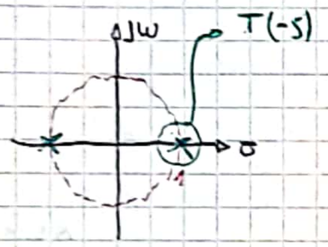
Videos clase 3-4

$$|T_b(\omega)|^2 = \frac{1}{1+\omega^{2n}} = T(\omega)\cdot T(\omega)^* = \overbrace{T(s)\cdot T(-s)}^{\text{Vamos a tener}}\Big|_{s=j\omega}$$ los polos de ambas

Si $n=1$

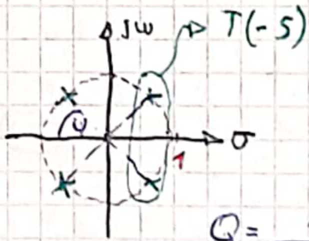
$$|T_b(\omega)|^2 = \frac{1}{1+\omega^2} \Rightarrow T_b(s)^2 = |T_b(\omega)|^2\Big|_{\omega=\frac{s}{j}} = \frac{1}{1+\left(\frac{s}{j}\right)^2} = \frac{1}{1-s^2}$$



$$T_{b1}(s) = \frac{1}{1+s}$$

Si $n=2$

$$|T_b(\omega)|^2 = \frac{1}{1+\omega^2} \Rightarrow T(s)^2 = \frac{1}{1+\frac{s^4}{j^4}} = \frac{1}{1+s^4}$$

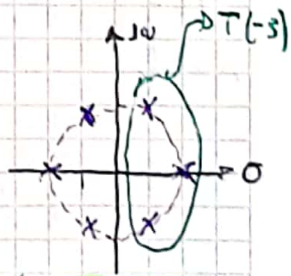


$$Q = \frac{1}{2\cos\psi} = \frac{\sqrt{2}}{2}$$

$$T_{b2}(s) = \frac{1}{s^2+s\cdot\frac{1}{Q}+1} = \frac{1}{s^2+s\sqrt{2}+1}$$

Si $n=3$

$$|T_b(\omega)|^2 = \frac{1}{1+\omega^6} \Rightarrow T(s)^2 = \frac{1}{1+\left(\frac{s}{j}\right)^6} = \frac{1}{1-s^6}$$



En todos los casos la fase entre los polos será $\Delta\theta = \frac{\pi}{n}$

$$T_b(s) = \frac{1}{(s+1)(s^2+s2\cos\psi+1)} = \frac{1}{(s+1)(s^2+s+1)}$$ Si n

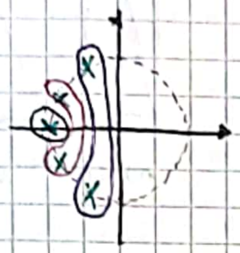
Si n impar $\Rightarrow$ polo en $\sigma=-1$

Si n par $\Rightarrow$ 1er polo en $s=1e^{j\pi/2n}$

$\Big\} \rightarrow$ desfas entre polos $= \frac{\pi}{n}$

NOTA

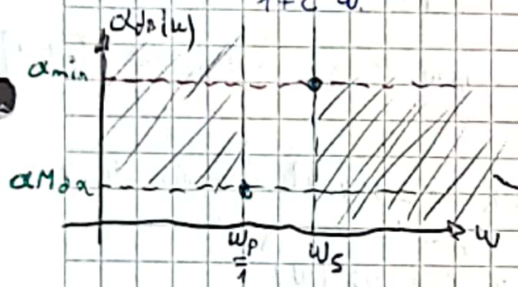
Si $n=5$



$$T_{bs}(s) = \frac{1}{s+1} \cdot \frac{1}{s^2+s\,2\cos^{\pi}/_5+1} \cdot \frac{1}{s^2+s\,2\cos^{2\pi}/_5+1}$$

Diseño desde plantilla:

$$|T(\omega)|^2 = \frac{1}{1+\varepsilon^2\omega^{2n}} \qquad \alpha_{dB}(\omega) = -20\log|T_{dB}(\omega)|$$



En $\omega_p \Rightarrow \alpha_{dB}(\omega_p=1) = -20\log\left|\frac{1}{\sqrt{1+\varepsilon^2}}\right| = -20\cdot\left(\frac{-1}{2}\right)\log(1+\varepsilon^2) = \alpha_{max}$

$$\varepsilon^2 = 10^{\alpha_{max}/10} - 1$$

Para $\omega = \omega_s$

$\alpha_{min} = 10\log(1+\varepsilon^2\omega_s^{2n}) \rightarrow$ por iteración podemos probar y encontrar el n que nos de un $\alpha_{min}|_n > \alpha_{min}$ requerido

$$|T_{dB}(\omega)| = \frac{1}{1+\varepsilon^2\omega_\text{*}^{2n}} \xrightarrow{\text{normalizando frecuencia}} \frac{1}{1+\varepsilon^2\left(\frac{\omega}{\omega_p}\right)^{2n}} = \frac{1}{1+\left(\frac{\omega}{\frac{\omega_p}{\varepsilon^{1/n}}}\right)^{2n}}$$

veníamos usando w normalizado

Nueva norma y encima tenemos un butterworth

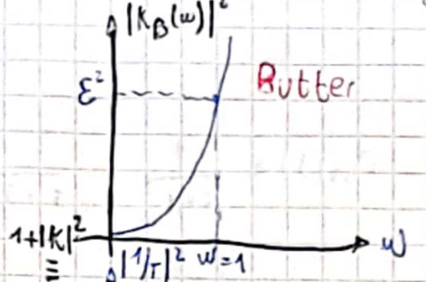
$$\Omega_\omega = \omega_p \cdot \underbrace{\varepsilon^{-1/n}}_{\text{coeficiente de renormalización}} = \omega_B$$

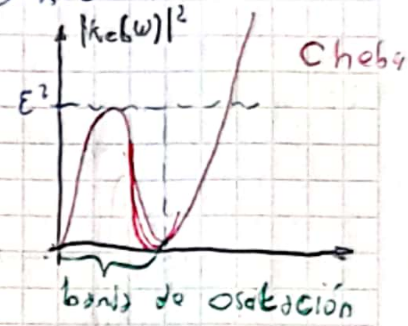
A mayor orden mayor es la distorsión del retardo de grupo

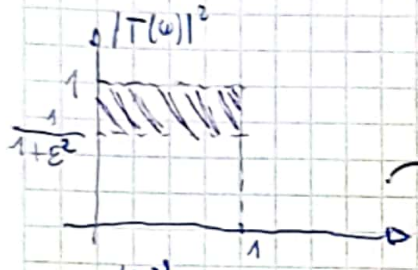
NOTA

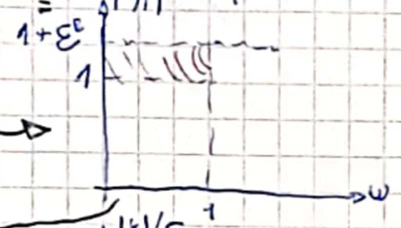
Clase 04: Aproximación de Chebyshev y Bessel

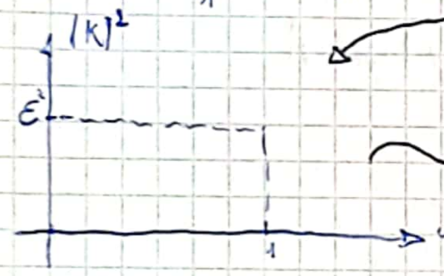
$$|T_B(w)|^2 = \frac{1}{1+\underbrace{\varepsilon^2 w^{2n}}_{|K(w)|^2}}$$

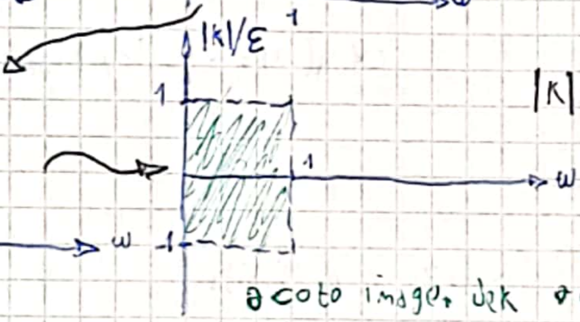












$|K|$ vive entre 0 y 1 y su imagen entre $[-1;1]$

Propongo: ε^2 ~~fosk~~ $|K| = y \Rightarrow \underbrace{y = \varepsilon\cos(nx)}_{\text{acoto imagen de k}} \Rightarrow \underbrace{x = \cos^{-1}(w)}_{\text{acoto el dominio de y}}$

$$|K| = \varepsilon\cos[n\cos^{-1}(w)]$$

$\cos^{-1}(w) = jz \quad ; \quad |w| > 1$

$$w = \cos jz = \frac{e^{j(jz)} + e^{-j(jz)}}{2} = \cosh(z)$$

$$w = \cosh(z) \Rightarrow z = \cosh^{-1}(w)$$

$$|K| = \varepsilon\cdot\cos[j\cosh^{-1}(w)\cdot n] = \varepsilon\cosh[n\cdot\cosh^{-1}(w)] \quad \forall w$$

↳ No sería para w>

$$\left[C_n(w) = 2w\cdot C_{n-1}(w) - C_{n-2}(w)\right]$$

$C_n = \varepsilon\cos[n\cos^{-1}(w)]$

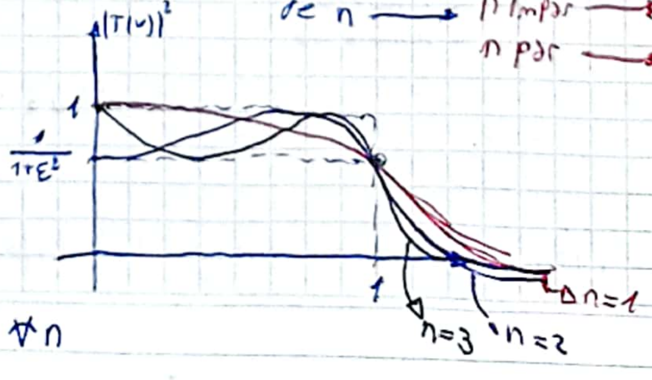
$C_0(w) = 1 \quad ; \quad C_1(w) = w$

EL punto de arranque depende de n → n impar → 1

n par → $\frac{1}{1+\varepsilon^2}$

$$C_2(w) = 2w\cdot w - 1 = 2w^2 - 1$$

$$|T_c(w)|^2 = \frac{1}{1 + C_n(w)^2}$$



NOTA: En $|T_c(1)|^2$ siempre es $1 \; \forall n$

Escaneado con CamScanner

La función toca los máximos y mínimos n veces contando el de salida.

Localización de polos y ceros

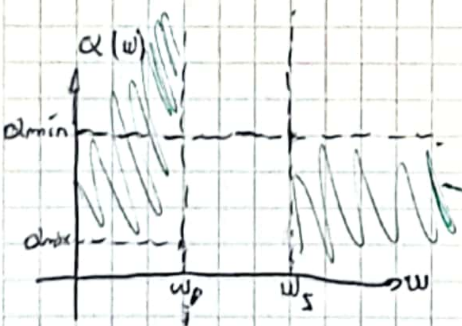
Resolver el denominador es engorroso y complicado

$$\left.\begin{aligned}\sigma_k &= -\operatorname{senh} a \,\operatorname{sen}\left(\frac{2k-1}{2n}\pi\right)\\ \omega_k &= \cosh a \,\cos\left(\frac{2k-1}{2n}\pi\right)\end{aligned}\right\} k=1,2,3,\dots,n$$

$$a = \frac{1}{n}\operatorname{senh}^{-1}\left(\frac{1}{\varepsilon}\right)$$

$$\overbrace{\frac{\sigma_k^2}{\operatorname{senh}^2 a} + \frac{\omega_k^2}{\cosh^2 a}}^{\text{elipse}} = 1$$

acá van a estar los polos



prohibido

$$\alpha_{max} = \frac{1}{|T|} = \sqrt{1+\varepsilon^2} \Rightarrow \alpha_{max\,dB} = 10\log(1+\varepsilon^2) \Rightarrow \varepsilon^2 = 10^{\alpha_{max}/10} - 1$$

$$\alpha_{min\,dB} = 10\log(1 + C_n^2(\omega))$$

$$\alpha_{min\,dB} = 10\log(1+\varepsilon^2\cosh^2[n\cosh^{-1}(\omega_s)])$$ ⇒ con esto saco n iterando

Para armar la $T(s)$:

$$|T_n(\omega)|^2 = \frac{1}{1+C_n^2(\omega)} \Rightarrow |T_n(s)|^2 = T_n(s)\,T_n(-s) = |T_n(\omega)|^2\Big]_{\omega=s/j}$$

y de ahí los polos en el semiplano izquierdo son los del $T(s)$