Note of Matlab File Exchange "PtInTriCheck"

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First, we know that a triangle area can be computed via the coordinates of its three nodal points.

$$\text{Area} = |A| \tag{1}$$

where

$$A = \frac{1}{2}\det\left(\begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}\right) \tag{2}$$

Similarly for higher dimensional cases, for example, in 3D case, volume of a 4-node tetrahedron

$$\text{Vol} = |V| \tag{3}$$

where

$$V = \frac{1}{6}\det\left(\begin{bmatrix} 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 \\ 1 & x_3 & y_3 & z_3 \\ 1 & x_4 & y_4 & z_4 \end{bmatrix}\right) \tag{4}$$

If we dont include the absolute function, and only look at function A, which can take either a positive or a negative value depending on the order of nodal points. for example, as shown in Fig.1 (a), A(pt1,pt2,pt3=A,B,C)=−A(pt1,pt2,pt3=B,A,C). Point "I" is inside triangle ΔABC, no matter A is positive or negative, what is sure is that A(A,B,C) and A(A,B,I) have the same sign. Similarly, A(I,B,C), A(A,I,C) all have same signs with A(A,B,C). If point "O" is outside of the triangle ΔABC, one of such A has different sign with A(A,B,C). For example, in (b), A(A,B,O) has different sign with A(A,B,C).

This result works for arbitrary dimension. So I compute the sum of the absolute differences of these signs to check whether a point is inside or outside a simplex. If all the sub-triangles have the same sign, which means point I is inside the triangle.

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Additional comment is that sometimes, we want to check lots of those points, and applying for-loops in Matlab is super slow. So its better to write everything in the vectorized form, thats what I did in the 2D and 3D case to help write down everything in the vectorized form.

If a point is located at the edges/surfaces, its also easy to do a little additional work to consider that special case. I still havent included that in the current version of this code for now.

There are other methods to achieve this function, too. Basically, this function is to check whether a point belongs to a convex hull of a few scattered points. Hope this helps and if there are still questions/comments or better suggestions. I appreciate you letting me know.

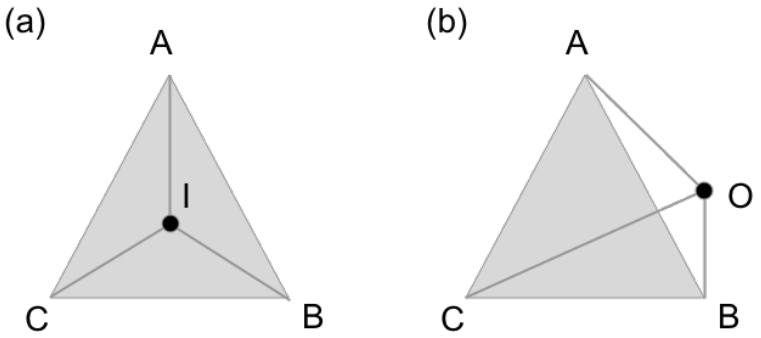

Figure 1