# Lateral Feedback and Lateral Feedforward Inhibition in Visual Processing (Equations)

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## 1 One-dimensional Stimuli

y = neural output from ganglion cell [$stimulus$]
x = stimuli received from photoreceptor [$stimulus$]
I = inhibition coefficient
n = number of neurons
i = index of non-edge neurons (2,3,4,5,... n-1)

### 1.1 Feed-forward

$$y_1 = x_1 - I * (x_2) \tag{1}$$
$$y_i = x_i - I * (x_{i-1} + x_{i+1}) \tag{2}$$
$$y_n = x_n - I * (x_{n-1}) \tag{3}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ ... \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & -I & 0 & 0 & ... & 0 & 0 & 0 \\ -I & 1 & -I & 0 & 0 & ... & 0 & 0 \\ 0 & -I & 1 & -I & 0 & 0 & ... & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... \\ 0 & 0 & ... & 0 & 0 & 0 & -I & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ ... \\ x_n \end{bmatrix}$$

### 1.2 Feed-back

$$y_1 = x_1 - I * (y_2) \tag{4}$$
$$y_i = x_i - I * (y_{i-1} + y_{i+1}) \tag{5}$$
$$y_n = x_n - I * (y_{n-1}) \tag{6}$$

Solving Linear system
$x_1 = y_1 + I * (y_2)$
$x_i = y_i + I * (y_{i-1} + y_{i+1})$
$x_n = y_n + I * (y_{n-1})$

$$\begin{bmatrix} 1 & I & 0 & 0 & ... & 0 & 0 & 0 \\ I & 1 & I & 0 & 0 & ... & 0 & 0 \\ 0 & I & 1 & I & 0 & 0 & ... & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... \\ 0 & 0 & ... & 0 & 0 & 0 & I & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ ... \\ y_n \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ ... \\ x_n \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ ... \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & I & 0 & 0 & ... & 0 & 0 & 0 \\ I & 1 & I & 0 & 0 & ... & 0 & 0 \\ 0 & 1 & I & 1 & 0 & 0 & ... & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... \\ 0 & 0 & ... & 0 & 0 & 0 & I & 1 \end{bmatrix}^{-1} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ ... \\ x_n \end{bmatrix}$$

# 2 Two-dimensional Stimuli

y = neural output from ganglion cell [*stimulus*]
x = stimuli received from photoreceptor [*stimulus*]
I = inhibition coefficient
n = number of neurons
i = index of non-edge neurons (2,3,4,5,... n-1)
r = # of rows
c = # of columns

$$Input = \begin{bmatrix} x_1 & x_2 & x_3 & \cdots & x_{c-1} & x_c \\ x_{(1)(c)+1} & x_{(1)(c)+2} & x_{(1)(c)+3} & \cdots & x_{(2)(c)-1} & x_{(2)(c)} \\ x_{(2)(c)+1} & x_{(2)(c)+2} & x_{(2)(c)+3} & \cdots & x_{(3)(c)-1} & x_{(3)(c)} \\ \ldots & \ldots & \ldots & \cdots & \ldots & \ldots \\ x_{(r-2)(c)+1} & x_{(r-2)(c)+2} & x_{(r-2)(c)+3} & \cdots & x_{(r-1)(c)-1} & x_{(r-1)(c)} \\ x_{(r-1)(c)+1} & x_{(r-1)(c)+2} & x_{(r-1)(c)+3} & \cdots & x_{(r)(c)-1} & x_{(r)(c)} \\ \end{bmatrix}$$

$$Inner = \begin{bmatrix} x_{(1)(c)+2} & x_{(1)(c)+3} & \cdots & x_{(2)(c)-1} \\ x_{(2)(c)+2} & x_{(2)(c)+3} & \cdots & x_{(3)(c)-1} \\ \ldots & \ldots & \ldots & \ldots \\ x_{(r-2)(c)+2} & x_{(r-2)(c)+3} & \cdots & x_{(r-1)(c)-1} \\ \end{bmatrix}$$

$$Output = \begin{bmatrix} y_1 & y_2 & y_3 & \cdots & y_{c-1} & y_c \\ y_{(1)(c)+1} & y_{(1)(c)+2} & y_{(1)(c)+3} & \cdots & y_{(2)(c)-1} & y_{(2)(c)} \\ y_{(2)(c)+1} & y_{(2)(c)+2} & y_{(2)(c)+3} & \cdots & y_{(3)(c)-1} & y_{(3)(c)} \\ \ldots & \ldots & \ldots & \cdots & \ldots & \ldots \\ y_{(r-2)(c)+1} & y_{(r-2)(c)+2} & y_{(r-2)(c)+3} & \cdots & y_{(r-1)(c)-1} & y_{(r-1)(c)} \\ y_{(r-1)(c)+1} & y_{(r-1)(c)+2} & y_{(r-1)(c)+3} & \cdots & y_{(r)(c)-1} & y_{(r)(c)} \\ \end{bmatrix}$$

Nine cases : inner matrix, left-top corner, right-top corner, left-bottom corner, right-bottom corner, top border, bottom border, left border, right border inner

## 2.1 Feed-forward

left-top corner

$$y_1 = x_1 - I(x_2 + x_{(1)(c)+1} + x_{(1)(c)+2}) \tag{7}$$

right-top corner

$$y_c = x_c - I(x_{c-1} + x_{(2)(c)-1} + x_{(2)(c)}) \tag{8}$$

left-bottom corner

$$y_{(r-1)(c)+1} = x_{(r-1)(c)+1} - I(x_{(r-1)(c)+2} + x_{(r-2)(c)+1} + x_{(r-2)(c)+2}) \tag{9}$$

right-bottom corner

$$y_{(r)(c)} = x_{(r)(c)} - I(x_{(r)(c)-1} + x_{(r-1)(c)-1} + x_{(r-1)(c)}) \tag{10}$$

top border (index e = 2,3,4 ... c-1)

$$y_e = x_e - I(x_{e-1} + x_{e+1} + x_{(1)(c)+(e-1)} + x_{(1)(c)+(e)} + x_{(1)(c)+(e+1)}) \tag{11}$$

bottom border (index e = [(r-1)(c)+2] ,[(r-1)(c)+3],[(r-1)(c)+4] ... [(r)(c)-1] )

$$y_e = x_e - I(x_{e-1} + x_{e+1} + x_{(e-c)-1} + x_{(e-c)} + x_{(e-c)+1}) \tag{12}$$

left border (index e = [(1)(c)+1],[(2)(c) + 1], ... [(r-2)(c) + 1] )

$$y_e = x_e - I(x_{e+1} + x_{(e-c)} + x_{(e-c)+1} + x_{(e+c)} + x_{(e+c)+1}) \tag{13}$$

right border (index e = [(2)(c)],[(3)(c)], ... [(r-1)(c)] )

$$y_e = x_e - I(x_{e-1} + x_{(e-c)} + x_{(e-c)-1} + x_{(e+c)} + x_{(e+c)-1}) \tag{14}$$

Inner matrix (index e = elements of Inner)

$$y_e = x_e - I(x_{(e-c)-1} + x_{(e-c)} + x_{(e-c)+1} + x_{e-1} + x_{e+1} + x_{(e+c)-1} + x_{(e+c)} + x_{(e+c)+1}) \tag{15}$$

e.g. Input is a 3x3 Matrix, r = 3, c = 3

$$\begin{bmatrix} y_1 \\ y_2 \\ y_c \\ y_{(1)(c)+1} \\ y_{(1)(c)+2} \\ y_{(2)(c)} \\ y_{(r-1)(c)+1} \\ y_{(r-1)(c)+2} \\ y_{(r)(c)} \end{bmatrix} = \begin{bmatrix} 1 & -I & 0 & -I & -I & 0 & 0 & 0 & 0 \\ -I & 1 & -I & -I & -I & -I & 0 & 0 & 0 \\ 0 & -I & 1 & 0 & -I & -I & 0 & 0 & 0 \\ -I & -I & 0 & 1 & -I & 0 & -I & -I & 0 \\ -I & -I & -I & -I & 1 & -I & -I & -I & -I \\ 0 & -I & -I & 0 & -I & 1 & 0 & -I & -I \\ 0 & 0 & 0 & -I & -I & 0 & 1 & -I & 0 \\ 0 & 0 & 0 & -I & -I & -I & -I & 1 & -I \\ 0 & 0 & 0 & 0 & -I & -I & 0 & -I & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_c \\ x_{(1)(c)+1} \\ x_{(1)(c)+2} \\ x_{(2)(c)} \\ x_{(r-1)(c)+1} \\ x_{(r-1)(c)+2} \\ x_{(r)(c)} \end{bmatrix}$$

## 2.2 Feed-back

left-top corner

$$y_1 = x_1 - I(y_2 + y_{(1)(c)+1} + y_{(1)(c)+2}) \tag{16}$$

right-top corner

$$y_c = x_c - I(y_{c-1} + y_{(2)(c)-1} + y_{(2)(c)}) \tag{17}$$

left-bottom corner

$$y_{(r-1)(c)+1} = x_{(r-1)(c)+1} - I(y_{(r-1)(c)+2} + y_{(r-2)(c)+1} + y_{(r-2)(c)+2}) \tag{18}$$

right-bottom corner

$$y_{(r)(c)} = x_{(r)(c)} - I(y_{(r)(c)-1} + y_{(r-1)(c)-1} + y_{(r-1)(c)}) \tag{19}$$

top border (index e = 2,3,4 ... c-1)

$$y_e = x_e - I(y_{e-1} + y_{e+1} + y_{(1)(c)+(e-1)} + y_{(1)(c)+(e)} + y_{(1)(c)+(e+1)}) \tag{20}$$

bottom border (index e = [(r-1)(c)+2] ,[(r-1)(c)+3],[(r-1)(c)+4] ... [(r)(c)-1] )

$$y_e = x_e - I(y_{e-1} + y_{e+1} + y_{(e-c)-1} + y_{(e-c)} + y_{(e-c)+1}) \tag{21}$$

left border (index e = [(1)(c)+1],[(2)(c) + 1], ... [(r-2)(c) + 1] )

$$y_e = x_e - I(y_{e+1} + y_{(e-c)} + y_{(e-c)+1} + y_{(e+c)} + y_{(e+c)+1}) \tag{22}$$

right border (index e = [(2)(c)],[(3)(c)], ... [(r-1)(c)] )

$$y_e = x_e - I(y_{e-1} + y_{(e-c)} + y_{(e-c)-1} + y_{(e+c)} + y_{(e+c)-1}) \tag{23}$$

Inner matrix (index e = elements of Inner)

$$y_e = x_e - I(y_{(e-c)-1} + y_{(e-c)} + y_{(e-c)+1} + y_{e-1} + y_{e+1} + y_{(e+c)-1} + y_{(e+c)} + y_{(e+c)+1}) \tag{24}$$

e.g. Input is a 3x3 Matrix, r = 3, c = 3

$$\begin{bmatrix} y_1 \\ y_2 \\ y_c \\ y_{(1)(c)+1} \\ y_{(1)(c)+2} \\ y_{(2)(c)} \\ y_{(r-1)(c)+1} \\ y_{(r-1)(c)+2} \\ y_{(r)(c)} \end{bmatrix} = \begin{bmatrix} 1 & I & 0 & I & I & 0 & 0 & 0 & 0 \\ I & 1 & I & I & I & I & 0 & 0 & 0 \\ 0 & I & 1 & 0 & I & I & 0 & 0 & 0 \\ I & I & 0 & 1 & I & 0 & I & I & 0 \\ I & I & I & I & 1 & I & I & I & I \\ 0 & I & I & 0 & I & 1 & 0 & I & I \\ 0 & 0 & 0 & I & I & 0 & 1 & I & 0 \\ 0 & 0 & 0 & I & I & I & I & 1 & I \\ 0 & 0 & 0 & 0 & I & I & 0 & I & 1 \end{bmatrix}^{-1} \begin{bmatrix} x_1 \\ x_2 \\ x_c \\ x_{(1)(c)+1} \\ x_{(1)(c)+2} \\ x_{(2)(c)} \\ x_{(r-1)(c)+1} \\ x_{(r-1)(c)+2} \\ x_{(r)(c)} \end{bmatrix}$$