

Deterministic epidemic model with n migratory neighboring populations

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1 Equations

c = central city

n = total neighbor cities

i = 1,2,3,4 ... n subscript for each neighboring city

T = n + 1 = Total number of cities

S = number of susceptible people [# of people]

I = number of infected people [# of people]

R = number of recovered people [# of people]

D = number of dead people [# of people]

N = number of alive people [# of people]

W = constant total number of existing people dead or alive [# of people]

a = infectivity rate constant [1/time]

b = recovery rate constant [1/time]

d = death rate constant [1/time]

m = migration rate constant [1/time]

Differential equations for Central City c

$$\frac{dS_c}{dt} = -(S_c * \frac{I_c}{N_c} * a_c) + (-(T-1)(m_{Sc} * S_c) + (m_{S1} * S_1) + (m_{S2} * S_2) + ... + (m_{Sn} * S_n)) \quad (1)$$

$$\frac{dI_c}{dt} = (S_c * \frac{I_c}{N_c} * a_c) - (I_c * b_c) - (I_c * d_c) + (-(T-1)(m_{Ic} * I_c) + (m_{I1} * I_1) + (m_{I2} * I_2) + ... + (m_{In} * I_n)) \quad (2)$$

$$\frac{dR_c}{dt} = (I_c * b_c) + (-(T-1)(m_{Rc} * R_c) + (m_{R1} * R_1) + (m_{R2} * R_2) + ... + (m_{Rn} * R_n)) \quad (3)$$

$$N_c = S_c + I_c + R_c \quad (4)$$

$$W_c = S_c + I_c + R_c + D_c = N_c + I_c \quad (5)$$

Differential equations for i Neighbor Cities

$$\frac{dS_i}{dt} = -(S_i * \frac{I_i}{N_i} * a_i) + ((m_{Sc} * S_c) - (m_{Si} * S_i)) \quad (6)$$

$$\frac{dI_i}{dt} = (S_i * \frac{I_i}{N_i} * a_i) - (I_i * b_i) - (I_i * d_i) + ((m_{Ic} * I_c) - (m_{Ii} * I_i)) \quad (7)$$

$$\frac{dR_i}{dt} = (I_i * b_i) + (m_{Rc} * R_c) - (m_{Ri} * R_i) \quad (8)$$

$$N_i = S_i + I_i + R_i \quad (9)$$

$$W_i = S_i + I_i + R_i + D_i = N_i + D_i \quad (10)$$

$$\begin{aligned}
\text{Migration Matrix} &= \begin{bmatrix} (m_{Sc}S_c) & (m_{S1}S_1) & \dots & (m_{Sn}S_n) \\ (m_{Ic}I_c) & (m_{I1}I_1) & \dots & (m_{In}I_n) \\ (m_{Rc}R_c) & (m_{R1}R_1) & \dots & (m_{Rn}R_n) \end{bmatrix} \\
\text{Flow Matrix} &= \begin{bmatrix} -(T-1) & 1 & 1 & 1 & \dots & 1 \\ 1 & -1 & 0 & 0 & \dots & 0 \\ 1 & 0 & -1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & 0 & 0 & 0 & \dots & -1 \end{bmatrix} \\
\text{Net Migration matrix} &= \text{Migration Matrix} * \text{Flow Matrix}
\end{aligned}$$

Simpler model

assume all satellite cities parameters are equal

$$\text{eg. } a_1 = a_2 = \dots = a_n, \quad b_1 = b_2 = \dots = b_n, \quad \dots$$

Central City

$$\frac{dS_c}{dt} = -(S_c * \frac{I_c}{N_c} * a_c) + (T-1)[(m_{S1} * S_1) - (m_{Sc} * S_c)] \quad (11)$$

$$\frac{dI_c}{dt} = (S_c * \frac{I_c}{N_c} * a_c) - (I_c * b_c) - (I_c * d_c) + (T-1)[(m_{I1} * I_1) - (m_{Ic} * I_c)] \quad (12)$$

$$\frac{dR_c}{dt} = (I_c * b_c) + (T-1)[(m_{R1} * R_1) - (m_{Rc} * R_c)] \quad (13)$$

Satellite cities

$$\frac{dS_1}{dt} = -(S_1 * \frac{I_1}{N_1} * a_1) + ((m_{Sc} * S_c) - (m_{S1} * S_1)) \quad (14)$$

$$\frac{dI_1}{dt} = (S_1 * \frac{I_1}{N_1} * a_1) - (I_1 * b_1) - (I_1 * d_1) + ((m_{Ic} * I_c) - (m_{I1} * I_1)) \quad (15)$$

$$\frac{dR_1}{dt} = (I_1 * b_1) + ((m_{Rc} * R_c) - (m_{R1} * R_1)) \quad (16)$$

general forward Euler's method

$$x_{t+1} = x(t + \Delta t) = x_t + f(x_t)\Delta t$$