《复杂系统》

Figure : 4.8/5.1/5.2/5.3

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Model Analysis I

- 人口增长模型：

$$x_t = ax_{t-1}$$

- 满足两个条件
  - 指数增长
  - 收敛到一定的极限
- 用 $f(x_{t-1})$ 来代替 a

$$x_t = f(x_{t-1})x_{t-1}$$

- 函数 f(x) 需要经过 (x,f(x)) = (0,a) 和 (K,1) 两点。

Model Analysis I

- 用线性函数表示 f(x)

$$f(x) = -\frac{a-1}{K}x + a$$
$$x_t = (-\frac{a-1}{K}x_{t-1} + a)x_{t-1}$$

这就是 *Logistic* 模型

- 令 $r = a - 1$, 则

$$\begin{aligned} x_t &= (-\frac{a-1}{K}x_{t-1} + a)x_{t-1} \\ &= (-\frac{r}{K}x_{t-1} + r + 1)x_{t-1} \\ &= x_{t-1} + rx_{t-1}(1 - \frac{x_{t-1}}{K}) \end{aligned}$$

Model Analysis I

- 如果 x 远小于 K

$$x_t \approx x_{t-1} + rx_{t-1}$$

以上就是单变量的模型建立

Model Analysis II

- 考虑一个典型场景——捕食者 (y) 与猎物 (x) 两个种群间相互作用根据一维的 *Logistic* 模型和指数衰减模型，分别针对猎物和捕食者建立同样的数学模型，将假设所得出的函数代入方程，可以得到

$$x_t = x_{t-1} + rx_{t-1}(1 - \frac{x_{t-1}}{K}) - (1 - \frac{1}{by_{t-1} + 1})$$

$$y_t = y_{t-1} - dyt - 1 + cx_{t-1}y_{t-1}$$

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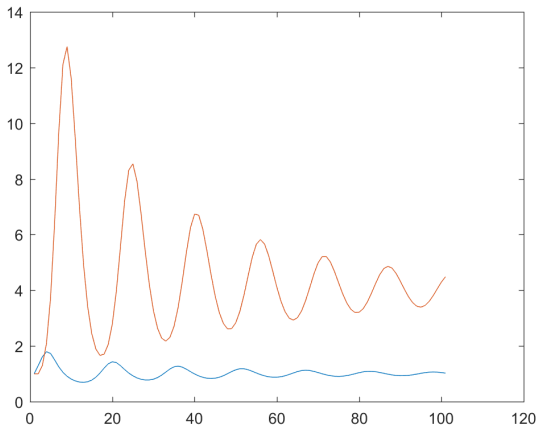
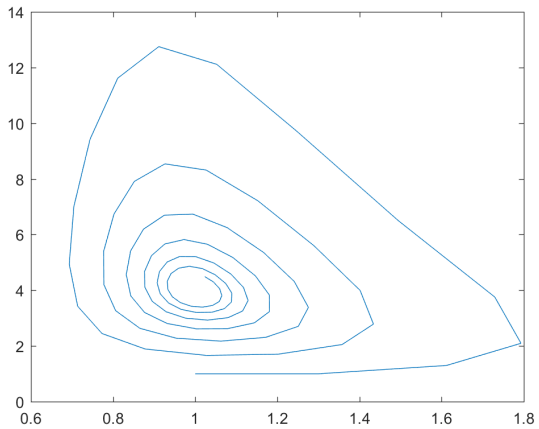
Building Your Own Model Equations with Multiple Variables

令 $r = b = d = c = 1, K = 5, x_0 = y_0 = 1$
此时的捕食者-猎物模型的模拟结果如下图 (1):



(a) State variables plotted over time



(b) Phase space visualization of the same result

图: Simulation results of the predator-prey mode

Matlab for 4.8 Left

```
r=1;
b=1;
c=1;
d=1;
s=100;
X=zeros(1,s);
Y=zeros(1,s);
X(1,1)=1;
Y(1,1)=1;
for t=1:1:100
    X(1,t+1)=X(1,t)+X(1,t).*(r-1+1./(b.*Y(1,t)+1)-X(1,t)./5);
    Y(1,t+1)=(1-d).*Y(1,t)+c.*Y(1,t).*X(1,t);
end
plot(X)
hold on
plot(Y)
hold off
```

Matlab for 4.8 Right

```
r=1;
b=1;
c=1;
d=1;
s=100;
X=zeros(1,s);
Y=zeros(1,s);
X(1,1)=1;
Y(1,1)=1;
for t=1:1:100
    X(1,t+1)=X(1,t)+X(1,t).*(r-1+1./(b.*Y(1,t)+1)-X(1,t)./5);
    Y(1,t+1)=(1-d).*Y(1,t)+c.*Y(1,t).*X(1,t);
end
plot(X,Y);
```

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绘制相空间

Matlab for 5.1

```
s=30;
X=zeros(1,s);
Y=zeros(1,s);
for x0=-2:0.5:2
    for y0=-2:0.5:2
        X(1,1)=x0;
        Y(1,1)=y0;
        for t=1:s
            X(1,t+1)=0.5.*X(1,t)+Y(1,t);
            Y(1,t+1)=-0.5.*X(1,t)+Y(1,t);
        end
        plot(X,Y)
        hold on;
    end
end
hold off;
```

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绘制三维相空间

Matlab for 5.2

```
s=30;
X=zeros(1,s);Y=zeros(1,s);Z=zeros(1,s);
for x0=-2:1:2
    for y0=-2:1:2
        for z0=-2:1:2
        X(1,1)=x0;
        Y(1,1)=y0;
        Z(1,1)=z0;
        for t=1:s
            X(1,t+1)=0.5.*X(1,t)+Y(1,t);
            Y(1,t+1)=-0.5.*X(1,t)+Y(1,t);
            Z(1,t+1)=-1*X(1,t)+(-1)*Y(1,t)+Z(1,t);
        end
        plot3(X,Y,Z)
        hold on;
        end
    end
end
hold off;
```

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轨迹相互交叉的离散时间模型相空间

Matlab for 5.3

```
s=30;
X=zeros(1,s);
Y=zeros(1,s);
for x0=-2:0.5:2
    for y0=-2:0.5:2
        X(1,1)=x0;
        Y(1,1)=y0;
        for t=1:s
            X(1,t+1)=-0.5.*X(1,t)+-0.7.*Y(1,t);
            Y(1,t+1)=X(1,t)+-0.5.*Y(1,t);
        end
        plot(X,Y)
        hold on;
    end
end
hold off;
```

Thanks for your Listening!