Question 4

$$\frac{\left|\left(fl(a) \oplus fl(b)\right) - (a+b)\right|}{|a+b|}$$

$$= \frac{\left|\left((a(1+\delta_1)) \oplus (b(1+\delta_2))\right) - (a+b)\right|}{|a+b|}$$

$$= \frac{\left|\left(a(1+\delta_1) + b(1+\delta_2)\right)(1+\delta_3) - (a+b)\right|}{|a+b|}$$

$$= \frac{\left|(a + a\delta_1 + b + b\delta_2)(1+\delta_3) - (a+b)\right|}{|a+b|}$$

$$= \frac{\left|(a + a\delta_1 + b + b\delta_2) + \delta_3(a + a\delta_1 + b + b\delta_2) - (a+b)\right|}{|a+b|}$$

$$= \frac{\left|a\delta_1 + b\delta_2 + a\delta_3 + a\delta_1\delta_3 + b\delta_3 + b\delta_2\delta_3\right|}{|a+b|}$$

$$= \frac{\left|a(\delta_1 + \delta_3 + \delta_1\delta_3) + b(\delta_2 + \delta_3 + \delta_2\delta_3)\right|}{|a+b|}$$

$$\leq \frac{\left|a(\delta_1 + \delta_3 + \delta_1\delta_3)\right| + \left|b(\delta_2 + \delta_3 + \delta_2\delta_3)\right|}{|a+b|} \qquad \textit{triangle inequality}$$

$$= \frac{|a||\delta_1 + \delta_3 + \delta_1\delta_3| + |b||\delta_2 + \delta_3 + \delta_2\delta_3|}{|a+b|}$$

$$\leq \frac{|a|(E + E + EE) + |b|(E + E + EE)}{|a+b|} \qquad \textit{since } |\delta_1|, |\delta_2|, |\delta_3| \leq E$$

$$= \frac{|a| + |b|}{|a+b|} E(2 + E)$$

Thus,

$$\frac{\left|\left(fl(a) \oplus fl(b)\right) - (a+b)\right|}{|a+b|} \leq \frac{|a| + |b|}{|a+b|} E(2 + E)$$