Q3

The condition number of M is

$$\kappa(M) = \|M\|\|M^{-1}\|$$

From slide FT-04, we know that

$$M = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \overline{W}_N^1 & \overline{W}_N^2 & \cdots & \overline{W}_N^{(N-1)} \\ 1 & \overline{W}_N^2 & \overline{W}_N^4 & \cdots & \overline{W}_N^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \overline{W}_N^{(N-1)} & \overline{W}_N^{2(N-1)} & \cdots & \overline{W}_N^{(N-1)(N-1)} \end{bmatrix}$$

Where W is the Nth root of unity

Since

$$M^{-1} = \frac{1}{N}\overline{M}$$

We have

$$M^{-1} = \frac{1}{N}\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N^1 & W_N^2 & \cdots & W_N^{(N-1)} \\ 1 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & W_N^{(N-1)} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix}$$

Since for any value k, $\left|\overline{W}_N^k\right| = \left|e^{\frac{-2\pi ik}{N}}\right| = 1$

Similarly, for any value k, $\left|W_N^k\right| = \left|e^{\frac{2\pi ik}{N}}\right| = 1$

Thus, using the 1-norm:

$$\kappa(M) = \|M\|_1\|M^{-1}\|_1 = \left(\max_c \sum_r \left|M_{r,c}\right|\right)\left(\max_c \sum_r \frac{1}{N}\left|\overline{M}_{r,c}\right|\right)$$
$$= (N)(1)$$
$$\kappa(M) = N$$

Thus, the condition number of M using the 1-norm is N.