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Assignment 8

2. $$\begin{cases} P_{t+1} = P_t e^{z_t} \\ X_{t+1} = \rho X_t + \eta_t \\ Q_t = P_t(1-\beta N_t - \theta X_t) \end{cases}$$

$z_t \sim N(\mu_z, \sigma_z^2)$ $\quad E\{\eta_t] = 0$ $\quad$ iid

$$V_t^\pi((P_t, \overset{x_t}{R_t})) = E_\pi\{\sum_{i=t}^{T} N_i P_i (1-\beta N_i - \theta X_i) \mid (P_t, \overset{x_t}{R_t})]$$

$$V_t^\pi((P_t, \overset{x_t}{R_t})) = \max_{N_t} \{ N_t P_t (1-\beta N_t - \theta X_t)$$
$$+ E\{V_{t+1}^\pi((P_{t+1}, R_{t+1}))]$$

$t = T-1$

$$V_{T-1}^\pi((P_{T-1}, \overset{X_{T-2}}{R_{T-1}})) = N_{T-1} P_{T-1}(1-\beta N_{T-1} - \theta X_{T-1})$$
$$= R_{T-1} P_{T-1}(1-\beta R_{T-1} - \theta X_{T-1})$$

$t = T-2$

$$V_{T-2}^\pi = \max_{N_{T-2}} \{ N_{T-2} P_{T-2}(1-\beta N_{T-2} - \theta X_{T-2})$$
$$+ E\{R_{T-1} P_{T-1}(1-\beta R_{T-1} - \theta X_{T-1})]\}$$

$$= \max_{N_{T-2}} \{ N_{T-2} P_{T-2} (1-\beta N_{T-2} - \theta X_{T-2})$$

$$+ E \{ (R_{T-2} - N_{T-2}) P_{T-2} e^{z_{T-2}} (1-\beta (R_{T-2} - N_{T-2})$$

$$- \theta (\rho X_{T-2} + \eta_{T-2}))]$$

$$\underset{z \sim N(\mu_z, \sigma_z^2)}{E} [e^{z}] = e^{\mu_z + \frac{1}{2}\sigma_z^2} \qquad E\{\eta_t] = 0$$

$$\Rightarrow = \max_{N_{T-2}} \{ N_{T-2} P_{T-2} (1-\beta N_{T-2} - \theta X_{T-2})$$

$$+ (R_{T-2} - N_{T-2}) P_{T-2} (1-\beta (R_{T-2} - N_{T-2})$$

$$- \theta \rho X_{T-2}) \cdot e^{\mu_z + \frac{1}{2}\sigma_z^2} \}$$

$$\phi(N_{T-2}) = N_{T-2} P_{T-2} (1-\beta N_{T-2} - \theta X_{T-2})$$

$$+ (R_{T-2} - N_{T-2}) P_{T-2} (1-\beta (R_{T-2} - N_{T-2}) - \theta \rho X_{T-2})$$

$$\times e^{\mu_z + \frac{1}{2}\sigma_z^2}$$

$$\frac{\partial \phi (N_{T-2})}{\partial N_{T-2}} = 0 \Rightarrow$$

$$N_{T-2}^{\gamma} = \frac{1}{2(1+e^{\mu_z+\frac{1}{2}\sigma_z^2})\beta} \times \Big(1 - e^{\mu+\frac{1}{2}\sigma_z^2}$$

$$+ (e^{\mu+\frac{1}{2}\sigma^2}\theta\rho - \theta) x_{T-2}$$

$$+ 2\beta e^{\mu_z+\frac{1}{2}\sigma_z^2} R_{T-2} \Big)$$

$$V_{T-2}^{\gamma} = \phi(N_{T-2}^{\gamma})$$

Follow the same method should give us

V_t^{γ} N_t^{γ}