4.4 Euler Characteristic, Poincaré Polynomial

The number of critical points modulo 2 of a Morse function depends only on the manifold and not on the function.

More precisely, we have

$$\#\,\mathrm{Crit}(f) \geq \sum_{k=0}^{n} (\dim\ker\partial_k - \dim\operatorname{im}\partial_{k+1}) = \sum_{k=0}^{n} \dim HM_k(V;\mathbb{Z}/2)$$

If we define $c_k(f)$ to be the number of critical points of index k of the Morse function f and let $\beta_k = \dim HM_k(V;\mathbb{Z}/2)$ (the kth Betti number of V), then the proposition can be written as

$$c_k(f) \geq \beta_k, \qquad k \geq 0$$

a series of inequalities known as the Morse inequalities.

6.1 The Arnold Conjecture

Let W be a compact symplectic manifold and let

$$H : W \times \mathbb{R} \to \mathbb{R}$$

be a time-dependent Hamiltonian. Suppose that the solutions of period 1 of the associated Hamiltonian system are nondegenerate. Then their number is greater than or equal to the sum

$$\sum_i \dim HM_i(W;\mathbb{Z}/2)$$