
Augmented Lagrangian Digital Image Correlation (AL-DIC) Code Manual (v3.8)

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Github page: https://github.com/jyang526843/2D_ALDIC
MATLAB FileExchange page: <https://www.mathworks.com/matlabcentral/fileexchange/70499-augmented-lagrangian-digital-image-correlation-and-tracking>

Last time updated on November 19, 2020

Contents

1	Introduction	3
2	Code installation	4
3	Code Section 1. MATLAB mex setup	4
3.1	Test MATLAB mex setup	4
3.2	Install mex C/C++ compiler	4
3.3	Execute code Section 1	6
4	Code Section 2: Load images and set up DIC parameters & mesh	6
4.1	Load DIC images	6
4.1.1	Load DIC images from certain folder	9
4.1.2	Load DIC images with prefix of image name	9
4.1.3	Load DIC images manually	9
4.2	Define region of interest (ROI)	10
4.3	Set up DIC parameters	11
4.4	Additional parameters setup when dealing with image sequence	13
5	Code Section 3: Computing initial guess from FFT-based cross correlation	15
5.1	FFT-based methods to compute initial guess	15
5.2	Remove noise in computed initial guess	17
6	Code Section 4: ALDIC Subproblem 1 (first local step)	19
6.1	Local subset IC-GN solver	19
6.2	Remove results of IC-GN bad subsets	21
7	Code Section 5: ALDIC Subproblem 2 (first global step)	22
7.1	Finite difference method	22
7.2	Finite element method	23
7.3	Comparison between finite difference and finite element methods in solving Subproblem 2	23
8	Code Section 6: ALDIC ADMM iterations	23
9	Code Section 7: Check convergence	24
10	Code Section 8: Compute strains	25
10.1	Smooth displacement field if needed	25
10.2	Compute strain field	25
10.3	Plot and save results	27
11	Summary of all the MATLAB command window screen outputs in the heterogeneous fracture case study	30
12	Frequently Asked Questions (FAQs)	33

12.1 About MATLAB mex set up	33
12.2 About MATLAB parallel computing	34
12.3 About ALDIC algorithm	34
Acknowledgements	36
References	36

1 Introduction

Digital image correlation (DIC) technique is a powerful experimental tool for measuring full-field displacements and strains. Most current DIC algorithms can be categorized into either *local* or finite-element-based *global* methods, see Fig. 1. As with most experimental approaches, there are drawbacks with each of these methods. In the local method the subset deformations are estimated independently and the computed displacement field may not necessarily be kinematically compatible. Thus, the deformation gradients can be noisy, especially when using small subsets. Although the global method often enforces kinematic compatibility, it generally incurs substantially greater computational costs than its local counterpart, which is especially significant for large data sets. Here we present a new hybrid DIC algorithm, called *augmented Lagrangian digital image correlation (ALDIC)* [1], which combines the advantages of both the local (fast computation times) and global (compatible displacement field) methods.

ALDIC code is freely available at Github and MATLAB File Exchange (link: [2]) and solves the general motion optimization problem by using the alternating direction method of multipliers (ADMM) [3]. Finite-element-based global DIC code is also available at MATLAB File Exchange [4].

We demonstrate that our ALDIC algorithm has high accuracy and precision while maintaining low computational cost, and is a significant improvement compared to current local and global DIC methods. For a review of both local and global DIC methods, and details of this new proposed ALDIC method, please see our paper [1] (full text can also be requested at [5]).

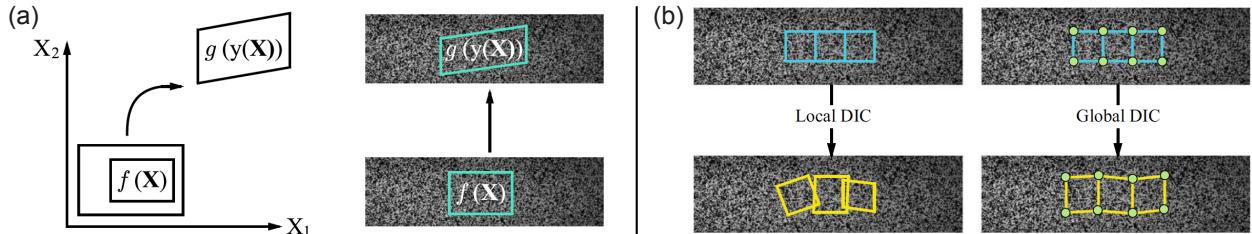


Figure 1: (a) Schematic showing a DIC reference image $f(\mathbf{X})$, with a general speckle pattern, deforming into the deformed image $g(\mathbf{y}(\mathbf{X}))$ under some mapping \mathbf{y} . \mathbf{X} and \mathbf{y} coordinates are in the reference and deformed images, respectively. (b) A schematic comparison between the local DIC method (left), where all the subsets are analyzed independently, and the finite-element-based global DIC method (right), where a global basis set is used to represent the full-field deformation.

Here some advantages of ALDIC algorithm are highlighted:

- (i) It's a fast algorithm using distributed parallel computing for a global nonconvex optimization.
- (ii) Global kinematic compatibility is added as a global constraint in the form of augmented Lagrangian, and solved using Alternating Direction Method of Multipliers (ADMM) scheme.
- (iii) Both displacement fields and affine deformation gradients are correlated at the same time.
- (iv) No need of much art-of-work about choosing displacement smoothing filters.
- (v) It works well with compressed images and adaptive mesh [6, 7].

- (vi) It can solve an image sequence with multiple frames. Both *cumulative* and *incremental* DIC modes are implemented where the latter one is quite useful for measuring very large deformations.
- (vii) If you are applying 3D volumetric digital volume correlation, please refer to our ALDVC paper [8, 9] and code [10, 11].

2 Code installation

ALDIC code can be downloaded at [2]. It has been tested on MATLAB versions later than R2018a. Parallel Computing Toolbox is highly recommended but not necessary to speed up the code.

To install the code, please download and unzip the code folder and put this folder on the MATLAB current working path.

After opening the main file "main_ALDIC.m", as shown in Fig. 2, ALDIC code can be executed by each section. Once you are familiar with the ALDIC code, you can execute the whole main file by clicking the "Run" button to run the whole file at once ("EDITOR >> RUN >> "). ALDIC code is easy to modify based on user's custom parameter choice. In this code manual, we will introduce the ALDIC code section by section.

3 Code Section 1. MATLAB mex setup

Execute this section and we will try to build "mex" functions from C/C++ source codes for image grayscale value interpolation, where linear, bi-cubic (by default) and bi-cubic splines interpolations are implemented in this code. For example, by default we use bi-cubic interpolations where the associated mex set up file is called "ba_interp2.cpp" [12].

3.1 Test MATLAB mex setup

First, we test whether there is already a C/C++ compiler installed on your computer by inputting `mex -setup` and press Enter key on the MATLAB command window. If an available C/C++ compiler is already installed, please skip Section 3.2 and jump to Section 3.3.

3.2 Install mex C/C++ compiler

The step of installing mex C/C++ compilers is a common step for users to run C/C++ codes with MATLAB. Mac users usually don't come across the error message from mex C/C++ compilers. For Windows users, you can follow these steps to install mex C/C++ compiler. More details can be found in [13, 14].

- Download: TDM-gcc compiler from: <http://tdm-gcc.tdragon.net/>

Run the whole mfile Run the selected section

```

1 %>----- Augmented Lagrangian DIC
2 %> Author: Jin Yang, PhD. Caltech
3 %> Contact: yangjin@caltech.edu
4 %> 2015.04.06,07; 2016.03.04
5 %-----%
6 %
7 %
8 %% Section 1: Clear MATLAB environment & mex set up Spline interpolation
9 close all; clear; clc; clearvars -global
10 fprintf('----- Section 1 Start ----- \n')
11 setenv('MW_MINGW64_LOC','C:\TDM-GCC-64')
12 % % cd("./Splines_interp/lib_matlab"); CompileLib; cd("../.."); % % mex bi-cubic spline interp
13 % % addpath("./Splines_interp/lib_matlab"); % dbstop if error % Old version codes.
14 mex -O ba_interp2.cpp;
15 addpath("./func"); addpath("./src"); addpath("./plotFiles/"); addpath("./plotFiles/export_fig-d96"
16 % addpath("./YOUR IMAGE FOLDER");
17 fprintf('----- Section 1 Done ----- \n \n')
18 %
19 %
20 %% Section 2: Load DIC parameters and set up DIC parameters
21 fprintf('----- Section 2 Start ----- \n')
22 % ===== Read images =====
23 [file_name,Img,DICpara] = ReadImage; close all;
24 % %%%% Uncomment the line below to change the DIC computing ROI manually %%%%%%
25 %gridxROIRange = [gridxROIRange1,gridxROIRange2]; gridyROIRange = [Val1, Val2];
26 %gridxROIRange = [224,918]; gridyROIRange = [787,1162];
27 % ===== Normalize images =====
28 [ImgNormalized,DICpara.gridxyROIRange] = funNormalizeImg(Img,DICpara.gridxyROIRange);
29 % ===== Initialize variable storage =====
30 ResultDisp = cell(length(ImgNormalized)-1,1);
31 ResultDefGrad = cell(length(ImgNormalized)-1,1);
32 ResultStrain = cell(length(ImgNormalized)-1,1);
33 ResultFEMesh = cell(ceil((length(ImgNormalized)-1)/DICpara.ImgSeqIncUnit),1); % For incremental D
34 fprintf('----- Section 2 Done ----- \n \n')
35

```

Figure 2: Main file of ALDIC code: “main_ALDIC.m”. Each section can be executed in order by clicking the “Run Section”.

The image shows a MATLAB Command Window titled "Command Window". The window displays the following text:

```

>> mex -setup
MEX configured to use 'MinGW64 Compiler (C)' for C language compilation.
Warning: The MATLAB C and Fortran API has changed to support MATLAB
variables with more than 2^32-1 elements. You will be required
to update your code to utilize the new API.
You can find more information about this at:
https://www.mathworks.com/help/matlab/matlab\_external/upgrading-mex-files-to-use-64-bit-api.html.

To choose a different language, select one from the following:
mex -setup C++
mex -setup FORTRAN

```

Figure 3: Message display on the command window when a mex C/C++ compiler is stalled successfully.

- Install TDM-gcc compiler on your computer. For example, I install it at 'C:\TDM-GCC-64' ¹.
- Restart MATLAB and input these codes on the command window:
`setenv('MW_MINGW64_LOC', 'YourTDMGCCPath'); mex -setup;`
 to check whether "mex" is set up successfully or not. Don't forget to replace the above
`'YourTDMGCCPath'` using your own installation location of TDM-gcc package in the last step.
 For example, if it's installed at 'C:\TDM-GCC-64', please replace previous
`'YourTDMGCCPath'` with 'C:\TDM-GCC-64'. If a mex C/C++ compiler is installed successfully, a message similar to (Fig. 3) will display on the MATLAB command window.

3.3 Execute code Section 1

Once a mex C/C++ compiler is installed, we can execute main_ALDIC.m code Section 1 and a successful message will display on the MATLAB command window, see Fig. 4.

4 Code Section 2: Load images and set up DIC parameters & mesh

This section is to load both DIC reference and deformed images and set up DIC parameters. First, please put your images on the MATLAB working path. After executing this section, all the ALDIC associated parameters will be stored in the workspace structure variable "DICpara". All the mesh properties will be stored in the structure variable "DICmesh" after executing code Section 3. Here we make a brief summary of both these two data structures in Table 1 and Table 2.

4.1 Load DIC images

MATLAB command screen displays these lines to allow user to select their choice to load DIC images, which will be explained in Section 4.1.1-4.1.3.

¹In practice, we find that this TDM-gcc compiler only works if installed on the first level main disks, such as 'C:\', 'D:\', 'E:\', etc.

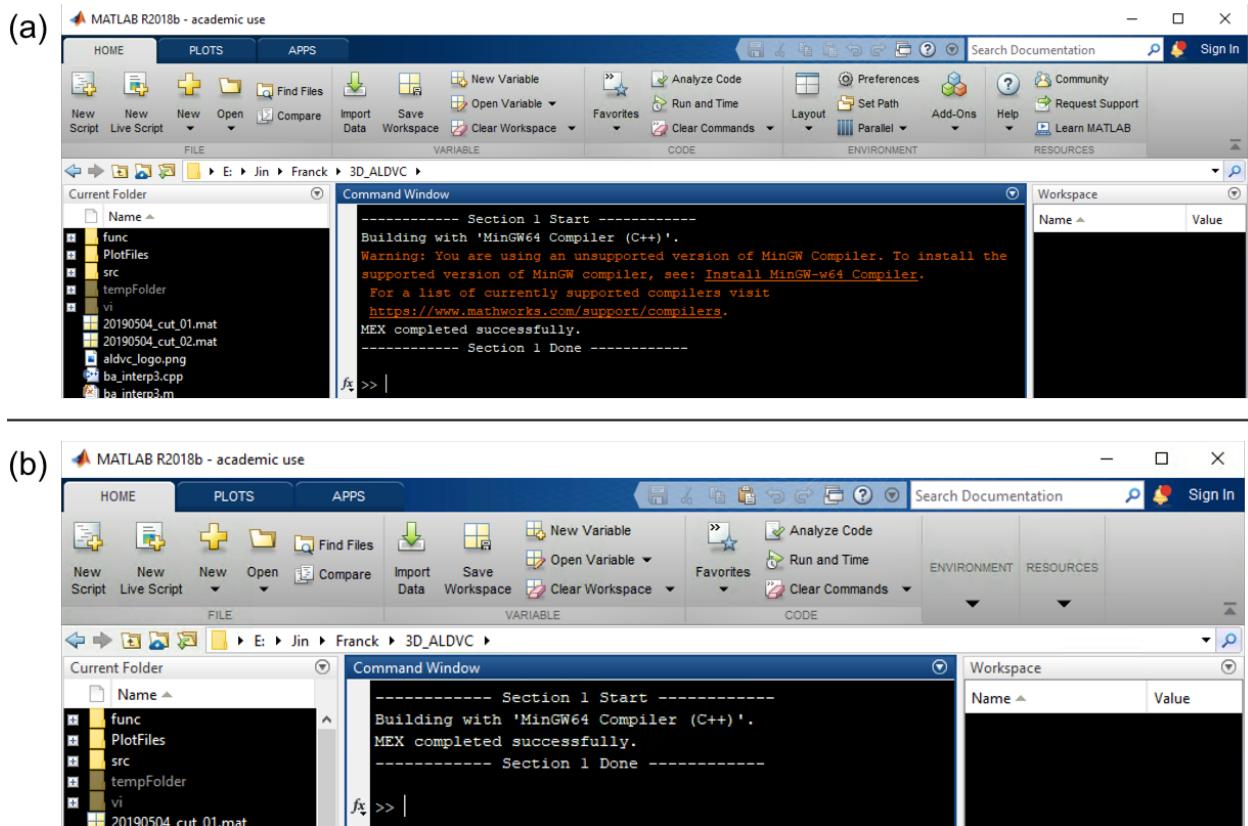


Figure 4: ALDIC code Section 1 is executed successfully and a message similar to (a) or (b) will display on the command window.

Table 1: Summary of DIC parameters in “DICpara” structure

DICpara variable	DIC parameter	Description and comments
winsize	Subset window size [winspace_x, winspace_y]	Local subset size in ALDIC Subproblem 1
winstepsize	Subset window step [winstepsize_x, winstepsize_y,	The distance between neighboring local subsets, or the finite element size in ALDIC Subproblem 2.
gridRange	DIC region of interest (ROI)	ROI can be defined by clicking top-left and bottom-right corner points.
Subpb2-FDOrFEM	0-’FD’: Finite difference method; 1-’FEM’: finite element method	Both finite different and finite element methods are implemented to solve ALDIC Subproblem 2.
ClusterNo	Number of threads to perform parallel computing	ALDIC Subproblem 1 can be sped up by applying parallel computing.
ImgSize	Image size	Size of 2D DIC images
ImgSeqIncUnit	To decide to perform cumulative or incremental DIC mode	Cumulative DIC is always to compare with the first reference frame; incremental DIC could update the reference frame.
ImgSeqInc-ROIUpdate-OrNot	In incremental mode, ROI can be updated at the same time of updating reference image.	It’s recommended to manually update ROI at the same time of updating reference image for measuring large deformations.
NewFFTSearched	Method to update initial guess to solve an image sequence	Result of last frame can be assigned as the initial guess for the next frame.
SizeOfFFT-SearchRegion	Size of FFT-cross correlation search region in the initial guess section	The search region size should be larger than the maximum x- and y-displacements.
displayIterOrNot	Display convergence details of Subproblem 1 IC-GN iterations	0 or 1: Don’t print or print IC-GN convergence info of each local subset.
Subpb1ICGN-MaxIterNum	Maximum IC-GN iterations to solve Subproblem 1	Subsets fail to converge within ‘Subpb1ICGNMaxIterNum’ iteration steps will be marked as bad subsets.
ICGNtol	Tolerance threshold of IC-GN iterations in solving Subproblem 1	E.g., ICGNtol takes value of 10^{-4} px by default.

Table 2: Summary of DIC mesh in “DICmesh” structure

DICmesh variable	DIC parameter	Description and comments
coordinatesFEM	Coordinates of nodal points in the finite element mesh and their connectivity	Linear 4-node quadrilateral (Q4) elements are used here. However, it can be extended to other type of finite elements with arbitrary shape function.
elementsFEM		
dirichlet	FE-mesh nodal points at the boundary	Indices of nodal points at ROI borders are assigned with Dirichlet or Neumann boundary conditions.
neumann		
x0, y0	Regular FE-mesh nodal grids	Here we use uniform regular Q4 FE-mesh.

```

1 ----- Section 2 Start -----
2 Choose method to load images:
3 0: Select images folder;
4 1: Use prefix of image names;
5 2: Manually select images.
6 Input here:

```

One comment for the ALDIC code is that it always manipulate the deformed images and tries to transform them back to the reference image to compute their deformation fields which is based on the *Lagrangian* description. If user wants to track the deformation field in the *Eulerian* description, user can select the reference image as the second image, and select the deformed image as the first image and manipulate the reference image to transform to the current deformed image.

4.1.1 Load DIC images from certain folder

If we select method 0: Select images folder to load DIC images, you will be asked to select the folder path, then all the images within this folder will be loaded automatically. In the *cumulative* DIC mode, the first frame is set to be the fixed reference image by default, while all the subsequent frames in the image sequence are set to be deformed images whose deformations will be tracked in the Lagrangian description. In the *incremental* DIC mode, the reference image can be updated after every certain number ("DICpara.ImgSeqIncUnit") of frames. After loading DIC images, please jump to Section 4.2.

4.1.2 Load DIC images with prefix of image name

If we select method 1: Use prefix of image names to load DIC images, user also needs to provide prefix text words of their DIC image frames (and all these images need to be added to the MATLAB path as well). For example, if all the DIC images are named in the format as: "img_0001.tif", "img_0002.tif", ..., user should input "img_0*.tif" on the MATLAB command screen to load all the DIC images started with prefix "img_0" and in the "tif" format. After loading DIC images, please jump to Section 4.2.

```

1 What is prefix of DIC images? E.g. img_0*.tif.
2 Input here:

```

4.1.3 Load DIC images manually

If we select method 2: Manually select images, user needs to load DIC images frame by frame manually, see Fig. 6. ALDIC code sets the first loaded image as the reference undeformed image and all other images as deformed images by default.

```

1 --- Please load first image ---
2 --- Please load next image ---
3 Do you want to load more deformed images? (0-yes; 1-no)

```

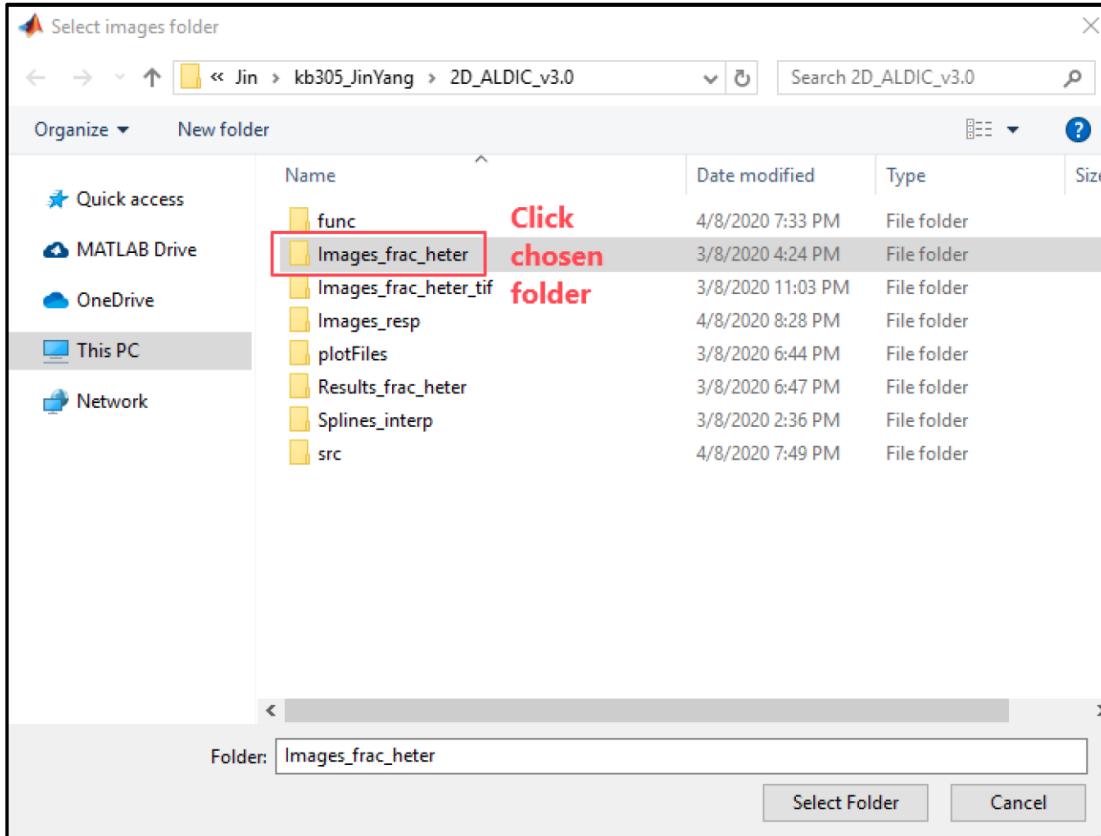


Figure 5: Load DIC images by selecting the folder including DIC images. For example, after clicking chosen folder "Images_frac_heter", then all the images in this folder will be loaded automatically.

Input "0" if you want to continue uploading images and input "1" if you want to stop uploading images. For example, we can select first reference image as "img_0000.tif", and select second image as "img_0570.tif".

4.2 Define region of interest (ROI)

Execute code Section 2, user can click both the top-left and bottom-right corner points on a popped-out image to define DIC region of interest (ROI). On the command window, it displays:

```
1 --- Define ROI corner points at the top-left and the bottom-right ---
```

Then user first click a left-top point and then click a right-bottom point to define ROI directly on the DIC image, see Fig. 7.

After clicking both the top-left and bottom-right corner points, the command window screen will display their coordinates in the unit of pixels. E.g., in the image shown in Fig. 7, I define a ROI with top-left and bottom-right corner points are:

```
1 Coordinates of top-left corner point are (322.786,74.730)
```

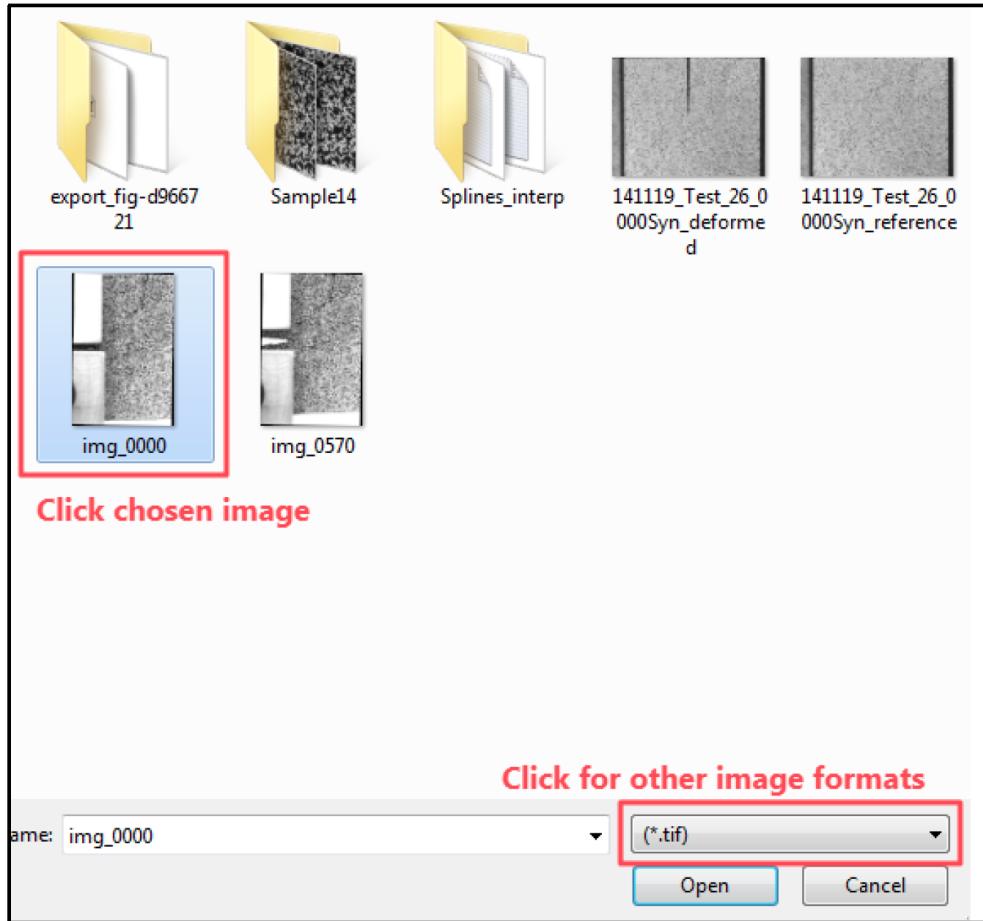


Figure 6: Manually load DIC images by clicking both reference and deformed images.

```
2 Coordinates of bottom-right corner point are (750.063, 1128.439)
```

Comment 1: If the top-left or bottom-right corner point is clicked out of the image, it will be adjusted to the nearest point on the original DIC image borders automatically.

Comment 2: If user prefers to use command lines to input ROI coordinates instead of directly clicking corner points on the DIC image. Please uncomment line `% gridxROIRange = [gridxROIRange1, gridxROIRange2];` `% gridyROIRange = [gridyROIRange1, gridyROIRange2];` and modify values of "gridxROIRange" and "gridyROIRange" there.

4.3 Set up DIC parameters

Then user will be asked to decide the local subset size and the subset step. “Subset size” is the edge length of local subset window (needs to be an even integer, 2M, and the subset set window is located from -Mpx to Mpx corresponding to the subset center); while the “subset step” is the

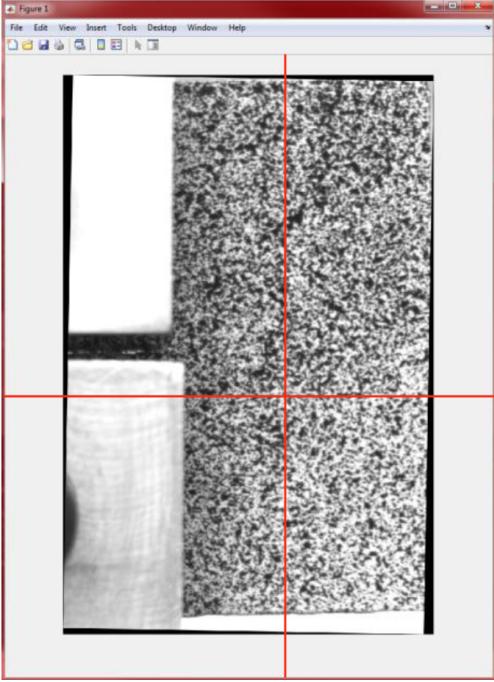


Figure 7: Manually click top-left and bottom-right corner points to define DIC region of interest (ROI).

distance between two neighboring subsets, and choice of “subset step” can be independent of choice of the subset size.

```

1 --- What is the subset size? ---
2 Input here: 20
3 --- What is the subset step? ---
4 Input here: 10

```

Subset size is the window size where we use to obtain initial guess through FFT cross correlation method, or through SSD minimization IC-GN iterations in the ALDIC local step. Subset step size is also the finite element mesh size in the ALDIC Subproblem 2 global step. Practically you can choose subset size from 10 px by 10 px to 60 px by 60 px to include 3~5 features of your DIC images; subset step size can be selected as 0.25~1 times of subset size. E.g., in the heterogeneous fracture example “img_0000.tif” and “img_0570.tif”, we select subset size as 20 px, and subset step is chosen as 10 px.

We also need to choose the solver method for ALDIC Subproblem 2 (global step). For this version of code, we can only deal with uniform grid mesh, and there is almost no difference whether you choose the “Finite difference method” or “Finite element method”. Even you choose “Finite difference method”, there are still finite element mesh generated (cf “DICmesh” in the Matlab workspace) which could help you conduct FEA analysis if you want to combine DIC with other FEA codes/software afterwards. My personal experience is that finite difference method is faster and a little bit more accurate than finite element method in the ALDIC Subproblem 2 (global step) because of the boundary effects.

```

1 --- Method to solve ALDIC global step Subproblem 2 ---
2   1: Finite difference(Recommended)
3   2: Finite element method
4 Input here: 1

```

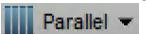
We need to set up parallel pools or tell MATLAB we don't want to use parallel pools.

```

1 --- Set up Parallel pool ---
2 How many parallel pools to open? (Put in 1 if no parallel computing)
3 Input here: 4

```

If we don't want to use parallel pools, input `0` or `1`. If we want to use parallel computing, put the number of your parallel pools. E.g., Input `4`.

MATLAB parallel computing environment can be set in the “Home  >> Environment >> Parallel  >> Parallel Preferences” (see Fig. 8 and [15] for more information about `parpool`).

4.4 Additional parameters setup when dealing with image sequence

If we upload an image sequence with more than two frames, as for each new frame, we can choose to use the displacement results in last frame as the initial guess for the new image frame, or we can just redo FFT initial guess for every new frame. This choice depends on how big relative displacements can be between two consecutive frames. Generally, if the relative displacement field between two consecutive frames is smaller than 5-7 voxels, we can use the deformation result of last frame as the initial guess displacement field for the new frame; otherwise it is suggested that we still need to redo the FFT initial guess process.

```

1 Since we are dealing with an image sequence with multiple frames, for each
   new frame, do we use the result of last frame as an initial guess or
   redo FFT initial guess for every new frame?
2 0: Use last frame;
3 1: Redo initial guess.
4 Input here:

```

When post-process image sequence with more than two frames, user could decide to perform either *cumulative* mode DIC or *incremental* mode DIC. The cumulative mode is the default setup and all the following frames will be compared with the first frame. However, incremental mode is preferred when dealing with extremely large deformations but may lose some accuracy because of the reference image has been updated. We recommend the user to try cumulative mode first, and if cumulative mode doesn't work very well, then try incremental mode. If you choose to use incremental mode, you will further be inquired to input how often you would like to update the reference image:

```

1 --- Choose cumulative or incremental mode ---
2   0: Cumulative(By default);
3   1: Incremental;
4 Input here: 1

```

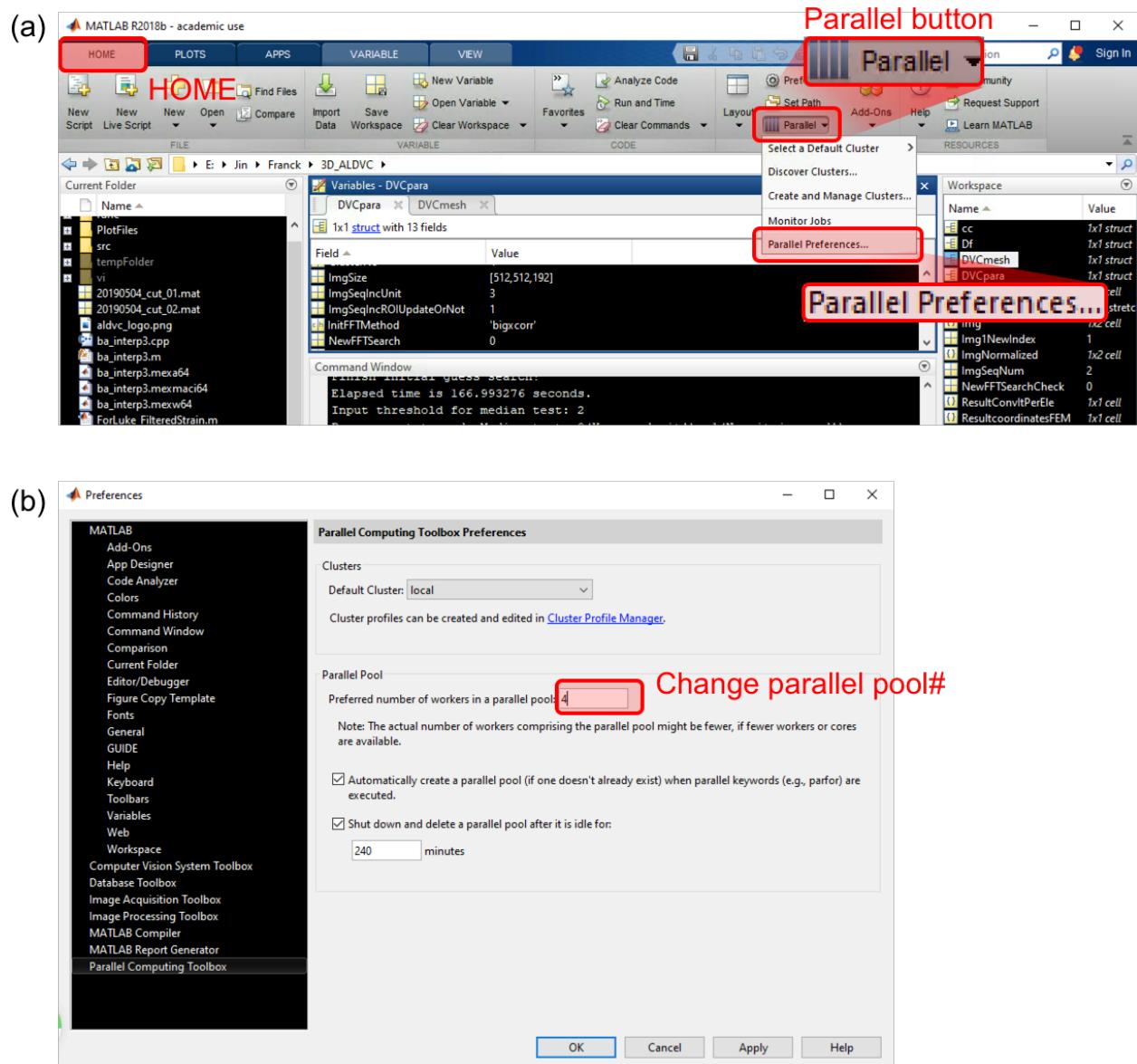


Figure 8: Set up MATLAB parallel preferences. See [15] for more information about `parpool`.

If you choose to use incremental mode, you will further be inquired to input how often you would like to update the reference image:

```
1 Incremental mode: How many frames to update reference image once?
2 Input here: 10
```

E.g., I want to update my reference image once every ten frames, so I input: `10`. The minimum number you can input is `1`, which means to update reference image every frame.

Every time the reference image is updated, you can choose to update the region of interest (ROI) at the same time or not. To achieve this, user will be asked as follows:

```
1 Update ROI at the same time of updating reference image?
2   0: Do not update ROI;
3   1: Manually(Recommended);
4   2: Automatically;
5 Input here: 1
```

E.g., Input `1` or `2` if you want to update ROI at the same time of updating reference image. Theoretically, this ROI update can be done automatically using the solved deformation of the last frame. However, we find it is most robust to update this ROI manually.

5 Code Section 3: Computing initial guess from FFT-based cross correlation

5.1 FFT-based methods to compute initial guess

In this section, an initial guess of the unknown deformation is computed using fast Fourier transform (FFT) based method to maximize zero normalized cross correlation function C_{ZNCC} , see Fig. 9(a-b).

$$C_{ZNCC} = \frac{\int(f - \mu_f)(g - \mu_g)}{\sigma_f \sigma_g} \quad (1)$$

where μ_f, μ_g are average grayscale values of DIC images f and g ; σ_f, σ_g are the standard deviation of image grayscale values. Above cross correlation can be fast computed using Fourier transform convolution theorem and fast Fourier transform (FFT). For example:

$$C_{CC} = \int f(\mathbf{x})g(\mathbf{x} + \mathbf{u})d\mathbf{x} = \mathcal{F}^{-1} \left[\overline{\mathcal{F}[f(\mathbf{x})]} \circ \mathcal{F}[g(\mathbf{x} + \mathbf{u})] \right] \quad (2)$$

where $\mathcal{F}[\cdot]$ denotes the Fourier transform, (\cdot) denotes the complex conjugate, and “ \circ ” denotes the Hadamard product (entry-wise product) and the absolute values are taken entry-wise as well.

The ALDIC code provides three methods to compute the initial guess for the unknown displacement fields, see Fig. 9(c-e), and following lines will display on the screen:

```
1 --- Method to compute an initial guess of displacements ---
2   0: Multigrid search based on an image pyramid
```

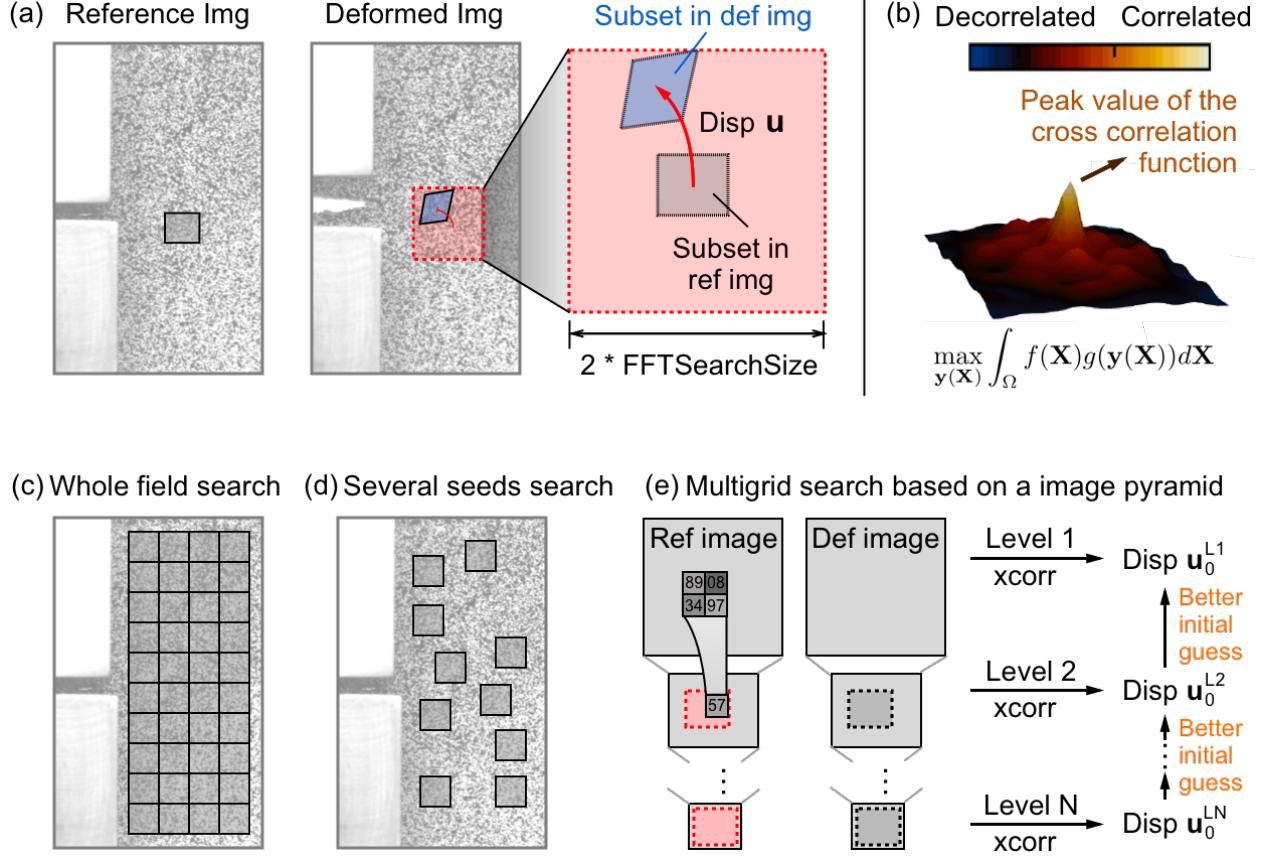


Figure 9: (a-b) Compute an initial guess for the unknown displacement of a local subset by maximizing the FFT-based (zero-normalized) cross correlation function. (c-d) All the local subsets, or several manually clicked seeds are analyzed with the FFT-based cross correlation calculations to compute initial guesses of their deformations. (e) A multigrid method based on a constructed image pyramid is used to compute the initial guess of the full-field deformation.

```

3 1: Whole field search for all the subsets
4 2: Search near manually clicked seeds and then interpolate for the
   full-field
5 Input here:

```

The **Method 1** (whole field search, Fig. 9(c)) is to perform the FFT-based cross correlation for all the local subsets (subsets can overlap), while the **Method 2** (Several seeds search, Fig. 9(d)) is to compute near several manually clicked local seeds. Both the **Methods 1 & 2** will also ask the user to input the size for the search area. User should try a small value of the search area size, and then gradually increase this value until it is larger than the maximum x- and y-displacements. For example, in the heterogeneous fracture example as shown in Fig. 11, the max magnitude of the displacements are around 60 px, thus this search area size can be chosen as an integer larger than 60. If the chosen FFT search area size is too small, for example, 30 or 50, the computed initial displacements will be super noisy (see Fig. 10(a-b)). If this happens, the user needs to keep increasing the search area size until a good initial guess is found. For example, search area size

equals 60 in Fig. 10(c).

```
1 --- What is your initial guess search zone size (pixels)? ---
2 User should start to try a small integer value, and gradually increase the
3 value of the search zone size until it is larger than the magnitudes of
4 |disp u| and |disp v|.
5 Input here: 60
6 Finish initial guess search!
```

To further speed up the above FFT-based optimization process, we also provide a multigrid **Method 0** based on a constructed image pyramid (see Fig. 9(e)). There is no need to provide the FFT search area size in this Method. For example, an initial guess of the above heterogeneous fracture deformation is solved as shown in Fig. 11.

5.2 Remove noise in computed initial guess

In practice, FFT-based method solved initial guess² may have large noise, user can further remove these bad points by applying a median filter, setting a q-factor threshold and setting both upper and lower bounds of displacements. User can also continue to remove bad points by directly clicking them on each image frame and then press "Enter" key. However, this step needs some manual work. If this noise removal is necessary, user could follow these steps and guide lines will display on MATLAB command window automatically:

```
1 Do you clear bad points by setting upper/lower bounds once more?
2 (0=yes; 1=no)
```

If user inputs "0", then he/she will be asked to set the upper and lower bounds of the x- and y-displacements. If user inputs "1", this process will be skipped.

```
1 % ===== Find bad initial guess points manually by setting bounds =====
2 What is your upper bound for x-displacement?
3 What is your lower bound for x-displacement?
4 What is your upper bound for y-displacement?
5 What is your lower bound for y-displacement?
6 Do you clear bad points by setting upper/lower bounds? (0=yes; 1=no)
```

Besides setting upper and lower bounds to remove local bad points, we can continue to remove bad points by clicking them directly.

```
1 % ===== Find bad initial guess points manually =====
2 'Do you clear bad points by directly pointing x-disp bad points?
3 (0=yes; 1=no)';
4 'Do you clear bad points by directly pointing y-disp bad points?
5 (0=yes; 1=no);'
```

²Some designed filters or iterative deformation method (IDM) can further improve the accuracy of initial guess, cf [16, 17, 18, 19]. These are beyond the scope of this code manual. Additionally, accuracy of computed initial guess will further be improved after executing ALDIC ADMM iterations (code Sections 4-6).

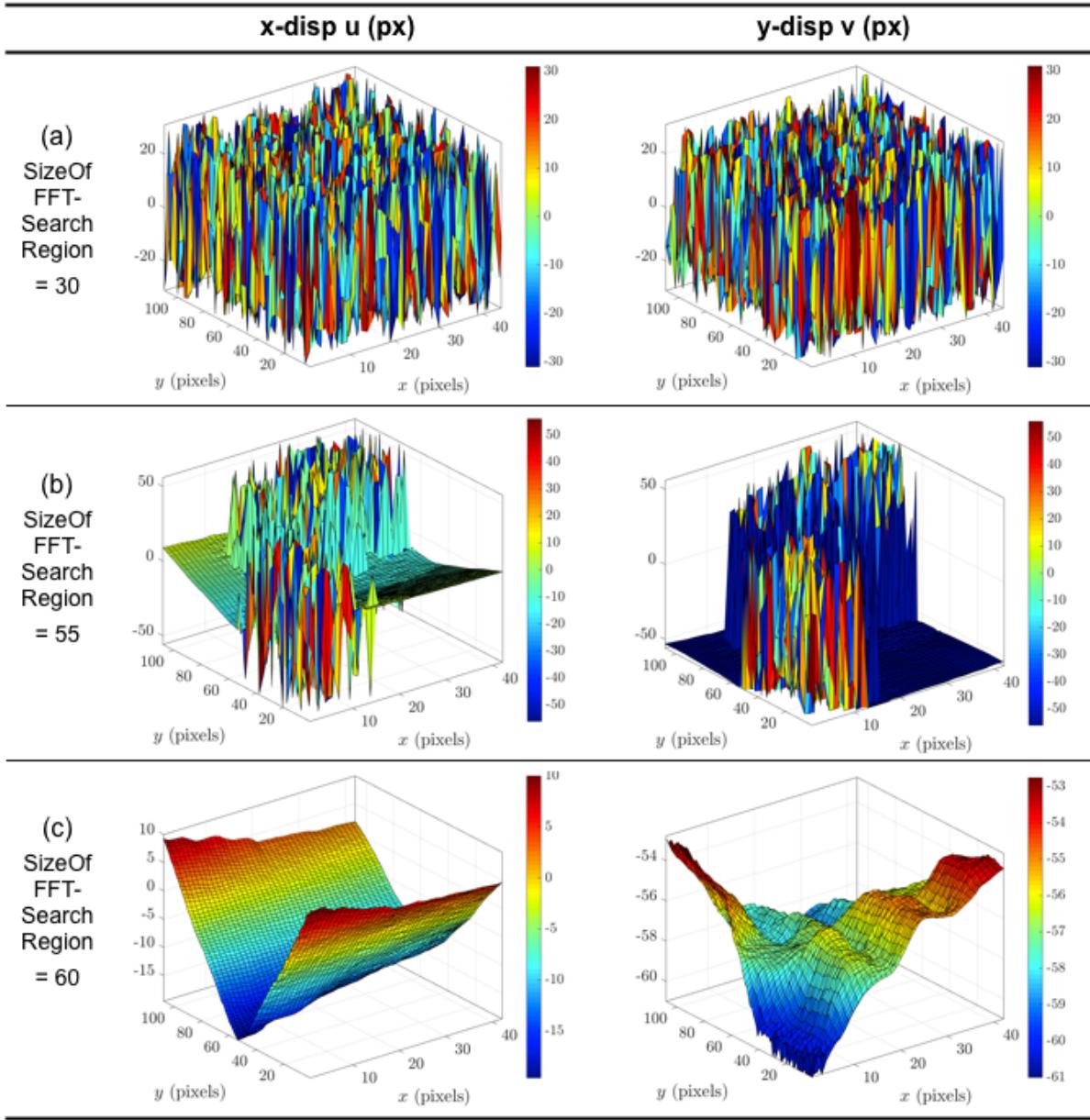


Figure 10: **Method 1** solved displacement fields by the FFT-based cross correlation method with different search area sizes. (a-b) If the chosen search area size is too small, the computed initial displacements will be super noisy. (c) A good search area size should not be smaller than the magnitude of the displacement components ($|dispu|$, $|dispv|$).

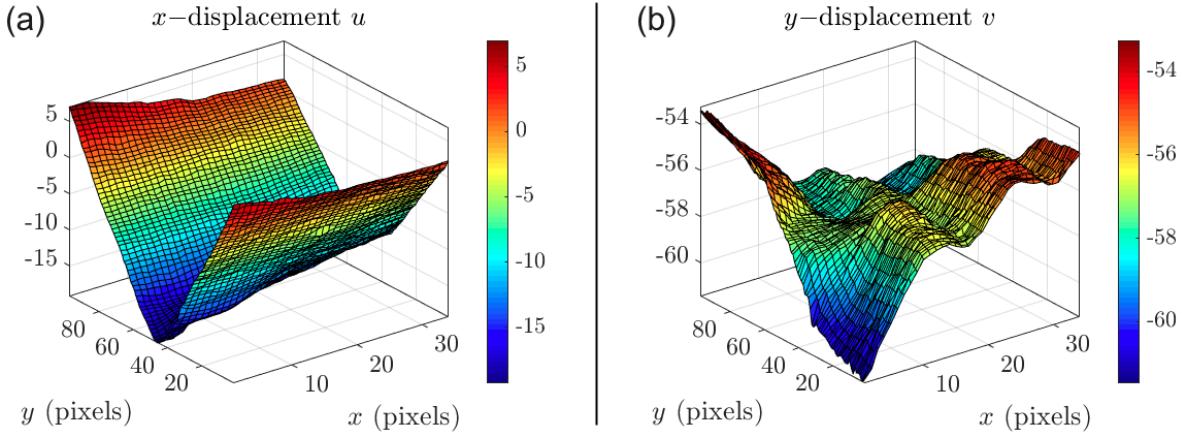


Figure 11: **Method 0** – Multigrid FFT-based cross correlation solver – computed initial guess of the unknown displacement field.

Directly clicking all the bad points and then press "Enter" key.

At the end of this section, a finite element mesh will be generated automatically.

```
1 --- Finish setting up mesh and assigning initial value! ---
```

Image pixel grayscale value gradients will also be computed very fast using finite difference operator and convolution operations.

```
1 --- Start to compute image gradients ---
2 --- Computing image gradients done ---
```

6 Code Section 4: ALDIC Subproblem 1 (first local step)

6.1 Local subset IC-GN solver

In this section, we solve ALDIC ADMM Subproblem 1 (local step) using IC-GN (inverse compositional Gauss Newton) scheme, where distributed parallel computing has been implemented. Execute this section, IC-GN solver will work and a wait-bar will pop out automatically and allows user to visualize when a program will finish solving ALDIC Subproblem 1, see Fig. 12. If you have very large DIC images, MATLAB parpool may spend several minutes before initializing the parallel computing and transferring all the image dataset. So please don't worry and wait a little bit more if you don't see a waitbar pops out immediately.

```
1 ----- Section 4 Start -----
2 **** Start step1 Subproblem1 ****
3 Local ICGN bad subsets %:
4 Elapsed time is xxx seconds.
```

(a) ----- Section 4 Start -----
***** Start step1 Subproblem1 *****
Starting parallel pool (parpool) using the 'local' profile ...
connected to 4 workers.

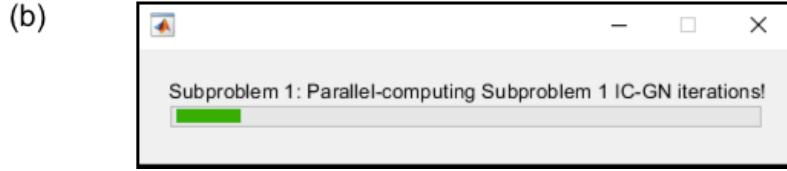


Figure 12: (a) Code section 4 starts and 4 threads are connected. (b) A waitbar pops out and allows user to visualize when a program will finish solving ALDIC Subproblem 1.

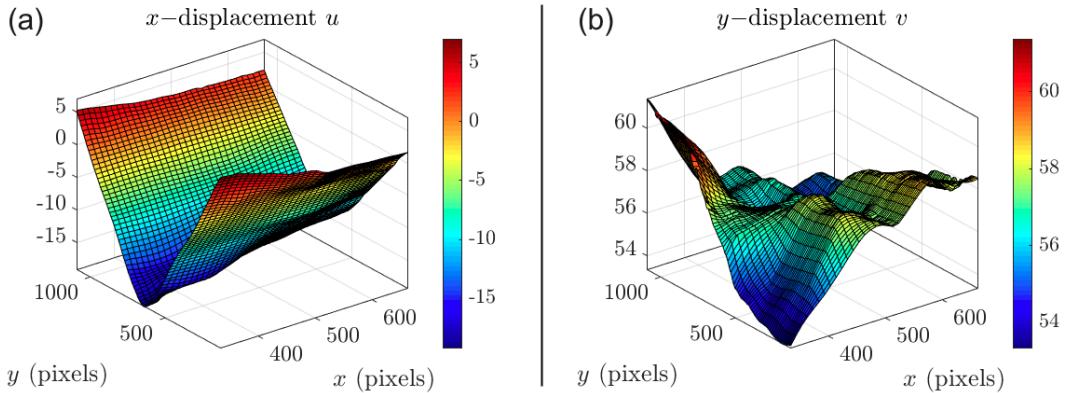


Figure 13: Solved displacements from ALDIC subproblem 1 first local step.

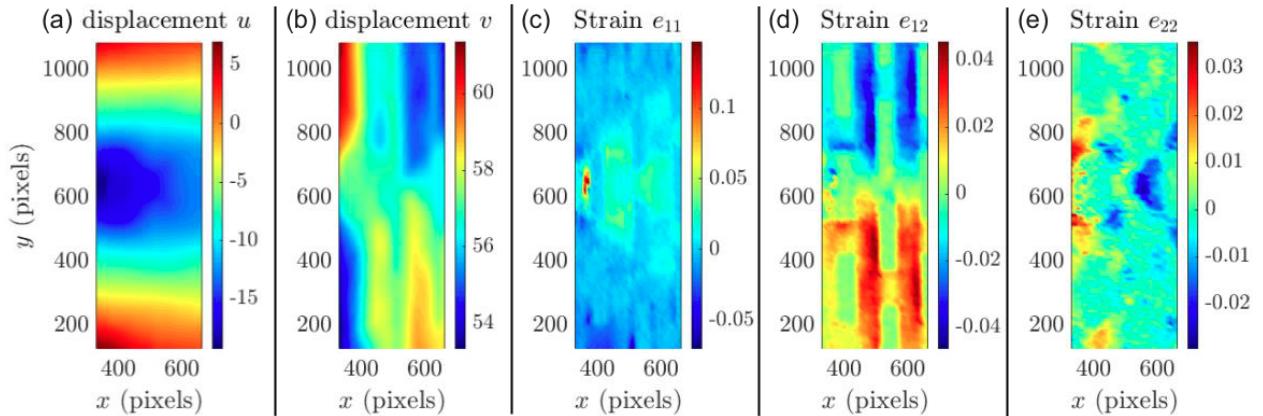


Figure 14: Contour plots of solved displacements and infinitesimal strains from ALDIC subproblem 1 first local step.

6.2 Remove results of IC-GN bad subsets

When this step finishes, a report will display on the command window. Same as Section 5, user can further remove these bad points by applying a median filter, setting a q-factor threshold and setting both upper and lower bounds of displacements. User can also continue to remove bad points by directly clicking them on each image stack and then press "Enter" key.

```

1 --- Start to remove bad local points ---
2 % ===== Find bad Local IC-GN step points manually by pointing them
3 % =====
4 'Do you clear bad points by directly clicking x-disp bad points? (0=yes;
5   1-no)';
6 'Do you clear bad points by directly clicking y-disp bad points? (0=yes;
7   1-no)';
8
9 % ===== Find bad guess points manually by setting bounds =====
10 What is your upper bound for x-displacement? % User input
11 What is your lower bound for x-displacement? % User input
12 What is your upper bound for y-displacement? % User input
13 What is your lower bound for y-displacement? % User input
14 Do you clear bad points by setting upper/lower bounds? (0=yes; 1-no) %
  Input "0" if you want to redo it.
15
16 --- Remove bad points done ---
17 ----- Section 4 Done -----

```

Comment: If you see this error message:

```

1 'Insufficient data for surface estimation.', '[uitemp] = gridfit(
2   coordinatesFEM(notnanindex,1), coordinatesFEM(notnanindex,2),U(2*
3     notnanindex-1), Coordxnodes, Coordynodes,'regularizer','springs');
4 uitemp = uitemp'; and 'main_ALDIC (line 106) LocalICGN(U0,DICmesh.
5   coordinatesFEM,Df,fNormalized,gNormalized,DICpara,'GaussNewton',tol);'.

```

Or if you see that all the local subsets diverged, in another word, the percentage of local ICGN bad subset is 100%, this usually happens because of the bad DIC parameter choice or bad DIC pattern quality. Here are a few steps that may help fix the poor convergence.

- (i) **Check DIC parameter.** In ALDIC code Section 2, each subset is expected to have enough features to track their deformations. With larger subset size ("DICpara.winsize"), the ALDIC ADMM Subproblem 1 will have higher convergence ratio. Typically, we hope there are at least 3~5 features (e.g. speckle dots, short fibers) within each local subset. Besides subset size, considering both computation speed and accuracy, the distance between neighboring subsets (DICpara.winstepsiz) is recommended to be 0.25~1 times of "DICpara.winsize".
- (ii) **Plot and check initial guess** Check the initial guess (around main_ALDIC.m lines 56 64), plot initial guess U0 to see whether this initial guess makes sense or not:
`U0 = Init(u,v,cc.max,DICmesh.x0,DICmesh.y0,0); PlotuvInit;`
 Here the newest version of code is to use the multiscale-method to search the initial guess in an adaptive way (line 60, `IntegerSearchMg`). If this initial guess doesn't look great, please

replace line (60) with line (58), where you can manually define the fixed size of FFT-search zone, also please make sure the size of search zone (`DICpara.SizeOfFFTSearcRegion`) is larger than the magnitude of displacement vector ($\max(u_x, u_y)$) if you are using non-multiscale " `IntegerSearch` ".

- (iii) **Change parameters of IC-GN solver.** For example, increasing `DICpara.Subpb1ICGNMaxIterum`, or decreasing `DICpara.ICGNtol` may help improve IC-GN iteration convergence.
- (iv) After checking above (i-iii), user can re-run code Section 4 to see whether most local subsets get convergence or not. User could uncomment `LocalICGN.m`: line 35 (`ClusterNo=1`) or line 58 `ClusterNo>1` to display this info message on the command window (However, this will slow down the code). So please still comment that display line after you fix this issue.

7 Code Section 5: ALDIC Subproblem 2 (first global step)

After solving ALDIC ADMM Subproblem 1 local step, we run Subproblem 2 global step in this section. Both finite difference and finite element methods are implemented. For uniform regular grid meshes, both finite difference and finite element method work very well. For arbitrary mesh, finite element method is much easier to implement and can be easily implemented with adaptive mesh technique (*adapt-ALDIC* code will be available soon).

7.1 Finite difference method

To solve Subproblem using finite difference method, a sparse finite difference operator **D** is generated by `funOperator3` satisfying:

$$\{\mathbf{F}\} = \mathbf{D}\{\mathbf{u}\} \quad (3)$$

where **u** is displacement, and **F** is “deformation gradient tensor minus identity”, whose associated components related to node (i) are

$$\begin{aligned} \mathbf{F} &= \left\{ \dots, F_{11}^{(i)}, F_{21}^{(i)}, F_{12}^{(i)}, F_{22}^{(i)}, \dots \right\}^T \\ \mathbf{u} &= \left\{ \dots, u_1^{(i)}, u_2^{(i)}, \dots \right\}^T \end{aligned} \quad (4)$$

```

1 ----- Section 5 Start -----
2 ***** Start step1 Subproblem2 *****
3 Assemble finite difference operator D
4 Elapsed time is xxxxxx seconds.
5 Finish assembling finite difference operator D
6 ***** Start step1 Subproblem2 *****
7 Elapsed time is xxxxxx seconds.
8 ----- Section 5 Done -----

```

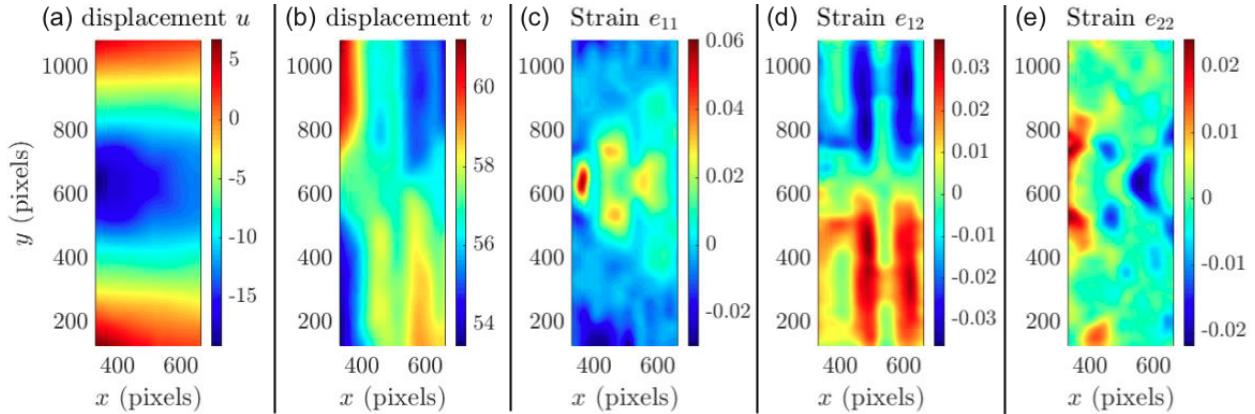


Figure 15: Contour plots of solved displacements and infinitesimal strains from ALDIC subproblem 2 first global step.

7.2 Finite element method

Subproblem 2 can also be solved using finite element method. In this code, we use an 4-node quadrilateral (Q4) uniform finite element, the same element type with our global DIC code (finite element based global DIC code is also available at MATLAB File Exchange [4]).

7.3 Comparison between finite difference and finite element methods in solving Subproblem 2

For uniform regular grid meshes, both finite difference and finite element method solve Subproblem 2 very well and provide accurate results. In practice, finite difference method behaves a little bit better than finite element method in the computation speed and has better accuracy near the ROI boundary. However, finite element method is much easier to implement with an arbitrary mesh, and can be easily combined with adaptive mesh technique.

8 Code Section 6: ALDIC ADMM iterations

Then ALDIC will do alternating direction method of multipliers (ADMM) iterations in this section. The tolerance threshold of ADMM iteration is set to be `1e-4` by default. But this threshold can be manually adjusted by modifying `tol2` in code Section 6. In practice, ALDIC ADMM usually converges within 3~5 iterations for the update less than `1e-2` px. For example, in the heterogeneous fracture example, ADMM iteration converges at the sixth step, but there is almost no difference between the solution at the third step, see Fig. 16.

```

1 ----- Section 6 Start -----
2 ***** Start step2 Subproblem1 *****
3 Local step bad subsets total # is: 0

```

```

4 Elapsed time is 6.078113 seconds.
5 ***** Start step2 Subproblem2 *****
6 Elapsed time is 0.007632 seconds.
7 Update local step = 0.018188
8 Update global step = 0.028851
9 ****
10
11 ***** Start step3 Subproblem1 *****
12 Local step bad subsets total # is: 0
13 Elapsed time is 6.036331 seconds.
14 ***** Start step3 Subproblem2 *****
15 Elapsed time is 0.007165 seconds.
16 Update local step = 0.0027562
17 Update global step = 0.025722
18 ****
19
20 ***** Start step4 Subproblem1 *****
21 Local step bad subsets total # is: 0
22 Elapsed time is 5.937378 seconds.
23 ***** Start step4 Subproblem2 *****
24 Elapsed time is 0.007198 seconds.
25 Update local step = 0.0019051
26 Update global step = 0.0018707
27 ****
28
29 ***** Start step5 Subproblem1 *****
30 Local step bad subsets total # is: 0
31 Elapsed time is 5.892075 seconds.
32 ***** Start step5 Subproblem2 *****
33 Elapsed time is 0.007503 seconds.
34 Update local step = 0.00015808
35 Update global step = 0.00027234
36 ****
37
38 ***** Start step6 Subproblem1 *****
39 Local step bad subsets total # is: 0
40 Elapsed time is 6.106291 seconds.
41 ***** Start step6 Subproblem2 *****
42 Elapsed time is 0.007841 seconds.
43 Update local step = 2.2404e-05
44 Update global step = 4.8639e-05
45 ****

```

9 Code Section 7: Check convergence

This section is able to check convergence of ALDIC ADMM iterations and delete some temporary variables in the workspace. This section can be skipped if you don't need it.

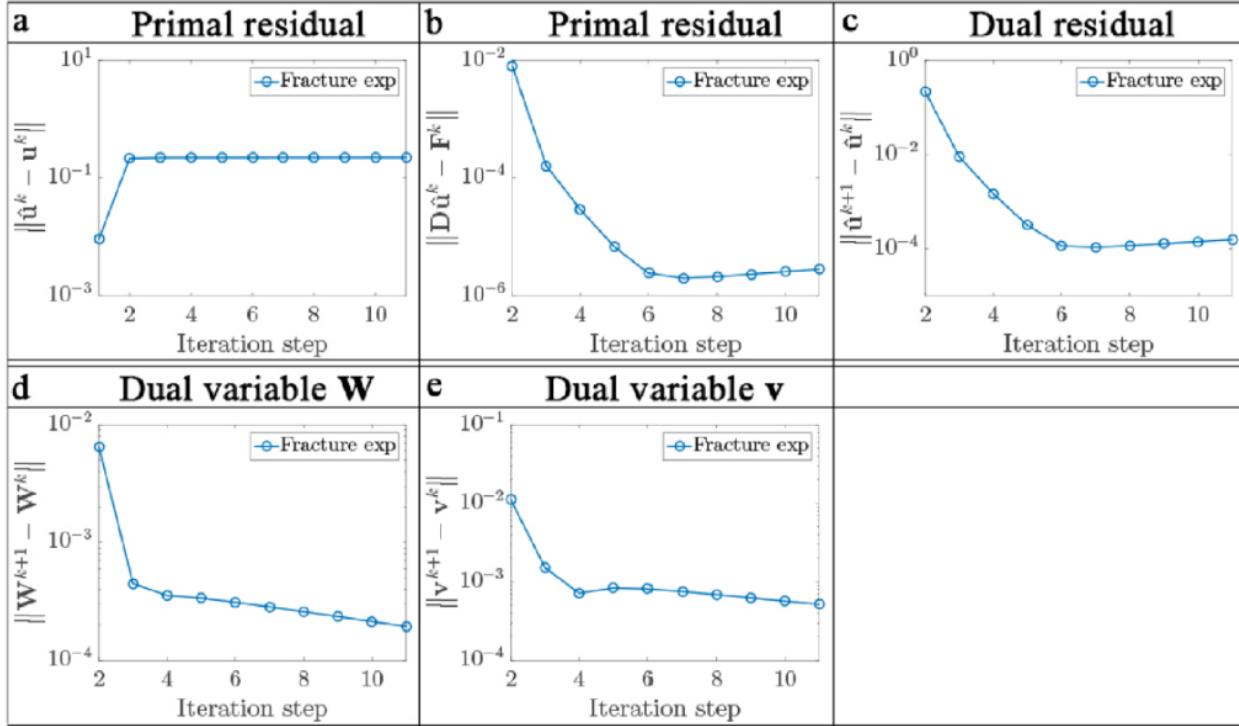


Figure 16: ALDIC ADMM algorithm usually converges after 3~5 iterations.

10 Code Section 8: Compute strains

10.1 Smooth displacement field if needed

Before computing strains, solved displacement fields can be further denoised if necessary. In most cases, ALDIC solved displacement fields are already denoised, so usually there is no need to further smooth solved displacement fields anymore.

```

1 ----- Section 8 Start -----
2 Do you want to smooth displacement? (0-yes; 1-no)1

```

If you put in “ 0 ”, a Gaussian smooth filter with standard deviation 0.5 will be applied. If a stronger smoothing filter is needed, please edit the Gaussian filter parameters “ `DispFilterSize` ”, and “ `DispFilterStd` ”.

10.2 Compute strain field

In ALDIC, “deformation gradient minus identity” \mathbf{F} is a direct output besides displacement field. Strain field can also be computed from numerically differentiating solved displacement field. We implement total four methods of computing the strain field.

- (0) First method is a **direct output** from ALDIC solved \mathbf{F} (deformation gradient minus identity);

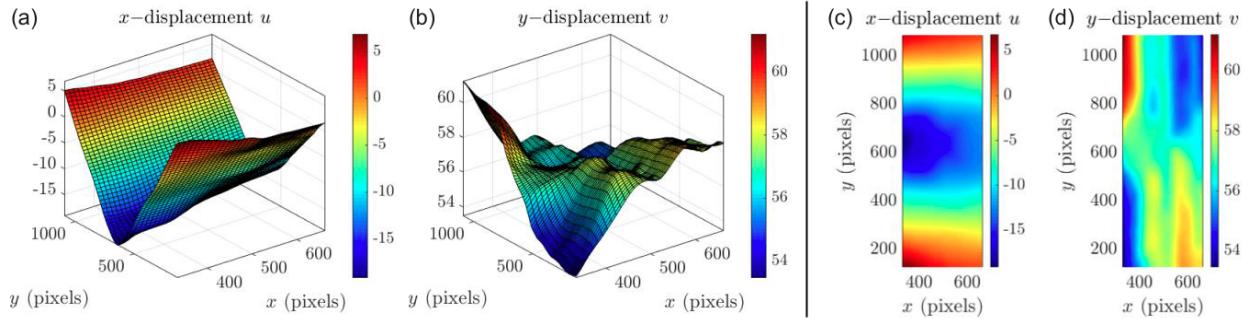


Figure 17: ALDIC solved displacement fields are much already denoised and usually there is no need to apply additional smoothing filters anymore.

- (1) **Central finite difference** of solved displacement field;
- (2) **Plane fitting method** to differentiate solved displacement field. In this method, you will be asked to input half plane width to define the size of the fitted plane;
- (3) Strain field can also be computed from **finite element Gauss points**.

```

1 What method to use to compute strain?
2   0: Direct output from ALDIC;
3     1: Finite difference(Recommended);
4     2: Plane fitting;
5     3: Finite element;
6 Input here: 1

```

If user inputs “ 2 ”, MATLAB command window will display following lines for continuing to define the plane size:

```
1 What is your half window size: % Input half window size for plane fitting
```

Three popular types of strains are implemented: infinitesimal strain, Eulerian strain based on deformed configuration, and finite Green-Lagrangian strain.

```

1 Infinitesimal stran or finite strain?
2   0: Infinitesimal stran;
3     1: Eulerian stran;
4     2: Green-Lagrangian stran;
5     3: Others: code by yourself;
6 Input here: 0

```

E.g., the comparison between different methods to compute strain fields for heterogeneous fracture data set is shown in Fig. 18, where results of this example are stored in “ ./results_frac_heter/ ”.

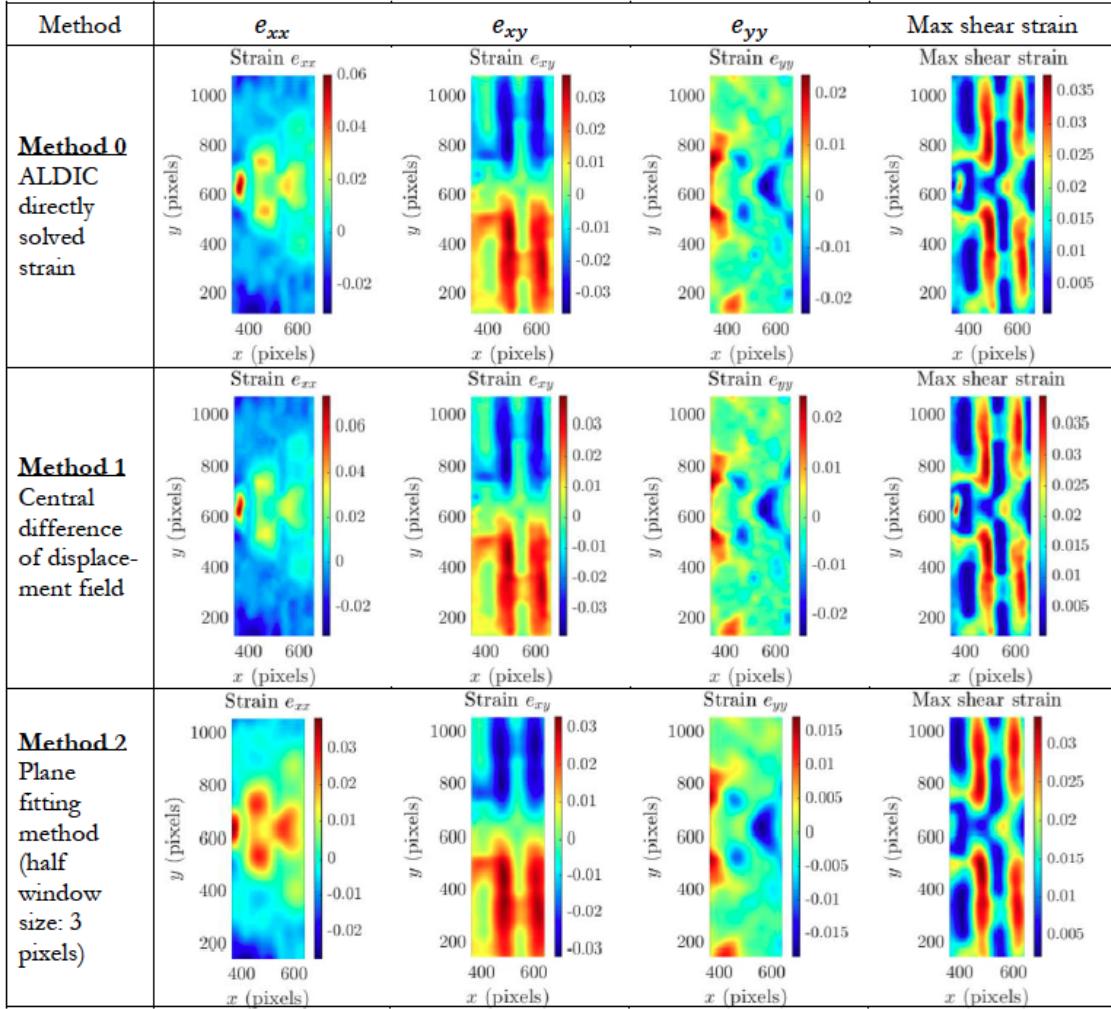


Figure 18: Comparison between different methods to compute strain fields for heterogeneous fracture example.

10.3 Plot and save results

Then the user will be asked to choose to plot the solved deformation fields on the undeformed, reference configuration (see Fig. 19), or to plot over the current, deformed configuration (see Fig. 20).

```

1 Over which image(s) you want to plot the results?
2   0: First image only (if you are using only two images);
3   1: Second and next images;
4 Input here: 1
5
6 Save figures into the format:
7 1: jpg(Choose transparency 0~1)
8 2: pdf(Choose transparency = 0)
9 3: Others: Edit codes in ./plotFiles/SaveFigFiles.m
10 Input here:
```

In the current version of the ALDIC code, to overlay with the original DIC images, final plots can only be saved in the “jpg” format.

```

1 Define transparency for overlaying original images:
2 Input a real number between 0(Only original images)
3 and 1(Non-transparent deformation results).
4 Input here(e.g. 0.5):

```

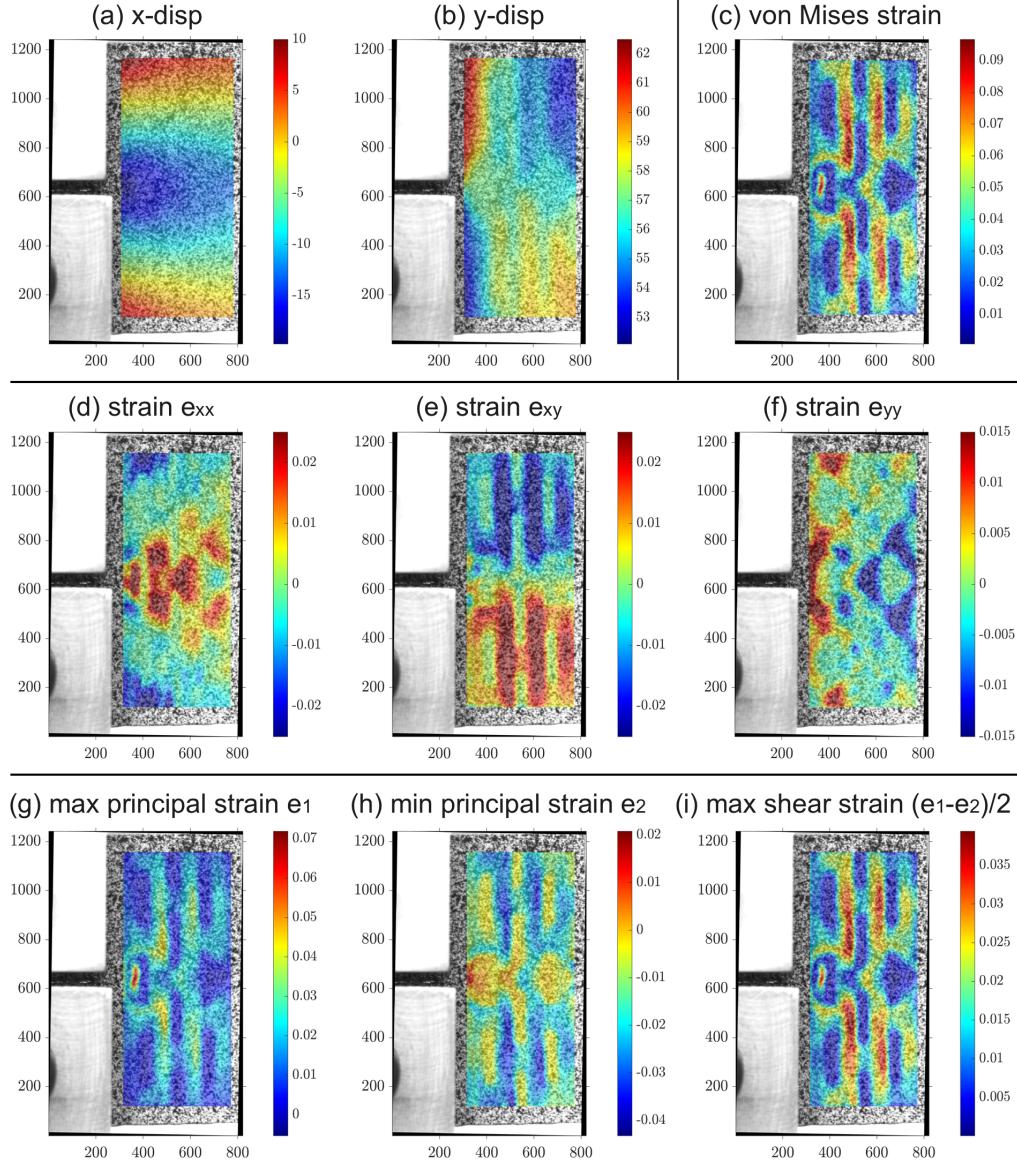


Figure 19: ALDIC tracked deformation fields with the original reference, undeformed image overlaid as the background image. (a-b) Solved x- and y-displacements (units: px). (c-i) Solved strain components: equivalent von Mises strain, e_{xx} , e_{xy} , e_{yy} , maximum principal strain, minimum principal strain, and maximum shear strain.

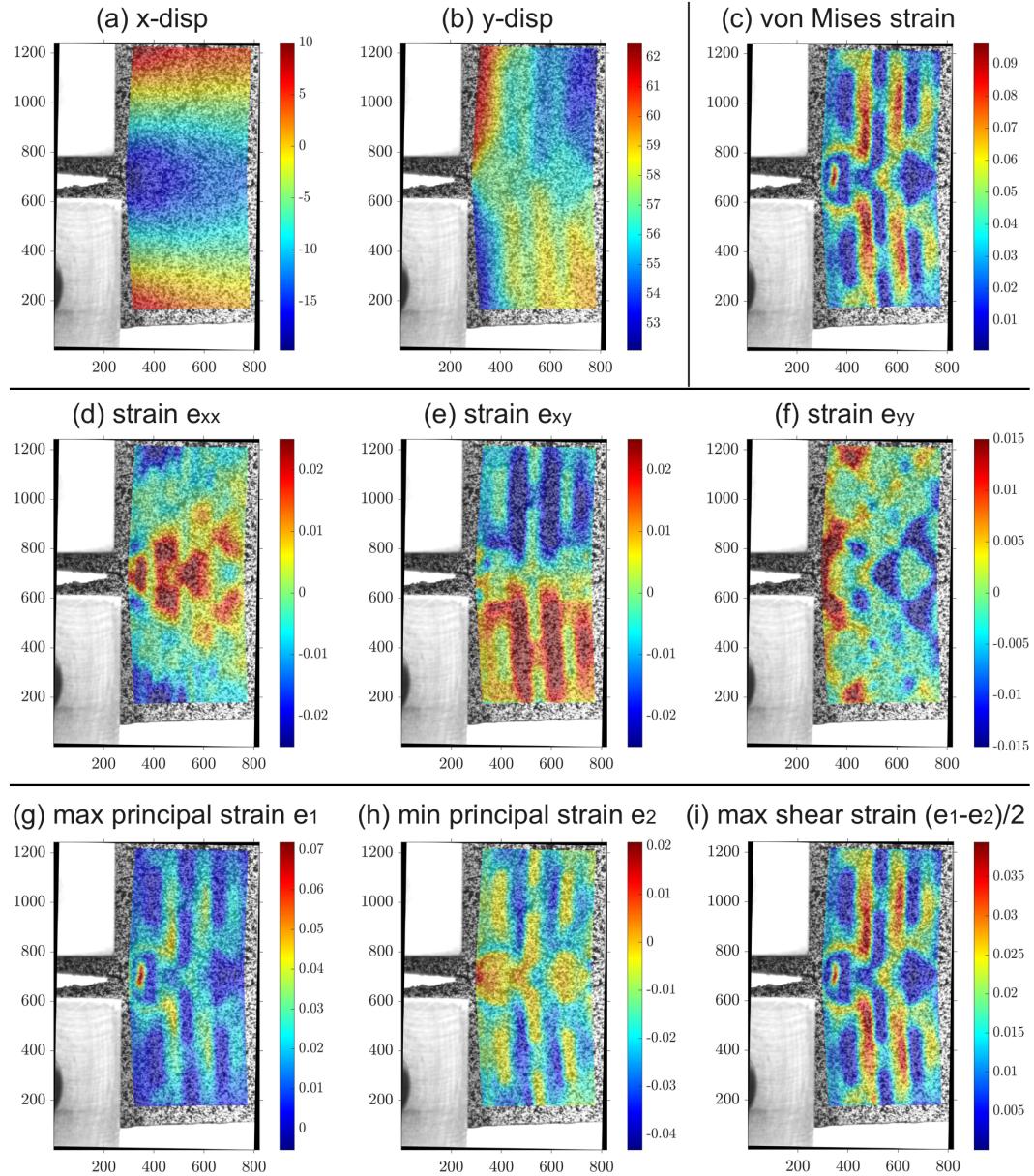


Figure 20: ALDIC tracked deformation fields with the current, deformed image overlaid as the background image. (a-b) Solved x- and y-displacements (units: px). c-i) Solved strain components: equivalent von Mises strain, e_{xx} , e_{xy} , e_{yy} , maximum principal strain, minimum principal strain, and maximum shear strain.

Finally, don't forget to save results for future use. All the DIC results will be saved in a Matlab matfile by executing following codes, see Table 3.

```

1 results_name = ['results_', imgname, '_ws', num2str(DICpara.winsize), '_st',
    num2str(DICpara.winstepsizesize), '.mat'];
2 save(results_name, 'file_name', 'DICpara', 'DICmesh', 'ResultDisp', '
```

Table 3: Summary of DIC results

Variable	Description
DICpara	ALDIC parameters, see Table 1
DICmesh	ALDIC finite element mesh details, see Table 2
ResultDisp	Solved displacements from ALDIC
ResultDefGrad	Solved “deformation gradient minus identity” \mathbf{F} from ALDIC
ResultStrain	Solved strains from ALDIC code Section 8
ResultFEMesh	Stored FE-mesh due to reference frame update in incremental mode
ALSub1Time, ALSub2Time	Computation time in solving Subproblems 1 & 2

```

3      ResultDefGrad', 'ResultStrain', 'ResultFEMesh', ...
      'ALSub1Time', 'ALSub2Time', 'ALSolveStep');

```

11 Summary of all the MATLAB command window screen outputs in the heterogeneous fracture case study

This simulation is performed in MATLAB (R2018b, 64-bit) and on a workstation with an Intel i5-4670X CPU with a base clock of 3.40 GHz (4 threads), 16.0 GB memory, and run under Windows 10.

```

1 ----- Section 1 Start -----
2 Building with 'MinGW64 Compiler (C++)'.
3 MEX completed successfully.
4 ----- Section 1 Done -----
5
6 ----- Section 2 Start -----
7 Choose method to load images:
8   0: Select images folder;
9   1: Use prefix of image names;
10  2: Manually select images.
11 Input here: 2
12 --- Please load first image ---
13 --- Please load next image ---
14 Do you want to load more deformed images? (0-Yes; 1-No)1
15
16 --- Define ROI corner points at the top-left and the bottom-right ---
17 Coordinates of top-left corner point are (306.000,76.000)
18 Coordinates of bottom-right corner point are (784.000,1128.000)
19
20 --- What is the subset size? ---
21 Input here: 20
22 --- What is the subset step? ---
23 Input here: 10
24
25 --- Method to solve ALDIC global step Subproblem 2 ---
26  1: Finite difference(Recommended)

```

```

27      2: Finite element method
28 Input here: 1
29
30 --- Set up Parallel pool ---
31 How many parallel pools to open? (Put in 1 if no parallel computing)
32 Input here: 2
33 ----- Section 2 Done -----
34
35 Current image frame #: 2/2
36
37 ----- Section 3 Start -----
38
39 --- Method to compute an initial guess of displacements ---
40     0: Multigrid search based on an image pyramid
41     1: Whole field search for all the subsets
42     2: Search near manually clicked seeds and then interpolate for the
        full-field
43 Input here: 0
44
45 Finish setting up mesh and assigning initial value!
46 --- Start to compute image gradients ---
47 --- Computing image gradients done ---
48 ----- Section 3 Done -----
49
50 ----- Section 4 Start -----
51 **** Start step1 Subproblem1 ****
52 --- Set up Parallel pool ---
53 Local ICGN bad subsets %: 130/5088=2.555%
54 Elapsed time is 9.112371 seconds.
55 --- Start to manually remove bad points ---
56 Do you clear bad points by setting upper/lower bounds once more? (0=yes;
    1=no)
57 Input here: 1
58 Do you clear bad points by directly pointing x-disp bad points? (0=yes; 1-
    no)
59 Input here: 0
60 Do you point out more x-disp bad points? (0=yes; 1-no) Input:
61 Do you clear bad points by directly pointing y-disp bad points? (0=yes; 1-
    no)
62 Input here: 0
63 Do you point out more y-disp bad points? (0=yes; 1-no) Input: 0
64 Do you point out more y-disp bad points? (0=yes; 1-no) Input: 1
65 --- Remove bad points done ---
66 ----- Section 4 Done -----
67
68 ----- Section 5 Start -----
69 **** Start step1 Subproblem2 ****
70 Assemble finite difference operator D
71 Elapsed time is 0.377279 seconds.
72 Finish assembling finite difference operator D
73 Elapsed time is 0.542143 seconds.

```

```

74 ----- Section 5 Done -----
75
76 ----- Section 6 Start -----
77 ***** Start step2 Subproblem1 *****
78 Local ICGN bad subsets %: 2/5088=0.039308%
79 Elapsed time is 7.321119 seconds.
80 ***** Start step2 Subproblem2 *****
81 Elapsed time is 0.032743 seconds.
82 Update local step = 0.027113
83 Update global step = 0.062093
84 ****
85
86 ***** Start step3 Subproblem1 *****
87 Local ICGN bad subsets %: 2/5088=0.039308%
88 Elapsed time is 5.284426 seconds.
89 ***** Start step3 Subproblem2 *****
90 Elapsed time is 0.032698 seconds.
91 Update local step = 0.015904
92 Update global step = 0.016563
93 ****
94
95 ***** Start step4 Subproblem1 *****
96 Local ICGN bad subsets %: 2/5088=0.039308%
97 Elapsed time is 5.025842 seconds.
98 ***** Start step4 Subproblem2 *****
99 Elapsed time is 0.042629 seconds.
100 Update local step = 0.010131
101 Update global step = 0.0043174
102 ****
103
104 ***** Start step5 Subproblem1 *****
105 Local ICGN bad subsets %: 2/5088=0.039308%
106 Elapsed time is 4.691986 seconds.
107 ***** Start step5 Subproblem2 *****
108 Elapsed time is 0.038495 seconds.
109 Update local step = 0.0088096
110 Update global step = 0.0033219
111 ****
112
113 ----- Section 6 Done -----
114
115 ----- Section 8 Start -----
116 Do you want to smooth displacement? (0-yes; 1-no)1
117 What method to use to compute strain?
118     0: Direct output from ALDIC;
119     1: Finite difference(Recommended);
120     2: Plane fitting;
121     3: Finite element;
122 Input here: 1
123 Infinitesimal stran or finite strain?
124     0: Infinitesimal stran;

```

```

125      1: Eulerian strain;
126      2: Green-Lagrangian strain;
127      3: Others: code by yourself;
128 Input here: 0
129 Over which image(s) you want to plot the results?
130     0: First image only (if you are using only two images);
131     1: Second and next images;
132 Input here: 1
133 Save figures into the format:
134     1: jpgg(Choose transparency 0~1)
135     2: pdf(Choose transparency = 1)
136     3: Others: Edit codes in ./plotFiles/SaveFigFiles.m
137 Input here: 1
138 Define transparency for overlaying original images:
139 Input a real number between 0(Only original images)
140 and 1(Non-transparent deformation results).
141 Input here(e.g., 0.5): .5
142 Current image frame #: 2/2
143 ----- Section 8 Done -----

```

12 Frequently Asked Questions (FAQs)

12.1 About MATLAB mex set up

[1] Where do I install TDM-gcc compiler?

This is a general question for MATLAB users to apply mex C/C++ compilers where more details can be found online, e.g., [13, 14]. User needs to install suitable mex C/C++ compiler based on their OS and MATLAB versions. Based on my experience, mex C/C++ compiler usually is already installed on MAC system. For Windows users, I usually install the TDM-gcc compiler to the first level main disks like “C:, D:, E:”, and higher level disks (like “C/FirstLevel/SecondLevel...”) may not work. If you have better suggestions, I appreciate you could let me know.

[2] What if I don't want to use mex compiler or “ba_interp2”? Can I still use ALDIC?

You can still use ALDIC MATLAB code! You need to convert the “ba_interp2” to other MATLAB interpolation functions. This change needs to be done in the “./func/funICGN.m” and “./func/funICGN_Subpb1.m” to change the “interpmethod”. For example, you can replace the function “ba_interp2” with “interp2”, where interp2 is the MATLAB built interpolation function. You can also try other interpolation schemes too. Based on my experience, “ba_interp2” runs much faster than the MATLAB built cubic interpolation script. So if “ba_interp2” still can be used, maybe that will be the best solution.

12.2 About MATLAB parallel computing

[3] Where to set up parallel computing preferences?

First, you need to make sure you have already installed parallel computing toolbox. Then all the parallel computing toolbox preferences can be modified by user as shown in Fig. 8. The maximum number of workers in a parallel pool is also limited by the computer hardware.

[4] What if I don't have a parallel computing toolbox?

You can easily install this toolbox. If you don't want to apply parallel computing, ALDIC still runs very fast, and the parameter "DICpara.ClusterNo" needs to be set as "0" or "1".

12.3 About ALDIC algorithm

[5] MATLAB reports error “Undefined function or variable”

Please make sure you have already added all the subfolders ('./src','./plotfiles', ...) on the MATLAB path.

[6] Which image to load as the first image?

One comment for the ALDIC code is that it always manipulate the deformed images and tries to transform them back to the reference image to compute their deformation fields which is based on the *Lagrangian* description. If user wants to track the deformation field in the *Eulerian* description, user can select the reference image as the second image, and select the deformed image as the first image and manipulate the reference image to transform to the current deformed image.

[7] Error “Insufficient data for surface estimation” when using “gridfit”

This problem usually happens when there are too few local subsets get convergence in Subproblem 1 (local step), which breaks down the gridfit function. Could you please check the bad subsets percentage which is expected to display on the command window. For example, if you have almost 100% bad subsets which means all the subsets are not converged in Subpb 1.

Here are three possible steps to help fix this problem. (i) In code Section 2, Check DIC parameters: subset size (DICpara.winsize). In 2D-DIC, each subset is expected to have at least 3~5 features (e.g. speckle dots) in your DIC pattern. Usually with larger subset size, Subpb 1 will have better convergence. But this subset size cannot be too large since it will also decrease the DIC overall spatial resolution. Theoretically, distance between neighboring subsets can be an arbitrary integer. Considering speed and accuracy, I recommend the winstepsize (DICpara.winstepsizes) to be (0.25~1)* winsize

(ii) In code Section 3, Check the initial guess (around main_ALIDC.m line 56 ~ 64), plot initial guess \mathbf{U}_0 to see whether this initial guess makes sense or not:

`U0 = Init(u,v,cc.max,DICmesh.x0,DICmesh.y0,0); PlotuvInit;` . The newest version of code is to use the multiscale method (line 60, “`IntegerSearchMg`”), if this initial guess doesn’t look great, please replace line (60) with line (58), where you can manually define the size of FFT-search zone.

(iii) After checking (i-ii), please re-run code Section 4. To see whether each local subset obtains convergence or not, you can uncomment (`LocalICGN.m`: line 35 (`ClusterNo=1`) or line 58 (`ClusterNo>1`)) to display this info message on the command window (However, this will slow down the code). If there are still problems or most subsets still get converged. Feel free to report the error message or send a pair of your image samples to (`aldicdvc@gmail.com`).

[8] How to compute other types of strains?

(i) What ALDIC solved are displacements ($\mathbf{U} = \mathbf{x}(\mathbf{X}) - \mathbf{X}$) and deformation gradients ($\mathbf{F} = \nabla_{\mathbf{X}}\mathbf{U} = \nabla_{\mathbf{X}}\mathbf{x} - \mathbf{I}$), where \mathbf{x} and \mathbf{X} are the coordinates in deformed and reference configurations, respectively. \mathbf{U} is stored in `ResultDisp`, and \mathbf{F} is stored in `ResultDefGrad`.

In ALDIC, length of the displacement \mathbf{U} vector is two times of the row number of the FE-mesh coordinates (`ResultFEMesh{1}.coordinatesFEM`), and \mathbf{U} vector is assembled by the nodal displacements $[u^1, v^1, u^2, v^2, \dots, u^N, v^N]^T$ with nodal indices as $[1, 2, \dots, N]$. The length of \mathbf{F} vector is four times of the row number of FE-mesh coordinates and is assembled in the order of $[F_{11}, F_{21}, F_{12}, F_{22}, F_{11}, F_{21}, F_{12}, F_{22}, \dots, F_{11}^N, F_{21}^N, F_{12}^N, F_{22}^N]^T$.

(ii) With ALDIC solved displacements \mathbf{U} and deformation gradients \mathbf{F} , we compute strains in code Section 8. For example, if strains are small ($< 5\%$), infinitesimal strain is a good option, where $e_{xx} = F_{11}$, $e_{yy} = F_{22}$, $e_{xy} = 0.5(F_{12} + F_{21})$. For engineering shear strains, $E_{xx} = F_{11}$, $E_{yy} = F_{22}$, $E_{xy} = (F_{12} + F_{21})$. With computed \mathbf{F} or `FStraintemp`, other types of finite strain measurements can be easily computed based on user’s choice.

(iii) Comment about difference between \mathbf{F} and `FStraintemp` . In code Section 8, you will be also be asked:

- 1 What method to use to compute strain?
- 2 0: Direct output from ALDIC;
- 3 1: Finite difference(Recommended);
- 4 2: Plane fitting;
- 5 3: Finite element;

If you choose method 0, `FStraintemp` is exactly the same with computed \mathbf{F} ; If you choose method 1-3, the `FStraintemp` is re-computed from the computed u . The size of `FStraintemp` is cropped by “Rad” since the strains near the edges are less accurate. The coordinates of `FStraintemp` are: $x0(1+Rad:M-Rad,1+Rad:N-Rad)$, $y0(1+Rad:M-Rad,1+Rad:N-Rad)$.

Acknowledgements

I am grateful to Dr. Louisa Avellar for sharing her unpublished images of fracture. I want to thank Prof. Can C. Aydiner, Dr. Alex K. Landauer and Jialiang Tao, and all the ALDIC users for lots of helpful feedback and discussions. I also want to thank Prof. Jose C. Outeiro for helpful suggestions to improve the ALDIC code to be used in the machining manufacturing processes. I also acknowledge the support of the US Air Force Office of Scientific Research through the MURI grant ‘Managing the Mosaic of Microstructure’ (FA9550-12-1-0458).

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