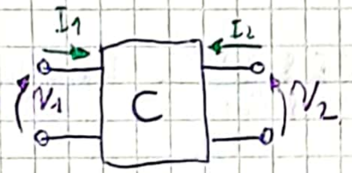
Clase 08: Cuadripolos



Matriz de parámetros

Respuesta $= C \cdot$ Excitación → Relación Lineal

$$\begin{cases} r_1 = e_1 c_{11} + e_2 c_{12} \\ r_2 = e_1 c_{21} + e_2 c_{22} \end{cases}$$

$$R^{N\times 1} = C^{N\times N} \cdot E^{N\times 1}$$

Las excitaciones pueden ser:

- $\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$
- $\begin{pmatrix} V_1 \\ I_2 \end{pmatrix}$
- $\begin{pmatrix} V_1 \\ I_1 \end{pmatrix}$
- $\begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$
- $\begin{pmatrix} I_1 \\ V_2 \end{pmatrix}$
- $\begin{pmatrix} V_2 \\ I_2 \end{pmatrix}$

para cada caso las respuestas son las otras 2

En A los casos: $E = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$; $R \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$ → parámetros Y

$E = \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$; $R \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$ → " Z

→ debido a q E adopta impedancia o admitancia

Si se eligen: $E = \begin{pmatrix} I_1 \\ V_2 \end{pmatrix}$; $R = \begin{pmatrix} V_1 \\ I_2 \end{pmatrix}$ → parámetros H

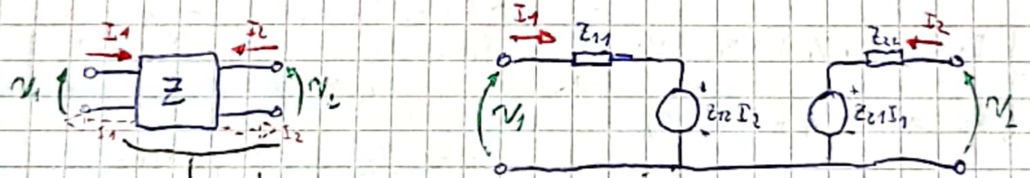
$E = \begin{pmatrix} V_1 \\ I_2 \end{pmatrix}$; $R = \begin{pmatrix} I_1 \\ V_2 \end{pmatrix}$ → parámetros G

Parámetros T o ABCD $\qquad T^{-1}$

$E = \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix}$; $R = \begin{pmatrix} V_1 \\ I_1 \end{pmatrix}$ | $E = \begin{pmatrix} V_1 \\ I_1 \end{pmatrix}$; $R = \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix}$

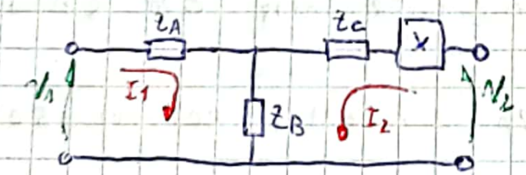
Parámetros Z

$V = Z \cdot I \longrightarrow \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \cdot \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} \longrightarrow \begin{cases} V_1 = Z_{11} I_1 + Z_{12} V_2 \\ V_2 = Z_{21} I_1 + Z_{22} V_2 \end{cases}$



Nunca se especifica pero tienen que existir

Para sacarnos los generadores controlados:



⊛ Por inspección podemos definir:

⊛ $Z_B = Z_{12}$; $Z_A = Z_{11} - Z_{12}$ ⊛

⊛ $Z_B + Z_C = Z_{21}$

$Z_C = Z_{22} - Z_{12}$

$V_1 = I_1 (Z_A + Z_B) + I_2 (Z_B)$ ⊛

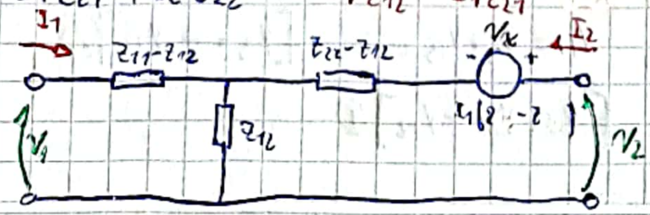
$V_2 = I_2 (Z_C + Z_B) + I_1 Z_B + X$ ⊛

Reescribiendo:

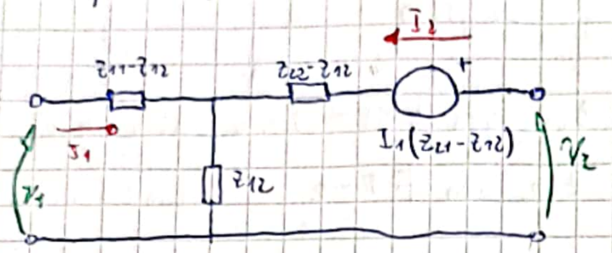
$V_1 = I_1 (Z_{11} - Z_{12} + Z_{12}) + I_2 \cdot Z_{12}$

$V_2 = I_1 Z_{12} + I_2 (Z_{22} - Z_{12} + Z_{12}) + \overbrace{I_1 Z_{21} - I_1 Z_{12}}^{V_x}$

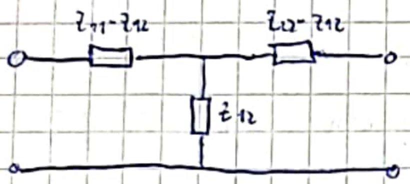
$V_2 = I_1 Z_{21} + I_2 Z_{22} + I_1 Z_{12} - I_1 Z_{21}$



de los parámetros z:

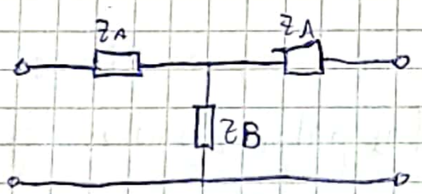


Si $Z_{12}=Z_{21}$



El cuadripolo es recíproco si $Z_{12}=Z_{12}$ y en esta materia, implica pasividad

Si $Z_{11}=Z_{22}$ → Cuadripolo simétrico



Los parámetros funcionan bajo excitaciones y respuestas

$$\left.\begin{aligned} V_1 &= I_1 Z_{11} + I_2 Z_{12} \\ V_2 &= I_1 Z_{21} + I_2 Z_{22} \end{aligned}\right\}$$

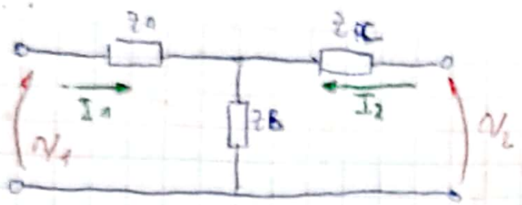
$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_2=0}$$

Todos los parámetros se miden con un circuito abierto

Si obtengo los parámetros z de cualquier red la podemos reducir a una red T

E) :



$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} = Z_A + Z_B \quad ; \quad Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} = Z_B$$

$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} = Z_B \quad ; \quad Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} = Z_C + Z_B$$

$$Z = \begin{pmatrix} Z_A+Z_B & Z_B \\ Z_B & Z_C+Z_B \end{pmatrix}$$

Z_{11} y Z_{22} son FRP → func reales y positivas

Z_{12} y Z_{21} son FT → " Transferencias

Los polos de Z_B van a aparecer en todos los parámetros
→ polos comunes

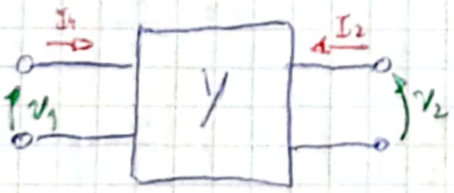
Los polos de Z_A y Z_C solo aparecen en Z_{11} y Z_{22} respectivamente
→ polos privados

Para que el circuito sea realizable:

$$K_{11} \cdot K_{22} - K_{12} \cdot K_{21} \geq 0$$

donde K_{ij} es el residuo de los polos comunes para cada parámetro Z_{ij}.

Parametros Y:



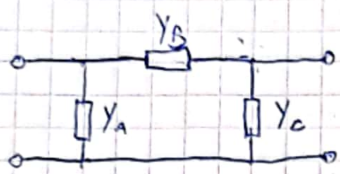
$$I_{11} = V_1 \cdot Y_{11} + V_2 \cdot Y_{12}$$
$$I_{22} = V_1 Y_{21} + V_2 Y_{22}$$

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} \quad ; \quad Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

$$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} \quad ; \quad Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$

Los parámetros Y se asocian a una red π

si la red es recíproca $Y_{12} = Y_{21}$



Las ecuaciones del los parámetros Y dan:

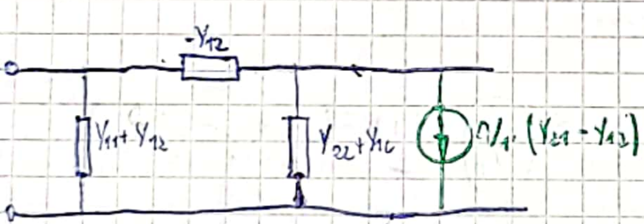
$I_1 = V_1(Y_A + Y_B) - V_2 Y_B \Rightarrow$ $Y_B = -Y_{12}$ $\quad Y_A = Y_{11} + Y_{12}$

$I_2 = V_1(-Y_B) + V_2 \cdot (Y_B + Y_C)$

$I_2 = -V_1 \cdot Y_{12} + V_2(Y_C - Y_{12}) \Rightarrow$ $Y_C = Y_{22} + Y_{12}$ y para llegar a la Ecuaciones iniciales:

$I_2 = -V_1 Y_{12} + V_2 Y_{22} + V_1(Y_{21} - Y_{12})$

El circuito resulta:



Nuevamente si: $Y_{21} = Y_{12} \rightarrow$ Red Recíproca y pasiva

y si $Y_A = Y_C \Rightarrow$ Red simétrica

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = Y_A + Y_B \quad ; \quad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -Y_B$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -Y_B \quad ; \quad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = Y_B + Y_C$$

$$Y^{-1} = Z = \frac{1}{\det(Y)} \cdot \mathrm{Adj}(Y)$$

$$\det(Y) = (Y_A + Y_B) \cdot (Y_C + Y_B) - Y_B^2 = Y_A Y_C + Y_A Y_B + Y_B Y_C$$

$$Z = \frac{1}{\det(Y)} \cdot \begin{pmatrix} Y_C+Y_B & Y_B \\ Y_B & Y_A+Y_B \end{pmatrix}$$

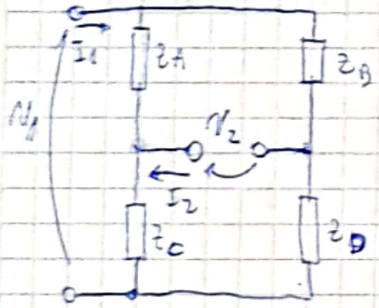
$$z_{11} = \frac{Y_C+Y_B}{Y_AY_B+Y_AY_C+Y_CY_B} \quad ; \quad z_{12} = \frac{Y_B}{Y_AY_B+Y_AY_C+Y_CY_B}$$

$$z_{21} = \frac{Y_B}{Y_AY_B+Y_AY_C+Y_CY_B} \quad ; \quad z_{22} = \frac{Y_A+Y_B}{Y_AY_B+Y_AY_C+Y_CY_B}$$

Otra vez tenemos los límites de realizabilidad con los Residuos

$$K_{11} \cdot K_{22} - K_{12}^2 \geq 0$$

Parámetros Z de un lattice



$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = (Z_A+Z_C) \,//\, (Z_B+Z_D)$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} =$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \left(\frac{V_2}{V_1}\right) \cdot z_{11} = \left(\frac{Z_C}{\cancel{Z_C+Z_A}} - \frac{Z_D}{\cancel{Z_B+Z_D}}\right) \cdot \frac{\cancel{(Z_A+Z_C)}\,\cancel{(Z_B+Z_D)}}{Z_A+Z_B+Z_C+Z_D}$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = (Z_A+Z_B) \,//\, (Z_C+Z_D)$$

$$z_{12} = z_{21} = \frac{Z_C \cdot (Z_B+Z_D) - Z_D(Z_A+Z_C)}{Z_A+Z_B+Z_C+Z_D} = \frac{Z_CZ_B - Z_DZ_A}{Z_A+Z_B+Z_C+Z_D}$$

Si el lattice es simétrico: $Z_C = Z_B$ y $Z_D = Z_A$

$$z_{11} = (Z_A+Z_B) \,//\, (Z_A+Z_B) = \frac{Z_A+Z_B}{2} \qquad z_{12} = z_{21} = \frac{Z_B^2 - Z_A^2}{2Z_B+2Z_A} = \frac{Z_B-Z_A}{2}$$

$$z_{22} = (Z_A+Z_B) \,//\, (Z_A+Z_B) = \frac{Z_A+Z_B}{2}$$

$$Z_{\text{Lattice sim}} = \begin{pmatrix} \frac{Z_A+Z_B}{2} & \frac{Z_B-Z_A}{2} \\ \frac{Z_B-Z_A}{2} & \frac{Z_A+Z_B}{2} \end{pmatrix}$$