# Assignment 3

2. $$V^* = \max_{a} \{ R(s,a) + \gamma \sum_{s' \in N} P(s,a,s') V^*(s') \}$$

$$= \max_{a \in [0,1]} \{ R(s,a) + \gamma \left( a V^*(s+1) + (1-a) V^*(s) \right)$$

$$R(s,a) = E\{ R_{t+1} \mid S_t = s, R_t = a \}$$

$$= a(1-a) + (1-a)(1+a)$$

$$= a - a^2 + 1 - a^2$$

$$= -2a^2 + a + 1$$

According to the symmetry (s goes from 1 to infinity), we should have that

$$V^*(s+1) = V^*(s) = V^*$$

$$V^* = -2a^2 + a + 1 + \gamma ( a V^*(s+1) + (1-a) V^*(s) )$$

$$V^{\gamma} = -2a^2 + a + 1 + \gamma V^{\gamma}$$

$$V^{\gamma} = \frac{-2a^2 + a + 1}{1 - \gamma}$$

$$\frac{\partial V^{\gamma}}{\partial a} = 0 \qquad a^{*} = \frac{1}{4} \qquad \pi^{\gamma}(s) = \frac{1}{4}$$

$$V^{\gamma} = \frac{-2 \times \frac{1}{16} + \frac{1}{4} + 1}{0.5} = \frac{9}{4}$$

3. $$R(s,a) = E[R_t \mid S_t = s, A_t = a]$$

$$= -E[e^{as'} \mid S_t = s, A_t = a]$$

$$= -\underset{s' \sim N(s, \sigma^2)}{E}[e^{as'}] = M[a]$$

$$= -e^{as + \frac{1}{2}\sigma^2 a^2}$$

$$V^{\gamma} = \max_{a \in R} \{-e^{as + \frac{1}{2}\sigma^2 a^2}\}$$

$$\frac{\partial}{\partial a}(-e^{as + \frac{1}{2}\sigma^2 a^2}) = -(s + \sigma^2 a) e^{as + \frac{1}{2}\sigma^2 a^2} = 0$$

$$a = -\frac{s}{\sigma^2}$$

$$\frac{\partial}{\partial^2 a}\left(-e^{as+\frac{1}{2}\sigma^2 a^2}\right) = -\left(\sigma^2 e^{as+\frac{1}{2}\sigma^2 a^2} + (s+\sigma^2 a)^2 e^{\cdots}\right)$$

$$= -\left(\sigma^2 + (s+\sigma^2 a)^2\right) e^{as+\frac{1}{2}\sigma^2 a^2} \Big|_{a=-\frac{s}{\sigma^2}}$$

$$= -\sigma^2 e^{as+\frac{1}{2}\sigma^2 a^2} < 0$$

$$\therefore V^* = -e^{-\frac{s^2}{\sigma^2}+\frac{s^2}{2\sigma^2}} = -e^{-\frac{s^2}{2\sigma^2}}$$

$$a^* = \pi(s) = -\frac{s}{\sigma^2}$$