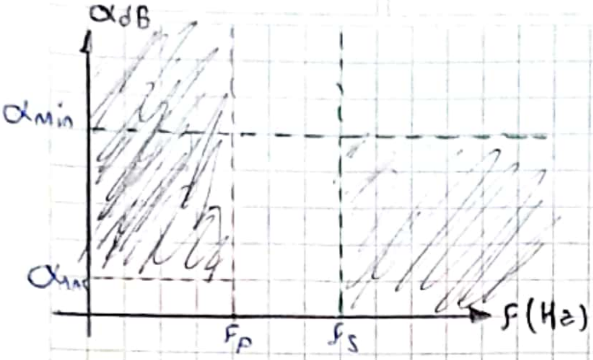
Tarea Semanal 3



$\alpha_{max} = 1dB$; $\alpha_{min} = 12 dB$

$f_p = 1500 \Rightarrow \omega_s = 9{,}424$ r/s

$f_S = 3000Hz \Rightarrow \omega_p = 18\,849{,}556$ r/s

$\Omega_\omega = \omega_s = 2\pi f_p \Rightarrow \omega_s' = 1$; $\omega_p' = 3$

$$|T_{MPn}(\omega)|^2 = \frac{1}{1+\varepsilon^2\omega^{2n}} \Rightarrow |T_{MPn}(\omega_p)|^2 = \frac{1}{1+\varepsilon^2}$$

$$\alpha(\omega_p)_{dB} = \alpha_{max} = 20\log\left(\sqrt{1+\varepsilon^2}\right) \Rightarrow 10\log(1+\varepsilon^2) = \alpha_{max}$$

$$1dB = 10\log(1+\varepsilon^2) \Rightarrow \varepsilon^2 = 10^{1dB/10dB} - 1 = 0{,}25$$

$$\alpha(\omega_s)dB = 10\log(1+\varepsilon^2\omega_s^{2n}) \Rightarrow 12dB = 10\log(1+\varepsilon^2 2^{2n})$$

Si $n = 1 \Rightarrow \alpha(\omega_s) = 3{,}08\,dB \rightarrow$ descartado No cumple con α_{min}

Si $n = 2 \Rightarrow \alpha(\omega_s) = 7{,}14\,dB \rightarrow$ " " " " "

Si $n = 3 \Rightarrow \alpha(\omega_s) = 12{,}45\,dB \rightarrow$ cumple con $\alpha_{min} \rightarrow$ Elijo $n = 3$

$\varepsilon^2 = 0{,}25$; $n = 3$

$\varepsilon = 0{,}5$

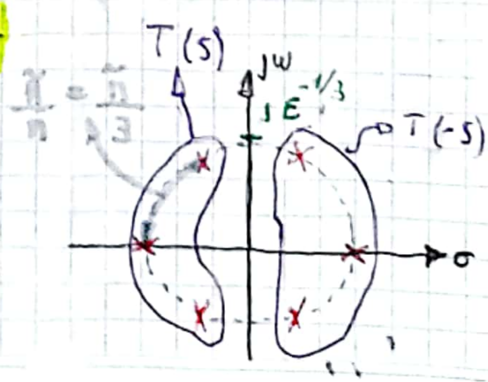
$$|T(\omega)|^2 = \frac{1}{1+\varepsilon^2\omega^6} \Rightarrow |T(s)|^2 = \frac{1}{1+\varepsilon^2(s/j)^6} = \frac{1}{1-\varepsilon^2 s^6}$$

$$|T(s)|^2 = \frac{-1/\varepsilon^2}{-1/\varepsilon^2 + s^6}$$

Polos:

$$s = \left(\frac{1}{\varepsilon^2}\right)^{1/6} \cdot e^{j\frac{2k\pi}{6}} \quad \text{con} \quad k = 0,1,2,\ldots,5$$

$$s = \varepsilon^{-1/n} e^{j\frac{2k\pi}{6}}$$



Obtención de la transferencia por polos de función.

$$|T(s)|^2 = \frac{1}{-\varepsilon^2 s^6 + 1} = \frac{1}{-A s^6 + 1} = T(s) \cdot T(-s)$$

$$T(s) \cdot T(-s) = \frac{1}{a s^3 + b s^2 + c s + 1} \cdot \frac{1}{-a s^3 + b s^2 - c s + 1}$$

$$-a^2 = -A \Rightarrow a = \sqrt{A} = \varepsilon \; (1)$$

$$-2ac + b^2 = 0 \qquad -c^2 + 2b = 0 \; (3)$$

$$c = \frac{b^2}{2a} \; (2) \qquad -\frac{b^4}{4a^2} + 2b = 0 \quad (2) \text{ en } (3)$$

$$b\left(\frac{b^3}{4a^2} + 2\right) = 0 \Rightarrow b = (-8a^2)^{1/3} = -2 \cdot a^{2/3}$$

b No puede ser 0 xq c sería 0 y me quedan polos en el semiplano derecho $\Rightarrow b = -2\,\varepsilon^{2/3}$

$$c = \frac{b^2}{2a} = \frac{4\varepsilon^{4/3}}{2\varepsilon} = 2\varepsilon^{1/3} = c$$

$$T(s) = \frac{1}{\varepsilon s^3 - 2\varepsilon^{4/3} s^2 + 2\varepsilon^{1/3} s + 1}$$

$$\left[\, T(s) = \frac{\varepsilon^{-1}}{s^3 - 2\varepsilon^{-1/3} s^2 + 2\varepsilon^{-2/3} s + \varepsilon^{-1}} \,\right]$$

Recordando que: $|T(s)|^2 = T(s) \cdot T(-s)$

y eligiendo los polos del semiplano izquierdo

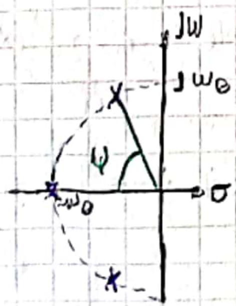
$$T(s) = \frac{1}{s+a} \cdot \frac{c}{s^2 + sb + c}$$

Los signos + se resultan ya que los polos deben estar en el semi plano izquierdo

El polo simple es real y sabemos que el radio del círculo es $\varepsilon^{-1/n}$:

$$\frac{1}{s+a} = \frac{1}{s+\varepsilon^{-1/3}} \Rightarrow a = \varepsilon^{-1/3}$$

para el par de polos complejos conjugados también se obtienen por inspección gráfica, sabemos el radio en el que se encuentran los polos y su apertura



$$\psi = \pi/3 \longrightarrow Q = \frac{1}{2\cos\pi/3} = 1$$

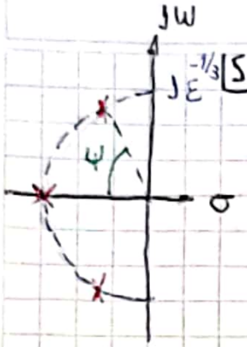
$$w_0 = a = \varepsilon^{-1/3}$$

Entonces los polos comp. conjugados:

$$\frac{w_0^2}{s^2 + s\frac{w_0}{Q} + w_0^2} = \frac{\varepsilon^{-2/3}}{s^2 + s\frac{\varepsilon^{-1/3}}{1} + \varepsilon^{-2/3}} = \frac{1{,}57}{s^2 + s\,1{,}25 + 1{,}57}$$

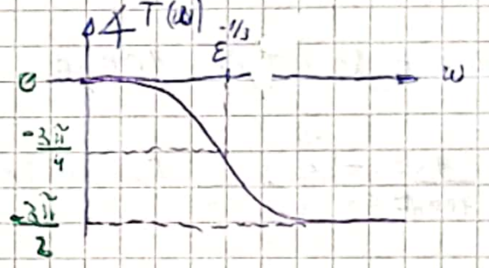
$$\left[T(s) = \frac{\varepsilon^{-1/3}}{s+\varepsilon^{-1/3}} \cdot \frac{\varepsilon^{-2/3}}{s^2 + s\,\varepsilon^{-1/3} + \varepsilon^{-2/3}} = \frac{\varepsilon^{-1}}{s^3 + s^2(\varepsilon^{-1/3} + \varepsilon^{-1/3}) + s(\varepsilon^{-2/3} + \varepsilon^{-2/3}) + \varepsilon^{-1}}\right]$$

En $w_0 \rightarrow 0\,dB$

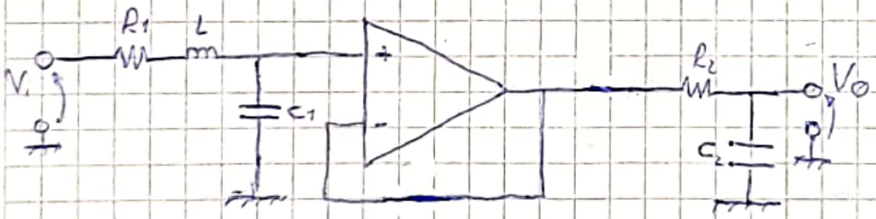


$\psi = \pi/3$

$\varepsilon^{-1/3} \cong 1,25$



Circuito pasivo:



Sabemos que

$$T(s) = \frac{1}{s+a} \cdot \frac{\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$a = \frac{1}{R_2 C_2} = \varepsilon^{-1/3}$$

Como no se afectan unas a otra:

$R_1 = R_2 = R$ y $C_1 = C_2 = C$

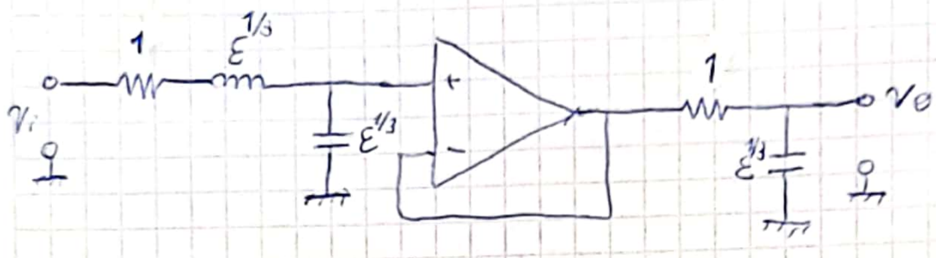
$$\frac{\omega_0}{Q} = \frac{R_1}{L} = \varepsilon^{-1/3}$$

$$\omega_0^2 = \frac{1}{LC_1} = \varepsilon^{-2/3}$$

$$\frac{1}{RC} = \varepsilon^{-1/3} \Rightarrow \left[C = \frac{\varepsilon^{1/3}}{R}\right] \; ; \; \frac{1}{LC} = \varepsilon^{-2/3} \Rightarrow \left[L = \frac{\varepsilon^{2/3}}{C} = \varepsilon^{1/3} R\right]$$

Tomando como norma de impedancia: $\Omega_z = R$

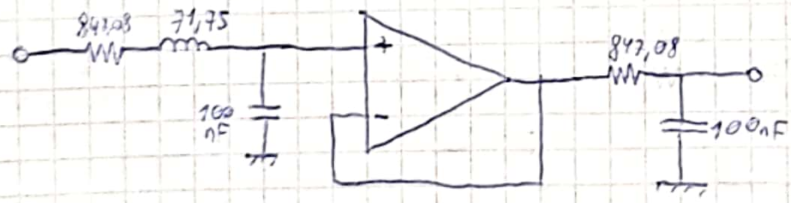
$R' = 1 \quad ; \quad L' = \varepsilon^{1/3} \quad ; \quad C' = \varepsilon^{1/3}$



Si tengo capacitores de 100nF

$C = \dfrac{C'}{\Omega_Z \Omega_\omega} \Rightarrow \Omega_Z = \dfrac{\varepsilon^{1/3}}{9\,424{,}77 \cdot 100nF} \quad ; \quad \Omega_\omega = 2\pi f_p \cong 9\,424{,}77$

$\Omega_Z = R = 847{,}08\,\Omega \quad ; \quad C = 100nF \quad ; \quad L = \dfrac{\varepsilon^{1/3} \cdot \Omega_Z}{\Omega_\omega} = 71{,}75 \text{ mHy}$



Recordando de la TS2, podemos utilizar la estructura ackerberg-Mosberg para generar los dos polos complejos conjugados:

En ackerberg-Mosberg:

$Q = \dfrac{R_2}{R_3} \quad , \quad \omega_0 = \dfrac{1}{CR_3} \quad , \quad k = \dfrac{R_3}{R_1}$

$1 = \dfrac{R_2}{R_3} \qquad \varepsilon^{1/3} = \dfrac{1}{CR_3} \qquad 1 = \dfrac{R_3}{R_1}$

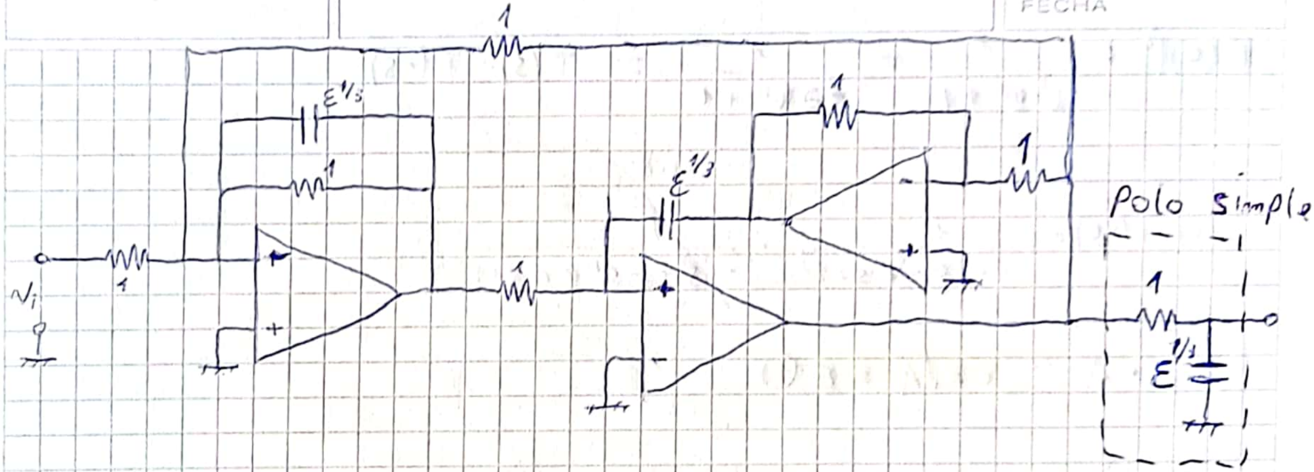
$[R_2 = R_3] \qquad \left[C = \dfrac{\varepsilon^{1/3}}{R_3} \right] \qquad [R_1 = R_3]$

Es claro que todas las resistencias deben ser iguales $\Rightarrow R_i = R$

$\Omega_Z = R_i \Rightarrow R' = 1 \;;\; C' = \varepsilon^{1/3}$

HOJA N°

FECHA



Desnormalizando para $\omega_s = 2\pi F_s$ y $C = 100nF$

$$C = \frac{C'}{\Omega_w \Omega_z} \Rightarrow \Omega_z = R = \frac{\varepsilon^{1/3}}{3000\pi \cdot 100nF} \cong 847,08$$

$C = 100nF$; $R = 847,08\Omega$