5a)

By solving

$$\begin{bmatrix} 2 & 3 & 1 \\ 6 & 8.9999 & 6 \\ 4 & 8 & 11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$$

We get

$$x^{(true)} = \begin{bmatrix} 1.416562515622656 \\ -0.499925011248312 \\ -0.333349997500375 \end{bmatrix}$$

b)

Where (i) is the ith row of A

$$\left[\begin{array}{ccc|c} 2 & 3 & 1 & 1 \\ 6 & 8.9999 & 6 & 2 \\ 4 & 8 & 11 & -2 \end{array}\right] \rightarrow \begin{array}{c} (2)-(6/2)(1) \\ (3)-(4/2)(1) \end{array} \rightarrow \left[\begin{array}{ccc|c} 2 & 3 & 1 & 1 \\ 0 & -0.0001 & 3 & -1 \\ 0 & 2 & 9 & -4 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 1 & 1 \\ 0 & -0.0001 & 3 & -1 \\ 0 & 2 & 9 & -4 \end{array}\right] \rightarrow \quad (3)-(2/-0.0001)(2) \quad \rightarrow \left[\begin{array}{ccc|c} 2 & 3 & 1 & 1 \\ 0 & -0.0001 & 3 & -1 \\ 0 & 0 & 60009 & -20004 \end{array}\right]$$

Using backward substitution

We get that $60009x_3 = -20004$, or $x_3 \approx -0.33335$

Then

$$(-0.0001)x_2 + (3)x_3 = -1 \quad \rightarrow \quad (-0.0001)x_2 + (3)(-0.33335) = -1 \quad \rightarrow \quad (-0.0001)x_2 - 1.0000 = -1$$
$$\rightarrow \quad (-0.0001)x_2 = 0.0000 \quad \rightarrow \quad x_2 = 0$$

Thus,

$$(2)x_1 + (3)x_2 + (1)x_3 = 1 \quad \rightarrow \quad (2)x_1 - 0.33335 = 1 \quad \rightarrow \quad (2)x_1 = 1.3334 \quad \rightarrow \quad x_1 = 0.6667$$

We get

$$x^{(no-povit)} = \begin{bmatrix} 0.6667 \\ 0 \\ -0.33335 \end{bmatrix}$$

c)

Where (i) is the ith row of A

$$\left[\begin{array}{ccc|c} 2 & 3 & 1 & 1 \\ 6 & 8.9999 & 6 & 2 \\ 4 & 8 & 11 & -2 \end{array}\right] \to \begin{array}{c}(2)\\(1)\end{array} \to \left[\begin{array}{ccc|c} 6 & 8.9999 & 6 & 2 \\ 2 & 3 & 1 & 1 \\ 4 & 8 & 11 & -2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 6 & 8.9999 & 6 & 2 \\ 2 & 3 & 1 & 1 \\ 4 & 8 & 11 & -2 \end{array}\right] \to \begin{array}{c}(2)-(2/6)(1)\\(3)-(4/6)(1)\end{array} \to \left[\begin{array}{ccc|c} 6 & 8.9999 & 6 & 2 \\ 0 & 0.0001 & -1.0000 & 0.33334 \\ 0 & 2.0000 & 7.0000 & -3.3333 \end{array}\right]$$

$$(2/6)(1) = (0.33333)(1) = [2.0000 \quad 2.9999 \quad 2.0000 \quad | \quad 0.66666\ ]$$

$$(4/6)(1) = (0.66667)(1) = [4.0000 \quad 6.0000 \quad 4.0000 \quad | \quad 1.3333\ ]$$

$$\left[\begin{array}{ccc|c} 6 & 8.9999 & 6 & 2 \\ 0 & 0.0001 & -1.0000 & 0.33334 \\ 0 & 2.0000 & 7.0000 & -3.3333 \end{array}\right] \to \begin{array}{c} \\ (3)-(2.0000/0.0001)(2)\end{array} \to \left[\begin{array}{ccc|c} 6 & 8.9999 & 6 & 2 \\ 0 & 0.0001 & -1.0000 & 0.33334 \\ 0 & 0 & 20007 & -6670.1 \end{array}\right]$$

$$(2.0000/0.0001)(2) = (20000)(2) = [0 \quad 2.0000 \quad -20000 \quad | \quad 6666.8\,]$$

Using backward substitution

We get that $20007x_3 = -6670.1$, or $x_3 \approx -0.33339$

Then

$$(0.0001)x_2 + (-1.0000)x_3 = 0.33334 \quad \to \quad (0.0001)x_2 + (-1.0000)(-0.33339) = 0.33334 \quad \to$$
$$(0.0001)x_2 + 0.33339 = 0.33334 \quad \to \quad (0.0001)x_2 = -0.00005 \quad \to \quad x_2 = -0.5$$

Thus,

$$(6)x_1 + (8.9999)x_2 + (6)x_3 = 2 \quad \to \quad (6)x_1 + (8.9999)(-0.5) + (6)(-0.33339) = 2$$
$$\to \quad (6)x_1 - (4.5000) - (2.0003) = 2 \quad \to \quad (6)x_1 = 8.5003 \quad \to \quad x_1 = 1.4167$$

We get

$$x^{(povit)} = \begin{bmatrix} 1.4167 \\ -0.5 \\ -0.33339 \end{bmatrix}$$

d)

$$x^{(true)} = \begin{bmatrix} 1.416562515622656 \\ -0.499925011248312 \\ -0.333349997500375 \end{bmatrix}$$

$$x^{(no-povit)} = \begin{bmatrix} 0.6667 \\ 0 \\ -0.33335 \end{bmatrix}$$

$$x^{(povit)} = \begin{bmatrix} 1.4167 \\ -0.5 \\ -0.33339 \end{bmatrix}$$

$$\left\| x^{(true)} \right\|_2 = \sqrt{\sum_{i=1}^{n} x^{(true)}{}_i^2} = 1.5387321399036$$

$$Reltive\ Error = \frac{\left\| x^{(true)} - x^{(no-povit)} \right\|_2}{\| x^{(true)} \|_2} = \frac{\left\| \begin{bmatrix} 0.7498625156 \\ -0.499925011248312 \\ 0.00000000249963 \end{bmatrix} \right\|_2}{1.5387321399036} = \frac{0.90123182876194}{1.5387321399036} \approx 0.5857$$

$$Reltive\ Error = \frac{\left\| x^{(true)} - x^{(no-povit)} \right\|_2}{\| x^{(true)} \|_2} = \frac{\left\| \begin{bmatrix} -0.00013748437735 \\ 0.00007498875169 \\ 0.00004000249963 \end{bmatrix} \right\|_2}{1.5387321399036} = \frac{0.000161633743}{1.5387321399036} \approx 0.0001$$