 | 2.8 |

Since we know

$$X^{2} = \sum_{i=1}^{10} \frac{(O_{i} - E_{i})^{2}}{E_{i}} = 9.83$$
(3.5)

follows a chi-squared distribution with N-1-m=10-1-1=8 degrees of freedom. Assume $\alpha=0.05$, we have

$$\chi_{0.05,8}^2 = 15.5 \ . \tag{3.6}$$

Therefore we are unable to reject H_0 at 5% level of significance.

From the table we have

$$\chi^2_{0.25,8} = 10.2, \qquad \chi^2_{0.1,8} = 13.4.$$
 (3.7)

Thus, the P-value of is approximately 0.2. In this case, we have reason to believe that the occurrence of police shootings in the US follows a poisson distribution in year 2015-2018.

4 Relationship Between Police Homicides & Weekday and Month

Figure (4.1) and (4.2) show the distribution of homicides by weekday and month.

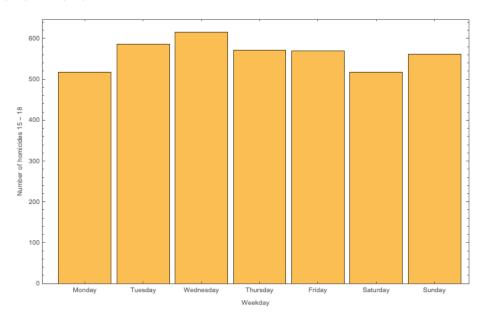


Figure 4.1: Number of occurrence of homicides on different weekdays in the US between January 1^{st} 2015 and December 31^{st} 2018

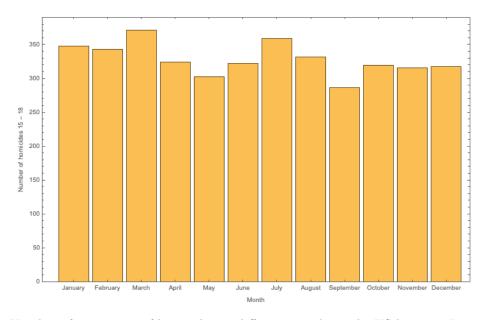


Figure 4.2: Number of occurrence of homicides on different months in the US between January 1^{st} 2015 and December 31^{st} 2018

It is intuitive to give a hypothesis that the occurrence of negative behaviors is related to the weekday. For example, it is seemingly reasonable to state that the cars produced on Monday will have higher

probability to have failure because workers may not be in the mood for working after a weekend. In the case of police homicides, we first give an intuitive guess and then test it using scientific way.

We are interested in whether the average numbers of police shootings depend on weekdays. So we perform an Pearson's Chi-squared Goodness-of-fit test to see whether these data conform to a discrete nearly uniform distribution on Ω =(Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday).

Let $(X_1, X_2, X_3, X_4, X_5, X_6, X_7)$ denotes the number of police shootings on Monday to Sunday and $X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 = 3943$. We would like to analyze the distribution of this random vector. Denote the probability of a police shooting occurring on the given day by p_i , i = 1, ..., 7 and $p_1 + ... + p_7 = 1$. Since the total number of Mondays in those three years may be different from that of Tuesdays (and analogously for other weekdays), the distribution is nearly but not uniform exactly. The total numbers of Mondays to Sundays between 2015 and 2018 are listed as follows.

| | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday | Total |
|----------------------|--------|---------|-----------|----------|--------|----------|--------|-------|
| Total number of days | 209 | 208 | 208 | 209 | 209 | 209 | 209 | 1461 |

Therefore, we set the null hypothesis to be

 H_0 : the data follow a multinomial distribution

with parameters
$$(p_1,..,p_7) = (\frac{209}{1461}, \frac{208}{1461}, \frac{208}{1461}, \frac{209}{1461}, \frac{209}{1461}, \frac{209}{1461}, \frac{209}{1461}).$$

The observed occurrence, observed frequency, expected frequency is listed as follow.

| | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday |
|------------------------------|--------|---------|-----------|----------|--------|----------|--------|
| Observed occurrence(O_i) | 517 | 586 | 616 | 573 | 572 | 518 | 561 |
| Expected occurrence(E_i) | 564.1 | 561.4 | 561.4 | 564.1 | 564.1 | 564.1 | 564.1 |

Table 4.1: Observed and expected frequency of police homicides in 2015-2018.

We see that $E[X_i] \ge 5$ for all i = 1, ..., 7, which means that the sample size is large enough to ensure the observed test statistic

$$X_{7-1}^2 = \sum_{i=1}^7 \frac{(O_i - E_i)^2}{E_i} = \frac{(517 - 564.1)^2}{564.1} + \frac{(586 - 561.4)^2}{561.4} + \dots + \frac{(561 - 564.1)^2}{564.1} = 14.357$$

follow a chi-squared distribution with 6 degree of freedom.

Since $\chi^2_{0.05,6} = 12.59$, the P-value is less than 5%. Therefore, we can reject H_0 at the 5% level of significance. There is reason to believe that the average number of police homicides depends on weekdays.

5 Confidence Interval for Parameter k of a Poisson Distribution

Let X_1, \cdot, X_n be a random sample of size n from a Poisson distribution of parameter k, that is to say with mean $\mu = k$ and variance $\sigma^2 = k$. Since the sample size n is large enough, we assume \overline{X} is normally distributed, with mean k and variance k/n. Therefore,

$$Z = \frac{\overline{X} - k}{\sqrt{k/n}}. (5.1)$$

So the $100(1-\alpha)\%$ confidence interval for \overline{X} (which is the estimator of k, denoted by \hat{k} form now on) is given by

$$\hat{k} \pm z_{\alpha/2} \sqrt{k/n}. \tag{5.2}$$

We can see that the interval depends on an unknown parameter k here. If we simple replace k by \hat{k} , the number $z_{\alpha/2}$ is no longer accurate. However, since the sample size n is large enough to allow the central limit theorem to hold, then the difference between $z_{\alpha/2}$ and the true value is negligible. Finally we obtain our $100(1-\alpha)\%$ confidence interval for k, given by

$$\hat{k} \pm z_{\alpha/2} \sqrt{\hat{k}/n}.\tag{5.3}$$

Now we calculate the interval using the data of the years 2015 to 2018. From Equation (3.2) gives k = 2.69884 and n = 1461. So a 95% confidence interval is given by

$$2.69884 \pm 1.96\sqrt{2.69884/1461}$$
, or 2.699 ± 0.084 . (5.4)

We estimate the data from Jan 1, 2019 to Feb 14, 2019, totally 45 days to test whether it follows a Poisson distribution. For the sample, the following number of deaths are observed:

| Number of deaths | Observed days |
|------------------|---------------|
| 0 | 3 |
| 1 | 14 |
| 2 | 9 |
| 3 | 7 |
| 4 | 6 |
| 5 | 5 |
| 6 | 0 |
| 7 | 0 |
| 8 | 0 |
| 9 | 1 |

The parameter k for the Poisson distribution is estimated using the sample

$$\hat{k} = \bar{X} = \frac{1}{45} (1 \times 14 + 2 \times 9 + 3 \times 7 + 4 \times 6 + 5 \times 5 + 9 \times 1) = 2.46667$$
 (5.5)

We combine the categories with 4 or more observed shootings.

$$P[X = 0] = \frac{e^{-\hat{k}}\hat{k}^0}{0!} = 0.0849$$

$$P[X = 1] = \frac{e^{-\hat{k}}\hat{k}^1}{1!} = 0.2093$$

$$P[X = 2] = \frac{e^{-\hat{k}}\hat{k}^2}{2!} = 0.2582$$

$$P[X = 3] = \frac{e^{-\hat{k}}\hat{k}^3}{3!} = 0.2123$$

$$P[X \ge 4] = \sum_{i=5}^{\infty} \frac{e^{-\hat{k}}\hat{k}^i}{i!} = 0.2353$$
(5.6)

Therefore we have the expected values.

| Number of deaths | Observed days | Expected days |
|------------------|---------------|---------------|
| 0 | 3 | 3.82 |
| 1 | 14 | 9.42 |
| 2 | 9 | 11.62 |
| 3 | 7 | 9.55 |
| 4 or more | 12 | 10.59 |

Then we have

$$X^{2} = \sum_{i=1}^{6} \frac{(O_{i} - E_{i})^{2}}{E_{i}} = 3.862$$
(5.7)

follows a chi-squared distribution with N-1-m=5-1-1=3 degrees pf freedom. Assume $\alpha=0.05$, we have $\chi^2_{0.05,3}=7.82$. Since 3.862<7.82, we are unable to reject H_0 at 5% level of significance. Then we may say that it follows a Poisson distribution.

6 Nelson's Formula and Prediction Intervals

Let X be the total count in a sample of size n from a Poisson distribution with mean λ . We note that $X \sim \text{Poisson } (n\lambda)$. Let Y denote the future total counts that can be observed in a sample size m from the same Poisson distribution so that $Y \sim \text{Poisson } (m\lambda)$. We first define the estimator of λ , $\hat{\lambda}$ as

$$\widehat{\lambda} = \frac{X}{n} \tag{6.1}$$

which can be proved to be an unbiased estimator as

$$E[\widehat{\lambda}] = \frac{n\lambda}{n} = \lambda. \tag{6.2}$$

Also, we have the estimator of Y as

$$\widehat{Y} = \frac{mX}{n} \tag{6.3}$$

which can also be proved to be an unbiased estimator as

$$Y - \mathbf{E}[\widehat{Y}] = m\lambda - \frac{m}{n} \mathbf{E}[X] = m\lambda - \frac{m}{n} \cdot n\lambda = 0.$$
 (6.4)

From the basic properties of Poisson distribution, we have

$$\mathrm{E}[X] = n\lambda \quad \& \quad \mathrm{E}(Y) = m\lambda \,,$$

 $\mathrm{Var}(X) = n\lambda \quad \& \quad \mathrm{Var}(Y) = m\lambda \,.$ (6.5)

Hence we can deduce that

$$E(m\hat{\lambda} - Y) = \frac{m}{n}n\lambda - m\lambda = 0 \tag{6.6}$$

and since X and Y are independent, i.e., Cov(X, Y) = 0, we have

$$\operatorname{Var}(m\widehat{\lambda} - Y) = \operatorname{Var}(\frac{m}{n}X - Y)$$

$$= \frac{m^2}{n^2}\operatorname{Var}(X) + \operatorname{Var}(Y)$$

$$= \frac{m^2}{n^2} \cdot n\lambda + m\lambda$$

$$= m^2\lambda(\frac{1}{n} + \frac{1}{m}),$$
(6.7)

and immediately we have $\widehat{\mathrm{Var}}(m\widehat{\lambda}) = m^2\widehat{\lambda}(1/n+1/m)$. From Central Limit Theorem, we have

$$\frac{m\widehat{\lambda} - Y}{\sqrt{\widehat{\text{Var}}(m\widehat{\lambda} - Y)}} \tag{6.8}$$

follows a standard normal distribution. Thus a $100(1-\alpha)\%$ prediction interval is given by

$$\widehat{Y} \pm z_{\alpha/2} \sqrt{m\widehat{Y}\left(\frac{1}{m} + \frac{1}{n}\right)},\tag{6.9}$$

which is valid under the assumption of large sample sizes. This prediction interval is called Nelson's prediction interval. Note that the prediction interval given above is not defined when X = 0. A commonly used adjustment to handle this extreme case is to set $\hat{Y} = 0.5m/n$ when X = 0.

Then, we can use the data from 2015 to 2018 to predict the number of police shooting in 2019. Using the method of maximum likelihood, we know that the estimator \hat{k} for a Poisson distribution is the mean \bar{x} . Therefore, given the data from 2015 to 2018, we can calculate the value of n and the estimate for k as

$$n = 365 \times 3 + 366 = 1461$$

 $\hat{k} = \frac{X}{n} = \frac{3943}{1461} = 2.69884,$ (6.10)

where k is just λ in Nelson's prediction interval. Also, we can express the estimator for Y by equation

$$\widehat{Y} = \widehat{\lambda} \, m \,. \tag{6.11}$$

We first plot the estimated data for the number of police shootings in 2019 as well as its 95% prediction interval. Note that since we are predicting a discrete random variable, the prediction interval is formed by the set of integer values and is given by

$$[\lceil L \rceil, \lfloor U \rfloor] \tag{6.12}$$

where L denotes the lower bound of Equation 6.9, U the upper bound of Equation 6.9, X is the smallest integer greater than or equal to X, Y is the largest integer less than or equal to X. Also, the estimated data is formed by the set of integer values, which are rounded to the nearest integer after being calculated by Equation 6.11. The graph is shown in Figure 6.1.

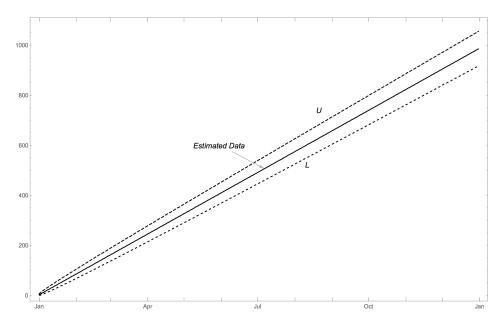


Figure 6.1: Estimated data for the number of police shootings in 2019 as well as its 95% prediction interval

The result shows that we are 95% sure that the true results will be within this interval. We can see from the graph that the estimated data, upper and lower bounds are all approximately straight lines, which is due to the fact that we have to round the number for murders of each day, since that number must be an integer. Moreover, as we have the real data from January 1st, 2019 to February 14th, 2019, we can compare them with our estimated data and the prediction interval, to see that whether our estimation

and prediction correspond to the reality. We plot the real data, our estimated data and the prediction interval together in Figure 6.2.

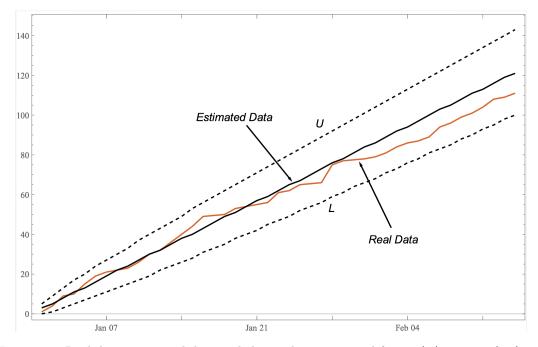


Figure 6.2: Real data, estimated data and the prediction interval from 1/1/2019 to 2/14/2019

The result shows that the real data roughly corresponds to our estimated data we obtain from Equation (6.11), although they have a trend to differ more greatly as time passes by. Besides, they are always within the prediction interval given by Equation (6.9). From the caluculation from Equation (5.5), we get k = 2.46667, which is smaller than the value we used here. Therefore, the estimated data is sightly larger than the real data. Hence, we can conclude that our estimation and prediction are appropriate so far. Further conclusion can be drawn if we have more real data in 2019.

7 Conclusion

Through this project, we practice the knowledge we learned in class. In the first six questions, we use the methodology learned from multiple sources to analyze the distribution of police homicides in US and we get the results that it follows a Poisson Distribution. In the last problem, we make a prediction of number of police homicides in 2019 in US and give a 95% prediction interval. During this whole project, we deepened our understanding of statistical methods in research and we made use of tools such as Mathematica to help us analyze data, which is a precious experience for us engineers.

What's more, from this project, we know that there are actually plenty of murders around the world. For example, in US, there are almost 3 people died a day just because of police homicide. This project definitely reminds us to cherish the priceless safety we have in China.

8 Reference

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9 Appendix

Python codes for data processing

```
import qrcode
import xlrd
from PIL import Image, ImageDraw, ImageFont
data = xlrd.open_workbook('fatal-police-shootings-data.xlsx')
table = data.sheets()[1]
dates = table.col_values(0, start_rowx=0, end_rowx=None)
date_num = {}
for date in dates:
   if (date not in date_num.keys()):
        date_num.update({date: dates.count(date)})
print(date_num)
numbers = list(date_num.values())
total = 0
for i in range(15):
   total += i * numbers.count(i)
print(total)
```

Mathematica codes for plotting

Plotting Fig 2.1

Plotting Fig 4.1

Plotting Fig 4.2

```
August, September, October, November, December}, Frame -> True, FrameLabel -> {"Month", "Number of homicides 15 - 18"}]
```

Plotting Fig 6.1 and 6.2

In[4]:= DateListPlot[{estimated, lowerbound, upperbound}]
 DateListPlot[{rawdata, estimated, lowerbound, upperbound}]

Data for plotting Figure (6.1) and (6.2)

| Date | Real | Estimated | Lowerbound | Upperbound |
|-----------|------|-----------|------------|------------|
| 2019/1/1 | 1 | 3 | 0 | 5 |
| 2019/1/2 | 4 | 5 | 1 | 9 |
| 2019/1/3 | 9 | 8 | 3 | 13 |
| 2019/1/4 | 10 | 11 | 5 | 17 |
| 2019/1/5 | 15 | 13 | 7 | 20 |
| 2019/1/6 | 19 | 16 | 9 | 24 |
| 2019/1/7 | 21 | 19 | 11 | 27 |
| 2019/1/8 | 22 | 22 | 13 | 30 |
| 2019/1/9 | 23 | 24 | 15 | 33 |
| 2019/1/10 | 26 | 27 | 17 | 37 |
| 2019/1/11 | 30 | 30 | 19 | 40 |
| 2019/1/12 | 32 | 32 | 22 | 43 |
| 2019/1/13 | 36 | 35 | 24 | 46 |
| 2019/1/14 | 40 | 38 | 26 | 49 |
| 2019/1/15 | 44 | 40 | 28 | 53 |
| 2019/1/16 | 49 | 43 | 31 | 56 |
| 2019/1/17 | 49 | 46 | 33 | 59 |
| 2019/1/18 | 50 | 49 | 35 | 62 |
| 2019/1/19 | 53 | 51 | 38 | 65 |
| 2019/1/20 | 54 | 54 | 40 | 68 |
| 2019/1/21 | 55 | 57 | 42 | 71 |
| 2019/1/22 | 56 | 59 | 45 | 74 |
| 2019/1/23 | 61 | 62 | 47 | 77 |
| 2019/1/24 | 62 | 65 | 49 | 80 |
| 2019/1/25 | 65 | 67 | 52 | 83 |
| 2019/1/26 | 65 | 70 | 54 | 86 |
| 2019/1/27 | 66 | 73 | 56 | 89 |
| 2019/1/28 | 75 | 76 | 59 | 92 |
| 2019/1/29 | 77 | 78 | 61 | 95 |
| 2019/1/30 | 77 | 81 | 64 | 98 |
| 2019/1/31 | 78 | 84 | 66 | 101 |
| 2019/2/1 | 79 | 86 | 68 | 104 |
| 2019/2/2 | 81 | 89 | 71 | 107 |
| 2019/2/3 | 84 | 92 | 73 | 110 |
| 2019/2/4 | 86 | 94 | 76 | 113 |
| 2019/2/5 | 87 | 97 | 78 | 116 |
| 2019/2/6 | 89 | 100 | 81 | 119 |
| 2019/2/7 | 94 | 103 | 83 | 122 |
| 2019/2/8 | 96 | 105 | 85 | 125 |
| 2019/2/9 | 99 | 108 | 88 | 128 |
| 2019/2/10 | 101 | 111 | 90 | 131 |

| 2010/2/11 | 104 | 113 | 93 | 134 |
|---|-----|-----------|----------|-----------|
| $\begin{array}{c c} 2019/2/11 \\ 2019/2/12 \end{array}$ | 104 | 116 | 95 95 | 137 |
| 2019/2/12 $2019/2/13$ | 109 | 110 | 98 98 | 140 |
| 2019/2/13 | 111 | 121 | 100 | 143 |
| 2019/2/14 $2019/2/15$ | 111 | 121 | 100 | 146 |
| 2019/2/15 2019/2/16 | | 124 127 | 102 | 149 |
| , , | | 130 | 103 | 149 152 |
| 2019/2/17 | | | | |
| 2019/2/18 | | 132 | 110 | 155 |
| 2019/2/19 | | 135 | 112 | 158 |
| 2019/2/20 | | 138 | 115 | 161 |
| 2019/2/21 | | 140 | 117 | 163 |
| 2019/2/22 | | 143 | 120 | 166 |
| 2019/2/23 | | 146 | 122 | 169 |
| 2019/2/24 | | 148 | 125 | 172 |
| 2019/2/25 | | 151 | 127 | 175 |
| 2019/2/26 | | 154 | 130 | 178 |
| 2019/2/27 | | 157 | 132 | 181 |
| 2019/2/28 | | 159 | 135 | 184 |
| 2019/3/1 | | 162 | 137 | 187 |
| 2019/3/2 | | 165 | 139 | 190 |
| 2019/3/3 | | 167 | 142 | 193 |
| 2019/3/4 | | 170 | 144 | 196 |
| 2019/3/5 | | 173 | 147 | 199 |
| 2019/3/6 | | 175 | 149 | 201 |
| 2019/3/7 | | 178 | 152 | 204 |
| 2019/3/8 | | 181 | 154 | 207 |
| 2019/3/9 | | 184 | 157 | 210 |
| 2019/3/10 | | 186 | 159 | 213 |
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| 2019/3/17 | | 205 | 177 | 233 |
| 2019/3/18 | | 208 | 179 | 236 |
| 2019/3/19 | | 211 | 182 | 239 |
| 2019/3/20 | | 213 | 184 | 242 |
| 2019/3/21 | | 216 | 187 | 245 |
| 2019/3/22 | | 219 | 189 | 248 |
| 2019/3/23 | | 221 | 192 | 251 |
| 2019/3/24 | | 224 | 194 | 254 |
| 2019/3/25 | | 227 | 197 | 257 |
| 2019/3/26 | | 229 | 199 | 259 |
| 2019/3/27 | | 232 | 202 | 262 |
| 2019/3/28 | | 235 | 204 | 265 |
| 2019/3/29 | | 237 | 207 | 268 |
| 2019/3/30 | | 240 | 209 | 271 |
| 2019/3/31 | | 243 | 212 | 274 |
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| 2019/4/4 | | 254 | 222 | 285 |

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| 2019/4/5 | 256 | 225 | 288 |
| 2019/4/6 | 259 | 227 | 291 |
| 2019/4/7 | 262 | 230 | 294 |
| 2019/4/8 | 264 | 232 | 297 |
| 2019/4/9 | 267 | 235 | 300 |
| 2019/4/10 | 270 | 237 | 303 |
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| 2019/4/12 | 275 | 242 | 308 |
| 2019/4/13 | 278 | 245 | 311 |
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| 2019/4/16 | 286 | 252 | 320 |
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| 2019/4/18 | 291 | 257 | 326 |
| 2019/4/19 | 294 | 260 | 329 |
| 2019/4/20 | 297 | 262 | 331 |
| 2019/4/21 | 300 | 265 | 334 |
| 2019/4/22 | 302 | 267 | 337 |
| 2019/4/23 | 305 | 270 | 340 |
| 2019/4/24 | 308 | 272 | 343 |
| 2019/4/25 | 310 | 275 | 346 |
| 2019/4/26 | 313 | 278 | 349 |
| 2019/4/27 | 316 | 280 | 351 |
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| 2019/5/3 | 332 | 295 | 369 |
| 2019/5/4 | 335 | 298 | 372 |
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| 2019/5/6 | 340 | 303 | 377 |
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| 2019/5/9 | 348 | 310 | 386 |
| 2019/5/10 | 351 | 313 | 389 |
| 2019/5/11 | 354 | 316 | 392 |
| 2019/5/12 | 356 | 318 | 394 |
| 2019/5/13 | 359 | 321 | 394 |
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| 2019/5/20 | 378 381 | 338 | 417 |
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| 2019/5/24 | 389 | 349 | 429 |
| 2019/5/25 | 391 | 351 | 431 |
| 2019/5/26 | 394 | 354 | 434 |
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|-----------|-----|-----|-----|
| 2019/5/29 | 402 | 361 | 443 |
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| 2019/6/4 | 418 | 377 | 460 |
| 2019/6/5 | 421 | 379 | 463 |
| 2019/6/6 | 424 | 382 | 466 |
| 2019/6/7 | 426 | 384 | 469 |
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| 2019/6/12 | 440 | 397 | 483 |
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| 2019/6/16 | 451 | 407 | 494 |
| 2019/6/17 | 453 | 410 | 497 |
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| 2019/6/30 | 488 | 443 | 534 |
| 2019/7/1 | 491 | 446 | 537 |
| 2019/7/2 | 494 | 448 | 540 |
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| 2019/7/8 | 510 | 464 | 557 |
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| 2019/7/10 | 515 | 469 | 562 |
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| 2019/7/13 | 524 | 476 | 571 |
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| 2019/7/15 | 529 | 481 | 576 |
| 2019/7/16 | 532 | 484 | 579 |
| 2019/7/17 | 534 | 487 | 582 |
| 2019/7/18 | 537 | 489 | 585 |
| 2019/7/19 | 540 | 492 | 588 |
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|---------------------|-----|------------|--------------|
| 2019/7/21 | 545 | 497 | 593 |
| 2019/7/22 | 548 | 499 | 596 |
| 2019/7/23 | 551 | 502 | 599 |
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| 2019/7/25 | 556 | 507 | 605 |
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| 2019/7/28 | 564 | 515 | 613 |
| 2019/7/29 | 567 | 517 | 616 |
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| | 572 | 520 522 | 622 |
| 2019/7/31 | 575 | 525 | 625 |
| 2019/8/1 | | | |
| 2019/8/2 | 578 | 528 520 | 627 |
| 2019/8/3 | 580 | 530 533 | $630 \\ 633$ |
| 2019/8/4 | 583 | 535 | 636 |
| 2019/8/5 | 586 | | |
| 2019/8/6 | 588 | 538 | 639 |
| 2019/8/7 | 591 | 540 | 642 |
| 2019/8/8 | 594 | 543 | 644 |
| 2019/8/9 | 596 | 546 | 647 |
| 2019/8/10 | 599 | 548 | 650 |
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| 2019/8/12 | 605 | 553 | 656 |
| 2019/8/13 | 607 | 556 | 659 |
| 2019/8/14 | 610 | 558 | 661 |
| 2019/8/15 | 613 | 561 | 664 |
| 2019/8/16 | 615 | 564 | 667 |
| 2019/8/17 | 618 | 566 | 670 |
| 2019/8/18 | 621 | 569 | 673 |
| 2019/8/19 | 623 | 571 | 676 |
| 2019/8/20 | 626 | 574 | 678 |
| 2019/8/21 | 629 | 576 | 681 |
| 2019/8/22 | 632 | 579 | 684 |
| 2019/8/23 | 634 | 582 | 687 |
| 2019/8/24 | 637 | 584 | 690 |
| 2019/8/25 | 640 | 587 | 693 |
| 2019/8/26 | 642 | 589 | 695 |
| 2019/8/27 | 645 | 592 | 698 |
| 2019/8/28 | 648 | 594 | 701 |
| 2019/8/29 | 650 | 597 | 704 |
| 2019/8/30 | 653 | 600 | 707 |
| 2019/8/31 | 656 | 602 | 710 |
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| 2019/9/2 | 661 | 607 | 715 |
| 2019/9/3 | 664 | 610 | 718 |
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| 2019/9/5 | 669 | 615 | 724 726 |
| 2019/9/6 | 672 | 618 | 726 |
| 2019/9/7 | 675 | 620 | 729 |
| 2019/9/8 | 677 | 623 | 732 |
| 2019/9/9 | 680 | 625 | 735 |
| 2019/9/10 | 683 | 628 | 738 |

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|----------------------|------------|------------|------------|
| 2019/9/12 | 688 | 633 | 743 |
| 2019/9/13 | 691 | 636 | 746 |
| 2019/9/14 | 694 | 638 | 749 |
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| | 702 704 | 646 648 | 758 760 |
| 2019/9/18 | 704 | | |
| 2019/9/19 | | 651 | 763 |
| 2019/9/20 | 710 | 654 | 766 760 |
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| 2019/9/22 | 715 | 659 | 772 |
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| 2019/9/24 | 721 | 664 | 777 |
| 2019/9/25 | 723 | 666 | 780 |
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| 2019/9/27 | 729 | 672 | 786 |
| 2019/9/28 | 731 | 674 | 789 |
| 2019/9/29 | 734 | 677 | 791 |
| 2019/9/30 | 737 | 679 | 794 |
| 2019/10/1 | 739 | 682 | 797 |
| 2019/10/2 | 742 | 684 | 800 |
| 2019/10/3 | 745 | 687 | 803 |
| 2019/10/4 | 748 | 690 | 806 |
| 2019/10/5 | 750 | 692 | 808 |
| 2019/10/6 | 753 | 695 | 811 |
| 2019/10/7 | 756 | 697 | 814 |
| 2019/10/8 | 758 | 700 | 817 |
| 2019/10/9 | 761 | 703 | 820 |
| 2019/10/10 | 764 | 705 | 822 |
| 2019/10/11 | 766 | 708 | 825 |
| 2019/10/12 | 769 | 710 | 828 |
| 2019/10/13 | 772 | 713 | 831 |
| 2019/10/14 | 775 | 715 | 834 |
| 2019/10/15 | 777 | 718 | 837 |
| 2019/10/16 | 780 | 721 | 839 |
| 2019/10/17 | 783 | 723 | 842 |
| 2019/10/18 | 785 | 726 | 845 |
| 2019/10/19 | 788 | 728 | 848 |
| 2019/10/20 | 791 | 731 | 851 |
| 2019/10/21 | 793 | 733 | 853 |
| 2019/10/22 | 796 | 736 | 856 |
| 2019/10/23 | 799 | 739 | 859 |
| 2019/10/24 | 802 | 741 | 862 |
| 2019/10/25 | 804 | 744 | 865 |
| 2019/10/26 | 807 | 746 | 868 |
| 2019/10/27 | 810 | 749 | 870 |
| 2019/10/28 | 812 | 752 | 873 |
| 2019/10/29 | 815 | 754 | 876 |
| 2019/10/29 | 818 | 757 | 879 |
| 2019/10/31 | 820 | 759 | 882 |
| 2019/10/31 2019/11/1 | 823 | 762 | 884 |
| 2019/11/1 | 826 | 764 | 887 |
| 2010/11/2 | 020 | 104 | 001 |

| 2019/11/3 | 829 | 767 | 890 |
|-------------------------|-----|------------|------------|
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| 2019/11/5 | 834 | 772 | 896 |
| 2019/11/6 | 837 | 775 | 899 |
| 2019/11/7 | 839 | 777 | 901 |
| 2019/11/8 | 842 | 780 | 904 |
| 2019/11/9 | 845 | 782 | 907 |
| 2019/11/10 | 847 | 785 | 910 |
| 2019/11/11 | 850 | 788 | 913 |
| 2019/11/11 | 853 | 790 | 915 |
| 2019/11/13 | 856 | 793 | 918 |
| 2019/11/14 | 858 | 795 | 921 |
| 2019/11/15 | 861 | 798 | 924 |
| 2019/11/16 | 864 | 801 | 927 |
| 2019/11/17 | 866 | 803 | 930 |
| 2019/11/18 | 869 | 806 | 932 |
| 2019/11/19 | 872 | 808 | 935 |
| 2019/11/19 | 874 | 811 | 938 |
| 2019/11/20 | 877 | 813 | 941 |
| 2019/11/21 | 880 | 816 | 944 |
| 2019/11/23 | 883 | 819 | 946 |
| 2019/11/23 | 885 | 821 | 949 |
| 2019/11/24 2019/11/25 | 888 | 824 | 952 |
| 2019/11/26 | 891 | 824 826 | 955 955 |
| 2019/11/20 | 893 | 829 | 958 |
| 2019/11/28 | 896 | 832 | 961 |
| 2019/11/28 2019/11/29 | 899 | 834 | 963 |
| 2019/11/29 2019/11/30 | 901 | 837 | 966 |
| ' ' | 901 | 839 | 969 |
| 2019/12/1 | 904 | 842 | 909 972 |
| 2019/12/2 2019/12/3 | 910 | 844 | 972 975 |
| 2019/12/3 | 910 | 847 | 973 977 |
| 2019/12/4 2019/12/5 | 912 | 850 | 980 |
| 2019/12/5 | 918 | 852 | 983 |
| 2019/12/0 | 920 | 855 | 986 986 |
| 2019/12/7 | 923 | 857 | 989 |
| 2019/12/8 2019/12/9 | 926 | 860 | 991 |
| 2019/12/9 | 928 | 863 | 994 |
| 2019/12/10 | 931 | 865 | 997 |
| 2019/12/11 | 934 | 868 | 1000 |
| 2019/12/13 | 936 | 870 | 1003 |
| 2019/12/14 | 939 | 873 | 1006 |
| 2019/12/14 | 942 | 875 | 1008 |
| 2019/12/16 | 945 | 878 | 1011 |
| 2019/12/17 | 947 | 881 | 1014 |
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| 2019/12/19 2019/12/20 | 955 | 888 | 1020 |
| 2019/12/20 | 958 | 891 | 1025 |
| 2019/12/21 | 961 | 894 | 1028 |
| 2019/12/23 | 963 | 896 | 1031 |
| 2019/12/24 | 966 | 899 | 1034 |
| 2019/12/24 | 969 | 901 | 1034 |
| 2010/12/20 | 505 | 501 | 1000 |

| 2019/12/26 | 972 | 904 | 1039 |
|------------|-----|-----|------|
| 2019/12/27 | 974 | 906 | 1042 |
| 2019/12/28 | 977 | 909 | 1045 |
| 2019/12/29 | 980 | 912 | 1048 |
| 2019/12/30 | 982 | 914 | 1051 |
| 2019/12/31 | 985 | 917 | 1053 |