Q6.

Solving $ABx = y$.

$ABx = y \Rightarrow P_A ABx = P_A y \Rightarrow (L_A U_A) Bx = P_A y$.

Since $B \in \mathbb{R}^{n \times n}$ and $x \in \mathbb{R}^{n \times 1}$

$Bx \in \mathbb{R}^{n \times 1}$. Let $s = Bx$ $s \in \mathbb{R}^{n \times 1}$

Let $z_A = U_A s$, since $U_A \in \mathbb{R}^{n \times n}$

$z_A \in \mathbb{R}^{n \times 1}$.

Thus, we can solve $L_A z_A = P_A y$ for z_A (N^2)
by using forward substitution

Then solve $U_A s = z_A$ for s. (N^2)
by using backward substitution

Now we know s, where $s = Bx$, solve $Bx = s$.

$Bx = s \Rightarrow P_B Bx = P_B s \Rightarrow L_B U_B x = P_B s$

Let $z_B = U_B x$

Thus we can solve $L_B z_B = P_B s$ for z_B (N^2)
by using forward sub.

Then solve $U_B x = z_B$ for x (N^2)

The result will be the solution to $ABx = y$.