

PARTICLE ACCELERATORS - THE ULTIMATE MICROSCOPES

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TRIUMF Student Seminar

<http://trshare.triumf.ca/~craddock/PartAccel.pdf>

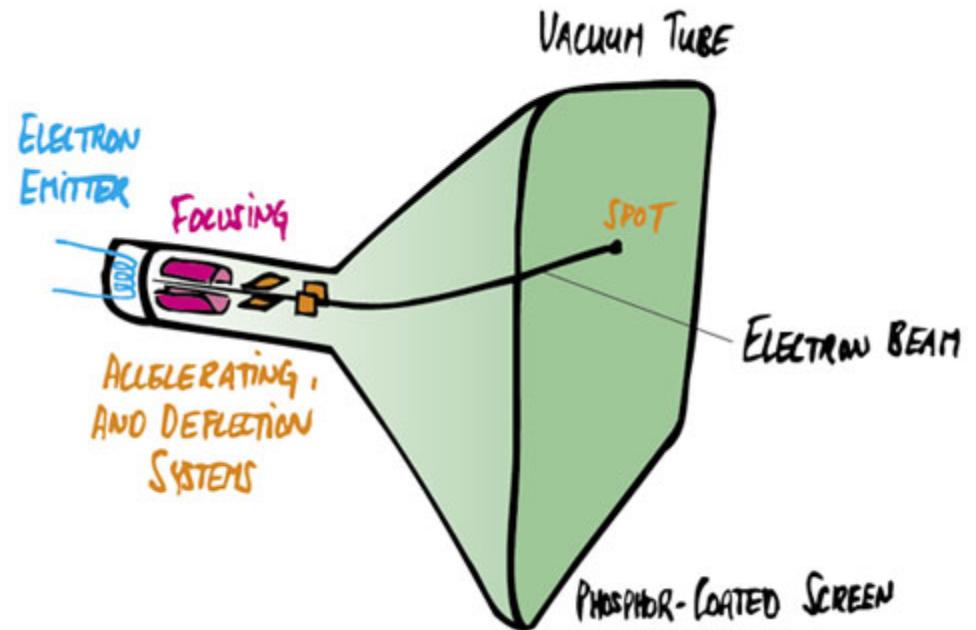
PART 1

1. Introduction and Classification
2. Electrostatic Accelerators
3. Radiofrequency (RF) Linear Accelerators
4. Classical Cyclotrons
5. Particle Beams & Weak Magnetic Focusing
6. The Relativity Challenge
7. Synchronous Acceleration
 - Synchrocyclotrons
 - Synchrotrons

THE ESSENTIALS

- Particle source (electrons, protons, ions)
- Vacuum
- Electric field for acceleration
- Magnetic and/or electric fields for focusing and steering
- Controls

A familiar particle accelerator:





CERN, Geneva - The LHC and SPS tunnels and the Swiss-French border

PARTICLE ACCELERATORS AS MICROSCOPES

Conventional microscopes can't resolve objects smaller than the wavelength of light ($\lambda \approx 0.6 \mu\text{m}$).

But the discovery of the **wave nature of particles**, with

$$E = h\nu \quad \text{and} \quad p = h/\lambda$$

showed that **high enough momentum p could give any desired resolution!**

The **optical limit of $\approx 0.6 \mu\text{m}$** corresponds to $p \approx 2 \text{ eV}/c$.

For **higher momenta**, particles are easier to produce & control than photons:

Electrons of 10 keV $\rightarrow p = 100 \text{ keV}/c$ ($\times 50,000$) $\rightarrow \lambda = 10^{-11} \text{ m}$ (0.1 Å)

- so **electron microscopes** can resolve **individual atoms**.

Electrons of 20 GeV $\rightarrow p = 20 \text{ GeV}/c$ ($\times 10^{10}$) $\rightarrow \lambda = 6 \times 10^{-17} \text{ m}$ (0.06 nucleon)

- enabling Taylor *et al.* to **observe quarks in protons at SLAC**.

SCIENTIFIC APPLICATIONS OF ACCELERATORS

Discipline

Particle Physics

Nuclear Physics

Atomic Physics

Molecular Physics

Chemistry

Condensed-Matter Physics

Materials Science

Surface Science

Geology

Biology

Archaeology

Tools

$e^\pm, p^\pm; \gamma, \nu, \mu, \pi, K, n$

$e, p, \text{ions}; \gamma, n, \pi$

) $e; \gamma, n, \beta, \mu$

) via

) electron microscopes

) X-ray tubes

) synchrotron rad'n sources

) spallation neutron sources

) β -NMR

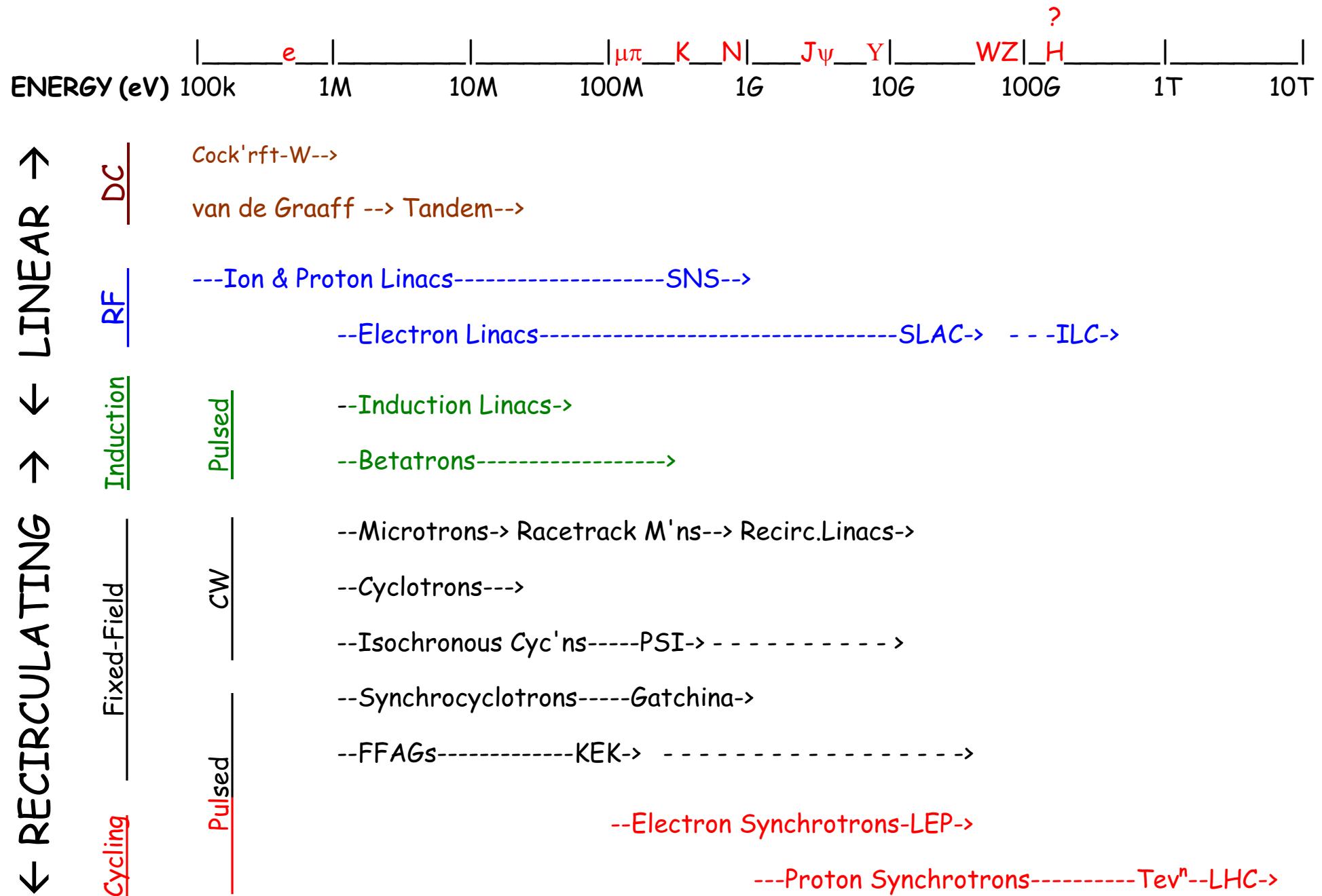
) μ SR

) mass spectrometry

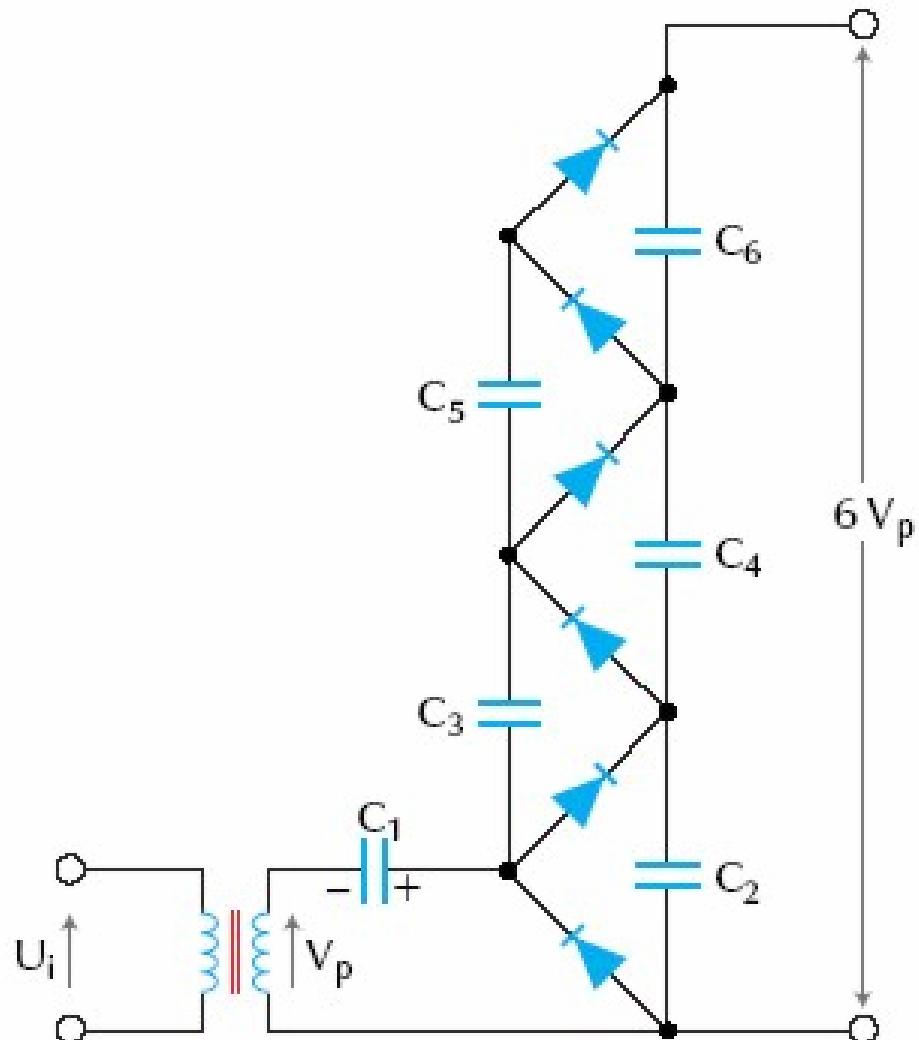
MEDICAL AND INDUSTRIAL APPLICATIONS

- Medical Diagnosis using X-rays, radioisotopes, PET scans)
- Cancer Therapy using X-rays, γ -rays, protons, C^+ ions) ~5,200
- Electron beam welding)
- Polymerization of plastics)
- Semiconductor circuit processing)
- Ion implantation) ~8,500
- Sterilization of food and waste)
- Borehole diagnostics)
- Dangerous material detectors)
- Plasma heating for fusion reactors
- Inertial fusion reactors
- Nuclear waste transmutation
- Electronuclear breeding
- Accelerator-Driven Subcritical Reactors

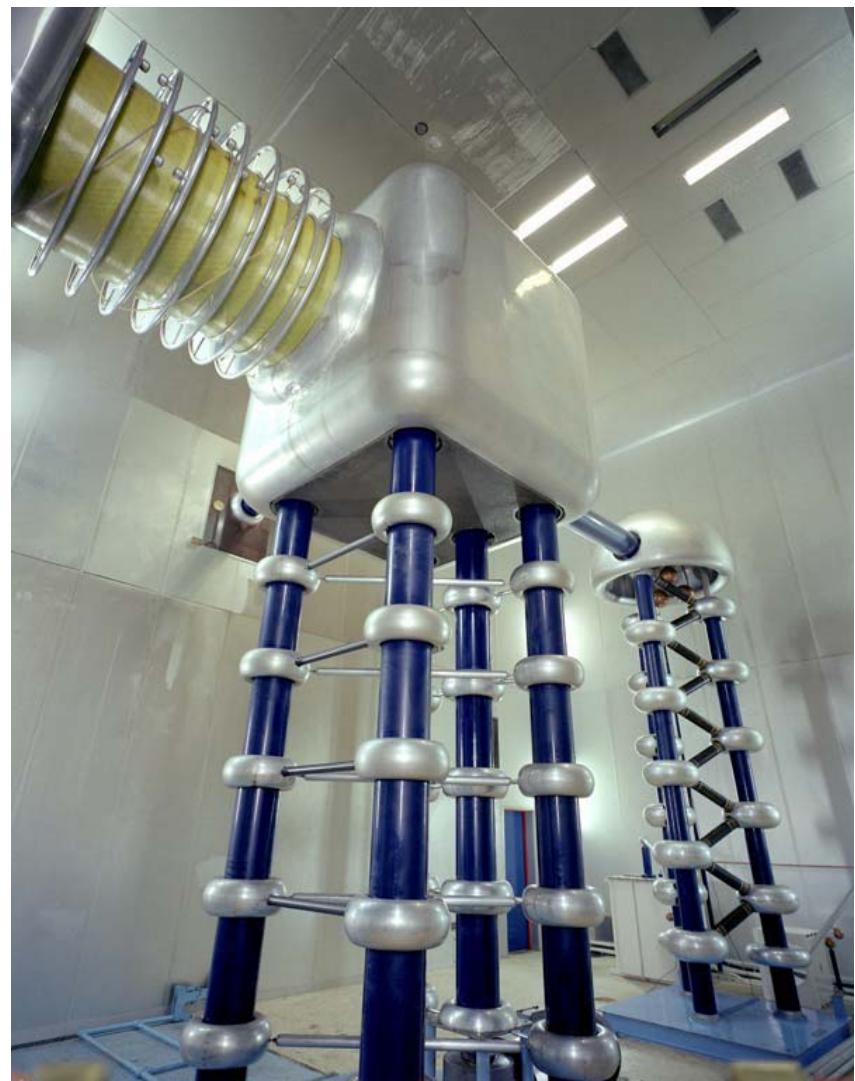
ENERGY RANGES OF DIFFERENT ACCELERATORS



DC HIGH-VOLTAGE ACCELERATORS - COCKCROFT-WALTON GENERATORS

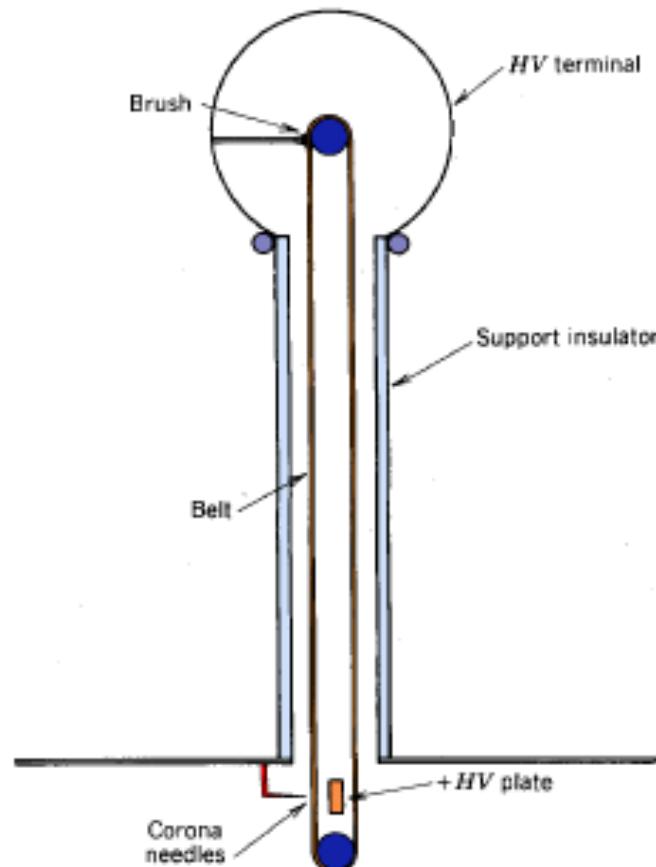
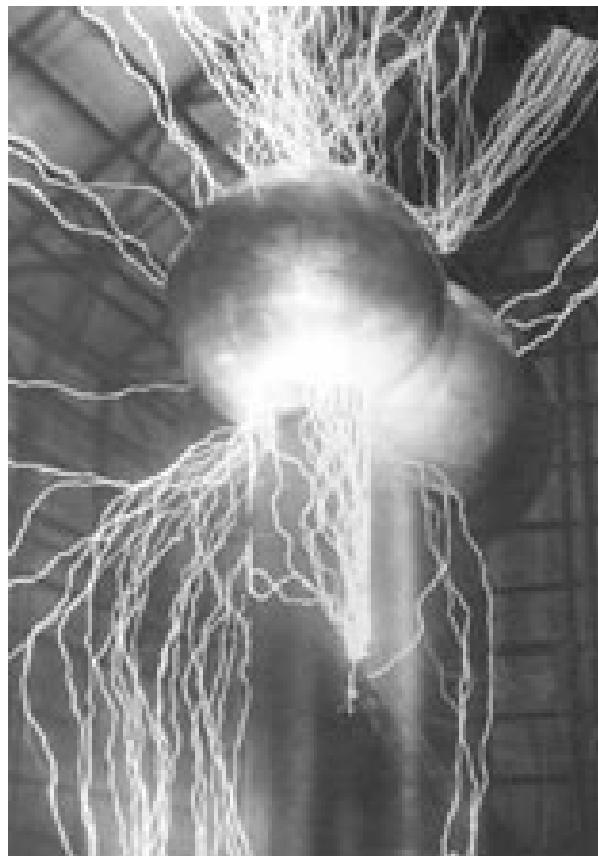


Cockcroft & Walton's cascade-rectifier voltage-multiplication circuit.

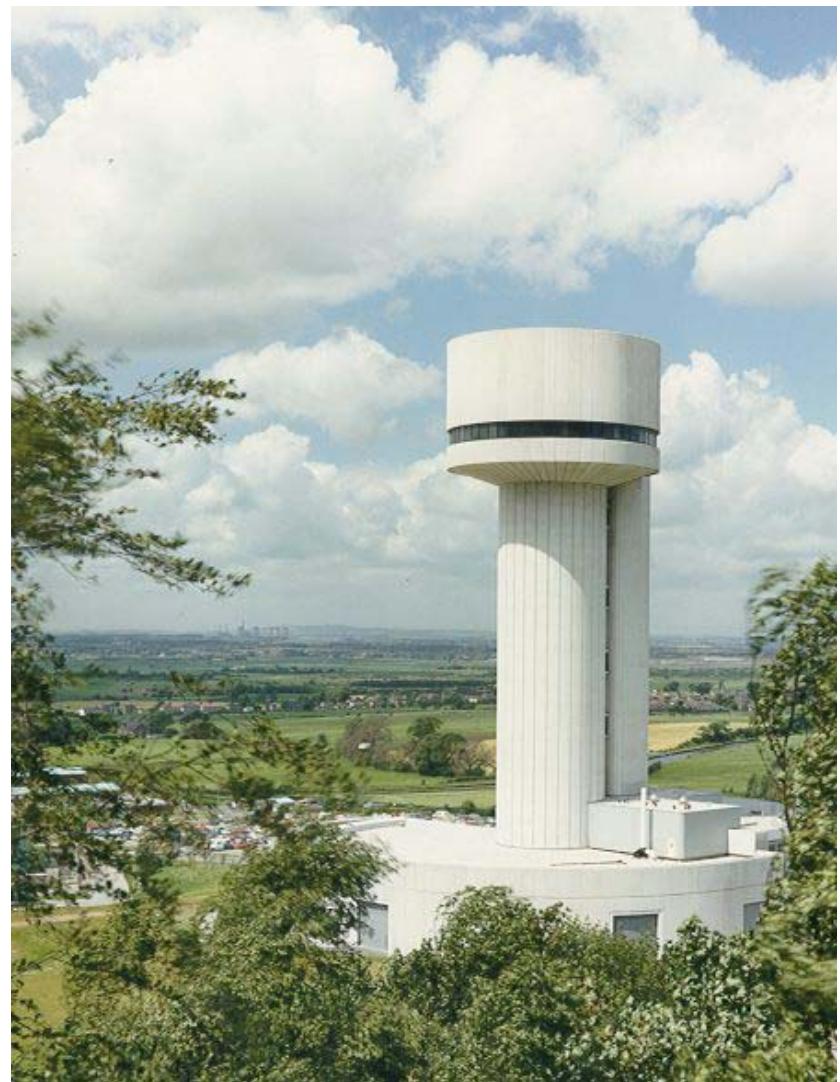
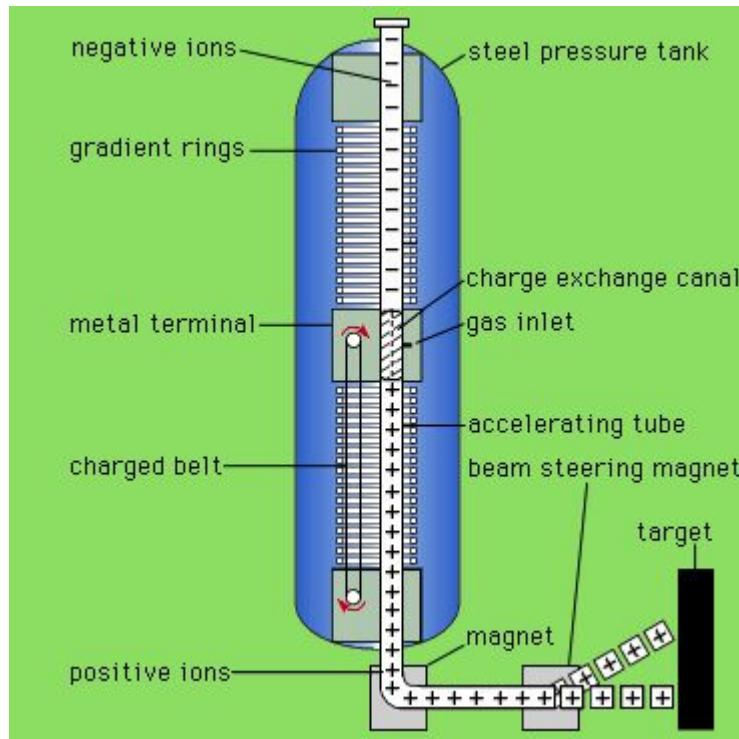


Fermilab 750-keV Cockcroft-Walton generator (R) & ion-source stand (L).

DC HIGH-VOLTAGE ACCELERATORS - VAN DE GRAAFF GENERATORS



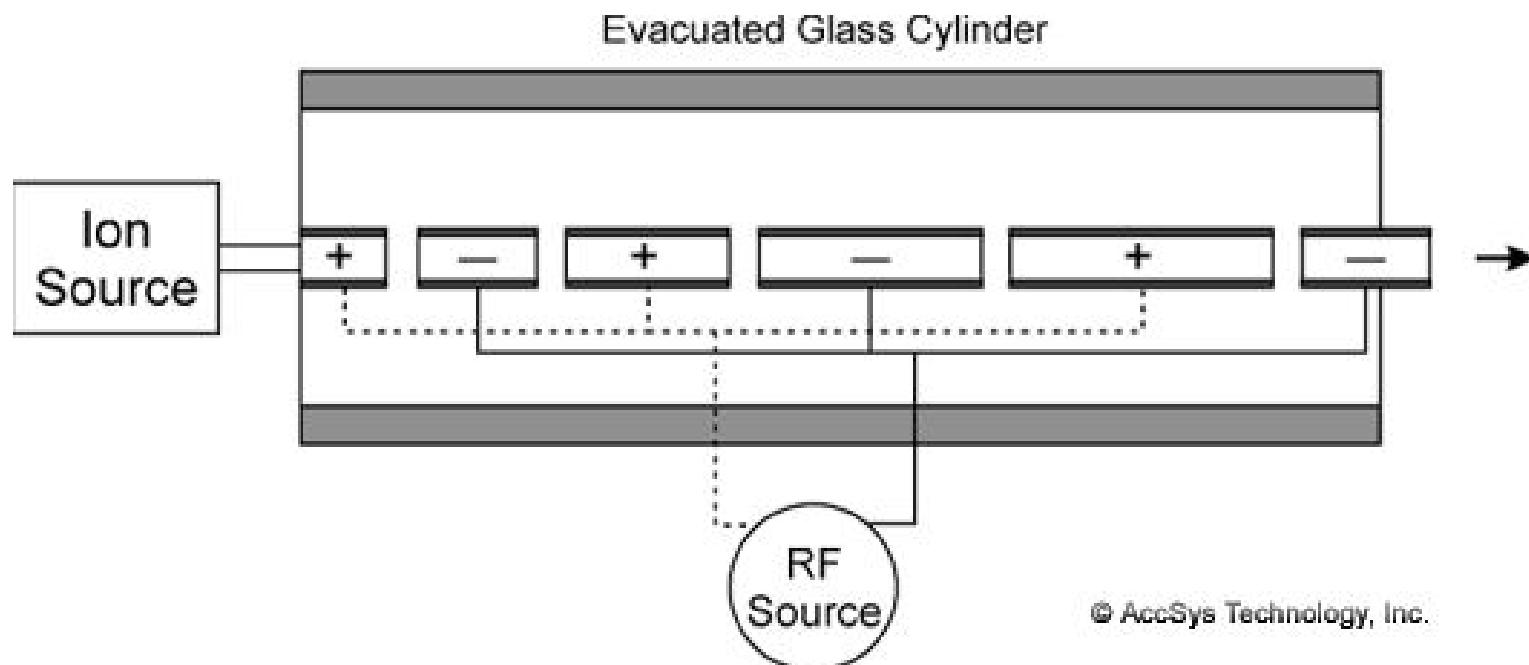
DC HIGH-VOLTAGE ACCELERATORS - TANDEM VAN DE GRAAFFS



Above: Daresbury folded tandem
(20 MV in a 230-ft tower).
Left: Yale 22-MV tandem.

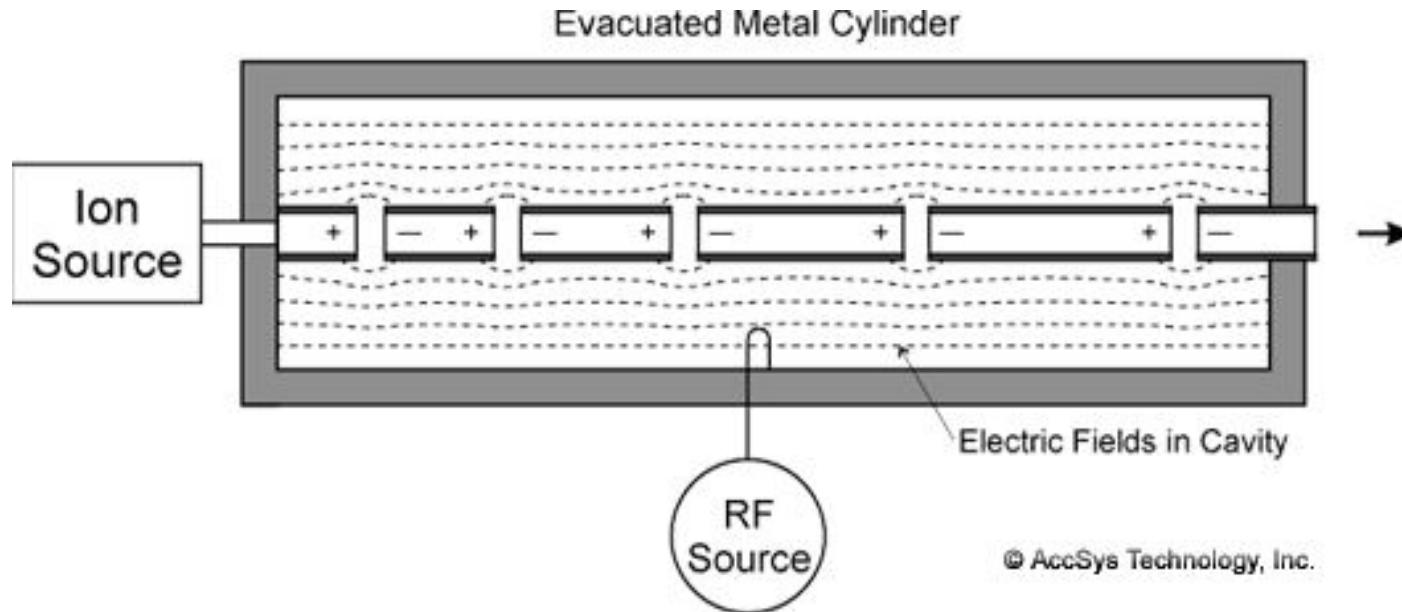
LOW-VOLTAGE AC (RF) LINEAR ACCELERATORS (LINACS)

- First suggested by **Ising** in 1924 (a travelling-wave proposal).
- **Wideröe** (1928) achieved 28-keV alkali ions, in a fixed-field version.
- **Lawrence & Sloan** accelerated Hg ions to 1-2 MeV (1930), 2.85 MeV (1934)
- The crucial innovation was the field-free **drift tubes**, shielding the ions from the electric field whenever it reversed direction.
- Note that the beam is **non-continuous** - a **stream of short pulses** - separated by the **rf period τ** .
- Also, the drift-tube **lengths ℓ_n** must **increase** with ion **velocity**: $\ell_n = v_n \tau / 2$

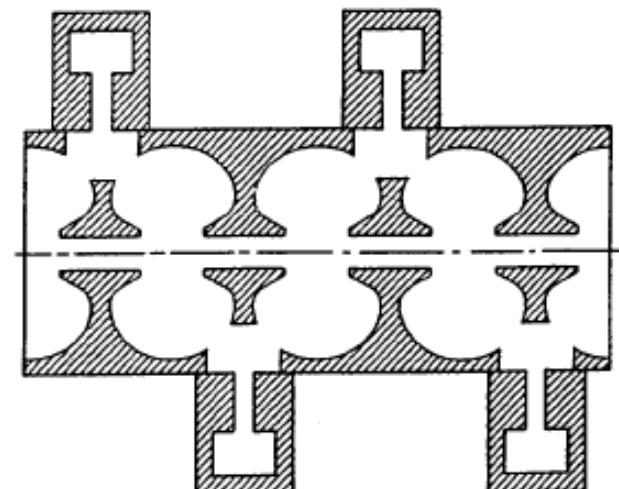


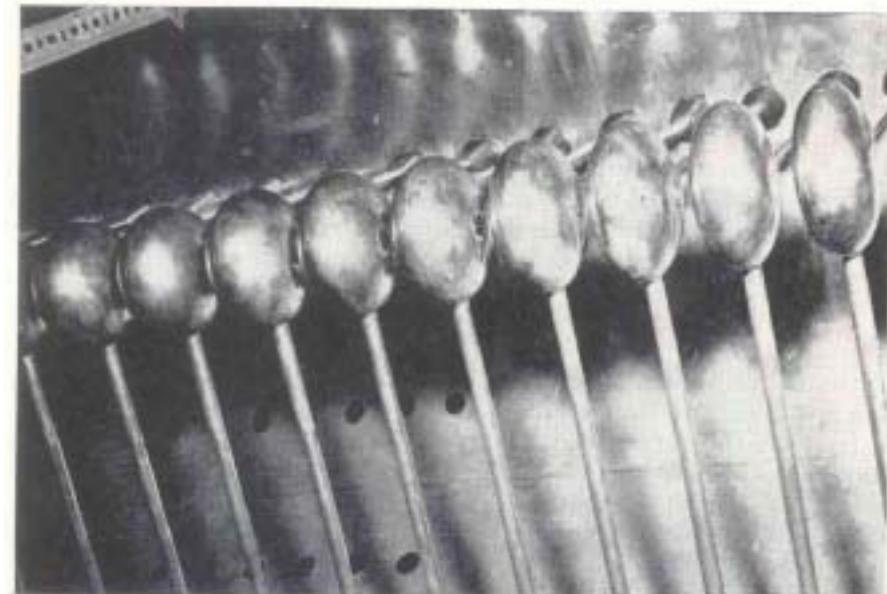
MODERN PROTON AND ION LINACS

Following Alvarez (1946), modern practice is to immerse the drift tubes in an **electromagnetic cavity**:



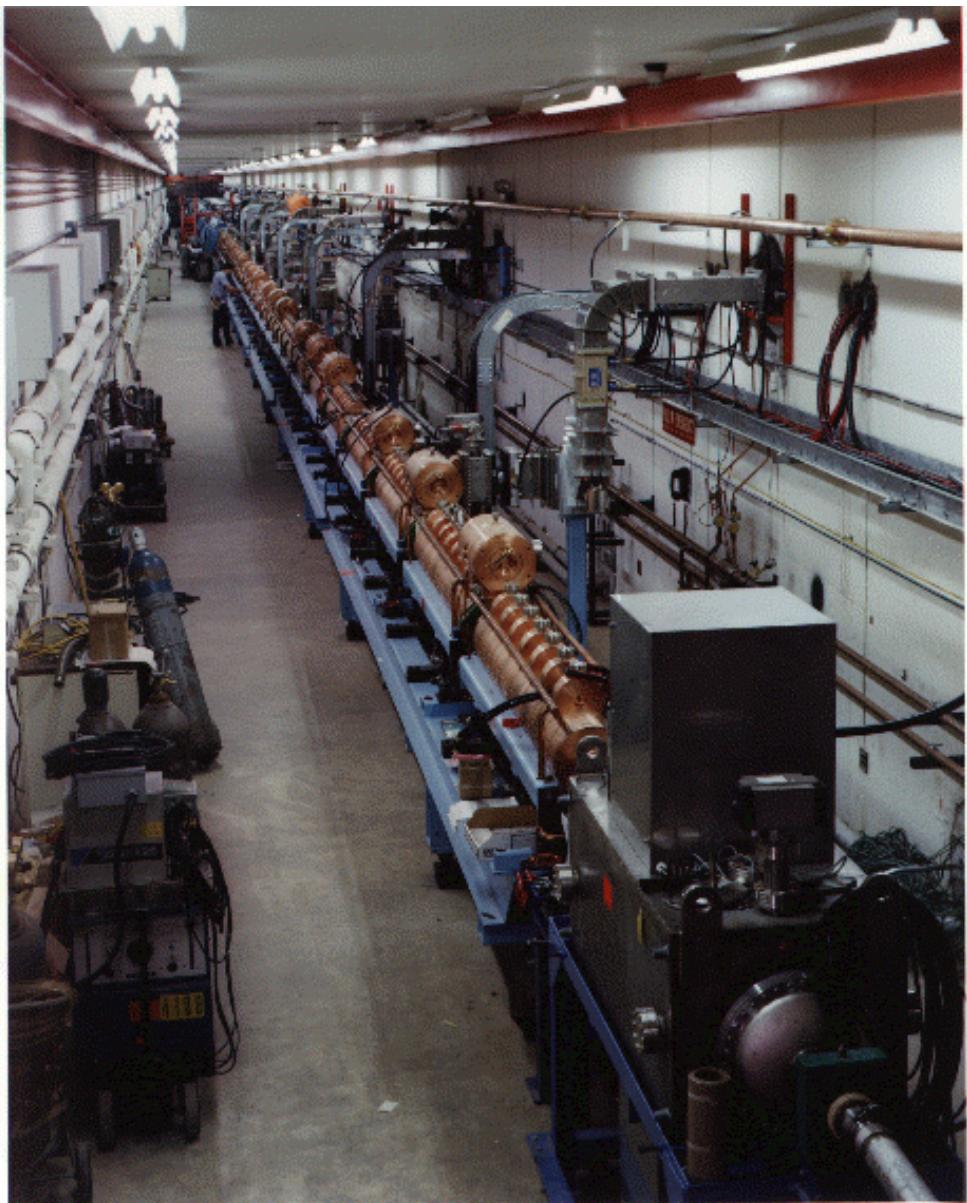
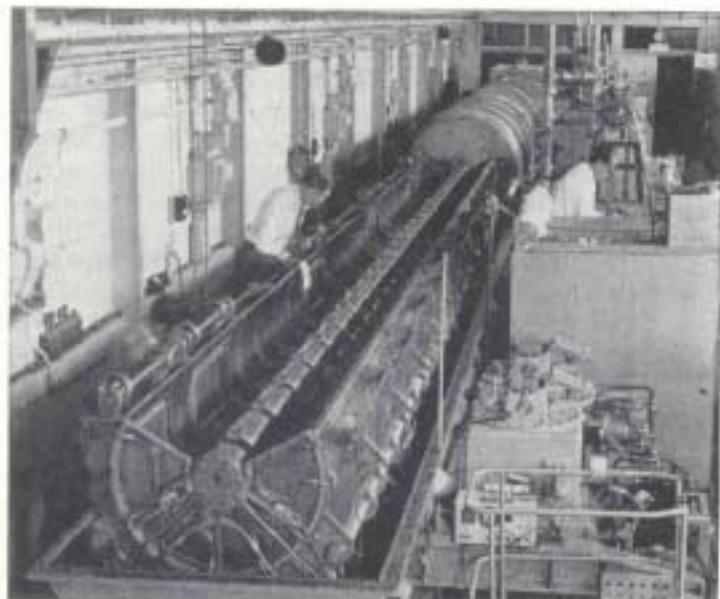
For **protons above 100 MeV**, the drift tubes are incorporated in a "**coupled-cavity**" design - introduced at **Los Alamos** in the 1960s.



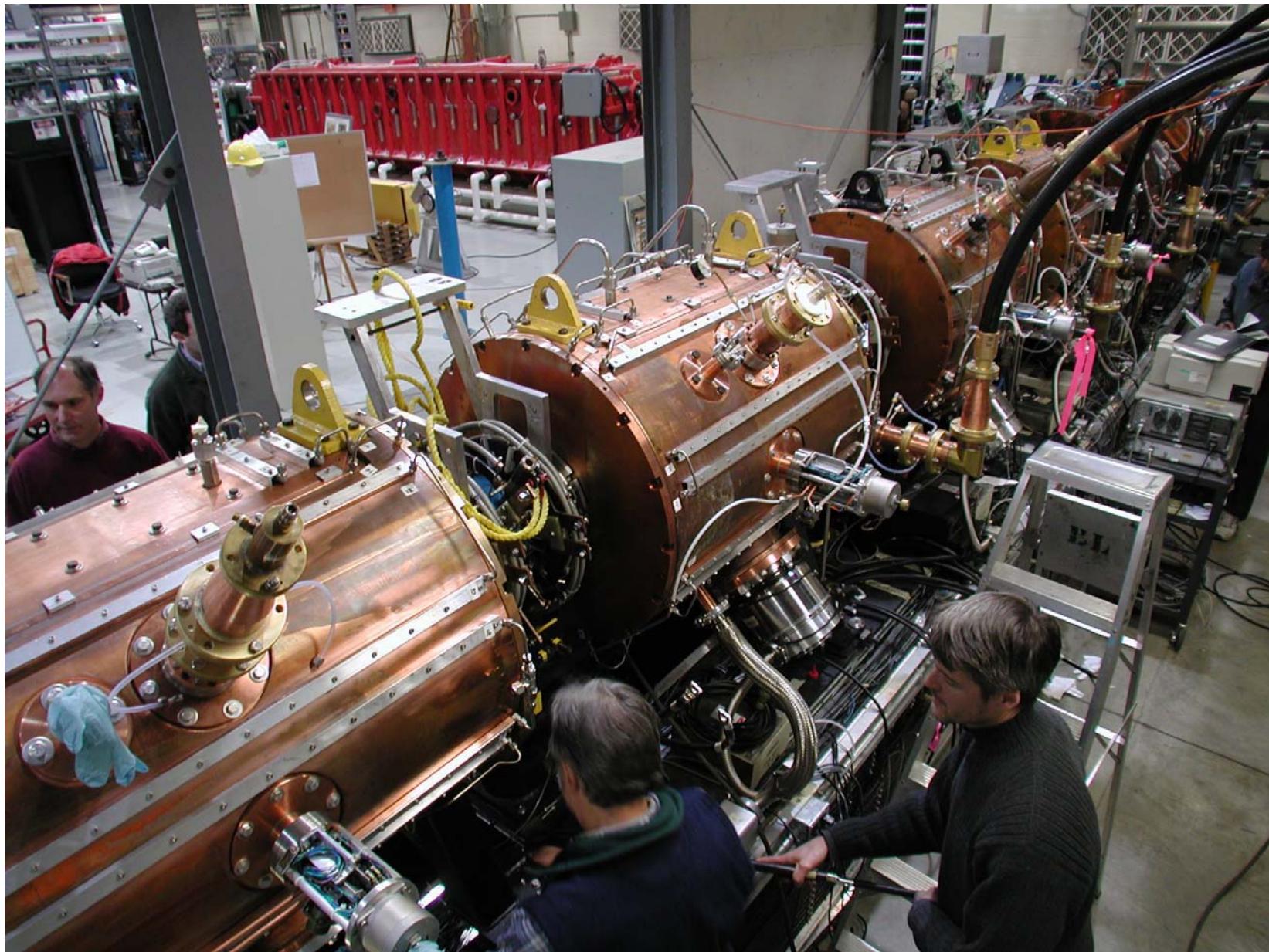


Courtesy of United Kingdom Atomic Energy Authority

Rutherford Lab proton linac drift tubes:
Top: at 50 MeV; Bottom: at 0.5 MeV.



Fermilab 400-MeV proton linac,
using side-coupled cavities



ISAC 1.5 MeV/u Drift-Tube Linac



The ISAC 150-keV/u RFQ linac

RFQ LINACS

Radio-frequency quadrupole linacs:

- were proposed by Kapchinsky and Teplyakov (1970)
- were demonstrated at Los Alamos (1980).
- have largely replaced DC accelerators as injectors of protons & ions.

The RFQ operates at low energy:

- accepting a continuous beam at a few keV
- bunching it with nearly 100% efficiency
- focusing and accelerating it to a few MeV.

Think of it as an electric quadrupole

- whose rf oscillations provide AG (Alternating Gradient) focusing
- whose pole-tip radius modulation provides E_z for acceleration.

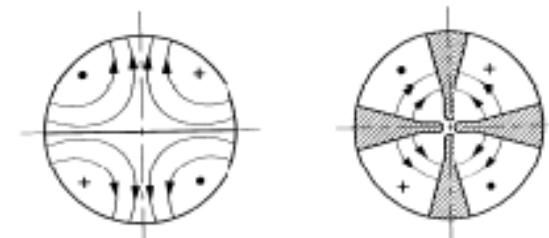
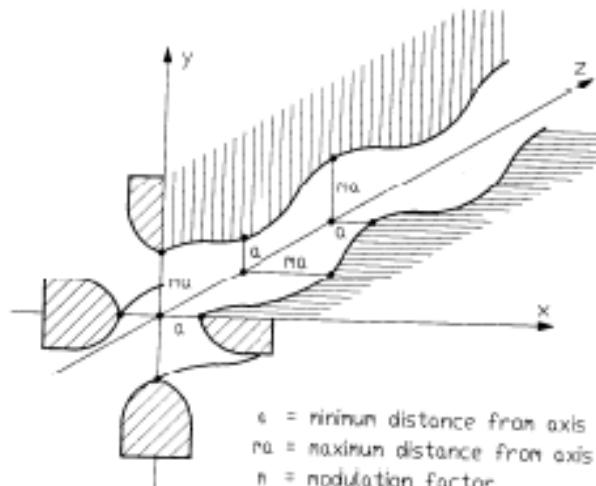
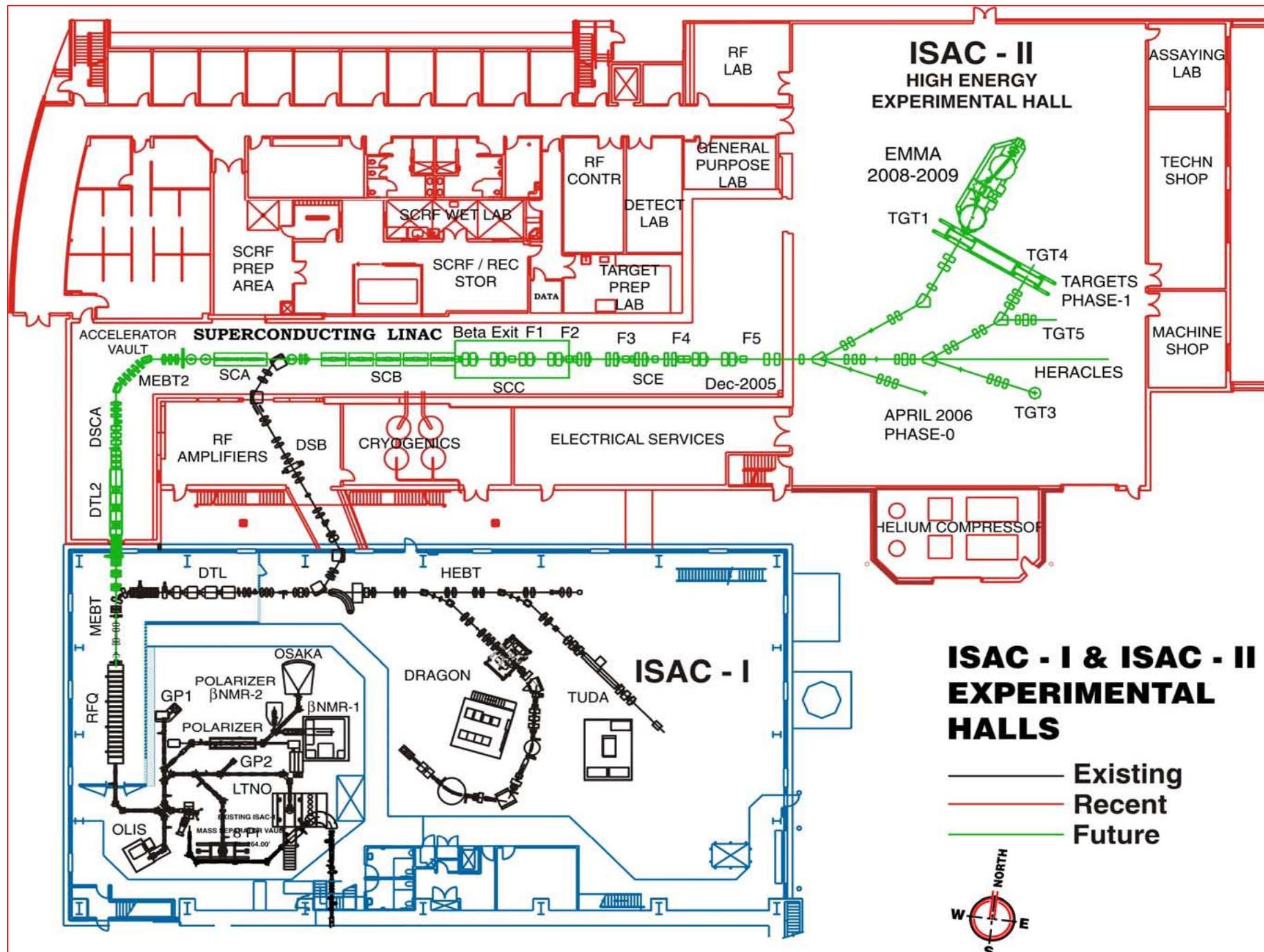


Fig. 17 TE₂₁₀ field pattern in empty and loaded cavities



SUPERCONDUCTING LINACS

Linacs need a **separate EM field** for each accelerating gap - so to reach **high energies** with **normal-conducting rf cavities** requires:

- **very high power** and ∴ usually **pulsed operation**;
- **very long machines**, as **field strength is limited**.

Thus LANSCE (800 MeV \times 1 mA = **0.8 MW** protons), operating on a **12% duty cycle**, needs **65 MW** rf power (150 MW mains?). Located in a remote part of New Mexico, it can afford to run only **3 mo/y**.

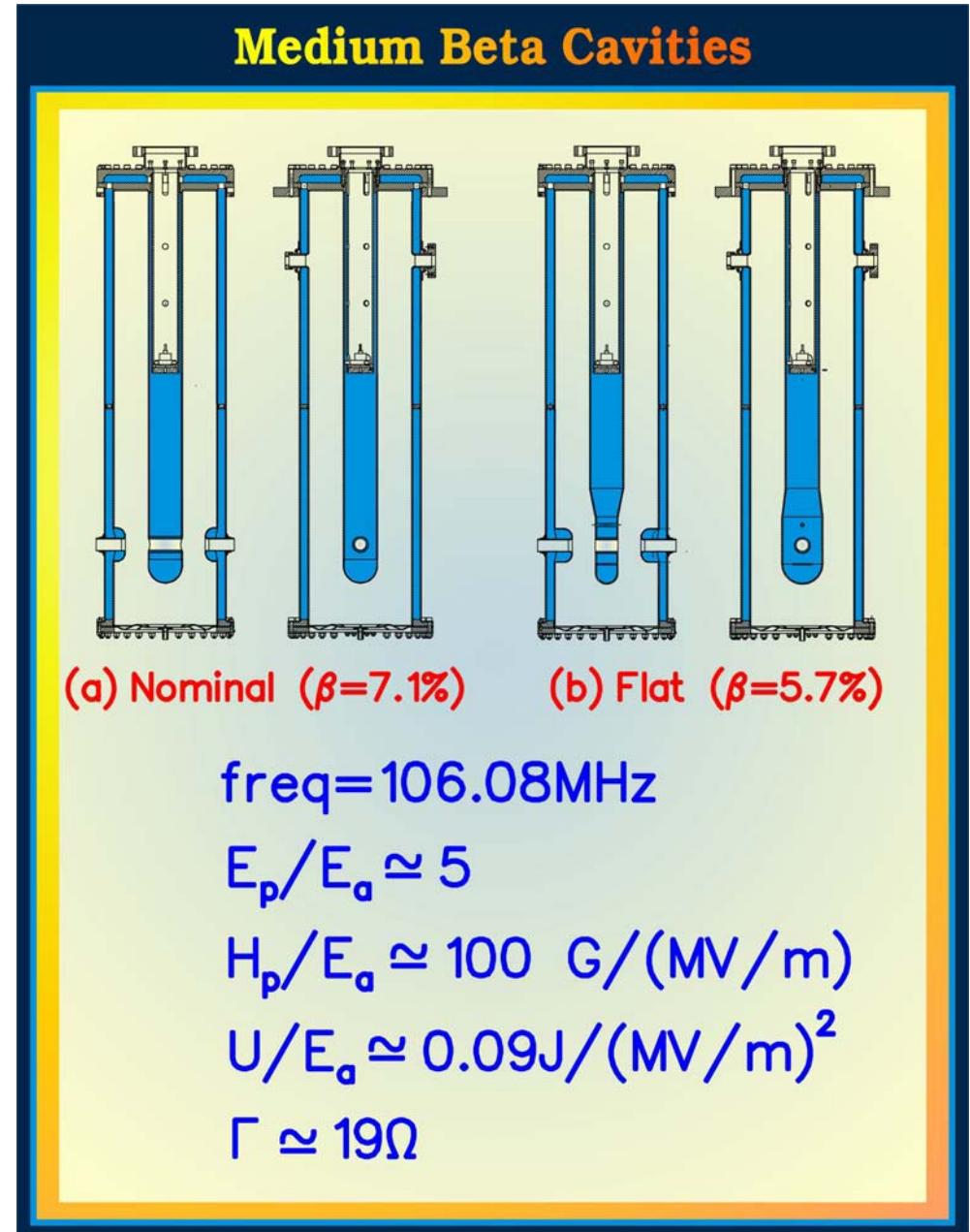
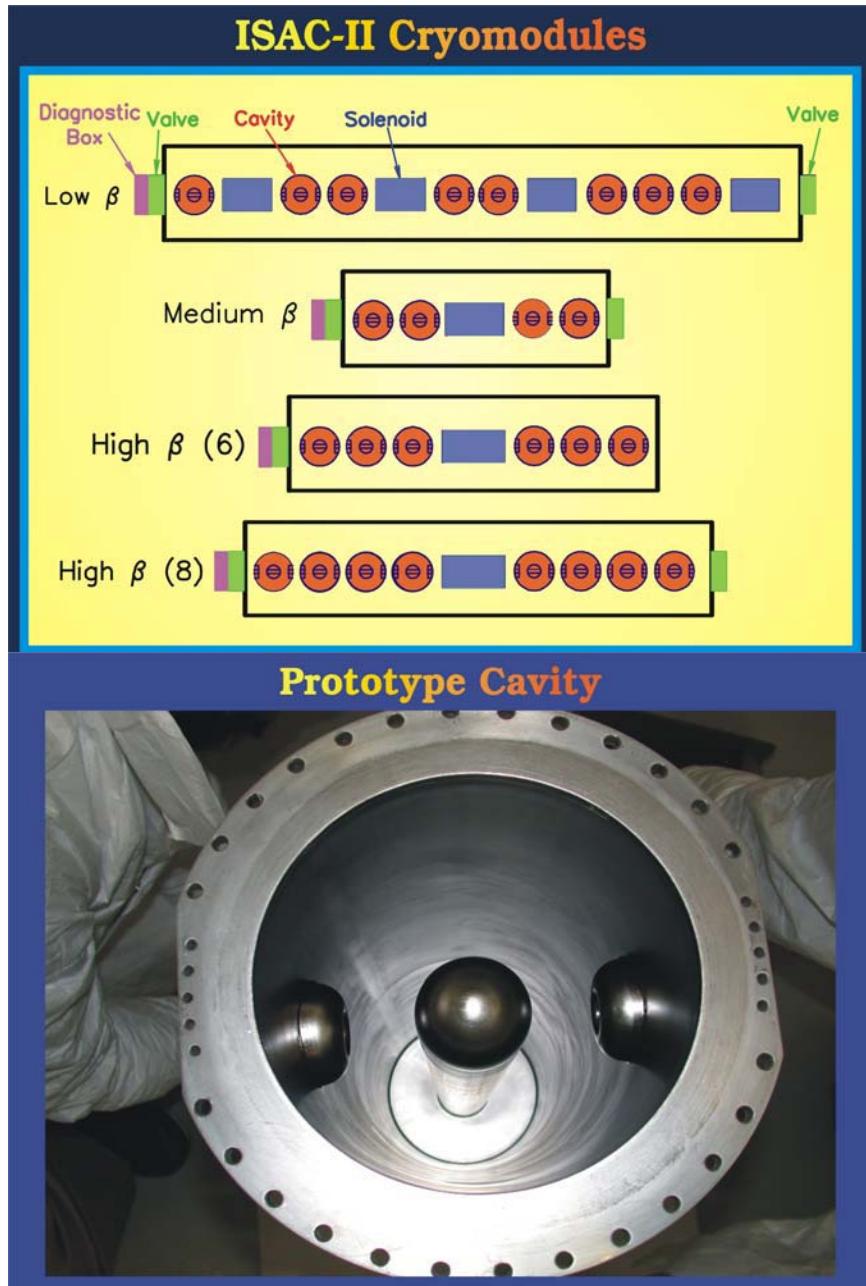
Superconducting cavities have therefore been pursued since ~1960 in the hope of **reducing the power dissipation in the walls to zero**.

Success came in the 70s and 80s using niobium:

- first **heavy ions** (range of v , few cavities - ATLAS....now ISAC-II)
- then **electrons** ($v=c$, many cavities same size - Cornell, CEBAF, LEP)
- now **protons** (wide range of v , many cavities - 1-GeV SNS).

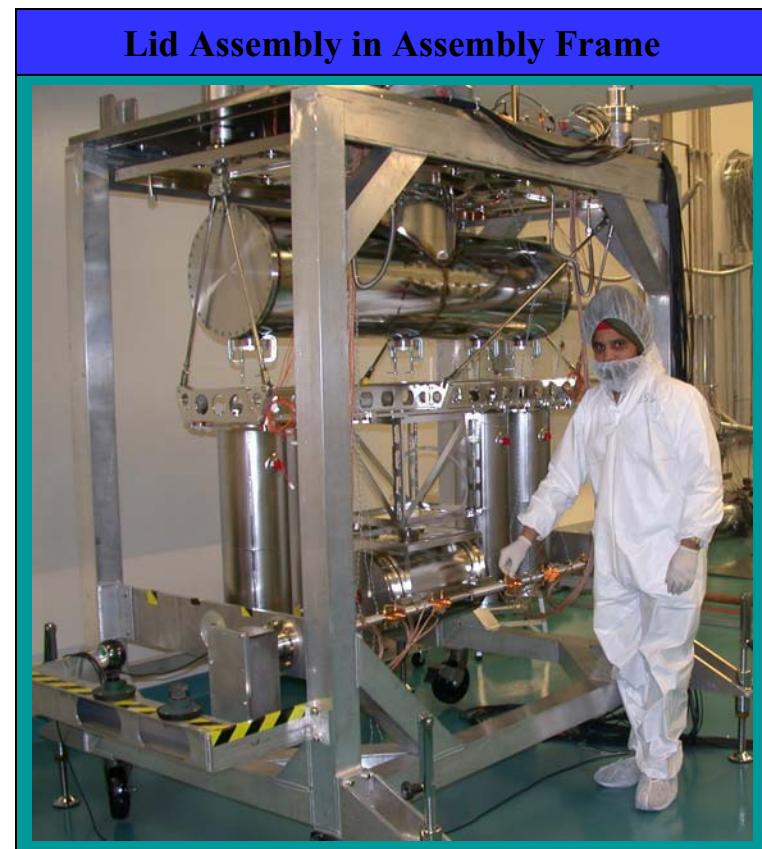
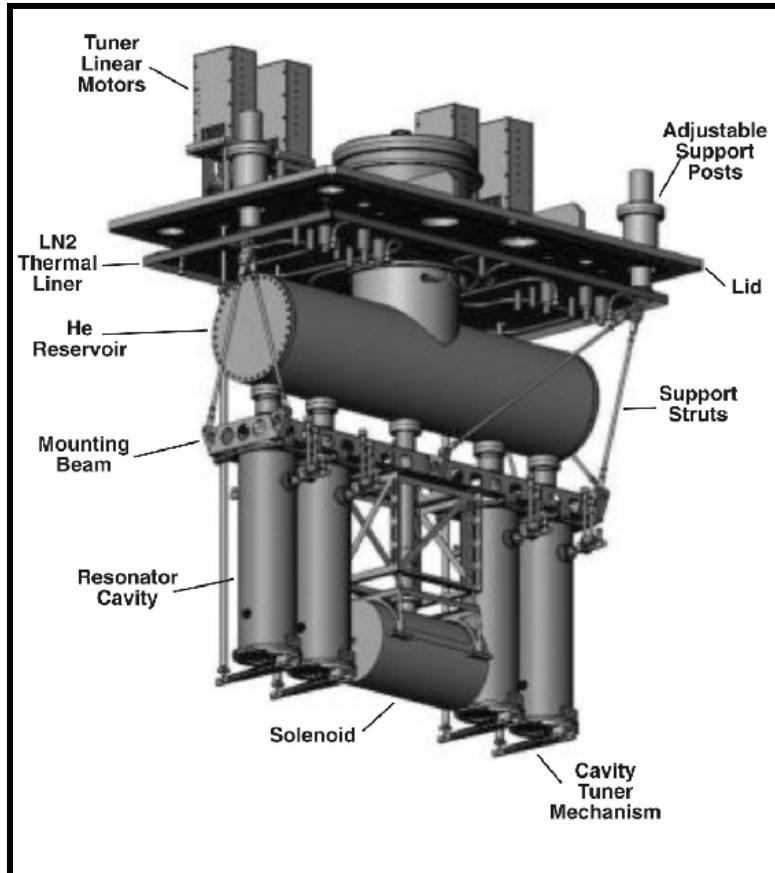
Furthermore, **much higher fields** can be produced - up to **30 MV/m**.

ISAC Superconducting RF Cavities



Medium Beta Cryomodule

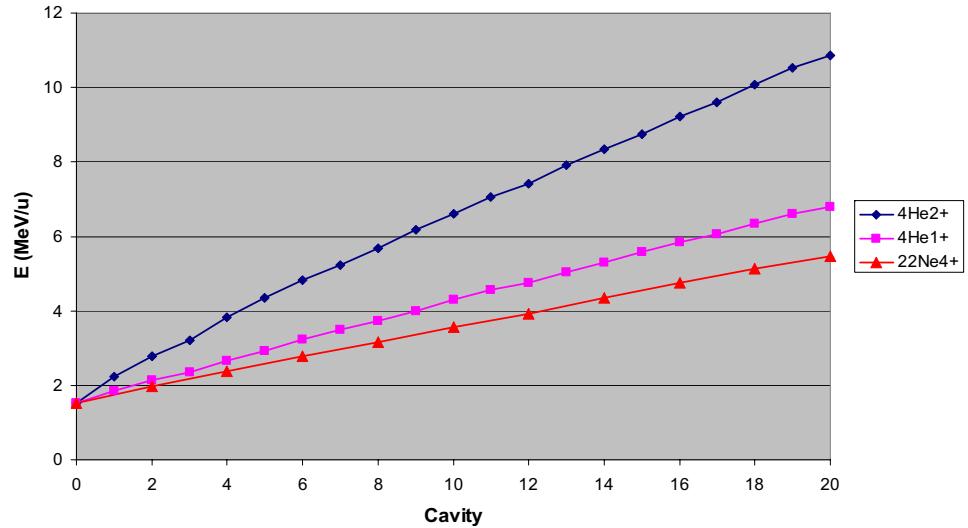
- 2x2x1m stainless steel box vacuum vessel
- LN2 cooled copper sheet used as thermal shield
- Mu metal between vacuum tank and LN2 shield
- Cold mass suspended from lid on three adjustable support pillars
- Four cavities $E_p=30\text{MV/m}$
- One SC solenoid @ 9T
- $V_{\text{eff}}=4.3\text{MV}$
- Single vacuum for thermal insulation and rf



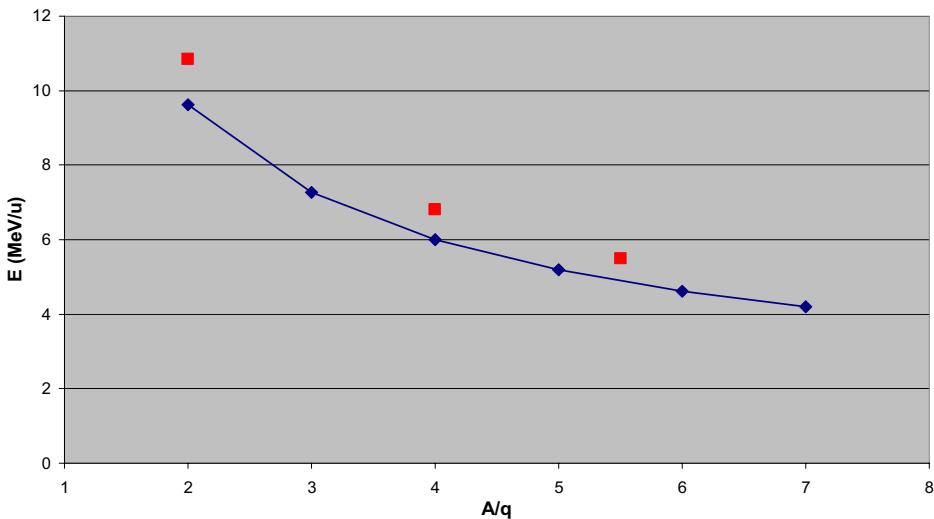
THE ISAC-II SUPERCONDUCTING LINAC



The last of the 8 cryomodules, containing 40 superconducting niobium cavities, was installed in March 2010. Radioactive ion beams can now be accelerated up to 6.5 MeV/u ($Q/A = 1/6$) or 16.5 MeV/u ($Q/A = \frac{1}{2}$).



Energy history during acceleration.



Expected E_{final} for 6MV/m and actual E_{final}

Commissioning beams

- $A/q=5.5$ (22Ne4+)
- $A/q=4$ (40Ca10+, 20Ne5+, 12C3+, 4He1+)
- $A/q=2$ (4He2+)

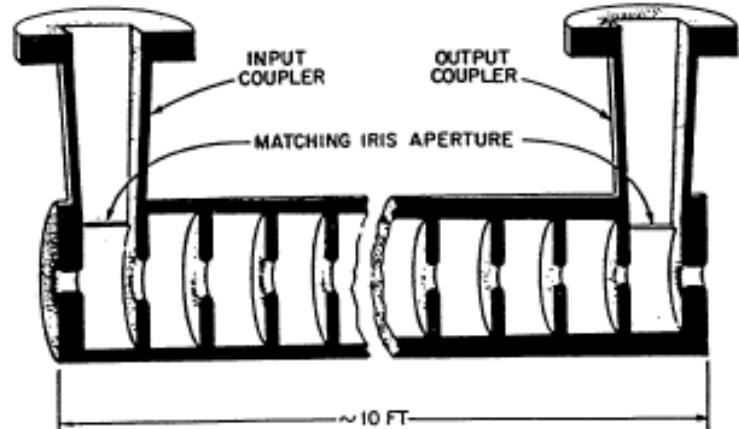
Performance

- Power @ 7W/cavity
- Design gradient is 6MV/m
- Average gradient is 7.2MV/m
- Final energy is 10.8, 6.8 and 5.5MeV/u for $A/q=2, 4, 5.5$ respectively
- Transmission >90%

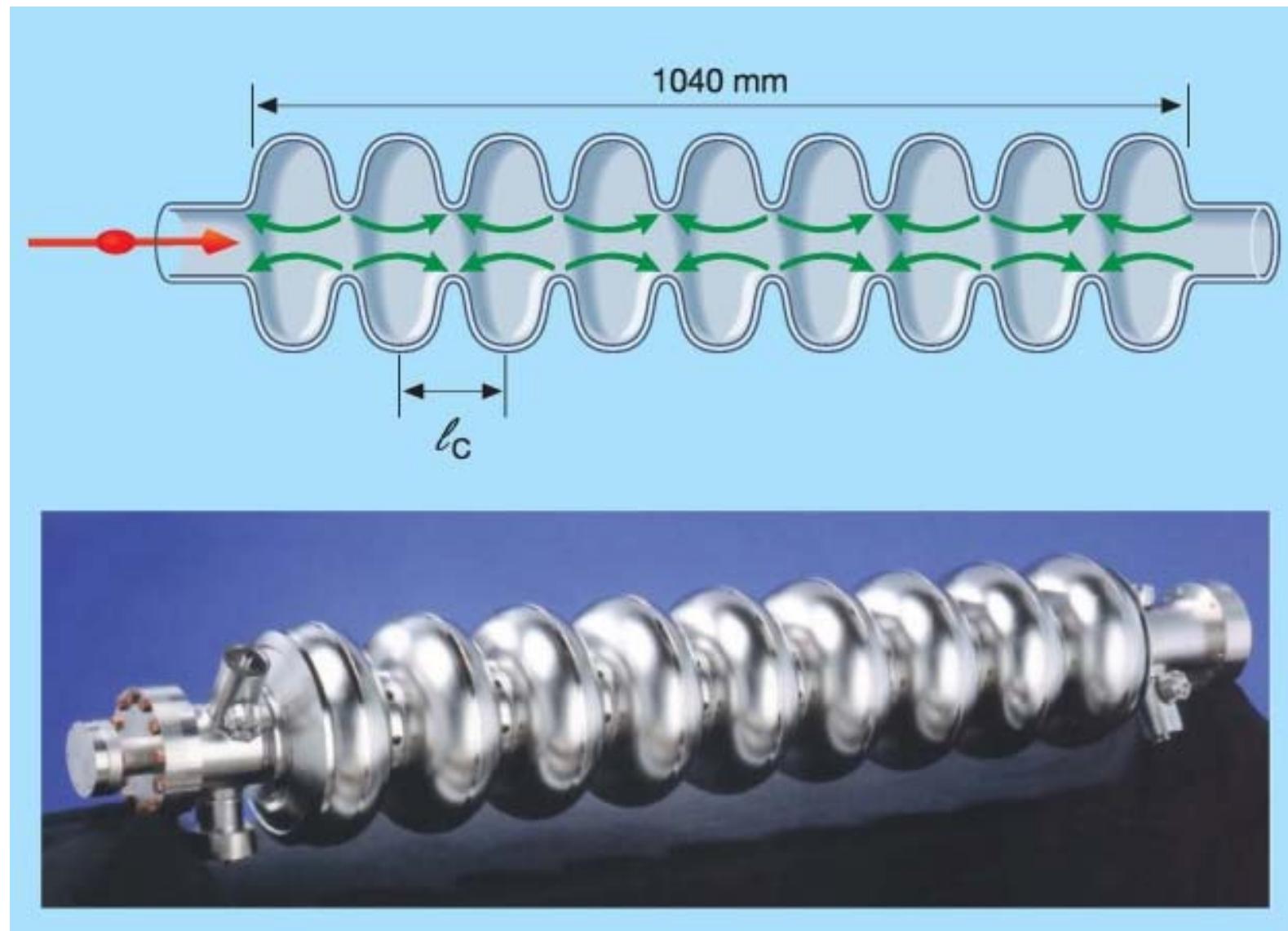
ELECTRON LINACS

Electrons are usually accelerated by travelling EM waves in disc-loaded waveguides, whose dimensions determine the wave's speed.

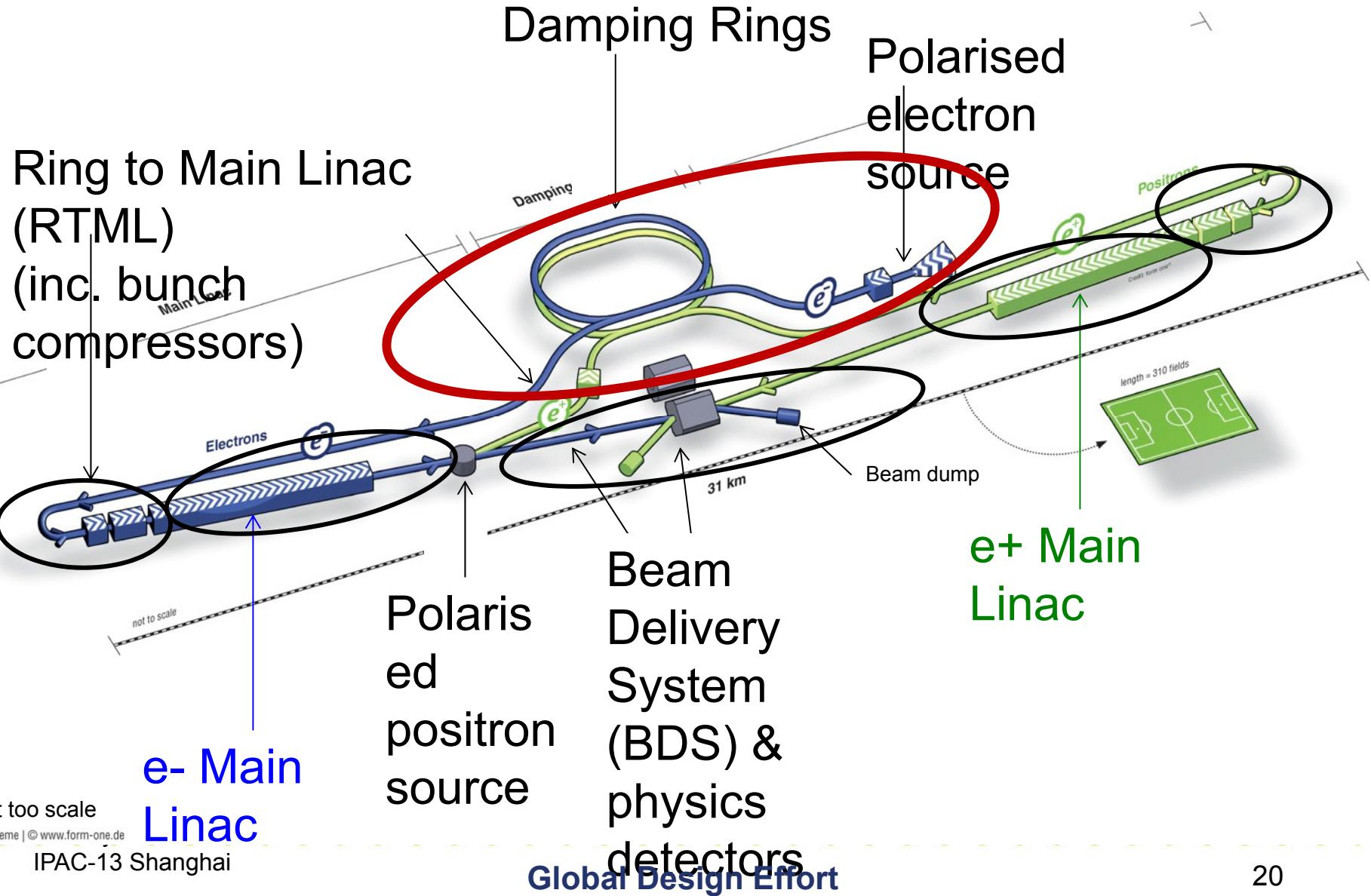
As an electron's speed $v \rightarrow c$, the speed of light, at relatively low energies (~ 500 keV), the waveguide dimensions can be kept constant for the higher energies, and those parts of the linac built from identical sections (as for the 50-GeV SLAC linac, below).



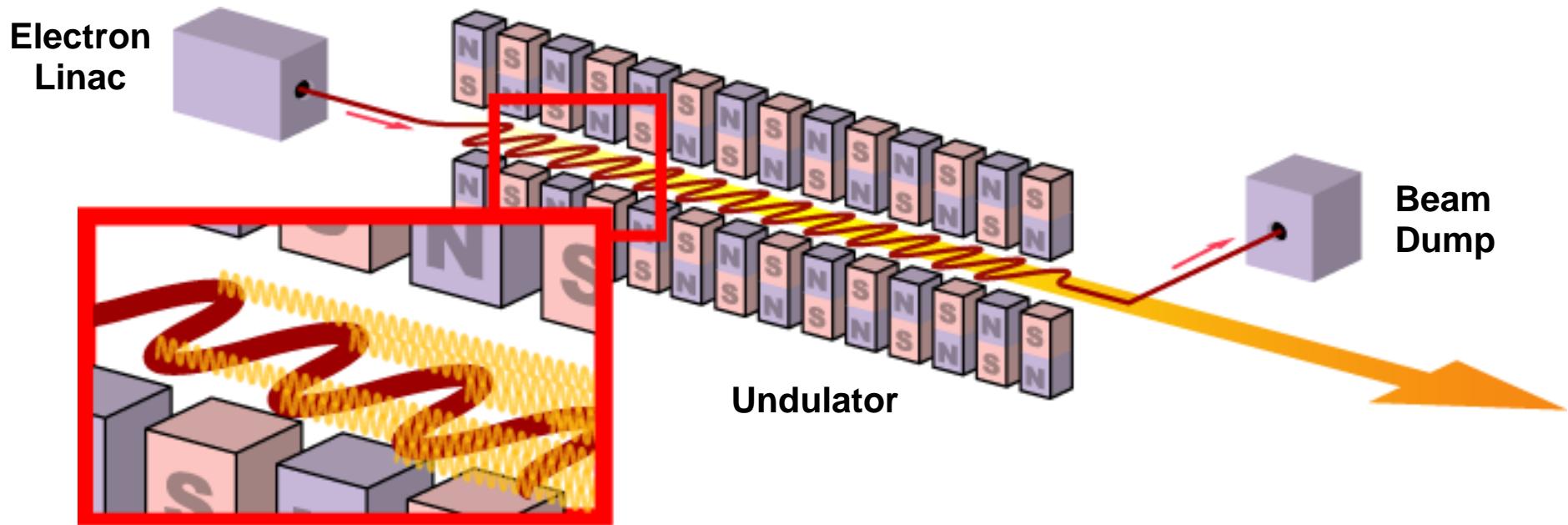
SUPERCONDUCTING CAVITIES FOR H.E. ELECTRONS



9-cell niobium cavity for the FLASH free electron laser linac at DESY



FREE ELECTRON LASER



Electron oscillations in the magnetic undulator produce **e-m radiation**, leading to further **stimulated emission** downstream.

If $eB\lambda_u \ll 2\pi\beta m_e c$, where λ_u is the undulator wavelength, the radiation will form a **narrow coherent beam**.

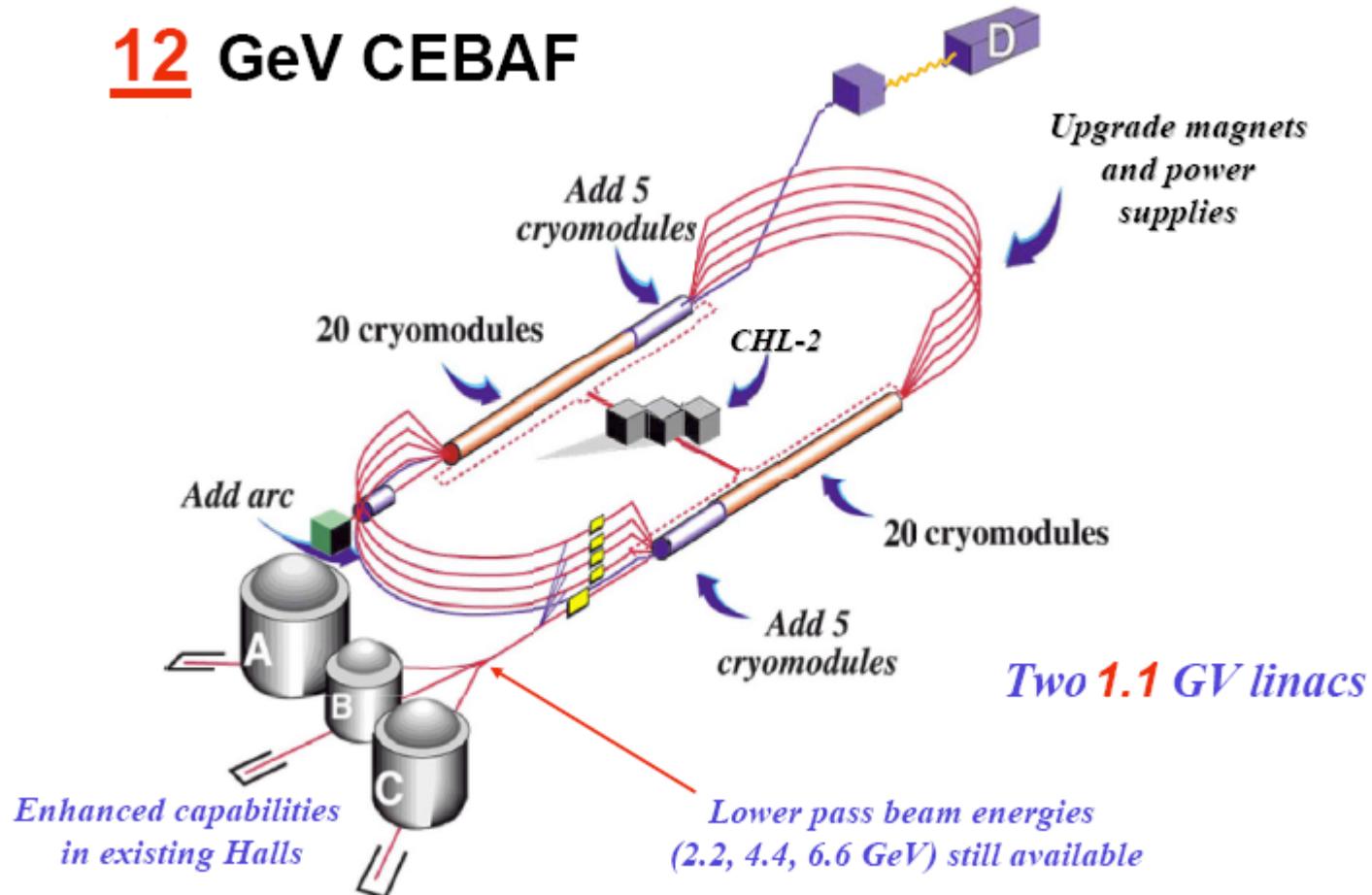
Adding a mirror at each end forms an optical cavity \rightarrow **laser action**.

Expensive, but: - **huge spectral range** - microwaves to X-rays ($\times 10^7$)
- **tunable** $\sim \pm 10\%$

RECIRCULATING LINACS

The first and best-known is **CEBAF** (Continuous Electron Beam Accelerator Facility) at Jefferson Lab (Virginia).

- currently $6 \text{ GeV} \times 200 \mu\text{A}$ (two 0.6-GeV linacs \times 5 separated orbits)
- upgrade to 12 GeV recently approved (add **more superconducting rf**)

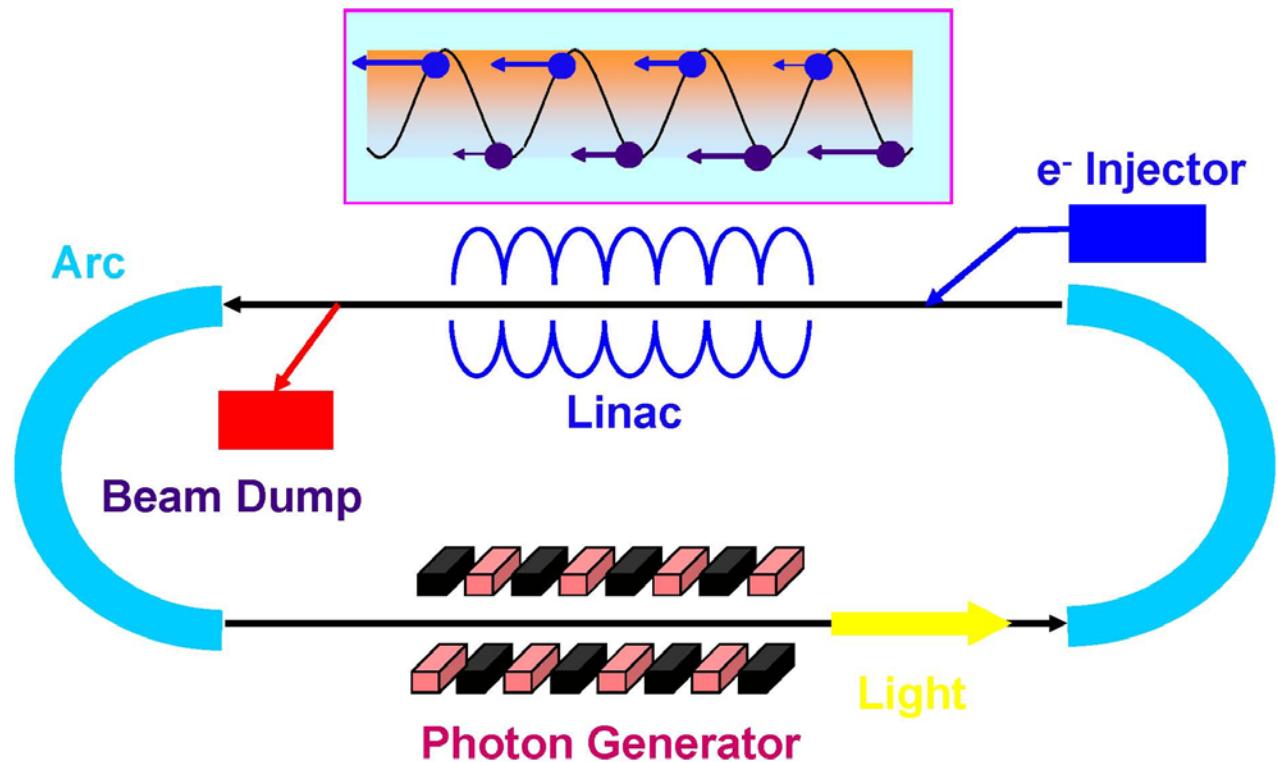


ENERGY RECOVERY LINAC (ERL)

2 passes through linac

- 1st accelerates
- 2nd decelerates
- energy returned to rf cavity.

This makes possible
high beam currents
with **modest rf power**.



The beam emittance is determined purely by the electron gun

- can be **very small** compared to circulating beams in **synchrotrons**, whose emittance is enlarged by random emission of radiation photons.

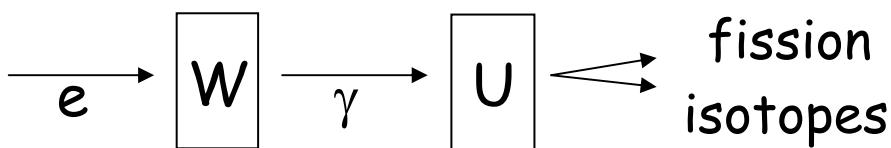
High current & small emittance → **very bright radiation source**.

Also low energy dump → **low waste heat & low radioactivity**.

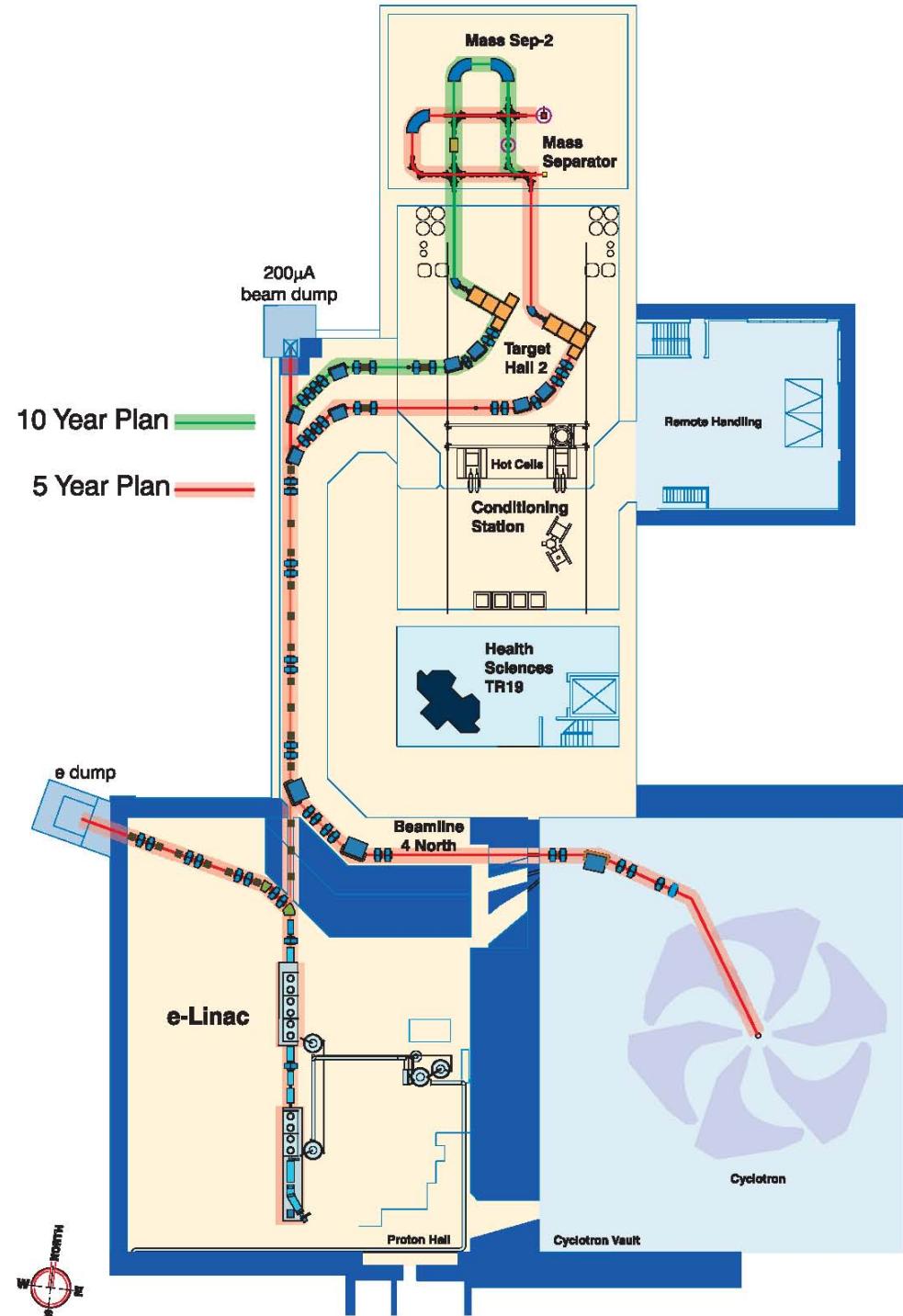
THE ARIEL PROJECT

To provide 3 independent radioactive-ion beams for experiments, TRIUMF plans to add 2 new production targets, fed by:

- $\leq 200 \mu\text{A}$ of $\sim 450\text{-MeV}$ protons (from the cyclotron via BL4N)
- 10 mA of 50-MeV electrons from a superconducting linac (via the **photofission** process)



- yielding a range of isotopes not available from proton reactions.



E-Linac Baseline Layout

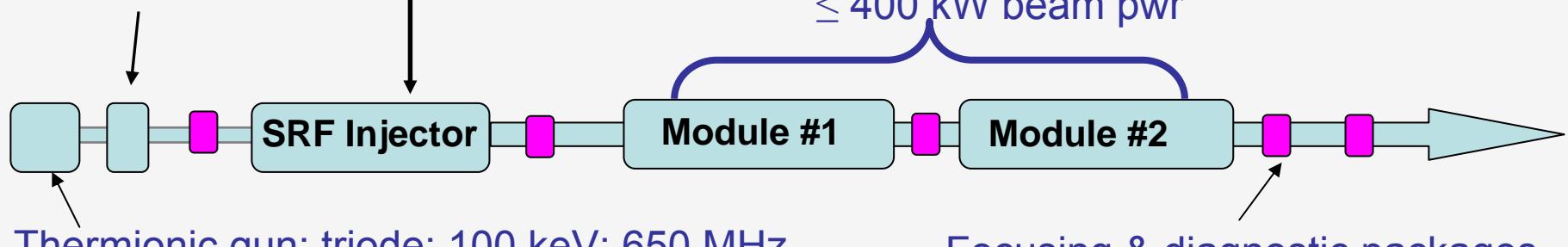
Injector linac

(acceleration & additional bunching)

10 MV/m, $Q=10^{10}$

10 mA, 5-10 MeV gain
 ≤ 100 kW beam pwr

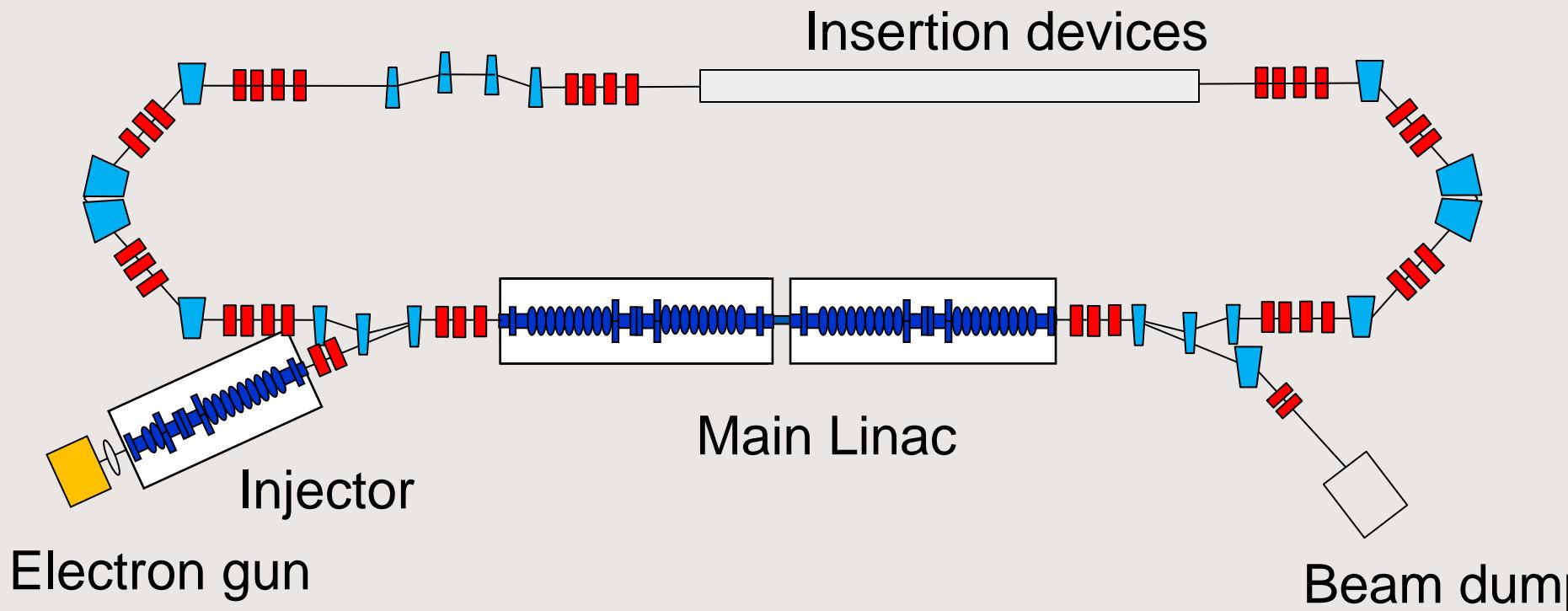
NC buncher



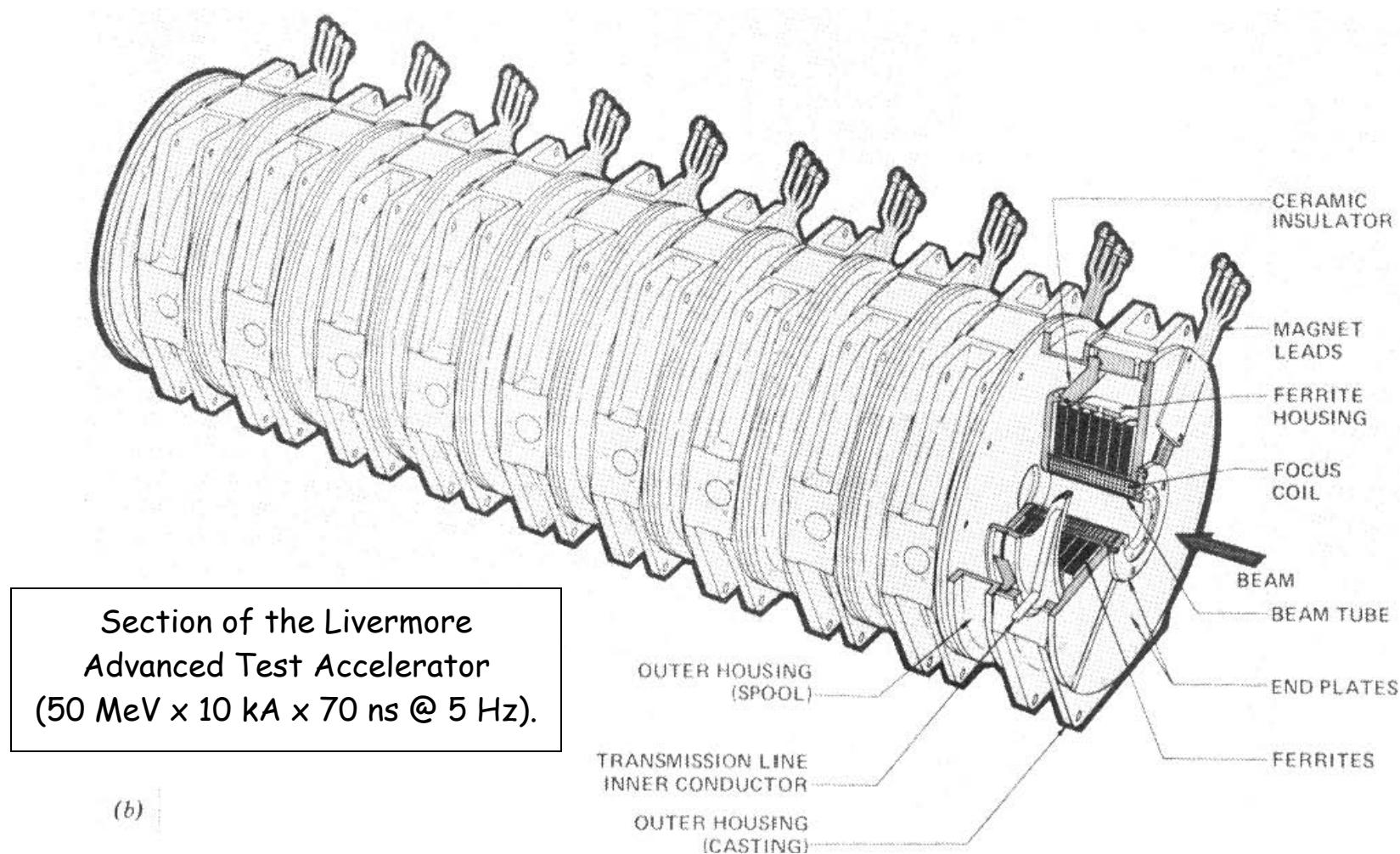
Division into injector & main linacs allows:

- Possible expansion path to test-bed for
 - Energy Recovery Linac (ERL) – e.g. 10 mA, 80 MeV
 - Recirculating Linear Accelerator (RLA) – e.g. 2 mA, 160 MeV

ERL Extension of the E-Linac



INDUCTION LINACS



Pulsed toroidal magnetic fields in a series of modules induce an accelerating electric field along the axis.

- Good for very high peak current (>10 kA) pulsed beams;
- Astron injector 3.7 MeV \times 350 A in 300 ns pulses @ 1 kHz.

MAGNETIC RESONANCE - THE CYCLOTRON



At Berkeley, the 27-year-old [Ernest Lawrence](#) saw Widerøe's linac article and set a grad student, [Dave Sloan](#), to repeat and improve on his work. By 1930 they had achieved 1-2 MeV Hg ions.

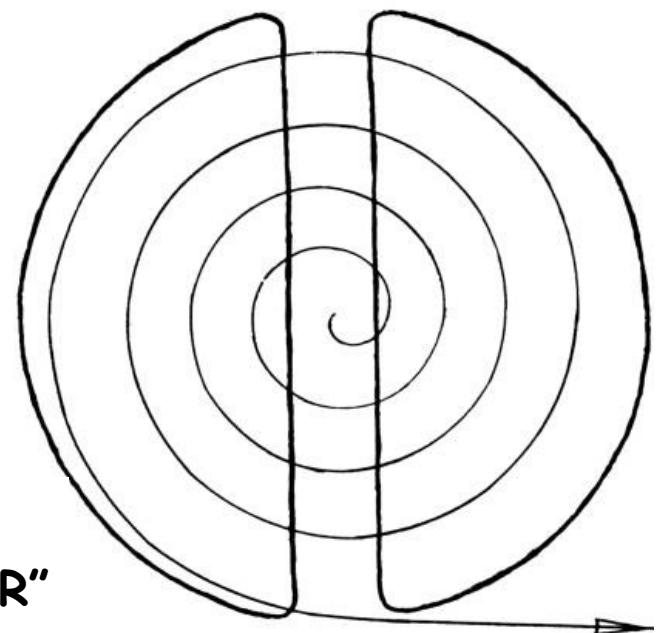
Widerøe's paper had also reported an unsuccessful attempt to build a "[beam transformer](#)" - a [betatron](#), where particles circulating in a rising magnetic field would be accelerated by the [induced electric field](#).

Perhaps it was this that led [Lawrence](#) to consider a [circular version of the drift-tube linac](#), using a uniform and constant magnetic field to return the particles repeatedly to the same accelerating gaps between two D-shaped electrodes.

[Animation](#)

Courtesy of ATOMKI Cyclotron Laboratory, Hungary

"THE CYCLOTRON AS
SEEN BY THE INVENTOR"



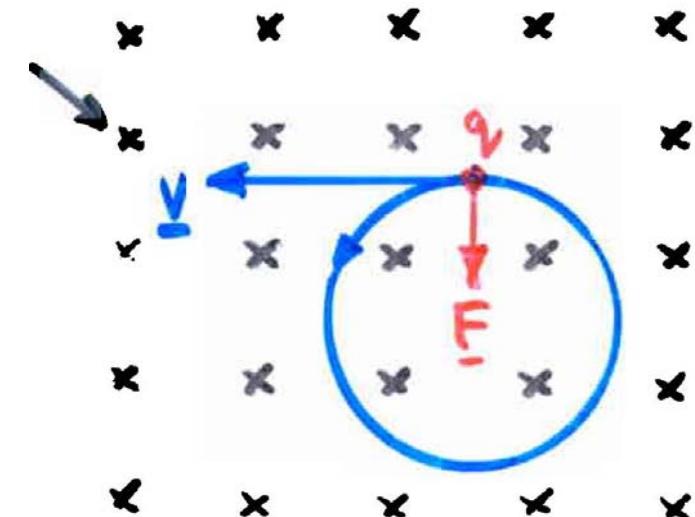
THE CYCLOTRON PRINCIPLE

When Lawrence worked out the particle dynamics, he found an unexpectedly favourable result:

Suppose uniform magnetic induction \underline{B} is applied perpendicular to the velocity \underline{v} of a particle with mass m , charge q :

The Lorentz Force $\underline{F} = q \underline{v} \times \underline{B}$ produces a circular track, and

$$q \underline{v} \times \underline{B} = m \underline{v}^2 / r = m \omega$$



\therefore "Cyclotron Frequency"

$$\omega = \frac{qB}{m}$$
 is independent of \underline{v}

- and the orbits are "isochronous".

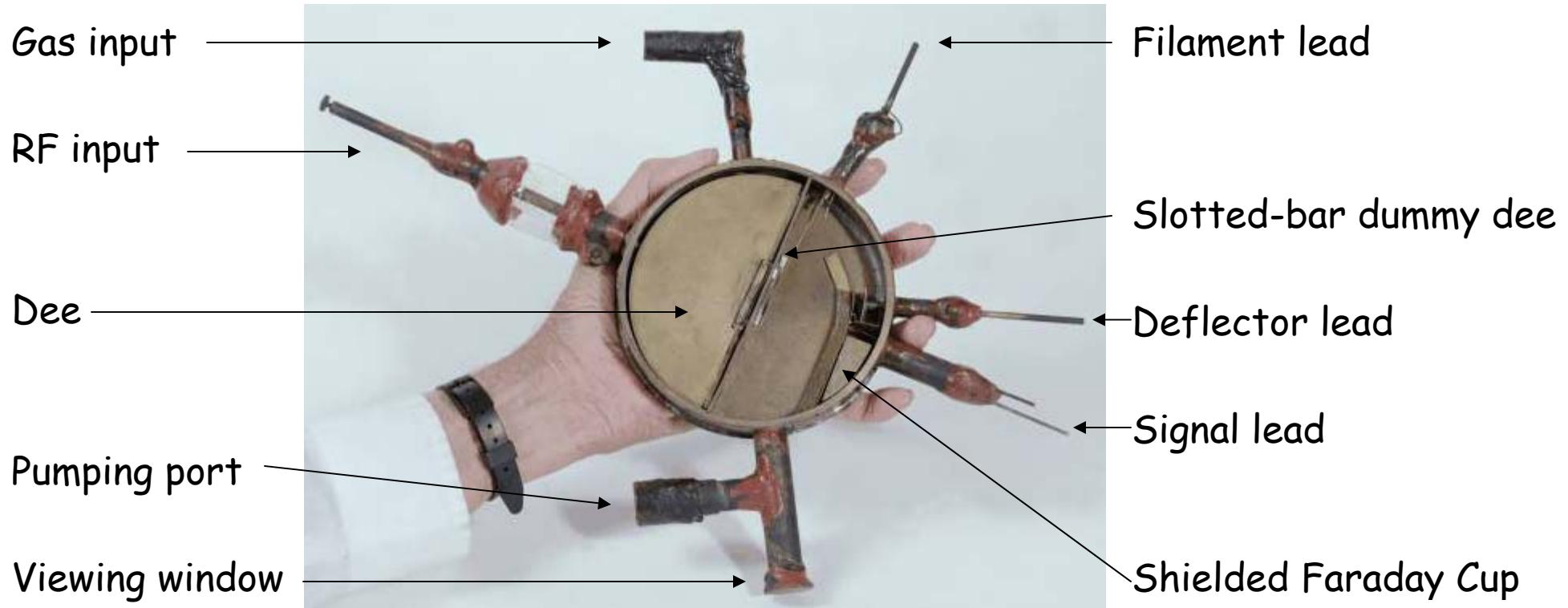
So:- the electrodes can be excited at a **fixed rf frequency**,

- the particles will remain in resonance throughout acceleration,
- and a new bunch can be accelerated on every rf voltage peak:

- "continuous-wave (cw) operation"

Note also that: $r = \frac{mv}{qB}$ radius \propto velocity v.

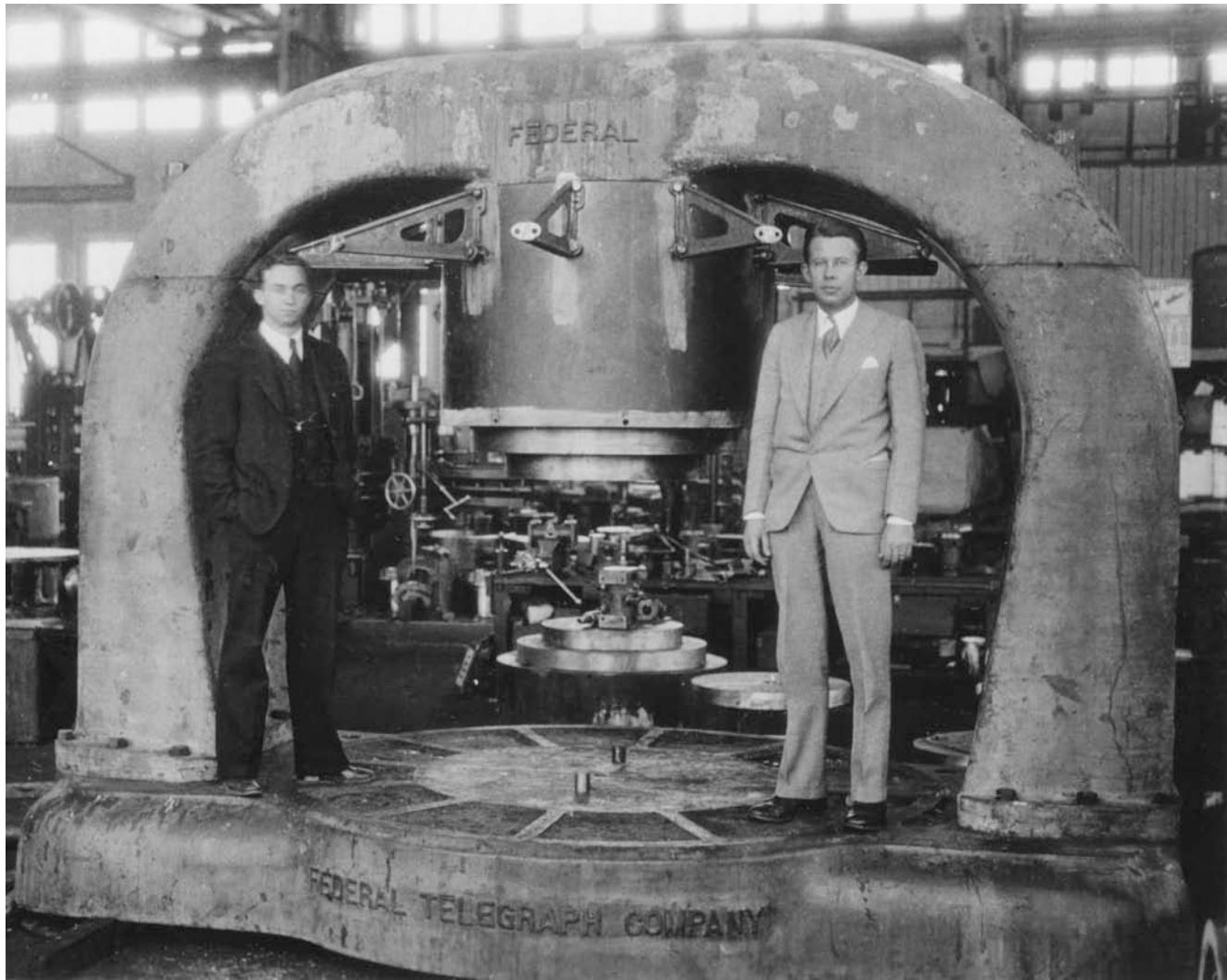
FIRST WORKING CYCLOTRON - Fall 1930



In late 1930, Lawrence's student, [Stanley Livingston](#), built a "4-inch" version in brass. Clear evidence of [magnetic field resonance](#) was found in November, and [in January 1931 they measured 80-keV protons](#).

Ions were produced from the residual gas by a heated filament at the centre. Note the liberally applied red sealing wax for vacuum tightness - and Glenn Seaborg's left hand.

THE 27-INCH (LATER 37-INCH) CYCLOTRON



Livingston (left) is said to have grumbled: "Lawrence got the Nobel Prize - and I got my Ph.D." - but it was awarded after just 8 months' research! Most of Berkeley's 1930s nuclear physics was performed with this machine.

TRANSVERSE FOCUSING

In accelerators and beam transport systems, "focusing":

- does **not** generally refer to creation of a **point focus**;
- it means **keeping a beam of neighbouring particles together** by an E and/or M field distribution that provides **restoring forces**, leading to **stable orbits**.

Similarly, "defocusing" refers to situations with **unstable orbits**.

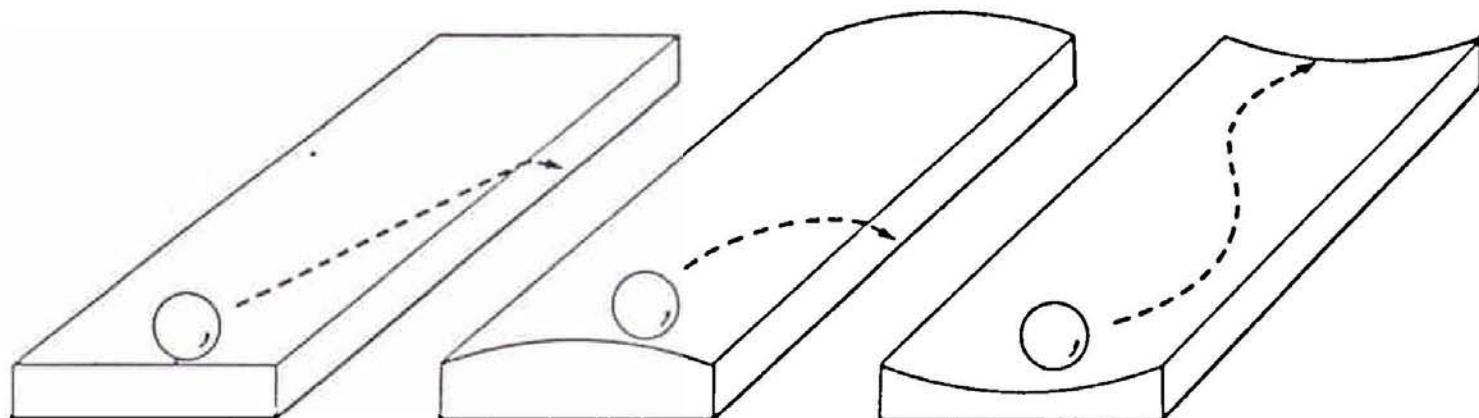
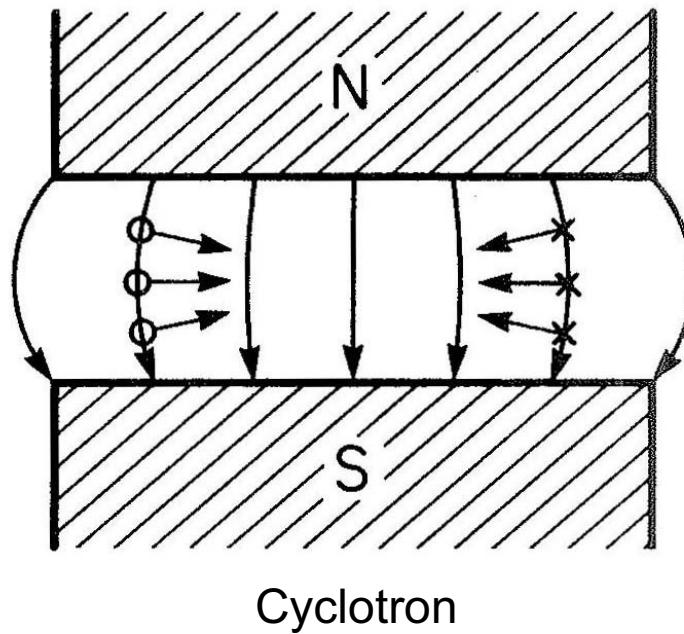
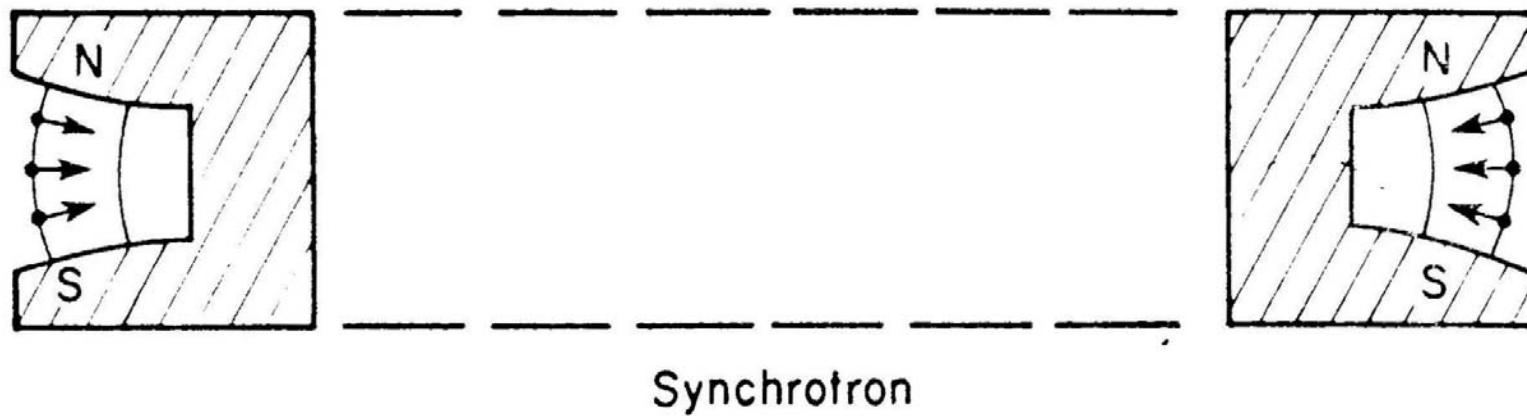


Fig. 2-1. Bowling alley analogy of systems of forces which produce neutral, unstable, or stable orbits.

WEAK (UNIFORM-GRADIENT) FOCUSING



Cyclotron



Synchrotron

An outwardly-decreasing (negative-gradient) field \Rightarrow vertical focusing.

MAGNETIC GRADIENT FOCUSING - AXIAL

Taking (r, θ, z) coordinates about the mid plane,

the axial (usually vertical) force $F_z = e v B_r$

& by Taylor expansion

$$= e v (\partial B_r / \partial z) z$$

& since $\text{curl } B = 0$

$$= e v (\partial B_z / \partial r) z .$$

Since F_z is linear in z we expect SHM - "betatron oscillations"

and identify

$$- m \omega_z^2 = e v (\partial B_z / \partial r)$$

The focusing strength is described by the "tune" v_z [Q_z in Europe]

- i.e. the number of oscillations per turn:

$$v_z = \frac{\omega_z}{\omega} = \sqrt{\frac{-e v}{m \omega} \frac{\partial B_z}{\partial r}} = \sqrt{\frac{-r}{B_z} \frac{\partial B_z}{\partial r}} \equiv \sqrt{-k}$$

where the logarithmic field gradient k is called the "field index".

Note that:- locally $B_z \sim r^k$

- many authors use $n \equiv -k$

- positive axial focusing requires $k < 0$ (B decreasing with r)
 - provided naturally by B fall-off towards pole edge.

For cyclotrons there are conflicting requirements:

- axial focusing requires $k < 0$, i.e. B_z falling with r ;
- fixed frequency $\omega = qB_z/m$ requires $B_z \propto m \propto E$, so rising with r !

If we choose isochronism, then $r \propto v \equiv \beta c$, $B_z \propto m = \gamma m_0$, resulting in axial defocusing:

$$v_z^2 = -k = -\frac{\beta}{\gamma} \frac{d\gamma}{d\beta} = -\beta^2 \gamma^2.$$

MAGNETIC GRADIENT FOCUSING - RADIAL

For radial motion (usually horizontal) we find:

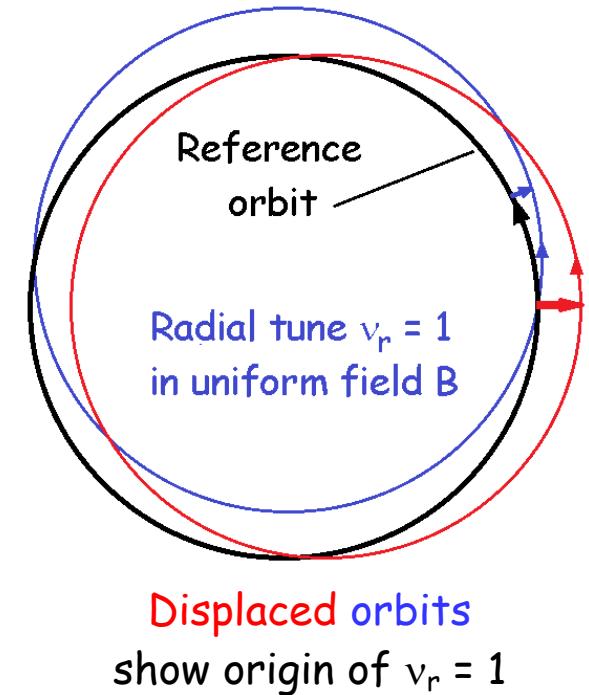
$$v_r = \sqrt{1+k} \quad \text{cf. } v_z = \sqrt{-k}$$

[$\text{curl } B = 0 \rightarrow$ complementary r & z focusing]

∴ For stability in both planes:

$$-1 < k < 0$$

- negative as expected,
but only allowing "weak focusing": $0 < v < 1$
 - <1 oscillation/turn !

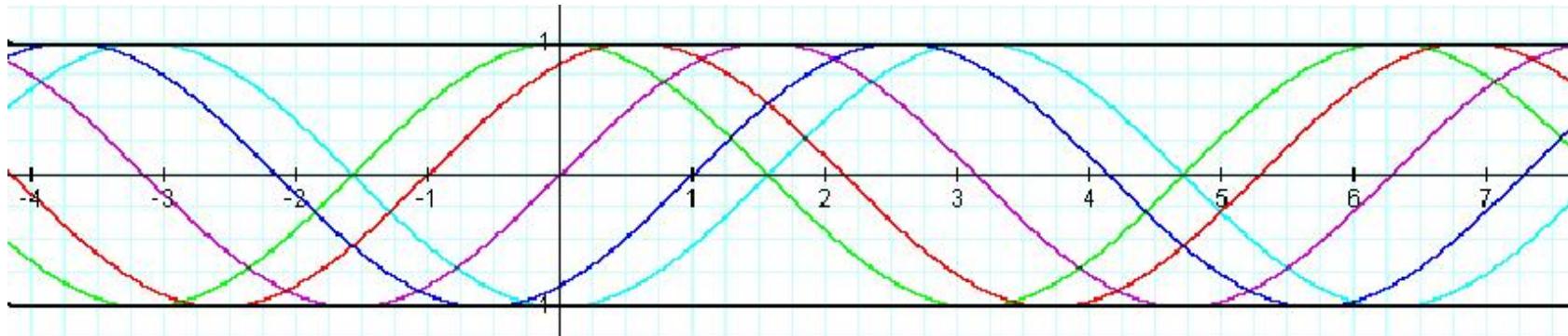


BETATRON OSCILLATIONS & EMITTANCE

With **uniform focusing**, charged particles will follow **sinusoidal paths** with **amplitude A_u** and **phase δ** (where u stands for transverse x or z):

$$u = A_u \cos(\nu_u \theta + \delta)$$

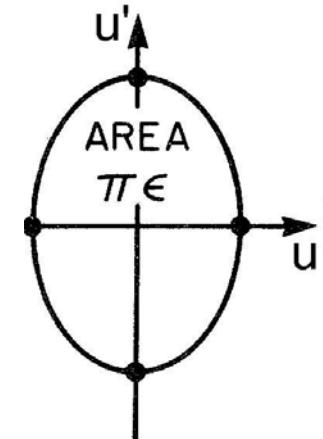
An aperture of half-height A can therefore accommodate a **beam** of particles with **any phase δ** and **all amplitudes $A_u \leq A$** , oscillating within a uniform envelope. [N.B. only $A_u = A$ orbits are shown in the sketch.]



Note that the divergence $u' = \frac{du}{rd\theta} = \frac{-\nu_u A_u}{r} \sin(\nu_u \theta + \delta)$.

So in $u-u'$ phase space, particles move in **elliptical paths** with $A_u \leq A$, whose **overall area** - a useful measure of **(range of displacement) \times (range of divergence)** - is known as the beam's "**emittance**"

$$E = \pi \epsilon = \pi \nu_u A^2 / r.$$



RELATIVISTIC PROBLEMS

Lawrence's group built a series of cyclotrons of increasing size in the 1930s, culminating in the 60-inch "Crocker" (10-MeV p, 20-MeV d, 40-MeV α) - and were imitated in labs all over the world.

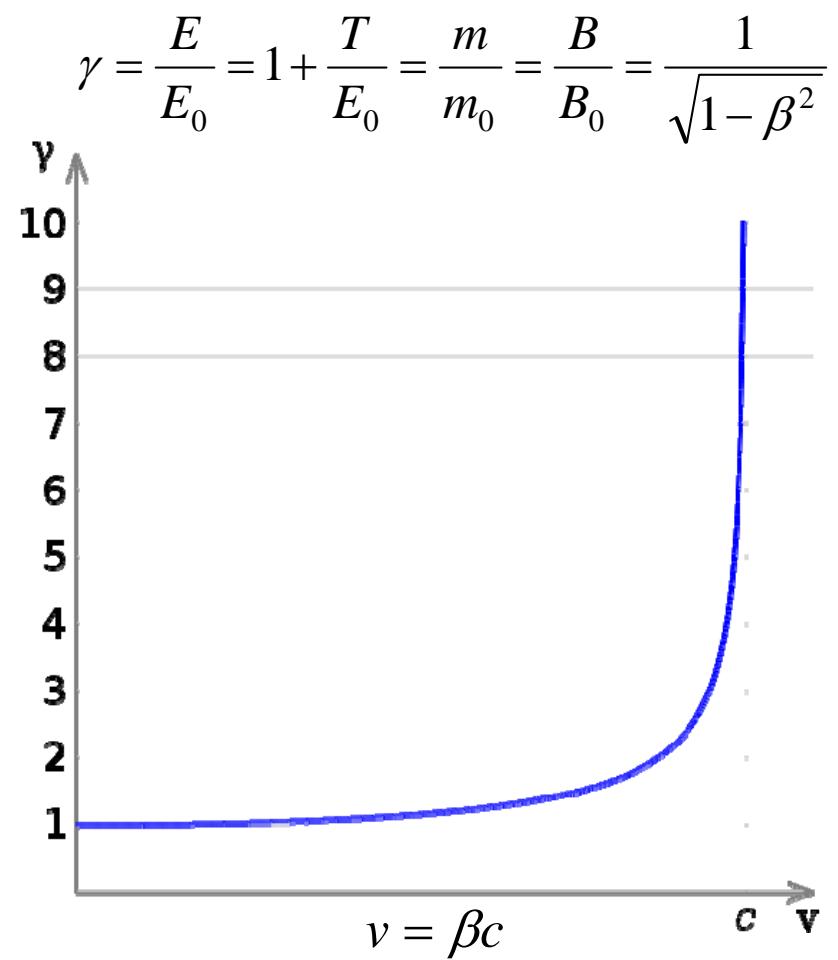
But the classical cyclotron quickly became a victim of its own success.

Special relativity tells us that $E = mc^2$

- so increasing E increases m , and in uniform B the angular frequency qB/m falls. Bethe & Rose (1937) predicted an 8-MeV limit for D^+ with $V_{rf} = 50$ kV.

[Isochronism might be recovered by making $B(r) \propto m(r)$

but a positive field gradient dB/dr
--> unacceptable vertical defocusing.]



SYNCHRONOUS ACCELERATION

Earliest options for higher E involved abandoning isochronism and

- Allowing frequency $\omega = q B/m$ (and ω_{rf}) to vary
- at the price of pulsed, rather than continuous operation
- and hence beam currents reduced $\times 1/1000$ to $\sim 0.1 \mu A$

Discovery of the Phase Stability Principle (Veksler '44, McMillan '45) gave confidence that particles would remain in phase with the rf.

Two approaches:

- constant B, changing r \rightarrow synchrocyclotron
- constant r, changing B \rightarrow synchrotron

SYNCHROCYCLOTRONS

Keep $B \approx \text{constant}$

$\therefore \omega \propto 1/m \propto 1/E$ - frequency decreases with E ;

and $r \propto v/\omega \propto vE$ - radius increases with E .

Demo by Richardson et al. (1946) on 37" cyclotron at Berkeley;

First full size was Berkeley 184" (190 MeV d^+ , 1946; later 700 MeV p)

Biggest : Gatchina : 1000 MeV, 7800 tons, 6.85 m pole diameter.

SYNCHROTRONS

Radius $r = \text{constant}$

$\therefore \omega = v/r$ increases to c/r

$B \propto m \omega \propto E v$ increases

(small diameter beam pipe

(ring of magnets

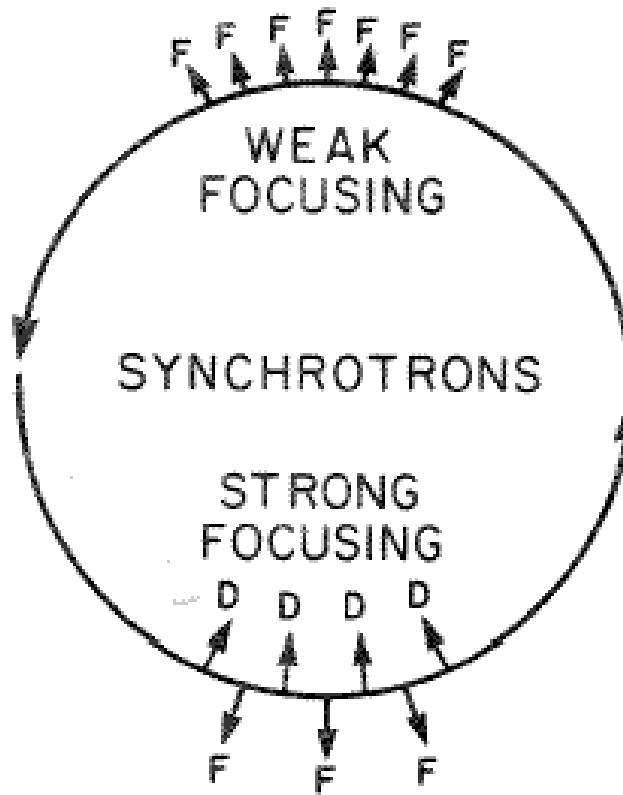
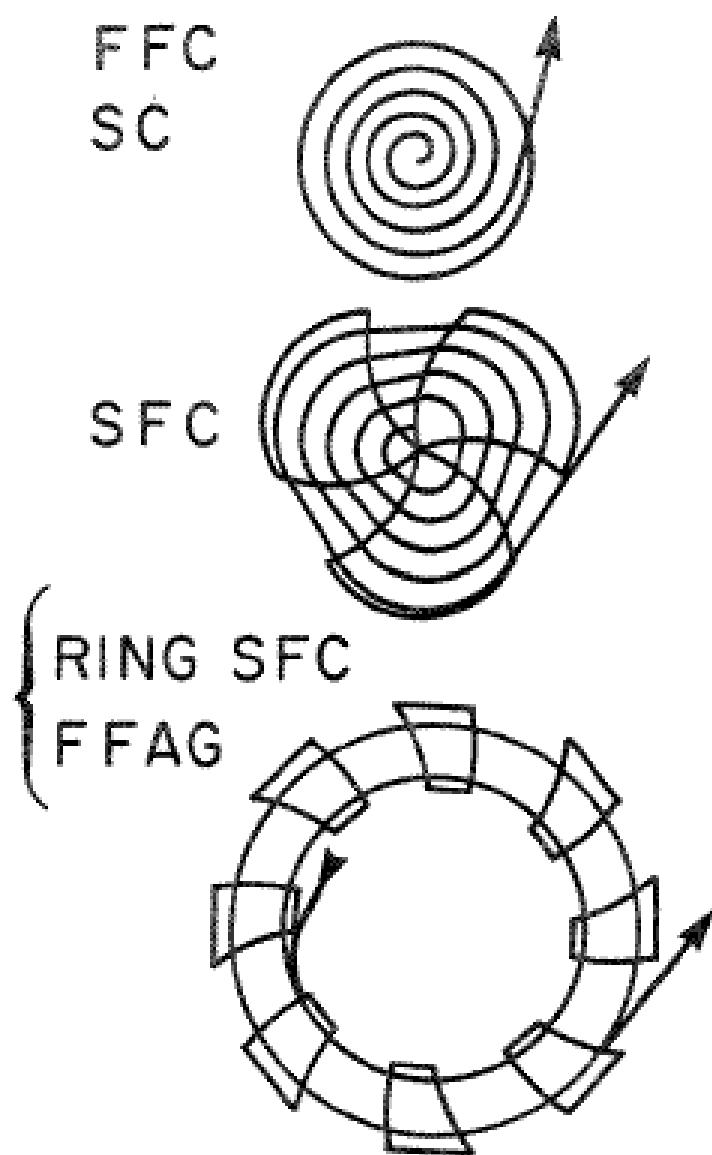
(ramping B is slow

First for e^- : Berkeley 300 MeV (1949)

First for p : Brookhaven 3 GeV Cosmotron (1952)

Biggest weak focusing synchrotron: Dubna, 10 GeV, 36000 tons

THE CYCLOTRON AND SYNCHROTRON FAMILIES



FFC = fixed frequency cyclotron
SC = synchrocyclotron
SFC = sector-focused cyclotron
FFAG = fixed field alternating gradient

THE 184-INCH SYNCHROCYCLOTRON



The Berkeley 184" was begun in 1939 as a classical cyclotron, to be operated with $V_{rf} = 1$ MV, but WWII interrupted rf installation and it was used to test mass spectrographic separation of uranium isotopes. FM rf was installed in 1946, yielding 190 MeV d^+ (700 MeV p in 1959).

SUPERCONDUCTING SYNCHROCYCLOTRONS (1)

A recent growth of interest in proton therapy - and desire to cut costs - has led to proposals for **superconducting synchrocyclotrons**:

higher magnetic fields

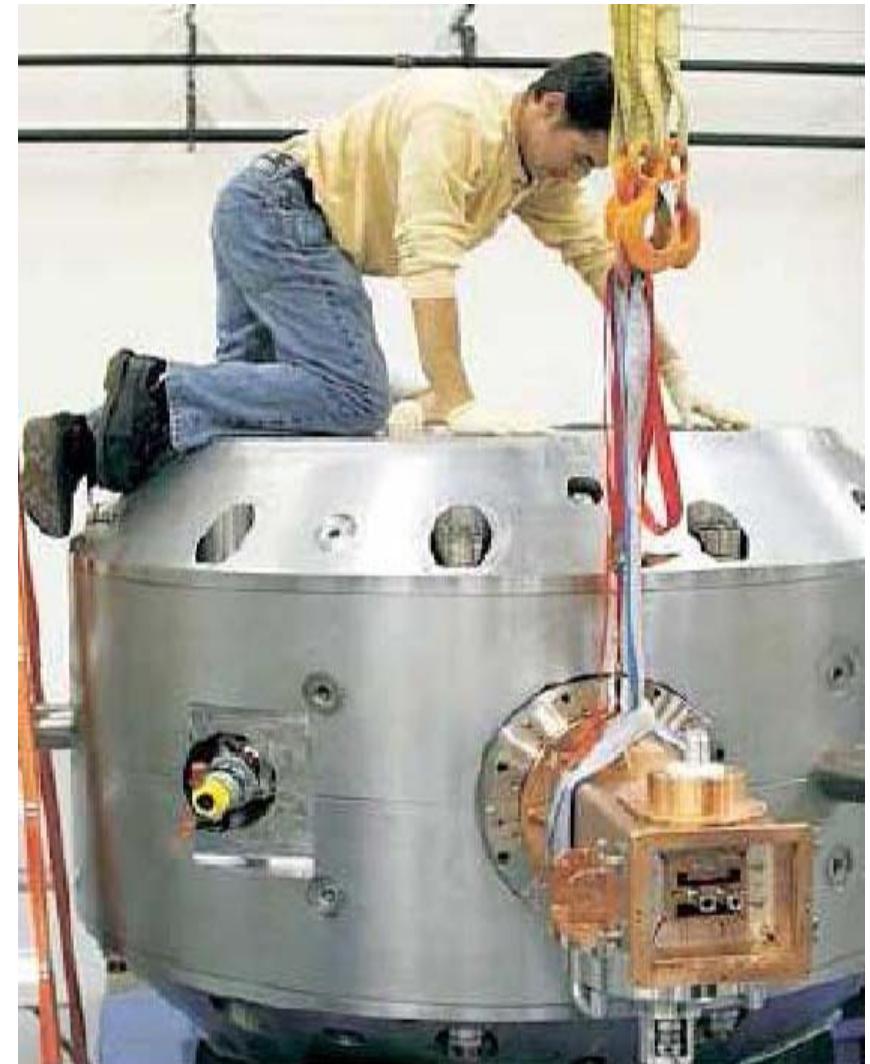
→ smaller radii r

→ lighter magnets $\propto r^3$.

Using **Nb_3Sn coils**, Tim Antaya (MIT) has built a **cryogen-free 9-T magnet** for the **Mevion** (formerly Still River) **S250 synchrocyclotron** - light enough to mount on a **rotating gantry**!

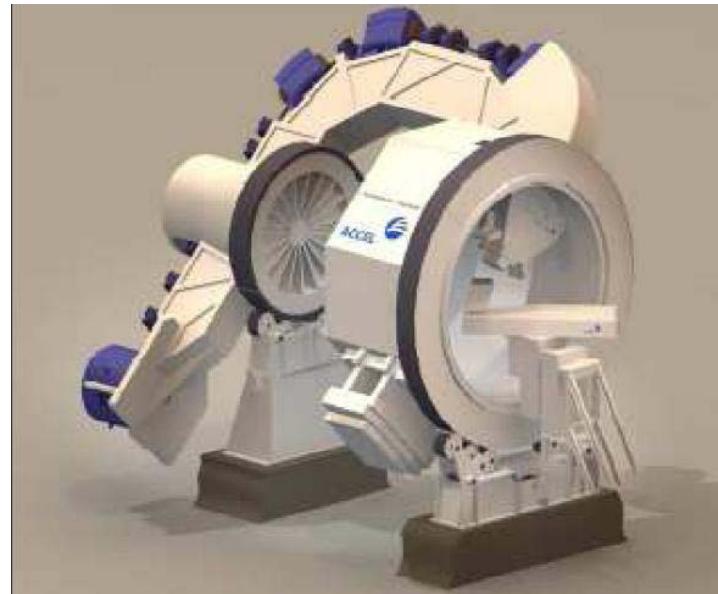
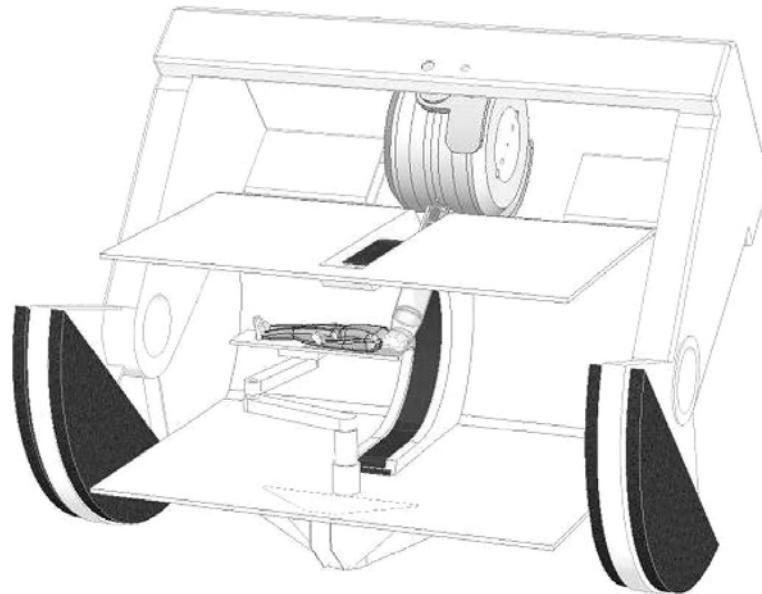
Compactness makes this challenging!

The **turn separation at 250 MeV** is **only $6 \mu m$** - so beam extraction relies on amplifying this to 1 cm by a local field perturbation! **One is in operation** (St. Louis), another delivered (NJ).



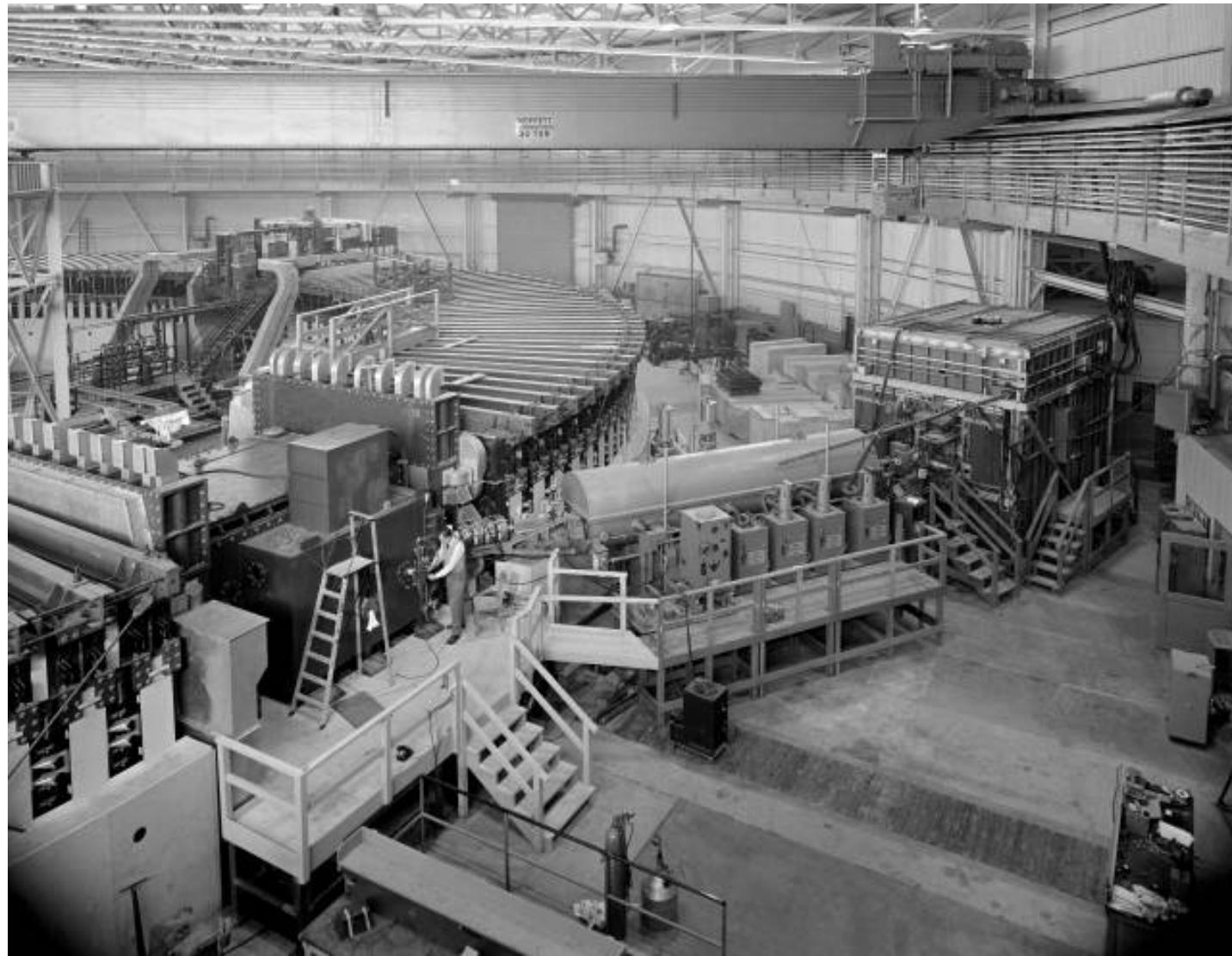
SUPERCONDUCTING SYNCHROCYCLOTRONS (2)

The **Mevion SC** is light enough to mount on a rotating gantry, and the whole system fits in a single room - but there are concerns about patients' exposure to stray radiation.



An alternative scheme adopted by **Varian-Accel** and **IBA** would place the **S-C** in a separate room, with beam delivery via a rotating gantry. The **IBA** design, with standard **NbTi coils**, achieves 5.6-T fields, and so is larger than Mevion's. Beam commissioning is under way. A superconducting **SC** is also planned for the **Italian CycLinac** project.

WEAK-FOCUSING SYNCHROTRONS - THE 6-GeV BERKELEY BEVATRON



Orbit radius = 19 m Magnet weight = 9,700 tons Pulse interval = 5 s
Note the 10-MeV linac injector, fed by a Cockcroft-Walton HT set.

PART 2

1. Sector Focusing - Radial-Sector Cyclotrons
2. Strong (i.e. Alternating) Focusing
3. Alternating-Gradient Synchrotrons
4. Spiral-Sector Cyclotrons
5. Fixed-Field Alternating-Gradient Accelerators
 - Scaling FFAGs
 - Non-Scaling FFAGs

SECTOR-FOCUSED CYCLOTRONS - I

Radial-sector pole pieces are used to give an azimuthal variation in field (AVF) strength - $B_h - B_v - B_h - B_v$ - and so a "scalloped" orbit, where ions cross the sector edges at a small "Thomas angle" κ

$$\kappa = \frac{\pi}{N} \frac{(B_h - \bar{B})(\bar{B} - B_v)}{(B_h - B_v)\bar{B}},$$

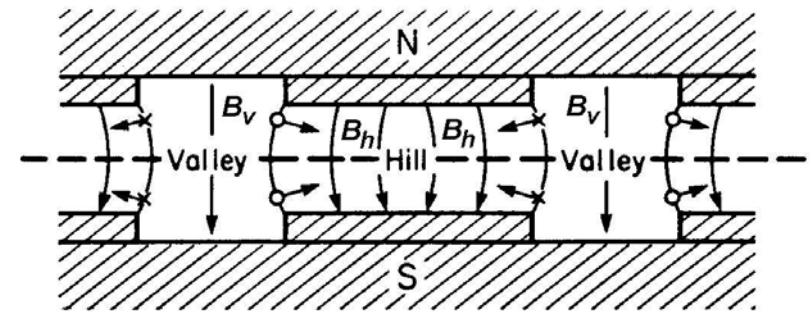
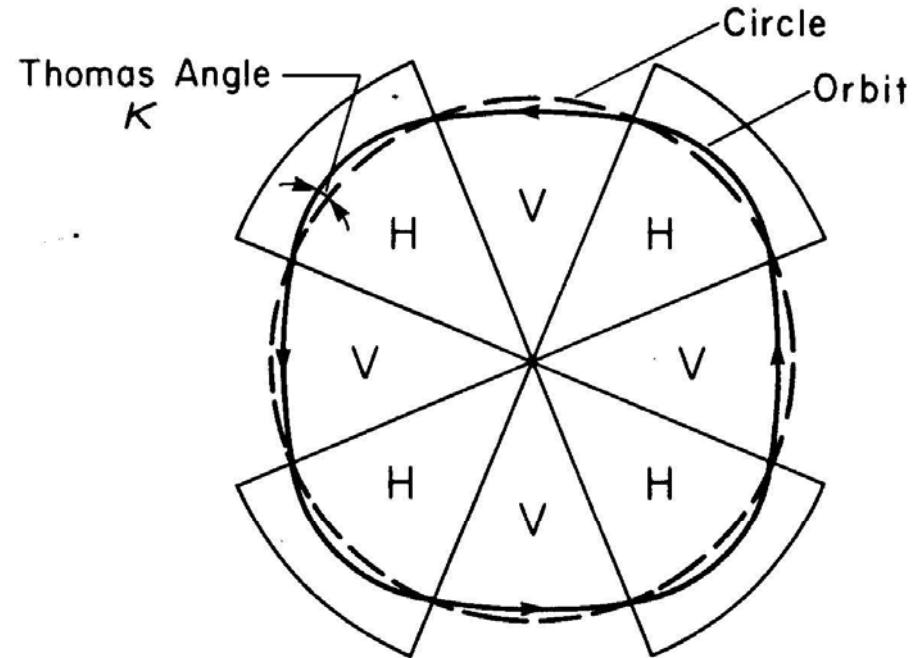
each time experiencing axial edge focusing - always positive.

Defining the magnetic "flutter" by

$$F^2 \equiv \left\langle \frac{(B - \bar{B})^2}{\bar{B}^2} \right\rangle = \frac{(B_h - \bar{B})(\bar{B} - B_v)}{\bar{B}^2},$$

and averaging the small impulses over the orbit, we find a contribution to the axial tune: $\Delta\nu_z^2 = F^2$.

Overall, for an isochronous magnetic field: $\nu_z^2 = -\beta^2\gamma^2 + F^2$, so that $\nu_z^2 > 0$ provided $F^2 > \beta^2\gamma^2$.



SECTOR-FOCUSED CYCLOTRONS - II

L.H. Thomas in 1938 was the first to show that AV Fields could be used to counter isochronous defocusing - proposing a $\cos N\theta$ variation with $N=3$ or 4 .

(A Welsh-born American, probably better known for

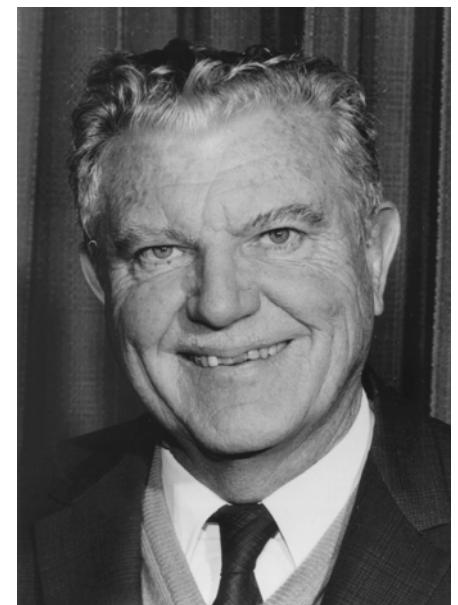
- Thomas precession
- Thomas-Fermi statistical model of the atom.)



His idea was neglected until Reg Richardson's team at Berkeley built two 3-sector electron models in 1950, reaching $\beta = 0.5$. Not declassified till 1956!

(Born in Edmonton, schooled in Vancouver & L.A.,

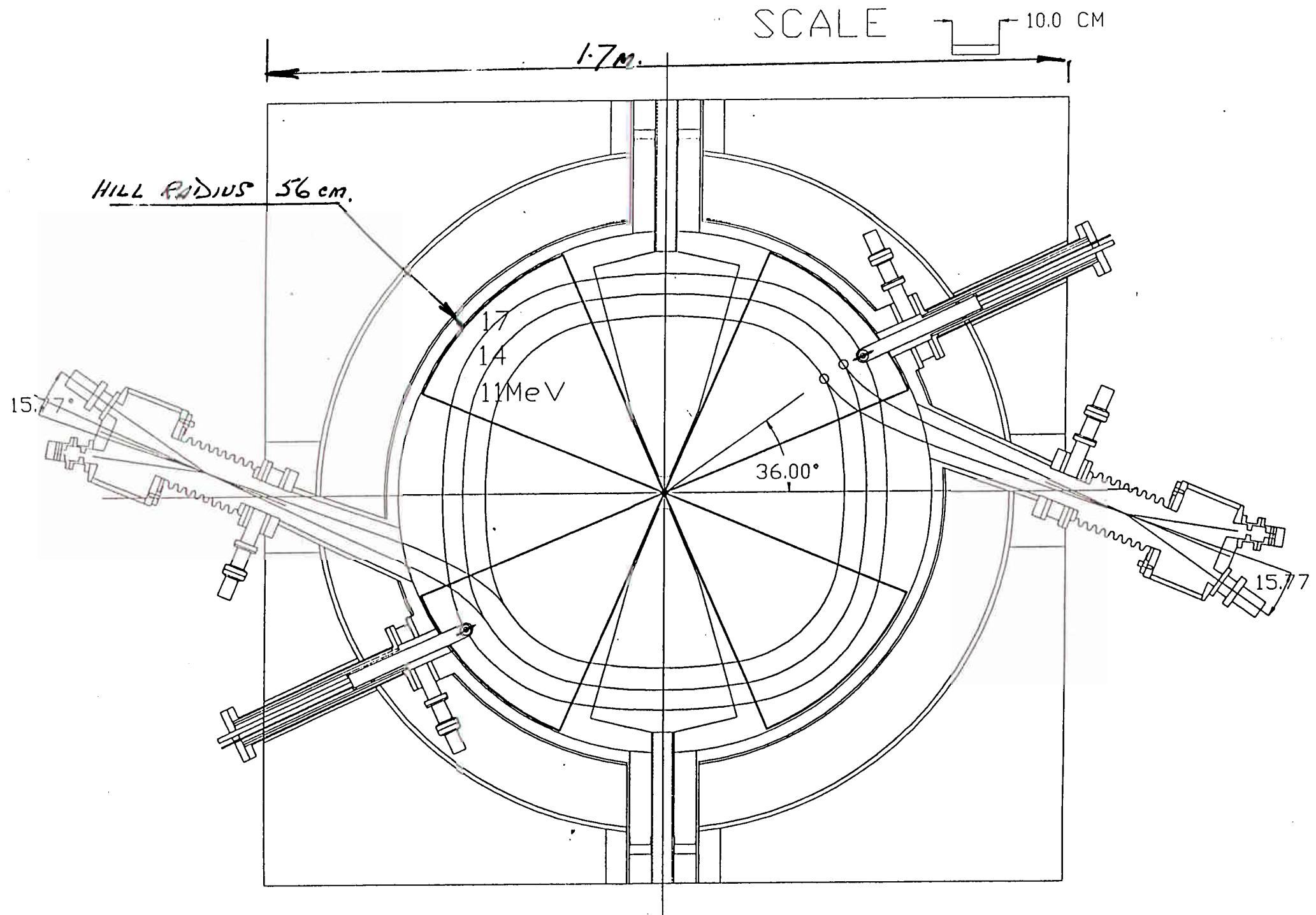
- first demonstration of phase stability
- first synchrocyclotron
- UCLA 50-MeV sector-focused cyclotron
- TRIUMF 500-MeV cyclotron meson factory.)



THE FIRST SECTOR-FOCUSING CYCLOTRON

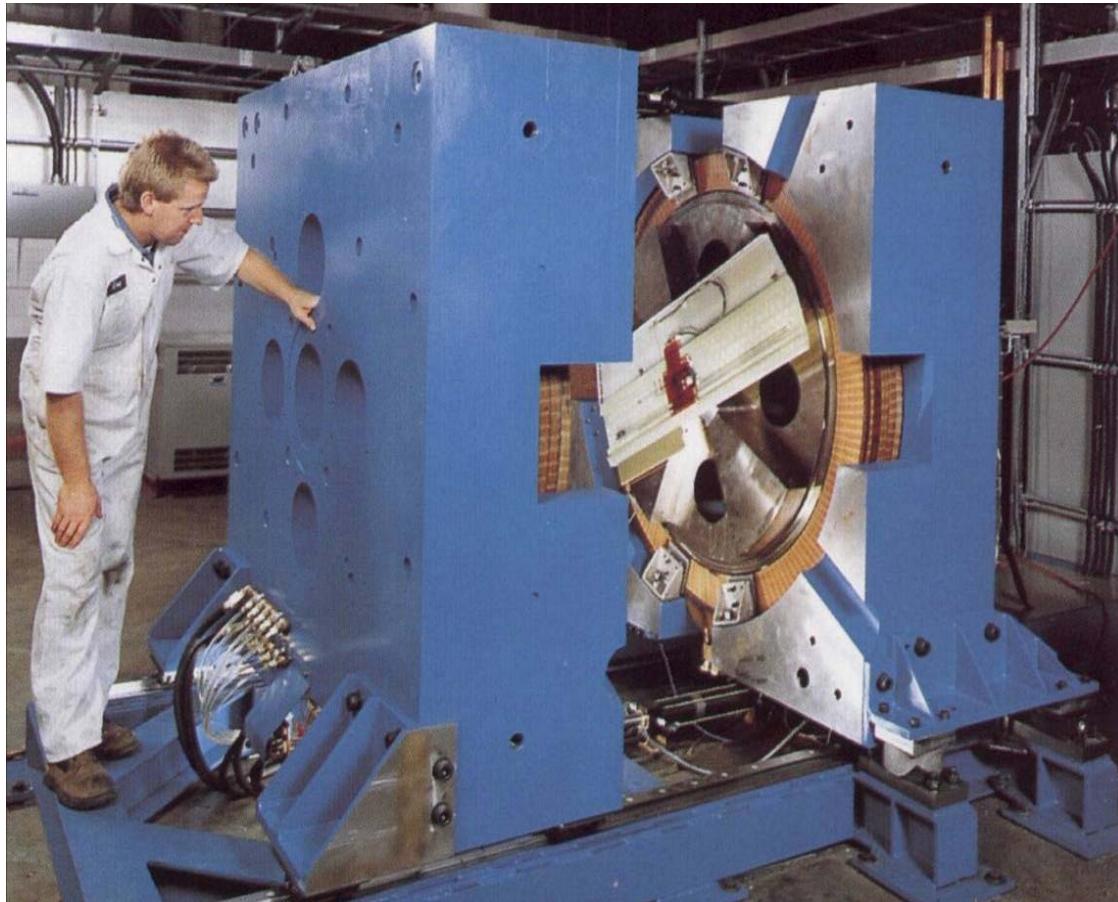


- One pole of a model built by J.R. Richardson et al. at Berkeley in 1950
- Electrons were accelerated to $\beta = 0.5$ (simulating 300-MeV deuterons)
- Based on L.H. Thomas's 1938 proposal
- Note the harmonic pole profile (à la Thomas)



TR13 General Layout

TR14/19 CYCLOTRON



The TR14/19 (originally TR13) accelerates $300 \mu\text{A}$ H^- to 14-19 MeV or D^- to 7-9.5 MeV. [Note the field-measuring apparatus.]

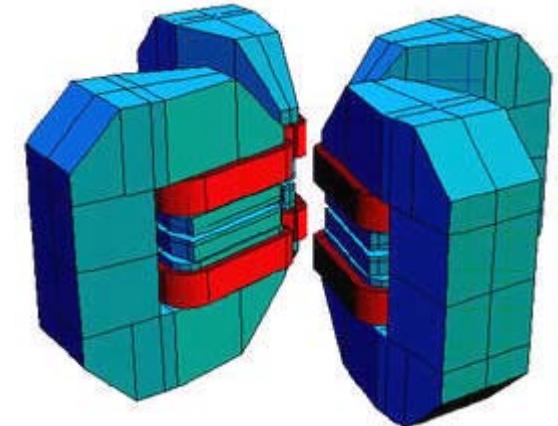
These energies are sufficient to produce the short-lived tracer isotopes needed for PET scans, such as ^{11}C ($\tau_{1/2} \approx 20 \text{ m}$), ^{13}N (10 m), ^{15}O (2 m) and ^{18}F (110 m), elements common in biological molecules.

RADIAL-SECTOR CYCLOTRONS

- There are ~800 small radial-sector cyclotrons (≤ 50 MeV)
- Mostly used for producing radioisotopes
- Hard to achieve $F^2 > 0.3$ with single poles and coils.

In separate-sector cyclotrons:

- the sectors have individual yokes and coils
- the valleys are:
 - magnetic field-free
 - available for rf, injection, extraction & diagnostics
- the flutter $F^2 = H^{-1} - 1$ can reach ≈ 1 (where H = hill fraction), making it possible to reach $\beta\gamma \approx 1$ (≈ 400 MeV/u).

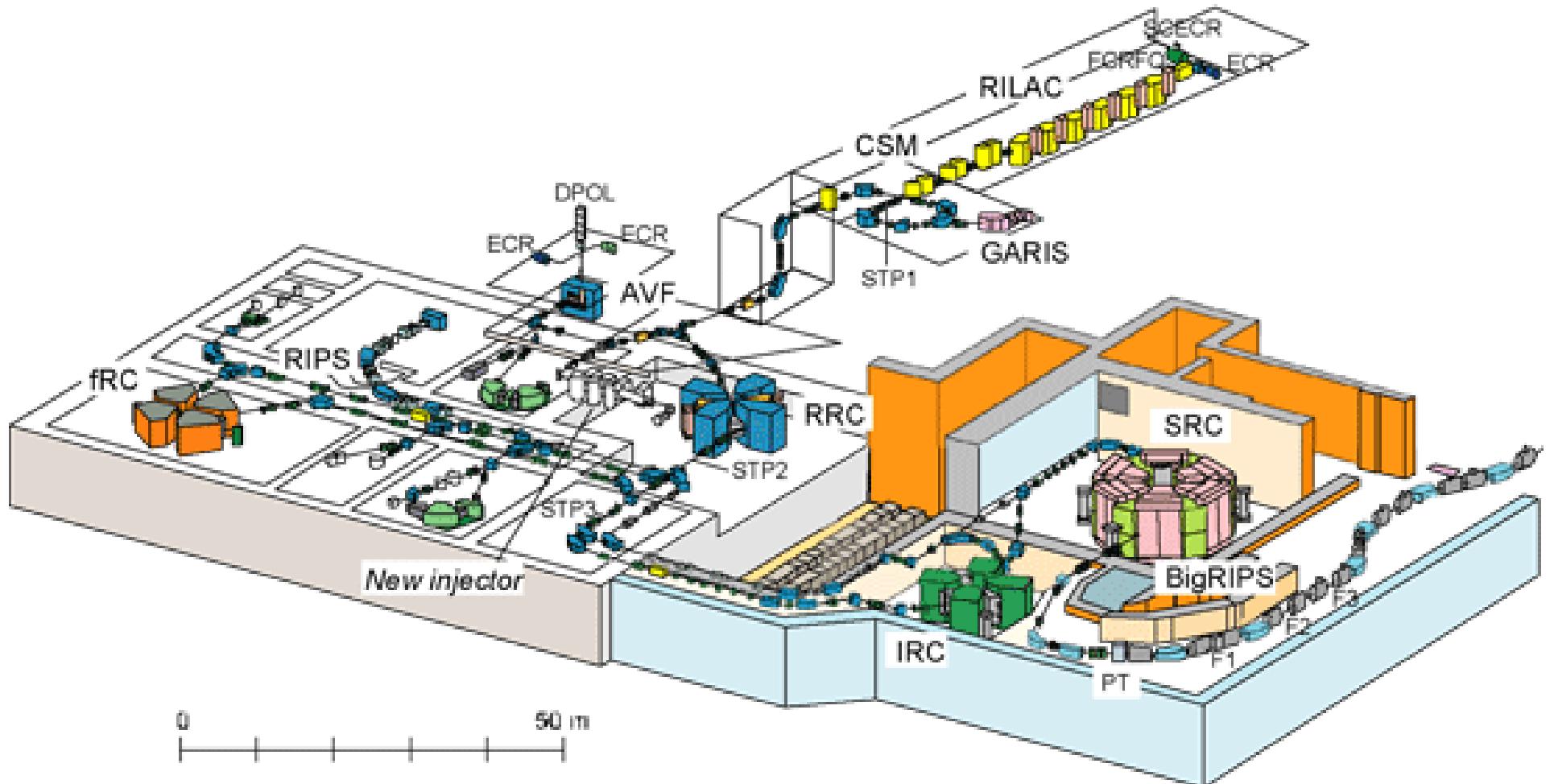


There are ~12 large separate-radial-sector cyclotrons:

- mainly designed for heavy ions (where high β is not desired)
- with energy capability up to $K = 2600$ (345 MeV/u U^{86+}),

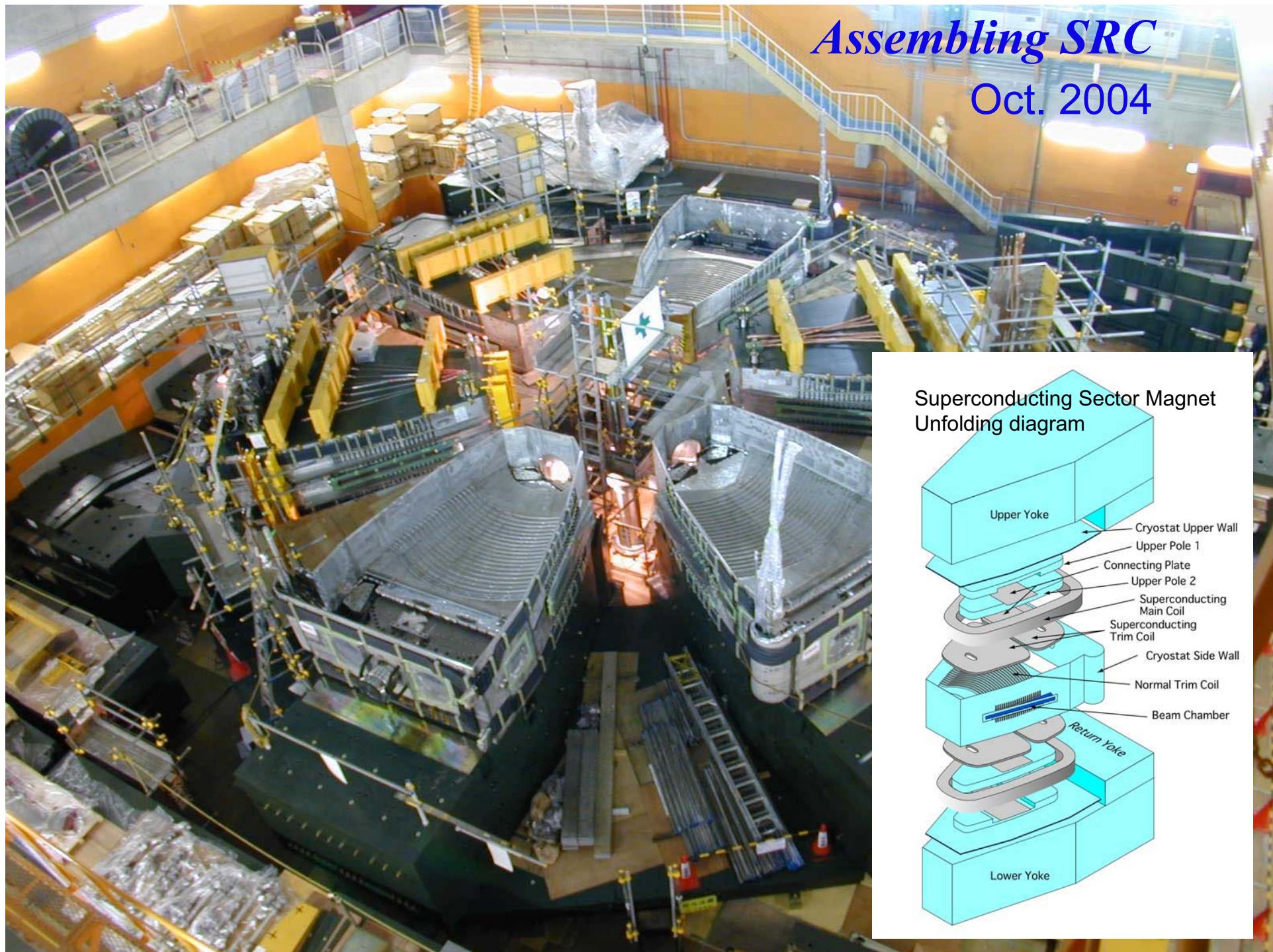
- where
$$K = \frac{(eB\rho)^2}{2m_u}$$
 and
$$\frac{T_{\max}}{A} = K \left(\frac{Q}{A} \right)^2$$
.

The RIKEN RadioIsotope Beam Factory (RIBF)

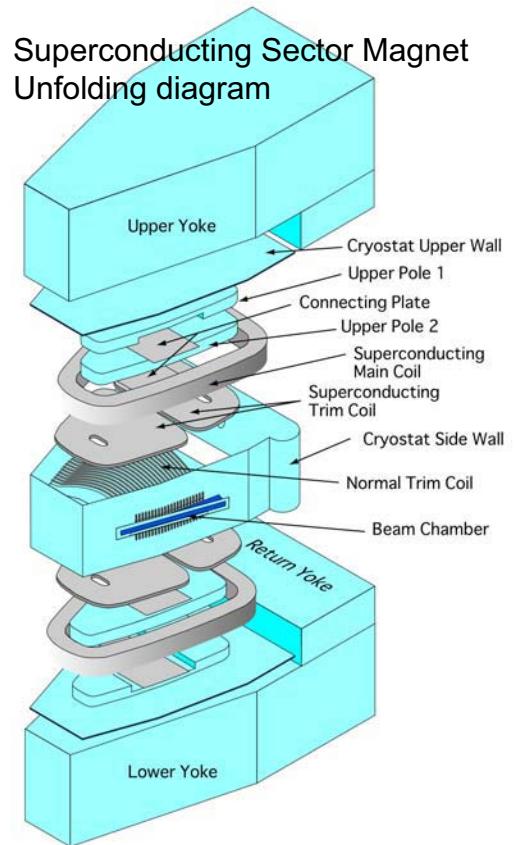


The RIBF involves a **chain of one linac and 5 cyclotrons**. It can deliver **345-MeV/u beams over the mass range $A = 50 - 92$** .

Of the **3 new ring cyclotrons**, the most novel and challenging - and the world's most massive - and most powerful - is the **8300-ton SRC**.



Superconducting Sector Magnet
Unfolding diagram



K2600-MeV SRC



On Nov. 7 2005 full excitation of sector magnets achieved.
A 140-ton cold mass cooled down to 4.5 K for 3 weeks.

"STRONG" FOCUSING FROM ALTERNATING LENSES

With the first Weak-Focusing Synchrotrons came a potential problem: the maximum vertical betatron oscillation amplitude is set by the magnet aperture A : $z = A \cos(\nu_z \theta + \psi)$.

$$\therefore \text{the maximum divergence} \quad \hat{z}' = \frac{1}{r} \frac{\hat{d}z}{d\theta} = \frac{\nu_z A}{r}$$

and the beam current accepted \propto beam emittance

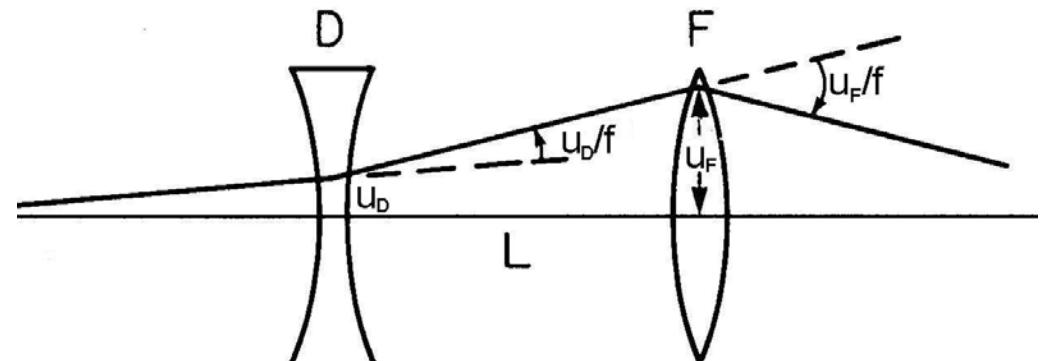
$$\pi A \hat{z}' = \frac{\pi \nu_z A^2}{r}.$$

So, as WFS (with $\nu_z \leq 1$) get larger, beam current decreases $\propto 1/r$.

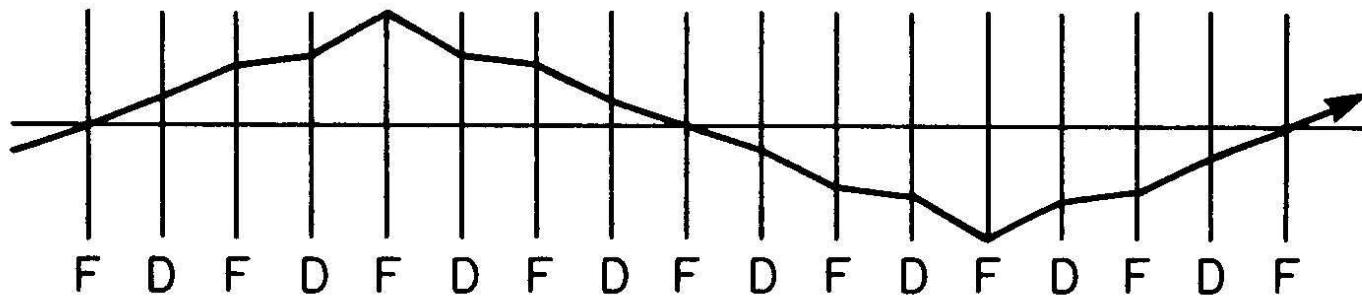
Solution is Alternating Gradient (AG) focusing:

Discovered by Christofilos (1950) & Courant, Livingston & Snyder ('52).

- alternating F and D lenses are overall focusing
- the deflexion ($\propto u$) is always greater at F than D.
- an old principle in optics.



ALTERNATING FOCUSING (continued)



N.B. - Net effect is focusing in both directions;

- it works for a chain of alternating F & D lenses;
- a vertically F gradient lens is horizontally D;
- and vice-versa - so it's focusing in both planes.
- no upper limit to lens strength $\rightarrow v_u > 1$

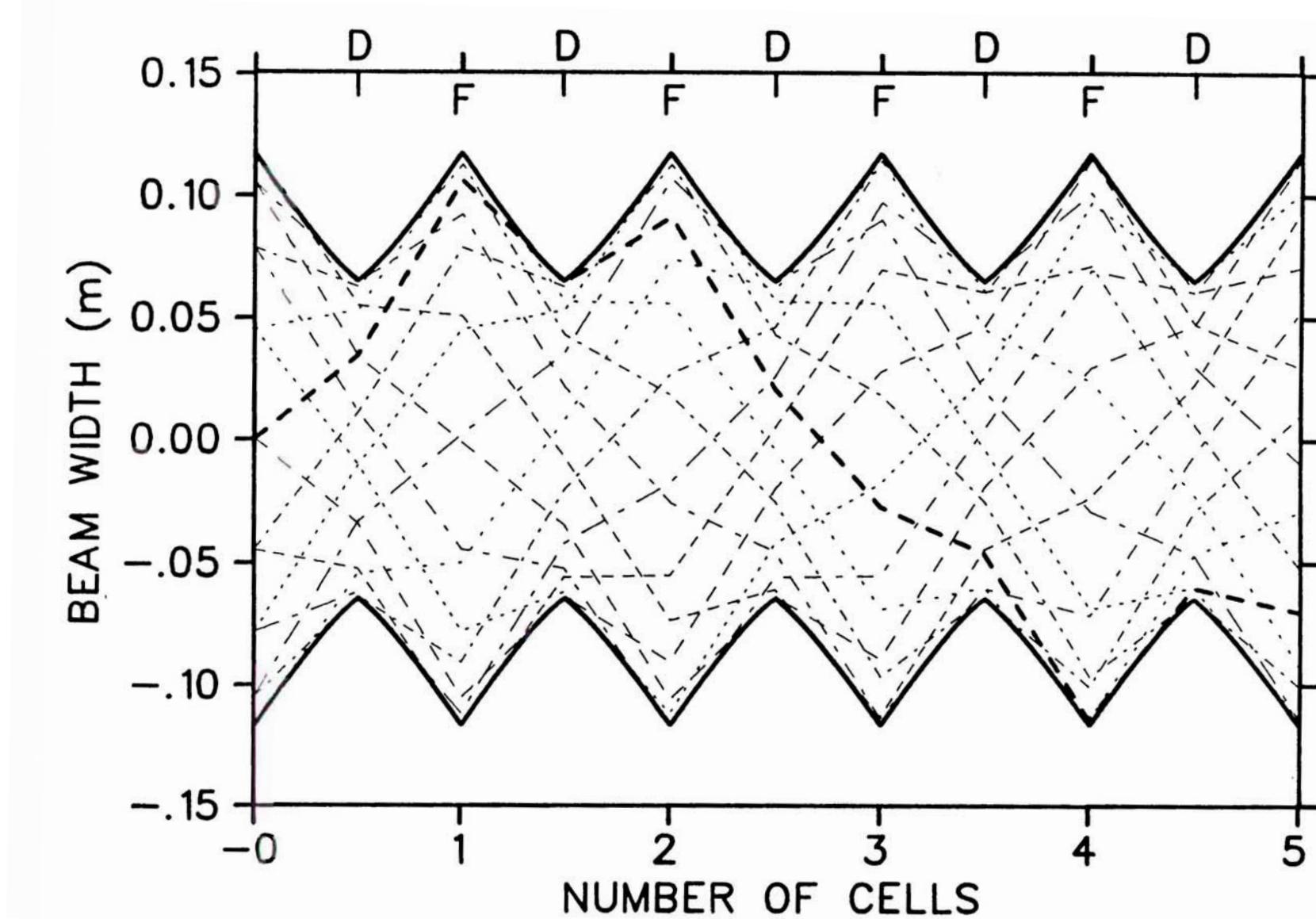
First used in "combined function" AG (bending and focusing) dipoles.

Now usually provided by "separated function" quadrupole magnets.

All high energy circular accelerators are now based on AG focusing.

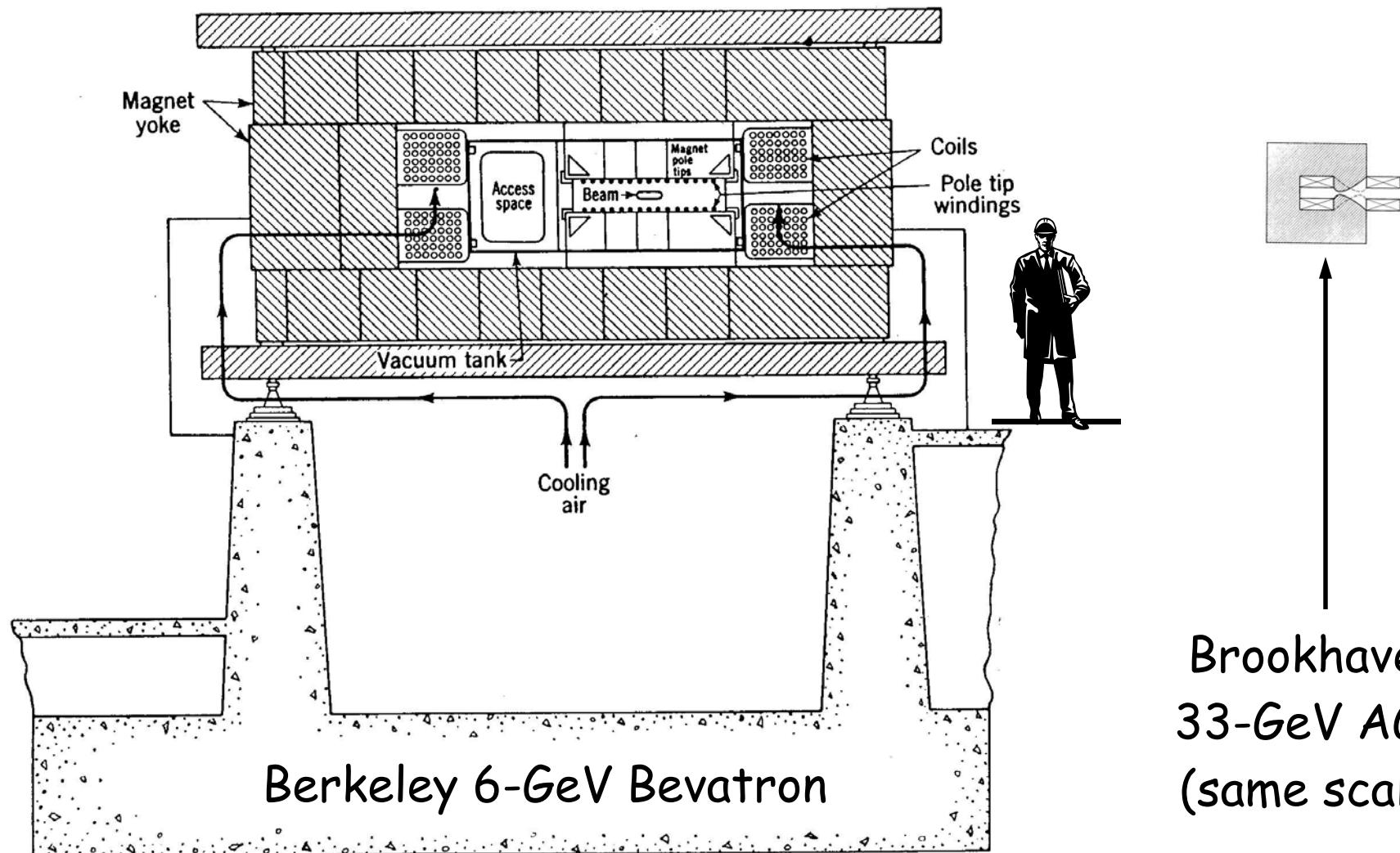
Note that transverse gradients are not essential

- it works for edge lenses too - as in spiral-sector cyclotrons.



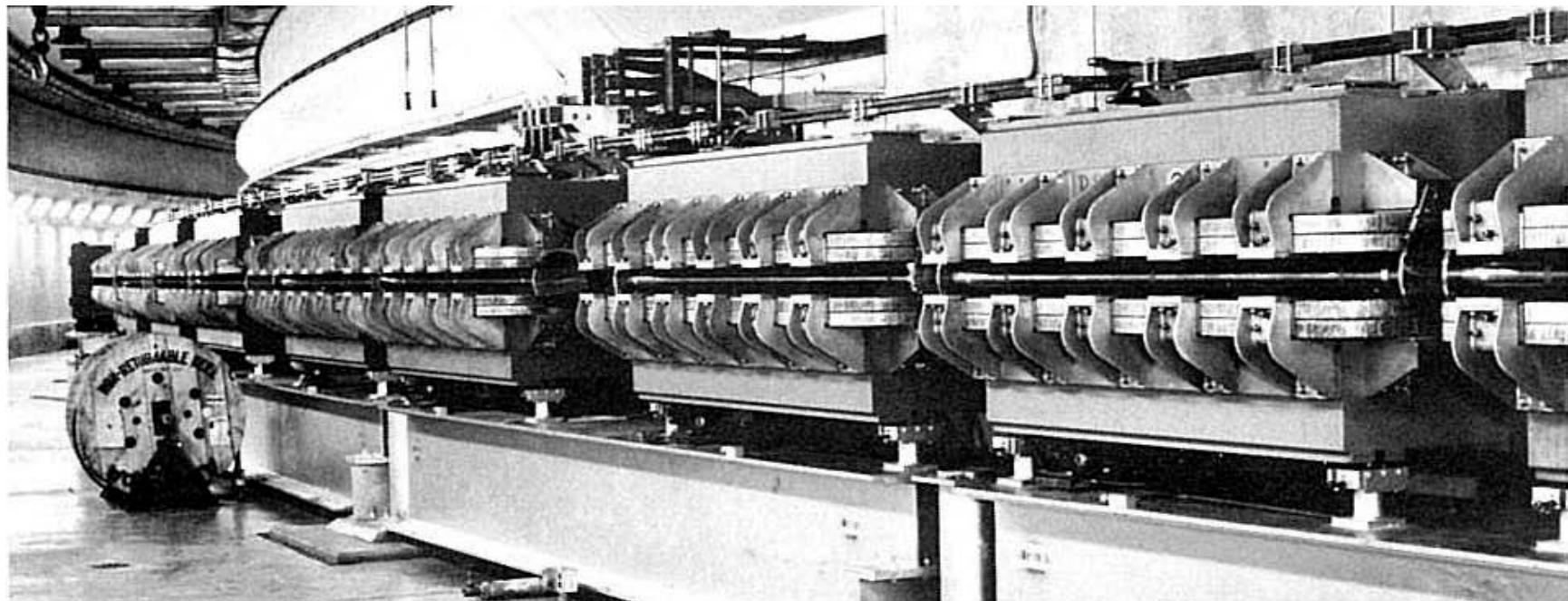
Alternating focusing in a FODO lattice, showing long wavelength betatron oscillations of various phases (but the same amplitude) within an overall envelope exhibiting the lattice periodicity.

WEAK & STRONG FOCUSING SYNCHROTRONS

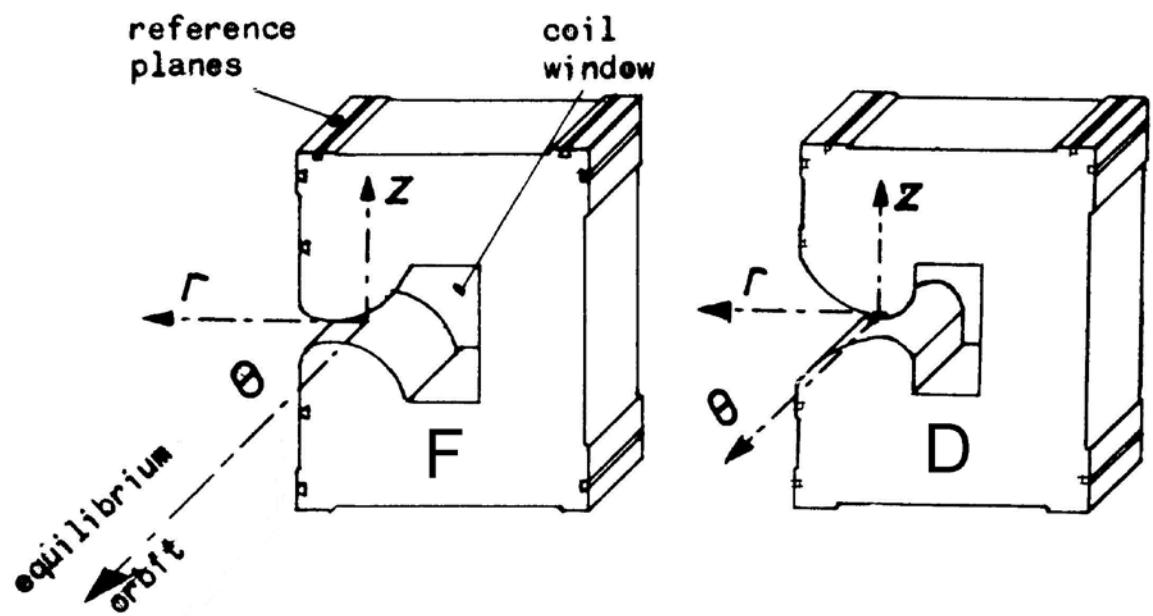


The tight focusing provided by high-gradient AG magnets enabled vacuum chambers and magnet cross-sections to be greatly reduced.

33-GeV BROOKHAVEN AG SYNCHROTRON



In the BNL AGS the horizontally focusing (F) and defocusing (D) magnets are arranged in pairs: -FF-DD-FF-DD-....



COMBINED FUNCTION - SEPARATED FUNCTION

The first AG synchrotrons used "combined-function" magnets:

- each magnet both bends and focuses the beam;
- wedge-shaped pole gap limits central field strength;
- fixed focusing - no tuning flexibility;
- lengthy magnets weaken alternating focusing effect.

Nowadays it's more usual to separate those functions:

- using dipole bending magnets with parallel polefaces - no focusing;
- and quadrupole focusing magnets - no bending;
- independent powering of quadrupoles allows changes in tune;
- space for extra components provided by stronger dipole fields;
- localized quadrupoles strengthen alternating focusing effect.

QUADRUPOLE MAGNET

$$B_x = g y$$

$$B_y = g x$$

$$|B| = g r$$

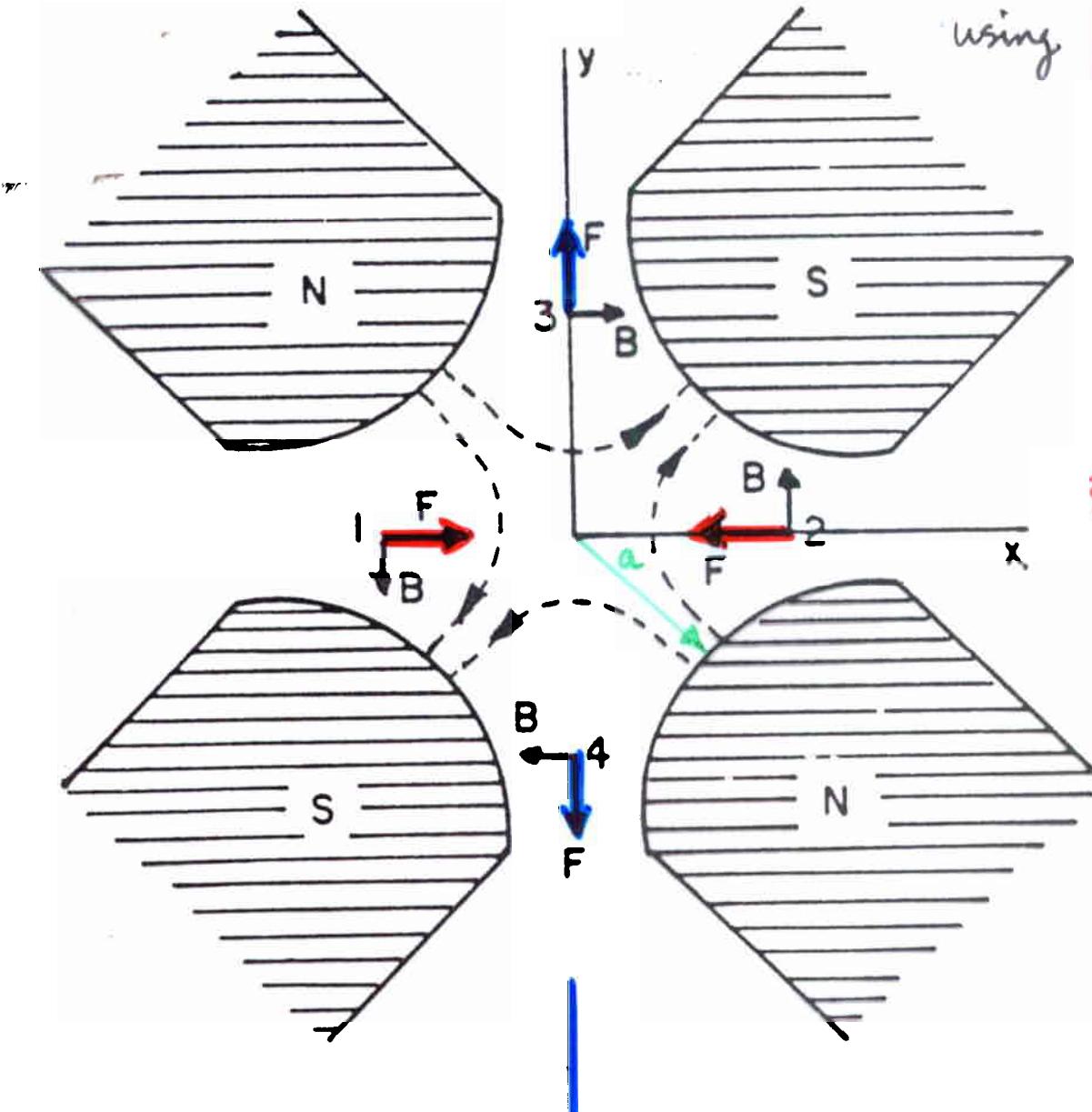
$$\frac{\partial B_x}{\partial y} = \frac{\partial B_y}{\partial x} = \frac{\partial |B|}{\partial r} = g = \frac{B_0}{a}$$

$$= B_0 @ r=a$$

- derivable from hyperbolic magnetic potential

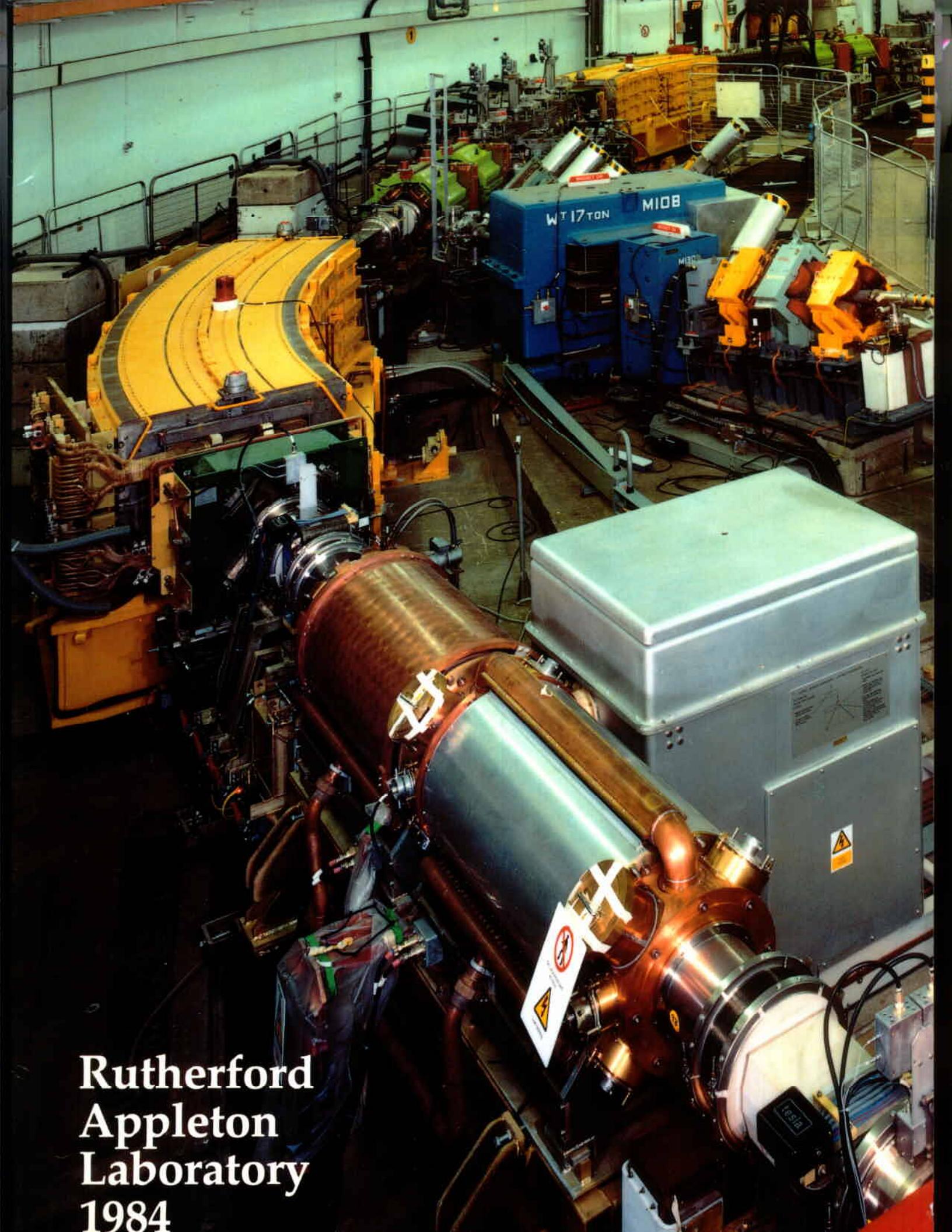
$$\phi = -g x y$$

$$\underline{B} = -\nabla \phi$$



Focusing
Plane

Defocusing
Plane



**Rutherford
Appleton
Laboratory
1984**

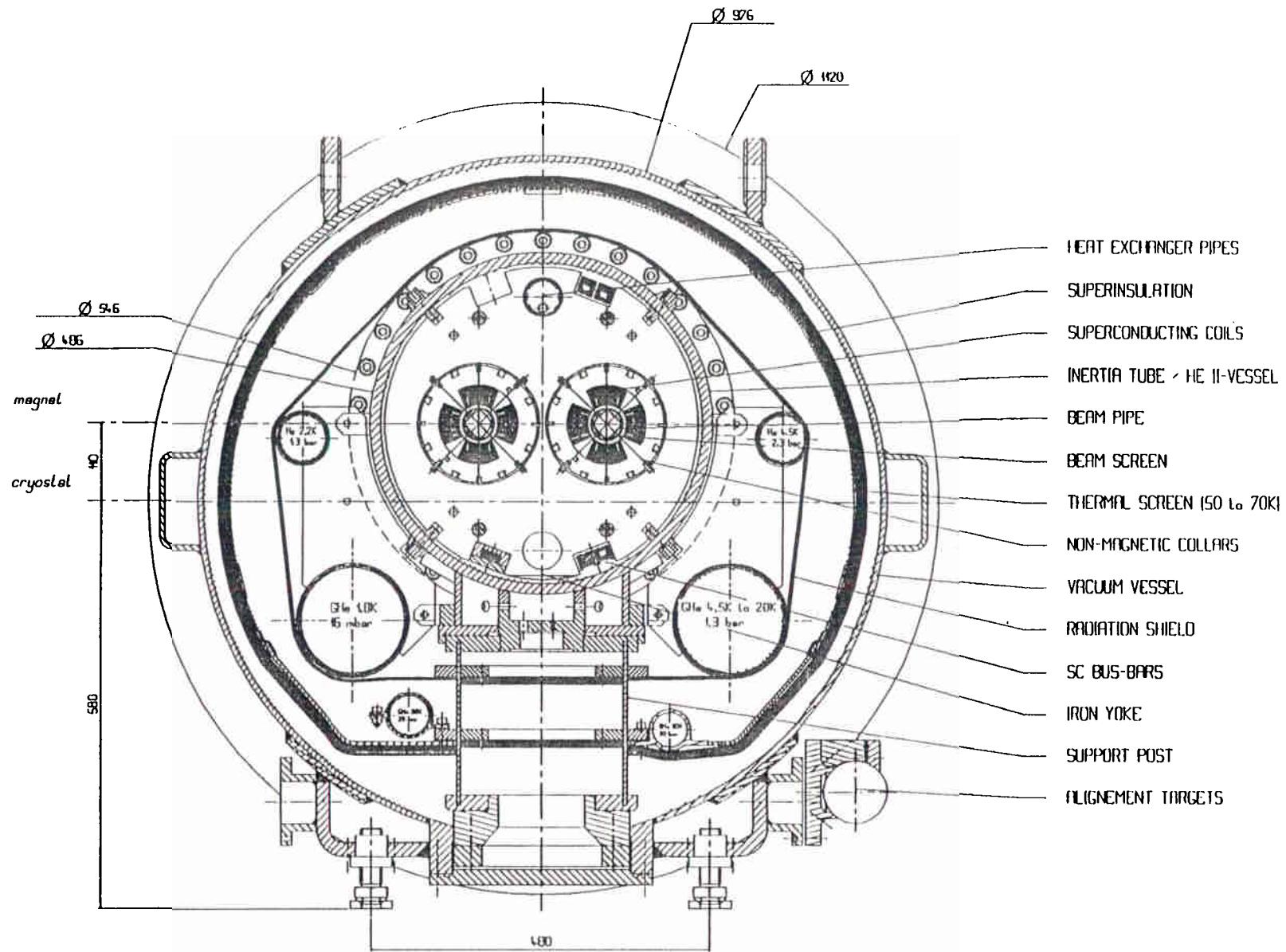


Fermilab - 450-GeV Main Ring (above) and 1-TeV Tevatron (below)



CERN, Geneva - The LHC and SPS tunnels and the Swiss-French border

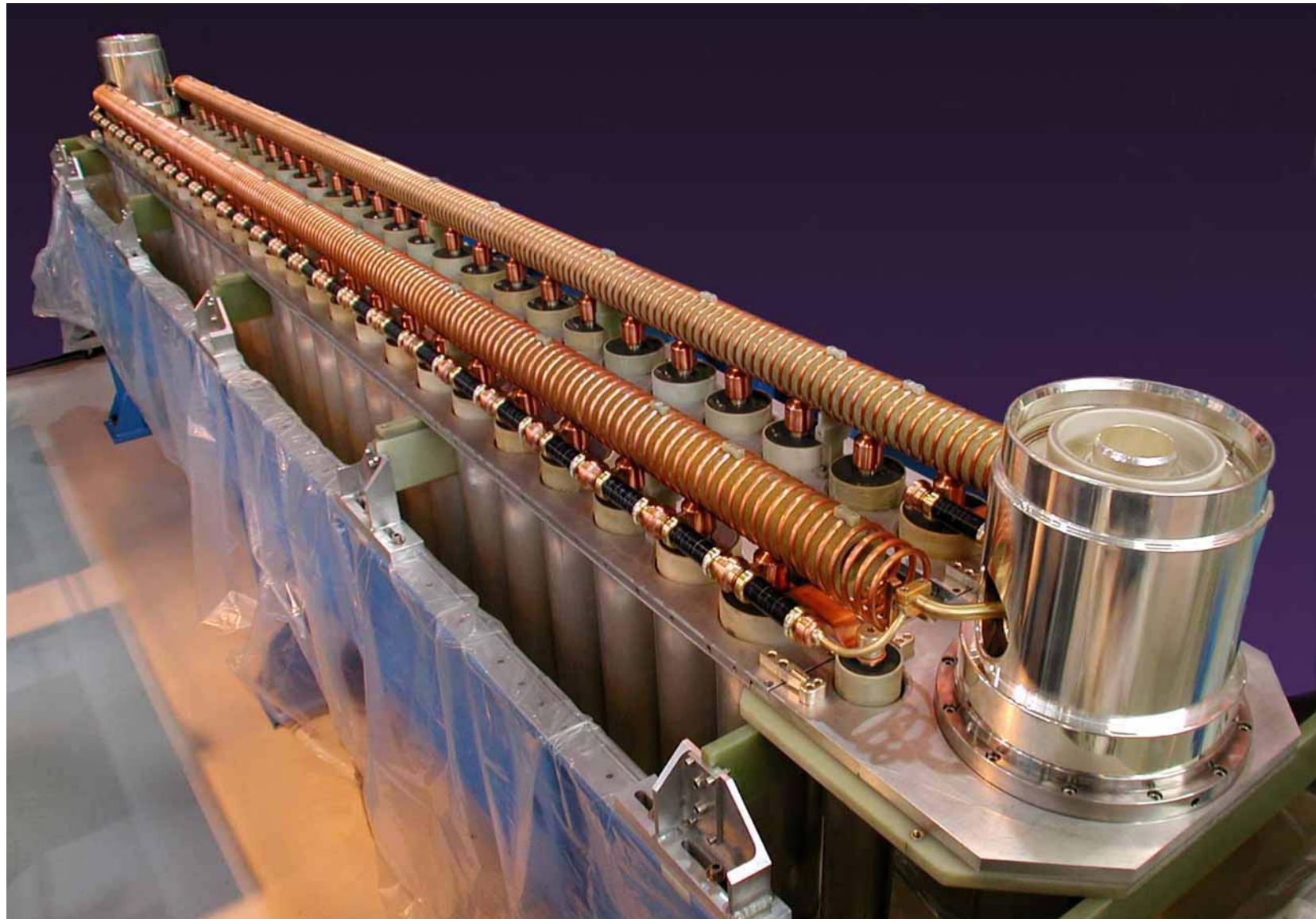
Figure 11: Schematic cross-section of the quadrupole





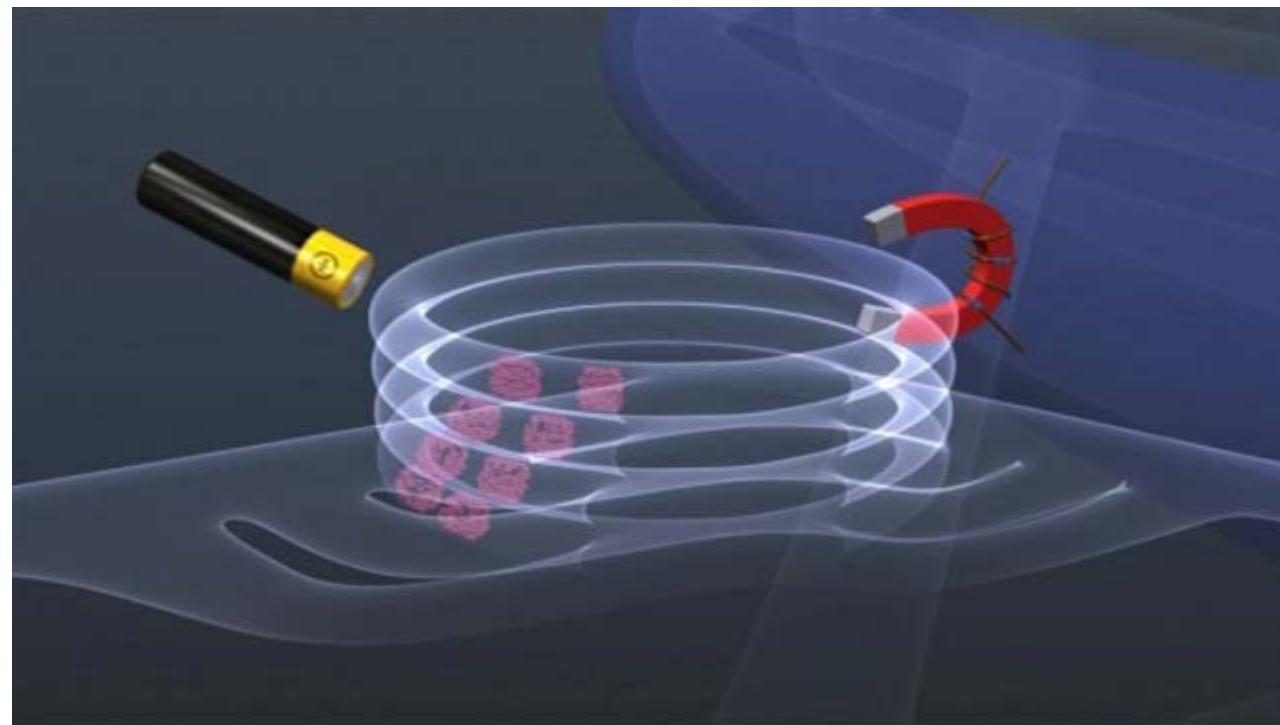
TRIUMF-CERN Collaboration - Twin-aperture warm quadrupole for the LHC

LHC KICKERS



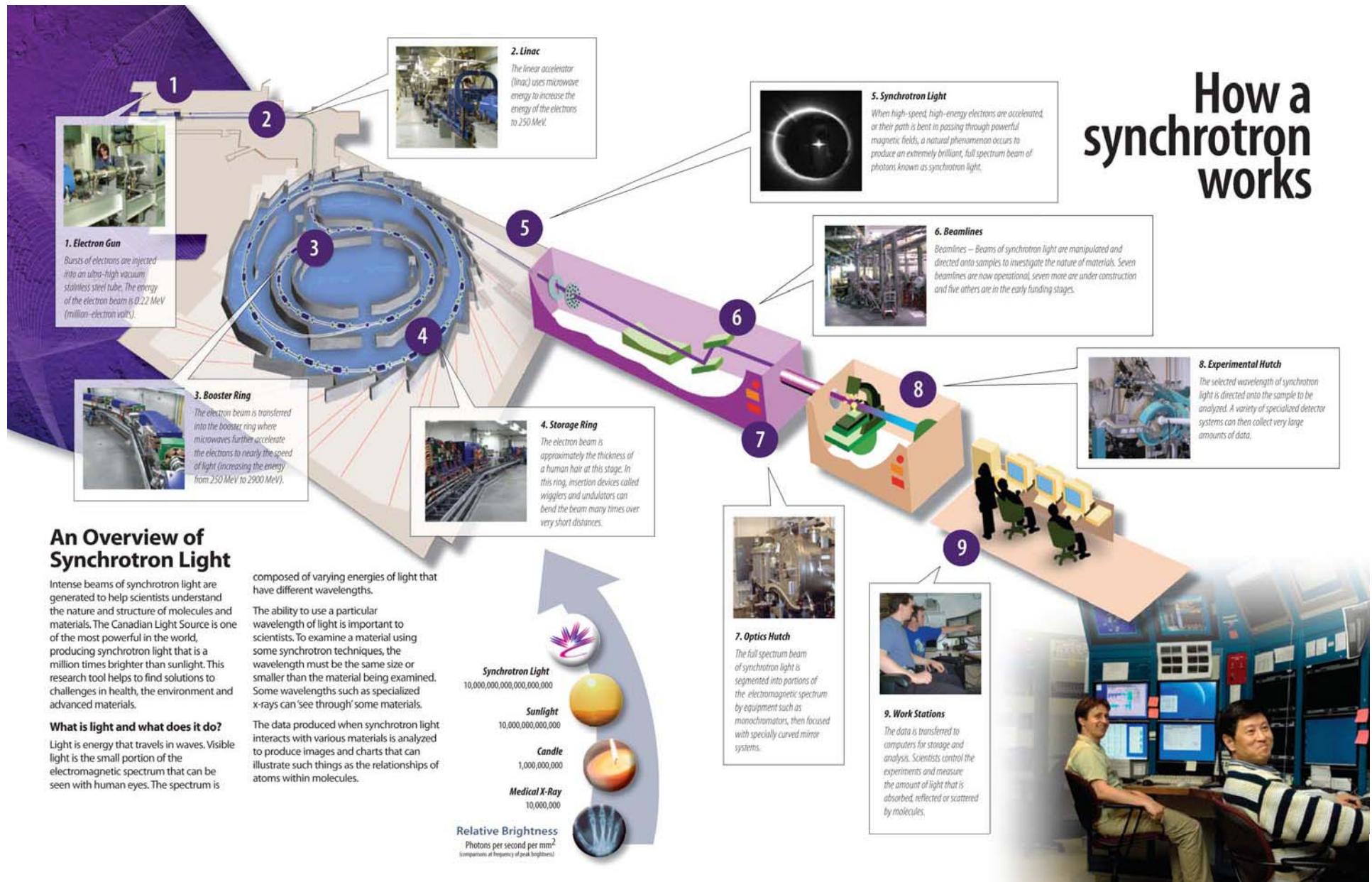
One of nine pulse forming networks built by TRIUMF for the LHC injection kickers

BOTTLE TO BANG



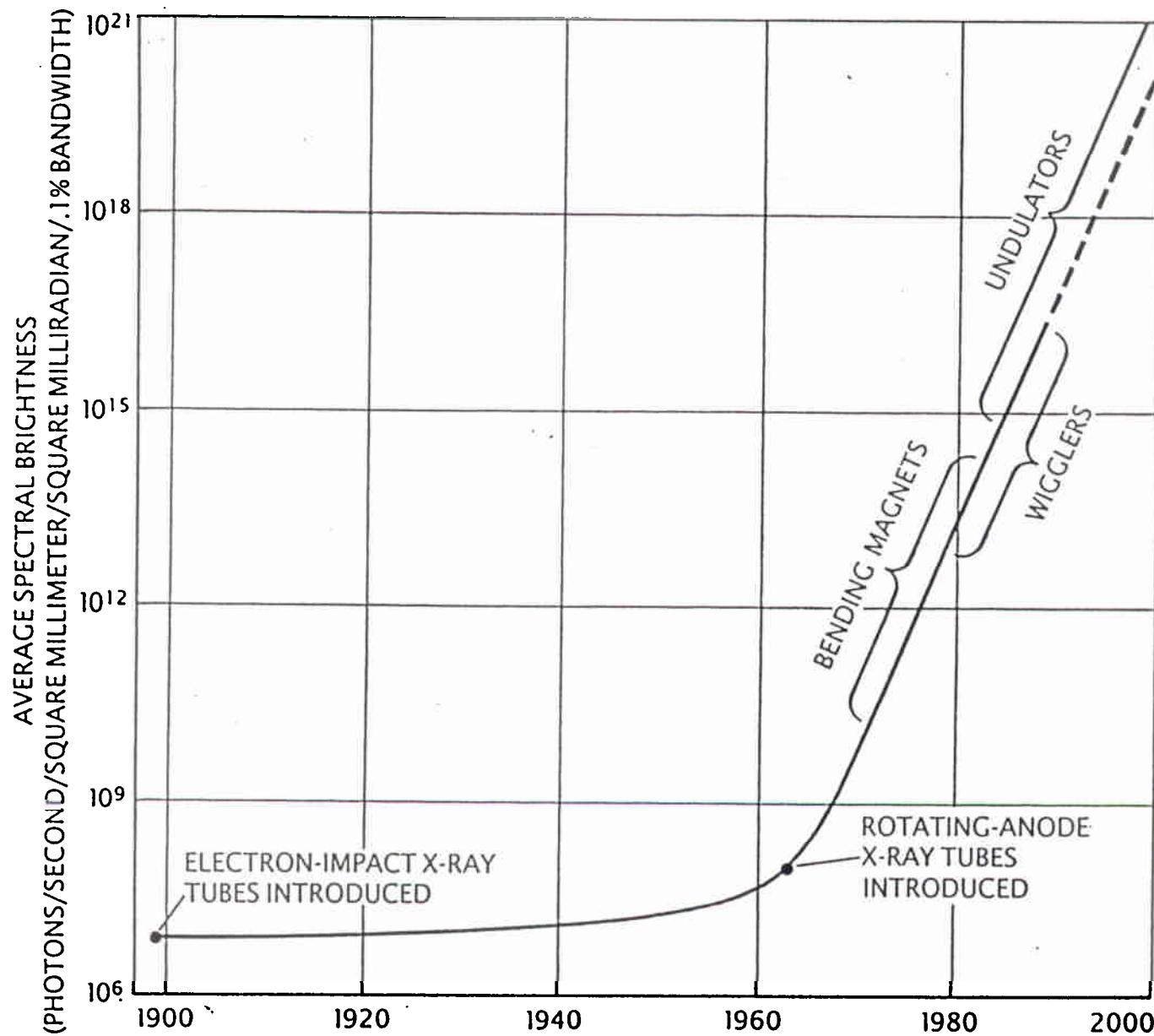
<http://cdsweb.cern.ch/record/1125472>

How a synchrotron works



THE CANADIAN LIGHT SOURCE IN SASKATOON

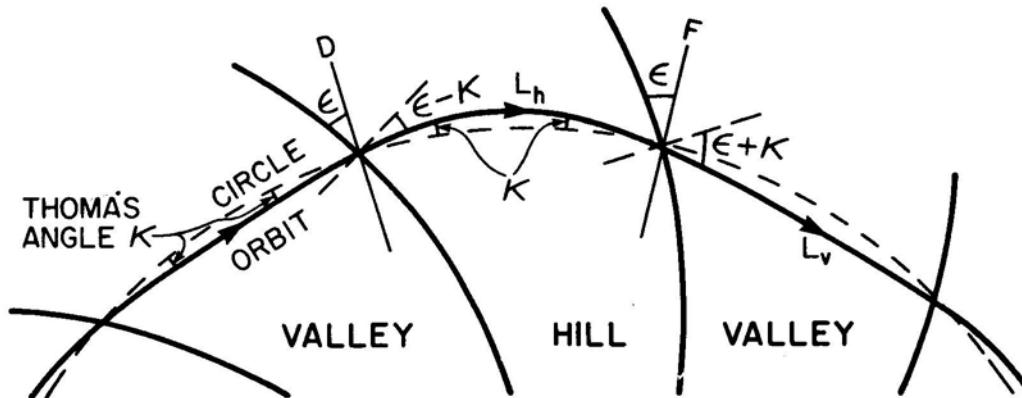
250-MeV electron linac + 2.9-GeV booster synchrotron + storage ring



BRIGHTNESS OF X RAYS has increased by many orders of magnitude since the advent of synchrotron-radiation sources. Undulators in storage rings are the brightest source.

Field of Research	Size and Range	bacterium	IR	Cell	Visible UV	Protein	VUV	molecule	Soft X-ray	atom	Hard X-ray	nucleus	γ -ray
Atomic Physics									Photo-electron spectroscopy				
Molecular Physics										Topography			
Solid state Physics										Chrystallography			
Surface Science									EXAFS, NEXAFS				
Material Science									Photo-emission, Absorption	X-ray diffraction			
Chemical Dynamics									Bond-selective, Photo chemistry				
Biology									X-ray microscopy	Radiography			
Medicine										Angiography			
Geology									Philography	Small angle scattering			
Industry									MOCVD	Lithography	Imaging		

ALTERNATING FOCUSING FOR CYCLOTRONS - SPIRAL SECTORS



Kerst (1956) suggested using **spiral sectors** to increase the **axial focusing**.

Spiral angle ϵ ($> \kappa$) \rightarrow edge-crossing angles $\kappa + \epsilon$ i.e. a strong **F** lens
 or $\kappa - \epsilon$ i.e. a less strong **D** lens.

The focal powers become

$$\frac{1}{f_z} = \pm \frac{e(B_h - B_v)}{mv} \tan(\epsilon \pm \kappa)$$

and overall

$$v_z^2 = -\beta^2 \gamma^2 + F^2 \left(1 + 2 \tan^2 \epsilon \right)$$

Spiralling is now used for most isochronous cyclotrons over 40 MeV.

The powerful $2 \tan^2 \epsilon$ term has enormously increased the energies attainable:

- TRIUMF: 70 - 520 MeV H^- , 250 μ A (\rightarrow 450 μ A); $\epsilon \leq 70^\circ$
- PSI: 590 MeV p, 2 mA (\rightarrow 3 mA); $\epsilon \leq 35^\circ$
- providing 1000 \times more intense beams for π , μ , n & RI production;
- designs exist up to 12 GeV!

TRIUMF 70-520 MeV H⁻ CYCLOTRON (in 1972)



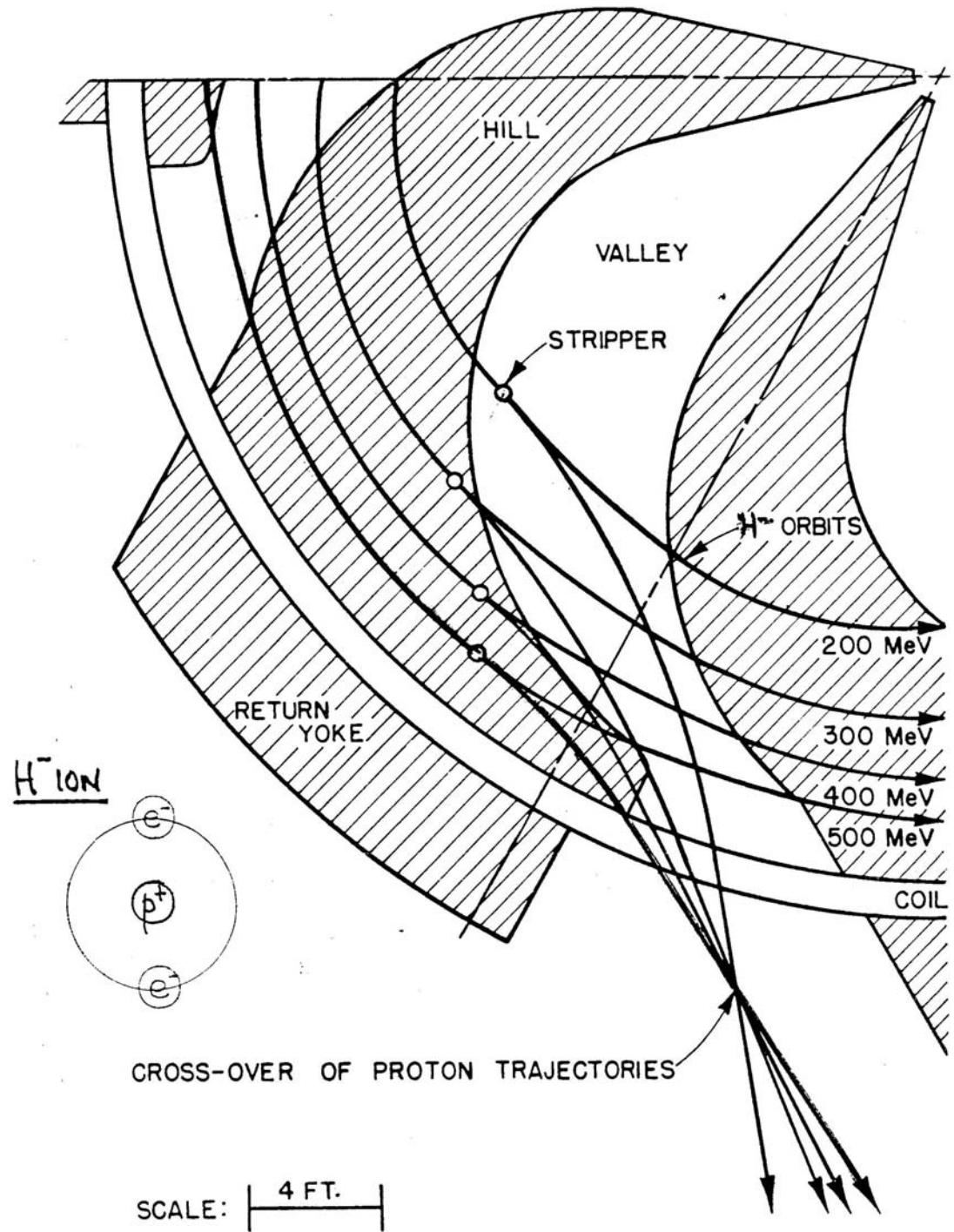
- Note iron-free valleys to maximize flutter
- Spiral angle increases with radius and energy from 0° to 70°
- H⁻ ions allow 4 separate extracted beams, but restrict hill field to 0.6 T

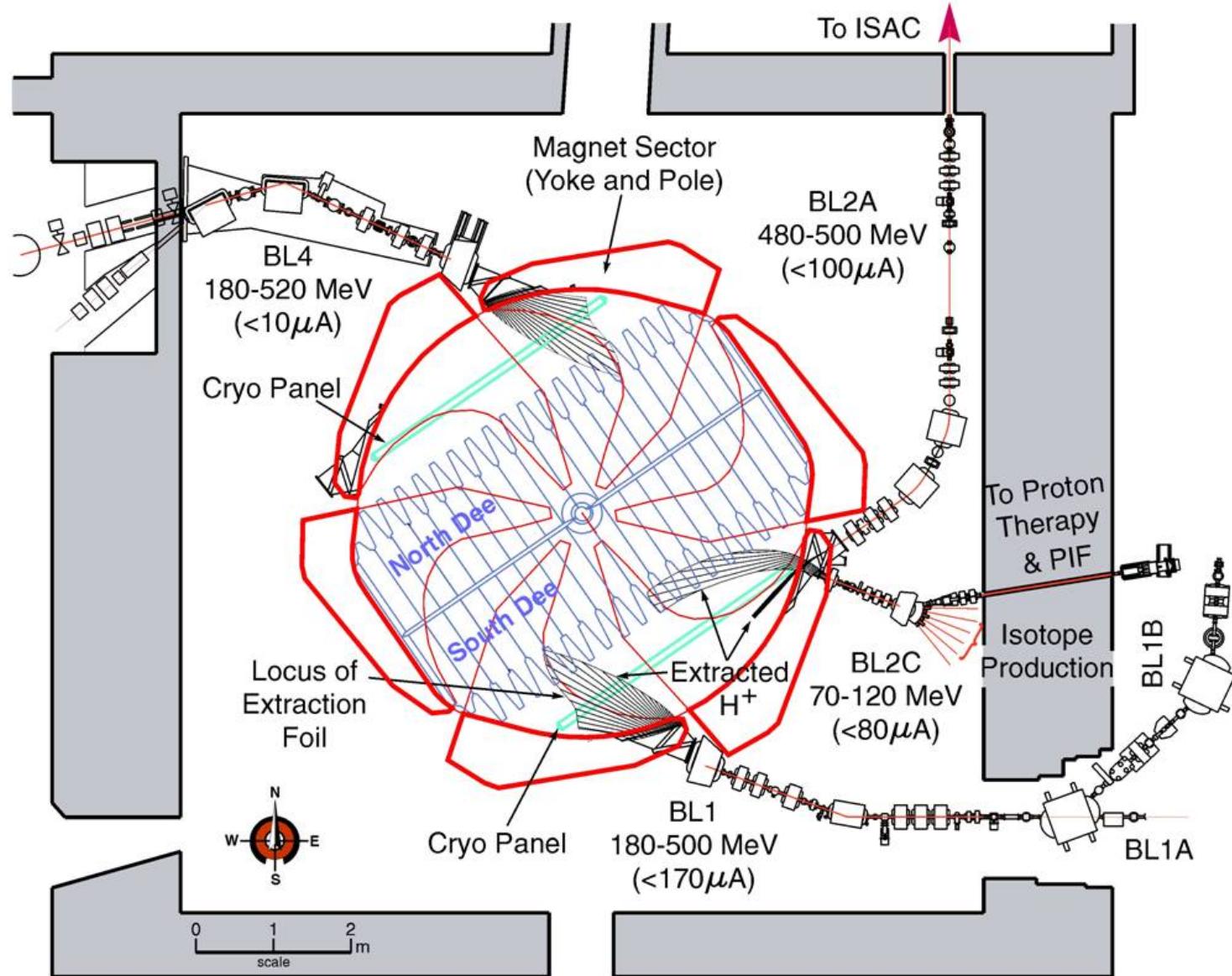
VARIABLE-ENERGY EXTRACTION BY H⁻ STRIPPING

When a high-energy H⁻ ion impinges on a thin foil, the **two electrons are stripped off**, leaving the positively charged proton bending in the opposite sense.

The foils are typically made of **25- μ m pyrolytic graphite**.

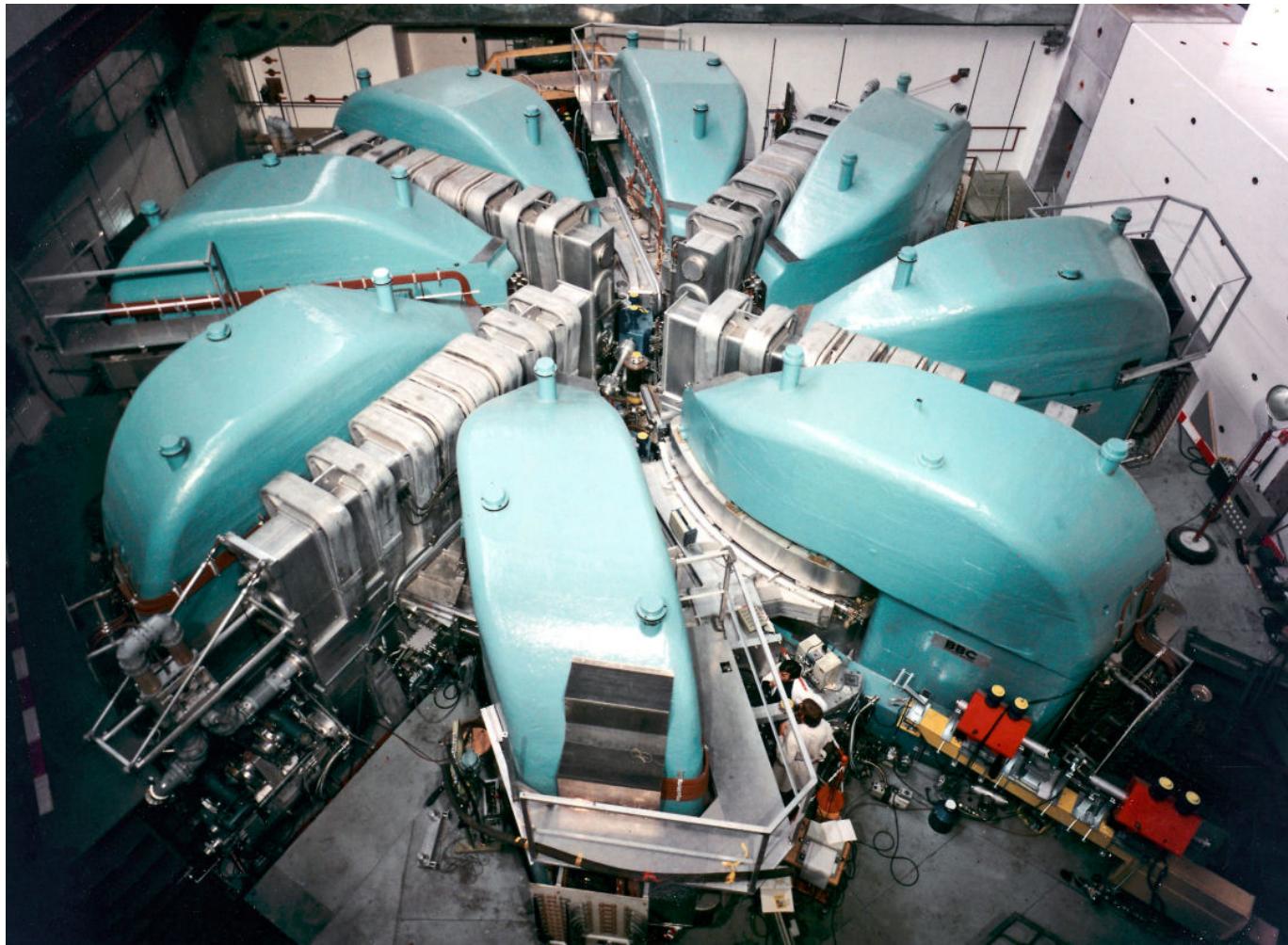
The foil azimuth for each radius is chosen so that the proton trajectories cross in a **"combination magnet"** that steers all into one beam line.



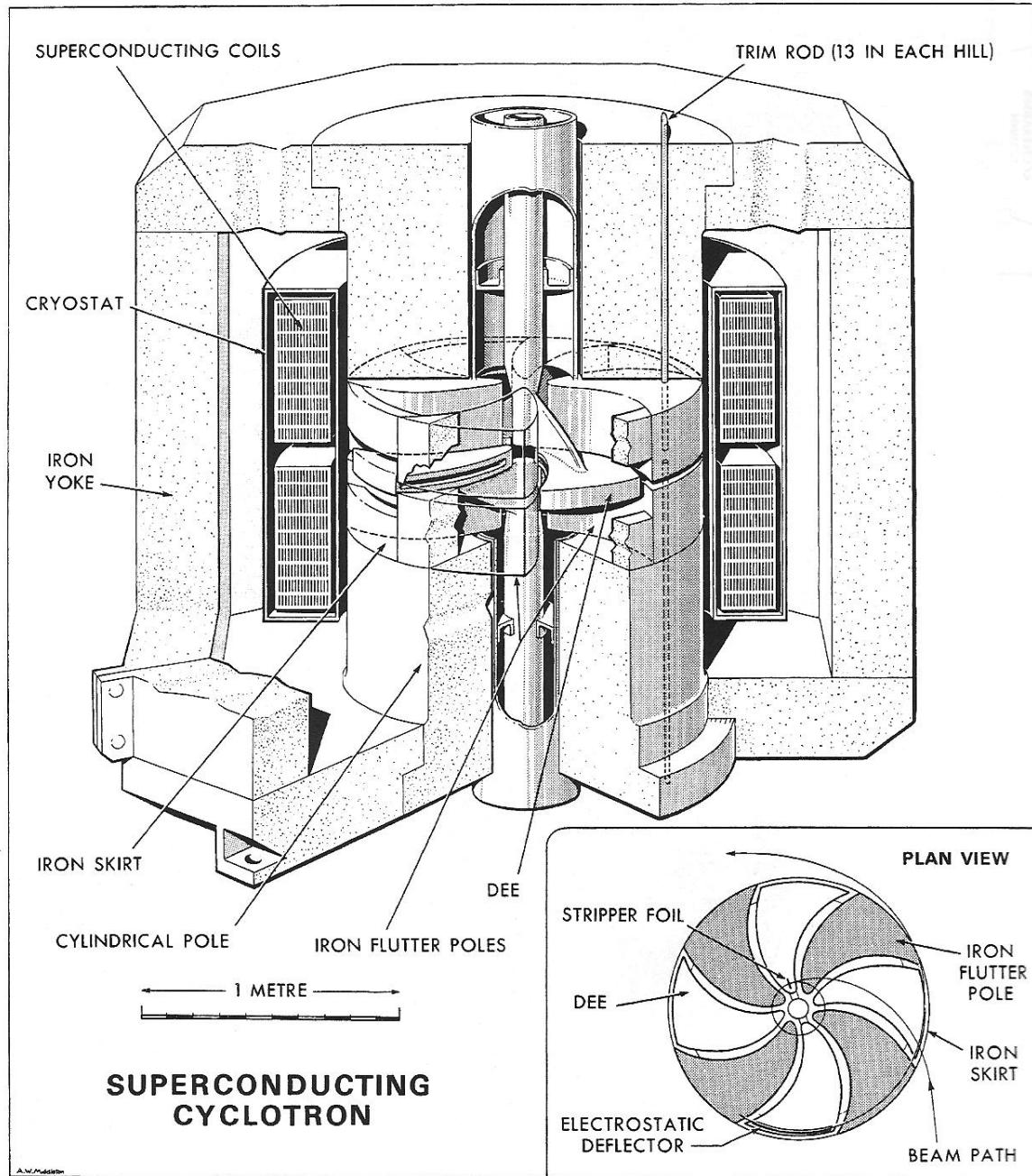


By using a **variety of foil shapes** for partial extraction at lower energies, TRIUMF currently extracts **3 beams of variable energy and intensity simultaneously** - and proposes to add a **4th**.

PSI 590-MeV RING CYCLOTRON

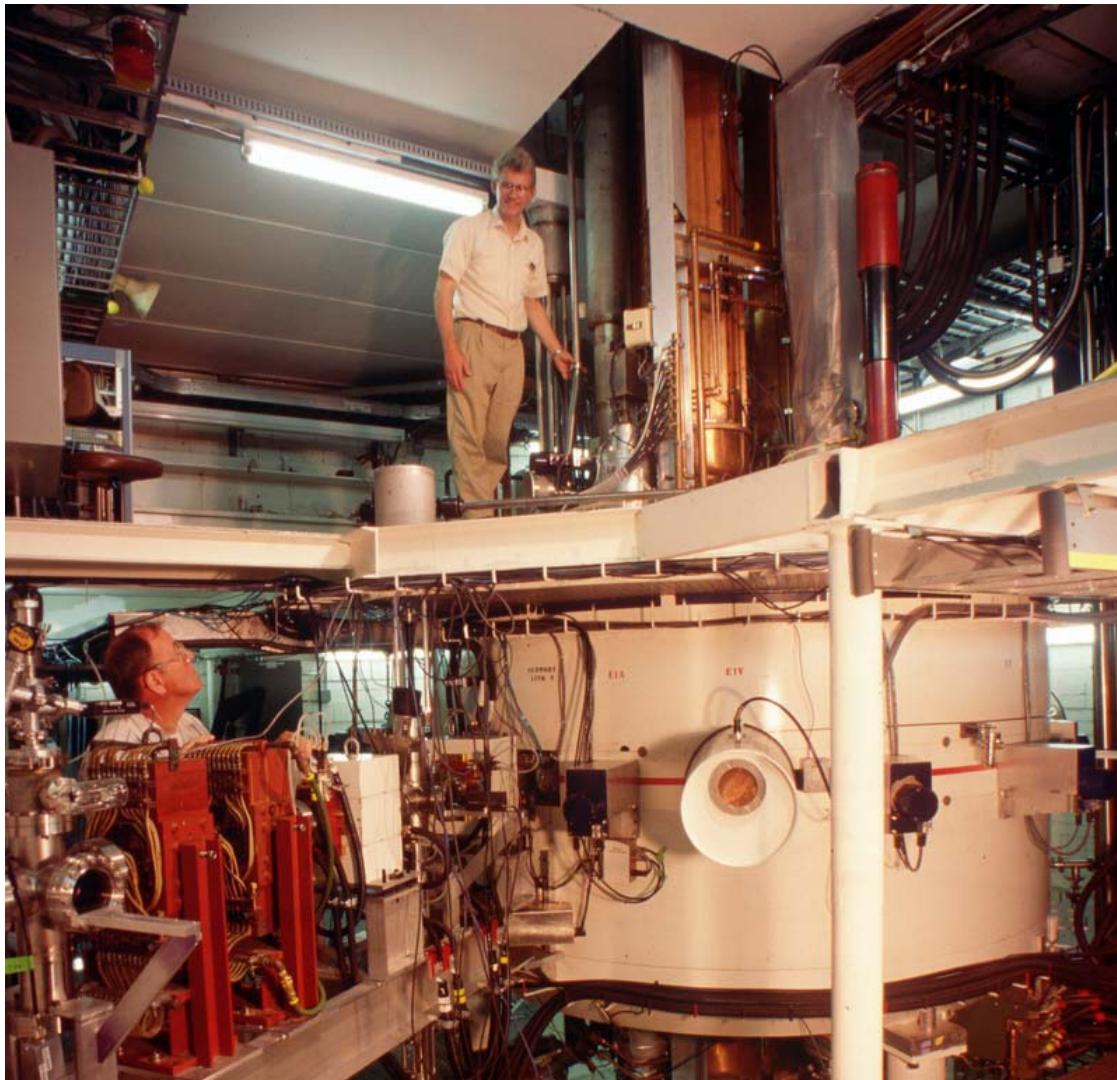


This separated-sector ring cyclotron, with 8 spiral magnets and four 1-MV rf cavities, delivers a 2.2-mA proton beam - the world's highest power particle beam (1.3 MW).



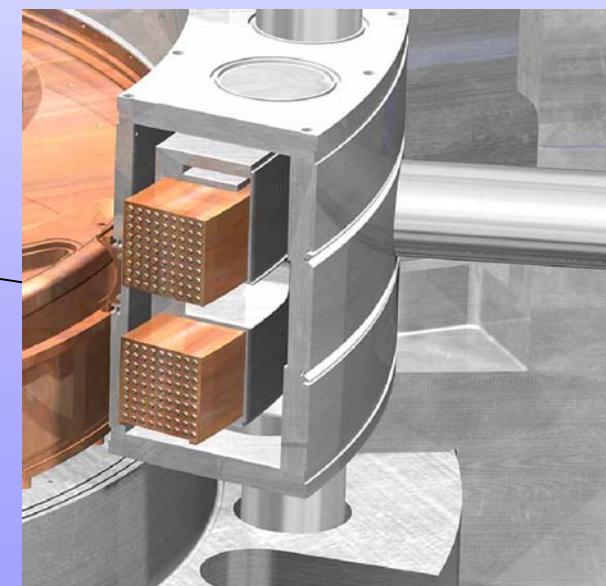
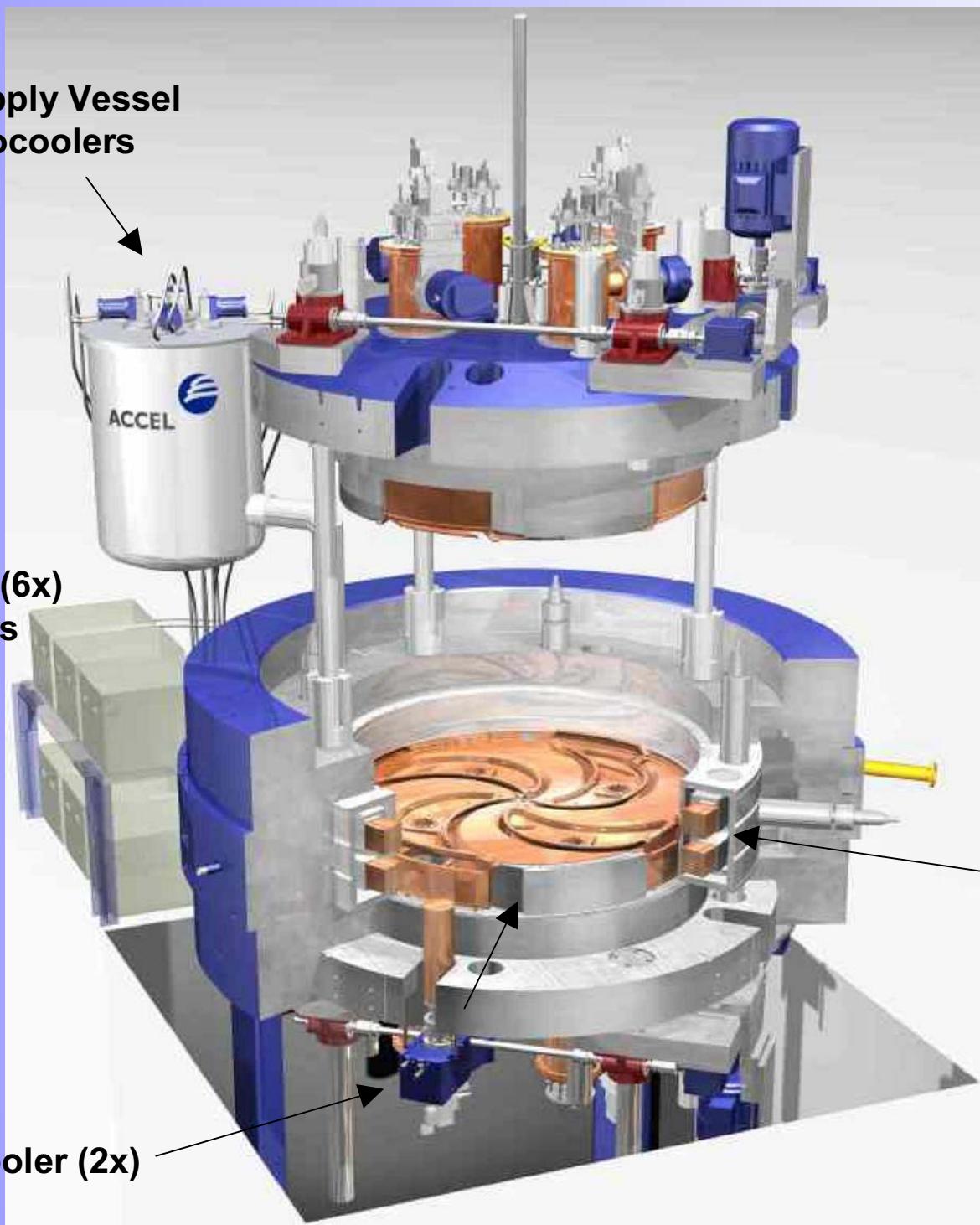
The first superconducting cyclotron design - the K520 at AECL Chalk River

K500 SUPERCONDUCTING CYCLOTRON AT MSU

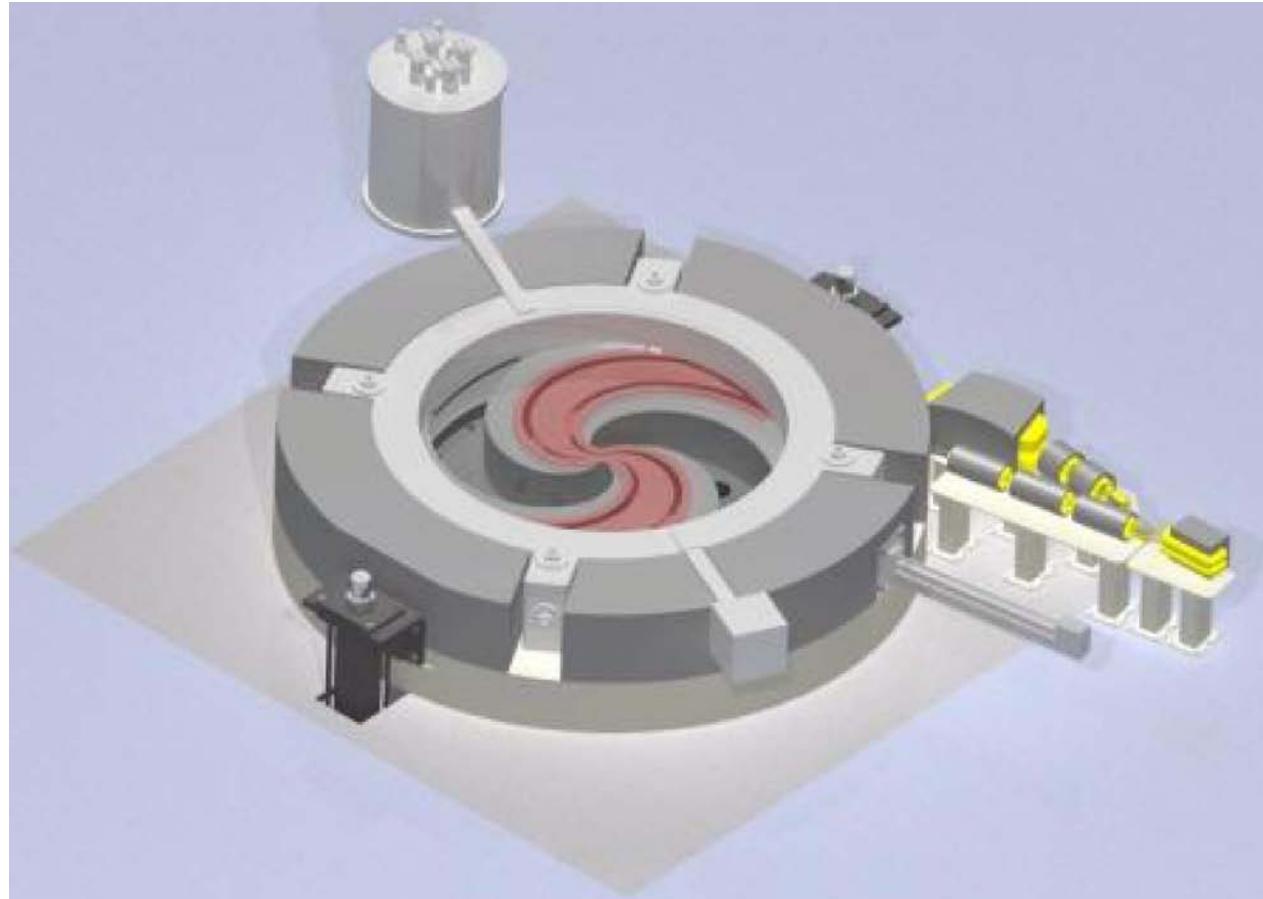


- The first cyclotron with superconducting coils to operate (Blosser 1982)
- can accelerate heavy ions (atomic number Z , mass A) to $500(Z/A)^2$ MeV/u.

250 MeV Superconducting Proton Cyclotron



IBA-JINR C400 CARBON/PROTON THERAPY CYCLOTRON



This joint IBA-Dubna design will provide a range of ions for therapy:

- 400-MeV/u ${}^4\text{He}^{2+}$, $({}^6\text{Li}^{3+})$, $({}^{10}\text{Be}^{5+})$, ${}^{12}\text{C}^{6+}$
- 265-MeV protons by stripping H^{2+} .

The outer diameter is 6.6 m, the magnet weight 700 tons.

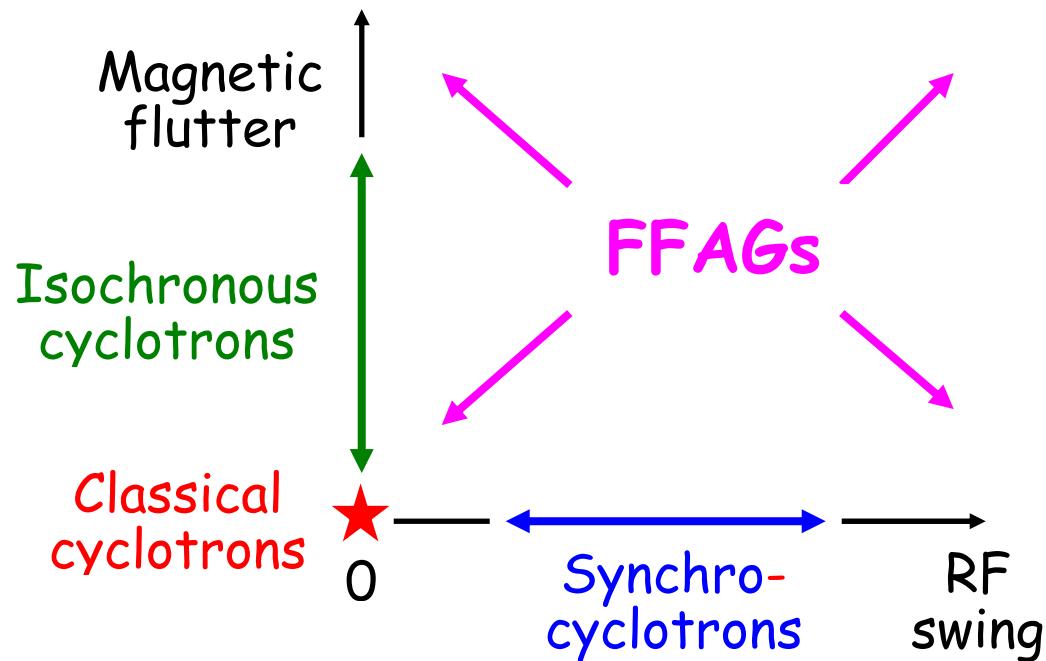
Construction will begin soon on the first C400, to be installed in Caen.

FFAGs - Fixed Field Alternating Gradient accelerators

Fixed Magnetic Field - members of the **CYCLOTRON** family¹

Magnetic field variation $B(\theta)$	Fixed Frequency (CW beam)	Frequency-modulated (Pulsed beam)
Uniform	Classical	Synchro-
Alternating	Isochronous	FFAG

But FFAG enthusiasts sometimes express an alternative view:
- cyclotrons are just special cases of the FFAG!



1. E.M. McMillan, *Particle Accelerators*, in *Experimental Nuclear Physics*, III, 639-786 (1959)

THE FFAG IDEA

- was to introduce alternating "strong" focusing to fixed-field accelerators (enabling higher rep rates and beam currents than in synchrotrons)
- either by alternating +ve and -ve bending magnets with radial edges, creating Alternating Gradient focusing (Ohkawa, Kolomensky, Symon, 1953-4)
- or by using spiral sector magnets (Kerst 1955) - as later used in cyclotrons.

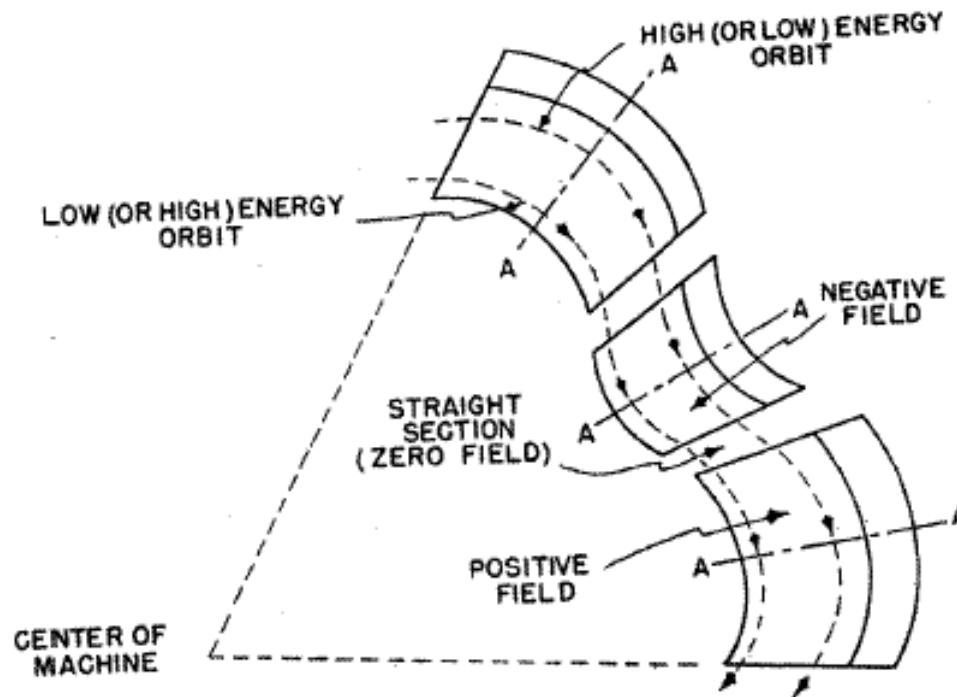


FIG. 2. Plan view of radial-sector magnets.

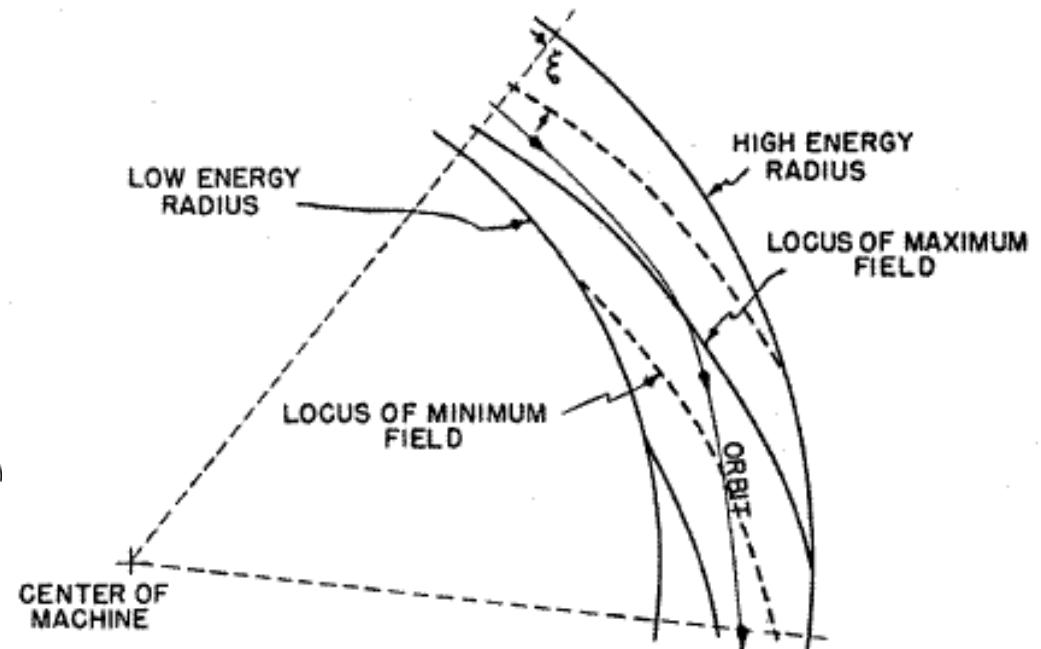


FIG. 3. Spiral-sector configuration.

BASIC CHARACTERISTICS OF FFAGs

are determined by their **FIXED MAGNETIC FIELD**

- **Spiral orbits**
 - needing **wider magnets, rf cavities** and **vacuum chambers** (compared to AG synchrotrons)
- **Faster rep rates (up to kHz?)** limited only by rf capabilities
 - not by magnet power supplies
- **Large acceptances**
- **High beam current**

The last 3 factors have fuelled the interest in FFAGs over 50 years!

The most intensive studies were carried out by Symon, Kerst, et al. at the **Mid-west Universities Research Association (MURA)** in the 1950s and 60s - who adopted the "**scaling**" principle

- and built several successful electron models.

SCALING DESIGNS - HORIZONTAL TUNE ν_r

Resonances were a worry in the 1950s, because of slow acceleration: if, at some energy, the betatron oscillation wavelength matches that of a harmonic component of the magnetic field, the ions may be driven into resonance, leading to loss of beam quality or intensity.

The general condition is $\ell \nu_x \pm m \nu_y = n$ where ℓ, m, n are integers.

So "**Scaling**" designs were used, with:

- the **same orbit shape at all energies**
- the **same optics** " " " " "
- the **same tunes** " " " " " \Rightarrow no crossing of resonances!

To 1st order, the (radial tune)² $\nu_r^2 \approx 1 + k$ (even with sector magnets)

where the **average field index** $k(r) \equiv \frac{r}{B_{av}} \frac{dB_{av}}{dr}$ and $B_{av} = \langle B(\Theta) \rangle$

So **large constant ν_r** requires **$k = \text{constant} \geq 0$**

$$\Rightarrow B_{av} = B_0 (r/r_0)^k \quad \text{and} \quad p = p_0 (r/r_0)^{(k+1)}$$

SCALING FFAGs - VERTICAL TUNE v_z

In the vertical plane, with **sector magnets** and to 1st order,

$$v_z^2 \approx -k + F^2(1 + 2\tan^2\varepsilon)$$

where the 2nd term describes the Thomas and spiral edge focusing effects.

Note $k > 0 \Rightarrow$ **vertical defocusing**

\therefore large constant, real v_z requires large, constant $F^2(1 + 2\tan^2\varepsilon)$

MURA kept (1) **magnetic flutter** $F^2 \equiv \left\langle \left(\frac{B(\theta) - B_{av}}{B_{av}} \right)^2 \right\rangle = \text{constant}$

(most simply achieved by using **constant profile** $B(\theta)/B_{av}$)

(2a) for spiral sectors,

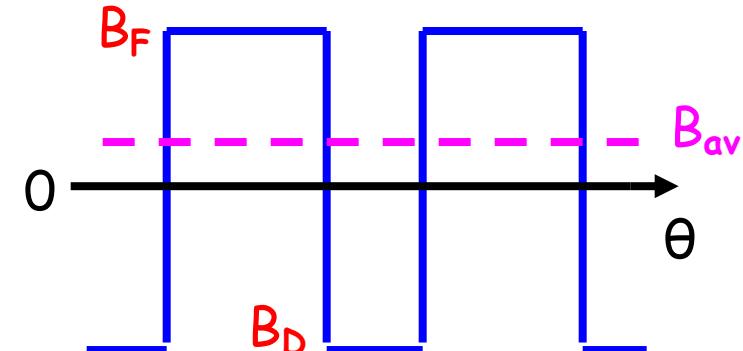
spiral angle $\varepsilon = \text{constant}$ (sector axis follows $R = R_0 e^{\Theta \cot \varepsilon}$)

(2b) for radial sectors,

$B_D = -B_F$ to boost F^2 .

Note - reverse fields increase average radius:

$\Rightarrow >4.5\times$ larger (Kerst & Symon '56 - no straights)



[Not so bad with straights: KEK 150-MeV FFAG has "circumference factor" 1.8]

MURA Electron FFAGs

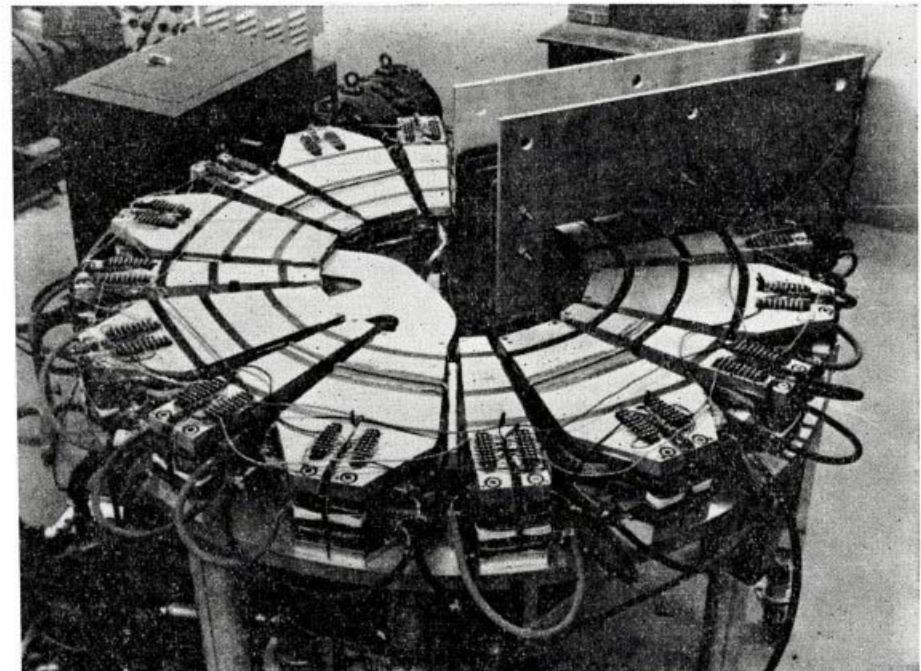
400keV radial sector →

50 MeV radial sector ↘

120 keV spiral sector ↘

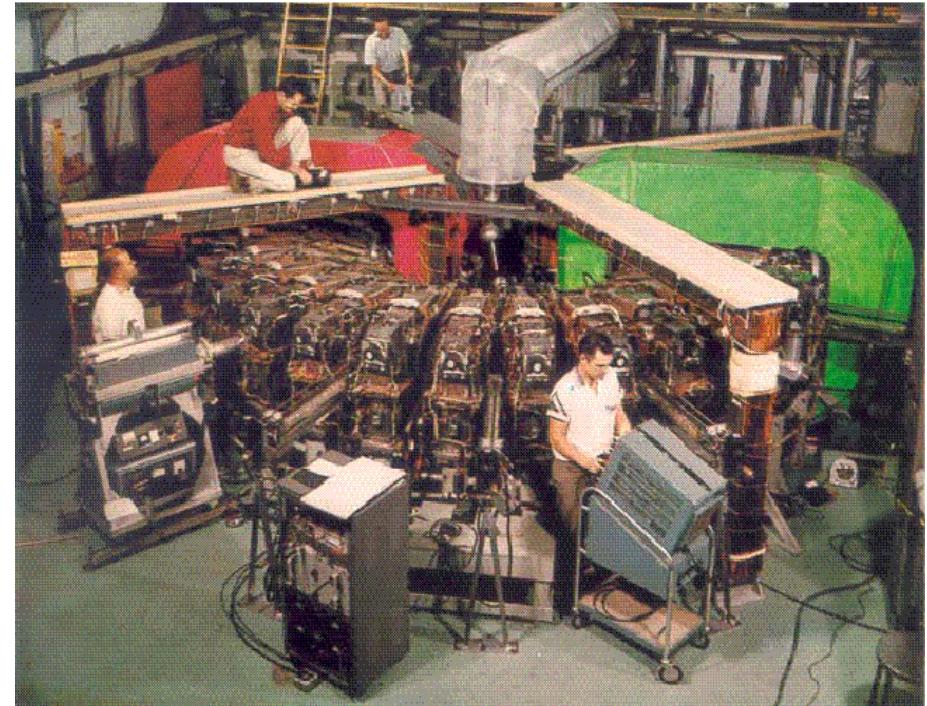


Courtesy of MURA



Courtesy of MURA

K.R. Symon, Proc PAC03, 452 (2003)



THE FFAG REVIVAL

In spite of their promise of **much higher beam intensity than AG synchrotrons** at GeV energies through their:

- higher repetition rates, and
- larger radial and momentum acceptance,

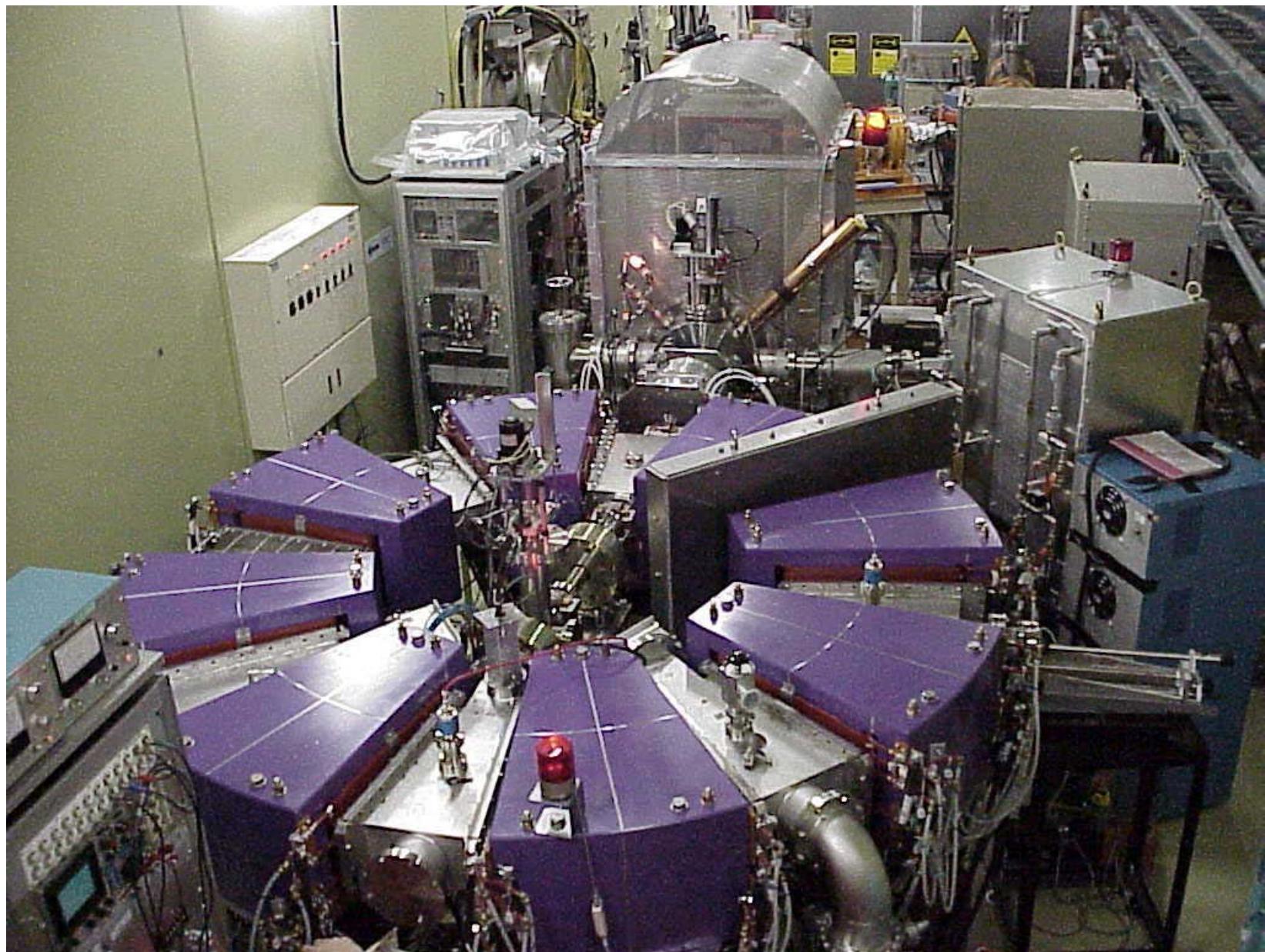
two major factors deterred construction of proton FFAGs until recently:

- **their complicated magnet shapes** (difficult to design and expensive to build)
- **the unreliability of the mechanical tuners** required for rapid frequency modulation for all particles heavier than electrons.

But in the late 1990s two independent new approaches were taken:

- **Mills & Johnstone** (Fermilab, 1997) proposed a **non-scaling lattice** for a muon accelerator
- **Mori** (KEK, 1999 - also with muons in view) started building a 1-MeV **proton scaling FFAG** - with solutions to the technical problems.

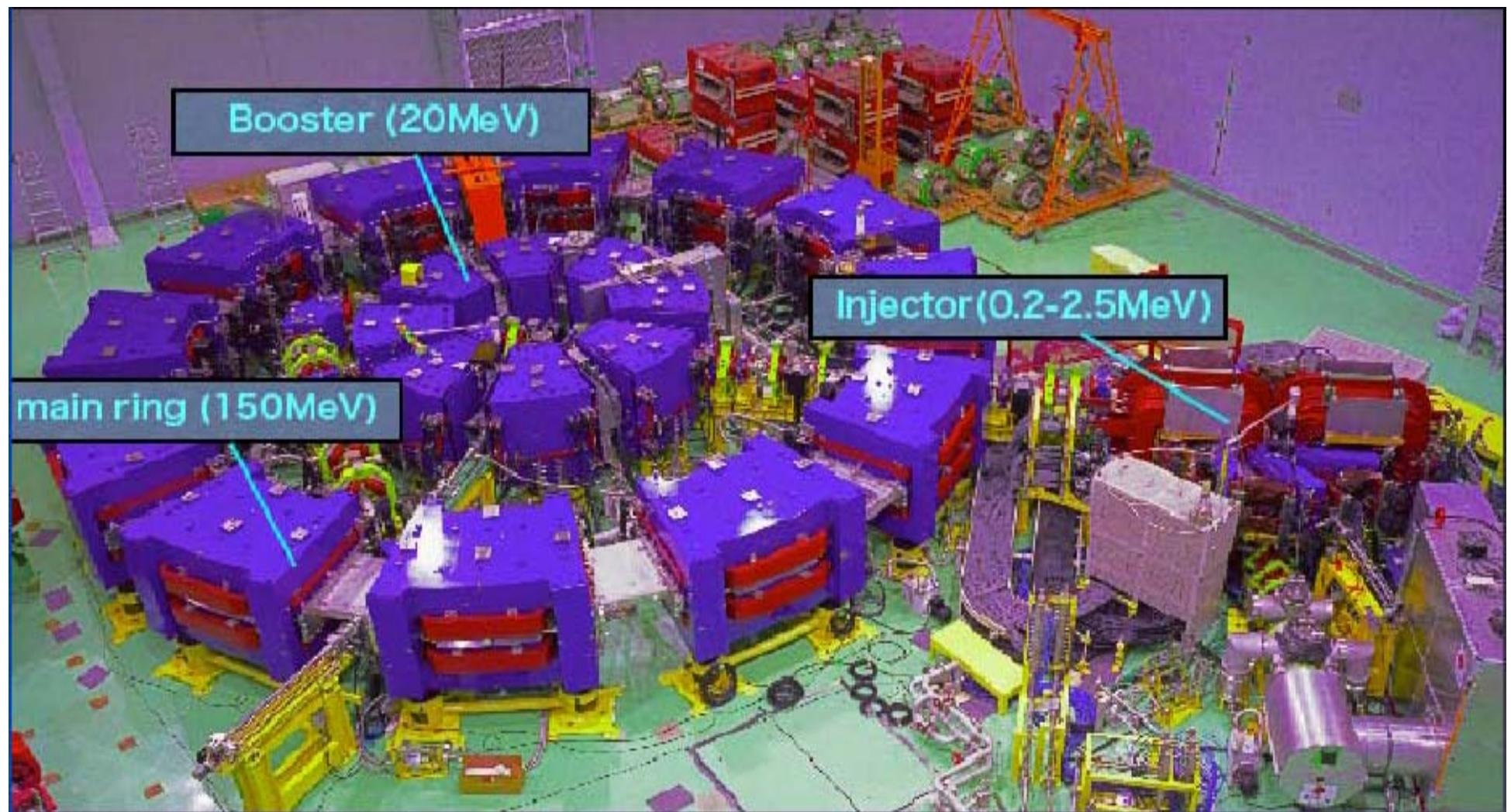
KEK Proof-of-Principle 1 MeV proton FFAG



KEK 150-MeV 12-Sector Proton FFAG



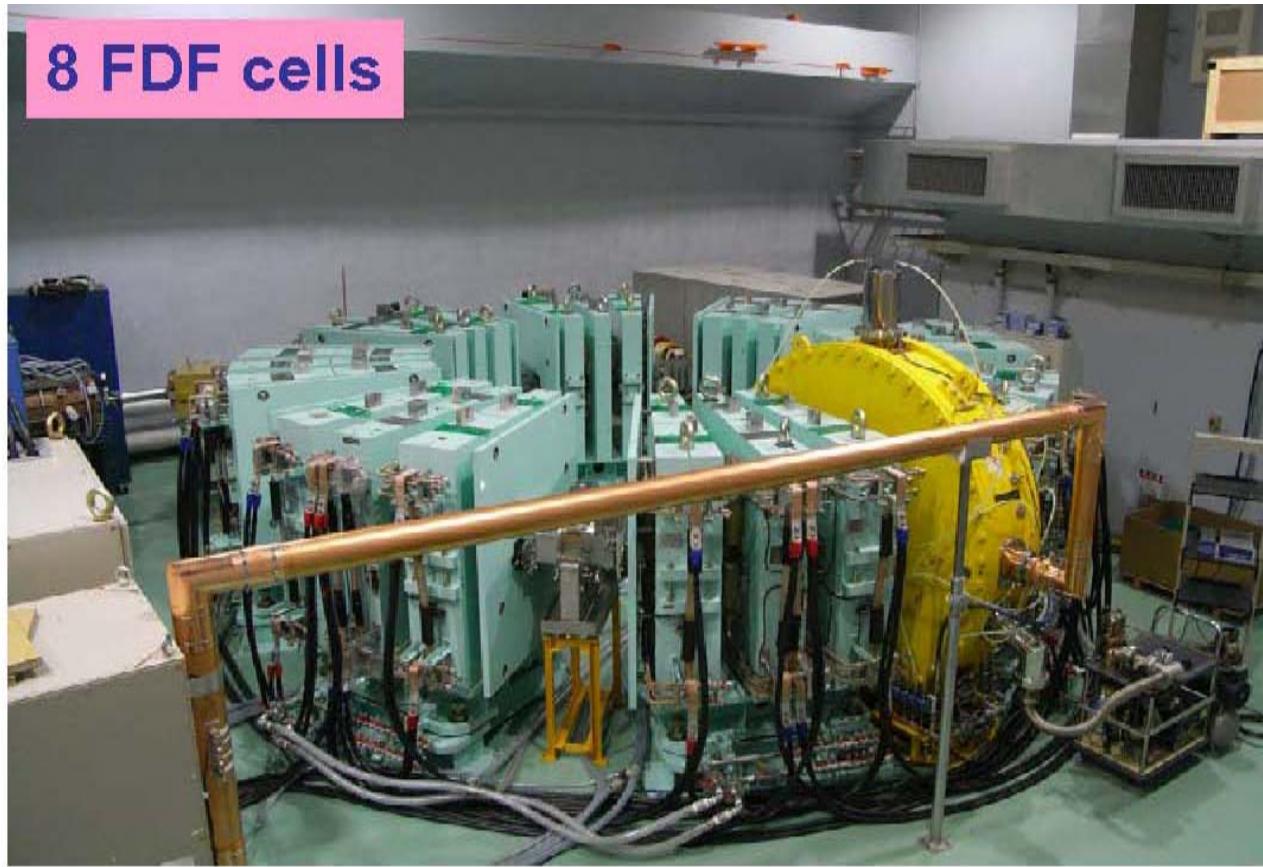
FFAG Complex at Kyoto University Research Reactor Inst.



The World's first test of Accelerator-Driven Sub-critical Reactor (ADSR) operation was performed in March 2009.

KURRI ERIT STORAGE RING FOR BNCT

(ERIT = Energy/Emittance Recovery Internal Target)



70-mA of circulating 11-MeV protons produce an **intense neutron beam** ($>10^9/\text{cm}^2/\text{s}$ at the patient) via the $\text{Be}(\text{p},\text{n})$ reaction.

$V_{\text{rf}} = 250 \text{ kV}$ plus large FFAG acceptances ($>3000 \text{ mm-mrad}$, $\pm 5\% \delta p/p$) allow **ionization cooling** to maintain stable beam over 1000 turns.

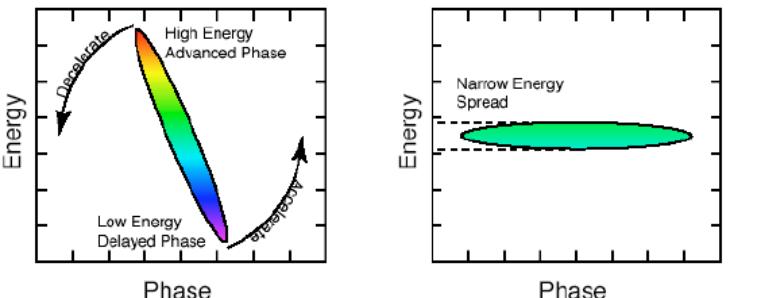
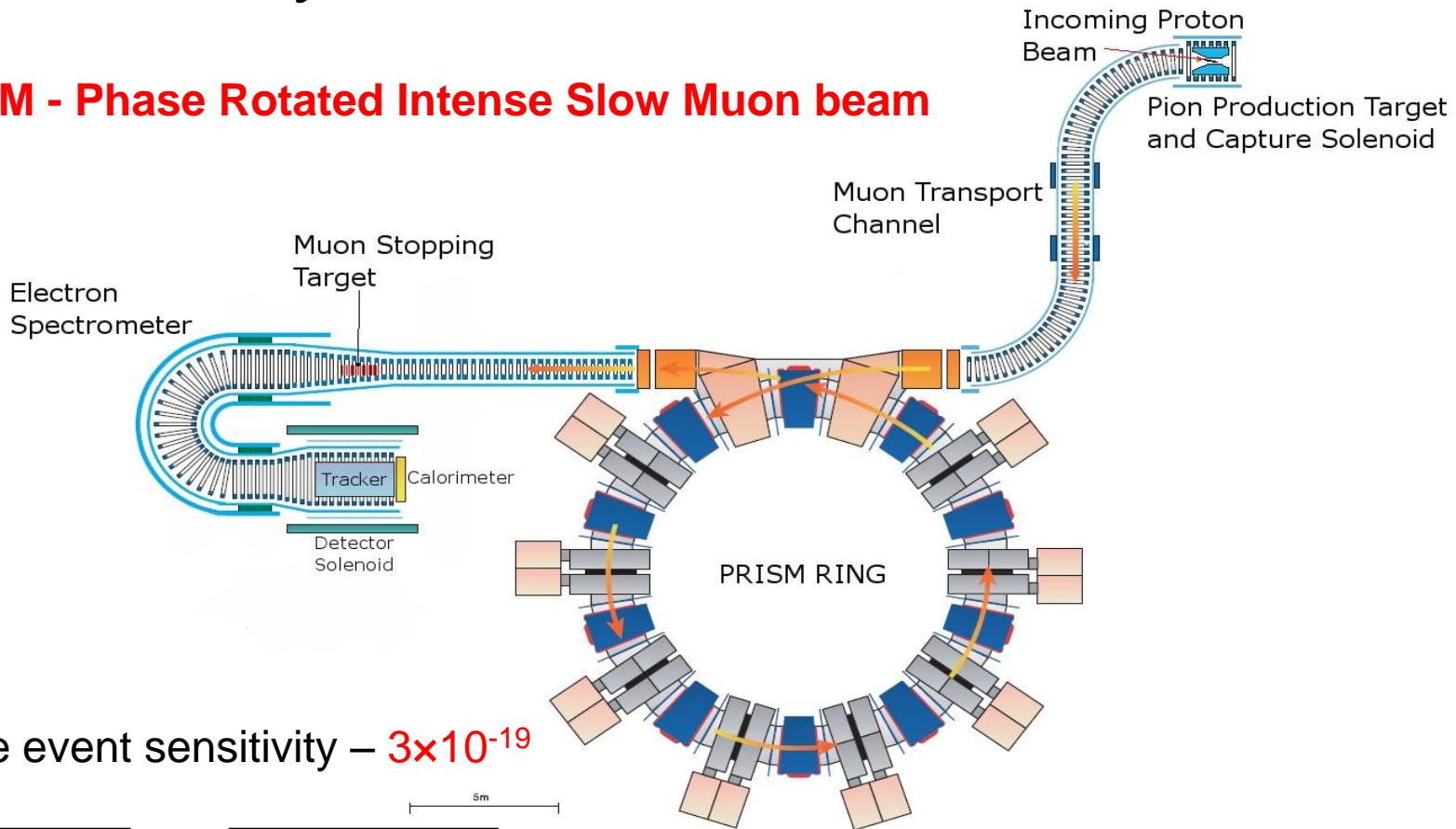
α -PARTICLE TEST RING FOR PRISM AT RCNP OSAKA



Using 6 of the PRISM muon storage ring's 10 sectors
to demonstrate bunch rotation in phase space

Layout of the PRISM/PRIME

PRISM - Phase Rotated Intense Slow Muon beam

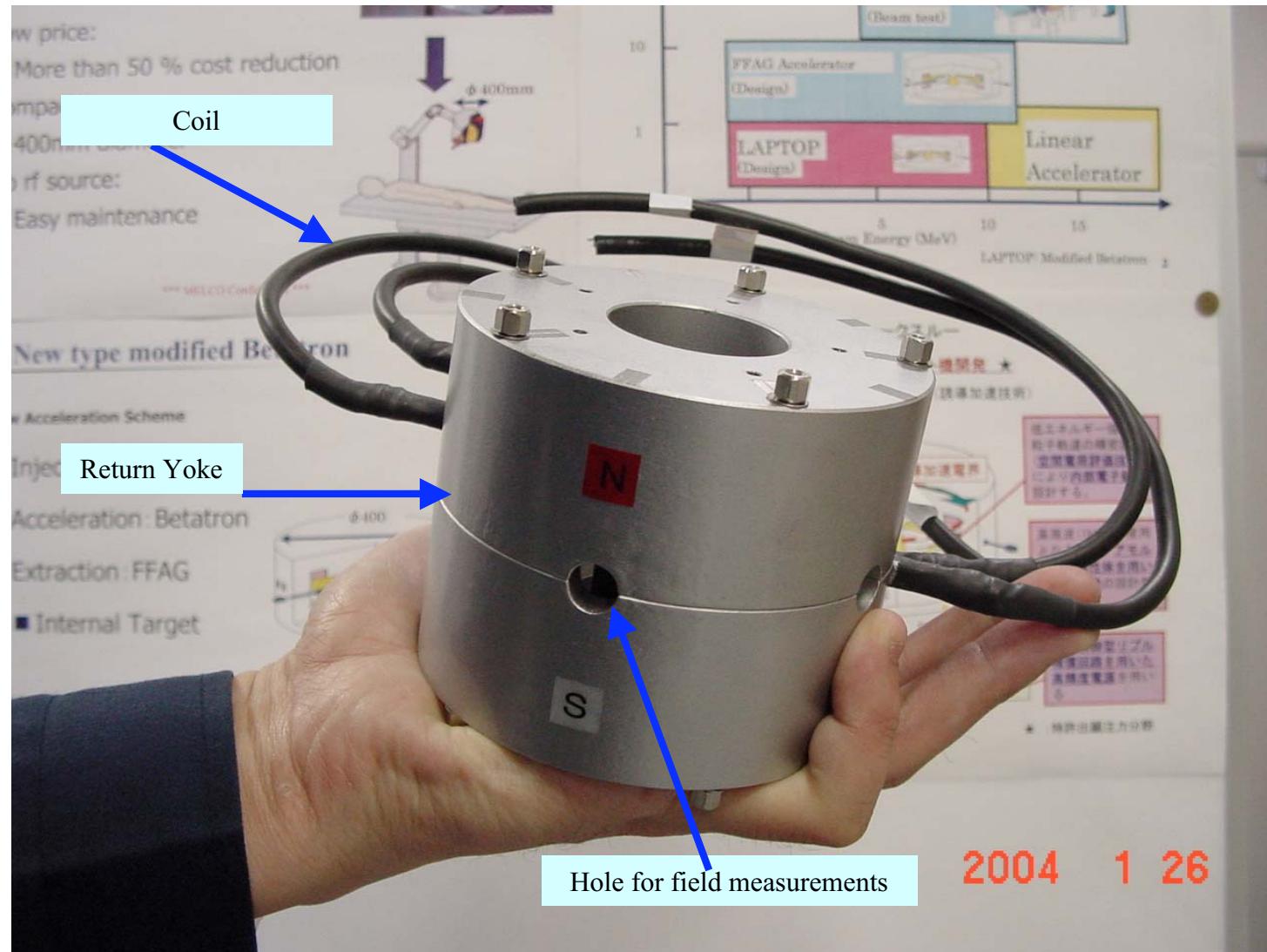


The PRISM/PRIME experiment based on FFAG ring was proposed (Y. Kuno, Y. Mori) for a next generation cLFV searches in order to:

- reduce the muon beam energy spread by **phase rotation**,
- **purify** the muon beam in the storage ring.

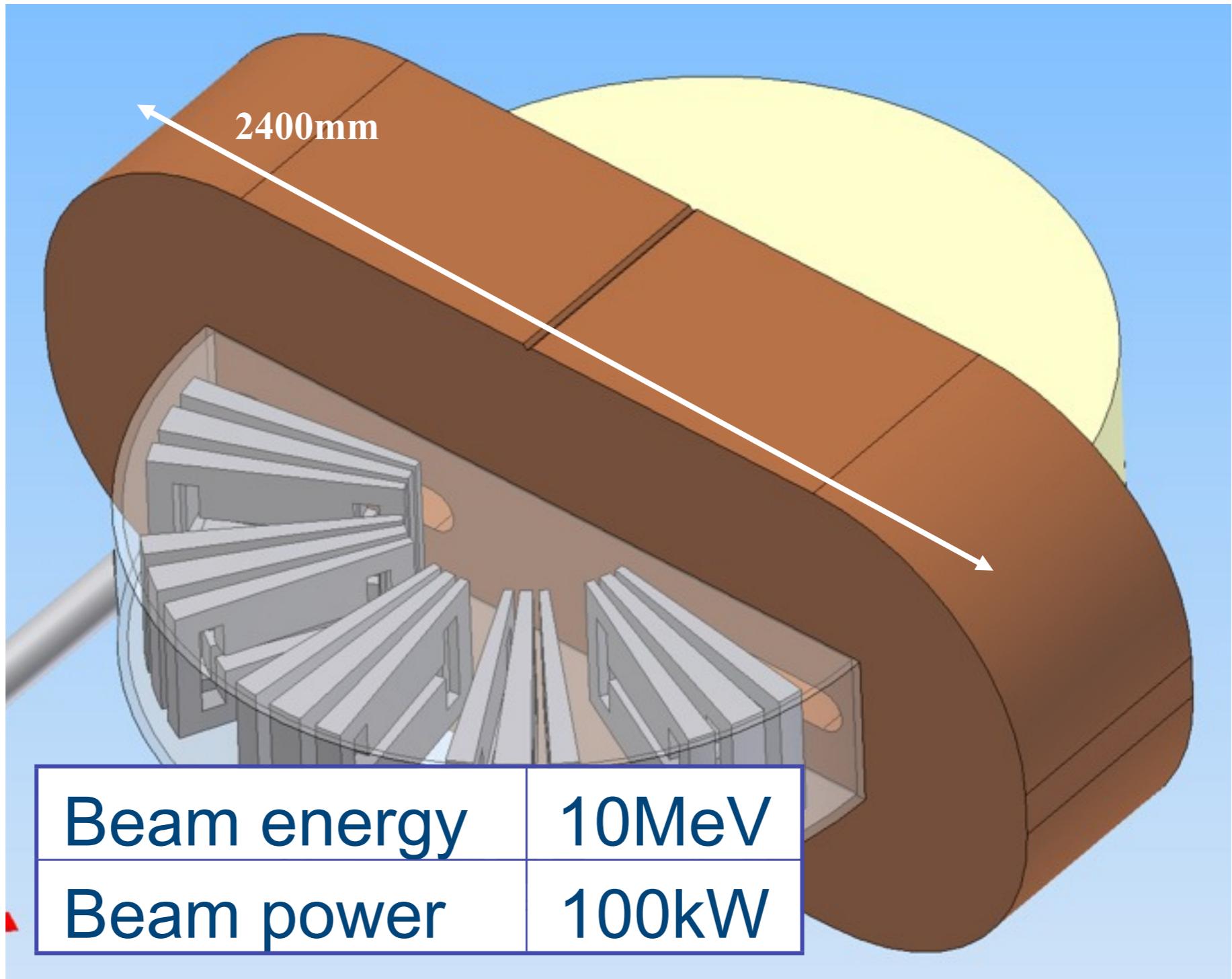
SCALING FFAGs - DESIGN STUDIES

	Energy (MeV/u)	Ion	Cells	Spiral angle	Radius (m)	Rep rate (Hz)	Comments
MEICo - Laptop	1	e	5	35°	.023 -.028	1,000	Hybrid - <u>Magnet built</u>
eFFAG	10	e	8	47°	0.26 - 1.0	5,000	20-100 mA
LPSC RACCAM	180	p	10	54°	3.2 - 3.9	>20	<u>Magnet sector 2008</u>
Ibaraki Med.Acc.	230	p	8	50°	2.2 - 4.1	20	0.1 μ A
MEICo - p Therapy	230	p	3	0°-60°	0 - 0.7	2,000	<u>SC</u> , Quasi-isochronous
MEICo - Ion Therapy (Mitsubishi Electric)	400	C^{6+}	16	64°	7.0 - 7.5	0.5	Hybrid (FFAG/synch")
	7	C^{4+}	8	0°	1.35 - 1.8	0.5	" " " "
NIRS Chiba - Hadron Therapy	{ 400 { 100 7	C^{6+} " C^{4+}	12 12 10	0° 0° 0°	10.1 - 10.8 5.9 - 6.7 2.1 - 2.9	200 " "	Compact radial sectors
Mu Cooling Ring	160	μ	12	0°	0.95 ± 0.08		Gas-filled
J-PARC Neutrino Factory	{ 20,000 10,000 3,000	μ " "	120 64 32	0° 0° 0°	200 90 30		<u>$\Delta r = 0.5$ m</u> , ~10 turns.
Accelerators	1,000	"	16	0°	10		<u>$Q \approx 1$</u> rf cavities allow broadband operation

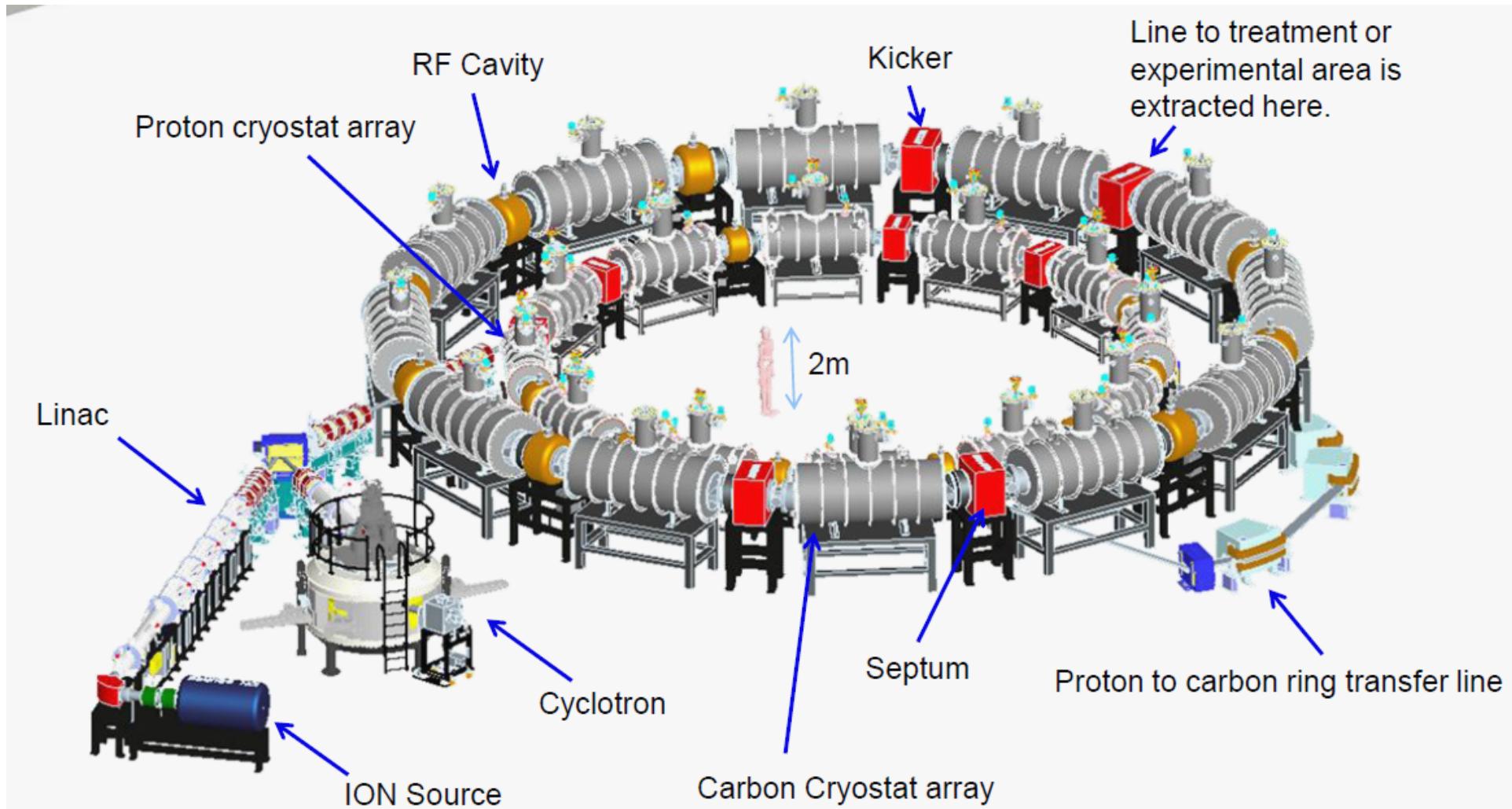


The present study is partially supported by the REIMEI Research Resources of Japan Atomic Energy Research Institute.

10MeV Electron Accelerator



PAMELA CANCER THERAPY FFAGs (Adams Inst. Oxford)

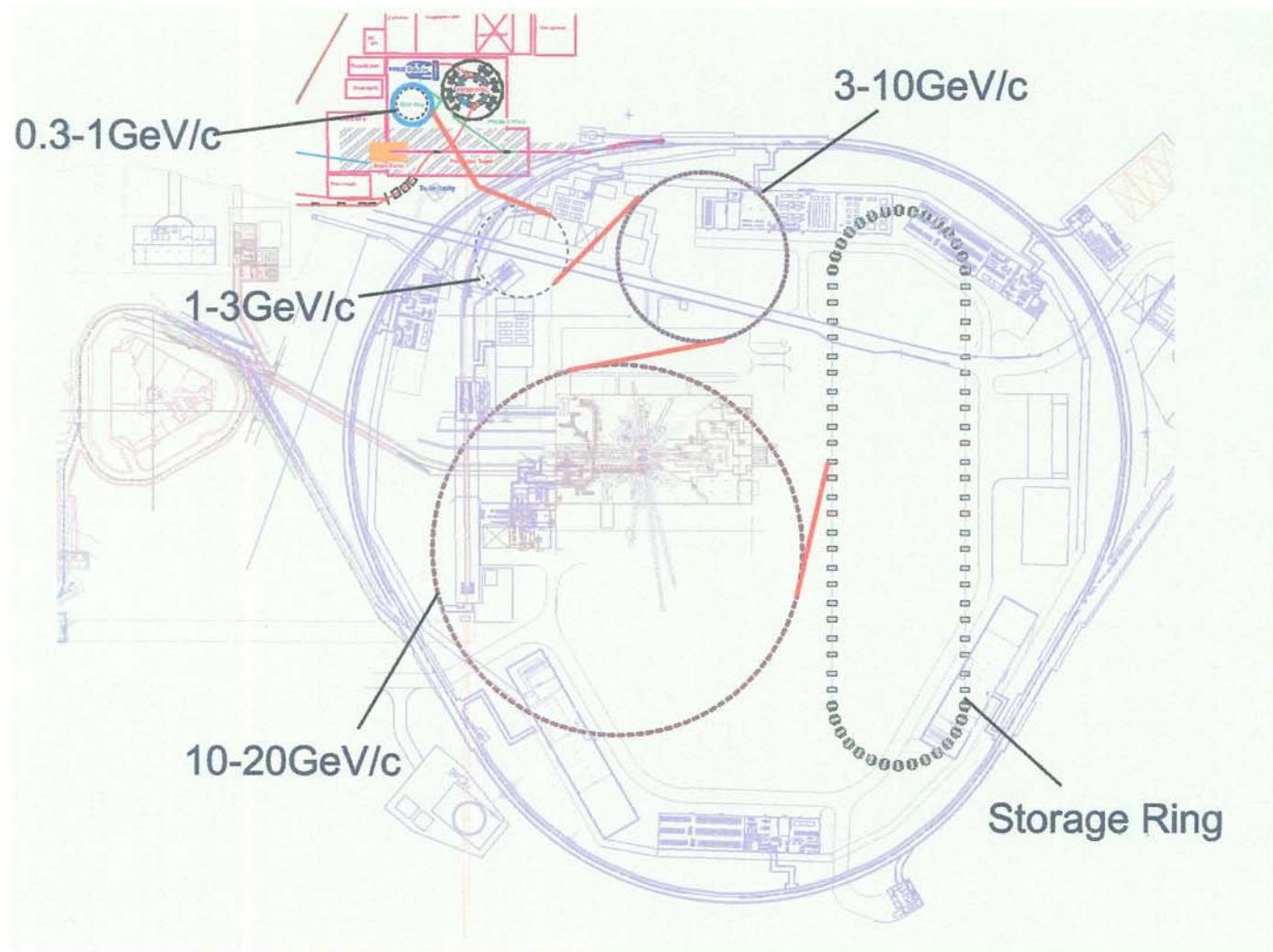


The small ring delivers 31-250 MeV protons

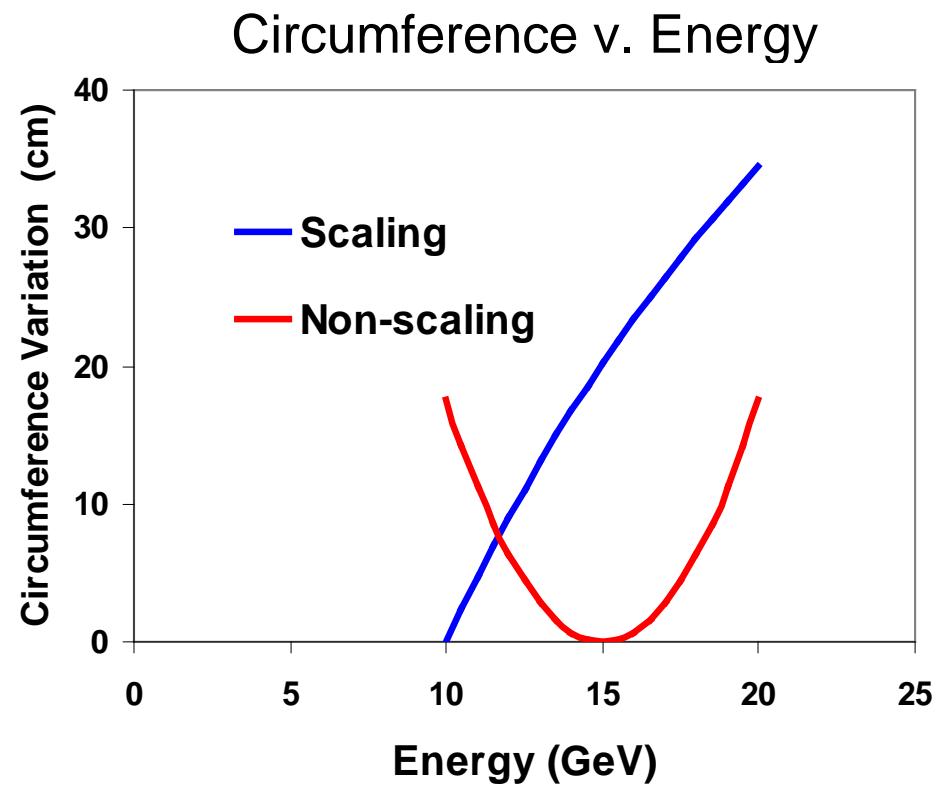
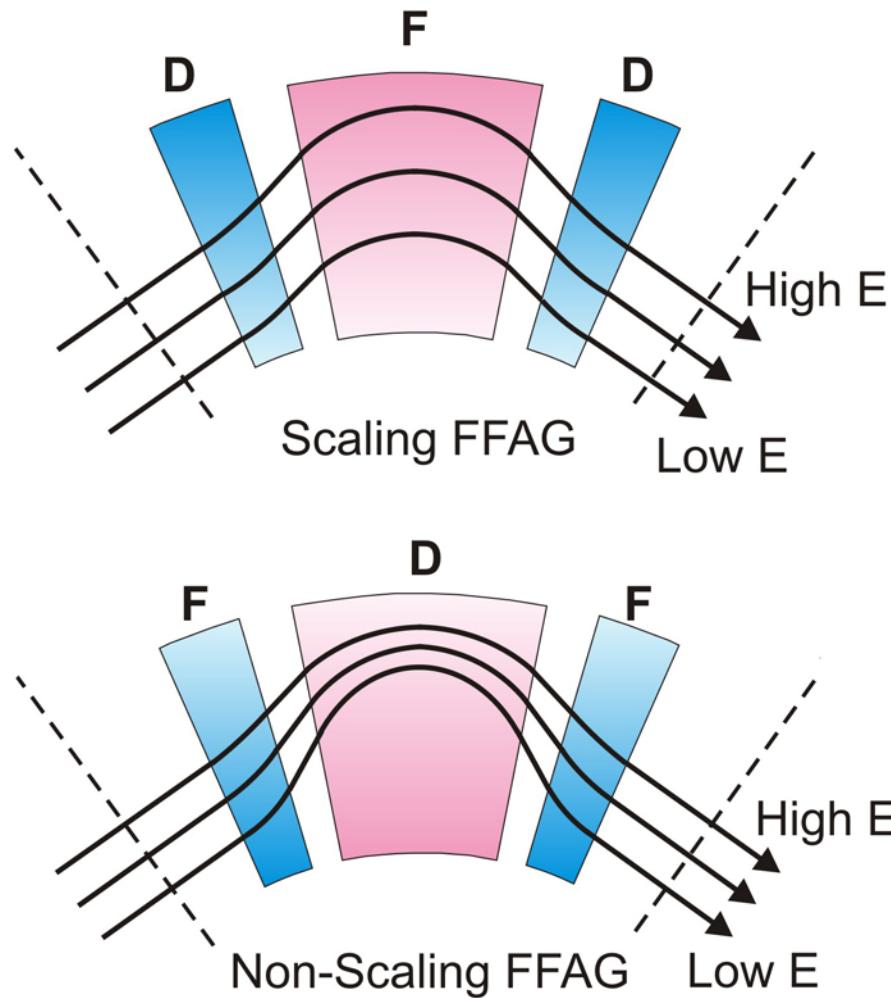
The large ring delivers 68-400 MeV/u C^{6+} ions

Superconducting 2-, 4-, 6- & 8-pole magnets keep the tunes constant.

Neutrino Factory : FFAG based



SCALING v. LINEAR NON-SCALING FFAGs



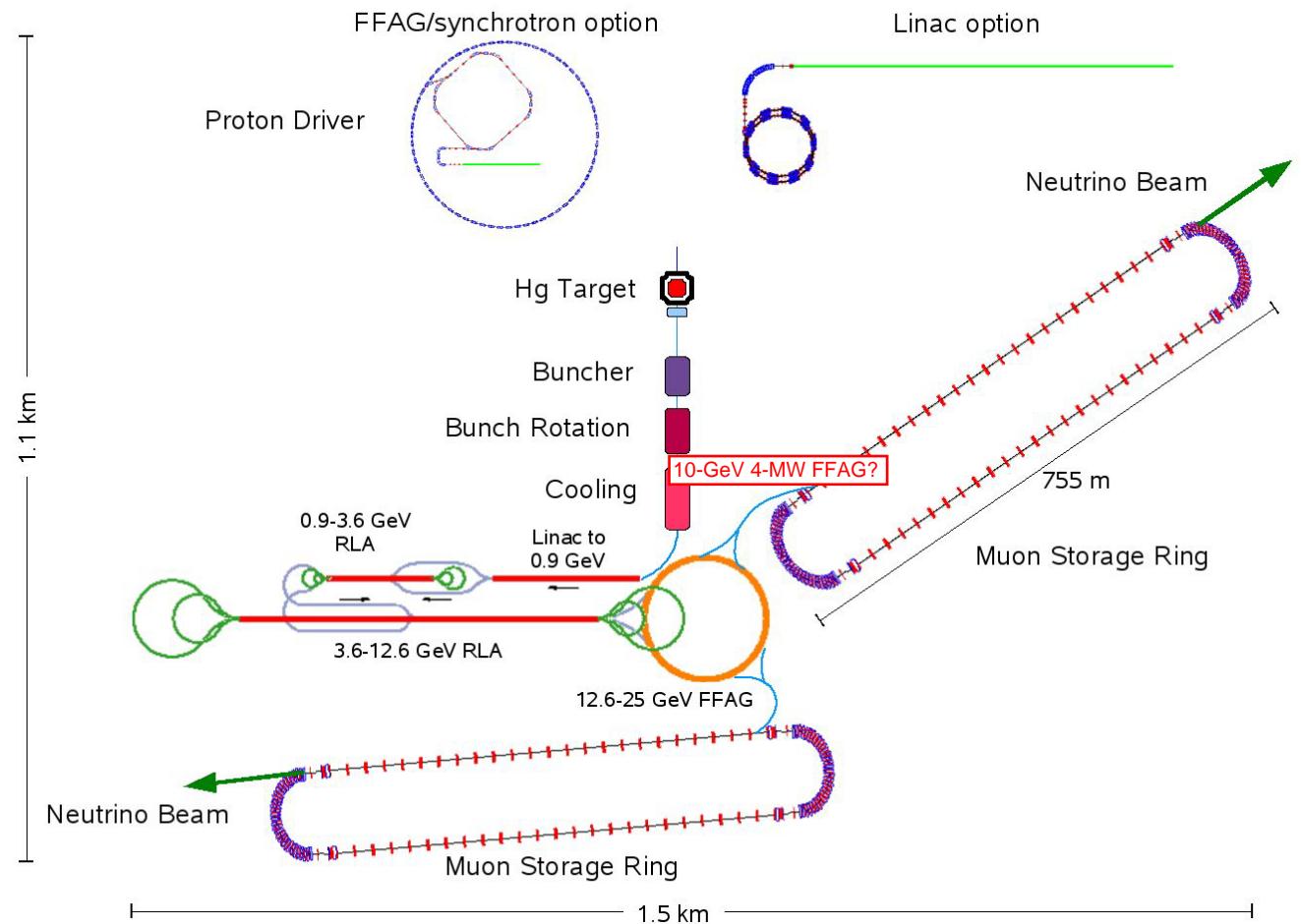
$$C(p) = C(p_m) + \frac{12\pi^2}{e^2 q^2 N L_{FD}} (p - p_m)^2$$

Non-scaling FFAGs using linear (i.e. constant-gradient) magnets offer :

- Greater momentum compaction (& hence narrower radial apertures);
- No high-order multipole field components to drive betatron resonances;
- Simpler construction.

NEUTRINO FACTORY - INTERNATIONAL DESIGN STUDY

Until March 2012
the IDS design
included **two non-**
scaling FFAGs:
- a high-intensity
10-GeV proton
"driver"
- a 12.6-25 GeV
muon accelerator



But then the Daya Bay reactors discovered a large ν mixing angle θ_{13} , lowering the optimum μ energy for ν production to 10 GeV - so for the final stage there's now a choice: 5-10 GeV FFAG, or 3-10 GeV RLA.
But for a few-TeV muon collider, NS-FFAGs are the only feasible option.

ELECTRON MODEL FFAG "EMMA"

A Proof of Principle machine for linear-B non-scaling FFAGs

to demonstrate their two novel features:

- Safe passage through many low-order structural resonances
- Acceleration outside buckets.

Studies have focused on an electron FFAG - with relativistic parameters similar to one for 10-20 GeV muons:

Energy	10-20 MeV
Circumference	15.9 m
Cells	42
B_{max}	0.2 T
F quad length	6.0 cm
D quad length	6.8 cm
RF frequency	1.3 GHz
Volts/cavity	19 kV



Daresbury Lab (UK) has offered lab space and a 7-35 MeV injector.

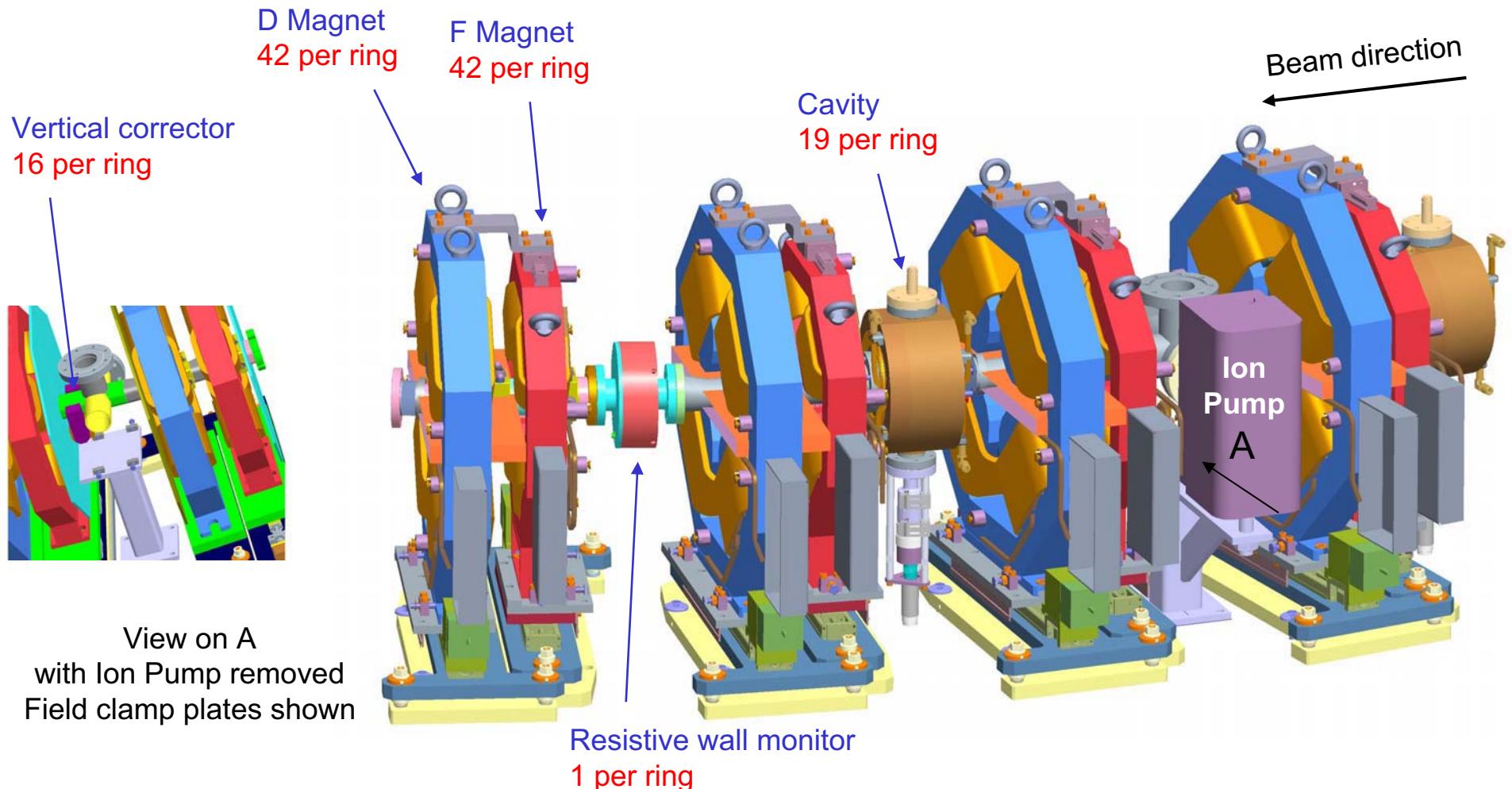
UK funding (\$16M) started April 2007.

NSERC grant awarded to Canadian collaborators.

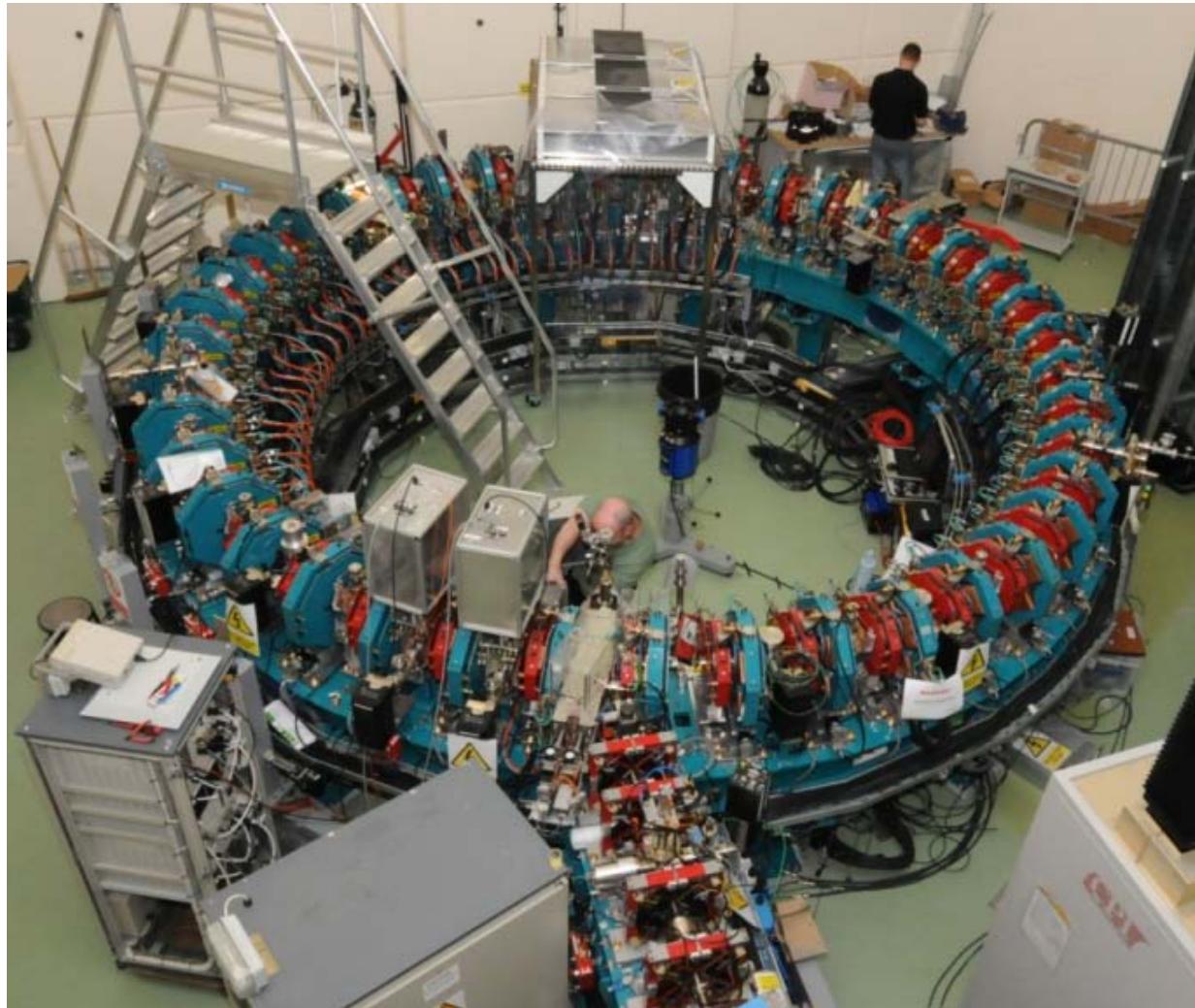


4 CELLS

Without field clamp plates shown

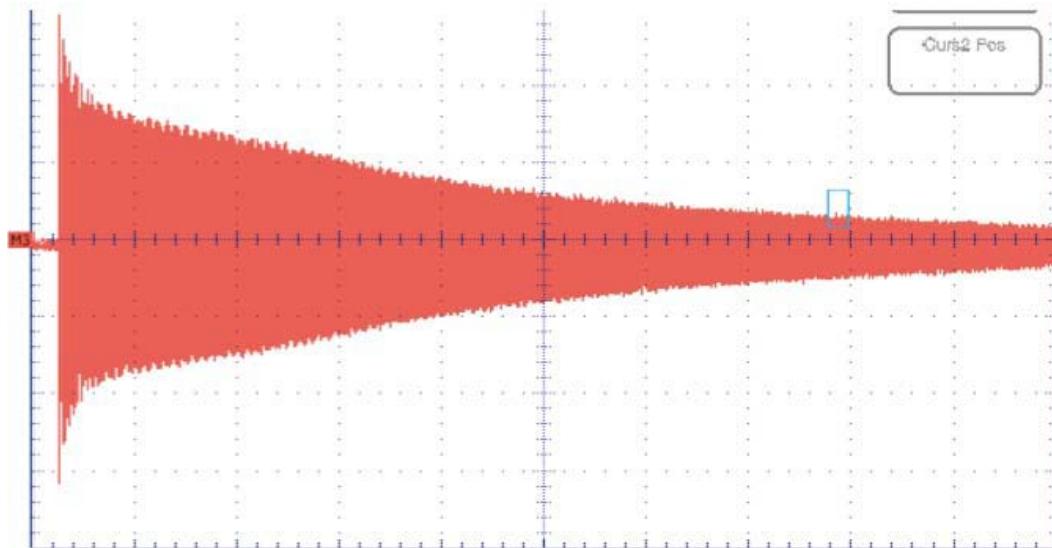


EMMA - THE FIRST NON-SCALING FFAG

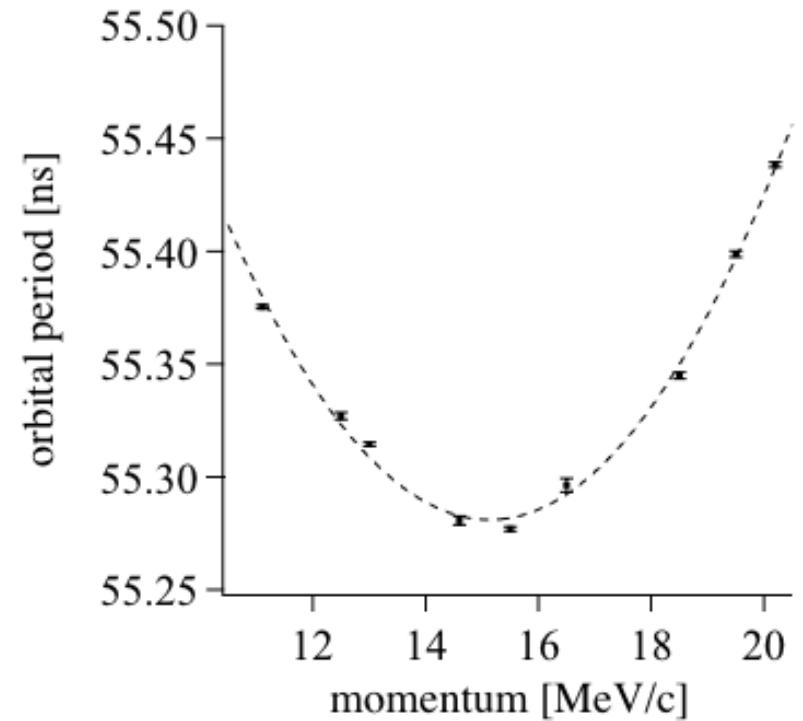


EMMA is a 10-20 MeV electron LNS-FFAG model
for a 10-20 GeV muon accelerator for a neutrino factory
- currently undergoing beam commissioning at Daresbury, UK.

EMMA COMMISSIONING - 1

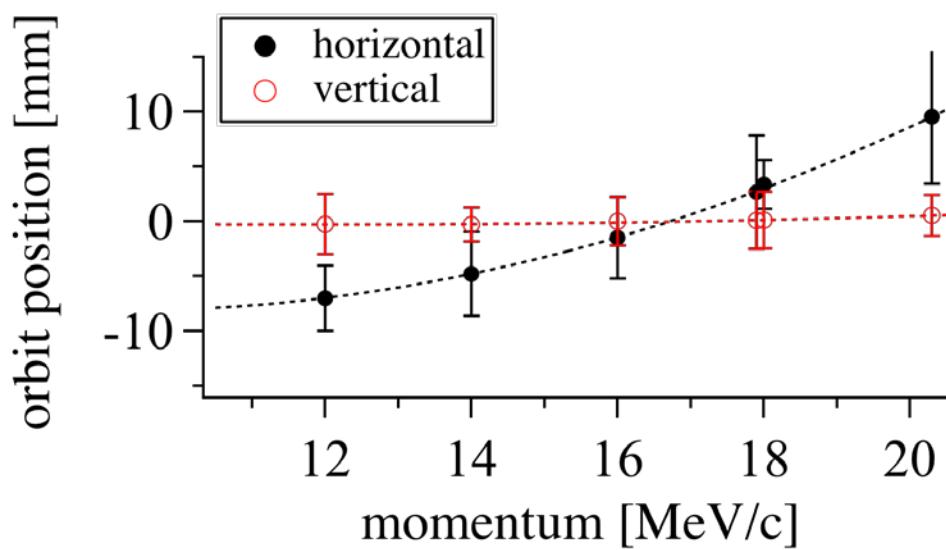


Left: Beam current v. time at fixed momentum (first observations of circulating beam show survival for > 1000 turns - August 2010).

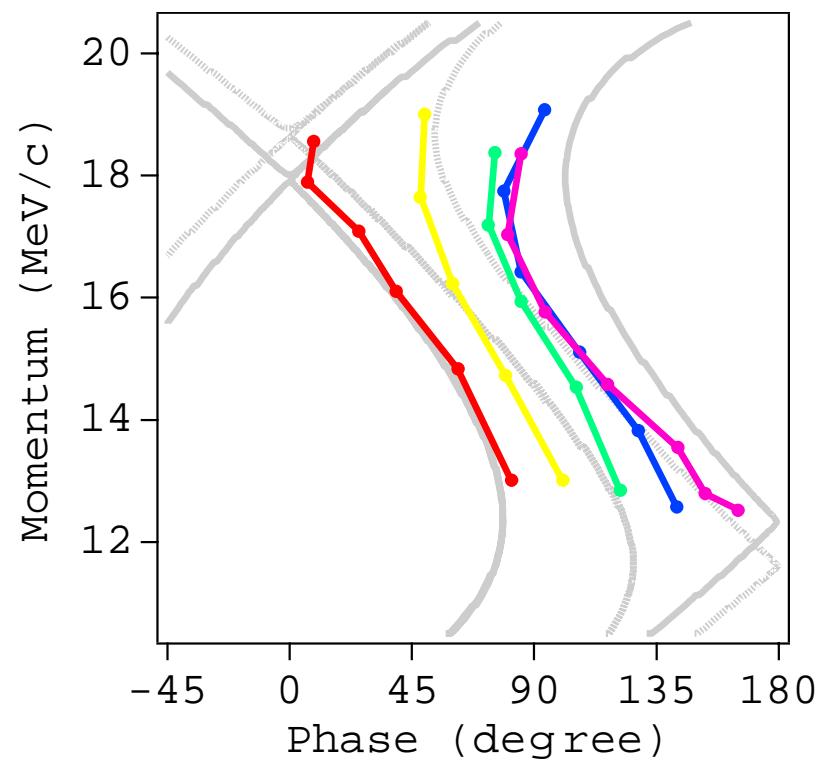


Right: Orbital period v. momentum, showing the expected parabolic dependence.

EMMA COMMISSIONING - 2

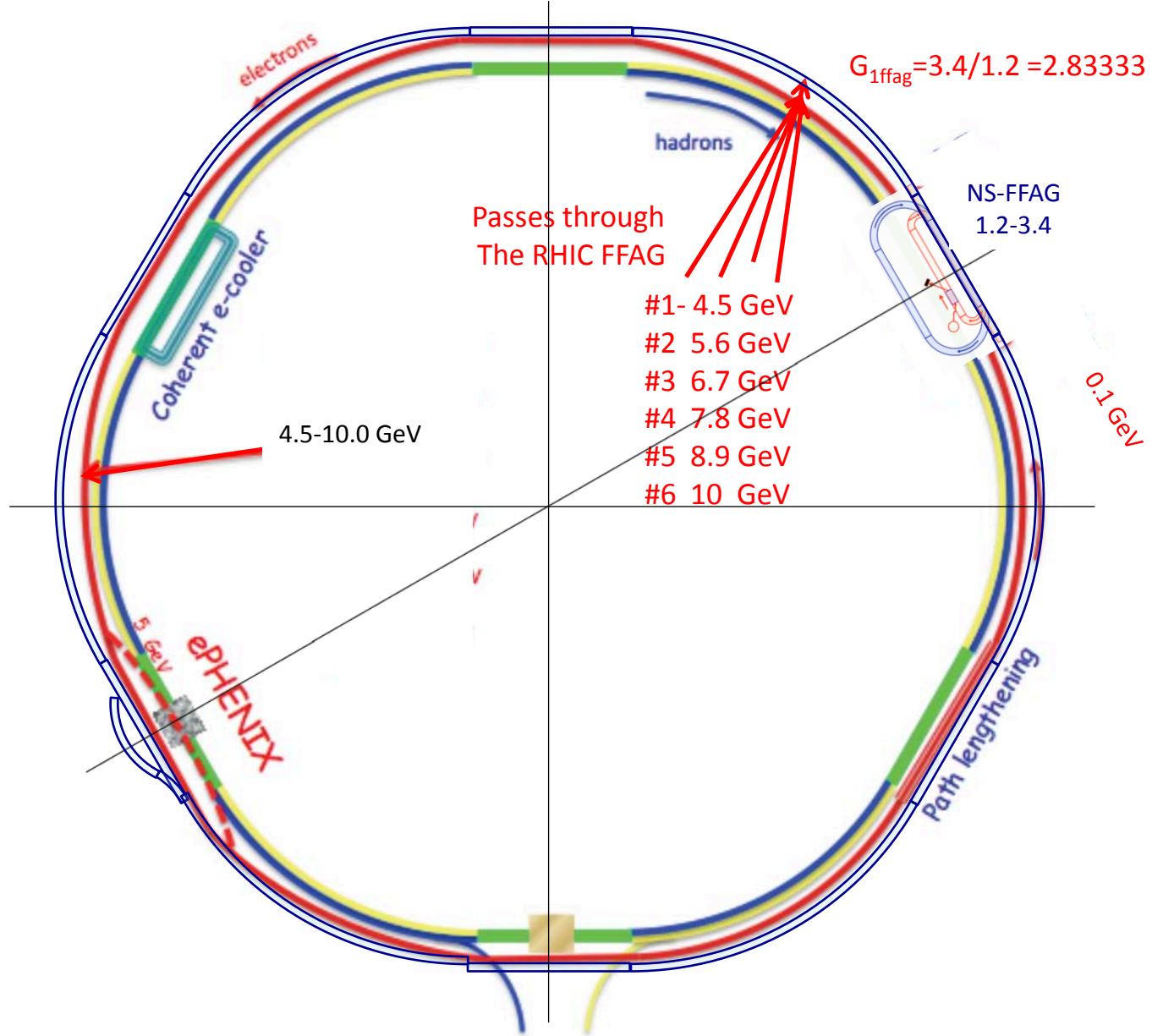


Left: Orbit position vs momentum



Right: Momentum versus rf phase, for 5 different initial phases (colour), compared to theoretical limits (grey).

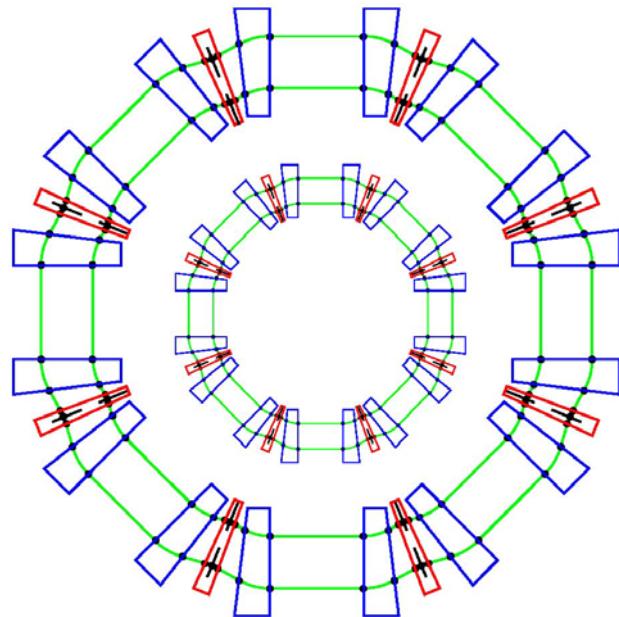
NS – FFAG's: 1.2→3.4 & 4.5→10 GeV



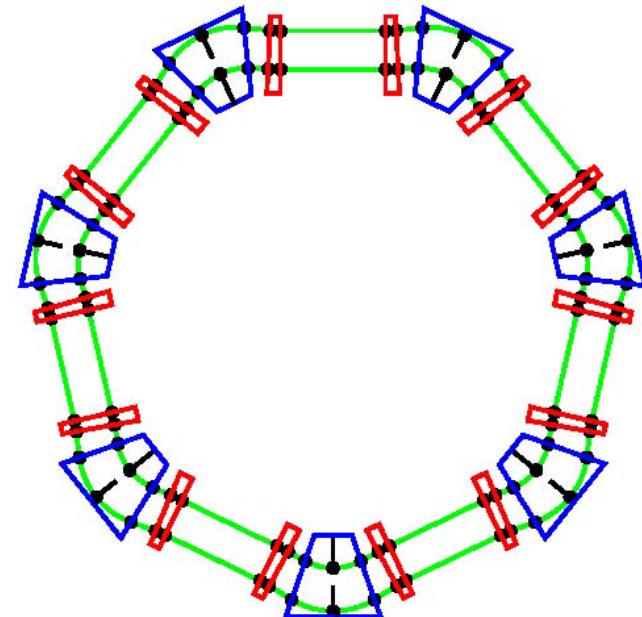
#2 FFAG	in li
11.1 GeV	→
12.2 GeV	→
13.3 GeV	→
14.4 GeV	→
15.6 GeV	→
16.7 GeV	→
17.8 GeV	→
18.9 GeV	→
20.0 GeV	→
21.1 Gev	→
22.2 GeV	→
23.3 GeV	→
24.4 GeV	→
25.5 GeV	→

NON-LINEAR NON-SCALING FFAGs

By using radial-sector magnets with non-linear fields, [Rees \(2004\)](#) and [Johnstone \(below, 2008-12\)](#) have been able to devise NS-FFAGs for GeV muons and ions that have **constant tunes** and are also **isochronous**.

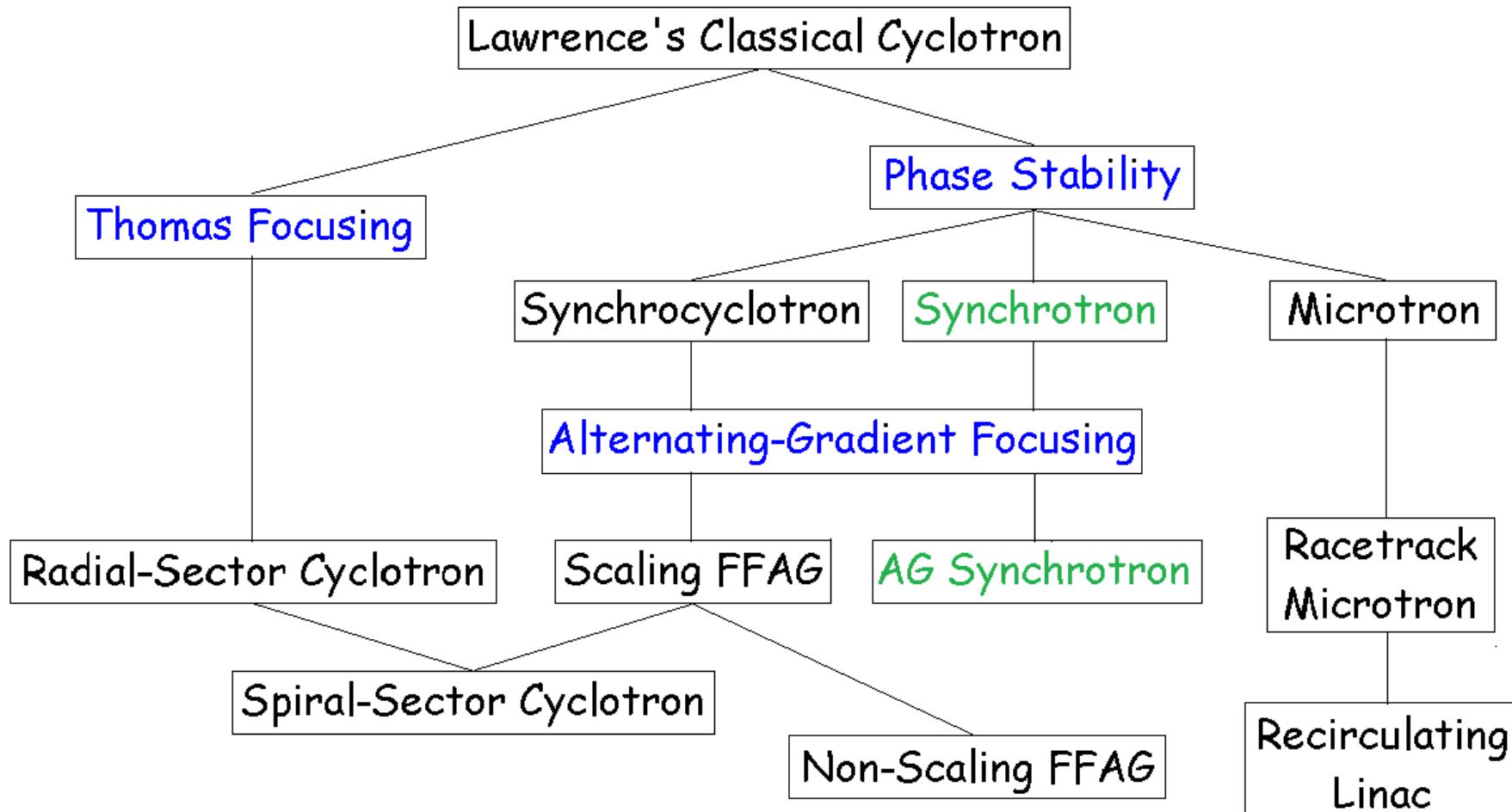


A pair of 8-cell-FDF rings form a multi-ion cancer treatment facility. The inner ring (orbit radii 2.75-3.39 m) takes protons from 30-250 MeV, and acts as injector to the outer ring (5.5-6.9 m) for carbon ions (68-400 MeV/u).



A 7-cell DFD ring for Accelerator-Driven Subcritical-Reactor (ADSR) operation. A high-intensity proton beam is accelerated from 0.33-1.0 GeV and from an injection radius of 4.35 m to an extraction radius of 5.13 m.

THE FAMILY TREE OF “CIRCULAR” RF ACCELERATORS



Fixed magnetic-field accelerators (cyclotron family) in black

Varying magnetic-field accelerators (synchrotron family) in green

<http://trshare.triumf.ca/~craddock/PartAccel.pdf>