Announcements

- Complete Homework 0 if you haven’t

- Teaching feedback welcome

Last Time: Introduction and Security Principles

- Course Logistics

- Security Principles

Today’s Cunning Plan: Cryptography

- Introduction to Cryptography
  - What is cryptography?
  - Definitions of key properties
  - IND-CPA security
  - A brief history of cryptography
  - Symmetric-key crypto
    - One-time pads

What is cryptography?

What is Cryptography?

- **Older definition**: The study of secure communication over insecure channels
- **Newer definition**: Provide rigorous guarantees on the security of data and computation in the presence of an attacker
  - Not just *confidentiality* but also *integrity* and *authenticity* (we'll see these definitions today)
- Modern cryptography is heavily based on math
  - The math is hard
  - We won't cover the details. Wikipedia is a great resource
  - Some system / algorithms are provably secure

Definitions of Key Properties

Meet Alice, Bob, Eve, and Mallory

- Alice and Bob: The main characters trying to send messages to each other over an insecure communication channel

- Eve: An **eavesdropper** who can read any data sent over the channel

- Mallory: A **manipulator** who can read and modify any data sent over the channel

Meet Alice, Bob, Eve, and Mallory

- We often describe cryptographic problems using a common cast of characters
- One scenario:
  - Alice wants to send a message to Bob.
  - However, Eve is going to *eavesdrop* on the communication channel.
  - How does Alice send the message to Bob without Eve learning about the message?
- Another scenario:
  - Bob wants to send a message to Alice.
  - However, Mallory is going to *tamper* with the communication channel.
  - How does Bob send the message to Alice without Mallory changing the message?

Three Goals of Cryptography

- In cryptography, there are three main properties that we want on our information
- **Confidentiality**: An adversary cannot *read* our messages.
- **Integrity**: An adversary cannot *change* our messages without being detected.
- **Authenticity**: I can prove that this message came from the person who claims to have written it.
  - Integrity and authenticity are closely related properties…
    - Before I can prove that a message came from a certain person, I have to prove that the message wasn't changed!

Keys

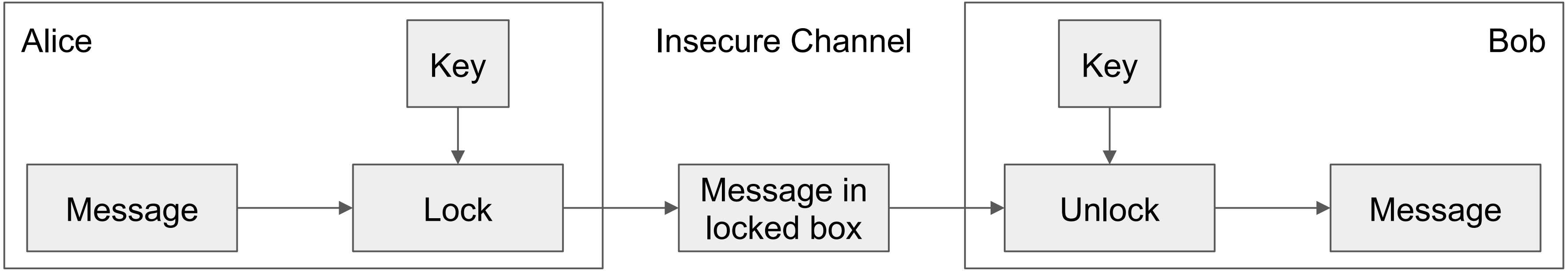
- The most basic building block of any cryptographic scheme: The **key**
- We use the key in our algorithms to secure messages
- Two models of keys:
  - **Symmetric key model**: Alice and Bob both know the value of the same secret key.
    - Example: One-time pad, AES encryption
  - **Asymmetric key model**: Everybody has two keys, a secret key and a public key.
    - Example: RSA encryption

Security Principle: Kerckhoff’s Principle

- This principle is closely related to Shannon’s Maxim
  - Don’t use security through obscurity. Assume the attacker knows the system.
- Kerckhoff’s principle says:
  - Cryptosystems should remain secure even when the attacker knows all internal details of the system
  - The key should be the only thing that must be kept secret
  - The system should be designed to make it easy to change keys that are leaked (or suspected to be leaked)
    - If your secrets are leaked, it is usually a lot easier to change the key than to replace every instance of the running software
- Our assumption: The attacker knows all the algorithms we use. The only information the attacker is missing is the secret key(s).

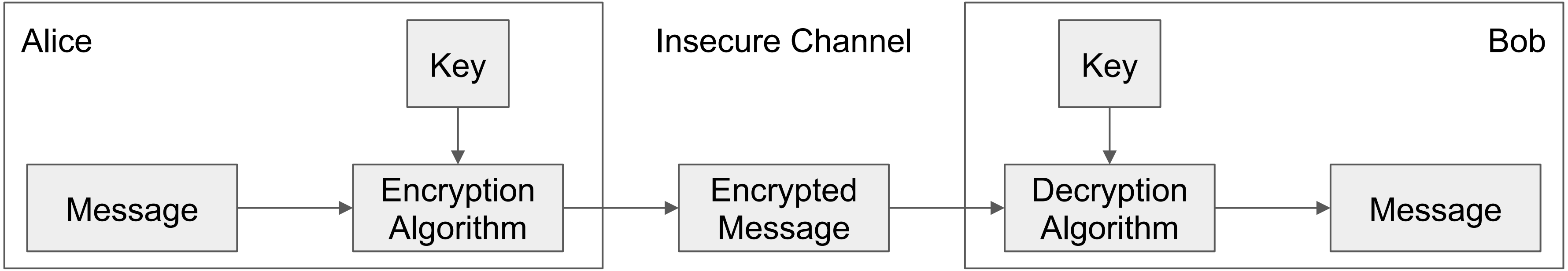
Confidentiality

- **Confidentiality**: An adversary cannot *read* our messages.
- Analogy: Locking and unlocking the message
  - Alice uses the key to lock the message in a box
  - Alice sends the message (locked in the box) over the insecure channel
  - Eve sees the locked box, but cannot access the message without the key
  - Bob receives the message (locked in the box) and uses the key to unlock the message



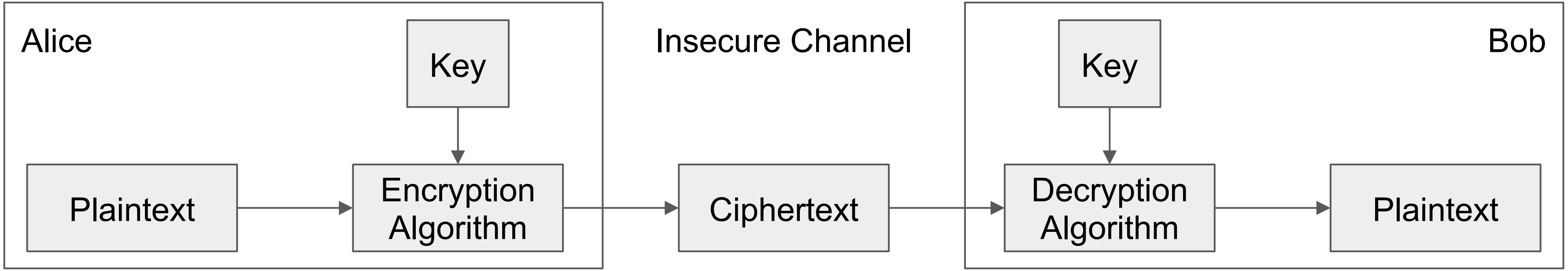
Confidentiality

- Schemes provide confidentiality by **encrypting** messages
  - Alice uses the key to **encrypt** the message: Change the message into a scrambled form
  - Alice sends the encrypted message over the insecure channel
  - Eve sees the encrypted message, but cannot figure out the original message without the key
  - Bob receives the encrypted message and uses the key to **decrypt** the message back into its original form



Confidentiality

- **Plaintext**: The original message
- **Ciphertext**: The encrypted message



Integrity (and Authenticity)

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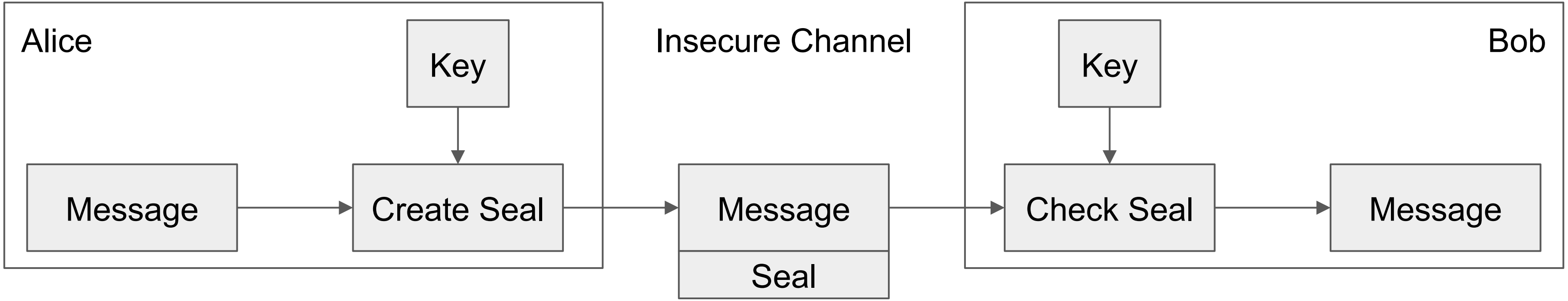
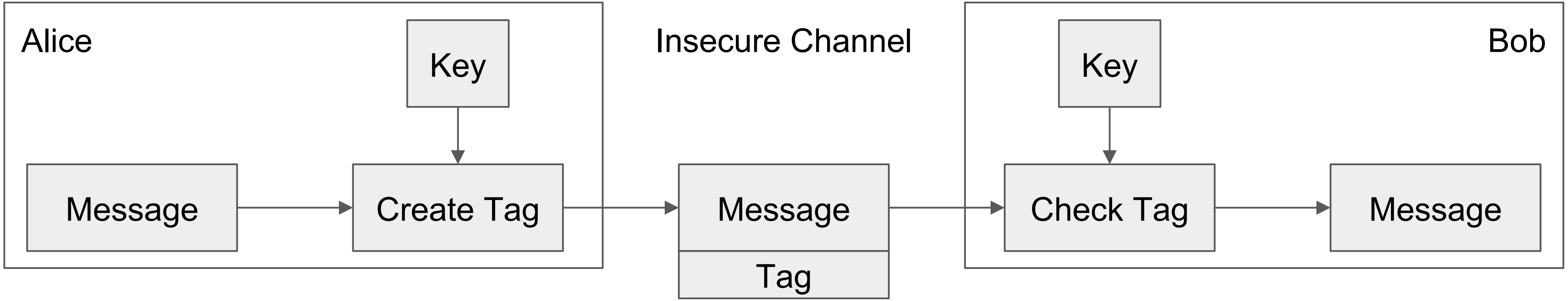
- **Integrity**: An adversary cannot *change* our messages without being detected.
- Analogy: Adding a seal on the message
  - Alice uses the key to add a special seal on the message (e.g. puts tape on the envelope)
  - Alice sends the message and the seal over the insecure channel
  - If Mallory tampers with the message, she'll break the seal (e.g. break the tape on the envelope)
  - Without the key, Mallory cannot create her own seal
  - Bob receives the message and the seal and checks that the seal has not been broken



Integrity (and Authenticity)

- Schemes provide integrity by adding a **tag** or **signature** on messages
    - Alice uses the key to generate a special tag for the message
    - Alice sends the message and the tag over the insecure channel
    - If Mallory tampers with the message, the tag will no longer be valid
    - Bob receives the message and the tag and checks that the tag is still valid



Threat Models (Types of Cryptanalysis)

- Ciphertext only
  - Attackers only have ciphertext, need to decrypt it
- Known-plaintext
  - Attackers know **some** ciphertext and its corresponding plaintext, needs to decrypt some other cyphertext
- Chosen-plaintext
  - Attackers can get **any** ciphertext they want encrypted, and need to decrypt some specific cyphertext
  - e.g., Eve can trick Alice into encrypting arbitrary messages of Eve's choice
- In this class, we often use the chosen plaintext attack (CPA) model

Cryptography Roadmap

	Symmetric-key	Asymmetric-key
Confidentiality	• One-time pads • Block ciphers with chaining modes (e.g. AES-CBC) • Stream ciphers	• RSA encryption • ElGamal encryption
Integrity, Authentication	• MACs (e.g. HMAC)	• Digital signatures (e.g. RSA signatures)

- Hash functions
- Pseudorandom number generators
- Public key exchange (e.g. Diffie-Hellman)
- Key management (certificates)
- Password management

Symmetric-Key Encryption

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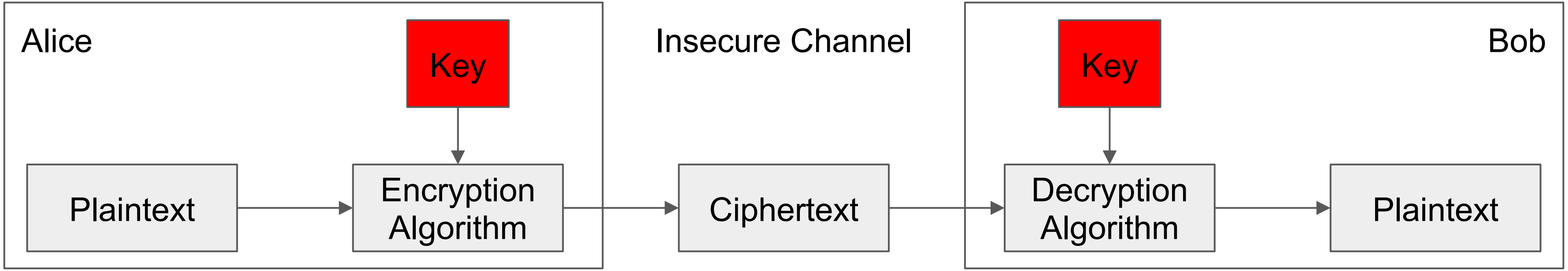
Symmetric-Key Encryption

- The next few schemes are symmetric-key encryption schemes
  - **Encryption schemes** aim to provide *confidentiality* (but not integrity or authentication)
  - **Symmetric-key** means Alice and Bob share the same secret key that the attacker doesn't know
    - Don't worry about how Alice and Bob share the key for now

- For modern schemes, we're going to assume that messages are *bitstrings*
  - **Bitstring**: A sequence of bits (0 or 1), e.g. `110101010010010`
  - Text, images, etc. can usually be converted into bitstrings before encryption, so bitstrings are a useful abstraction. After all, everything in a computer is just a sequence of bits!

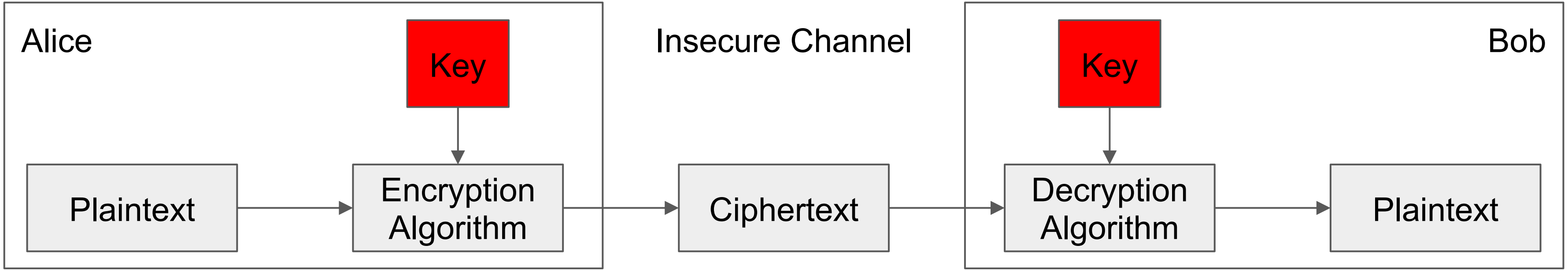
Symmetric-Key Encryption: Definition

- A symmetric-key encryption scheme has three algorithms:
  - KeyGen() → K: Generate a key K
  - Enc(K, M) → C: Encrypt a **plaintext** M using the key K to produce **ciphertext** C
  - Dec(K, C) → M: Decrypt a ciphertext C using the key K



Symmetric-Key Encryption: Definition

- What properties do we want from a symmetric encryption scheme?
  - **Correctness**: Decrypting a ciphertext should result in the message that was originally encrypted
    - $\mathrm{Dec}(K, \mathrm{Enc}(K, M)) = M$ for all $K \leftarrow \mathrm{KeyGen}()$ and M
  - **Efficiency**: Encryption/decryption algorithms should be fast: >1 Gbps on a standard computer
  - **Security**: Confidentiality



Defining Confidentiality

- Recall our definition of confidentiality from earlier: “An adversary cannot read our messages”
  - This definition isn’t very specific
    - What if Eve can read the first half of Alice’s message, but not the second half?
    - What if Eve figures out that Alice’s message starts with “Dear Bob”?
  - This definition doesn’t account for prior knowledge
    - What if Eve already knew that Alice’s message ends in “Sincerely, Alice”?
    - What if Eve knows that Alice’s message is “BUY!” or “SELL” but doesn't know which?

Defining Confidentiality

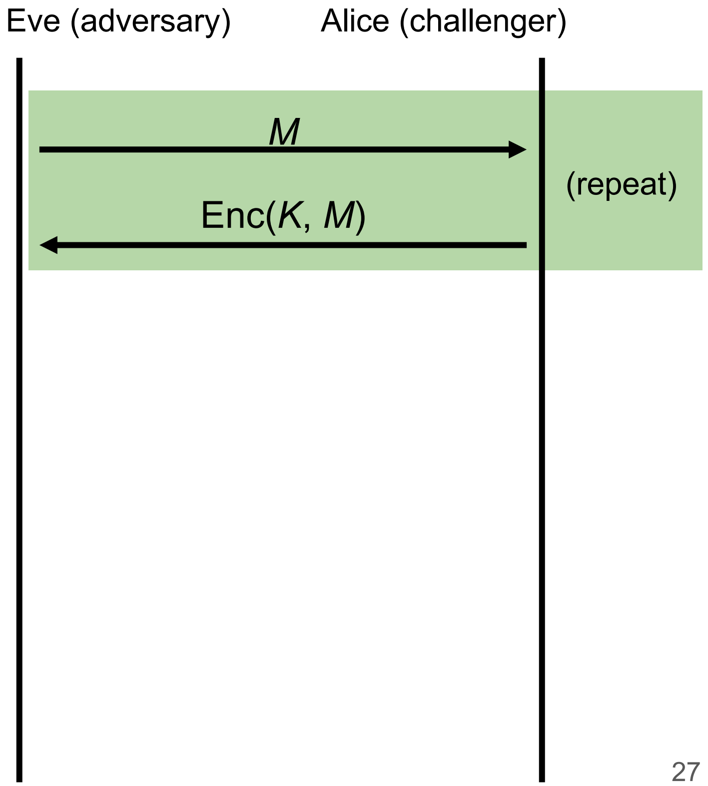
- A better definition of confidentiality: The ciphertext should not give the attacker *any **additional** information* about the plaintext.
- Let's design an experiment/security **game** to test our definition:
  - Eve chooses two messages M_0 and M_1 of the same length
  - Alice chooses one message at random M_b, encrypts it, and sends the ciphertext
  - Eve knows either M_0 or M_1 was sent, but doesn't know which
  - Eve reads the ciphertext and tries to guess which message was sent
  - If the probability that Eve correctly guesses which message was sent is 1/2, then the encryption scheme is confidential
- Intuition
  - If the scheme is confidential, Eve can only guess with probability 1/2, which is no different than if Eve hadn't sent the ciphertext at all
  - In other words: the ciphertext gave Eve no ***additional*** information about which plaintext was sent!

Defining Confidentiality: IND-CPA

- Recall our threat model: Eve can also perform a **chosen plaintext attack (CPA)**
  - It means, Eve can trick Alice into encrypting arbitrary messages of Eve's choice
  - We can adapt our experiment to account for this threat model
- A better definition of confidentiality: Even if Eve is able to trick Alice into encrypting messages, Eve can still only guess what message Alice sent with probability 1/2.
  - This definition is called **IND-CPA** (indistinguishability under chosen plaintext attack)
- Cryptographic properties are often defined in terms of “games” that an adversary can either “win” or “lose”

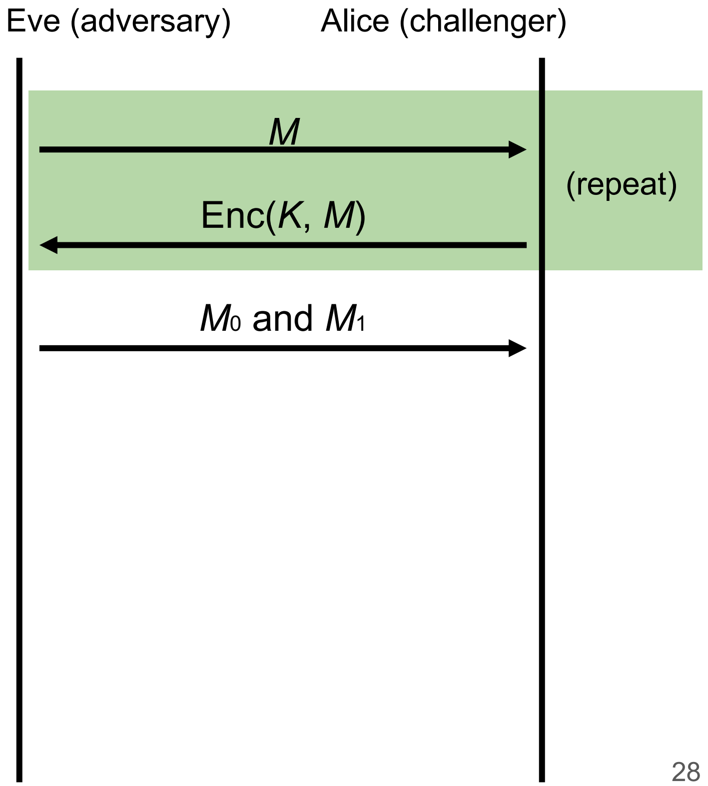
Defining Confidentiality: IND-CPA

1. Eve may choose plaintexts to send to Alice and receives their ciphertexts



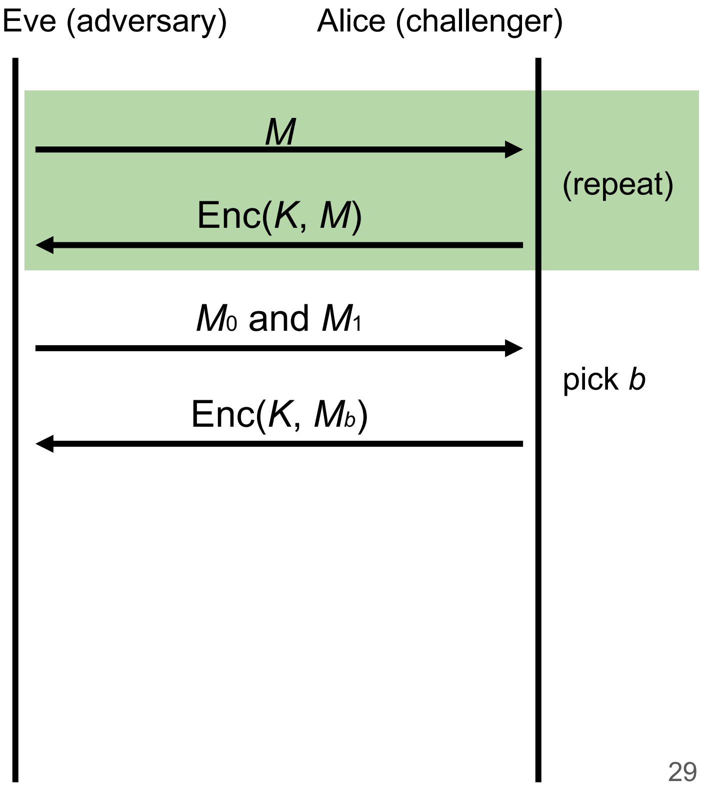
Defining Confidentiality: IND-CPA

1. Eve may choose plaintexts to send to Alice and receives their ciphertexts
2. Eve issues a pair of plaintexts M_0 and M_1 to Alice



Defining Confidentiality: IND-CPA

1. Eve may choose plaintexts to send to Alice and receives their ciphertexts
2. Eve issues a pair of plaintexts M_0 and M_1 to Alice
3. Alice randomly chooses either M_0 or M_1 to encrypt and sends the encryption back
    a. Alice does not tell Eve which one was encrypted!



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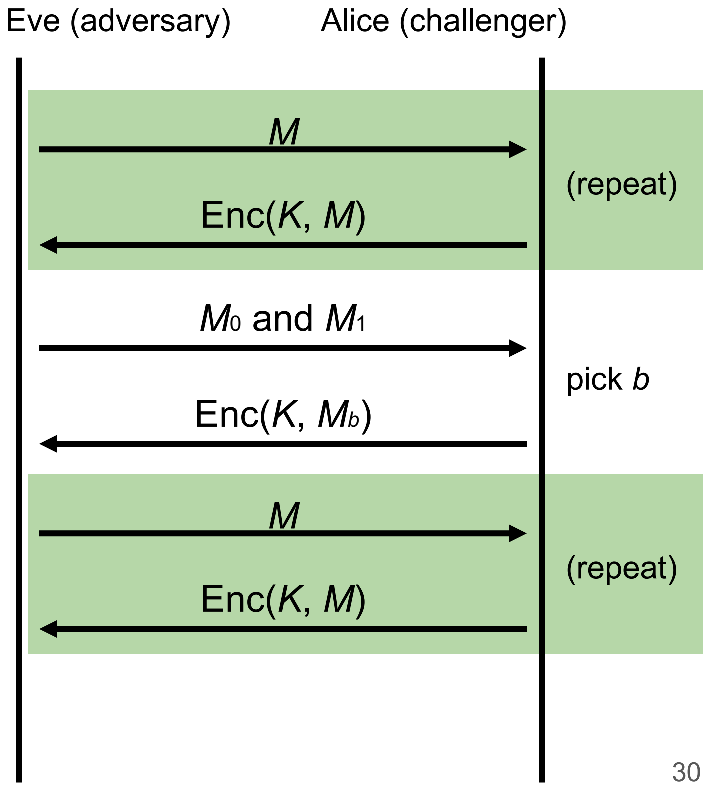
4. Eve may again choose plaintexts to send to Alice and receives their ciphertexts



Defining Confidentiality: IND-CPA

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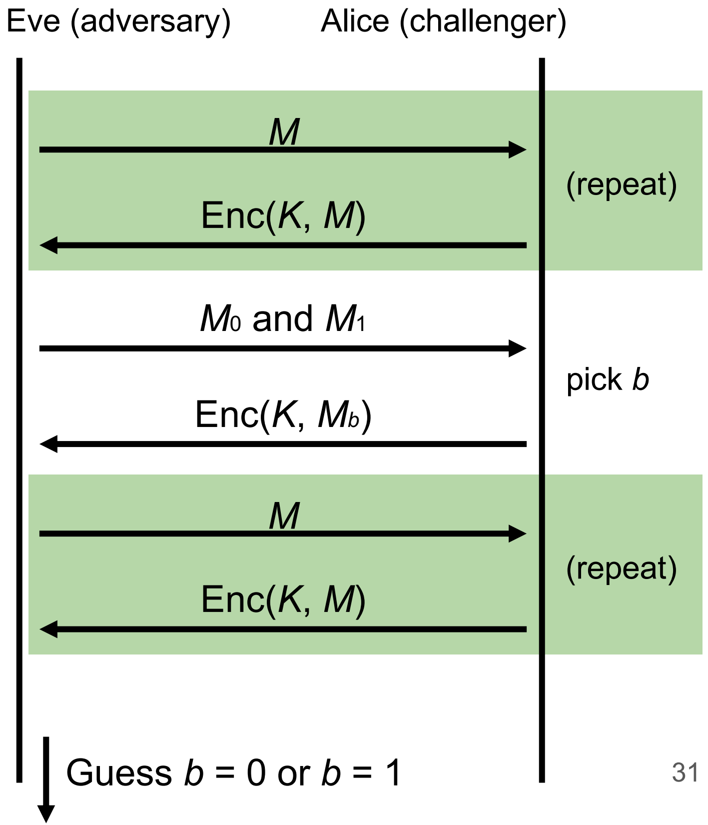
5. Eventually, Eve outputs a guess as to whether Alice encrypted M_0 or M_1



Defining Confidentiality: IND-CPA

- If Eve correctly guesses which message Alice encrypted, then Eve wins. Otherwise, she loses.
- How does Eve guess whether M_0 or M_1 was encrypted? What strategy does she use?
  - We don't *assume* she uses a particular strategy; Eve represents all possible strategies
- Proving insecurity: There exists at least *one* strategy that can win the IND-CPA game with probability > 1/2
  - 1/2 is the probability of winning by random guessing
  - If you can be better than random, then the ciphertext has leaked information, and Eve is able to learn it and use it to gain an advantage!
- Proving security: For *all* attackers/Eve-s, the probability of winning the IND-CPA game is at most 1/2

Edge Cases: Length

- Cryptographic schemes are (usually) allowed to leak the length of the message
  - To hide length: All messages must always be the same length
    - 16-byte messages: We can’t encrypt large messages (images, videos, etc.)!
    - 1-GB messages: Sending small messages (text, Tweets, etc.) needs 1 GB of bandwidth!
    - This is unpractical
- In the IND-CPA game: M_0 and M_1 must be the same length
  - To break IND-CPA, Eve must learn something other than message length

Edge Cases: Attacker Runtime

- Some schemes are theoretically vulnerable, but secure in any real-world setting
  - If an attack takes longer than the life of the solar system to complete, it probably won't happen!
  - Or if it would require a computer made out of a literal galaxy worth of science-fiction nanotech
- In the IND-CPA game: Eve is limited to a practical runtime
  - One common practical limit: Eve is limited to polynomial runtime algorithms (no exponential-time algorithms)

Edge Cases: Negligible Advantage

- Sometimes it's possible for Eve to win with probability $1/2 + 1/2^{128}$
  - This probability is greater than 1/2, but it's so close to 1/2 that it's as good as 1/2.
  - Eve's advantage is so small that she can't use it for any practical attacks
- In the IND-CPA game: The scheme is secure even if Eve can win with probability $\leq 1/2 + \varepsilon$, where ε is *negligible*
  - The actual mathematical definition of negligible is out of scope
  - Example: $1/2 + 1/2^{128}$: Negligible advantage
  - Example: 2/3: Non-negligible advantage

IND-CPA: Putting it together

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1. Eve may choose plaintexts to send to Alice and receives their ciphertexts
2. Eve issues a pair of plaintexts M_0 and M_1 to Alice
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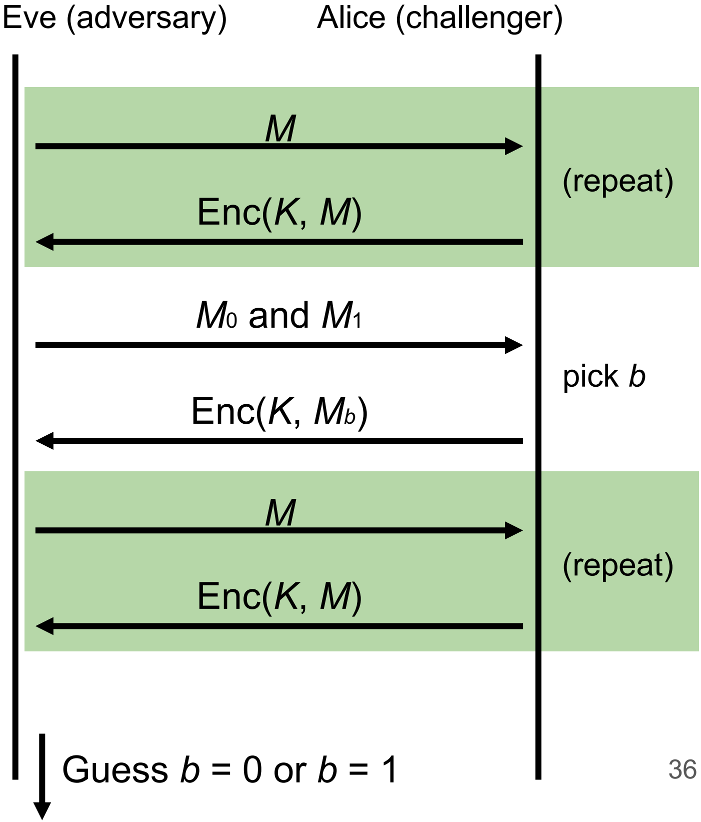
4. Eve may again choose plaintexts to send to Alice and receives their ciphertexts
5. Eventually, Eve outputs a guess as to whethe
encrypted M_0 or M_1

- An encryption scheme is IND-CPA secure if for all polynomial time attackers Eve:
    - Eve can win with probability ≤ 1/2 + Ɛ, where Ɛ is *negligible.*



A Brief History of Cryptography

Cryptography by Hand: Caesar Cipher

- One of the earliest cryptographic schemes was the **Caesar cipher**
  - Used by Julius Caesar over 2,000 years ago
- KeyGen():
  - Choose a key *K* randomly between 0 and 25
- Enc(*K*, *M*):
  - Replace each letter in *M* with the letter *K* positions later in the alphabet
  - If *K* = 3, plaintext DOG becomes GRJ
- Dec(*K*, *C*):
  - Replace each letter in *C* with the letter *K* positions earlier in the alphabet
  - If *K* = 3, ciphertext GRJ becomes DOG

K = 3			
M	*C*	*M*	*C*
A	D	N	Q
B	E	O	R
C	F	P	S
D	G	Q	T
E	H	R	U
F	I	S	V
G	J	T	W
H	K	U	X
I	L	V	Y
J	M	W	Z
K	N	X	A
L	O	Y	B
M	P	Z	C

Cryptography by Hand: Attacks on the Caesar Cipher

- Eve sees the ciphertext JCKN ECGUCT, but doesn’t know the key *K*
- If you were Eve, how would you try to break this algorithm?
- Brute-force attack: Try all 26 possible keys!
- Use existing knowledge: Assume that the message is in English

+1	IBJM DBFTBS	+9	ATBE VTXLTK	+17	SLTW NLPDLC
+2	$\boxed{\text{HAIL CAESAR}}$	+10	ZSAD USWKSJ	+18	RKSV MKOCKB
+3	GZHK BZDRZQ	+11	YRZC TRVJRI	+19	QJRU LJNBJA
+4	FYGJ AYCQYP	+12	XQYB SQUIQH	+20	PIQT KIMAIZ
+5	EXFI ZXBPXO	+13	WPXA RPTHPG	+21	OHPS JHLZHY
+6	DWEH YWAOWN	+14	VOWZ QOSGOF	+22	NGOR IGKYGX
+7	CVDG XVZNVM	+15	UNVY PNRFNE	+23	MFNQ HFJXFW
+8	BUCF WUYMUL	+16	TMUX OMQEMD	+24	LEMP GEIWEV
				+25	KDLO FDHVDU

Cryptography by Hand: Attacks on the Caesar Cipher

- Eve sees the ciphertext JCKN ECGUCT, but doesn't know the key K
- Chosen-plaintext attack: Eve tricks Alice into encrypting plaintext of her choice
  - Eve sends a message M = AAA and receives C = CCC
  - Eve can deduce the key: C is 2 letters after A, so K = 2
  - Eve has the key, so she can decrypt the ciphertext

Cryptography by Hand: Substitution Cipher

- A better cipher: create a mapping of each character to another character.
  - Example: A = N, B = Q, C = L, D = Z, etc.
  - Unlike the Caesar cipher, the shift is no longer constant!
- KeyGen():
  - Generate a random, one-to-one mapping of characters
- Enc(*K*, *M*):
  - Map each letter in *M* to the output according to the mapping *K*
- Dec(*K*, *C*):
  - Map each letter in *C* to the output according to the *reverse* of the mapping *K*

K			
M	*C*	*M*	*C*
A	N	N	G
B	Q	O	P
C	L	P	T
D	Z	Q	A
E	K	R	J
F	R	S	O
G	V	T	D
H	U	U	I
I	E	V	C
J	S	W	F
K	B	X	M
L	W	Y	X
M	Y	Z	H

Cryptography by Hand: Attacks on Substitution Ciphers

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- Does the brute-force attack still work?
  - There are $26! \approx 2^{88}$ possible mappings to try
    - Too much for most modern computers… for now
- How about the chosen-plaintext attack?
  - Trick Alice into encrypting ABCDEFGHIJKLMNOPQRSTUVWXYZ, and you'll get the whole mapping!
- Another strategy: *cryptanalysis*
  - The most common English letters in text are E, T, A, O, I, N

K			
M	*C*	*M*	*C*
A	N	N	G
B	Q	O	P
C	L	P	T
D	Z	Q	A
E	K	R	J
F	R	S	O
G	V	T	D
H	U	U	I
I	E	V	C
J	S	W	F
K	B	X	M
L	W	Y	X
M	Y	Z	H

Takeaways

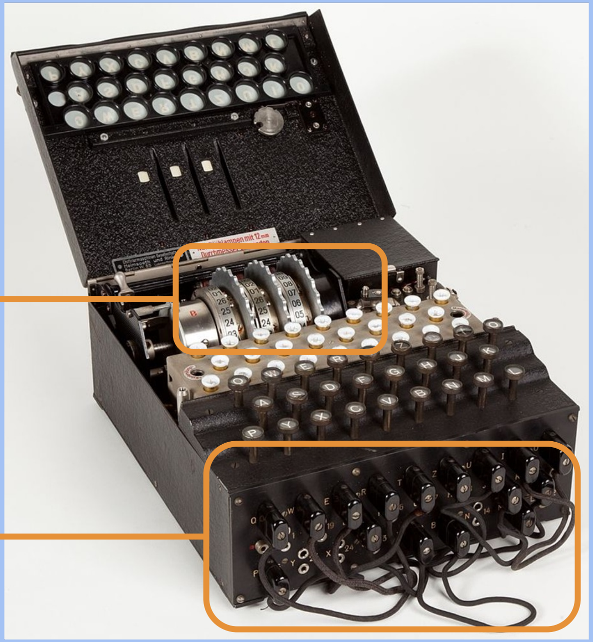
- Cryptography started with paper-and-pencil algorithms (Caesar cipher)
- Then cryptography moved to machines (Enigma)
- Finally, cryptography moved to computers (which we're about to study)
- Hopefully you gained some intuition for some of the cryptographic definitions

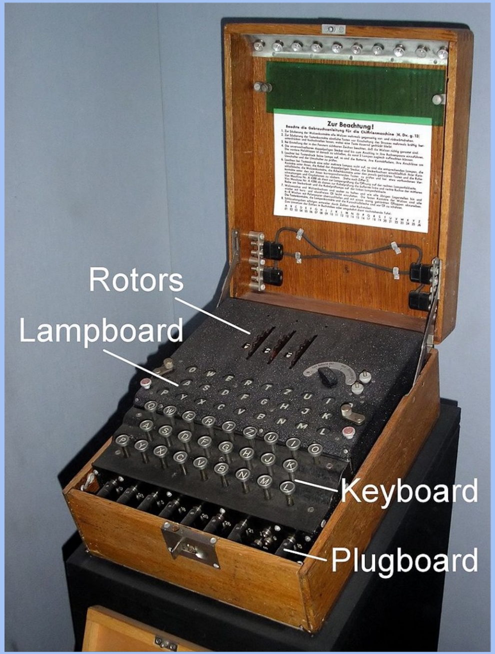
Cryptography by Machines: Enigma

- A mechanical encryption machine used by the Germans in WWII

Rotors

Plugboard



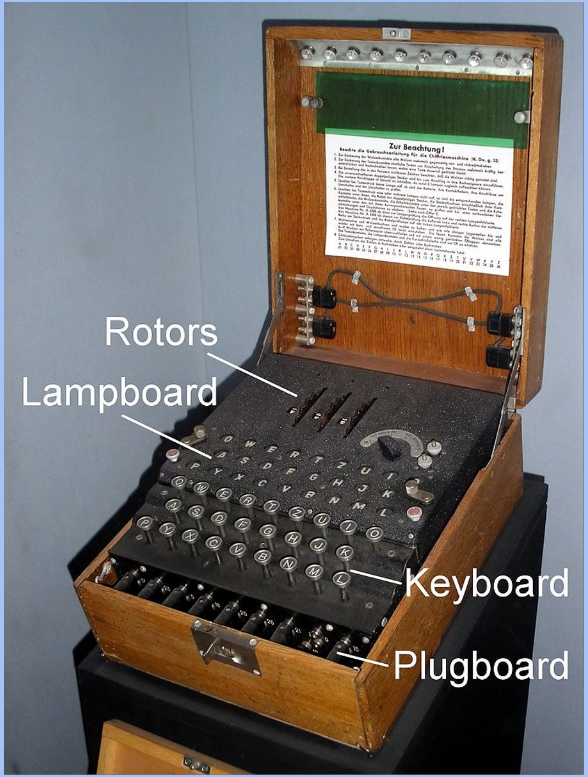


Enigma Operating Principle: Rotor Machine

- The encryption core was composed of 3 or 4 rotors
  - Each rotor was a fixed permutation (e.g. A maps to F, B maps to Q...)
  - And the end was a "reflector", a rotor that sent things backwards
  - Plus a fixed-permutation plugboard
- A series of rotors were arranged in a sequence
  - Each keypress would generate a current from the input to one light for the output
  - Each keypress also advanced the first rotor
    - When the first rotor makes a full rotation, the second rotor advances one step
    - When the second rotor makes a full rotation, the third rotor advances once step

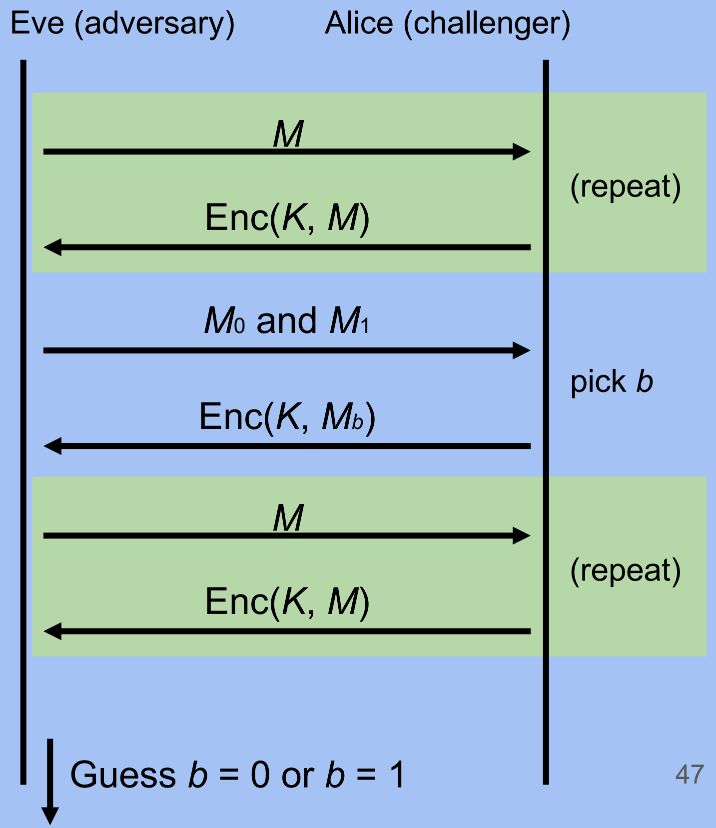
Cryptography by Machines: Enigma

- KeyGen():
  - Choose rotors, rotor orders, rotor positions, and plugboard settings
  - 158,962,555,217,826,360,000 possible keys
- Enc(*K*, *M*) and Dec(*K*, *C*):
  - Input the rotor settings *K* into the Enigma machine
  - Press each letter in the input, and the lampboard will light up the corresponding output letter
  - Encryption and decryption are the same algorithm!
- Germans believed that Enigma was an "unbreakable code"



Cryptography by Machines: Enigma

- Enigma has a significant weakness: a letter never maps to itself!
    - No rotor maps a letter to itself
    - The reflector never maps a letter to itself
    - This property is necessary for Enigma's mechanical system to work
- What pair of messages should Eve send to Alice in the challenge phase?
    - Send $M_0 = A^k$, $M_1 = B^k$
    - M_0 is a string of k 'A' characters, M_1 is a string of k 'B' characters
- How can Eve probably know which message Alice encrypted?
    - If there are no 'A' characters, it was M_0
    - If there are no 'B' characters, it was M_1



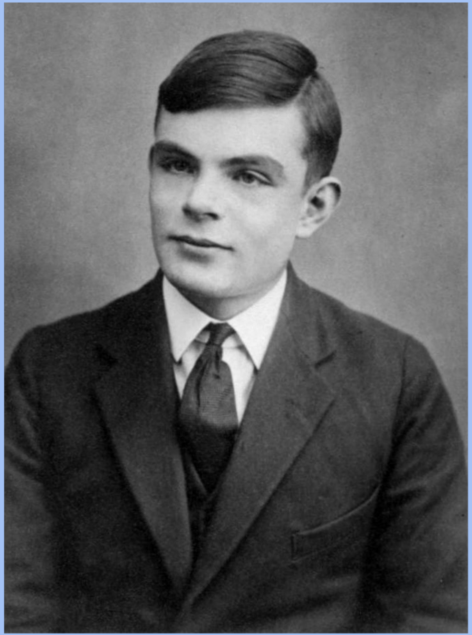
Cryptography by Machines: Attack on Enigma

- Polish and British cryptographers built BOMBE, a machine to brute-force Enigma keys
- Why was Enigma breakable?
  - Kerckhoff's principle: The Allies stole Enigma machines, so they knew the algorithm
  - Known plaintext attacks: the Germans often sent predictable messages (e.g. the weather report every morning)
  - Chosen plaintext attacks: the Allies could trick the Germans into sending a message (e.g. "newly deployed minefield")
  - Brute-force: BOMBE would try many keys until the correct one was found
    - Plus a weakness: You'd be able to try multiple keys with the same hardware configuration

BOMBE machine

Cryptography by Machines: Legacy of Enigma

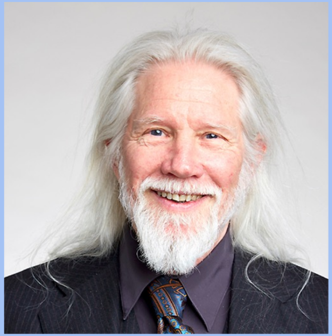
- Alan Turing, one of the cryptographers who broke Enigma, would go on to become one of the founding fathers of computer science
- Most experts agree that the Allies breaking Enigma shortened the war in Europe by about a year



Alan Turing

Cryptography by Computers

- The modern era of cryptography started after WWII, with the work of Claude Shannon
- “New Directions in Cryptography” (1976) showed how number theory can be used in cryptography
  - Its authors, Whitfield Diffie and Martin Hellman, won the Turing Award in 2015 for this paper
- This is the era of cryptography we’ll be focusing on



One of these is Diffie, and the other one is Hellman.

One-Time Pads

Cryptography Roadmap

	Symmetric-key	Asymmetric-key
Confidentiality	• One-time pads • Block ciphers • Stream ciphers	• RSA encryption
Integrity, Authentication	• MACs	• Digital signatures

- Hash functions
- Pseudorandom number generators
- Public key exchange (e.g. Diffie-Hellman)
- Key management (certificates)
- Password management

Review: XOR

The XOR operator takes two bits and outputs one bit:

$0 \oplus 0 = 0$
$0 \oplus 1 = 1$
$1 \oplus 0 = 1$
$1 \oplus 1 = 0$

Useful properties of XOR:

$x \oplus 0 = x$
$x \oplus x = 0$
$x \oplus y = y \oplus x$
$(x \oplus y) \oplus z = x \oplus (y \oplus z)$
$(x \oplus y) \oplus x = y$

One-Time Pads: Key Generation

Alice

K	0	1	1	0	0	1	0	1	0	1	1	1

The key *K* is a randomly-chosen bitstring.

Recall: We are in the symmetric-key setting, so we'll assume Alice and Bob both know this key.

One-Time Pads: Encryption

Alice

K	0	1	1	0	0	1	0	1	0	1	1	1
M	1	0	0	1	1	0	0	1	0	1	0	0

The plaintext M is the bitstring that Alice wants to encrypt.

Idea: Use XOR to scramble up M with the bits of K.

One-Time Pads: Encryption

Alice

K	0	1	1	0	0	1	0	1	0	1	1	1
	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$
M	1	0	0	1	1	0	0	1	0	1	0	0
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
C	1	1	1	1	1	1	0	0	0	0	1	1

Encryption algorithm: XOR each bit of *K* with the matching bit in *M*.

The ciphertext *C* is the encrypted bitstring that Alice sends to Bob over the insecure channel.

One-Time Pads: Decryption

Bob

K	0	1	1	0	0	1	0	1	0	1	1	1
C	1	1	1	1	1	1	0	0	0	0	1	1

Bob receives the ciphertext C. Bob knows the key K. How does Bob recover M?

One-Time Pads: Decryption

Bob

K	0	1	1	0	0	1	0	1	0	1	1	1
	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$
C	1	1	1	1	1	1	0	0	0	0	1	1
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
M	1	0	0	1	1	0	0	1	0	1	0	0

Decryption algorithm: XOR each bit of K with the matching bit in C.

One-Time Pad

- KeyGen()
  - Randomly generate an *n*-bit key, **where *n* is the length of your message**
    - Recall: For today, we assume that Alice and Bob can securely share this key
    - For one-time pads, we generate a *new* key for every message
- Enc(K, M) = $K \oplus M$
  - Bitwise XOR M and K to produce C
    - In other words: XOR the *i*th bit of the plaintext with the *i*th bit of the key.
    - $C_i = K_i \oplus M_i$
  - Alice and Bob use a different key (or pad) for each encryption (this is the "one-time" in one-time pad).
- Dec(K, C) = $K \oplus C$
  - Bitwise XOR C and K to produce M
    - $M_i = K_i \oplus C_i$

One-Time Pad: Correctness

- Correctness: If we encrypt and then decrypt, we should get the original message back

$\text{Enc}(K, M)$	$= K \oplus M$	Definition of encryption
$\text{Dec}(K, \text{Enc}(K, M))$	$= \text{Dec}(K, K \oplus M)$	Decrypting the ciphertext
	$= K \oplus (K \oplus M)$	Definition of decryption
	$= M$	XOR property

One-Time Pad: Security

- Recall our definition of confidentiality: The ciphertext should not give the attacker any additional information about the plaintext
- Recall our experiment to test confidentiality from earlier:
  - Alice has encrypted and sent either $M0$ or $M1$
  - Eve knows either $M0$ or $M1$ was sent, but doesn't know which
  - Eve reads the ciphertext and tries to guess which message was sent
  - If the probability that Eve correctly guesses which message was sent is 1/2, then the encryption scheme is confidential

One-Time Pad: Security

Possibility 0: Alice sends Enc(K, M_0)

The ciphertext is $C = K \oplus M_0$

Therefore, $K = C \oplus M_0$

Possibility 1: Alice sends Enc(K, M_1)

The ciphertext is $C = K \oplus M_1$

Therefore, $K = C \oplus M_1$

K was chosen randomly, so both possibilities are equally possible

Eve has learned no new information, so the scheme is *perfectly secure*

One-Time Pad is IND-CPA secure if K was randomly chosen for every encryption!

One-Time Pad: Security

Possibility 0: Alice sends Enc(K, M_0)

The ciphertext is $C = K \oplus M_0$

Therefore, $K = C \oplus M_0$

Possibility 1: Alice sends Enc(K, M_1)

The ciphertext is $C = K \oplus M_1$

Therefore, $K = C \oplus M_1$

What if:

We reuse the same key for different messages? Do we still have IND-CPA?

Answer: No. Eve can derive the key easily, thus all the plaintext.

Impracticality of One-Time Pads

- Problem #1: Key generation
  - For security to hold, keys must be randomly generated for every message, and never reused
  - Randomness is expensive, as we'll see later
- Problem #2: Key distribution
  - To communicate an *n*-bit message, we need to securely communicate an *n*-bit key first
  - But if we have a way to securely communicate an *n*-bit key, we could have communicated the message directly!
- Only practical application: Communicate keys in advance
  - You have a secure channel now, but you won't have it later
  - Use the secure channel now to communicate keys in advance
  - Use one-time pad later to communicate over the insecure channel
  - And people can compute this by hand without computers!

Summary

- What is cryptography?
- Definitions of key properties
- A better definition of confidentiality: IND-CPA security
- A brief history of cryptography
- Symmetric-key crypto