Assignment 4

1. Initialize Value Function

$$V_0(S_1) = 10.0 \quad , \quad V_0(S_2) = 1.0 \quad V_0(S_3) = 0.0$$

$$Q_k(s,a) = R(s,a) + \Sigma P(s,a,s') V_{k-1}(s')$$

$k=1$

$$Q_1(S_1, a_1) = 8.0 + 0.2 \times 10 + 0.6 \times 1.0 = 10.6$$
$$Q_1(S_1, a_2) = 10.0 + 0.1 \times 10 + 0.2 \times 1.0 = 11.2$$
$$Q_1(S_2, a_1) = 1.0 + 0.3 \times 10 + 0.3 \times 1.0 = 4.3$$
$$Q_1(S_2, a_2) = -1.0 + 0.5 \times 10 + 0.3 \times 1.0 = 4.3$$

$$\pi_1(S_1) = a_2 \qquad \pi_1(S_2) = a_1$$

$$V_1(S_1) = 11.2 \qquad V_2(S_1) = 4.3$$

$k=2$

$$Q_2(S_1, a_1) = 8.0 + 0.2 \times 11.2 + 0.6 \times 4.3 = 12.82$$
$$Q_2(S_1, a_2) = 10.0 + 0.1 \times 11.2 + 0.2 \times 4.3 = 11.98$$

$$Q_2(S_2, a_1) = 1.0 + 0.3 \times 11.2 + 0.3 \times 4.3 = 5.65$$
$$Q_2(S_2, a_2) = -1.0 + 0.5 \times 11.2 + 0.3 \times 4.3 = 5.89$$

$$\pi_2(S_1) = a_1 \qquad \pi_2(S_2) = a_2$$

$$V_2(S_1) = 12.82 \qquad V_2(S_2) = 5.89$$

2. $\because P(s_1, a_1, s') > P(s_1, a_2, s')$ for $\forall s' \in \{s_1, s_2\}$

$P(s_2, a_1, s') \leq P(s_2, a_2, s')$

$$Q_k(s_1, a_1) - Q_k(s_1, a_2)$$

$$= -2.0 + 0.1 V_{k-1}(s_1) + 0.4 V_{k-1}(s_2)$$

We know that $V_k(s) \geq V_{k'}(s)$ for $k > k'$

$\therefore \quad Q_2(s_1, a_1) - Q_2(s_1, a_2) > 0$

$\Rightarrow Q_k(s_1, a_1) - Q_k(s_1, a_2) > 0$ for $\forall k > 2$

Similarly

$$Q_k(s_2, a_1) - Q_k(s_2, a_2) < 0 \text{ for } \forall k > 2$$

$\therefore \quad \pi_k(s_1) = a_1 \quad \pi_k(s_2) = a_2$ for $\forall k > 2$