HW2: 3.29, 3.30, 3.35, 3.36, 3.43, 3.52, 3.53, 3.57

3.29

A mechatronic assembly is subjected to a final functional test. Suppose that defects occur at random in these assemblies, and that defects occur according to a Poisson distribution with parameter $\lambda = 0.02$.

(a) What is the probability that an assembly will have exactly one defect?

This is a Poisson distribution with parameter $\lambda = 0.02$, $x \sim \text{POI}(0.02)$.

$$\Pr\{x=1\} = p(1) = \frac{e^{-0.02}(0.02)^1}{1!} = 0.0196$$

(b) What is the probability that an assembly will have one or more defects?

$$\Pr\{x \geq 1\} = 1 - \Pr\{x=0\} = 1 - p(0) = 1 - \frac{e^{-0.02}(0.02)^0}{0!} = 1 - 0.9802 = 0.0198$$

(c) Suppose that you improve the process so that the occurrence rate of defects is cut in half to $\lambda = 0.01$. What effect does this have on the probability that an assembly will have one or more defects?
This is a Poisson distribution with parameter $\lambda = 0.01$, $x \sim \text{POI}(0.01)$.

$$\Pr\{x \geq 1\} = 1 - \Pr\{x=0\} = 1 - p(0) = 1 - \frac{e^{-0.01}(0.01)^0}{0!} = 1 - 0.9900 = 0.0100$$

Cutting the rate at which defects occur reduces the probability of one or more defects by approximately one-half, from 0.0198 to 0.0100.

3.30.

The probability distribution of x is $f(x) = ke^{-x}, 0 \leq x \leq \infty$. Find the appropriate value of k. Find the mean and variance of x.

For $f(x)$ to be a probability distribution, $\int_{-\infty}^{+\infty} f(x)dx$ must equal unity.

$$\int_0^{\infty} ke^{-x}dx = [-ke^{-x}]_0^{\infty} = -k[0-1] = k \Rightarrow 1$$

This is an exponential distribution with parameter $\lambda = 1$.

$\mu = 1/\lambda = 1$ (Eqn. 3.32)

$\sigma^2 = 1/\lambda^2 = 1$ (Eqn. 3.33)

3.35.

A production process operates with 1% nonconforming output. Every hour a sample of 25 units of product is taken, and the number of nonconforming units counted. If one or more nonconforming units are found, the process is stopped and the quality control technician must search for the cause of nonconforming production. Evaluate the performance of this decision rule.

This is a binomial distribution with parameter $p = 0.01$ and $n = 25$. The process is stopped if $x \geq 1$.

$$\Pr\{x \geq 1\} = 1 - \Pr\{x < 1\} = 1 - \Pr\{x = 0\} = 1 - \binom{25}{0}(0.01)^0(1-0.01)^{25} = 1 - 0.78 = 0.22$$

This decision rule means that 22% of the samples will have one or more nonconforming units, and the process will be stopped to look for a cause. This is a somewhat difficult operating situation.

This exercise may also be solved using Excel (Excel Function BINOMDIST(x, n, p, TRUE)) or Minitab.

MTB > Calc > Probability Distribution > Binomial

Cumulative Distribution Function

```
Binomial with n = 25 and p = 0.01

x  P( X <= x )
0     0.777821
```

3.36.

Continuation of Exercise 3.35. Consider the decision rule described in Exercise 3.35. Suppose that the process suddenly deteriorates to 4% nonconforming output. How many samples, on average, will be required to detect this?

$x \sim \text{BIN}(25, 0.04)$ Stop process if $x \geq 1$.

$$\Pr\{x \geq 1\} = 1 - \Pr\{x < 1\} = 1 - \Pr\{x = 0\} = 1 - \binom{25}{0}(0.04)^0(1-0.04)^{25} = 1 - 0.36 = 0.64$$

MTB > Calc > Probability Distribution > Binomial

Cumulative Distribution Function

```
Binomial with n = 25 and p = 0.04

x  P( X <= x )
0     0.360397
```

3.43.

An electronic component for a medical X-ray unit is produced in lots of size N = 25. An acceptance testing procedure is used by the purchaser to protect against lots that contain too many nonconforming components. The procedure consists of selecting five components at random from the lot (without replacement) and testing them. If none of the components is nonconforming, the lot is accepted.

(a) If the lot contains two nonconforming components, what is the probability of lot acceptance?

This is a hypergeometric distribution with N = 25 and n = 5, without replacement.
Given D = 2 and x = 0:

$$\Pr\{\text{Acceptance}\} = p(0) = \frac{\binom{2}{0}\binom{25-2}{5-0}}{\binom{25}{5}} = \frac{(1)(33{,}649)}{(53{,}130)} = 0.633$$

This exercise may also be solved using Excel (Excel Function HYPGEOMDIST(x, n, D, N)) or Minitab.

MTB > Calc > Probability Distribution > Hypergeometric

Cumulative Distribution Function

```
Hypergeometric with N = 25, M = 2, and n = 5

x  P( X <= x )
0     0.633333
```

(b) Calculate the desired probability in (a) using the binomial approximation. Is this approximation satisfactory? Why or why not?

For the binomial approximation to the hypergeometric, $p = D/N = 2/25 = 0.08$ and n = 5.

$$\Pr\{\text{acceptance}\} = p(0) = \binom{5}{0}(0.08)^0(1-0.08)^5 = 0.659$$

This approximation, though close to the exact solution for x = 0, violates the rule-of-thumb that n/N = 5/25 = 0.20 be less than the suggested 0.1. The binomial approximation is not satisfactory in this case.

(c) Suppose the lot size was N = 150. Would the binomial approximation be satisfactory in this case?

For N = 150, $n/N = 5/150 = 0.033 \leq 0.1$, so the binomial approximation would be a satisfactory approximation the hypergeometric in this case.

(d) Suppose that the purchaser will reject the lot with the decision rule of finding one or more nonconforming components in a sample of size n, and wants the lot to be rejected with probability at least 0.95 if the lot contains five or more nonconforming components. How large should the sample size n be?

Find n to satisfy $\Pr\{x \geq 1 \mid D \geq 5\} \geq 0.95$, or equivalently $\Pr\{x = 0 \mid D = 5\} < 0.05$.

$$p(0) = \frac{\binom{5}{0}\binom{25-5}{n-0}}{\binom{25}{n}} = \frac{\binom{5}{0}\binom{20}{n}}{\binom{25}{n}}$$

try $n = 10$

$$p(0) = \frac{\binom{5}{0}\binom{20}{10}}{\binom{25}{10}} = \frac{(1)(184,756)}{(3,268,760)} = 0.057$$

try $n = 11$

$$p(0) = \frac{\binom{5}{0}\binom{20}{11}}{\binom{25}{11}} = \frac{(1)(167,960)}{(4,457,400)} = 0.038$$

Let sample size n = 11.

3.52.

The output voltage of a power supply is normally distributed with mean 5 V and standard deviation 0.02 V. If the lower and upper specifications for voltage are 4.95 V and 5.05 V, respectively, what is the probability that a power supply selected at random will conform to the specifications on voltage?

$x \sim N(5, 0.02^2)$; LSL = 4.95 V; USL = 5.05 V

$$\Pr\{\text{Conformance}\} = \Pr\{\text{LSL} \le x \le \text{USL}\} = \Pr\{x \le \text{USL}\} - \Pr\{x \le \text{LSL}\}$$
$$= \Phi\left(\frac{5.05-5}{0.02}\right) - \Phi\left(\frac{4.95-5}{0.02}\right) = \Phi(2.5) - \Phi(-2.5) = 0.99379 - 0.00621 = 0.98758$$

3.53.

Continuation of Exercise 3.52. Reconsider the power supply manufacturing process in Exercise 3.52. Suppose we wanted to improve the process. Can shifting the mean reduce the number of nonconforming units produced? How much would the process variability need to be reduced in order to have all but one out of 1000 units conform to the specifications?

The process, with mean 5 V, is currently centered between the specification limits (target = 5 V). Shifting the process mean in either direction would increase the number of nonconformities produced. Desire Pr{Conformance} = 1 / 1000 = 0.001. Assume that the process remains centered between the specification limits at 5 V. Need $\Pr\{x \le \text{LSL}\}$ = 0.001 / 2 = 0.0005.

$$\Phi(z) = 0.0005$$
$$z = \Phi^{-1}(0.0005) = -3.29$$
$$z = \frac{\text{LSL}-\mu}{\sigma}, \quad \text{so } \sigma = \frac{\text{LSL}-\mu}{z} = \frac{4.95-5}{-3.29} = 0.015$$

Process variance must be reduced to 0.015^2 to have at least 999 of 1000 conform to specification.

3.57.

The specifications on an electronic component in a target-acquisition system are that its life must be between 5000 and 10,000 h. The life is normally distributed with mean 7500 h. The manufacturer realizes a price of $10 per unit produced; however, defective units must be replaced at a cost of $5 to the manufacturer. Two different manufacturing processes can be used, both of which have the same mean life. However, the standard deviation of life for process 1 is 1000 h, whereas for process 2 it is only 500 h. Production costs for process 2 are twice those for process 1. What value of production costs will determine the selection between processes 1 and 2?

$x_1 \sim N(7500, \sigma_1^2 = 1000^2)$; $x_2 \sim N(7500, \sigma_2^2 = 500^2)$; LSL = 5,000 h; USL = 10,000 h

sales = \$10/unit, defect = \$5/unit, profit = \$10 $\times$ Pr{good} + \$5 $\times$ Pr{bad} $- c$

For Process 1

$$\begin{aligned}
\text{proportion defective} = p_1 &= 1 - \Pr\{\text{LSL} \le x_1 \le \text{USL}\} = 1 - \Pr\{x_1 \le \text{USL}\} + \Pr\{x_1 \le \text{LSL}\} \\
&= 1 - \Pr\left\{z_1 \le \frac{10{,}000 - 7{,}500}{1{,}000}\right\} + \Pr\left\{z_1 \le \frac{5{,}000 - 7{,}500}{1{,}000}\right\} \\
&= 1 - \Phi(2.5) + \Phi(-2.5) = 1 - 0.9938 + 0.0062 = 0.0124
\end{aligned}$$

profit for process 1 = 10 (1 – 0.0124) + 5 (0.0124) – c_1 = 9.9380 – c_1

For Process 2

$$\begin{aligned}
\text{proportion defective} = p_2 &= 1 - \Pr\{\text{LSL} \le x_2 \le \text{USL}\} = 1 - \Pr\{x_2 \le \text{USL}\} + \Pr\{x_2 \le \text{LSL}\} \\
&= 1 - \Pr\left\{z_2 \le \frac{10{,}000 - 7{,}500}{500}\right\} + \Pr\left\{z_2 \le \frac{5{,}000 - 7{,}500}{500}\right\} \\
&= 1 - \Phi(5) + \Phi(-5) = 1 - 1.0000 + 0.0000 = 0.0000
\end{aligned}$$

profit for process 2 = 10 (1 – 0.0000) + 5 (0.0000) – c_2 = 10 – c_2

If $c_2 > c_1 + 0.0620$, then choose process 1