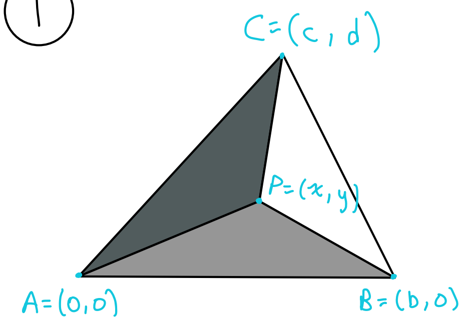
1




$$A = (x_a y_b + x_b y_c + x_c y_a - x_a y_c - x_b y_a - x_c y_b)$$

$$A = \frac{1}{2}\left(0(0) + b(d) + c(0) - 0(d) - b(0) - c(0)\right) = \frac{bd}{2}$$

$$A_a = \frac{1}{2}\left(x(0) + b(d) + c(y) - x(d) - b(y) - c(0)\right) = \frac{bd + cy - xd - by}{2}$$

$$A_b = \frac{1}{2}\left(0(y) + x(d) + c(0) - 0(d) - x(0) - c(y)\right) = \frac{xd - cy}{2}$$

$$A_c = \frac{1}{2}\left(0(0) + b(y) + x(0) - 0(y) - b(0) - x(0)\right) = \frac{by}{2}$$

$$\alpha = \frac{A_a}{A} = \frac{\frac{(b-x)d + (c-b)y}{\cancel{2}}}{\frac{bd}{\cancel{2}}} = \alpha = \frac{bd + cy - xd - by}{bd}$$

$$\beta = \frac{A_b}{A} = \frac{\frac{xd - cy}{\cancel{2}}}{\frac{bd}{\cancel{2}}} = \beta = \frac{xd - cy}{bd}$$

$$\gamma = \frac{A_c}{A} = \frac{\frac{by}{2}}{\frac{bd}{\cancel{2}}} = \gamma = \frac{y}{d}$$

$$\delta + \beta + \gamma = \frac{bd + \cancel{cy} - \cancel{xd} - \cancel{by}}{bd} + \frac{\cancel{xd} - \cancel{cy}}{bd} + \frac{\cancel{by}}{bd} = \frac{bd}{bd} = 1$$

We know that $0<\delta<1$, $0<\beta<1$, and $0<\gamma<1$, because $\delta+\beta+\gamma=1$. This means that $P$ is inside the triangle thus proving that $P=\delta A+\beta B+\gamma C$.

(2) $P' = MP + b \quad A' = MA + b \quad B' = MB + b \quad C' = MC + b$

$$P = \delta A + \beta B + \gamma C$$

$$P' = M(\delta A + \beta B + \gamma C) + b$$

$$\Leftrightarrow b = (\delta + \beta + \gamma) b = \delta b + \beta b + \gamma b$$

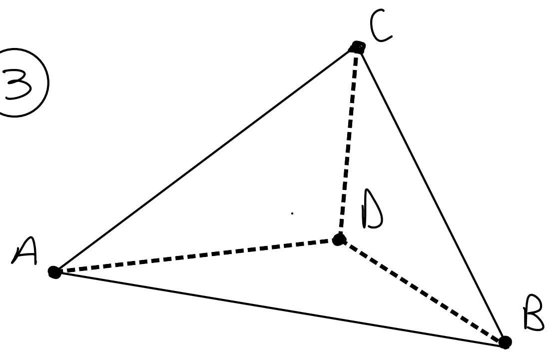
$$P' = \underline{\delta MA} + \underline{\beta MB} + \underline{\gamma MC} + \underline{\delta b} + \underline{\beta b} + \underline{\gamma b}$$

$$= \delta(MA + b) + \beta(MB + b) + \gamma(MC + b)$$

$$P' = \delta A' + \beta B' + \gamma C'$$

$P'$ uses the same barycentric coordinats, with respect to $A'B'C'$, as $P$, with respect to $ABC$.

3




Since a tetrahedron has 4 vertices rather than 3, we need to introduce a fourth variable, delta $\delta$.

To find the barycentric coordinates of a point $P$ inside the tetrahedron, we can use:

$$\alpha = \frac{V_A}{V}, \quad \beta = \frac{V_B}{V}, \quad \gamma = \frac{V_C}{V}, \quad \text{and} \quad \delta = \frac{V_D}{V},$$

where $V$ is total volume of tetrahedron, $V_A$ is the volume of subtetrahedron $PBCD$, $V_B$ is the volume of subtetrahedron $APCD$, $V_C$ is the volume of subtetrahedron $ABPD$, and $V_D$ is the volume of subtetrahedron $ABCP$.