Q2a) To interpolate horizontally between $f_{0,1}$ and $f_{1,1}$, we use linear interpolation to get

$$f_{x,1} = Interpolate(f_{0,1}, f_{1,1}) = \frac{8+10}{2} = 9$$

To interpolate horizontally between $f_{0,0}$ and $f_{1,0}$, we use linear interpolation to get

$$f_{x,0} = Interpolate(f_{0,0}, f_{1,0}) = \frac{12+15}{2} = 13.5$$

Finally, to interpolate vertically between $f_{x,1}$ and $f_{x,0}$, we use linear interpolation to get

$$f_{x,y} = Interpolate(f_{x,1}, f_{x,0}) = \frac{9+13.5}{2} = 11.25$$

<u>Using method a), we get $f_{x,y} = 11.25$</u>.

b) To interpolate vertically between $f_{0,0}$ and $f_{0,1}$, we use linear interpolation to get

$$f_{0,y} = Interpolate(f_{0,0}, f_{0,1}) = \frac{12+8}{2} = 10$$

To interpolate vertically between $f_{1,0}$ and $f_{1,1}$, we use linear interpolation to get

$$f_{1,y} = Interpolate(f_{1,0}, f_{1,1}) = \frac{15+10}{2} = 12.5$$

Finally, to interpolate horizontally between $f_{0,y}$ and $f_{1,y}$, we use linear interpolation to get

$$f_{x,y} = Interpolate(f_{0,y}, f_{1,y}) = \frac{10+12.5}{2} = 11.25$$

<u>Using method b), we get $f_{x,y} = 11.25$</u>.

c) Weight for $f_{0,1}$ is $w_{0,1} = (1-x)y$,
Similarly,

$$\begin{aligned} w_{0,0} &= (1-x)(1-y) = 1 - x - y + xy \\ w_{0,1} &= (1-x)y = y - xy \\ w_{1,0} &= x(1-y) = x - xy \\ w_{1,1} &= xy \end{aligned}$$

Thus, the weighted sum of the f-values is:

$$\begin{aligned} f_{x,y} &= Weighted\ sum = w_{0,0}f_{0,0} + w_{0,1}f_{0,1} + w_{1,0}f_{1,0} + w_{1,1}f_{1,1} \\ &= 12(1 - x - y + xy) + 8(y - xy) + 15(x - xy) + 10(xy) \\ &= 12 + 3x - 4y - xy \\ f_{x,y} &= (x+4)(3-y) \end{aligned}$$

To interpolate $f_{0,0}, f_{0,1}, f_{1,0}, f_{1,1}$, it means we want to get a value $f_{x,y}$ where both x and y are between 0 and 1, and one efficient way is to take linear interpolation, that is $x = (1-0)/2 = 0.5,\ and\ y = (1-0)/2 = 0.5$
Thus, for

$$\begin{aligned} f_{x,y} &= f_{0.5,0.5} = (x+4)(3-y) \\ &= (0.5+4)(3-0.5) \\ &= 4.5 \times 2.5 \\ f_{x,y} &= 11.5 \end{aligned}$$

<u>Using method c), we get $f_{x,y} = 11.25$</u>.
Therefore, this proves that all three methods give the same value $f_{x,y}$.