Question 3

a)

The largest value in F should be in the form $+0.6666 \times 7^8$ or 66660000 in base-7.

b)

$$26530_7 = 0.2653 \times 7^5$$
$$10000_7 = 0.1000 \times 7^5$$
$$26530_7 \times 10000_7 = (0.2653 \times 0.1000) \times 7^{(5+5)} = 0.2653 \times 7^9$$

Since p = 9 > 8, the exponent is higher than the upper bound,

Thus, the value of $26530_7 \times 10000_7$ would be an Overflow Error.

c)

$$\mathrm{E} = \left\lceil \frac{\beta}{2} \right\rceil \beta^{-t} = \left\lceil \frac{7}{2} \right\rceil 7^{-4} = 4 \times 7^{-4}$$

OR

E is defined to be the smallest number such that fl(1+E) > 1, thus

$$1 = 0.1000 \times 7^{-1} = 1.000 \times 7^0$$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | . | 0 | 0 | 0 | | * | $7^0$ |
| + | 0 | . | 0 | 0 | 0 | 4 | * | $7^0$ |
| | 1 | . | 0 | 0 | 1 | | * | $7^0$ |

If we use 0.0003, then adding 0.0003 would not round up to 1.001

Thus, the smallest value of $\mathrm{E} \in \mathrm{F}$, machine epsilon, is $4 \times 7^{-4}$ or 0.0004 in base-7.

d)

$$(0.1000 \ldots 0.6666) \times 7^{-8}$$
$$(0.1000 \ldots 0.6666) \times 7^{-7}$$
$$\ldots$$
$$(0.1000 \ldots 0.6666) \times 7^{-1}$$
$$(0.1000 \ldots 0.6666) \times 7^{0}$$
$$(0.1000 \ldots 0.6666) \times 7^{1}$$
$$(0.1000 \ldots 0.6666) \times 7^{2}$$
$$\ldots$$
$$(0.1000 \ldots 0.6666) \times 7^{8}$$

Notice that each row has 10000 in base-7 numbers

- The rows marked in yellow have the numbers smaller in magnitude than 1, and there are 12 in base-7 rows.
- The rows marked in green have the numbers equal or larger in magnitude than 1, and there are 11 in base-7 rows.
- In total, there are 23 in base-7 rows.

Notice that the number 0, which is an exception of the number system, is smaller in magnitude than 1

- Since 0 is the only exception, it has little impact to the fraction, so the approximation of the fraction is:

$$\text{Fraction smaller} = \frac{12_7 \times 10000_7 + 1_7}{23_7 \times 10000_7 + 1_7} \approx \frac{12_7}{23_7} \approx 0.3464_7$$

Thus, the fraction of the values are smaller in magnitude than 1 is 0.3464 in base-7, or 0.5294 in base-10