# Simplex Implementation in Python

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## 1 Overview

This document presents two linear optimization test cases. Included are the problem formulations and results from my algorithm as well as a commercial linear programming software (Gurobi). Code for the two test cases can be found in the respository.

## 2 Problem Formulation

Problems chosen for this model were Models 7 & 11. Source is Prof. John Burke's Linear Optimization Course (link).

**Model 7 Formulation: Electronics Company**

Variable Representation:

- $r_n$ is numbers of radios produced in week n
- $w_{t,\ n}$ is total workers training in week n
- $w_{r,\ n}$ is total workers making radios in week n
- $t_n$ is total trainees in week n

Problem formulation includes sale price of radios, worker salaries, and trainee salaries. Note that the $5 radio production cost is accounted for by subtracting $5 from each of the sale prices

max $15r_1 + 13r_2 + 11r_3 + 9r_4 - 200(w_{r,1} + w_{r,2} + w_{r,3} + w_{r,4} + w_{t,1} + w_{t,2} + w_{t,3} + w_{t,4}) - 100(t_1 + t_2 + t_3)$
subject to

Radio Manufacturing Constraints (each worker makes 50 radios per week):
$r_1 = 50w_{r,1}$
$r_2 = 50w_{r,2}$
$r_3 = 50w_{r,3}$
$r_4 = 50w_{r,4}$

Initial Workforce Constraint:
$w_{r,1} + w_{t,1} = 50$

New Workforce Constraint (each worker trains three people per week):
$w_{r,2} + w_{t,2} = w_{r,1} + 4w_{t,1}$
$w_{r,3} + w_{t,3} = w_{r,2} + 4w_{t,2}$
$w_{r,4} + w_{t,4} = w_{r,2} + 4w_{t,3}$

Radio Production Constraint:
$r_1 + r_2 + r_3 + r_4 = 21,475$

Trainee Constraint:
$t_1 = 3w_{t,1}$
$t_2 = 3w_{t,2}$
$t_3 = 3w_{t,3}$

**Model 11 Formulation: A Packing Problem**

Variable Representation: $t_{i,\,j}$ is tons of cargo j in compartment i (front, middle, or back)

Profit for each cargo type is \$280, \$360, \$320, and \$250, respectively

max $280(t_{f,1} + t_{m,1} + t_{b,1}) + 360(t_{f,2} + t_{m,2} + t_{b,2}) + 320(t_{f,3} + t_{m,3} + t_{b,3}) + 250(t_{f,4} + t_{m,4} + t_{b,4})$ subject to

Weight Capacity Constraints:
$t_{f,1} + t_{f,2} + t_{f,3} + t_{f,4} \leq 12$
$t_{m,1} + t_{m,2} + t_{m,3} + t_{m,4} \leq 18$
$t_{b,1} + t_{b,2} + t_{b,3} + t_{b,4} \leq 10$

Volume Capacity Constraints:
$500t_{f,1} + 700t_{f,2} + 600t_{f,3} + 400t_{f,4} \leq 7000$
$500t_{m,1} + 700t_{m,2} + 600t_{m,3} + 400t_{m,4} \leq 9000$
$500t_{b,1} + 700t_{b,2} + 600t_{b,3} + 400t_{b,4} \leq 5000$

Cargo Availability Constraints:
$t_{f,1} + t_{m,1} + t_{b,1} \leq 20$
$t_{f,2} + t_{m,2} + t_{b,2} \leq 16$
$t_{f,3} + t_{m,3} + t_{b,3} \leq 25$
$t_{f,4} + t_{m,4} + t_{b,4} \leq 13$

Weight Proportionality Constraints (note W is total weight):
$t_{f,1} + t_{f,2} + t_{f,3} + t_{f,4} = \frac{12}{40}W$
$t_{m,1} + t_{m,2} + t_{m,3} + t_{m,4} = \frac{18}{40}W$
$t_{b,1} + t_{b,2} + t_{b,3} + t_{b,4} = \frac{10}{40}W$

# 3 Cargo Model Output

Note that Gurobi does not accept equality constraints - to ensure that the algorithms receive identical inputs, all equalities are expressed as two constraints (a $\leq$ and $\geq$) for both algorithms.

Code output shown below:

```
Optimal phase 1 solution acheived...
proceeding to phase 2

=========
SOLUTION
=========
final_values
{'tf4': 4.67, 'tm1': 5.5, 'tb1': 10.0, 'tf2': 7.33, 'tm2': 4.17, 'tb2': 0.0, 'tm4': 8.33}
objective value:  11730.0
```

Figure 1: Simplex algorithm output for cargo problem

And the gurobi solution:

Note that though the optimal values and variables are different, the objective value for both functions is the same.

```
Solved in 13 iterations and 0.00 seconds
Optimal objective  1.173000000e+04
tf1 1
tf2 7
tf3 0
tf4 4
tm1 0
tm2 0
tm3 9
tm4 9
tb1 10
tb2 0
tb3 0
tb4 0
w 40
Obj: 11730
```

Figure 2: Gurobi output for cargo problem

# 4 Radio Model Output

Code output shown below:

```
Optimal phase 1 solution acheived...
proceeding to phase 2

=========
SOLUTION
=========
final_values
{'t2': 0.0, 'r4': 7053.13, 'wr1': 6.31, 'r3': 7053.12, 'wt1': 33.69, 'r2': 7053.12, 'wr2': 141.06, 'wr3': 141.06, 't
3': 0.0, 'wr4': 141.06, 'r1': 315.63, 't1': 101.06}
objective value:  134743.75
```

Figure 3: Simplex algorithm output for radio problem

And the gurobi solution:

```
Solved in 4 iterations and 0.00 seconds
Optimal objective  1.347437500e+05
r1 315.625
r2 7053.12
r3 7053.12
r4 7053.12
wr1 6.3125
wr2 141.062
wr3 141.062
wr4 141.062
wt1 33.6875
wt2 0
wt3 0
wt4 0
t1 101.062
t2 0
t3 0
Obj: 134744
```

Figure 4: Gurobi output for radio problem

Note in this case both the variable values and objective values are the same. One disadvantage to this method is that unlike the cargo, the variables in this problem are not actually continuous. It's impossible to manufacture half a radio or train one third of a worker. For this reason, the true optimal value is likely lower than the reported one as values would be restricted to integers. However, this example illustrates that my code output is identical to the commercial solver in a continuous situation.