### 《复杂系统》

Figure: 4.8/5.1/5.2/5.3

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- Mathematical modeling
- 2 4.8-Simulation results of the predator-prey model
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### Model Analysis I

• 人口增长模型:

$$x_t = ax_{t-1}$$

- 满足两个条件
  - 指数增长
  - 收敛到一定的极限
- 用 f(x<sub>t-1</sub>) 来代替 a

$$x_t = f(x_{t-1})x_{t-1}$$

函数 f(x) 需要经过 (x,f(x)) = (0,a) 和 (K,1) 两点。



## Model Analysis I

• 用线性函数表示 f(x)

$$f(x) = -\frac{a-1}{K}x + a$$

$$x_t = (-\frac{a-1}{K}x_{t-1} + a)x_{t-1}$$

#### 这就是 Logistic 模型

•  $\diamondsuit$  r = a - 1, 则

$$x_{t} = \left(-\frac{a-1}{K}x_{t-1} + a\right)x_{t-1}$$
$$= \left(-\frac{r}{K}x_{t-1} + r + 1\right)x_{t-1}$$
$$= x_{t-1} + rx_{t-1}\left(1 - \frac{x_{t-1}}{K}\right)$$

### Model Analysis I

● 如果 × 远小于 K

$$x_t \approx x_{t-1} + rx_{t-1}$$

以上就是单变量的模型建立





### Model Analysis II

考虑一个典型场景——捕食者(y)与猎物(x)两个种群间相互作用根据一维的 Logistic 模型和指数衰减模型,分别针对猎物和捕食者建立同样的数学模型,将假设所得出的函数代入方程,可以得到

$$\begin{aligned} x_t &= x_{t-1} + rx_{t-1}(1 - \frac{x_{t-1}}{K}) - (1 - \frac{1}{by_{t-1} + 1}) \\ y_t &= y_{t-1} - dyt - 1 + cx_{t-1}y_{t-1} \end{aligned}$$





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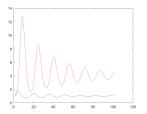




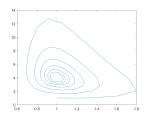
#### 4.8

#### Building Your Own Model Equations with Multiple Variables

令 
$$r = b = d = c = 1, K = 5, x_0 = y_0 = 1$$
  
此时的捕食者-猎物模型的模拟结果如下图 (1):



(a) State variables plotted over time



(b) Phase space visualization of the same result

图: Simulation results of the predator-prey mode



### Matlab for 4.8 Left

```
1 r=1;
^{2} b=1;
3 c=1:
4 d=1;
s = 100;
6 X=zeros(1,s);
7 Y=zeros(1,s);
8 X(1,1)=1;
9 Y (1,1)=1;
10 for t=1:1:100
X(1,t+1)=X(1,t)+X(1,t).*(r-1+1./(b.*Y(1,t)+1)-X(1,t)./5);
  Y(1,t+1)=(1-d).*Y(1,t)+c.*Y(1,t).*X(1,t);
12
13 end
14 plot(X)
15 hold on
16 plot(Y)
17 hold off
```



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### Matlab for 4.8 Right

```
1 r=1;
^{2} b=1;
3 c=1;
4 d=1;
5 s=100;
6 X = zeros(1,s);
7 Y = zeros(1,s);
8 \times (1,1) = 1;
9 Y (1,1)=1;
10 for t=1:1:100
11
  X(1,t+1)=X(1,t)+X(1,t).*(r-1+1./(b.*Y(1,t)+1)-X(1,t)./5);
  Y(1,t+1)=(1-d).*Y(1,t)+c.*Y(1,t).*X(1,t);
12
13 end
14 plot(X,Y);
```





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# 绘制相空间

#### Matlab for 5.1

```
1 s = 30;
2 X=zeros(1,s);
_{3} Y=zeros(1,s);
4 \text{ for } x0 = -2:0.5:2
      for y0 = -2:0.5:2
           X(1,1)=x0;
           Y(1,1) = v0;
           for t=1:s
                X(1,t+1)=0.5.*X(1,t)+Y(1,t);
                Y(1,t+1) = -0.5.*X(1,t)+Y(1,t);
           end
11
           plot(X,Y)
           hold on;
13
      end
14
15 end
16 hold off;
```

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### 绘制三维相空间

#### Matlab for 5.2

```
1 s = 30;
2 X=zeros(1,s); Y=zeros(1,s); Z=zeros(1,s);
3 \text{ for } x0=-2:1:2
      for y0 = -2:1:2
           for z0=-2:1:2
           X(1,1)=x0;
           Y(1,1) = y0;
           Z(1,1)=z0;
8
           for t=1:s
                X(1,t+1)=0.5.*X(1,t)+Y(1,t);
10
                Y(1,t+1) = -0.5.*X(1,t)+Y(1,t);
                Z(1,t+1) = -1 * X(1,t) + (-1) * Y(1,t) + Z(1,t);
12
           end
13
           plot3(X,Y,Z)
14
           hold on;
15
           end
16
       end
17
18 end
```

19 hold off;

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# 轨迹相互交叉的离散时间模型相空间

#### Matlab for 5.3

```
1 s = 30;
2 X=zeros(1,s);
_{3} Y=zeros(1,s);
4 \text{ for } x0 = -2:0.5:2
      for y0 = -2:0.5:2
           X(1,1)=x0;
           Y(1,1) = v0;
           for t=1:s
                X(1,t+1) = -0.5.*X(1,t) + -0.7.*Y(1,t);
                Y(1,t+1)=X(1,t)+-0.5.*Y(1,t);
           end
11
           plot(X,Y)
           hold on;
13
      end
14
15 end
16 hold off;
```

# Thanks for your Listening!



