Q2

Let f be a real-valued signal, $f_n \in \mathbb{R} \;\; for \;\; n = 0, \dots, N-1$

$$F_k = \sum_{n=0}^{N-1} f_n \exp\left(\frac{-2\pi ink}{N}\right) \qquad for\; k = 0, \dots, N-1$$

$$g_n = f_{N-n}$$

Assume the Fourier coefficients of g are G_k

$$G_k = \sum_{n=0}^{N-1} g_n \exp\left(\frac{-2\pi ink}{N}\right) \qquad for\; k = 0, \dots, N-1$$

$$G_k = \sum_{n=0}^{N-1} f_{N-n} \exp\left(\frac{-2\pi ink}{N}\right)$$

Change of variables: Let $m = N - n,$ so that $n = N - m,$

$$when\, n = 0, \qquad f_{N-n} = f_N, \;\; m = N$$

$$when\, n = 1, \qquad f_{N-n} = f_{N-1}, \;\; m = N-1$$

$$when\, n = 2, \qquad f_{N-n} = f_{N-2}, \;\; m = N-2$$

$$\dots$$

$$when\, n = N-1, \qquad f_{N-n} = f_1, \;\; m = 1$$

Thus, when substituting $m = N - n$, the summation can start from $m = 1$ to $m = N$

$$G_k = \sum_{m=1}^{N} f_m \exp\left(\frac{-2\pi i(N-m)k}{N}\right)$$

$$G_k = \sum_{m=1}^{N} f_m \exp(-2\pi ik) \cdot \exp\left(\frac{2\pi i(m)k}{N}\right)$$

$$G_k = \sum_{m=1}^{N} f_m \exp\left(\frac{2\pi imk}{N}\right)$$

$$G_k = \sum_{m=0}^{N} f_m \exp\left(\frac{2\pi imk}{N}\right) - \sum_{m=0}^{0} f_m \exp\left(\frac{2\pi imk}{N}\right)$$

$$G_k = \sum_{m=0}^{N} f_m \exp\left(\frac{2\pi imk}{N}\right) - f_0 \exp\left(\frac{2\pi i(0)k}{N}\right)$$

$$G_k = \sum_{m=0}^{N} f_m \exp\left(\frac{2\pi imk}{N}\right) - f_0$$

$$G_k = \left(f_N \exp\left(\frac{2\pi iNk}{N}\right) + \sum_{m=0}^{N-1} f_m \exp\left(\frac{2\pi imk}{N}\right)\right) - f_0$$

$$G_k = \left(f_N + \sum_{m=0}^{N-1} f_m \exp\left(\frac{2\pi imk}{N}\right)\right) - f_0$$

Since f is a signal, it is periodic, thus $f_N = f_0$, this implies:

$$G_k = \sum_{m=0}^{N-1} f_m \exp\left(\frac{2\pi imk}{N}\right)$$

Since f is real-valued, $f_m = \overline{f_m}$, thus:

$$G_k = \overline{\sum_{m=0}^{N-1} \overline{f_m} \exp\left(\frac{-2\pi \imath mk}{N}\right)}$$

$$G_k = \overline{\sum_{m=0}^{N-1} f_m \exp\left(\frac{-2\pi \imath mk}{N}\right)}$$

$$G_k = \overline{F_k}$$

Therefore, it has been proven that $G_k = \overline{F_k}$.