Q1

$$f = [1,3,2,4,5,4,2,3]^T$$

$W_8^0 = \bar{W}_8^0 = 1$	$W_8^1 = \bar{W}_8^7 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$	$W_8^2 = \bar{W}_8^6 = i$	$W_8^3 = \bar{W}_8^5 = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$
$W_8^4 = \bar{W}_8^4 = -1$	$W_8^5 = \bar{W}_8^3 = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$	$W_8^6 = \bar{W}_8^2 = -i$	$W_8^7 = \bar{W}_8^1 = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

$W_4^0 = \bar{W}_4^0 = 1$	$W_4^1 = \bar{W}_4^3 = i$	$W_4^2 = \bar{W}_4^2 = -1$	$W_4^3 = \bar{W}_4^1 = -i$

$W_2^0 = \bar{W}_2^0 = 1$	$W_2^1 = \bar{W}_2^1 = -1$

Decompose stage 1: $f = \begin{bmatrix}1\\3\\2\\4\\5\\4\\2\\3\end{bmatrix} \begin{bmatrix}1+5\\3+4\\2+2\\4+3\\(1-5)\bar{W}_8^0\\(3-4)\bar{W}_8^1\\(2-2)\bar{W}_8^2\\(4-3)\bar{W}_8^3\end{bmatrix} = \begin{bmatrix}6\\7\\4\\7\\-4\\-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\\0\\-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\end{bmatrix}$

Decompose stage 2: $\begin{bmatrix}6\\7\\4\\7\end{bmatrix} \begin{bmatrix}6+4\\7+7\\(6-4)\bar{W}_4^0\\(7-7)\bar{W}_4^1\end{bmatrix} = \begin{bmatrix}10\\14\\2\\0\end{bmatrix}$

$$\begin{bmatrix}-4\\-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\\0\\-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\end{bmatrix} \begin{bmatrix}-4+0\\\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i+\left(-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right)\right)\bar{W}_4^0\\-4-0\\\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i-\left(-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right)\right)\bar{W}_4^1\end{bmatrix} = \begin{bmatrix}-4\\(-\sqrt{2})(1)\\-4\\(\sqrt{2}i)(-i)\end{bmatrix} = \begin{bmatrix}-4\\-\sqrt{2}\\-4\\\sqrt{2}\end{bmatrix}$$

Decompose stage 3: $\begin{bmatrix}10\\14\end{bmatrix} \begin{matrix}[10+14]\\[(10-14)\bar{W}_2^0]\end{matrix} \rightarrow \begin{matrix}[24]\\[-4]\end{matrix}$

$$\begin{bmatrix}2\\0\end{bmatrix} \begin{matrix}[2+0]\\[(2-0)\bar{W}_2^0]\end{matrix} \rightarrow \begin{matrix}[2]\\[2]\end{matrix}$$

$$\begin{bmatrix}-4\\-\sqrt{2}\end{bmatrix} \begin{matrix}[-4+(-\sqrt{2})]\\\left[\left(-4-(-\sqrt{2})\right)\bar{W}_2^0\right]\end{matrix} \rightarrow \begin{matrix}[-4-\sqrt{2}]\\[-4+\sqrt{2}]\end{matrix}$$

$$\begin{bmatrix}-4\\\sqrt{2}\end{bmatrix} \begin{matrix}[-4+(\sqrt{2})]\\\left[\left(-4-(\sqrt{2})\right)\bar{W}_2^0\right]\end{matrix} \rightarrow \begin{matrix}[-4+\sqrt{2}]\\[-4-\sqrt{2}]\end{matrix}$$

Recombine Stage 1:

$$
\begin{matrix} [24] \\ [-4] \end{matrix} \rightarrow \begin{bmatrix} 24 \\ -4 \end{bmatrix}
$$

$$
\begin{matrix} [2] \\ [2] \end{matrix} \rightarrow \begin{bmatrix} 2 \\ 2 \end{bmatrix}
$$

$$
\begin{matrix} [-4-\sqrt{2}] \\ [-4+\sqrt{2}] \end{matrix} \rightarrow \begin{bmatrix} -4-\sqrt{2} \\ -4+\sqrt{2} \end{bmatrix}
$$

$$
\begin{matrix} [-4+\sqrt{2}] \\ [-4-\sqrt{2}] \end{matrix} \rightarrow \begin{bmatrix} -4+\sqrt{2} \\ -4-\sqrt{2} \end{bmatrix}
$$

Recombine Stage 2:

$$
\begin{matrix} \begin{bmatrix} 24 \\ -4 \end{bmatrix} \\ \begin{bmatrix} 2 \\ 2 \end{bmatrix} \end{matrix} \rightarrow \begin{bmatrix} 24 \\ 2 \\ -4 \\ 2 \end{bmatrix}
$$

$$
\begin{matrix} \begin{bmatrix} -4-\sqrt{2} \\ -4+\sqrt{2} \end{bmatrix} \\ \begin{bmatrix} -4+\sqrt{2} \\ -4-\sqrt{2} \end{bmatrix} \end{matrix} \rightarrow \begin{bmatrix} -4-\sqrt{2} \\ -4+\sqrt{2} \\ -4+\sqrt{2} \\ -4-\sqrt{2} \end{bmatrix}
$$

Recombine Stage 2:

$$
\begin{matrix} \begin{bmatrix} 24 \\ 2 \\ -4 \\ 2 \end{bmatrix} \\ \begin{bmatrix} -4-\sqrt{2} \\ -4+\sqrt{2} \\ -4+\sqrt{2} \\ -4-\sqrt{2} \end{bmatrix} \end{matrix} \rightarrow \begin{bmatrix} 24 \\ -4-\sqrt{2} \\ 2 \\ -4+\sqrt{2} \\ -4 \\ -4+\sqrt{2} \\ 2 \\ -4-\sqrt{2} \end{bmatrix}
$$

Thus, the DFT of f is

$$
\begin{bmatrix} 24 \\ -4-\sqrt{2} \\ 2 \\ -4+\sqrt{2} \\ -4 \\ -4+\sqrt{2} \\ 2 \\ -4-\sqrt{2} \end{bmatrix}
$$