15.1 Double Integrals Over Rectangles

Iterated Integrals

Fubini's Theorem allows us to switch the order of integration. For $a \leq x \leq b, c \leq y \leq d$,

$$\iint_R f(x,y)\,dA = \int_a^b \int_c^d f(x,y)\,dy\,dx = \int_c^d \int_c^d f(x,y)\,dx\,dy \tag{1}$$

15.2 Double Integrals Over General Regions

Integrals Between Curves

Type I Region lies between two functions of x, that is

$$a \leq x \leq b, \qquad g_1(x) \leq y \leq g_2(x)$$

Type II Region lies between two functions of y, that is

$$c \leq y \leq d, \qquad h_1(x) \leq x \leq h_2(x)$$

To solve these, make sure the function bounds are in the inner integral. See textbook for images.
Area is defined as $\iint 1\,dA$.

Switching Order of Integration

Integrals can be switched as long as the region is the same. For example, the region defined by

$$0 \leq x \leq 4, \sqrt{x} \leq y \leq 2$$

is the same region as

$$0 \leq y \leq 2, 0 \leq x \leq y^2$$

Draw a picture!

15.3 Double Integrals in Polar Coordinates

$$r^2 = x^2 + y^2 \qquad x = r\cos\theta \qquad y = r\cos\theta \tag{1}$$

$$\iint_R f(x,y)\,dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\cos\theta) r\,dr\,d\theta \tag{2}$$

The "infinitesimal rectangle" $dA = dx\,dy = r\,dr\,d\theta$.

15.4 Applications of Double Integrals

Omitted.

15.5 Surface Area

$$A = \iint_D \sqrt{f_x(x,y)^2 + f_y(x,y)^2 + 1}\, dA \tag{1}$$

Note the similarity to the arc length formula.

15.6 Triple Integrals

Iterated Integrals

Fubini's Theorem Allows us to switch the order of integration. There are different types of regions as well, defined between two functions. Make sure the functions are in the inner integrals.