Tarea Semanal 5

Filtro máxima planicidad pasa altos $F_p = 300\,Hz$
y cero de transmisión en $F_{0z} = 100\,Hz$.

De la plantilla prototipo normalizada podemos sacar varios datos:

① en $\Omega = 1$ $\alpha \cong 3\,dB \rightarrow$ Butterworth

② en $\Omega = 1$ se desarrolla la mitad de la fase, la cual es $\varphi(\Omega)\big|_{\Omega=1} = -135°$ por lo que en $\Omega \rightarrow \infty$ $\varphi(\Omega)\big|_{\Omega\rightarrow\infty} = -270°$. Se sabe que cada polo aporta $-90°$ en $\Omega \rightarrow \infty$ por lo que $n = 3$

Por lo tanto: $\Omega_\omega = 2\pi f_p$

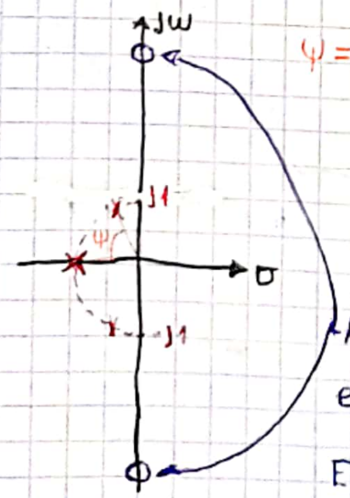
$\omega_p = 2\pi f_p \longrightarrow \omega_p' = 1$

$\omega_{0z} = 2\pi f_{0z}$ $\quad \omega_{0z}' = \frac{1}{3}$

Aplicando el núcleo de transformación $\Omega = \frac{1}{\omega}$

$$\Omega_p = \frac{1}{\omega_p} = 1 \quad ; \quad \Omega_{0z} = \frac{1}{\omega_{0z}} = 3$$

Como es butter de orden 3:



$$\psi = \frac{\pi}{3} \rightarrow Q = \frac{1}{2\cos\psi} = 1 \qquad \Omega_0 = 1$$

$$T_{lp}'(\$) = \frac{1}{\$+1} \cdot \frac{1}{\$^2 + s\,\frac{1}{Q} + 1}$$

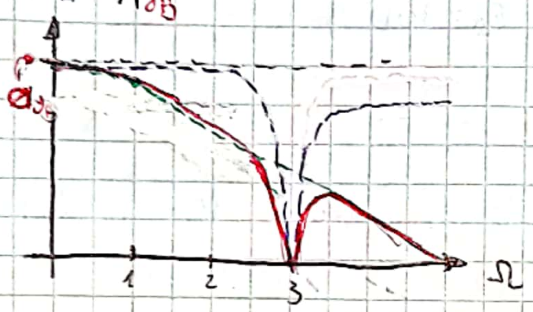
Ademas se nos piden dos ceros de transferencia en $\Omega = \Omega_{0z} = 3$

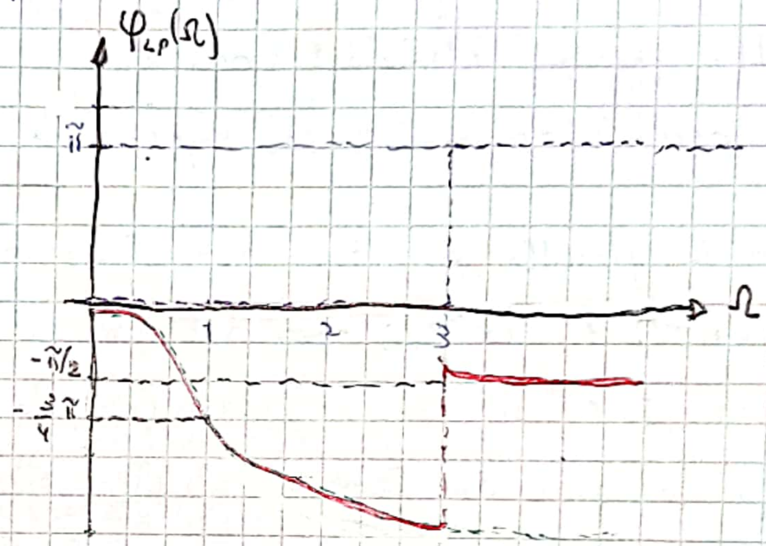
Estos ceros entran en juego en el numerador por lo que:

$$T_{lp}(\$) = \frac{1}{\$+1} \cdot \frac{1}{\$^2 + \$ + 1} \cdot (\$^2 + \Omega_{0z}^2) \cdot k$$

Para que sea de 0dB $\rightarrow K = 1/\Omega_{oz}^2 = 1/9$

--- aporte del notch





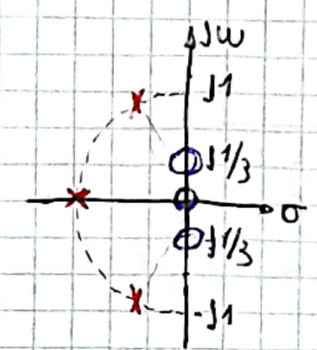
Transformación al Filtro objetivo:

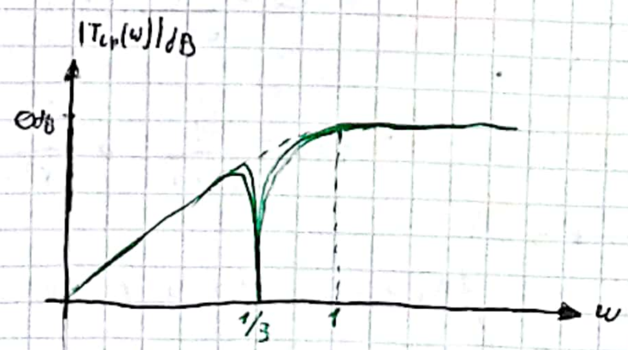
$$T_{hp}(S) = T_{Lp}(\$)\Big|_{\$ \to 1/S} = \frac{1}{\frac{1}{S}+1} \cdot \frac{\left(\frac{1}{S}\right)^2 + \Omega_{oz}^2}{\left(\frac{1}{S}\right)^2 + \frac{1}{S} + 1} \cdot K$$

$$T_{hp}(S) = \frac{S}{1+S} \cdot \frac{\frac{1+\Omega_{oz}^2 S^2}{S^2}}{\frac{1+S+S^2}{S^2}} \cdot K$$

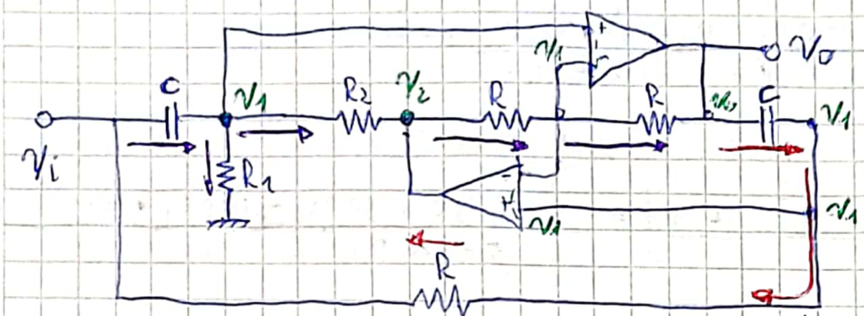
$$T_{hp}(S) = \frac{S}{1+S} \cdot \Omega_{oz}^2 \, \frac{1/\Omega_{oz}^2 + S^2}{S^2+S+1} \cdot \underbrace{\frac{1}{\Omega_{oz}^2}}_{K}$$

$$\left[T_{hp}(S) = \cdot \frac{S}{S+1} \cdot \frac{S^2 + 1/9}{S^2+S+1} \right]$$





Análisis de la estructura propuesta:



$$(V_i - V_1) \cdot SC = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} \quad (1)$$

$$\frac{V_2 - V_1}{R} = \frac{V_1 - V_0}{R}$$

$$V_2 + V_0 = 2V_1 \quad (2)$$

$$V_2 = 2V_1 - V_0$$

$$(V_0 - V_1) SC = \frac{V_1 - V_i}{R}$$

$$V_0 SCR + V_i = V_1 \cdot (1 + SCR)$$

$$V_1 = \frac{V_0 \, SCR + V_i}{1 + SCR} \quad (3)$$

de 2 en 4 :

$$(V_i - V_1) sC = \frac{V_1}{R_1} + \frac{V_1 - 2V_1 + V_0}{R_2}$$

$$V_i \, sC = \frac{V_1}{R_1} + V_1 \, sC - \frac{V_1}{R_2} + \frac{V_0}{R_2}$$

$$V_i \cdot sC = V_1 \cdot \left(sC + \frac{1}{R_1} - \frac{1}{R_2} \right) + V_0 \cdot \frac{1}{R_2} \rightarrow V_i \, sCR_2 - V_0 = V_1 \cdot \left(sCR_2 + \frac{R_2}{R_1} - 1 \right) \quad 4$$

Escaneado con CamScanner

Reemplazando (3) en (4)

$$V_i \cdot sCR_2 \mp V_0 = \left(sCR_2 + \frac{R_2}{R_1} - 1\right) \cdot \frac{V_0 sCR + V_i}{sCR + 1}$$

$$V_i \cdot \left[sCR_2 \mp \frac{sCR_1R_2 + R_2 - R_1}{R_1 \cdot (sCR+1)}\right] = \left[\left(\frac{sCR_1R_2 + R_2 - R_1}{R_1}\right) \cdot \frac{sCR}{sCR+1} + 1\right] V_0$$

$$V_i \left[\frac{sCR_1R_2 \cdot (sCR+1) - sCR_1R_2 \mp R_2 + R_1}{R_1(sCR+1)}\right] = \frac{s^2C^2R_1R_2R + sCRR_2 - \cancel{sCRR_1} + \cancel{sCRR_1} + R_1}{(sCR+1)R_1} V_0$$

$$V_i \left[s^2C^2RR_1R_2 + R_1 - R_2\right] = \left[s^2C^2RR_1R_2 + sCRR_2 + R_1\right] V_0$$

$$\left[\frac{V_0}{V_i} = \frac{s^2C^2RR_1R_2 + (R_1 - R_2)}{s^2C^2RR_1R_2 + sCRR_2 + R_1} = \frac{s^2 + \frac{R_1 - R_2}{R_1} \cdot \frac{1}{C^2RR_2}}{s^2 + s\frac{1}{CR_1} + \frac{1}{C^2RR_2}}\right]$$

Necesito cumplir

$$\frac{s^2 + 1/9}{s^2 + s + 1} \Rightarrow \frac{1}{CR_1} = 1 \Rightarrow \left[R_1 = \frac{1}{C}\right]$$

$$\frac{1}{C^2RR_2} = 1 \Rightarrow R = \frac{1}{C^2 \cdot R_2}$$

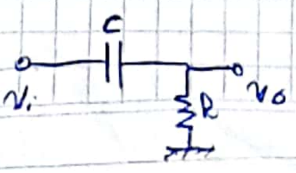
$$\frac{R_1 - R_2}{R_1} = \frac{1}{9} \Rightarrow R_1 - \frac{R_1}{9} = R_2$$

$$\left[R_2 = \frac{8}{9} R_1\right]$$

Normalizando $C = 1$:

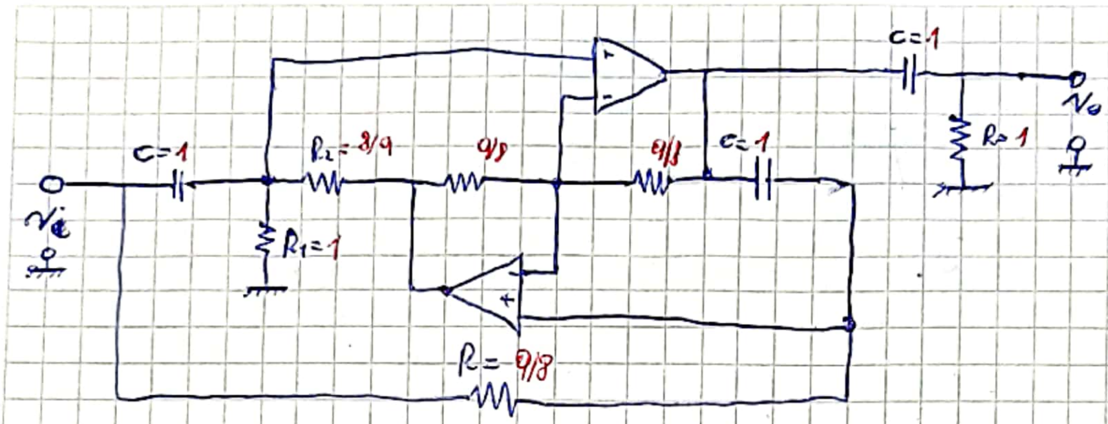
$$R_1 = \frac{1}{C} = 1 \; ; \quad R_2 = \frac{8}{9} R_1 = \frac{8}{9} \; ; \quad R = \frac{1}{C^2 R_2} = \frac{9}{8}$$

El polo simple $\frac{s}{s+1}$ lo consigo con un RC

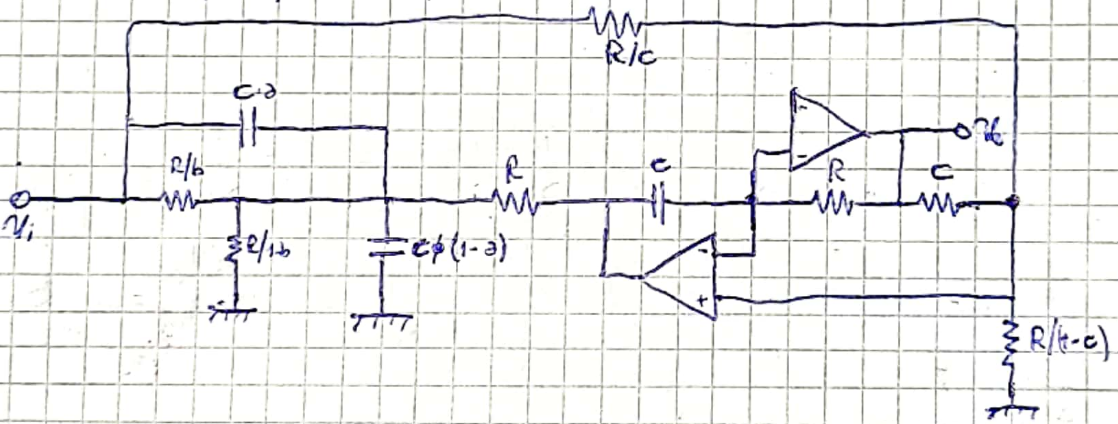


$$\frac{V_0}{V_i} = \frac{R}{\frac{1}{sC} + R} = \frac{sCR}{1 + sCR} = \frac{s}{s + 1/RC}$$

como $\frac{1}{RC} = 1 \Rightarrow R = \frac{1}{C} = 1$



Circuito propuesto por Schaumann



La transferencia de este sistema es la siguiente

$$\frac{V_o}{V_i} = \frac{s^2(2a-c) + s(2b-c)\frac{\omega_0}{Q} + c\,\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2} \qquad \text{donde} \quad \omega_0 = \frac{1}{RC}$$

Si queremos lograr nuestro filtro con este circuito:

$$\left\{\begin{array}{l} c\,\omega_0^2 = \omega_{0z}^2 \\ 2b - c = 0 \\ 2a - c = 1 \\ \omega_0^2 = 1 \end{array}\right. \Rightarrow \left.\begin{array}{l} c = \left(\frac{1}{3}\right)^2 \\ b = c/2 \\ a = \frac{c+1}{2} \end{array}\right\}$$

$$\frac{1}{RC} = 1 \Rightarrow \left[R = \frac{1}{C} = 1\right]$$