CASE 6.2, Farm Management

a. Identify verbally the components of a linear programming model for this problem.

- Objective
  i. Net income from the livestock
  ii. Net income from the crops
  iii. Remains from the investment
  iv. Value of livestock
  v. Income from working on a neighboring farm
  vi. Living expenses, $40,000
- Variables
  i. x_1: acreage for soybeans, x_2: acreage for corn, x_3: acreage for wheat;
  ii. y_1: number of new cows, y_2: number of new hens
  iii. z_1: working time for neighbor in W/Sp, z_2: working time for neighbor in Su/F
- Constraints
  i. Acre amount, 640
  ii. Person-hours, 4000 in W/Sp, 4500 in Su/F
  iii. Investment fund, $20,000
  iv. Chicken house limit, 5,000 hens
  v. Cow barn limits, 42 cows
  vi. Acre of corn for each cow, 1
  vii. Acre of wheat for each hen, 0.05

b. Formulate this model.

- Objective function

$$(30 + y_1) \times 850 + (2000 + y_2) \times 4.25 + 70x_1 + 60x_2 + 40x_3 + 20000 - 1500y_1 - 3y_2 + (35000 + 1500y_1) \times 0.9 + (5000 + 3y_2) \times 0.75 + 5z_1 + 5.5z_2 - 40000$$

- Constraints

$$2(30 + y_1) + x_1 + x_2 + x_3 \leq 640$$
$$10(30 + y_1)6 + 0.05(2000 + y_2)6 + x_1 + 0.9x_2 + 0.6x_3 + z_1 = 4000$$
$$10(30 + y_1)6 + 0.05(2000 + y_2)6 + 1.4x_1 + 1.2x_2 + 0.7x_3 + z_2 = 4500$$
$$1500y_1 + 3y_2 \leq 20000$$
$$2000 + y_2 \leq 5000$$
$$30 + y_1 \leq 42$$
$$x_2 \geq 30 + y_1$$
$$x_3 \geq 0.05(2000 + y_2)$$
$$all\ x, y, z\ non\ negative$$

c. Obtain an optimal solution.

The best estimation of the family's monetary worth at the end of the coming year is 99367. See details in the attached GAMS solution screenshot.

```
                    LOWER     LEVEL     UPPER    MARGINAL

---- VAR TOT           -INF  99367.000     +INF       .

  TOT  income

---- VAR X  acre of each grain

          LOWER     LEVEL     UPPER    MARGINAL

soybean       .     450.000     +INF       .
corn          .      30.000     +INF       .
wheat         .     100.000     +INF       .

---- VAR Z  hours working for neighbor

      LOWER     LEVEL     UPPER    MARGINAL

WS        .    1063.000     +INF       .
SF        .    1364.000     +INF       .

---- VAR Y  number of new purchased livestock

       LOWER     LEVEL     UPPER    MARGINAL

cow        .         .       +INF    -53.000
hen        .         .       +INF     -0.858
```

d. Find the allowable range to stay optimal for the net value per acre planted for each of the three crops.

From sensitivity analysis results of GAMS, the allowable range is, soybean net income is from 61.6 to inf, corn net income is from –inf to 68.4, wheat net income is from –inf to 64.15.

VARIABLE NAME	LOWER	CURRENT	UPPER
TOT	-INF	1	+INF
X(soybean)	61.6	70	+INF
X(corn)	-INF	60	68.4
X(wheat)	-INF	40	64.15
Z(WS)	-INF	5	62.3
Z(SF)	-INF	5.5	40
Y(cow)	-INF	700	+INF
Y(hen)	-INF	3.5	+INF

e. Find otimal solution under each scenario after making the necessary adjustments to part b.

```
                    LOWER     LEVEL     UPPER    MARGINAL

---- VAR TOT           -INF  67859.550     +INF       .

  TOT  income

---- VAR X  acre of each grain

          LOWER     LEVEL     UPPER    MARGINAL

soybean       .         .       +INF    -22.700
corn          .      42.000     +INF       .
wheat         .     105.000     +INF       .

---- VAR Z  hours working for neighbor

      LOWER     LEVEL     UPPER    MARGINAL

WS        .     749.200     +INF       .
SF        .    1226.100     +INF       .

---- VAR Y  number of new purchased livestock

       LOWER     LEVEL     UPPER    MARGINAL

cow        .      12.000     +INF     43.900
hen        .     100.000     +INF      0.007
```

Figure 1: Drought Scenario, optimal value is 67859

```
---- VAR TOT            -INF  74019.650     +INF

  TOT  income

---- VAR X  acre of each grain

           LOWER     LEVEL     UPPER    MARGINAL

soybean      .         .        +INF     -6.600
corn         .      451.000     +INF       .
wheat        .      105.000     +INF       .

---- VAR Z  hours working for neighbor

      LOWER     LEVEL     UPPER    MARGINAL

WS      .      381.100     +INF       .
SF      .      735.300     +INF       .

---- VAR Y  number of new purchased livestock

       LOWER     LEVEL     UPPER    MARGINAL

cow      .       12.000     +INF     52.200
hen      .      100.000     +INF      0.062
```

Figure 2: Flood Scenario, optimal value is 74019

```
---- VAR TOT            -INF  88767.000     +INF

  TOT  income

---- VAR X  acre of each grain

           LOWER     LEVEL     UPPER    MARGINAL

soybean      .      450.000     +INF       .
corn         .       30.000     +INF       .
wheat        .      100.000     +INF       .

---- VAR Z  hours working for neighbor

      LOWER     LEVEL     UPPER    MARGINAL

WS      .     1063.000     +INF       .
SF      .     1364.000     +INF       .

---- VAR Y  number of new purchased livestock

       LOWER     LEVEL     UPPER    MARGINAL

cow      .         .        +INF    -13.000
hen      .         .        +INF     -0.358
```

Figure 3: Early Frost Scenario, optimal value is 88767

```
---- VAR TOT            -INF  66648.800     +INF

  TOT  income

---- VAR X  acre of each grain

           LOWER     LEVEL     UPPER    MARGINAL

soybean      .         .        +INF    -27.700
corn         .       42.000     +INF       .
wheat        .      100.000     +INF       .

---- VAR Z  hours working for neighbor

      LOWER     LEVEL     UPPER    MARGINAL

WS      .      782.200     +INF       .
SF      .     1259.600     +INF       .

---- VAR Y  number of new purchased livestock

       LOWER     LEVEL     UPPER    MARGINAL

cow      .       12.000     +INF     38.900
hen      .         .        +INF     -0.493
```

Figure 4: Drought and early frost Scenario, optimal value is 66648

```
---- VAR TOT            -INF  69434.550     +INF

  TOT  income

---- VAR X  acre of each grain

           LOWER     LEVEL     UPPER    MARGINAL

soybean      .         .        +INF     -2.700
corn         .       42.000     +INF       .
wheat        .      105.000     +INF       .

---- VAR Z  hours working for neighbor

      LOWER     LEVEL     UPPER    MARGINAL

WS      .      749.200     +INF       .
SF      .     1226.100     +INF       .

---- VAR Y  number of new purchased livestock

       LOWER     LEVEL     UPPER    MARGINAL

cow      .       12.000     +INF     68.900
hen      .      100.000     +INF      0.257
```

Figure 5: Flood and early frost Scenario, optimal value is 69434.55

f. For the optimal solution under each of the 6 scenarios, calculate the monetary worth would be at the end of year if each of the other five scenarios occur instead. In your judgment, which solution provides the best balance between large monetary worth under good condition, and avoid large loss under adverse conditions.

	Expected Scenario	Good	Drought	Flood	Early Frost	Drought and Early Fr	Flood and Early Frost
	Soybean	450	0	0	450	0	0
	Corn	30	42	451	30	42	42
	Wheat	100	105	105	100	100	105
	Cow	0	12	12	0	12	12
	Hen	0	100	100	0	0	100
	Hr in WS	1063	749	381	1063	782	749
	Hr in SF	1364	1226	735.3	1364	1259	1226
	Actual Scenario						
	Good	99367	75208	95209.15	99367	75004.5	75208
	Drought	57117	67858	57184.15	57117	67854.5	67858
	Flood	70417	70378	74019.15	70417	70324.5	70378
	Early Frost	88767	73318	85139.15	88767	73164.5	73318
	Drought and Early Frost	53717	66598	53879.15	53717	66644.5	66598
	Flood and Early Frost	67367	69433	68984.15	67367	69404.5	69433
	Mean	72792	70465.5	72402.48333	72792	70399.5	70465.5
Same Prob	Variance	17922.40916	3269.813068	15946.01444	17922.40916	3175.655838	3269.813068

From this results table, according to minimum variance, it is easily found that solution of drought and early frost provide the best balance.

g. Modify the linear programming model to fit this new approach

The difference is the change of net value for three corps, which means it is equal to the estimation according to the estimated weather condition. Then net value table is

Soybean: 34; Corn: 27.5; Wheat: 20.75

h. Repeat part c for this modified model

```
---- VAR TOT            -INF  80537.000     +INF        .

  TOT  income

---- VAR X  acre of each grain

            LOWER     LEVEL     UPPER    MARGINAL

soybean       .     414.000     +INF        .
corn          .      42.000     +INF        .
wheat         .     100.000     +INF        .

---- VAR Z  hours working for neighbor

      LOWER     LEVEL     UPPER    MARGINAL

WS      .     368.200     +INF        .
SF      .     680.000     +INF        .

---- VAR Y  number of new purchased livestock

       LOWER     LEVEL     UPPER    MARGINAL

cow      .      12.000     +INF      22.500
hen      .         .       +INF      -0.020
```

From this table, the family's monetary worth at the end of the coming year is 80537.

i. Use a shadow price to analyze whether it would be worthwhile to obtain a bank loan with a 10% interest rate.

```
                        LOWER     LEVEL     UPPER    MARGINAL

---- EQU INVEST          -INF  18000.000 20000.000      .

  INVEST  money limit
```

Since the marginal (shadow price) is 0, which is less than 10%, it is not worthwhile to make a bank loan.

j. Sensitivity analysis for part h

VARIABLE NAME	LOWER	CURRENT	UPPER
TOT	-INF	1	+INF
X(soybean)	29.1	34	+INF
X(corn)	-INF	27.5	32.4
X(wheat)	-INF	20.75	28.15
Z(WS)	-INF	5	23.5
Z(SF)	-INF	5.5	16.07
Y(cow)	-INF	700	+INF
Y(hen)	-INF	3.5	+INF

Net values of soybean and corn should be estimated more precisely.

If changes are simultaneous, for example, coef of soybean goes down from 34 to $x > 29.1$, the percentage of change is $\frac{(34-x)}{(34-29.1)} = a$. According to 100 percentage rule for objective function coefficients, the allowable change percentage for corn or wheat is $1 - a$, which means $(1 - a) \times (32.4 - 27.5)$ for corn or $(1 - a) \times (28.15 - 20.75)$ for what.

k. Describe one situation outside of farm management that fit this description.

Think about a multinational corporation, like Walmart, that import the merchandise from various nations, like China, India, and Thailand, etc. The different objective national condition and the relationship between the U.S. and the objective nation could affect the cost and profit of Walmart. Hence Walmart should decide how much good A should be imported from nation a, and how much good B should be imported from nation b, with consideration of the average of circumstance.