

Q1

$$f = [1, 3, 2, 4, 5, 4, 2, 3]^T$$

$W_8^0 = \bar{W}_8^0 = 1$	$W_8^1 = \bar{W}_8^7 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$	$W_8^2 = \bar{W}_8^6 = i$	$W_8^3 = \bar{W}_8^5 = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$
$W_8^4 = \bar{W}_8^4 = -1$	$W_8^5 = \bar{W}_8^3 = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$	$W_8^6 = \bar{W}_8^2 = -i$	$W_8^7 = \bar{W}_8^1 = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

$W_4^0 = \bar{W}_4^0 = 1$	$W_4^1 = \bar{W}_4^3 = i$	$W_4^2 = \bar{W}_4^2 = -1$	$W_4^3 = \bar{W}_4^1 = -i$
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$W_2^0 = \bar{W}_2^0 = 1$	$W_2^1 = \bar{W}_2^1 = -1$
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Decompose stage 1:  $f =$

$$f = \begin{bmatrix} 1 \\ 3 \\ 2 \\ 4 \\ 5 \\ 4 \\ 2 \\ 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1+5 \\ 3+4 \\ 2+2 \\ 4+3 \\ (1-5)\bar{W}_8^0 \\ (3-4)\bar{W}_8^1 \\ (2-2)\bar{W}_8^2 \\ (4-3)\bar{W}_8^3 \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 4 \\ 7 \\ -4 \\ -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \\ 0 \\ -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i \end{bmatrix}$$

Decompose stage 2:

$$\begin{bmatrix} 6 \\ 7 \\ 4 \\ 7 \end{bmatrix} \rightarrow \begin{bmatrix} 6+4 \\ 7+7 \\ (6-4)\bar{W}_4^0 \\ (7-7)\bar{W}_4^1 \end{bmatrix} = \begin{bmatrix} 10 \\ 14 \\ 2 \\ 0 \end{bmatrix}$$

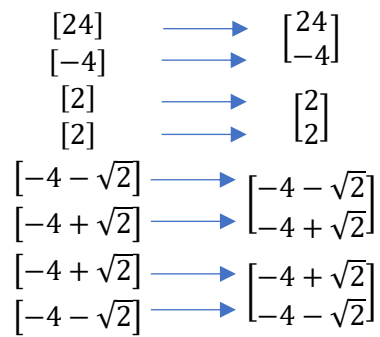
$$\begin{bmatrix} 10 \\ 14 \\ 2 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} -4+0 \\ \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i + \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)\bar{W}_4^0\right) \\ -4-0 \\ \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)\bar{W}_4^1\right) \end{bmatrix} = \begin{bmatrix} -4 \\ (-\sqrt{2})(1) \\ -4 \\ (\sqrt{2}i)(-i) \end{bmatrix} = \begin{bmatrix} -4 \\ -\sqrt{2} \\ -4 \\ \sqrt{2} \end{bmatrix}$$

Decompose stage 3:

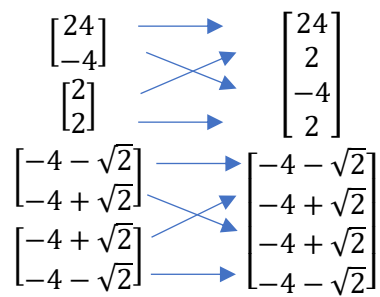
$$\begin{bmatrix} 10 \\ 14 \\ 2 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 10+14 \\ (10-14)\bar{W}_2^0 \\ 2+0 \\ (2-0)\bar{W}_2^0 \end{bmatrix} \rightarrow \begin{bmatrix} 24 \\ -4 \\ 2 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} -4 \\ -\sqrt{2} \\ -4 \\ \sqrt{2} \end{bmatrix} \rightarrow \begin{bmatrix} -4+(-\sqrt{2}) \\ ((-4-(-\sqrt{2}))\bar{W}_2^0) \\ -4+(\sqrt{2}) \\ ((-4-(\sqrt{2}))\bar{W}_2^0) \end{bmatrix} \rightarrow \begin{bmatrix} -4-\sqrt{2} \\ -4+\sqrt{2} \\ -4+\sqrt{2} \\ -4-\sqrt{2} \end{bmatrix}$$

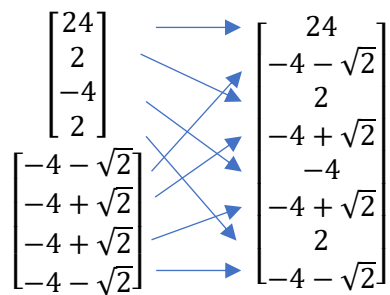
Recombine Stage 1:



Recombine Stage 2:



Recombine Stage 2:



Thus, the DFT of f is

$$\begin{bmatrix} 24 \\ -4 - \sqrt{2} \\ 2 \\ -4 + \sqrt{2} \\ -4 \\ -4 + \sqrt{2} \\ 2 \\ -4 - \sqrt{2} \end{bmatrix}$$