More room for Sterile Neutrinos?

A cosmology final project

In this project, we explore the potential of an expansion to the ΛCDM model wherein massive neutrinos and a light sterile neutrino with a non-zero chemical potential are considered. This paper will explain the motivating physics behind the model choice, how the computation was performed, and remark on the results.

I. Model Choices

I 1. Choice of N effective

N effective parameterizes the number of ultra-relativistic species in the early universe. The Standard Model (SM) value of N_{eff} is 3.045, accounting for the three known neutrino species, photons and the non-instantaneous decoupling times of neutrinos. Any extra relativistic radiation prior to decoupling could raise Neff, making it an extremely powerful metric for constraining exotic particle theories like sterile neutrinos, axions and other dark matter candidates, and more.

Because $\Omega_{rad} = (1 + (7/8)N_{eff}(4/11)^{(4/3)})\Omega_{\gamma}$, a change in N_{eff} (holding all other parameters constant) would affect the matter composition (and therefore the expansion rate) of the universe. Any change in that rate will alter the CMB power spectrum - in this case, most notably by increasing damping a the high-l tail. That being said, holding all other parameters constant is not realistic (nor is it good science), and the model assumptions and even computational method choices all impact the best-fit values of N_{eff} on the same data in a non-trivial way. [1] Interestingly, N_{eff} is not particularly sensitive to changes in neutrino masses. In the above study, the Hubble parameter was most impacted by changes to N_{eff}.

To explore the influence N_{eff} has on other cosmological parameters, for this analysis we fix N_{eff} to be 4.044 by adding an additional neutrino species.

I 2. Choice of summed neutrino masses

Once neutrino masses were discovered via oscillations in the latter half of the 20th century, the problem of measuring those masses has been a focus on the neutrino physics community. Leading limits from direct detection currently come from KATRIN, which places its limits at 0.8 eV. [2] These limits pale in comparison to those obtained by Planck, which places a limit of 0.24 eV at the 95% confidence level. [3]

The matter power spectrum and (for neutrino masses below 1 eV) angular diameter distance are the cosmological parameters most sensitive to neutrino masses. [4]

For this analysis, which focuses on the temperature anisotropies, we choose a neutrino mass compatible with the Planck data and terrestrial measurements of neutrino mass difference-squared.[5] This allows us to incorporate the massive neutrinos favored by terrestrial experiments, but it not expected to alter the power spectrum significantly.

I 3. Choice of non-zero chemical potential

The most 'exotic' choice made in this model is the inclusion of a sterile neutrino with a non-zero chemical potential. This choice is inspired by an early paper seeking to explain the miniBooNE anomaly and its' favored explanation, the light sterile neutrino, with the cosmological data favoring a lower N_{eff}.[6] In that work, Shi notes that an ~1 eV sterile neutrino, together with a chemical potential of about 0.1, can significantly lower N_{eff}. Though the precise parameters noted in that work have now been disfavored by terrestrial neutrino experiments, the ability of specific neutrino properties to alter N_{eff} is intriguing, so I decided to include a chemical potential in the model for this work.

II Parameters to Fit

Once we've chosen our extension to the ΛCDM model, we need to choose which of the original parameters we will fit. As noted above, the influence of the changes we've made in the neutrino sector are likely to be strongly felt in the values of $\Omega_{CDM}h^2$ and h_0, so choosing those is natural. $\Omega_b h^2$ (though in constrained by the other two), is less directly influenced by our neutrino choices, and so is selected for comparison.

III Analysis procedure

This analysis uses the CLASS Boltzmann solver to generate power spectra for a given set of parameters and cosmological models.[7] CLASS is extremely flexible, allowing for the modification of cosmological parameters and a variety of models without modifying the codebase. We use the dataset provided by the Planck collaboration for the CMB power spectrum. [3]

The analysis plan was as follows:
1) Fit Planck data to the ΛCDM in order to understand how to run class and estimate its efficiency.
2) Run an MCMC algorithm on our expanded sterile neutrino with chemical potential (SNCP) model
3) Examine the posterior distributions and likelihoods to determine what we can learn from testing this expanded model.

III 1. Fit the ΛCDM model

In the course of fitting CMB data to the Planck best fit data, we compared the results of running CLASS on the Planck best-fit values, and obtained a spectrum which differed by an amplitude factor. This was also reflected in the initial MCMC fits, which were roughly in agreement with the Planck best-fit results. To correct for this, an amplitude parameter was added to the model, resulting in much better agreement with the data and the CLASS default settings (fig 1).

The suggestion that this amplitude difference is due to a difference in A_s does not at first appear consistent with these results - including A_s in this fitted parameters should have been adequate to address the issue if that were the case. That being said, the posterior distribution of A_s does indicate that the bounds on the prior may have been too strict, and the corner plot of A_s and amplitude, though containing too little data to be conclusive, indicates that the two may be anti-correlated (fig 2).

Simulated power spectra for the ΛCDM model

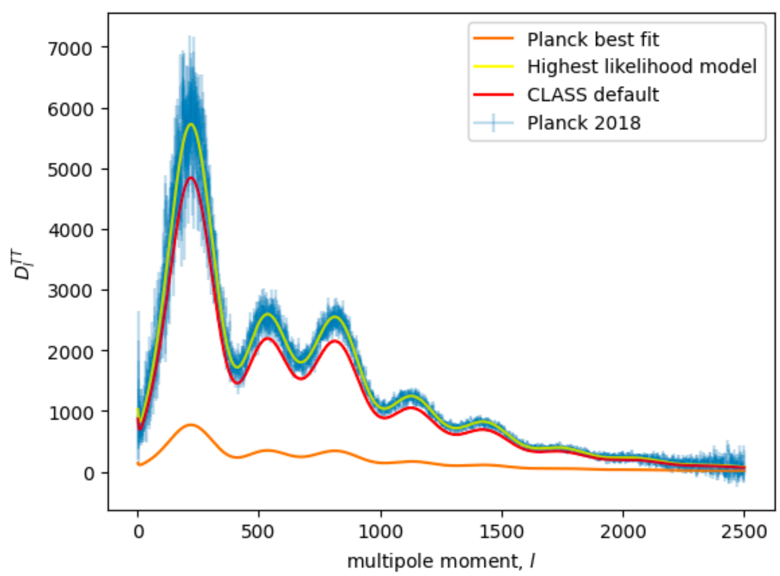

Fig. 1 The CLASS fits for the Planck best fit parameters and CLASS's default settings given the standard ΛCDM model, along with the highest-likelihood model from the ΛCDM + amplitude correction model.

The choice made for this analysis was to proceed with an amplitude correction, and the expanded model we tested therefore fit the four parameters $\Omega_{CDM}h^2$, $\Omega_b h^2$, h_0, and the amplitude. We use the CLASS default value for $A_{s.}$, in part because the CLASS default values did not exhibit the same amplitude disagreement as the fit without an amplitude parameter or the simulated Planck best-fit model. (In doing so, we eliminate the ability to further investigate whether A_s is the cause of this amplitude disagreement.)

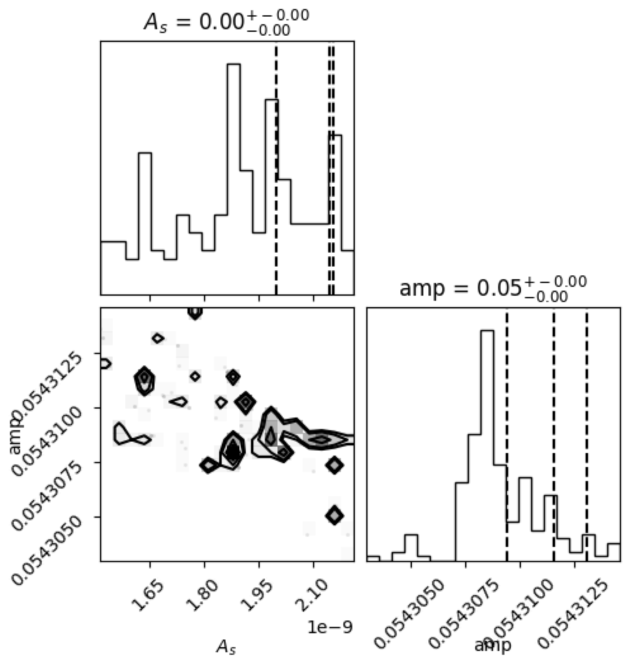

Fig 2. Posterior distributions for and correlation between A_s and amplitude, indicating that the bounds on A_s may have been too restrictive, and there may be some anti-correlation between the two parameters

III 2. Fit the SNCP model

The procedure followed for fitting the SNCP model is a standard MCMC fitting procedure, defining the likelihood and model manually, and using python library emcee to handle the MCMC procedure.[8] Priors for the fit parameters were chosen by comparison to the variations between models and fitting methods as summarized in [1] (see notebook for more details).

Despite having fewer parameters, this model took much longer per iteration than our test model, which introduced a few time- and resource-based restrictions on obtaining a sufficient number of samples for a statistically significant analysis. (A correction to this was attempted, and a faster sample with a few improvements as will be suggested below was sent to the Harvard computing cluster, but the job terminated right before completion due to an unexpected time-out error. There was not enough time before this paper was do to repeat the job, but I'm happy to do so and expand the analysis if necessary.)

We assign 64 walkers (ie, separate Monte Carlo chains) and were able to complete 275 iterations without issue. This was sufficient to pass the autocorrelation time for each chain (about 35), but not to obtain a statistically significant sample. Figure 3 shows the traceplots for the fit parameters; only the amplitude appears to have converged. The true model parameters all appear too restricted by the priors to reach their true preferred values.

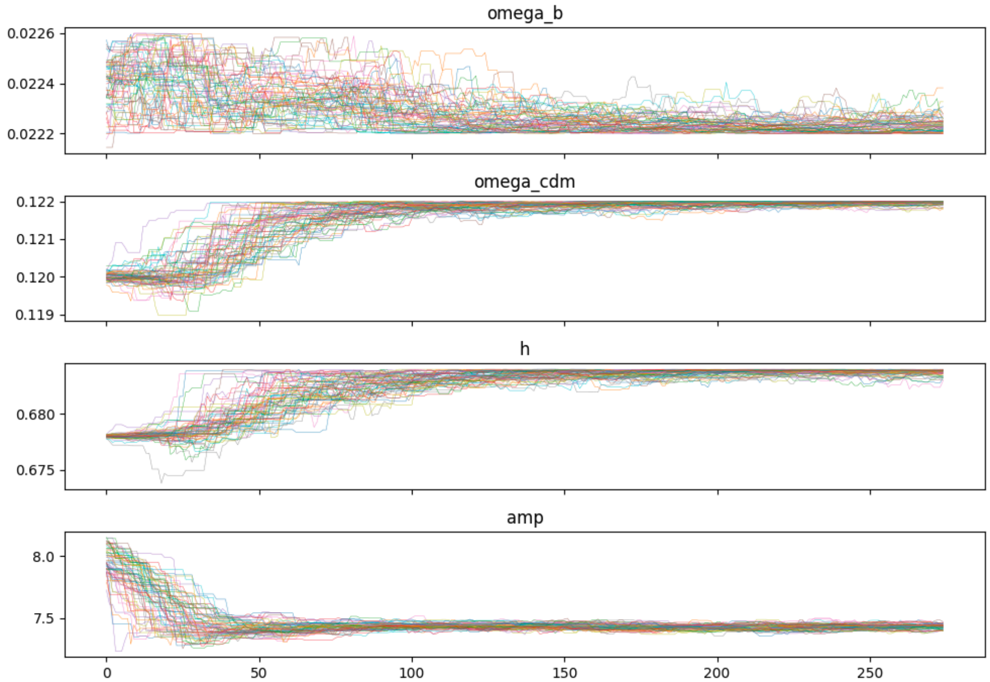

Fig. 3 Traceplots for fit parameters. The autocorrelation times were (top to bottom) 32, 34, 35, and 16 iterations. No parameter obtained the emcee recommended 50*correlation time iterations (about 1570)

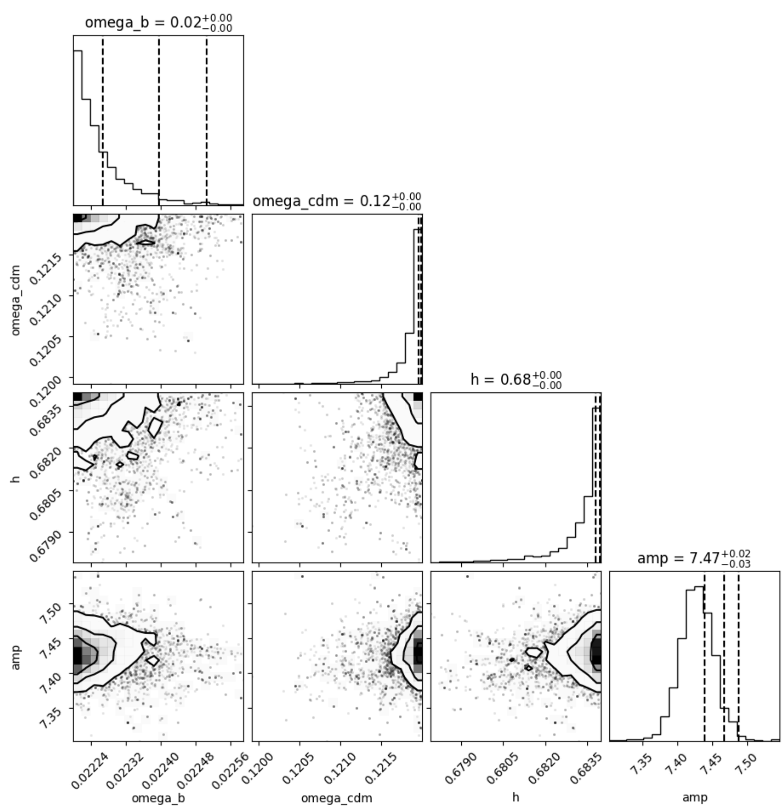

Fig. 4 Posterior distributions and correlations between our fit parameters for the SNCP model. Note the drift to the edges of allowed parameter space by the cosmological parameters shown.

This restriction is further visible in the posterior distributions for the parameters, where the distributions for $\Omega_{CDM}h^2$, $\Omega_b h^2$, and h_0 are located at the very edges of the allowed parameter space, showing that the preferred values may lie completely outside the allowed region. (Fig. 4)

III 3. See what we can learn

Despite not obtaining reliable best-fit values for our model parameters, this test still gives use useful information on the potential of our SNCP mode. Because the deviation in cosmological parameters induced by the SNCP model is far greater than what can be accounted for in known variations of those parameters based on existing model differences. Therefore, non-CMB based measurements of the cosmological parameters could put strong constraints on this model.

Furthermore, the values obtained by the least-unlikely proposal in the MCMC chain was less unlikely, given the data, than both the default CLASS parameters, the default CLASS parameters with an additional neutrino, and the least-unlikely cosmological parameters obtained from our model, used to simulate a more standard sterile neutrino model (Fig. 5). Furthermore, the SNCP highest-likelihood model differs less from the Planck 2018 data than the CLASS 4-neutrino model (Fig. 6). Taking all this together, we can conclude that including a non-zero chemical potential may decrease tensions between sterile neutrino models and cosmological data.

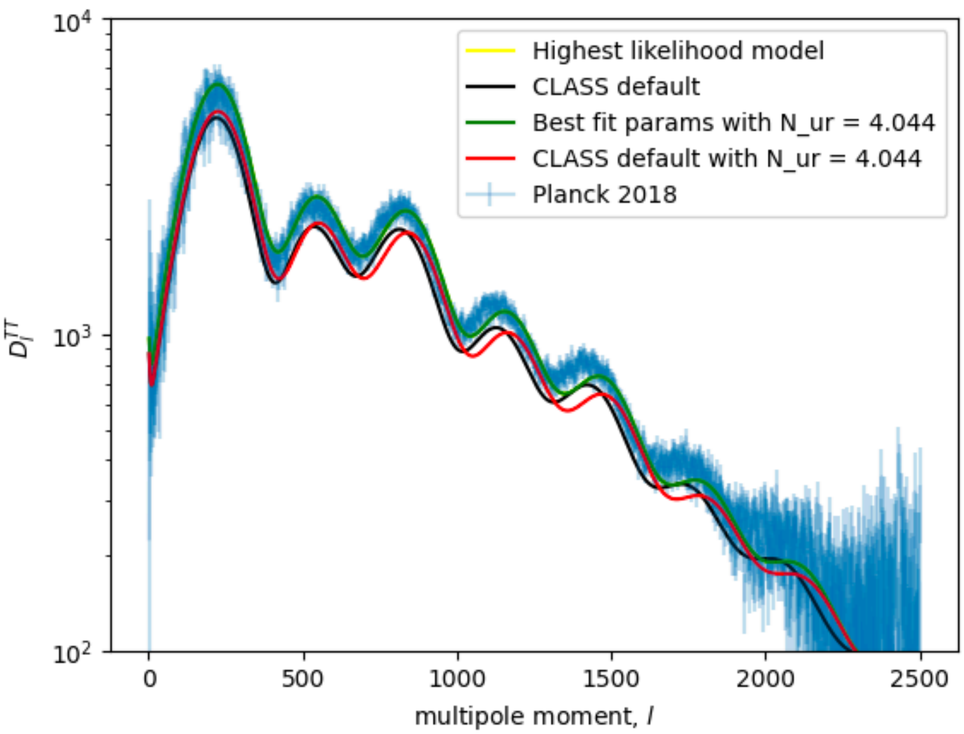

Fig 5. Visual comparison between the SNCP highest likelihood model, parameters obtained by that model used in a standard 4-neutrino model (yellow and green, visually indistinguishable, thought the SNCP highest likelihood model has a relative likelihood 4 times that of the 4-neutrino model), the CLASS default parameters (black), and the CLASS defaults cosmological parameters in a standard 4-neutrino model.

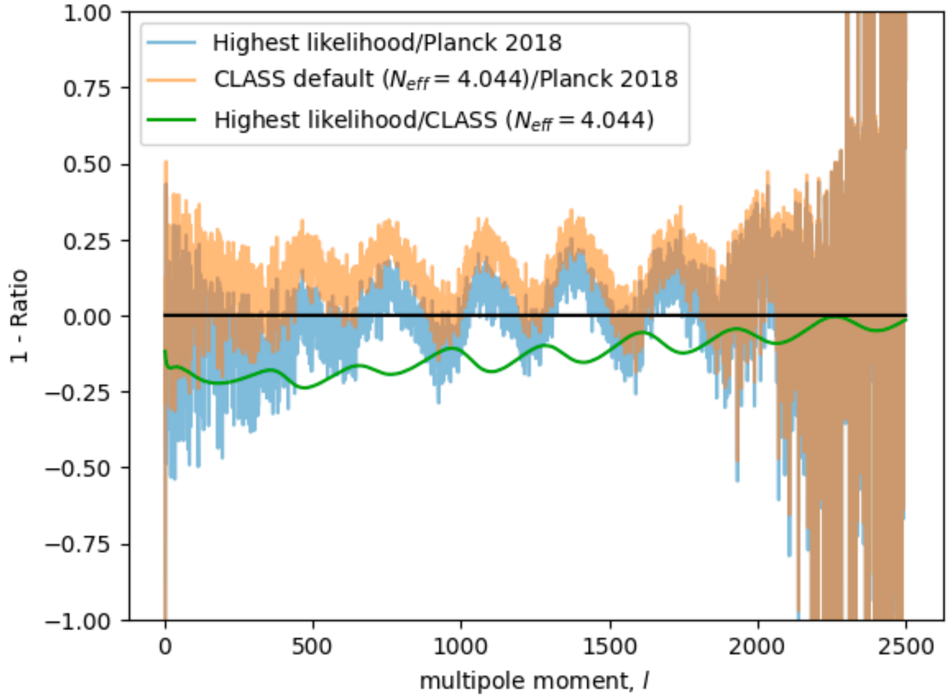

Fig. 6 Model deviations from data

IV Conclusions and next steps

I'm hesitant to draw too strong a conclusion from this analysis given the small number of samples obtained with which to test the data against our model. It's clear that rigorous analysis of the SNCP model required a more robust simulation with much broader priors. Additionally, it would be ideal to determine the precise cause of the amplitude disagreement and remove that parameter from the model.

To work towards this, I ran a short MCMC on slightly broadened priors. To speed up the model, we ran 20 walkers with 400 steps each. Even in this small sample, we see that the trends from our analysis continue, with our energy density parameters climbing out of the allowed space. Here the priors on the Hubble parameter were expanded enough for it to (start to) converge (Fig. 7).

Additionally, the new highest-likelihood model is preferred over our initial guess by a factor of two. This distribution is shown in figure 8.

Overall, I find the potential of a sterile-neutrino chemical potential intriguing, and am curious what the exploration of a similar model in the current preferred light sterile neutrino phase space would yield, especially given that recent work on sterile neutrinos with additional interactions have shown promise in resolving tensions between those experiments which do and do not see electron-neutrino oscillation anomalies [9]. Additionally, I would further explore the influence this model may have on loosening cosmological limits on heavy sterile neutrinos, the topic of my dissertation research.

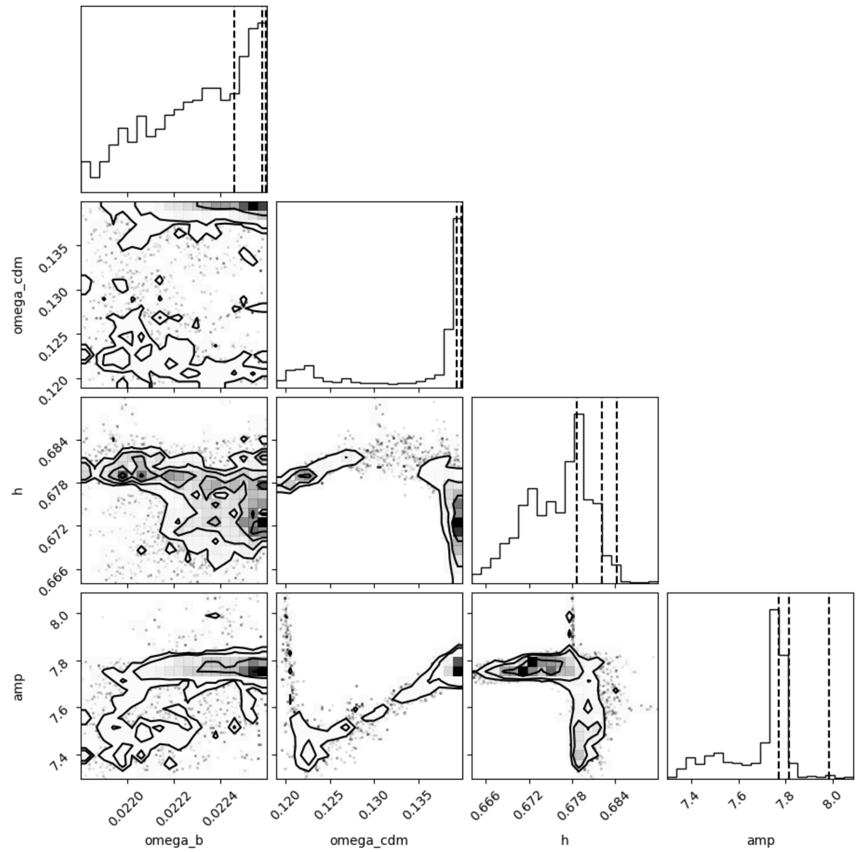

Fig. 7 Posterior distributions and correlations for cosmological parameters in the broader-parameter SNCP model

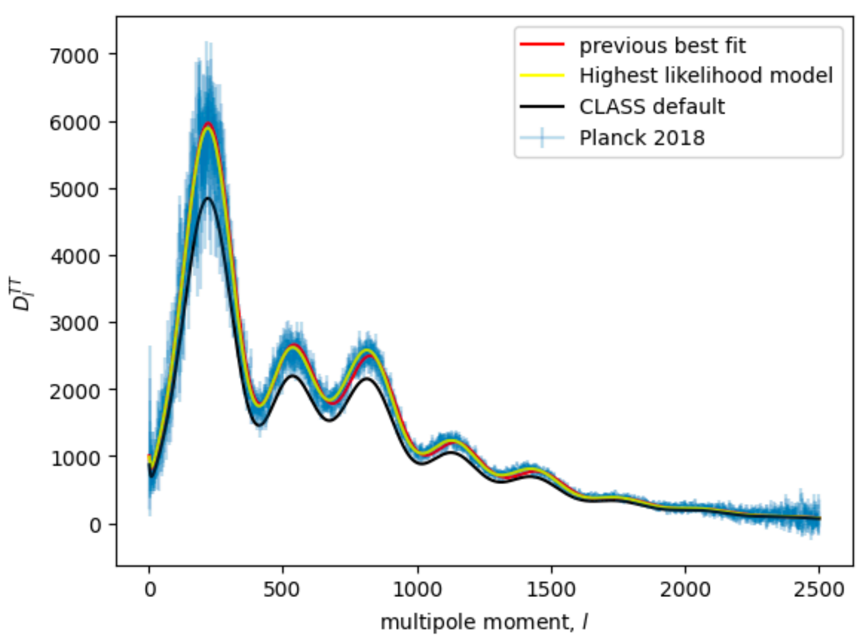

Fig. 8 Calculated power spectra for the original SNCP model (red), the new preferred model (yellow), compared to the CLASS default and Planck data.

Code:

The work for step 1 as described in this analysis can be found in Step1.ipynb
The main analysis can be found in cosmology_project.ipynb, with helper functions described both in the notebook and in cosmo_helpers.py (required for parallelization purposes)
The short analysis mentioned in the conclusion makes use of a modification to cosmo_helpers.py, cosmo_optimized.py, and Addendum.ipynb

Also found in the GitHub are various files created for grid submission and other attempts at optimization, which were unfortunately not successful.

Works Cited

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