

Basic R: Matrices

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Matrix problems

1. Suppose

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

- (a) Check that $A^3 = \mathbf{0}$
- (b) Replace the third column of A by the sum of the second and third columns

(a) First, produce A, Then Set $A^3 = 0$

```
A <- matrix(c(1,1,3,5,2,6,-2,-1,-3), nrow = 3, byrow = TRUE)
A
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    3
## [2,]    5    2    6
## [3,]   -2   -1   -3
```

```
A%%A%%A
```

```
##      [,1] [,2] [,3]
## [1,]    0    0    0
## [2,]    0    0    0
## [3,]    0    0    0
```

(b) Then, add the columns 2 and 3 and assign the sum to the third column

```
A[,3] <- A[,2] + A[,3]
```

```
A
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    4
## [2,]    5    2    8
## [3,]   -2   -1   -4
```

2. Create the following matrix B with 15 rows

$$B = \begin{bmatrix} 10 & -10 & 10 \\ 10 & -10 & 10 \\ \dots & \dots & \dots \\ 10 & -10 & 10 \end{bmatrix}$$

Calculate the 3x3 matrix $B^T B$. You can make this calculation with the function `crossprod()`. See the documentaion.

```
B <- matrix(c(10,-10,10), b=T,nc=3,nr=15)
crossprod(B)
```

```
##      [,1] [,2] [,3]
## [1,] 1500 -1500 1500
## [2,] -1500 1500 -1500
## [3,] 1500 -1500 1500
```

B

```
##      [,1] [,2] [,3]
## [1,] 10 -10 10
## [2,] 10 -10 10
## [3,] 10 -10 10
## [4,] 10 -10 10
## [5,] 10 -10 10
## [6,] 10 -10 10
## [7,] 10 -10 10
## [8,] 10 -10 10
## [9,] 10 -10 10
## [10,] 10 -10 10
## [11,] 10 -10 10
## [12,] 10 -10 10
## [13,] 10 -10 10
## [14,] 10 -10 10
## [15,] 10 -10 10
```

3. Create a 6 x 6 matrix `matE` with every element equal to 0. check what the functions `row()` and `col()` return when applied to `matE`.

Now, create the 6 x 6 matrix:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Here is `matE`, a 6x6 matrix of 0's followed by `row(matE)` and `col(matE)`

```
matE <- matrix(0, nrow = 6, ncol = 6)
```

```
# With a little experimentation you would see  
# that the specified pattern is in the |1|'s  
row(matE)-col(matE)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]  
## [1,]    0   -1   -2   -3   -4   -5  
## [2,]    1    0   -1   -2   -3   -4  
## [3,]    2    1    0   -1   -2   -3  
## [4,]    3    2    1    0   -1   -2  
## [5,]    4    3    2    1    0   -1  
## [6,]    5    4    3    2    1    0
```

```
# so you use the locations of the 1's to modify matE
matE[abs(row(matE)-col(matE))==1] <- 1
matE
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    0    1    0    0    0    0
## [2,]    1    0    1    0    0    0
## [3,]    0    1    0    1    0    0
## [4,]    0    0    1    0    1    0
## [5,]    0    0    0    1    0    1
## [6,]    0    0    0    0    1    0
```

4. Look at the help for the function `outer()`. Now, create the following patterned matrix:

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \end{bmatrix}$$

```
a <- 0:4
A <- outer(a,a,"+")
A
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    1    2    3    4
## [2,]    1    2    3    4    5
## [3,]    2    3    4    5    6
## [4,]    3    4    5    6    7
## [5,]    4    5    6    7    8
```

5. Create the following patterned matrices. Your solutions should be generalizable to enable creating larger matrices with the same structure.

(a)

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 0 \\ 2 & 3 & 4 & 0 & 1 \\ 3 & 4 & 0 & 1 & 2 \\ 4 & 0 & 1 & 2 & 3 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 8 & 9 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 9 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 0 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\ 1 & 0 & 8 & 7 & 6 & 5 & 4 & 3 & 2 \\ 2 & 1 & 0 & 8 & 7 & 6 & 5 & 4 & 3 \\ 3 & 2 & 1 & 0 & 8 & 7 & 6 & 5 & 4 \\ 4 & 3 & 2 & 1 & 0 & 8 & 7 & 6 & 5 \\ 5 & 4 & 3 & 2 & 1 & 0 & 8 & 7 & 6 \\ 6 & 5 & 4 & 3 & 2 & 1 & 0 & 8 & 7 \\ 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 8 \\ 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{bmatrix}$$

(a)

```
x <- 0:4
Y <- outer(x,x,"+")%%5
Y
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]  0    1    2    3    4
## [2,]  1    2    3    4    0
## [3,]  2    3    4    0    1
## [4,]  3    4    0    1    2
## [5,]  4    0    1    2    3
```

(b)

```
s <- 0:9
T <- outer(s,s,"+")%%10
T
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
## [1,]  0    1    2    3    4    5    6    7    8    9
## [2,]  1    2    3    4    5    6    7    8    9    0
## [3,]  2    3    4    5    6    7    8    9    0    1
## [4,]  3    4    5    6    7    8    9    0    1    2
## [5,]  4    5    6    7    8    9    0    1    2    3
## [6,]  5    6    7    8    9    0    1    2    3    4
## [7,]  6    7    8    9    0    1    2    3    4    5
## [8,]  7    8    9    0    1    2    3    4    5    6
## [9,]  8    9    0    1    2    3    4    5    6    7
## [10,]  9    0    1    2    3    4    5    6    7    8
```

(c)

```
p <- 0:8
q <- outer(p,p,"-")%%9
q
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
## [1,]  0    8    7    6    5    4    3    2    1
## [2,]  1    0    8    7    6    5    4    3    2
## [3,]  2    1    0    8    7    6    5    4    3
## [4,]  3    2    1    0    8    7    6    5    4
## [5,]  4    3    2    1    0    8    7    6    5
## [6,]  5    4    3    2    1    0    8    7    6
## [7,]  6    5    4    3    2    1    0    8    7
## [8,]  7    6    5    4    3    2    1    0    8
## [9,]  8    7    6    5    4    3    2    1    0
```

6. Solve the following system of linear equations by setting up and solving the matrix equation $Ax = y$.

$$\begin{aligned}x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 &= 7 \\2x_1 + x_2 + 2x_3 + 3x_4 + 4x_5 &= -1 \\3x_1 + 2x_2 + x_3 + 2x_4 + 3x_5 &= -3 \\4x_1 + 3x_2 + 2x_3 + x_4 + 2x_5 &= 5 \\5x_1 + 4x_2 + 3x_3 + 2x_4 + x_5 &= 17\end{aligned}$$

```
y <- c(7,-1,-3,5,17)
AMat <- matrix(0,nr=5, ncol=5)
AMat <- abs(col(AMat)-row(AMat))+1
x = solve(AMat)%*%y
rownames(x) <-c("X1","X2","X3","X4","X5")
colnames(x) <-c("Value")
x
```

```
##      Value
## X1      -2
## X2       3
## X3       5
## X4       2
## X5      -4
```

7. Create a 6 x 10 matrix of random integers chosen from 1,2,...,10 by executing the following two lines of code:

```
set.seed(75)
aMat <- matrix(sample(10, size=60, replace=TRUE), nr=6)
```

Use the matrix you have created to answer these questions:

- Find the number of entries in each row which are greater than 4.
- Which rows contain exactly two occurrences of the number seven?
- Find those pairs of columns whose total (over both columns) is greater than 75. The answer should be a matrix with two columns; so, for example, the row (1,2) in the output matrix means that the sum of columns 1 and 2 in the original matrix is greater than 75. Repeating a column is permitted; so, for example, the final output matrix could contain the rows (1,2), (2,1), and (2,2).

What if repetitions are not permitted? Then only (1,2) from (1,2),(2,1) and (2,2) would be permitted.

(a)

```
set.seed(75)
aMat <- matrix(sample(10, size=60, replace=TRUE), nr=6)
apply(aMat, 1, function(x){sum(x>4)})
```

```
## [1] 4 7 6 2 6 7
```

(b)

```
set.seed(75)
aMat <- matrix(sample(10, size=60, replace=TRUE), nr=6)
which( apply(aMat,1,function(x){sum(x==7)==2}) )
```

```
## [1] 5
```

(c)

```
set.seed(75)
aMat <- matrix(sample(10, size=60, replace=TRUE), nr=6)
aMatColSums <- colSums(aMat)
cbind( rep(1:10,rep(10,10)), rep(1:10,10) ) [outer(aMatColSums,aMatColSums,"+")>75,]
```

```
##      [,1] [,2]
## [1,]    2    2
## [2,]    2    6
## [3,]    2    8
## [4,]    6    2
## [5,]    6    8
## [6,]    8    2
## [7,]    8    6
## [8,]    8    8
```

8. Calculate

(a) $\sum_{i=1}^{20} \sum_{j=1}^5 \frac{i^4}{(3+j)}$

```
sum((1:20)^4 * sum(1/(3+(1:5))))
```

```
## [1] 639215.3
```

```
# or
```

```
sum(outer((1:20)^4, (3+(1:5)), "/"))
```

```
## [1] 639215.3
```

(b) $\sum_{i=1}^{20} \sum_{j=1}^5 \frac{i^4}{(3+ij)}$

(c)

```
sum( (1:20)^4 / (3 + outer(1:20,1:5,"*")) )
```

```
## [1] 89912.02
```

(c) $\sum_{i=1}^{10} \sum_{j=1}^i \frac{i^4}{(3+ij)}$

(d)

```
sum( outer(1:10,1:10,function(i,j){ (i>=j)*i^4/(3+i*j) }) )
```

```
## [1] 6944.743
```