CASE 6.2, Farm Management

- a. Identify verbally the components of a linear programming model for this problem.
 - Objective
 - i. Net income from the livestock
 - ii. Net income from the crops
 - iii. Remains from the investment
 - iv. Value of livestock
 - v. Income from working on a neighboring farm
 - vi. Living expenses, \$40,000
 - Variables
 - i. x_1 : acreage for soybeans, x_2 : acreage for corn, x_3 : acreage for wheat;
 - ii. y_1 : number of new cows, y_2 : number of new hens
 - iii. z_1 : working time for neighbor in W/Sp, z_2 : working time for neighbor in Su/F
 - Constraints
 - i. Acre amount, 640
 - ii. Person-hours, 4000 in W/Sp, 4500 in Su/F
 - iii. Investment fund, \$20,000
 - iv. Chicken house limit, 5,000 hens
 - v. Cow barn limits, 42 cows
 - vi. Acre of corn for each cow, 1
 - vii. Acre of wheat for each hen, 0.05
- b. Formulate this model.
 - Objective function

$$(30 + y_1) \times 850 + (2000 + y_2) \times 4.25 +$$
 $70x_1 + 60x_2 + 40x_3 +$
 $20000 - 1500y_1 - 3y_2 +$
 $(35000 + 1500y_1) \times 0.9 + (5000 + 3y_2) \times 0.75 +$
 $5z_1 + 5.5z_2 - 40000$

Constraints

$$\begin{aligned} &2(30+y_1)+x_1+x_2+x_3\leq 640\\ &10(30+y_1)6+0.05(2000+y_2)6+x_1+0.9x_2+0.6x_3+z_1=4000\\ &10(30+y_1)6+0.05(2000+y_2)6+1.4x_1+1.2x_2+0.7x_3+z_2=4500\\ &1500y_1+3y_2\leq 20000\\ &2000+y_2\leq 5000\\ &30+y_1\leq 42\\ &x_2\geq 30+y_1\\ &x_3\geq 0.05(2000+y_2)\\ &all\ x,y,z\ non\ negative \end{aligned}$$

c. Obtain an optimal solution.

The best estimation of the family's monetary worth at the end of the coming year is 99367. See details in the attached GAMS solution screenshot.

```
LOWER LEVEL UPPER MARGINAL
---- VAR TOT
                 -INF 99367.000 +INF .
 TOT income
---- VAR X acre of each grain
        LOWER
               LEVEL UPPER MARGINAL
              450.000
                450.000 +inf .
30.000 +inf .
+inf .
corn
               100.000
---- VAR Z hours working for neighbor
           LEVEL
                   UPPER MARGINAL
     . 1063.000 ....
1364.000 +INF
---- VAR Y number of new purchased livestock
    LOWER LEVEL UPPER MARGINAL
    . +INF
                           -53.000
                     +INF
                             -0.858
```

d. Find the allowable range to stay optimal for the net value per acre planted for each of the three crops.

From sensitivity analysis results of GAMS, the allowable range is, soybean net income is from 61.6 to inf, corn net income is from –inf to 68.4, wheat net income is from –inf to 64.15.

VARIABLE NAME	LOWER	CURRENT	UPPER
TOT	-INF	1	+INF
X(soybean)	61.6	70	+INF
X(corn)	-INF	60	68.4
X(wheat)	-INF	40	64.15
Z(WS)	-INF	5	62.3
Z(SF)	-INF	5.5	40
Y(cow)	-INF	700	+INF
Y(hen)	-INF	3.5	+INF

e. Find otimal solution under each scenario after making the necessary adjustments to part b.

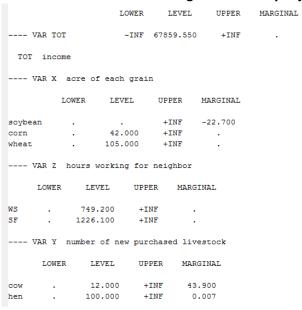


Figure 1: Drought Scenario, optimal value is 67859

TOT income VAR X acre of each grain LOWER LEVEL UPPER MARGINAL soybean +INF -6.600 corn . 451.000 +INF wheat . 105.000 +INF VAR Z hours working for neighbor LOWER LEVEL UPPER MARGINAL WS . 381.100 +INF SF . 735.300 +INF VAR Y number of new purchased livestock	VAR	тот	-INF	74019.650	+INF	
LOWER LEVEL UPPER MARGINAL soybean +INF -6.600 corn . 451.000 +INF . wheat . 105.000 +INF . VAR Z hours working for neighbor LOWER LEVEL UPPER MARGINAL WS . 381.100 +INF . SF . 735.300 +INF VAR Y number of new purchased livestock	TOT in	come				
soybean . +INF -6.600 corn . 451.000 +INF . wheat . 105.000 +INF . VAR Z hours working for neighbor LOWER LEVEL UPPER MARGINAL WS . 381.100 +INF . SF . 735.300 +INF . VAR Y number of new purchased livestock	VAR	X acre of	each grai	n		
Corn		LOWER	LEVEL	UPPER	MARGINAL	
wheat . 105.000 +INF	soybean			+INF	-6.600	
VAR Z hours working for neighbor LOWER LEVEL UPPER MARGINAL WS . 381.100 +INF . SF . 735.300 +INF . VAR Y number of new purchased livestock	corn		451.000	+INF		
LOWER LEVEL UPPER MARGINAL WS . 381.100 +INF . SF . 735.300 +INF . VAR Y number of new purchased livestock	wheat		105.000	+INF		
LOWER LEVEL UPPER MARGINAL WS . 381.100 +INF . SF . 735.300 +INF . VAR Y number of new purchased livestock						
WS . 381.100 +INF . SF . 735.300 +INF VAR Y number of new purchased livestock	VAR	Z hours we	orking for	neighbor		
WS . 381.100 +INF . SF . 735.300 +INF VAR Y number of new purchased livestock						
SF . 735.300 +INF VAR Y number of new purchased livestock	LOW	ER LEVI	EL UPP	ER MARC	SINAL	
SF . 735.300 +INF VAR Y number of new purchased livestock		000				
VAR Y number of new purchased livestock						
-	SF .	735.	300 +1	NF .		
	VAR Y number of new purchased livestock					
LOWER LEVEL UPPER MARGINAL	LO	WER LE	VEL UP	PER MAI	RGINAL	
cow . 12.000 +INF 52.200	COW	. 12	.000 +	INF 52	2.200	
hen . 100.000 +INF 0.062						

Figure 2: Flood Scenario, optimal value is 74019

VAR TO	T	-INF	88767.000	+INF
TOT inco	ome			
VAR X	acre of e	each grai	n	
	LOWER	LEVEL	UPPER	MARGINAL
soybean		150.000	+INF	
corn		30.000	+INF	
wheat	. 1	100.000	+INF	
VAR Z	hours wor	rking for	neighbor	
LOWER	LEVE!	L UPP	ER MARG	SINAL
ws .	1063.00	00 +I	NF .	
SF .	1364.00	00 +I	NF .	
VAR Y	number of	f new pur	chased liv	restock
LOWE	R LEVI	EL UP	PER MAR	RGINAL
cow .		+	INF -13	3.000
hen .		+	INF -0	.358

Figure 3: Early Frost Scenario, optimal value is 88767

```
---- VAR TOT -INF 66648.800
                               +INF
 TOT income
---- VAR X acre of each grain
        LOWER LEVEL UPPER MARGINAL
soybean .
                       +INF -27.700
               42.000
                       +INF
wheat
             100.000 +INF
---- VAR Z hours working for neighbor
    LOWER
          LEVEL UPPER MARGINAL
          782.200 +INF
        1259.600 +INF
---- VAR Y number of new purchased livestock
     LOWER
            LEVEL
                    UPPER MARGINAL
           12.000 +INF 38.900
                    +INF
                           -0.493
            .
```

Figure 4: Drought and early frost Scenario, optimal value is 66648

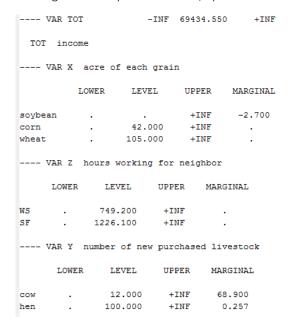


Figure 5: Flood and early frost Scenario, optimal value is 69434.55

f. For the optimal solution under each of the 6 scenarios, calculate the monetary worth would be at the end of year if each of the other five scenarios occur instead. In your judgment, which solution provides the best balance between large monetary worth under good condition, and avoid large loss under adverse conditions.

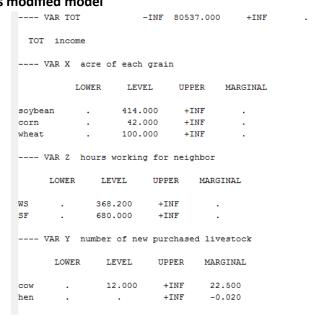
Expected Scenario	Good	Drought	Flood	Early Frost	Drought and Earl	y Fr(Flood and Early Fros
Soybean	450	0	0	450	0	0
Corn	30	42	451	30	42	42
Wheat	100	105	105	100	100	105
Cow	0	12	12	0	12	12
Hen	0	100	100	0	0	100
Hr in WS	1063	749	381	1063	782	749
Hr in SF	1364	1226	735. 3	1364	1259	1226
Actual Scenario						
Good	99367	75208	95209.15	99367	75004.5	75208
Drought	57117	67858	57184.15	57117	67854.5	67858
Flood	70417	70378	74019.15	70417	70324.5	70378
Early Frost	88767	73318	85139.15	88767	73164.5	73318
Drought and Early Fro	st 53717	66598	53879.15	53717	66644.5	66598
Flood and Early Frost	67367	69433	68984.15	67367	69404.5	69433
Mean	72792	70465.5	72402. 48333	72792	70399.5	70465. 5
Same Prob Variance	17922. 40916	3269. 813068	15946.01444	17922. 40916	3175. 655838	3269. 813068

From this results table, according to minimum variance, it is easily found that solution of drought and early frost provide the best balance.

g. Modify the linear programming model to fit this new approach

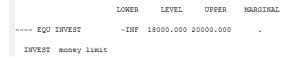
The difference is the change of net value for three corps, which means it is equal to the estimation according to the estimated weather condition. Then net value table is

Soybean: 34; Corn: 27.5; Wheat: 20.75 **h. Repeat part c for this modified model**



From this table, the family's monetary worth at the end of the coming year is 80537.

i. Use a shadow price to analyze whether it would be worthwhile to obtain a bank loan with a 10% interest rate.



Since the marginal (shadow price) is 0, which is less than 10%, it is not worthwhile to make a bank loan.

j. Sensitivity analysis for part h

VARIABLE NAME	LOWER	CURRENT	UPPER
TOT	-INF	1	+INF
X(soybean)	29.1	34	+INF
X(corn)	-INF	27.5	32.4
X (wheat)	-INF	20.75	28.15
Z(WS)	-INF	5	23.5
Z(SF)	-INF	5.5	16.07
Y(cow)	-INF	700	+INF
Y(hen)	-INF	3.5	+INF

Net values of soybean and corn should be estimated more precisely.

If changes are simultaneous, for example, coef of soybean goes down from 34 to x>29.1, the percentage of change is $\frac{(34-x)}{(34-29.1)}=a$. According to 100 percentage rule for objective function coefficients, the allowable change percentage for corn or wheat is 1-a, which means $(1-a)\times(32.4-27.5)$ for corn or $(1-a)\times(28.15-20.75)$ for what.

k. Describe one situation outside of farm management that fit this description.

Think about a multinational corporation, like Walmart, that import the merchandise from various nations, like China, India, and Thailand, etc. The different objective national condition and the relationship between the U.S. and the objective nation could affect the cost and profit of Walmart. Hence Walmart should decide how much good A should be imported from nation a, and how much good B should be imported from nation b, with consideration of the average of circumstance.