We have the function:

$$x(t) = 3e^t - 2e^{2t}$$

$$y(t) = 3e^t - 4e^{2t}$$

To prove that this is a solution to the IVP,

We need to show that the solution satisfies the dynamics equations and the initial state:

$$x'(t) = 3e^{t} - 2(2)e^{2t} = 3e^{t} - 4e^{2t} = y(t)$$

$$y'(t) = 3e^{t} - 4(2)e^{2t} = 3e^{t} - 8e^{2t} = -6e^{t} + 9e^{t} + 4e^{2t} - 12e^{2t}$$

$$= (-6e^{t} + 4e^{2t}) + (9e^{t} - 12e^{2t}) = -2(3e^{t} - 2e^{2t}) + 3(3e^{t} - 4e^{2t}) = -2x(t) + 3y(t)$$

$$x(0) = 3e^{0} - 2e^{2(0)} = 3 - 2 = 1$$

$$y(0) = 3e^{0} - 4e^{2(0)} = -1$$

Therefore, since the functions

$$x(t) = 3e^t - 2e^{2t}$$

$$v(t) = 3e^t - 4e^{2t}$$

Satisfies

$$x'(t) = y(t)$$

$$y'(t) = -2x(t) + 3y(t)$$

$$x(0) = 1$$

$$y(0) = -1$$

Thus, this is a solution to the IVP.