

Precalculus Problem Set 1

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Contents

0	Preface	1
1	Composite and Inverse Functions	1

0 Preface

I wrote this collection of problems in order to help my student's develop creative mathematical thinking and master their ability to solve Pre Calculus Problems. All problems should be solved without a calculator unless otherwise specified.

1 Composite and Inverse Functions

Problem 1.1

Let $f(x) = x^2$ and $g(x) = \sqrt{x+1}$. Write what each composition of functions is equal to

i) $f \circ g$

ii) $g \circ f$

iii) $g \circ g$

iv) $f \circ f$

v) $f \circ (g \circ f)$

Problem 1.2

Let $f(x) = ax + \frac{b}{x}$ and $g(x) = \frac{1}{x}$. Write what each composition of functions is equal to:

i) $f(g(x))$

ii) $g(g(x))$

iii) $g(f(x))$

iv) $f(f(x))$

Problem 1.3

Write each of the following functions as a composition of functions:

i) $h(x) = (x-1)^2$

ii) $h(x) = (x+1)^2 + (x+1)$

$$iii) \ h(x) = \sqrt{x^3 + x + 1}$$

$$iv) \ h(x) = x$$

Problem 1.4

Determine if the following pairs of functions are each other's inverses by composition of functions:

$$i) \ f(x) = (x - 1)^3 \text{ and } g(x) = x^{\frac{1}{3}} + 1$$

$$ii) \ f(x) = x + 1 \text{ and } g(x) = x - 1$$

$$iii) \ f(x) = \frac{x+1}{3} \text{ and } g(x) = 3x - 3$$

$$iv) \ f(x) = \frac{x-1}{x} \text{ and } g(x) = \frac{1}{1-x}$$

Problem 1.5

Find the inverses of the following functions. Once you find the inverse, verify that you are correct by composition of functions.

$$i) \ f(x) = 3x + 1$$

$$ii) \ f(x) = (4x + 1)^5$$

$$iii) \ f(x) = 1/x$$

$$iv) \ f(x) = x$$

Problem 1.6

Come up with three functions $f(x), g(x), h(x)$ such that the following holds:

$$f(g(h(x))) = \sqrt{\sin(\sqrt{x+1})}$$

Problem 1.7

Come up with two function $g(x)$ and $h(x)$ such that the following is true

$$g(g(h(x))) = \sin(x)$$

Problem 1.8

If $f(x)$ and $g(x)$ are inverses, what is the inverse of $g(f(g(x)))$?

Problem 1.9

Come up with two function $f(x)$ and $g(x)$ such that $g(f(x)) = 0$ and $f(g(x)) = 1$.

Problem 1.10

Given that $A(x) = \frac{1+x}{1-x}$, what is $A^{-1}(x)$? Verify your answering by showing that

$$A(A^{-1}(x)) = A^{-1}(A(x)) = x$$