14.1 Functions of Several Variables

Level Curves $f(x, y) = k$ where k is a constant.

Level Surfaces $f(x, y, z) = k$

14.2 Limits and Continuity

Definition of limits

Let f be a function of two variables whose domain D includes points arbitrarily close to (a, b).

$$\lim_{(x,y)\to(a,b)} f(x, y) = L \tag{1}$$

if for every number $\varepsilon > 0$ there is a corresponding number $\delta > 0$ such that

$$\text{if} \quad (x, y) \in D \text{ and } 0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta \quad \text{then} \quad |f(x, y) - L| < \varepsilon \tag{2}$$

When the Limit Does Not Exist

If we can find two different paths of approach along which the function $f(x, y)$ has different limits, then it follows that $\lim_{(x,y)\to(a,b)} f(x, y)$ does not exist.

Limit Laws Approaching along axes and a constant

$$\lim_{(x,y)\to(a,b)} x = a \qquad \lim_{(x,y)\to(a,b)} y = b \qquad \lim_{(x,y)\to(a,b)} c = c \tag{3}$$

The **Squeeze Theorem** also holds.

Continuity

The direct substitution property: A function f is called
continuous at (a, b) if

$$\lim_{(x,y)\to(a,b)} f(x, y) = f(a, b) \tag{4}$$

We say f is **continuous on** D if f is continuous at every point (a, b) in D.

14.3 Partial Derivatives

Partial Derivatives

$$f_x(a, b) = g'(a) \quad \text{where} \quad gx = f(x, b) \tag{1}$$