Assignment 8

2. $$g(S) = p\, g_1(S) + h\, g_2(S)$$

$$g_1(S) = \int_S^{\infty} (x-S) f(x)\, dx$$

$$= \int_{-\infty}^{\infty} (x-S) f(x)\, dx - \int_{-\infty}^{S} (x-S) f(x)\, dx$$

$$= \mu - S - \int_{-\infty}^{S} (x-S) f(x)\, dx$$

$$= \mu - S - \int_{-\infty}^{S} x f(x)\, dx + S \int_{-\infty}^{S} f(x)\, dx$$

$$= \mu - S - \int_{-\infty}^{S} x f(x)\, dx + S F(S)$$

$$= \mu - S - \int_{-\infty}^{S} x\, dF(x) + S F(S)$$

$$= \mu - S - x F(x) \Big|_{-\infty}^{S} + \int_{-\infty}^{S} F(x)\, dx + S F(S)$$

$$= \mu - S + \int_{-\infty}^{S} F(x)\, dx$$

Similarly,

$$g_2(S) = \int_{-\infty}^{S} (S-x) f(x)\, dx$$

$$= S\int_{-\infty}^{S} f(x)\, dx - \int_{-\infty}^{S} x f(x)\, dx$$

$$= SF(S) - \int_{-\infty}^{S} x\, dF(x)$$

$$= SF(S) - xF(x)\Big|_{-\infty}^{S} + \int_{-\infty}^{S} F(x)\, dx$$

$$= \int_{-\infty}^{S} F(x)\, dx$$

$$g(S) = p\, g_1(S) + h\, g_2(S)$$

$$= p\left(\mu - S + \int_{-\infty}^{S} F(x)\, dx\right) + h\int_{-\infty}^{S} F(x)\, dx$$

$$= p(\mu - S) + (p+h)\int_{-\infty}^{S} F(x)\, dx$$

$$\frac{dg(S)}{dS} = 0 \Rightarrow -p + (p+h)F(S) = 0$$

$$F(S) = \frac{p}{p+h}$$

$$S = F^{-1}\left(\frac{p}{p+h}\right)$$

We can frame this problem in terms of call/put option.

$g_1(S) = E\{\max(x-S, 0)\}$ is the price of call option

$g_2(S) = E\{\max(S-x, 0)\}$ is the price of put option

p and h are shares of put/call option in the portfolio

$g(S)$ is the price of the portfolio