Homework-2

DATA 255 : Deep Learning

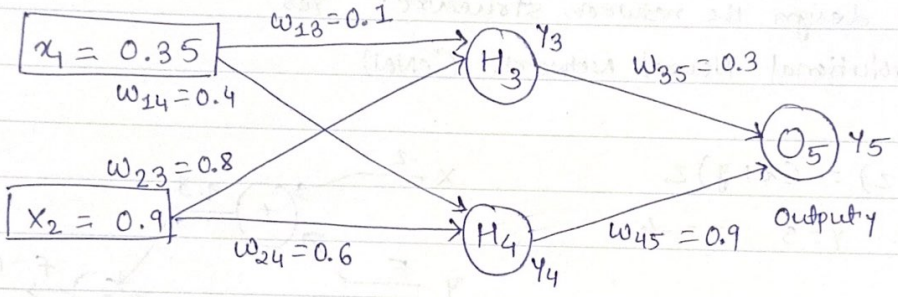
Back Propagation Example

Github Link
https://github.com/jyanimaulik/Deep-Learning/tree/main/Homework%202

Back Propagation Example

→ Assume that the neurons have a sigmoid activation function

→ The actual output of y is 0.5 and learning rate is 1.



⇒ Forward Pass

Calculating output for y_3, y_4 and y_5

we have $a_j = \sum_j (w_{ij} \times x_i)$ $\quad y_j = F(a_j) = \frac{1}{1+e^{-a_j}}$

$a_1 = w_{13} * x_1 + w_{23} * x_2$
$= 0.1 \times 0.35 + 0.8 \times 0.9$
$= 0.755$

$y_3 = f(a_1)$
$= 1/(1+e^{-0.755})$
$= 0.68$

$a_2 = w_{14} * x_1 + w_{24} * x_2$
$= 0.4 \times 0.35 + 0.6 * 0.9$
$= 0.68$

$y_4 = f(a_2)$
$= 1/(1+e^{-0.68})$
$= 0.6637$

$$a_3 = w_{35} * y_3 + w_{45} * y_4$$
$$= 0.3 * 0.68 + 0.9 * 0.6637$$
$$= 0.801$$

$$y_5 = f(a_3) = \frac{1}{(1+e^{-0.801})} = 0.69 \quad (\text{Network output } y_5) = y$$

⟹ Computing Error

$$\text{Error} = y_{target} - y_5$$
$$= 0.5 - 0.69$$
$$= \underline{\underline{-0.19}}.$$

⟹ Backward Pass

Computing Errors δ_3, δ_4 and δ_5

For output layer

$$\delta_5 = y(1-y)(y_{target} - y)$$
$$= 0.69\ (1-0.69)\ (0.5-0.69)$$
$$= -0.0406$$

For hidden layer

$$\delta_3 = y_3(1-y_3)\, w_{35} * \delta_5$$
$$= 0.68 * (1-0.68) * (0.3 * -0.0406)$$
$$= -0.00265$$

$\delta_4 = y_4(1-y_4) w_{45} * \delta_5$

$= 0.6637 * (1 - 0.6637) * (0.9 * -0.0406)$

$= -0.0082$

$\Rightarrow$ Computing new weights

① $\Delta w_{45} = \eta \delta_5 y_4 = 1 * (-0.0406) * 0.6637$

$= -0.0269$

w_{45} (new) $= \Delta w_{45} + w_{45}$ (old)

$= -0.0269 + 0.9$

$= 0.8731$

② $\Delta w_{14} = \eta \delta_4 x_1 = 1 * (-0.0082) * (0.35)$

$= -0.00287$

w_{14} (new) $= \Delta w_{14} + w_{14}$ (old)

$= -0.00287 + 0.4$

$= 0.3971$

③ $\Delta w_{35} = \eta \delta_5 y_3 = 1 * (-0.0406) * 0.68$

$= -0.027608$

Δw_{35} (new) $= \Delta w_{35} + w_{35}$ (old)

$= -0.027608 + 0.3$

$= 0.272392$

④ $\Delta w_{13} = \eta \delta_3 x_1 = 1 * (-0.00265) * (0.35)$

$= -0.0009275$

$w_{13}(new) = \Delta w_{13} + w_{13}(old)$

$= -0.0009275 + 0.1$

$= 0.0990725$

⑤ $\Delta w_{23} = \eta \delta_3 x_2 = 1 * (-0.00265) * 0.9$

$= -0.002385$

$w_{23}(new) = \Delta w_{23} + w_{23}(old)$

$= -0.002385 + 0.8$

$= 0.797615$

⑥ $\Delta w_{24} = \eta \delta_4 x_2 = 1 * (-0.0082) * (0.9)$

$= -0.00738$

$w_{24}(new) = \Delta w_{24} + w_{24}(old)$

$= -0.00738 + 0.6$

$= 0.59262$

Now we have

old weights	Updated weights
$w_{13} = 0.1$	$w_{13} = 0.0991$
$w_{14} = 0.4$	$w_{14} = 0.3971$
$w_{23} = 0.8$	$w_{23} = 0.7976$
$w_{24} = 0.6$	$w_{24} = 0.5926$
$w_{35} = 0.3$	$w_{35} = 0.2724$
$w_{45} = 0.9$	$w_{45} = 0.8731$

⟹ Let's calculate Forward Pass again and compute y_3, y_4 and y_5 output

$$a_1 = w_{13} * x_1 + w_{23} * x_2$$
$$= 0.0991 \times 0.35 + 0.7976 * 0.9$$
$$= 0.7525$$
$$y_3 = f(a_1) = \frac{1}{1+e^{-0.7525}} = 0.6797$$

$$a_2 = w_{14} * x_1 + w_{24} * x_2$$
$$= 0.3971 * 0.35 + 0.5926 * 0.9$$
$$= 0.6723$$
$$y_4 = f(a_2) = \frac{1}{1+e^{-0.6723}} = 0.6620$$

$$a_3 = w_{35} * y_3 + w_{45} * y_4$$
$$= 0.2724 * 0.6797 + 0.8731 * 0.6620$$
$$= 0.7631$$
$$y_5 = f(a_3) = \frac{1}{1 + e^{-0.7631}} = 0.6820$$

y_5 = Network output = 0.6820

$$\text{Error} = y_{\text{target}} - y_5$$
$$= 0.5 - 0.6820$$
$$= -0.182.$$

So new error is reduced from −0.19 to −0.182 we will continue this forward and backward pass till we reach to our expected goal such as minimum error or near to our target output.