6.1. ☺

A manufacturer of component for automobile transmissions wants to use control charts to monitor a process producing a shaft. The resulting data from 20 samples of 4 shaft diameters that have been measured are:

$$\sum_{i=1}^{20} \overline{x}_i = 10.275, \quad \sum_{i=1}^{20} R_i = 1.012$$

(a) Find the control limits that should be used on the $\overline{x}$ and R control charts.

For $n = 4$, $A_2 = 0.729$, $D_4 = 2.282$, $D_3 = 0$

$$\overline{\overline{x}} = \frac{\sum_{i=1}^{m} \overline{x}_i}{m} = \frac{10.275}{20} = 0.5138; \quad \overline{R} = \frac{\sum_{i=1}^{m} R_i}{m} = \frac{1.012}{20} = 0.0506$$

$$\text{UCL}_{\overline{x}} = \overline{\overline{x}} + A_2\overline{R} = 0.5138 + 0.729(0.0506) = 0.5507$$

$$\text{CL}_{\overline{x}} = \overline{\overline{x}} = 0.5138$$

$$\text{LCL}_{\overline{x}} = \overline{\overline{x}} - A_2\overline{R} = 0.5138 - 0.729(0.0506) = 0.4769$$ (Equations 6.2, 6.3, 6.4, 6.5)

$$\text{UCL}_{R} = D_4\overline{R} = 2.282(0.0506) = 0.1155$$

$$\text{CL}_{R} = \overline{R} = 0.0506$$

$$\text{LCL}_{R} = D_3\overline{R} = 0(0.0506) = 0.00$$

(b) Assume that the 20 preliminary samples plot in control on both charts. Estimate the process mean and standard deviation.

Process mean = $\overline{\overline{x}} = 0.5138$

For $n = 4$, $d_2 = 2.059$

Process standard deviation = $\hat{\sigma} = \overline{R}/d_2 = 0.0506 / 2.059 = 0.0246$ (Equation 6.6)

6.3. ☺

Reconsider the situation described in Exercise 6.1. Suppose that several of the preliminary 20 samples plot out of control on the *R* chart. Does this have any impact on the reliability of the control limits on the $\bar{x}$ chart?

Yes. Out-of-control samples on the *R* chart signal a potential problem with process variability, resulting in an unreliable estimate of the process standard deviation, and consequently impacting the accuracy of the upper and lower control limits on the $\bar{x}$ chart.

6.4. ☺

Discuss why it is important to establish control on the *R* chart first when using $\overline{x}$ and *R* control charts to bring a process into statistical control

If the *R* chart is out of control, the process variability is unstable, and the control limits on the $\overline{x}$ chart (which requires an estimate of process variability) are unstable. Without valid control limits, it is difficult to judge whether the process average is actually in control, and improvement attempts could actually result in a waste of resources and time.

6.6. ☺

Components used in a cellular telephone are manufactured with nominal dimension of 0.3 mm and lower and upper specification limits of 0.295 mm and 0.305 mm respectively. The $\bar{x}$ and R control charts for this process are based on subgroups of size 3 and they exhibit statistical control, with the center line on the $\bar{x}$ at 0.3015 mm and the center line on the R chart at 0.00154 mm.

(a) Estimate the mean and standard deviation of this process.

Process mean = $\bar{\bar{x}} = \mathrm{CL}_{\bar{x}} = 0.3015$

For $n = 3$, $d_2 = 1.693$

Process standard deviation = $\hat{\sigma} = \bar{R}/d_2 = \mathrm{CL}_{\bar{R}}/d_2 = 0.00154/1.693 = 0.00091$ (Equation 6.6)

(b) Suppose that parts below the lower specification limits can be reworked, but parts above the upper specification limit must be scrapped. Estimate the proportion of scrap and rework produced by this process.

$$p_{rework} = P\{x > 0.305\} = 1 - \Phi\left(\frac{\mathrm{USL} - \mathrm{CL}_{\bar{x}}}{\hat{\sigma}}\right) = 1 - \Phi\left(\frac{0.305 - 0.3015}{0.00091}\right) = 1 - \Phi(3.846) = 1 - 0.99994 = 0.00006$$

(p. 242)

$$p_{scrap} = P\{x < 0.295\} = \Phi\left(\frac{\mathrm{LSL} - \mathrm{CL}_{\bar{x}}}{\hat{\sigma}}\right) = \Phi\left(\frac{0.295 - 0.3015}{0.00091}\right) = \Phi(-7.143) = 0$$

(c) Suppose that the mean of this process can be reset by fairly simple adjustments. What value of the process mean would you recommend?

Recommend centering at the midpoint of the specification, Target = (USL + LSL) / 2 = 0.300

6.23.

Consider the $\bar{x}$ and R charts you established in Exercise 6.7 using $n = 5$.

$n_{\text{old}} = 5;\ \bar{\bar{x}}_{\text{old}} = 34.00;\ \bar{R}_{\text{old}} = 4.7$

(a) Suppose that you wished to continue charting this quality characteristics using $\bar{x}$ and R charts based on a sample size of n =3. What limits would be used on the $\bar{x}$ and R charts?

for $n_{\text{new}} = 3$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}}_{\text{old}} + A_{2(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 34 + 1.023\left[\frac{1.693}{2.326}\right](4.7) = 37.50$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}}_{\text{old}} - A_{2(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 34 - 1.023\left[\frac{1.693}{2.326}\right](4.7) = 30.50$$

$$\text{UCL}_{R} = D_{4(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 2.574\left[\frac{1.693}{2.326}\right](4.7) = 8.81$$

$$\text{CL}_{R} = \bar{R}_{\text{new}} = \left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = \left[\frac{1.693}{2.326}\right](4.7) = 3.42$$

$$\text{LCL}_{R} = D_{3(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 0\left[\frac{1.693}{2.326}\right](4.7) = 0$$

(b) What would be the impact of the decision you made in part (a) on the ability of the $\bar{x}$ chart to detect a 2σ shift in the mean?

The $\bar{x}$ control limits for $n = 5$ are "tighter" (31.29, 36.72) than those for $n = 3$ (30.50, 37.50). This means a 2σ shift in the mean would be detected more quickly with a sample size of $n = 5$.

6.23. continued

(c) Suppose you wished to continue charting this quality characteristic using $\bar{x}$ and R charts based on a sample size of n = 8. What limits would be used on the $\bar{x}$ and R charts?

$$\mathrm{UCL}_{\bar{x}} = \bar{\bar{x}}_{\text{old}} + A_{2(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 34 + 0.373\left[\frac{2.847}{2.326}\right](4.7) = 36.15$$

$$\mathrm{LCL}_{\bar{x}} = \bar{\bar{x}}_{\text{old}} - A_{2(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 34 - 0.373\left[\frac{2.847}{2.326}\right](4.7) = 31.85$$

$$\mathrm{UCL}_{R} = D_{4(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 1.864\left[\frac{2.847}{2.326}\right](4.7) = 10.72$$

$$\mathrm{CL}_{R} = \bar{R}_{\text{new}} = \left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = \left[\frac{2.847}{2.326}\right](4.7) = 5.75$$

$$\mathrm{LCL}_{R} = D_{3(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 0.136\left[\frac{2.847}{2.326}\right](4.7) = 0.78$$

(d) What is the impact of using n = 8 on the ability of the $\bar{x}$ chart to detect a 2σ shift in the mean?

The $\bar{x}$ control limits for n = 8 are even "tighter" (31.85, 36.15), increasing the ability of the chart to quickly detect the 2σ shift in process mean.

6.25.

Control charts for $\overline{x}$ and R are maintained for an important quality characteristic. The sample size is n= 7; $\overline{x}$ and R are computed for each sample. After 35 samples we have found that

$\sum_{i=1}^{35} \overline{x}_i = 7{,}805$ and $\sum_{i=1}^{35} R_i = 1{,}200$

(a) Set up $\overline{x}$ and R charts using these data.

$$\overline{\overline{x}} = \frac{\sum_{i=1}^{35} \overline{x}_i}{m} = \frac{7805}{35} = 223; \quad \overline{R} = \frac{\sum_{i=1}^{35} R_i}{m} = \frac{1200}{35} = 34.29$$

$$\text{UCL}_{\overline{x}} = \overline{\overline{x}} + A_2\overline{R} = 223 + 0.419(34.29) = 237.37$$

$$\text{LCL}_{\overline{x}} = \overline{\overline{x}} - A_2\overline{R} = 223 - 0.419(34.29) = 208.63$$

$$\text{UCL}_R = D_4\overline{R} = 1.924(34.29) = 65.97$$

$$\text{LCL}_R = D_3\overline{R} = 0.076(34.29) = 2.61$$

(b) Assuming that both charts exhibit control, estimate the process mean and standard deviation.

$$\hat{\mu} = \overline{\overline{x}} = 223; \quad \hat{\sigma}_x = \overline{R}/d_2 = 34.29/2.704 = 12.68$$

(c) If the quality characteristic is normally distributed and if the specifications are 220 ± 35, can the process meet the specifications? Estimate the fraction nonconforming.

$\hat{C}_P = \dfrac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \dfrac{+35-(-35)}{6(12.68)} = 0.92$, the process is not capable of meeting specifications.

$$\hat{p} = \Pr\{x > \text{USL}\} + \Pr\{x < \text{LSL}\} = 1 - \Pr\{x < \text{USL}\} + \Pr\{x < \text{LSL}\} = 1 - \Pr\{x \le 255\} + \Pr\{x \le 185\}$$

$$= 1 - \Phi\left(\frac{255-223}{12.68}\right) + \Phi\left(\frac{185-223}{12.68}\right) = 1 - \Phi(2.52) + \Phi(-3.00) = 1 - 0.99413 + 0.00135 = 0.0072$$

(d) Assuming the variance to remain constant, state where the process mean should be located to minimize the fraction nonconforming. What would be the value of the fraction nonconforming under these conditions?

The process mean should be located at the nominal dimension, 220, to minimize non-conforming units.

$$\hat{p} = 1 - \Phi\left(\frac{255-220}{12.68}\right) + \Phi\left(\frac{185-220}{12.68}\right) = 1 - \Phi(2.76) + \Phi(-2.76) = 1 - 0.99711 + 0.00289 = 0.00578$$

6.39.

An $\bar{x}$ chart has a center line of 100, uses three-sigma control limits, and is based on a sample size of four. The process standard deviation is known to be six. If the process mean shifts from 100 to 92, what is the probability of detecting this shift on the first sample following the shift?

$\mu_0 = 100;\quad L = 3;\quad n = 4;\quad \sigma = 6;\quad \mu_1 = 92$

$k = (\mu_1 - \mu_0)/\sigma = (92 - 100)/6 = -1.33$

$$
\begin{aligned}
\Pr\{\text{detecting shift on 1st sample}\} &= 1 - \Pr\{\text{not detecting shift on 1st sample}\} \\
&= 1 - \beta \\
&= 1 - \left[\Phi\left(L - k\sqrt{n}\right) - \Phi\left(-L - k\sqrt{n}\right)\right] \\
&= 1 - \left[\Phi\left(3 - (-1.33)\sqrt{4}\right) - \Phi\left(-3 - (-1.33)\sqrt{4}\right)\right] \\
&= 1 - [\Phi(5.66) - \Phi(-0.34)] \\
&= 1 - [1 - 0.37] \\
&= 0.37
\end{aligned}
$$

6.60.

The following $\bar{x}$ and s charts based on n = 4 have shown statistical control:

$\bar{x}$ Chart	s Chart
UCL = 710	UCL = 18.08
Center line = 700	Center line = 7.979
LCL = 690	LCL = 0

(a) Estimate the process parameters μ and σ.

$$\hat{\mu} = \bar{\bar{x}} = 700; \;\; \hat{\sigma}_x = \bar{S} / c_4 = 7.979 / 0.9213 = 8.661$$

(b) If the specifications are at 705 ± 15, and the process output is normally distributed, estimate the fraction nonconforming.

$$\hat{p} = \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} = \Phi\left(\frac{\text{LSL} - \bar{\bar{x}}}{\hat{\sigma}_x}\right) + 1 - \Phi\left(\frac{\text{USL} - \bar{\bar{x}}}{\hat{\sigma}_x}\right) = \Phi\left(\frac{690 - 700}{8.661}\right) + 1 - \Phi\left(\frac{720 - 700}{8.661}\right)$$

$$= \Phi(-1.15) + 1 - \Phi(2.31) = 0.1251 + 1 - 0.9896 = 0.1355$$

(c) For the $\bar{x}$ chart, find the probability of a type I error, assuming σ is constant.

$$\alpha = \Pr\{\bar{x} < \text{LCL}\} + \Pr\{\bar{x} > \text{UCL}\} = \Phi\left(\frac{\text{LCL} - \bar{\bar{x}}}{\sigma_{\bar{x}}}\right) + 1 - \Phi\left(\frac{\text{UCL} - \bar{\bar{x}}}{\sigma_{\bar{x}}}\right) = \Phi\left(\frac{690 - 700}{8.661/\sqrt{4}}\right) + 1 - \Phi\left(\frac{710 - 700}{8.661/\sqrt{4}}\right)$$

$$= \Phi(-2.31) + 1 - \Phi(2.31) = 0.0104 + 1 - 0.9896 = 0.0208$$

(d) Suppose the process mean shifts to 693 and the standard deviation simultaneously shifts to 12. Find the probability of detecting this shift on the $\bar{x}$ chart on the first subsequent sample.

$$\Pr\{\text{detect on 1st sample}\} = \Pr\{\bar{x} < \text{LCL}\} + \Pr\{\bar{x} > \text{UCL}\}$$

$$= \Phi\left(\frac{\text{LCL} - \mu_{\text{new}}}{\sigma_{\bar{x},\text{new}}}\right) + 1 - \Phi\left(\frac{\text{UCL} - \mu_{\text{new}}}{\sigma_{\bar{x},\text{new}}}\right) = \Phi\left(\frac{690 - 693}{12/\sqrt{4}}\right) + 1 - \Phi\left(\frac{710 - 693}{12/\sqrt{4}}\right)$$

$$= \Phi(-0.5) + 1 - \Phi(2.83) = 0.3085 + 1 - 0.9977 = 0.3108$$

(e) For the shift of part (d), find the average run length.

$$\text{ARL}_1 = \frac{1}{1-\beta} = \frac{1}{1 - \Pr\{\text{not detect}\}} = \frac{1}{\Pr\{\text{detect}\}} = \frac{1}{0.3108} = 3.22$$