International Journal of Forecasting 31 (2015) 33–50

Contents lists available at ScienceDirect

International Journal of Forecasting

journal homepage: www.elsevier.com/locate/ijforecast

Forecasting GDP growth using mixed-frequency models with switching regimes

Fady Barsoum, Sandra Stankiewicz *

University of Konstanz, Department of Economics, Chair of Statistics and Econometrics, Box 129, 78457 Konstanz, Germany

ARTICLE INFO

Keywords:
Markov-switching
Business cycle
Mixed-frequency data analysis
Forecasting

ABSTRACT

For modelling mixed-frequency data with a business cycle pattern, we introduce the Markov-switching Mixed Data Sampling model with unrestricted lag polynomial (MS-U-MIDAS). Usually, models of the MIDAS-class use lag polynomials of a specific function which impose some structure on the weights of the regressors included in the model. This may lead to a deterioration in the predictive power of the model if the structure imposed differs from the data generating process. When the difference between the available data frequencies is small and there is no risk of parameter proliferation, using an unrestricted lag polynomial might not only simplify the model estimation, but also improve its forecasting performance. We allow the parameters of the MIDAS model with an unrestricted lag polynomial to change according to a Markov-switching scheme in order to account for the business cycle pattern observed in many macroeconomic variables. Thus, we combine the unrestricted MIDAS with a Markov-switching approach and propose a new Markov-switching MIDAS model with unrestricted lag polynomial (MS-U-MIDAS). We apply this model to a large dataset with the help of factor analysis. Monte Carlo experiments and an empirical forecasting comparison carried out for the U.S. GDP growth show that the models of the MS-U-MIDAS class exhibit nowcasting and forecasting performances which are similar to or better than those of their counterparts with restricted lag polynomials.

1. Introduction

Forecasting GDP growth is important for the decision making process, at both the central administrative (central bank, government) and industry levels. Due to difficulties in the measurement of GDP, it is published with a delay of a couple of months and revised repeatedly. This creates an obstacle for policy makers and market participants, who need to be ahead of changes in the economy, or at least to adjust quickly. Thus, reliable predictions are needed badly, but most of the existing forecasting models do not perform satisfactorily. This might be due to the fact that these models often ignore the non-linearities in the data (e.g., business cycle patterns), and/or fail to explore the informational content of the data published more frequently than the GDP or with a shorter lag. In addition, many models fail to make use of the informational content of large datasets, due to the problem of parameter proliferation.

Many approaches fail to account for the fact that macroeconomic variables often behave differently in different phases of the business cycle. Thus, a model with constant parameters might not reflect the present situation well, let alone be useful for forecasting. Furthermore, most models cannot include time series of different frequencies within the same regression. Instead, they require the data to be transformed (through either aggregation or interpolation), so that left- and right-hand side variables are of the same frequency. However, that might lead to a loss of

* Corresponding author.
E-mail addresses: fady.nagy-barsoum@uni-konstanz.de (F. Barsoum), sandra.stankiewicz@uni-konstanz.de (S. Stankiewicz).

http://dx.doi.org/10.1016/j.ijforecast.2014.04.002
0169-2070/

information. In addition, many models are not suitable for dealing with large datasets, which forces the forecaster to limit the number of explanatory variables and ignore useful information from other potential regressors. Finally, in most models it is not possible to include the most recent observations of the higher-frequency variable when the corresponding data on the dependent variable are not yet available. Thus, these models cannot be used for nowcasting. This is a major drawback, as higher-frequency variables are useful indicators of the current state of the economy. They are also published relatively often and with a small delay, which makes them potentially very useful for forecasting lower-frequency variables, such as GDP growth.

Various different approaches can be used to solve the above-mentioned problems, including regime-switching models, introduced by Hamilton (1989), Mixed Data Sampling Regressions (MIDAS), recently developed by Ghysels, Santa-Clara, and Valkanov (2002), and dynamic factor analysis (see e.g. Stock & Watson, 2002a). Regime-switching models allow the parameters of the model to change according to the current state of the economy (e.g., different parameters for the expansion and recession periods), and accounting for business cycle patterns in macroeconomic variables might improve the forecasting performance of the model. MIDAS models, on the other hand, can include time series of different frequencies in the same regression without transforming them through aggregation or interpolation. They are also very useful in nowcasting (MIDAS with leads), as they can make use of the observations of higher-frequency variables even if the data from lower-frequency variables for the corresponding period are not yet available.[1] Finally, dynamic factor analysis helps to exploit the informational content of large datasets by summarizing the variation in the observed variables using just a few unobserved factors. Thus, using a single factor that explains a large part of the dataset variation, instead of a single observed variable, may capture more information from the available dataset and ensure parsimony of the model.

A vast body of literature on Markov-switching models is available, usually in the context of modelling the business cycle patterns of macroeconomic data. Anas, Billio, Ferrara, and Duca (2007) explore the use of multivariate Markov-switching models for analysing the relationship between the phases of the business cycle in the United States and the Euro zone. Krolzig (2000) investigates the forecasting performance of the multivariate Markov-switching processes through Monte Carlo experiments and an empirical application to the United States business cycle. In addition, Clements and Krolzig (1998) study the forecasting performances of Markov-switching models through Monte Carlo simulations and an empirical study for the US GNP. Lahiri and Wang (1994) use the Markov-switching framework to predict turning points in the US business cycle. Cheung and Erlandsson (2005), Engel (1994) and Frömmel, MacDonald, and Menkhoff (2005), use Markov-switching models to explain and predict the fluctuations in exchange rates, whereas Evans and Wachtel (1993), Pagliacci and Barraez (2010) and Simon (1996) use the Markov-switching framework to analyse the past dynamics of inflation in Venezuela, the United States and Australia respectively.

MIDAS models, which were introduced to the literature only recently, have already found a number of interesting applications in both macroeconomics and finance. Kuzin, Marcellino, and Schumacher (2011) investigate the performance of the MIDAS model for nowcasting and forecasting GDP in the Euro area, relative to a mixed-frequency VAR (with missing values of the lower frequency variables interpolated using Kalman filter). They conclude that the two approaches seem to be complementary, as MIDAS performs better for short forecast horizons, whereas the mixed-frequency VAR does better for longer ones. Marcellino and Schumacher (2010) undertake a similar study, investigating the abilities of factor MIDAS models vs. state space factor models for forecasting German GDP. They find that factor MIDAS models usually outperform their state-space counterparts in forecasting, and that the most parsimonious MIDAS regression performs best overall.

Bai, Ghysels, and Wright (2013) compare MIDAS regressions with state space models through Monte Carlo simulations and an empirical exercise focusing on predicting GDP growth in the United States. They conclude that the two approaches are comparable in terms of forecasting performances. Clements and Galvão (2008, 2009) use MIDAS regressions of monthly and quarterly data for forecasting the GDP growth of the United States and obtain promising results, especially for MIDAS with leads. Andreou, Ghysels, and Kourtellos (2013) test the suitability of MIDAS factor models with leads for forecasting quarterly GDP growth in the United States with a large dataset of daily financial and quarterly macroeconomic indicators. They find such models to perform relatively well, especially in the crisis periods. Barsoum (2011) carries out a similar analysis for the United Kingdom, comparing MIDAS and factor-augmented MIDAS (both with and without leads) with a bunch of benchmark models. He obtains mixed results on the performances of MIDAS models in general, but promising results for MIDAS with leads.

Although they are useful, Markov-switching and MIDAS models can only address one problem at a time: either the issue of business cycle patterns or the difference in data frequencies. Guérin and Marcellino (2013) therefore combine the two approaches, introducing a Markov-switching Mixed Data Sampling model (MS-MIDAS).[2] They assess its forecasting performance through Monte Carlo simulations and carry out empirical studies on forecasting GDP growth in the United States and the United Kingdom, and show that MS-MIDAS is a useful approach. In their version of the model, Guérin and Marcellino (2013) use the so-called restricted lag polynomial, which is based on a specific function (e.g., exponential function). Depending on this function, a particular structure is imposed on the weights of the regressors in the model. This prevents parameter proliferation, but at the same time restricts the values that

[1] This feature also makes MIDAS models useful for dealing with ragged-edge data (though this is outside the scope of this paper).

[2] More recently, Bessec and Bouabdallah (2013) used a factor-augmented MS-MIDAS model for forecasting.

those weights can take. Although it is quite flexible, this approach might not reflect the data generating process fully, possibly leading to a deterioration in the forecasting performance of the model. Foroni, Marcellino, and Schumacher (in press) compare the forecasting performances of MIDAS and U-MIDAS (a MIDAS model with unrestricted lag polynomial weights) by means of Monte Carlo simulations. They show that, for small differences in frequencies of the variables analysed, the U-MIDAS performs better than the restricted MIDAS for most of the cases examined. For the rest of the cases, the models' performances are comparable both in-sample and out-of-sample. For most macroeconomic applications, quarterly and monthly data are used, so the difference in the frequencies of the variables is small and parameter proliferation is not a serious problem. Thus, the unrestricted version of MIDAS might be very useful for forecasting variables such as GDP growth. Since accounting for the business cycle pattern in macroeconomic data might improve the forecasting performance of the model, we extend the U-MIDAS approach by incorporating it into a Markov-switching framework in order to allow for changes in parameters according to the business cycle state of the economy. Thus, we combine the MS-MIDAS of Guérin and Marcellino (2013) with the U-MIDAS of Foroni et al. (in press), and propose a Markov-switching Mixed Data Sampling model with an unrestricted lag polynomial (MS-U-MIDAS).

We evaluate the usefulness of the MS-U-MIDAS model via a Monte Carlo study and an empirical forecasting comparison. We first investigate the qualities of the MS-U-MIDAS model with autoregressive dynamics (MS-U-MIDAS-AR) through Monte Carlo experiments. For different data generating processes (DGPs), we compare the in-sample and out-of-sample performances of the MS-U-MIDAS-AR model to those of its restricted counterparts (MS-MIDAS-AR and MS-ADL-MIDAS) in terms of Root Mean Squared Errors (RMSE) and Quadratic Probability Scores (QPS). For the in-sample analysis in terms of both the RMSE and QPS, we find that MS-U-MIDAS-AR performs better than its counterparts on average. For the out-of-sample analysis, the three models all perform comparably in terms of RMSEs, although MS-U-MIDAS-AR beats MS-MIDAS-AR in the case when the simulated data are highly persistent, which is a result that is consistent with the findings of Foroni et al. (in press). In terms of predicting the true regime, however, the MS-U-MIDAS-AR performs better than its restricted counterparts on average.

In our empirical forecast comparison, we use the MS-U-MIDAS model to forecast GDP growth in the United States using a large dataset of monthly macroeconomic and financial indicators. To reduce the dimension of the data and at the same time use the available information efficiently, we extract factors from the dataset using Principal Component Analysis (PCA). These factors are then used as regressors for forecasting the GDP growth. First, we investigate the in-sample properties of the MS-U-MIDAS model. Then, the out-of-sample forecasting performance of the MS-U-MIDAS class of models is compared with the performances of a wide range of Markov-switching and MIDAS type models, as well as benchmark models such as the random walk (RW), autoregressive (AR) and (autoregressive) distributed lag models ((A)DL). As has been mentioned, the method of construction of the MIDAS-class of models makes it easy to include data of higher frequencies, even if the corresponding data of a lower frequency are not available. Thus, one can use these models for nowcasting by including data on monthly variables corresponding to the quarter for which the forecast of a lower frequency variable (such as GDP growth) is made. That quality makes these models a very useful forecasting tool for policy makers, and we explore this feature in our analysis, using models with leads whenever possible. We find that, in most of the cases analysed, the forecasting performance of the MS-U-MIDAS class of models is comparable to or better than those of their restricted counterparts, which is probably due to the fact that the former model does not impose any structure on the weights of the regressors, and thus has more flexibility in adjusting to the true data generating process.

The paper is structured as follows. Section 2 presents the class of MIDAS models used in the analysis. Section 3 describes the design and results of Monte Carlo simulations. In Section 4, we apply the MS-U-MIDAS model for predicting the US GDP growth and present the results of our forecasting comparison. Section 5 concludes.

2. Forecasting models

2.1. The MIDAS model

The MIDAS model was introduced to the literature recently by Ghysels et al. (2002). The basic version of the MIDAS regression used to obtain an h-step-ahead forecast can be written using notation based on that of Clements and Galvão (2008):

$$y_t^Q = \beta_0 + \beta_1 B(L^{1/m}; \boldsymbol{\theta}) x_{t-h}^M + \varepsilon_t, \tag{2.1}$$

where $B(L^{1/m}; \boldsymbol{\theta}) = \sum_{k=1}^{K} b(k; \boldsymbol{\theta}) L^{(k-1)/m}$ is the sum of weights assigned to K lags of the independent variable (the lag polynomial). $b(k; \boldsymbol{\theta})$ is the kth weight of the K-lag polynomial, shaped by a certain function of $\boldsymbol{\theta}$ parameters (such as the exponential function described below). L denotes the lag operator, such that $L^{s/m} x_{t-h}^M = x_{t-h-s/m}^M$. t is the time index for the lower frequency variable y, whereas m is the time index for the higher frequency variable x. Q and M describe variables observed on a quarterly and a monthly basis, respectively.

MIDAS models are useful for nowcasting, as they allow for the inclusion of any higher-frequency data available when the corresponding observations of the lower-frequency variable are not yet known. In this case, one can include the most recent observations of the explanatory variable in the lag polynomial of Eq. (2.1).

There are a couple of different lag polynomials that are used in MIDAS regressions (e.g., Beta and Almon lag polynomials). We focus on the most commonly used function for the lag polynomial in MIDAS models: the exponential function. The so-called Exponential Almon lag polynomial parametrizes $b(k; \boldsymbol{\theta})$ according to the following scheme:

$$b(k; \boldsymbol{\theta}) = \frac{\exp(\theta_1 k + \theta_2 k^2 + \cdots + \theta_Q k^Q)}{\sum_{k=1}^{K} \exp(\theta_1 k + \theta_2 k^2 + \cdots + \theta_Q k^Q)}. \tag{2.2}$$

This specification ensures that the lag coefficients are positive and sum to one, which is the necessary condition for the identification of the parameters of the model. The parameter estimation is done by the non-linear least squares method. Empirical applications are usually based on two parameters of the function described above, and thus $\boldsymbol{\theta} = (\theta_1, \theta_2)'$, which simplifies the model, but still ensures flexibility in the specification of the shape of the polynomial. A wide variety of shapes of lag coefficients can be obtained, ranging from equal weights for all lags, to weights that decline at a given pace, to weights that form a hump shape.

2.2. Unrestricted MIDAS

In some cases, the shape of the lag polynomial determined by the function in Eq. (2.2), for example, may not reflect the underlying data generating process well. Therefore, a model without restrictions on the weights of the lag polynomial was introduced by Foroni et al. (in press) and called the unrestricted MIDAS (U-MIDAS):

$$y_t^Q = \beta_0 + \sum_{j=0}^{J-1} \beta_{j+1} x_{t-h-j/m}^M + \varepsilon_t.$$

The notation here is consistent with that in Eq. (2.1).

What distinguishes the above regression from Eq. (2.1) is the fact that no structure is imposed on the shape of the weights of the lag polynomial. That means that all $J+1$ parameters of this model need to be estimated, whereas in the case of the restricted MIDAS model, the number of parameters to be estimated is limited to four by construction: $\beta_0, \beta_1, \theta_1$ and θ_1. However, when the difference in frequencies in the analysed data is small, as is the case in many macroeconomic applications, the issue of parameter proliferation due to the use of the U-MIDAS model is not especially problematic. In a series of Monte Carlo experiments, Foroni et al. (in press) show that the performance of the U-MIDAS model is comparable to or better than that of the restricted MIDAS both in-sample and out-of-sample, when the difference in frequencies between the dependent and independent variables is small (e.g., quarterly vs. monthly data). In addition, the U-MIDAS model can be estimated by ordinary least squares, which simplifies the estimation relative to the MIDAS model. The main drawback of the U-MIDAS model is that, due to parameter proliferation, its performance declines dramatically when the difference in frequencies between the variables in the model is large. Thus, this approach is not suitable for all kinds of analyses. However, for many macroeconomic applications the use of functional lag polynomials in MIDAS does not seem to be necessary, and using the U-MIDAS model instead may be beneficial.

2.3. Markov-switching U-MIDAS

Macroeconomic data often exhibit a business cycle pattern. Therefore, it is reasonable to assume that the parameters of the model change according to the business cycle phase of the economy. One possible way to account for this data behaviour is the use of the Markov-switching model of Hamilton (1989). The parameters of this model depend on the current economic regime (e.g., the parameters differ in the recession and expansion phases). Guérin and Marcellino (2013) combined the Markov-switching approach with the MIDAS framework and introduced the Markov-switching MIDAS regression (MS-MIDAS):

$$y_t^Q = \beta_0(S_t) + \beta_1(S_t)B(L^{1/m}; \boldsymbol{\theta})x_{t-h}^M + \varepsilon_t(S_t),$$

where $\varepsilon_t|S_t \sim NID(0, \sigma^2(S_t))$; that is, the error terms are normally and identically distributed with mean zero and variance $\sigma^2(S_t)$, which varies with changing states of the world. $S_t = \{1, \dots, R\}$ denotes different states of the world (regimes) which are present in the data generating process. The MS-MIDAS model is similar in a way to Eq. (2.1). However, S_t in the brackets indicates the parameters that change according to different regimes. In the above regression, the intercept β_0, the slope parameter β_1 and the variance of the error term $\sigma^2(S_t)$ are allowed to change.

The probability of transition from the current regime a to regime b is defined as follows:

$$p_{ab} = \Pr(S_{t+1} = b | S_t = a).$$

All possible transition probabilities form a matrix P with probabilities of staying in the same regime in the next period on the diagonal, and probabilities of switching to another state in the next period below and above the diagonal. For example, for two regimes a and b:

$$P = \begin{pmatrix} p_{aa} & p_{ab} \\ p_{ba} & p_{bb} \end{pmatrix}, \tag{2.3}$$

where p_{ab} is the probability of switching from state a to state b in the next period, p_{ba} is the probability of changing from regime b to a in the following period, and p_{aa} and p_{bb} are the probabilities of staying in the same regime in the next period. The sums of the probabilities in each row add up to one. Thus, in the case of two regimes, it is sufficient to determine p_{aa} and p_{bb}, for example, to obtain the whole matrix. We assume that the transition probabilities stay constant over time, which is a standard approach in the Markov-switching applications.

As was explained above, MIDAS models with the lag polynomials restricted by some specific function might not be flexible enough to reflect the true data generating process well. The same applies to the Markov-switching version of the model that was introduced by Guérin and Marcellino (2013). Since Foroni et al. (in press) found that using the unrestricted version of the model could potentially improve its forecasting performance, we incorporate the unrestricted lag polynomial into the Markov-switching framework and introduce the unrestricted Markov-switching MIDAS model (MS-U-MIDAS):

$$y_t^Q = \beta_0(S_t) + \sum_{j=0}^{J-1} \beta_{j+1}(S_t) x_{t-h-j/m}^M + \varepsilon_t(S_t).$$

To account for the business cycle pattern of the data, the parameters of the above equation, that is, the intercept β_0, the slope parameters β_{j+1}, and the variance of the error term σ_ε^2, can change according to different regimes. Note that while the parameters $\boldsymbol{\theta}$ stay fixed at their estimated values in the MS-MIDAS of Guérin and Marcellino

(2013), all of the parameters in the MS-U-MIDAS may switch, giving the latter model more flexibility. With the help of information criteria, one can decide on the number of regimes present in the data generating process and on the parameters that should be allowed to switch. For example, one can take into consideration a model with all of the above-mentioned parameters switching, or consider a model with, say, only the intercept and/or the variance of the error term switching. Thus, the model presented above offers great flexibility in modelling the available data, and may be very useful for forecasting purposes.

All of the variations of the Markov-switching models presented in this paper are estimated by the Maximum-Likelihood method. Thus, an assumption about the normality of the error terms is required. Following the procedure described by Hamilton (1994), we maximize the following log-likelihood function:

$$L = \sum_{t=1}^{T} \log f(y_t^Q | \Omega_{t-1}),$$

where $f(y_t^Q|\Omega_{t-1})$ denotes the density of y_t^Q conditional on Ω_{t-1}, the information given up to time $t-1$. The conditional density $f(y_t^Q|\Omega_{t-1})$ can be rewritten as:

$$f(y_t^Q|\Omega_{t-1}) = \sum_{i=1}^{R} P(S_t = i|\Omega_{t-1}) f(y_t^Q|S_t = i, \Omega_{t-1}).$$

The maximization of the log-likelihood function is carried out with the help of the Expectation Maximization algorithm, as described by Hamilton (1994). We use MATLAB for all computations.[3]

2.4. MIDAS models with autoregressive dynamics

Many empirical studies show that adding an autoregressive term to a model improves its forecasting performance significantly. Therefore, we also include autoregressive dynamics in the models considered in this paper.

Andreou et al. (2013) introduce autoregressive dynamics into the MIDAS regression in a straightforward way, calling their model an ADL-MIDAS:

$$y_t^Q = \beta_0 + \sum_{i=0}^{p-1} \lambda_{i+1} y_{t-d-i}^Q + \beta_1 B(L^{1/m}; \boldsymbol{\theta}) x_{t-h}^M + \varepsilon_t. \quad (2.4)$$

When h is an integer, $d = h$, but when some information on the regressors is available for the quarter for which the forecast is calculated, e.g., when $h = 1/3$ (data on two months within the quarter is available), then $d = 1$.

Clements and Galvão (2008) propose a different solution, introducing the autoregressive term into the MIDAS model as a common factor, so that the response of y to x remains non-seasonal. Their model is called MIDAS-AR:

$$y_t^Q = \beta_0 + \lambda y_{t-d}^Q + \beta_1 B(L^{1/m}; \boldsymbol{\theta})(1 - \lambda L^d) x_{t-h}^M + \varepsilon_t. \quad (2.5)$$

The introduction of the autoregressive dynamics into the U-MIDAS model (U-MIDAS-AR) is straightforward, and the model is given by:

$$y_t^Q = \beta_0 + \sum_{i=0}^{p-1} \lambda_{i+1} y_{t-d-i}^Q + \sum_{j=0}^{J-1} \beta_{j+1} x_{t-h-j/m}^M + \varepsilon_t.$$

The Markov-switching versions of MIDAS-AR, ADL-MIDAS and U-MIDAS-AR used for h-step-ahead forecasts are given by:

$$\text{MS-MIDAS-AR: } y_t^Q = \beta_0(S_t) + \lambda y_{t-d}^Q + \beta_1(S_t) B(L^{1/m}; \boldsymbol{\theta}) \times (1 - \lambda L^d) x_{t-h}^M + \varepsilon_t(S_t), \quad (2.6)$$

$$\text{MS-ADL-MIDAS: } y_t^Q = \beta_0(S_t) + \sum_{i=0}^{p-1} \lambda_{i+1} y_{t-d-i}^Q + \beta_1(S_t) \times B(L^{1/m}; \boldsymbol{\theta}) x_{t-h}^M + \varepsilon_t(S_t), \quad (2.7)$$

and

$$\text{MS-U-MIDAS-AR: } y_t^Q = \beta_0(S_t) + \sum_{i=0}^{p-1} \lambda_{i+1} y_{t-d-i}^Q + \sum_{j=0}^{J-1} \beta_{j+1}(S_t) x_{t-h-j/m}^M + \varepsilon_t(S_t). \quad (2.8)$$

3. Monte Carlo experiment

We investigate the in-sample fit and forecasting performance of the MS-U-MIDAS with autoregressive dynamics through Monte Carlo experiments for different data generating processes (DGPs). The first DGP is an extended version of the process used by Foroni et al. (in press), and is given as a bivariate Markov-switching VAR(1):

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} \rho & \delta_l(S_t) \\ \delta_h & \rho \end{pmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} e_{y,t}(S_t) \\ e_{x,t}(S_t) \end{pmatrix}. \quad (3.1)$$

We assume a business cycle pattern in the DGP by allowing some of the parameters of the above-described model to switch between regimes. In our simulations, we allow for two regimes. We assume that y_t depends on x_t, but y_t has no influence on x_t. Thus, we set the parameter δ_h to zero.

For the sake of comparison, we use a similar set of possible parameter values to that of Foroni et al. (in press). We run the simulations for various values of the parameter ρ, to account for different possible degrees of persistence of y_t and x_t : $\rho = \{0.1; 0.5; 0.9\}$. We assume that the degree of persistence of the variables stays constant across regimes, which is an assumption consistent with the models presented in Section 2.4. Unlike ρ, the parameter δ_l, which determines how strongly x_t influences y_t, can take different values for different regimes: $\delta_l(S_t = 1) = \{0.1; 0.5\}$ and $\delta_l(S_t = 2) = 1$. The error terms also change their characteristics across the regimes. We assume that $e_{y,t}$ and $e_{x,t}$ are independently and normally distributed with mean zero. Their variances switch between regimes and are chosen in such a way that the unconditional variance of y_t equals 1 in the first regime and 2 in the second. Similarly, we also consider a bivariate MS-VAR(4) as a DGP. The exact details are shown in Eq. (A.1) in the online Appendix.

---

[3] We gratefully acknowledge the help of Pierre Gurin, who provided us with his GAUSS code for the MS-MIDAS estimation as a robustness check for our code. For the estimation of the MS-U-MIDAS class of models, we modified Perlin's (2011) Toolbox for Markov-switching models, available from the website: http://www.mathworks.com/matlabcentral/fileexchange/authors/21596.

Table 1
In-sample root mean squared errors of MS-U-MIDAS-AR relative to MS-MIDAS-AR and MS-ADL-MIDAS (where the DGP is a bivariate MS-VAR(1) with two regimes).

ρ	δ_l		Transition probabilities		RMSE (25th, 50th and 75th percentiles)					
	$S_t = 1$	$S_t = 2$	p_{11}	p_{22}	MS-U-MIDAS-AR vs. MS-MIDAS-AR			MS-U-MIDAS-AR vs. MS-ADL-MIDAS		
					25th	50th	75th	25th	50th	75th
0.1	0.1	1	0.95	0.85	0.95	0.98	1.01	0.97	0.99	1.00
0.1	0.5	1	0.95	0.85	0.95	0.98	1.01	0.97	0.99	1.00
0.5	0.1	1	0.95	0.85	0.95	0.98	1.01	0.96	0.98	1.00
0.5	0.5	1	0.95	0.85	0.93	0.96	0.99	0.94	0.96	0.99
0.9	0.1	1	0.95	0.85	0.79	0.85	0.91	0.96	1.00	1.04
0.9	0.5	1	0.95	0.85	0.43	0.50	0.55	0.95	0.99	1.02
0.1	0.1	1	0.95	0.95	0.96	0.99	1.01	0.97	0.99	1.00
0.1	0.5	1	0.95	0.95	0.95	0.98	1.01	0.97	0.99	1.00
0.5	0.1	1	0.95	0.95	0.94	0.97	1.00	0.93	0.96	0.98
0.5	0.5	1	0.95	0.95	0.92	0.96	0.99	0.90	0.93	0.96
0.9	0.1	1	0.95	0.95	0.71	0.78	0.85	0.95	0.99	1.02
0.9	0.5	1	0.95	0.95	0.37	0.44	0.51	0.90	0.98	1.04

The table presents the summary of the results of 1000 Monte Carlo simulations. For each replication, 600 observations of a higher-frequency variable x_t and 200 observations of a lower-frequency variable y_t are generated according to a bivariate MS-VAR(1) model with two regimes. Then, the MS-U-MIDAS-AR, MS-MIDAS-AR and MS-ADL-MIDAS models are estimated and their in-sample fits are measured using the Root Mean Squared Error. The table presents the 25th, 50th and 75th percentiles of the ratio of the RMSE of the MS-U-MIDAS-AR to the RMSE of MS-MIDAS-AR and MS-ADL-MIDAS over all 1000 simulations. Values below/above one indicate a better/worse in-sample performance of the MS-U-MIDAS-AR model relative to the MS-MIDAS-AR or MS-ADL-MIDAS model. The analysis is done for different transition probabilities between the two regimes and for different values of the parameters of the MS-VAR model (see Eq. (3.1)).

The second DGP we consider for the out-of-sample analysis is an autoregressive Markov-switching MIDAS process (MS-MIDAS-AR), given by Eq. (2.6). The high-frequency variable x_t is generated according to Eq. (3.2). We use six lags of the high-frequency variable in the DGP. We also use an exponential Almon lag polynomial, specified in Eq. (2.2), with $\boldsymbol{\theta} = (\theta_1, \theta_2)'$. We account for two regimes and allow the intercept β_0, the slope parameter β_1 and the variance of the error term σ^2 to switch between the regimes. The parameters $\boldsymbol{\theta}$ and the autoregressive parameter λ stay constant across the regimes. We make the experiment for two different sets of parameters, the exact values of which are given in Table 5. The parameter values are chosen according to Foroni et al. (in press) and the estimation results for our dataset.

For both DGPs, we assume two different possible matrices of transition probabilities. For the first case, $p_{11} = 0.95$ and $p_{22} = 0.85$, whereas for the second case, $p_{11} = p_{22} = 0.95$. The high probabilities of staying in the same regime in the next period reflect the high persistence of the business cycle regimes observed in reality. As both DGPs are limited to two regimes, the above-described probabilities fully specify the matrix P given in Eq. (2.3).

We compare the in-sample and forecasting performances of the MS-U-MIDAS-AR model relative to MS-MIDAS-AR and MS-ADL-MIDAS, in order to see which of them performs best for each of the DGPs. In all simulations we account for the start-up effect; that is, we delete the first 100 simulated values of the variables y_t and x_t. During estimation, the Akaike criterion (AIC) is used to determine the lag order for each simulation. Up to six lags of the high frequency variable are used for the estimation.

3.1. In-sample analysis

Our main focus is on the out-of-sample performance of the MS-U-MIDAS model. Thus, for the in-sample analysis we only consider the first DGP, that is, a bivariate Markov-switching VAR (MS-VAR). We generate 600 observations of y_t and x_t, following Eq. (3.1). This can be thought of as using 600 monthly observations. We then assume that y_t is observed only every third period, which corresponds to observing y_t on a quarterly basis. Thus, we obtain 200 low-frequency observations. Then, for each replication, we estimate MS-U-MIDAS-AR, MS-MIDAS-AR and MS-ADL-MIDAS, and investigate the in-sample fits of these models by means of the Root Mean Squared Error (RMSE) and Quadratic Probability Score (QPS). The RMSE is defined as a square root of the average squared differences between the estimated values of the dependent variable $\hat{y}_t$ and the actually observed values of y_t:

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{t=\tau}^{T} (\hat{y}_t - y_t)^2}{T - \tau + 1}},$$

where τ denotes the beginning of the evaluation period.

The Quadratic Probability Score is a measure of the accuracy of regime prediction, and is defined as follows:

$$\mathrm{QPS} = \frac{2 \cdot \sum_{t=\tau}^{T} (P(S_t = 1) - S_t)^2}{T - \tau + 1},$$

where S_t is a dummy variable which takes the value of 1 for the first regime and 0 for the second regime, and $P(S_t = 1)$ is the predicted probability of being in the first regime in period t. For the in-sample analysis, $\tau = 1$.

We replicate the process described above 1000 times and obtain the results given in Tables 1 and 2. The tables show the 25th, 50th, 75th percentiles of the ratio of the RMSE/QPS of MS-U-MIDAS-AR to that of MS-MIDAS-AR or MS-ADL-MIDAS, calculated over 1000 simulations so as to provide an overview of the distribution of the results for all replications. Values of the ratio that are below/above one mean that MS-U-MIDAS-AR performs better/worse

Table 2
In-sample quadratic probability scores of MS-U-MIDAS-AR relative to MS-MIDAS-AR and MS-ADL-MIDAS (where the DGP is a bivariate MS-VAR(1) with two regimes).

ρ	δ_l		Transition probabilities		QPS (25th, 50th and 75th percentiles)					
	$S_t = 1$	$S_t = 2$	p_{11}	p_{22}	MS-U-MIDAS-AR vs. MS-MIDAS-AR			MS-U-MIDAS-AR vs. MS-ADL-MIDAS		
					25th	50th	75th	25th	50th	75th
0.1	0.1	1	0.95	0.85	0.97	1.00	1.08	0.98	1.00	1.03
0.1	0.5	1	0.95	0.85	0.97	1.01	1.06	0.98	1.00	1.03
0.5	0.1	1	0.95	0.85	0.93	1.00	1.09	0.95	1.00	1.06
0.5	0.5	1	0.95	0.85	0.91	0.97	1.06	0.93	0.98	1.05
0.9	0.1	1	0.95	0.85	0.23	0.94	1.02	0.90	0.99	1.04
0.9	0.5	1	0.95	0.85	0.88	1.00	1.00	1.00	1.00	1.03
0.1	0.1	1	0.95	0.95	0.97	1.01	1.04	0.98	1.00	1.02
0.1	0.5	1	0.95	0.95	0.97	1.00	1.03	0.98	1.00	1.03
0.5	0.1	1	0.95	0.95	0.89	0.98	1.04	0.95	0.98	1.03
0.5	0.5	1	0.95	0.95	0.87	0.95	1.00	0.92	0.98	1.02
0.9	0.1	1	0.95	0.95	0.11	0.73	1.00	0.88	1.00	1.00
0.9	0.5	1	0.95	0.95	0.16	0.98	1.06	0.85	1.00	1.08

The table presents a summary of the results of 1000 Monte Carlo simulations. For each replication, 600 observations of a higher-frequency variable x_t and 200 observations of a lower-frequency variable y_t are generated according to a bivariate MS-VAR(1) model with two regimes. Then the MS-U-MIDAS-AR, MS-MIDAS-AR and MS-ADL-MIDAS models are estimated and their accuracy in predicting regimes is measured using the Quadratic Probability Score. The table presents the 25th, 50th and 75th percentiles of the ratio of the QPS of the MS-U-MIDAS-AR to the QPS of MS-MIDAS-AR and MS-ADL-MIDAS over all 1000 simulations. Values below/above one indicate a better/worse in-sample performance of the MS-U-MIDAS-AR model relative to the MS-MIDAS-AR or MS-ADL-MIDAS model. The analysis is done for different transition probabilities between the two regimes and for different values of the parameters of the MS-VAR model (see Eq. (3.1)).

than MS-MIDAS-AR or MS-ADL-MIDAS in terms of the in-sample performance.

Simulations carried out for DGPs generated by different sets of parameters, show that, in terms of the median of the RMSE, the MS-U-MIDAS-AR outperforms the MS-MIDAS-AR in all cases considered, and also performs better than the MS-ADL-MIDAS model in all but one case. However, in most cases, the differences between the models are small. The only exceptions are the cases when $\rho = 0.9$, when the MS-U-MIDAS-AR model outperforms the MS-MIDAS-AR considerably. However, no big differences between the MS-U-MIDAS-AR and MS-ADL-MIDAS are reported in those cases. This reveals that the MS-U-MIDAS-AR and MS-ADL-MIDAS show better in-sample performances than the MS-MIDAS-AR model when the persistence of the variable y_t is high and the case analysed is close to the unit root.

In terms of the QPS, the in-sample performance of the MS-U-MIDAS-AR is comparable to that of the MS-ADL-MIDAS, and better than that of the MS-MIDAS-AR, in most cases considered.

3.2. *Out-of-sample analysis*

For the study of the out-of-sample performance of the MS-U-MIDAS-AR model vs. MS-MIDAS-AR and MS-ADL-MIDAS, we generate an additional 30 out-of-sample low-frequency observations from a specific DGP. In other words, we consider 200 low-frequency observations for the in-sample estimation, and 30 for the forecasting evaluation. We then compare the forecasting performance of MS-U-MIDAS-AR to those of MS-MIDAS-AR and MS-ADL-MIDAS for one-step-ahead forecasts by means of the RMSE and QPS. We replicate the procedure 1000 times for two of the DGPs that were described in the earlier sections.

3.2.1. *DGP generated as a two-regime MS-VAR*

The first out-of-sample experiment is carried out for the DGP generated as a MS-VAR(1) (see Eq. (3.1)) according to the procedure described above. The results of the forecasting evaluation of the MS-U-MIDAS-AR model vs. MS-MIDAS-AR and MS-ADL-MIDAS can be found in Tables 3 and 4. For different combinations of the parameters of the DGP, the tables contain the 25th, 50th and 75th percentiles of the ratio of the RMSE/QPS of MS-U-MIDAS-AR to the RMSE/QPS of MS-MIDAS-AR and to the RMSE/QPS of MS-ADL-MIDAS, calculated over all 1000 replications. Values below/above one indicate that MS-U-MIDAS-AR performs better/worse than MS-MIDAS-AR or MS-ADL-MIDAS in the out-of-sample analysis.

When the true DGP is MS-VAR(1), there is no clear winner of the out-of-sample comparison. Looking at the median of the results for the RMSE, the MS-U-MIDAS-AR clearly performs better than the MS-MIDAS-AR for the cases when the persistence of the low-frequency variable is high ($\rho = 0.9$), and thus when more information that is relevant for the future can be exploited from the past data. This confirms the results of the in-sample analysis. However, for other cases, the performance of the MS-U-MIDAS is comparable to or slightly worse than those of the MS-MIDAS-AR and the MS-ADL-MIDAS. On the other hand, looking at the QPS, we can observe that MS-U-MIDAS outperforms the MS-MIDAS-AR in predicting the true regime in almost all cases. It also performs at least as well as the MS-ADL-MIDAS model.

As a robustness check, we repeat the out-of-sample comparison for the case when the true DGP is an MS-VAR(4), thus allowing for more dynamics. The results of this exercise can be found in Tables A.3 and A.4 in the online Appendix. In terms of the RMSE, the performance of the MS-U-MIDAS-AR deteriorates slightly relative to the MS-MIDAS-AR and MS-ADL-MIDAS. One possible explanation of this deterioration is that the AIC suggests a large number of lags, and since the unrestricted model is less parsimonious, its forecasting performance worsens. However, in terms of predicting the regime changes, the MS-U-MIDAS-AR still performs better than both the MS-MIDAS-AR and MS-ADL-MIDAS on average.

Table 3
Out-of-sample root mean squared errors of MS-U-MIDAS-AR relative to MS-MIDAS-AR and MS-ADL-MIDAS (where the DGP is a bivariate MS-VAR(1) with two regimes).

ρ	δ_l		Transition probabilities		RMSE (25th, 50th and 75th percentiles)					
					MS-U-MIDAS-AR vs. MS-MIDAS-AR			MS-U-MIDAS-AR vs. MS-ADL-MIDAS		
	$S_t = 1$	$S_t = 2$	p_{11}	p_{22}	25th	50th	75th	25th	50th	75th
0.1	0.1	1	0.95	0.85	0.98	1.03	1.09	0.99	1.01	1.04
0.1	0.5	1	0.95	0.85	0.98	1.03	1.10	0.99	1.01	1.04
0.5	0.1	1	0.95	0.85	0.97	1.03	1.09	0.97	1.03	1.12
0.5	0.5	1	0.95	0.85	0.93	1.00	1.08	0.99	1.06	1.17
0.9	0.1	1	0.95	0.85	0.76	0.90	1.03	0.88	0.99	1.11
0.9	0.5	1	0.95	0.85	0.47	0.56	0.67	0.88	1.01	1.16
0.1	0.1	1	0.95	0.95	0.98	1.03	1.10	0.99	1.00	1.03
0.1	0.5	1	0.95	0.95	0.98	1.04	1.10	0.99	1.00	1.02
0.5	0.1	1	0.95	0.95	0.94	1.01	1.09	0.98	1.04	1.12
0.5	0.5	1	0.95	0.95	0.92	1.00	1.08	0.99	1.06	1.14
0.9	0.1	1	0.95	0.95	0.63	0.79	0.98	0.84	1.01	1.21
0.9	0.5	1	0.95	0.95	0.42	0.51	0.63	0.90	1.06	1.28

The table presents a summary of the results of 1000 Monte Carlo simulations. For each replication, 600 observations of a higher-frequency variable x_t and 200 observations of a lower-frequency variable y_t are generated for the in-sample period, whereas 30 lower-frequency (90 higher-frequency) observations are generated for the out-of-sample evaluation according to a bivariate MS-VAR(1) model with two regimes. Then, the MS-U-MIDAS-AR, MS-MIDAS-AR and MS-ADL-MIDAS models are estimated and their out-of-sample performances are measured using the root mean squared error. The table presents the 25th, 50th and 75th percentiles of the ratio of the RMSE of the MS-U-MIDAS-AR to the RMSE of MS-MIDAS-AR and MS-ADL-MIDAS over all 1000 simulations. Values below/above one indicate a better/worse out-of-sample performance of the MS-U-MIDAS-AR model relative to the MS-MIDAS-AR or MS-ADL-MIDAS model. The analysis is done for different transition probabilities between the two regimes and for different values of the parameters of the MS-VAR model (see Eq. (3.1)).

Table 4
Out-of-sample quadratic probability scores of MS-U-MIDAS-AR relative to MS-MIDAS-AR and MS-ADL-MIDAS (where the DGP is a bivariate MS-VAR(1) with two regimes).

ρ	δ_l		Transition probabilities		QPS (25th, 50th and 75th percentiles)					
					MS-U-MIDAS-AR vs. MS-MIDAS-AR			MS-U-MIDAS-AR vs. MS-ADL-MIDAS		
	$S_t = 1$	$S_t = 2$	p_{11}	p_{22}	25th	50th	75th	25th	50th	75th
0.1	0.1	1	0.95	0.85	0.87	1.00	1.14	0.97	1.00	1.07
0.1	0.5	1	0.95	0.85	0.82	0.98	1.12	0.96	1.00	1.06
0.5	0.1	1	0.95	0.85	0.79	1.04	1.78	0.66	0.97	1.30
0.5	0.5	1	0.95	0.85	0.61	0.95	1.36	0.54	0.91	1.34
0.9	0.1	1	0.95	0.85	0.79	0.99	1.25	0.71	1.00	1.32
0.9	0.5	1	0.95	0.85	0.78	0.99	1.17	0.69	1.00	1.37
0.1	0.1	1	0.95	0.95	0.84	0.97	1.10	0.97	1.00	1.03
0.1	0.5	1	0.95	0.95	0.80	0.95	1.07	0.97	1.00	1.03
0.5	0.1	1	0.95	0.95	0.62	0.97	1.06	0.66	0.98	1.06
0.5	0.5	1	0.95	0.95	0.54	0.93	1.03	0.52	0.96	1.05
0.9	0.1	1	0.95	0.95	0.67	0.94	1.20	0.71	1.01	1.39
0.9	0.5	1	0.95	0.95	0.71	0.96	1.20	0.76	1.01	1.41

The table presents a summary of the results of 1000 Monte Carlo simulations. For each replication, 600 observations of a higher-frequency variable x_t and 200 observations of a lower-frequency variable y_t are generated for the in-sample period, whereas 30 lower-frequency (90 higher-frequency) observations are generated for the out-of-sample evaluation according to a bivariate MS-VAR(1) model with two regimes. Then, the MS-U-MIDAS-AR, MS-MIDAS-AR and MS-ADL-MIDAS models are estimated and their accuracy in predicting regimes is measured by the quadratic probability score. The table presents the 25th, 50th and 75th percentiles of the ratio of the QPS of the MS-U-MIDAS-AR to the QPS of MS-MIDAS-AR and MS-ADL-MIDAS over all 1000 simulations. Values below/above one indicate a better/worse out-of-sample performance of the MS-U-MIDAS-AR model in comparison to the MS-MIDAS-AR or MS-ADL-MIDAS model. The analysis is done for different transition probabilities between the two regimes and for different values of the parameters of the MS-VAR model (see Eq. (3.1)).

3.2.2. DGP generated as a two-regime MS-MIDAS-AR

In order to investigate the out-of-sample performance of MS-U-MIDAS-AR vs. MS-MIDAS-AR and MS-ADL-MIDAS in the case when MS-MIDAS-AR is the true DGP, we generate data according to the MS-MIDAS-AR model with an exponential Almon lag (with six lags of the higher-frequency variable[4]) and two regimes (see Eq. (2.6)). The higher frequency variable x_t is generated from the following AR(1) process:[5]

$$x_t = 0.025 + 0.9 \cdot x_{t-1} + \varepsilon_t, \quad \text{where } \varepsilon_t \sim N(0, 1). \quad (3.2)$$

We allow the intercept β_0, the slope parameter β_1 and the variance of the error term σ^2 to switch. We consider two possible sets of parameters for the DGP. The details of this parametrization can be found in Table 5. The transition probabilities remain the same as for the first DGP.

4 We also carried out the exercise for three lags of the higher-frequency variable, but the results were similar to those presented here, so we refrain from including them in the paper.

5 We also conducted the exercise for a different degree of persistence of variable x_t, namely for $\rho = 0.3$, but the results were similar to those presented here, so we refrain from including them in the paper.

Table 5
Choice of the parameters for the Monte Carlo simulations, when the data generating process is a MS-MIDAS-AR with two regimes.

Set	Regime	β_0	θ_1	θ_2	β_1	λ	σ^2
1	$S_t = 1$	-1	$2 \cdot 10^{-1}$	$3 \cdot 10^{-2}$	0.6	0.2	1
	$S_t = 2$	1	$2 \cdot 10^{-1}$	$3 \cdot 10^{-2}$	0.2	0.2	0.67
2	$S_t = 1$	-0.5	0.7	-0.5	0.8	0.3	1
	$S_t = 2$	0.5	0.7	-0.5	0.1	0.3	0.67

Table 6
Out-of-sample root mean squared error of MS-U-MIDAS-AR relative to MS-MIDAS-AR and MS-ADL-MIDAS (where the DGP is a MS-MIDAS-AR with two regimes).

Set	Transition probabilities		MS-U-MIDAS-AR vs. MS-MIDAS-AR			MS-U-MIDAS-AR vs. MS-ADL-MIDAS		
	p_{11}	p_{22}	25th	50th	75th	25th	50th	75th
1	0.95	0.85	0.97	1.03	1.09	0.96	1.00	1.05
1	0.95	0.95	0.96	1.02	1.08	0.95	1.00	1.05
2	0.95	0.85	0.98	1.04	1.11	0.98	1.03	1.09
2	0.95	0.95	0.98	1.03	1.09	0.98	1.02	1.07

The table presents a summary of the results of 1000 Monte Carlo simulations. For each replication, 600 observations of a higher-frequency variable x_t and 200 observations of a lower-frequency variable y_t are generated for the in-sample period, whereas 30 lower-frequency (90 higher-frequency) observations are generated for the out-of-sample evaluation according to a MS-MIDAS-AR model with two regimes. Then, the MS-U-MIDAS-AR, MS-MIDAS-AR and MS-ADL-MIDAS models are estimated and their out-of-sample performances are measured by the root mean squared error. The table presents the 25th, 50th and 75th percentiles of the ratio of the RMSE of the MS-U-MIDAS-AR to those of MS-MIDAS-AR or MS-ADL-MIDAS, calculated over all 1000 simulations. Values below/above one indicate a better/worse out-of-sample performance of the MS-U-MIDAS-AR model relative to its restricted counterpart. The analysis is done for different transition probabilities between the two regimes and for different values of the parameters of the data generating process (see Eq. (2.6) and Table 5).

Table 7
Out-of-sample quadratic probability score of MS-U-MIDAS-AR relative to MS-MIDAS-AR and MS-ADL-MIDAS (where the DGP is a MS-MIDAS-AR with two regimes).

Set	Transition probabilities		MS-U-MIDAS-AR vs. MS-MIDAS-AR			MS-U-MIDAS-AR vs. MS-ADL-MIDAS		
	p_{11}	p_{22}	25th	50th	75th	25th	50th	75th
1	0.95	0.85	0.69	0.96	1.08	0.63	0.95	1.08
1	0.95	0.95	0.63	0.98	1.09	0.71	0.99	1.10
2	0.95	0.85	0.73	0.97	1.14	0.68	0.94	1.12
2	0.95	0.95	0.76	0.98	1.09	0.76	0.98	1.09

The table presents a summary of the results of 1000 Monte Carlo simulations. For each replication, 600 observations of a higher-frequency variable x_t and 200 observations of a lower-frequency variable y_t are generated for the in-sample period, whereas 30 lower-frequency (90 higher-frequency) observations are generated for the out-of-sample evaluation according to a MS-MIDAS-AR model with two regimes. Then, the MS-U-MIDAS-AR, MS-MIDAS-AR and MS-ADL-MIDAS models are estimated and their out-of-sample performances in predicting regimes are measured using the quadratic probability score. The table presents the 25th, 50th and 75th percentiles of the ratio of the QPS of the MS-U-MIDAS-AR to those of MS-MIDAS-AR and MS-ADL-MIDAS, calculated over all 1000 simulations. Values below/above one indicate a better/worse out-of-sample performance of the MS-U-MIDAS-AR model relative to its restricted counterpart. The analysis is done for different transition probabilities between the two regimes and for different values of the parameters of the data generating process (see Eq. (2.6) and Table 5).

The evaluation of the out-of-sample performances of the models is done in terms of their forecasting accuracy and ability to predict regimes. Thus, we consider the 25th, 50th and 75th percentiles of the ratio of the RMSE of MS-U-MIDAS-AR relative to those of MS-MIDAS-AR and MS-ADL-MIDAS, calculated over 1000 replications (Table 6). We also calculate the quadratic probability score of the MS-U-MIDAS-AR relative to those of the MS-MIDAS-AR and MS-ADL-MIDAS models (Table 7). For both tables, values below/above one indicate that MS-U-MIDAS-AR performs better/worse than MS-MIDAS-AR or MS-ADL-MIDAS.

Looking at the median of the results, for most combinations of parameters considered, the MS-U-MIDAS-AR model is slightly outperformed both by the MS-MIDAS-AR and MS-ADL-MIDAS in terms of forecasting accuracy. However, in predicting regimes, the MS-U-MIDAS-AR model outperforms both of its counterparts for all parameter combinations considered. Therefore, it seems that even in the case when MS-MIDAS-AR is favoured (as this is the true DGP), MS-U-MIDAS-AR outperforms its restricted counterparts in regime prediction.

4. Forecasting GDP growth in the United States

In our empirical study, we investigate the in- and out-of-sample performances of the class of MS-U-MIDAS models in forecasting quarterly GDP growth in the United States with the help of monthly macroeconomic and financial variables. For horizons $h = \left\{\frac{1}{3}, \frac{2}{3}, 1, 2, 4\right\}$ (fractions denote the cases of nowcasting), we compare the forecasting performances of the MS-U-MIDAS(-AR) models with those of the corresponding models of the MS-MIDAS(-AR) and MS-ADL-MIDAS class. In addition, for one-, two- and four-step-ahead forecasts, we also include further Markov-switching models in the comparison: the Markov-Switching Distributed Lag model (MS-DL) and the Markov-Switching Autoregressive Distributed Lag model (MS-ADL). The MS-DL and MS-ADL models are equivalent to MS-MIDAS and MS-ADL-MIDAS respectively, with all lags of the explanatory variable weighted equally. Thus, it is reasonable to compare these models with MS-MIDAS regressions in order to see whether the use of models with estimated lag polynomial weights is necessary. However, a major drawback of the distributed lag class of models is that they do not allow for leads, and thus they cannot use the available data on monthly regressors for the quarter for which the forecast is calculated. That is why these models cannot be used for nowcasting.

In our comparison, we also include benchmark models without Markov-switching, in particular the random walk and AR(2) models, U-MIDAS and MIDAS (with and without autoregressive dynamics), the Distributed Lag model (DL) and the Autoregressive Distributed Lag model (ADL). These constant-parameter models are restricted versions of the models with Markov-switching, so it is reasonable to use them for the sake of a comparison of the model performance. An overview of the models we use in the analysis can be found in Table A.1 in the online Appendix.

The first part of the empirical study concentrates on the description and introductory analysis of the dataset, as well as on the extraction of factors from the data. Further parts include the analysis of the in-sample fit of the MS-U-MIDAS-AR model and its forecasting performance (in terms of the RMSE and QPS) relative to a wide spectrum of other models mentioned above.

4.1. Data

For the empirical exercise on forecasting quarterly GDP growth in the United States, we use a set of 156 monthly macroeconomic and financial variables, yielding information on i.a.: industrial production, employment, inflation, federal debt, bank assets, interest rates, government bonds, stock prices and some macroeconomic leading indicators. The data come from Datastream. The data on output cover the period from 1959:Q1 to 2011:Q3 (211 quarters). The monthly data cover the period from June 1958 to September 2011. The information on business cycles in the United States comes from the National Bureau of Economic Research,[6] and is used to assess the performance of the model in predicting regime changes in the economy.

The dataset we use in our analysis is qualitatively similar to other forecasting studies on the US GDP growth (e.g., Giannone & Small, 2008), but includes fewer variables (156 compared to 200 variables in the cited paper). This is due to the fact that Markov-switching models involve the estimation of a large number of parameters. Thus, the use of such models requires a long sample, which means that fewer variables are available throughout the analysed period than is the case in some other studies. Although our sample includes a wide spectrum of macroeconomic and financial variables, our data selection is limited by the availability of time series.

The dataset we use is a final vintage, balanced dataset, collected in September 2012. Thus, we do not account for data revisions and publication lags of the monthly variables. As the dataset includes more than 150 variables, this would simply complicate the analysis greatly. Besides, we do not know the exact publication pattern for all of the variables we use in the analysis. We also do not want to create any kind of artificial pattern, especially when we cannot realistically assume that this pattern will remain stable over the whole period we consider in our analysis. However, as a robustness check, we repeat the forecasting exercise for real-time data on GDP growth.[7] The results of this real-time analysis can be found in the online Appendix (Tables A.10–A.13).

We carried out Augmented Dickey–Fuller and Philips–Perron unit root tests for all time series included in our data set, to investigate their order of integration. Based on the results, we then transformed the data so as to ensure stationarity. Depending on the characteristics and behaviour of the data, we used either first/second differences or first/second differences of logarithms of the time series to make them stationary. The GDP growth was calculated as: $100(\ln(GDP_t) - \ln(GDP_{t-1}))$. The details of the transformation for specific variables and an exact description of all time series used in the analysis can be found in Table A.2 in the online Appendix.

4.2. Extracting factors from the dataset

MIDAS models are usually applied within a univariate framework, that is, with only one explanatory variable included in the model. This ensures parsimony and simplifies the estimation, but does not seem to be the optimal solution, as it is difficult to select a single variable that summarizes the overall economic situation accurately and preserves the same level of relevance for the economic activity over a long period of time. A reasonable approach should explore the informational content of a wide range of available macroeconomic and financial time series, which can reflect the current state of the economy only as a whole, not as individual variables. However, the inclusion of many regressors in the model results in parameter proliferation, and is therefore very problematic. A possible solution to that issue is the use of factor analysis. There is a vast body of literature available on factor extraction techniques, e.g. Bai (2003), Bai and Ng (2002), Forni, Hallin, Lippi, and Reichlin (2000, 2005) or Stock and Watson (2002a). It is assumed that an N-dimensional set of explanatory variables $\boldsymbol{X}_t$ can be represented by a few common latent factors $\boldsymbol{F}_t$, which are then used as regressors in the main analysis:

$$\boldsymbol{X}_t = \Delta \boldsymbol{F}_t + \boldsymbol{e}_t,$$

where $\boldsymbol{e}_t$ is a vector of idiosyncratic disturbances, and Δ is the loading matrix.

The main idea behind factor analysis is to transform the available dataset in such a way that a substantial part of the variation in the observed variables can be explained by just a few unobserved factors. Thus, using a single factor

6 Data available at: http://www.nber.org/cycles/cyclesmain.html.

7 The real-time data on the GDP come from the website: http://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data/data-files/ROUTPUT/.

that explains a large part of the variation, instead of a single observed variable, may capture more information from the available dataset and ensure the parsimony of the model. However, useful though this method is, one should use it with some caution. Factor analysis is a purely statistical way of summarizing the variation in the dataset in an efficient way, in order to reduce the dimension of the system. The factors which explain the biggest part of the overall variation in the dataset are not always the best predictors of the dependent variable. Consequently, the choice of factors may play a crucial role in the analysis, and the forecasting performance of the model might depend heavily on it. To achieve good forecasting results and at the same time maintain parsimony of the model, one may use forecast combinations of single-factor regressions. We follow this approach later in the paper.

Following the above-described way of reducing the dimension of the dataset, we use Principal Component Analysis to extract factors from our set of monthly macroeconomic and financial variables for the United States. For the introductory analysis, we extract factors using the whole available sample of monthly data; that is, from June 1958 to September 2011. However, for a further analysis of the forecasting performances of the models, we first extract factors for the in-sample period only, then factors are extracted recursively for the out-of-sample period; that is, they are updated with each forecast calculation. For the full sample, we find that the first factor explains about 10% of the variation in the dataset, whereas the first five factors together account for about 27% of the variation. These numbers might not sound impressive, but they are not uncommon for large datasets which include a wide spectrum of different variables (see e.g. Stock & Watson, 2002b). Fig. A.1 in the online Appendix provides a graphical illustration of the percentage of the total variation in the dataset that is explained by each of the first ten factors.

The choice of the optimal number of factors for further analysis is not a trivial issue. We follow the approach of Bai and Ng (2002), who treat the selection of the number of factors as a problem of optimizing the trade-off between the goodness-of-fit of the model and its parsimony. We apply the first and second criteria proposed by these authors and find that the two criteria point consistently to four and five factors respectively. Thus, considering five factors for further analysis seems to be a reasonable approach. One can make use of the informational content of each of these factors either by considering them together as regressors within one model or by combining the forecasts obtained through models which use them as single explanatory variables. To avoid parameter proliferation, we focus on the latter option, and go on to investigate the forecasting accuracy of the single-factor models, as well as their combinations.

It turns out that factor 1 loads mainly on variables such as industrial production, the industrial confidence indicator, leading macroeconomic indicators and manufacturers' survey data. Factor 2 loads mainly on interest rates, bond yields and stock market indices. Factor 3 loads mainly on inflation data (CPI, PPI, oil price index), whereas factor 4 loads mainly on bond yields, industrial production and numbers of working hours. Finally, factor 5 loads on stock prices, employment and manufacturers' survey data. Further analysis shows how useful each of these factors is in forecasting GDP growth and the extent to which the forecasting performances of the tested models can be enhanced by combining the forecasts obtained from single factors. A graphical representation of the five factors, together with the GDP growth, can be found in Fig. A.2 in the online Appendix.

4.3. Further issues concerning the MS-MIDAS class of models

An issue of the utmost importance for the Markov-switching type models is the choice of the optimal number of regimes and the determination of the parameters that are allowed to switch in the model. For the model specification, we use the Akaike (AIC) and Bayesian (BIC) Information Criteria. We test the cases where the numbers of regimes are two (recession and expansion) and three (recession, stable growth and rapid expansion). However, the information criteria do not deliver unanimous results on the optimal number of regimes. The AIC points towards three regimes (three regimes were also used in a similar study by Guérin & Marcellino, 2013, for the US GDP, although that result was obtained using the BIC), whereas the BIC, which penalizes the growth in the number of parameters in the model more strongly, points towards a model with two regimes. Thus, for the sake of robustness, we present results for models with two and three regimes. In addition, we also find it important to see how the performance of the MS-U-MIDAS model changes relative to its restricted counterpart in the case when the number of parameters grows (two regimes vs. three).

Furthermore, with the help of the AIC and BIC, we investigate models that allow for a switch only in the intercept β_0; in the intercept β_0 and the variance of the error term σ^2; and in the intercept β_0, variance σ^2 and slope parameter β_1 (the coefficient of the autoregressive term λ and the parameters of the lag polynomial $\boldsymbol{\theta}$ do not switch between regimes). Both criteria favour models with all parameters switching. Since the volatility of the GDP growth time series analysed changes over time, and increases especially dramatically during the crisis periods, we do not think that it is reasonable to consider a model without a switch in the variance of the error term, so this case is not examined further in our paper. However, we do consider the other cases; that is, for the sake of completeness, we investigate the performances of models with a switch in the intercept and variance only, as well as with all of the parameters being allowed to switch between the regimes.

We also use the above-mentioned information criteria to determine the optimal number of lags of the explanatory variables included in the analysed models, setting the maximum number of autoregressive lags to four (four quarters can be taken into account) and the maximum number of lags of the other explanatory variables to six (six for the monthly data, that is, the information from the last two quarters can be included) in order to avoid parameter proliferation. In most cases, the information criteria opt for different numbers of lags, but we find hardly any differences between the performances of the models estimated according to the AIC and BIC. In this paper, we only present results for the number of lags chosen according to the AIC, as this criterion tends to overestimate the optimal number of lags, thus creating a potentially more difficult test for the unrestricted class of MIDAS models, which are more prone to parameter proliferation than their restricted counterparts.

4.4. In-sample fit of the MS-U-MIDAS model

In this paper, we consider many different models with a lot of parameters, so it is difficult to present the estimation results for all of them. Generally, for regime-switching models, one can observe that the parameters vary depending on the states of the world: a small intercept β_0 and high variance σ^2 indicate a recession, whereas a high intercept β_0 and low variance σ^2 point to expansion. A useful feature of Markov-switching models is that, in addition to the estimated parameters and fitted values of the dependent variable, one can also obtain the probabilities of being in a specific regime at a given point in time. Thus, one can calculate the probabilities of being in a period of recession or expansion. As this feature is particularly important for the policy makers and provides an opportunity to show the in-sample performance of the model in a graphical way, we present the smoothed probabilities of being in a specific regime in relation to the actual crisis periods (taken from the NBER database).

We consider the performances of the five factors taken for further analysis and present the results for the MS-U-MIDAS model with two and three regimes (with the intercept, slope and variance of the error term switching). For the case of two regimes, all five of the factors perform quite well in detecting the recent periods of crisis (after 1990). However, most of them (especially factors 1, 4 and 5) face substantial problems in matching the earlier recessions, due mainly to the fact that the overall volatility of the series before 1990 is considerably higher than that after 1990. On the other hand, factor 2 seems to match the crisis periods before 1990 especially well, while the spike representing the dot-com bubble from 2000–2001 is less conspicuous (see Fig. 1). This points to the fact that some of the factors (factors 2 and 3) perform better in matching the recessions before 1990, while others excel at detecting the more recent crises. This might be an argument in favour of either using multiple factors within the same model or combining the results obtained from single-factor regressions.

The unsatisfactory performance of the model in identifying recessionary periods before 1990 could also be a point in favour of a model with three regimes. Due to the greater parameter flexibility, such a model might be able to deal with the change in volatility of the GDP growth of the United States across the analysed period better than the model with two regimes. The estimation of the MS-U-MIDAS model with three regimes and all parameters switching yields slightly better in-sample results for the period before 1990 than that of the model with two regimes (see Fig. 2), although the results for factors 1, 4 and 5 are still not satisfactory for this time period. Again, the recent recessions are matched more accurately. However, a relative in-sample performance improvement for some factors in the case of three regimes does not guarantee that models with three regimes will perform better than those with two regimes in the out-of-sample analysis, which is the main purpose of this work.

4.5. Out-of-sample performance of the MS-U-MIDAS model

In order to investigate the forecasting performance of the MS-U-MIDAS model, we divide our original sample into two parts: the time between 1959:Q1 and 1999:Q4 is considered the in-sample period (160 quarters), whereas the time from 2000:Q1 to 2011:Q3 (47 quarters) is used for the forecasting exercise. The choice of the in- and out-of-sample periods is arbitrary. However, the in-sample period should be long enough to ensure precise parameter estimation, whereas the out-of-sample period, which encompasses some periods of both expansion and recession, is chosen to be long enough for a reliable assessment of the forecasting performance of the model. For the exercise presented here we use final-vintage data, but we also repeat the analysis for the real-time dataset for the US GDP. The results of this exercise for horizons $\left\{h = \frac{1}{3}, \frac{2}{3}, 1\right\}$ can be found in the online Appendix (see Tables A.10–A.13), and are comparable overall to the results for the final vintage data.

We calculate forecasts recursively for horizons $\{h = \frac{1}{3}, \frac{2}{3}, 1, 2, 4\}$. We also extract factors from the explanatory dataset in a recursive way. For nowcasting, the horizon is defined as a fraction and the forecasts are calculated using leads; that is, in the MIDAS class of models, we include monthly observations from the quarter for which the forecast is calculated.

The forecasting exercise is carried out not only for the MS-U-MIDAS class of models, but also for a wide range of other models: MS-MIDAS(-AR) models, the MS-ADL-MIDAS model, the Markov-switching Distributed Lag model (MS-DL), the Markov-switching Autoregressive Distributed Lag model (MS-ADL), and some models without Markov-switching: MIDAS and U-MIDAS (with and without autoregressive dynamics), the Distributed Lag model (DL), the Autoregressive Distributed Lag model (ADL), an AR(2) and a random walk (RW). In the case of models that do not allow for mixed frequencies, it is not possible to use leads. For this reason, these models are only included in the comparison for horizons $h = \{1, 2, 4\}$, not for nowcasting.

First, the forecasts are calculated separately for models that use each of the five single factors considered in the analysis, then those forecasts are combined according to five different schemes in order to enhance the forecasting performances of the single factors. We first consider the mean (Mean) and median (Med.) of the forecasts from all single-factor models. Furthermore, we use a scheme, proposed by Stock and Watson (2004), that assigns weights to single-factor forecasts according to their Squared Discounted Mean Forecast Error (S.D.E.), thus accounting for the past forecasting performance of the model, and at the same time giving more weight to more recent observations. The forecast weights w_{it} in this method are calculated as follows:

$$w_{it} = \frac{m_{it}^{-1}}{\sum_{j=1}^{n} m_{jt}^{-1}},$$

$$\text{where } m_{it} = \sum_{s=\tau}^{t-h} \delta^{t-h-s} (y_{s+h} - \hat{y}_{i,s+h|s})^2.$$

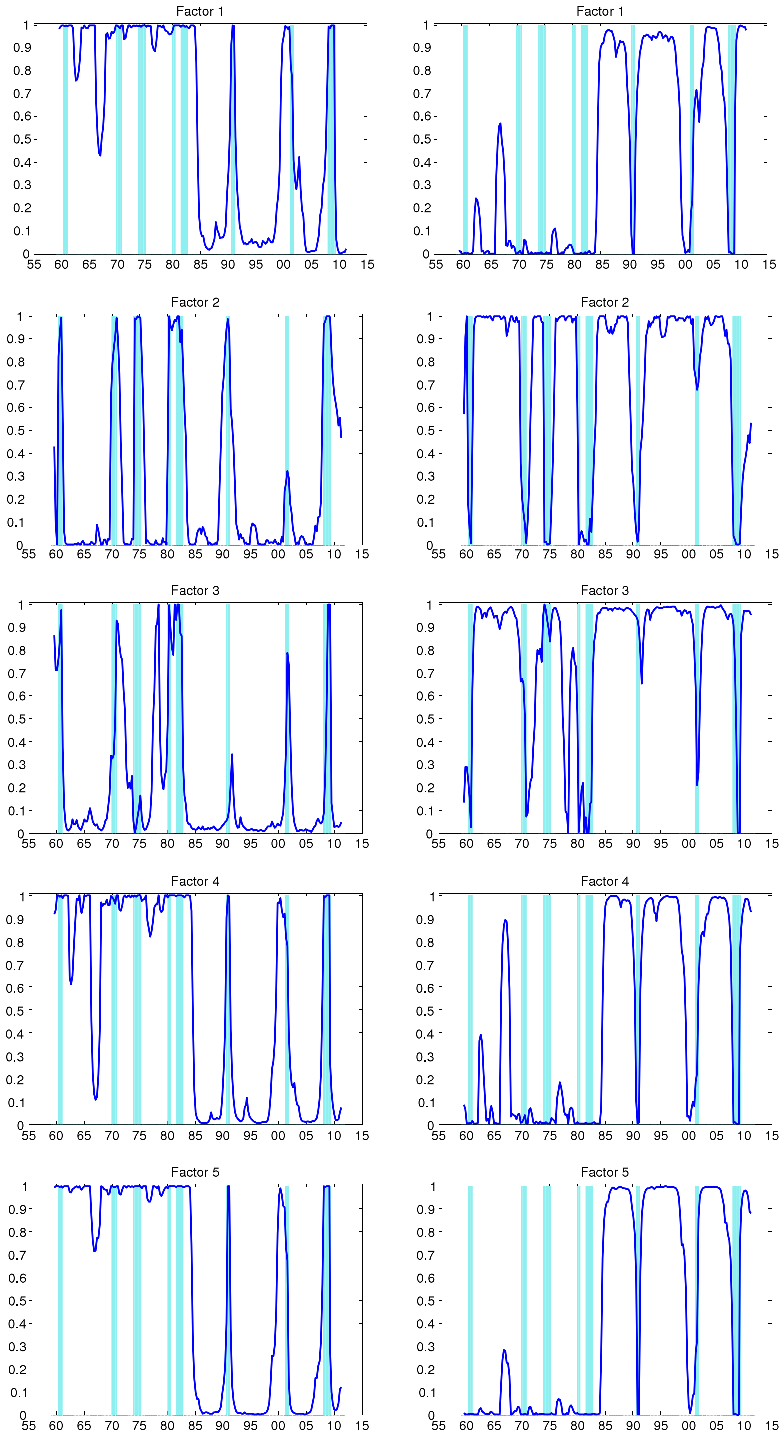

Fig. 1. Smoothed probabilities (dark line) of a crisis (left) and expansion (right), calculated for the period from 1959:Q1 to 2011:Q3 for the MS-U-MIDAS model with two regimes (where all parameters are allowed to switch). The probabilities are compared with the actual recessionary periods according to the NBER (bars).

Here, δ denotes the discount factor, which is equal to 0.9 in our analysis, h is the forecast horizon, y is the actual and $\hat{y}$ the estimated GDP growth, n is the number of forecasts combined, t is the time index, and i is the index denoting the forecast of the ith model considered for the combination.

Finally, we also consider a forecast combination scheme which trims the recently worst model, where we take into account both the last quarter (Tr. (1)) and the last four quarters (Tr. (4)).

The forecasts obtained through the above-described combination methods are then compared in terms of RMSE

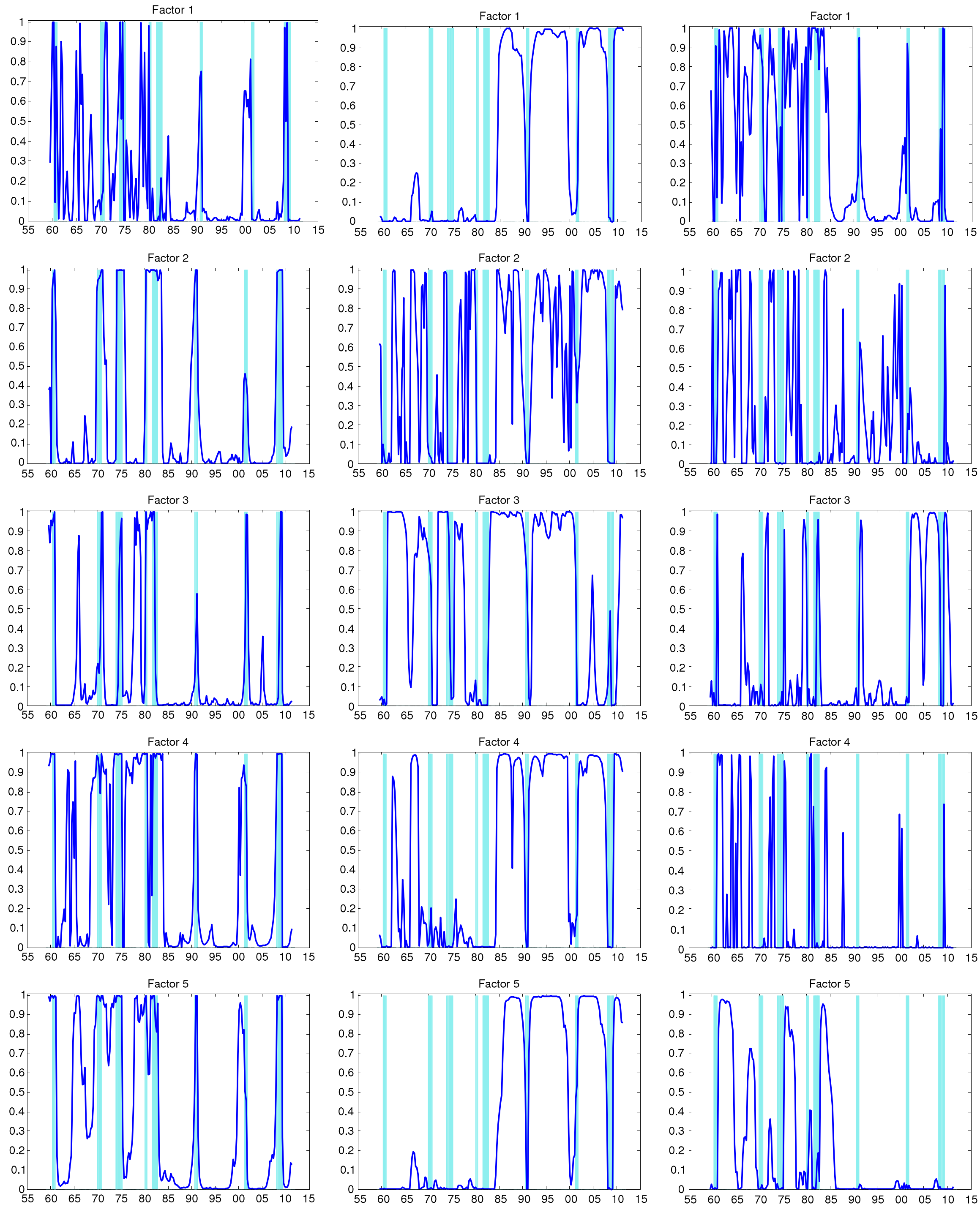

Fig. 2. Smoothed probabilities (dark line) of a crisis (left), stable growth (middle) and expansion (right), calculated for the period from 1959:Q1 to 2011:Q3 for the MS-U-MIDAS model with three regimes (where all parameters are allowed to switch). The probabilities are compared with the actual recessionary periods according to the NBER (bars).

and QPS values. We also compare the forecasting performances of the combination schemes with those of the single-factor models. The results of such exercises for different forecasting horizons can be found in Tables A.5–A.9 in the online Appendix. To save space, we present results in Table 8 in the main part of the paper for different forecasting horizons, but for only one forecast combination scheme (Mean). The RMSEs are presented relative to the benchmark model, which is an AR(2). Values below/above one indicate a better/worse performance of the specific model

Table 8
Root mean squared errors relative to an AR(2) model. Final vintage data are used. The numbers of lags in the models are chosen according to the AIC.

Model	Switch	$h = 1/3$	$h = 2/3$	$h = 1$	$h = 2$	$h = 4$
Benchmarks						
RW	–	X	X	1.11	1.09	1.23
AR(2)	–			1.00	1.00	1.00
Two regimes						
MS-U-MIDAS	I, V	0.93*	0.93	1.04	1.06	1.00
MS-U-MIDAS	I, S, V	0.91*	0.96	1.04	1.04	0.96
MS-U-MIDAS-AR	I, V	0.83***	0.83**	0.90	0.96	0.98
MS-U-MIDAS-AR	I, S, V	0.83***	0.81***	0.93	0.97	0.98
MS-MIDAS	I, V	0.89**	0.91	1.04	1.06	1.00
MS-MIDAS	I, S, V	0.86**	0.90*	1.05	1.05	1.00
MS-MIDAS-AR	I, V	0.87***	0.86*	0.94	0.96	0.97
MS-MIDAS-AR	I, S, V	0.85***	0.85***	0.94	0.97	0.99
MS-ADL-MIDAS	I, V	0.81***	0.84***	0.92	0.97	1.01
MS-ADL-MIDAS	I, S, V	0.79***	0.83***	0.96	1.00	1.02
MS-ADL	I, V	X	X	0.93	0.96	0.97
MS-ADL	I, S, V			0.93	0.96	0.96
MS-DL	I, V	X	X	1.05	1.05	0.99
MS-DL	I, S, V			1.02	1.05	0.96
Three regimes						
MS-U-MIDAS	I, V	0.92*	0.91	0.97	1.01	0.94
MS-U-MIDAS	I, S, V	0.85**	0.92**	0.97	0.99	0.95
MS-U-MIDAS-AR	I, V	0.80***	0.87*	0.94	0.95	0.95
MS-U-MIDAS-AR	I, S, V	0.83***	0.81***	0.95	0.97	0.97
MS-MIDAS	I, V	0.90**	0.90**	0.97	1.01	0.96
MS-MIDAS	I, S, V	0.89**	0.86**	0.96	1.00	0.96
MS-MIDAS-AR	I, V	0.84***	0.87**	0.94	0.98	0.95
MS-MIDAS-AR	I, S, V	0.83***	0.84**	0.92	0.99	0.94
MS-ADL-MIDAS	I, V	0.77***	0.85***	0.91	0.97	0.99
MS-ADL-MIDAS	I, S, V	0.84**	0.85***	0.94	0.98	1.00
MS-ADL	I, V	X	X	0.93	0.98	0.96
MS-ADL	I, S, V			0.93	0.97	0.94
MS-DL	I, V	X	X	1.00	1.00	0.95
MS-DL	I, S, V			0.98	0.96	0.94
Models without Markov-switching						
U-MIDAS	–	0.93*	0.95	1.04	1.05	1.01
U-MIDAS-AR	–	0.82***	0.85***	0.92***	0.97	1.00
MIDAS	–	0.85**	0.91	1.02	1.07	1.01
MIDAS-AR	–	0.82***	0.85***	0.94***	0.98	0.99
ADL-MIDAS	–	0.79***	0.84***	0.92	0.97	1.01
DL	–	X	X	1.00	0.97	1.08
ADL	–			0.94***	0.98	0.98

Column "Switch" gives information on the parameters that are allowed to switch in the model: I: intercept β_0, V: variance of the error term σ^2, S: slope β_1. The table presents the out-of-sample RMSEs for the models compared in this paper. The original sample is divided into an in-sample period between 1959:Q1 and 1999:Q4 (160 quarters) and an out-of-sample period between 2000:Q1 and 2011:Q3 (47 quarters). For the out-of-sample period, the forecasts are calculated recursively. In order to evaluate the forecasting performances of the models, we calculate the RMSE for each model for the whole out-of-sample period. The RMSEs are calculated for the forecast combination scheme that involves taking the mean of the forecasts from single factor models.

* Indicates significance at the 10% level.

** Indicates significance at the 5% level.

*** Indicates significance at the 1% level. (Significance of the forecasting performance relative to that of the random walk model was investigated via the Diebold–Mariano test.)

relative to an AR(2). We also present the out-of-sample results for the QPS in Table 9. A concise summary of all of the results from Tables A.5–A.9 from the online Appendix can be found in Table 10.

There is no clear pattern which could help us to rank the factors in terms of their forecasting suitability. However, it seems that factor 1 in particular, and factors 3 and 5 to a lesser extent, perform decently in forecasting GDP growth for some of the cases considered, whereas factor 2 deals with the task considerably worse. This might seem surprising, as factor 2 performs very well in detecting periods of recession in the in-sample analysis. One reason for such a situation might be the fact that factor 2 loads mainly on financial variables, which are helpful in detecting the periods of financial crises, but are not particularly useful in forecasting GDP growth during periods of stable growth. This results in an overall unsatisfactory forecasting performance. On the other hand, factor 1, and factors 3, 4 and 5

Table 9
Quadratic probability scores calculated for filtered probabilities of the crisis regime for the out-of-sample period. Final vintage data are used.

Model	Switch	$h = 1/3$	$h = 2/3$	$h = 1$
Two regimes				
MS-U-MIDAS	I, V	0.360^{**}	0.345^{*}	0.392
MS-U-MIDAS	I, S, V	0.314^{**}	0.442	0.505
MS-U-MIDAS-AR	I, V	0.295^{***}	0.381^{**}	0.450
MS-U-MIDAS-AR	I, S, V	0.381^{**}	0.324^{***}	0.383
MS-MIDAS	I, V	0.345^{**}	0.391	0.443
MS-MIDAS	I, S, V	0.372	0.503	0.528
MS-MIDAS-AR	I, V	0.331^{***}	0.388^{**}	0.437
MS-MIDAS-AR	I, S, V	0.332	0.479	0.514
MS-ADL-MIDAS	I, V	0.405^{**}	0.410^{**}	0.452
MS-ADL-MIDAS	I, S, V	0.368^{**}	0.271^{***}	0.490
MS-ADL	I, V	X	X	0.497
MS-ADL	I, S, V			0.473
MS-DL	I, V	X	X	0.441
MS-DL	I, S, V			0.481
Three regimes				
MS-U-MIDAS	I, V	0.261^{***}	0.297^{***}	0.296^{***}
MS-U-MIDAS	I, S, V	0.316^{**}	0.297^{***}	0.276^{***}
MS-U-MIDAS-AR	I, V	0.353^{*}	0.342^{**}	0.308^{***}
MS-U-MIDAS-AR	I, S, V	0.320^{**}	0.344^{*}	0.335^{**}
MS-MIDAS	I, V	0.349^{*}	0.290^{***}	0.305^{**}
MS-MIDAS	I, S, V	0.342^{*}	0.295^{***}	0.279^{***}
MS-MIDAS-AR	I, V	0.327^{**}	0.319^{**}	0.312^{**}
MS-MIDAS-AR	I, S, V	0.301^{***}	0.285^{***}	0.327^{**}
MS-ADL-MIDAS	I, V	0.354^{*}	0.337^{**}	0.341^{*}
MS-ADL-MIDAS	I, S, V	0.300^{**}	0.352^{*}	0.316^{**}
MS-ADL	I, V	X	X	0.355^{*}
MS-ADL	I, S, V			0.349^{*}
MS-DL	I, V	X	X	0.264^{***}
MS-DL	I, S, V			0.306^{**}

Column "Switch" gives information on the parameters that are allowed to switch in the model: I: intercept β_0, V: variance of the error term σ^2, S: slope β_1.
The table presents the out-of-sample QPS for the models compared in this paper. The original sample is divided into an in-sample period between 1959:Q1 and 1999:Q4 (160 quarters) and an out-of-sample period between 2000:Q1 and 2011:Q3 (47 quarters). For the out-of-sample period, the predicted probabilities of the crisis regime are calculated recursively for each single factor model. These probability forecasts are then combined by taking the mean of the single forecasts. The combined probability forecasts are then compared with the actual recessionary periods according to NBER.
* Indicates significance at the 10% level.
** Indicates significance at the 5% level.
*** Indicates significance at the 1% level. (Significance of the regime predicting performance relative to that of the MS-ADL model with two regimes was investigated via the Diebold–Mariano test.)

to a lesser extent, load on macroeconomic variables, which prevents them from being very accurate in detecting financial crises, but adds to their relatively good performance in forecasting GDP growth.

In general, our results confirm the findings of many previous forecasting exercises, which have concluded that combining forecasts gives more robust, and in many cases also better, results than using forecasts from single-variable models only (Timmermann, 2006). Not only do forecast combinations usually display a better forecasting accuracy, they also tend to show more reliability than the individual forecasts. For most of the models analysed, the out-of-sample performances of individual forecasts depend heavily on the single factor used for forecasting. Thus, the same model may display either a very good or a very poor performance, depending on which of the five factors is used for forecasting. Moreover, the differences in the average performances of the five combination schemes (Mean, Med., S.D.E., Tr. (1) and Tr. (4)) are small for each of the regressions considered.

It is worth noting that, in some cases, models without Markov-switching perform surprisingly well after combining the forecasts from different factors. However, if we take a closer look at their performances in forecasting using single factors, we can observe that they perform worse than their Markov-switching counterparts. Thus, in many cases, models without Markov-switching parameters gain slightly more from forecast combination than the class of Markov-switching models that we consider in this paper. In several cases, the performances of the Markov-switching models after combining the forecasts for different factors are worse than for the best performing factor. On the other hand, in most of the cases, the forecasting performances of the models without Markov-switching parameters after forecast combination are better than that of

Table 10
Summary of the results.

Model	$h = 1/3$	$h = 2/3$	$h = 1$	$h = 2$	$h = 4$
In terms of RMSE					
MS-U-MIDAS	0.0	0.0	0.0	0.0	47.2
MS-U-MIDAS-AR	25.0	66.6	66.7	41.7	5.6
MS-MIDAS	0.0	0.0	0.0	0.0	0.0
MS-MIDAS-AR	0.0	16.7	0.0	25.0	47.2
MS-ADL-MIDAS	75.0	16.7	33.3	33.3	0.0
In terms of QPS					
MS-U-MIDAS	66.7	16.7	50.0		
MS-U-MIDAS-AR	33.3	16.7	8.3		
MS-MIDAS	0.0	33.3	25.0		
MS-MIDAS-AR	0.0	0.0	16.7		
MS-ADL-MIDAS	0.0	33.3	0.0		

The table contains a summary of the results from Tables A.5–A.9 in the online Appendix, and shows the proportion of times (in %) when a specific type of model is better than the other models. The comparison is done in terms of the RMSE and QPS, and includes the specifications for single factor models, as well as for a forecast combination made by taking the mean of the single forecasts. The models considered in the comparison are the models that are the main scope of this paper: MS-U-MIDAS(-AR), MS-MIDAS(-AR) and MS-ADL-MIDAS. The models with all parameters switching (I, V, S) and the models without the slope parameters switching (I, V) are considered as one type of model and classified together.

the best performing factor. Therefore, when comparing the forecasting performances of the models, one should not focus solely on the combined forecasts, but should also consider the forecasting accuracy of the single factors.

Overall, the results show that the MS-U-MIDAS-AR models perform especially well in terms of the RMSE for horizons $h = 2/3$ and $h = 1$ (they are better than other models in more than 60% of the cases considered). Only for $h = 1/3$ does the MS-ADL-MIDAS model show a better performance than the MS-U-MIDAS-AR, while the MS-U-MIDAS model beats its counterpart with autoregressive dynamics for $h = 4$. The MS-MIDAS(-AR) types of models are clearly beaten by both the MS-U-MIDAS(-AR) and MS-ADL-MIDAS. However, one has to admit that the differences are rather small and statistically insignificant in most cases. In fact, the results of an equal predictive accuracy test (the Diebold–Mariano test), conducted relative to the benchmark model, show that, in terms of both forecasting performance and regime predicting performance, most of the results are insignificant for horizons $h = 2$ and $h = 4$. For shorter horizons $h = \left\{\frac{1}{3}, \frac{2}{3}, 1\right\}$, most of the results are highly significant. This shows that the MIDAS type of models are especially useful for nowcasting.

In terms of the QPS, the MS-U-MIDAS model performs well, especially for horizons $h = 1/3$ and $h = 1$. For horizon $h = 2/3$, the performances of MS-U-MIDAS(-AR), MS-MIDAS and MS-ADL-MIDAS are comparable. Although adding autoregressive dynamics is very important in improving the forecasting accuracy in terms of RMSE for all models considered and for all forecasting horizons, this is not the case when our interest is in forecasting the true regime. The forecasting results for QPS show that, in most of the cases, adding autoregressive dynamics leads to a deterioration of the regime predicting ability of the models. It is also worth noting that, in most of the cases, models with three regimes outperform their equivalents with two regimes in predicting the true regime, whereas the forecasting accuracy in terms of RMSE is robust to the choice of the number of regimes.

5. Conclusion

The class of MIDAS models opens new possibilities for researchers to use the available data of different frequencies in forecasting, and to deal with the problem of different publication lags, found with many macroeconomic variables. In addition, the Markov-switching MIDAS offers the possibility of modelling the business cycle pattern that is present in many macroeconomic time series. However, one of the issues that is still open for research on MIDAS is the optimal choice of the shape of weights in the lag polynomial used in this class of models. In most applications, this choice is restricted by a specific function shaping the lag polynomial, and thus might not reflect the true data generating process fully.

In this paper, we have investigated the usefulness of factor MIDAS models with unrestricted lag polynomials and a Markov-switching component for modelling large datasets. We show that it can be implemented with good results for forecasting tasks, when the differences between the frequencies of the analysed variables are small (e.g., quarterly–monthly data). We compare the MS-U-MIDAS(-AR), MS-MIDAS(-AR) and MS-ADL-MIDAS models through Monte Carlo simulations for different DGPs and through an empirical forecasting exercise for GDP growth in the United States. In both cases, the performance of the MS-U-MIDAS class of models is comparable to or better than those of its restricted counterparts. This makes the unrestricted Markov-switching MIDAS a useful alternative to the restricted MS-MIDAS model for many macroeconomic applications, especially when the difference in frequencies is small. Furthermore, we show that using forecast combinations of models with different factors often provides better results than using forecasts from single factor models. To sum up, the methods presented and applied in this paper help to make better use of the informational content of the available datasets, and thus, in many cases, can obtain better forecasts of the variables of interest.

Acknowledgments

We would like to thank Prof. Ralf Brüggemann, the Associate Editor and two anonymous referees, as well as the participants at the 67th European Meeting of the Econometric Society (Gothenburg 2013), the 17th Spring Meeting of Young Economists at the Centre for European Economic Research, the Marie Curie ITN Conference on Financial Risk Management and Risk Reporting, the DFH Königsfeld workshop on Applied Econometrics, and the Doctoral Seminar on Econometrics at the University of Konstanz for useful comments that helped us to improve our paper. The first author gratefully acknowledges funding from the European Community's Seventh Framework Programme FP7-PEOPLE-ITN-2008 under grant agreement number PITN-GA-2009-237984.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.ijforecast.2014.04.002.

References

Anas, J., Billio, M., Ferrara, L., & Duca, M. L. (2007). *Business cycle analysis with multivariate Markov switching models* (pp. 249–260). Palgrave Macmillan.

Andreou, E., Ghysels, E., & Kourtellos, A. (2013). Should macroeconomic forecasters use daily financial data and how? *Journal of Business and Economic Statistics*, *31*(2), 240–251.

Bai, J. (2003). Inferential theory for factor models of large dimensions. *Econometrica*, *71*(1), 135–171.

Bai, J., Ghysels, E., & Wright, J. H. (2013). State space models and MIDAS regressions. *Econometric Reviews*, *32*(7), 779–813.

Bai, J., & Ng, S. (2002). Determining the number of factors in approximate factor models. *Econometrica*, *70*(1), 191–221.

Barsoum, F. (2011). *How useful is financial market data in forecasting UK real GDP?* Mimeo.

Bessec, M., & Bouabdallah, O. (2013). *Forecasting GDP over the business cycle in a multi-frequency and data-rich environment*. Working paper.

Cheung, Y.-W., & Erlandsson, U. G. (2005). Exchange rates and Markov switching dynamics. *Journal of Business and Economic Statistics*, *23*(3), 314–320.

Clements, M., & Galvão, A. B. (2008). Macroeconomic forecasting with mixed-frequency data: forecasting output growth in the United States. *Journal of Business and Economic Statistics*, *26*(4), 546–554.

Clements, M., & Galvão, A. B. (2009). Forecasting US output growth using leading indicators: An appraisal using MIDAS models. *Journal of Applied Econometrics*, *24*(7), 1187–1206.

Clements, M., & Krolzig, H.-M. (1998). A comparison of the forecast performance of Markov-switching and threshold autoregressive models of US GNP. *The Econometrics Journal*, *1*, 47–75.

Engel, C. (1994). Can the Markov switching model forecast exchange rates? *Journal of International Economics*, *36*, 151–165.

Evans, M., & Wachtel, P. (1993). Inflation regimes and the sources of inflation uncertainty. *Journal of Money, Credit and Banking*, *25*, 475–511.

Forni, M., Hallin, M., Lippi, M., & Reichlin, L. (2000). The generalized dynamic-factor model: identification and estimation. *The Review of Economics and Statistics*, *82*(4), 540–554.

Forni, M., Hallin, M., Lippi, M., & Reichlin, L. (2005). The generalized dynamic factor model: one-sided estimation and forecasting. *Journal of the American Statistical Association*, *100*(471), 830–840.

Foroni, C., Marcellino, M., & Schumacher, C. (in press). Unrestricted mixed data sampling (MIDAS): MIDAS regressions with unrestricted lag polynomials. *Journal of the Royal Statistical Society*.

Frömmel, M., MacDonald, R., & Menkhoff, L. (2005). Markov switching regimes in a monetary exchange rate model. *Economic Modelling*, *22*(3), 485–502.

Ghysels, E., Santa-Clara, P., & Valkanov, R. (2002). *The MIDAS touch: mixed data sampling regression models*. Working Paper, UNC and UCLA.

Giannone, D., & Small, L. R. D. (2008). Nowcasting: the real-time informational content of macroeconomic data. *Journal of Monetary Economics*, *55*, 665–676.

Guérin, P., & Marcellino, M. (2013). Markov switching MIDAS models. *Journal of Business and Economic Statistics*, *31*(1), 45–56.

Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica*, *57*(2), 357–384.

Hamilton, J. D. (1994). *Time series analysis*. Princeton University Press.

Krolzig, H.-M. (2000). *Predicting Markov-switching vector autoregressive processes*. Working paper.

Kuzin, V., Marcellino, M., & Schumacher, C. (2011). MIDAS vs mixed-frequency VAR for nowcasting GDP in the Euro area. *International Journal of Forecasting*, *27*, 529–542.

Lahiri, K., & Wang, J. G. (1994). Predicting cyclical turning points with leading index in a Markov switching model. *Journal of Forecasting*, *13*, 245–263.

Marcellino, M., & Schumacher, C. (2010). Factor MIDAS for nowcasting and forecasting with ragged-edge data: a model comparison for German GDP. *Oxford Bulletin of Economics and Statistics*, *72*(4), 518–550.

Pagliacci, C., & Barraez, D. (2010). A Markov-switching model of inflation: looking at the future during uncertain times. *Analisis Economico*, *XXV*(59), 25–46.

Perlin, M. (2011). *MS regress—the MATLAB package for Markov regime switching models*. Working paper.

Simon, J. (1996). *A Markov-switching model of inflation in Australia*. Research discussion paper.

Stock, J., & Watson, M. (2002a). Forecasting using principal components from a large number of predictors. *Journal of the American Statistical Association*, *97*(460), 1167–1179.

Stock, J., & Watson, M. (2002b). Macroeconomic forecasting using diffusion indexes. *Journal of Business and Economic Statistics*, *20*(2), 147–162.

Stock, J., & Watson, M. (2004). Combination forecasts of output growth in a seven-country data set. *Journal of Forecasting*, *23*, 405–430.

Timmermann, A. (2006). *Handbook of economic forecasting*. Amsterdam: North Holland.

Fady Barsoum is currently a Ph.D. student at the University of Konstanz and a former researcher in the Marie Curie Initial Training Network. His main research interests are in time series analysis, particularly mixed-frequency models, Markov-switching models, dynamic factor models, forecasting, and Bayesian vector autoregression models.

Sandra Stankiewicz studied quantitative methods in economics at the Warsaw School of Economics and the University of Mainz. She is currently a Ph.D. student of the Doctoral Programme in Quantitative Economics and Finance at the University of Konstanz. Her main research interests are in time series analysis, especially mixed-frequency, Markov-switching and VAR models.