8.7.

A process is in control with $\bar{\bar{x}} = 100, \bar{s} = 1.05$, and n = 5. The process specifications are at 95 ± 10. The quality characteristic has a normal distribution.

$$\hat{\mu} = \bar{\bar{x}} = 100; \bar{s} = 1.05; \hat{\sigma}_x = \bar{s}/c_4 = 1.05/0.9400 = 1.117$$

(a) Estimate the potential capability.

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{(95+10)-(95-10)}{6(1.117)} = 2.98$$

(b) Estimate the actual capability.

$$\hat{C}_{pl} = \frac{\hat{\mu} - \text{LSL}_x}{3\hat{\sigma}_x} = \frac{100-(95-10)}{3(1.117)} = 4.48; \quad \hat{C}_{pu} = \frac{\text{USL}_x - \hat{\mu}}{3\hat{\sigma}_x} = \frac{(95+10)-100}{3(1.117)} = 1.49$$

$$\hat{C}_{pk} = \min(\hat{C}_{pl}, \hat{C}_{pu}) = 1.49$$

(c) How much could the fallout in the process be reduced if the process were corrected to operate at the nominal specification?

$$\begin{aligned}
\hat{p}_{\text{Actual}} &= \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} \\
&= \Pr\{x < \text{LSL}\} + \left[1 - \Pr\{x \le \text{USL}\}\right] \\
&= \Pr\left\{z < \frac{\text{LSL} - \hat{\mu}}{\hat{\sigma}}\right\} + \left[1 - \Pr\left\{z \le \frac{\text{USL} - \hat{\mu}}{\hat{\sigma}}\right\}\right] \\
&= \Pr\left\{z < \frac{85-100}{1.117}\right\} + \left[1 - \Pr\left\{z \le \frac{105-100}{1.117}\right\}\right] \\
&= \Phi(-13.429) + \left[1 - \Phi(4.476)\right] \\
&= 0.0000 + \left[1 - 0.999996\right] \\
&= 0.000004
\end{aligned}$$

$$\begin{aligned}
\hat{p}_{\text{Potential}} &= \Pr\left\{z < \frac{85-95}{1.117}\right\} + \left[1 - \Pr\left\{z \le \frac{105-95}{1.117}\right\}\right] \\
&= \Phi(-8.953) + \left[1 - \Phi(8.953)\right] \\
&= 0.000000 + \left[1 - 1.000000\right] \\
&= 0.000000
\end{aligned}$$

8.12.

Suppose that 20 of the parts manufactured by the processes in Exercise 8.11 were assembled so that their dimensions were additive; that is, $x = x_1 + x_2 + \cdots + x_{20}$. Specifications on x are 2,000 ± 200. Would you prefer to produce the parts using process A or process B? Why? Do the capability ratios computed in Exercise 8.11 provide any guidance for process selection?

Process A: $\hat{\mu}_A = 20(100) = 2000; \hat{\sigma}_A = \sqrt{20\hat{\sigma}^2} = \sqrt{20(3.191)^2} = 14.271$

Process B: $\hat{\mu}_B = 20(105) = 2100; \hat{\sigma}_B = \sqrt{20\hat{\sigma}^2} = \sqrt{20(1.064)^2} = 4.758$

Process B will result in fewer defective assemblies. For the parts $\left(\hat{C}_{pk,A} = 1.045\right) < \left(1.566 = \hat{C}_{pk,B}\right)$ indicates that more parts from Process B are within specification than from Process A.

9.4.

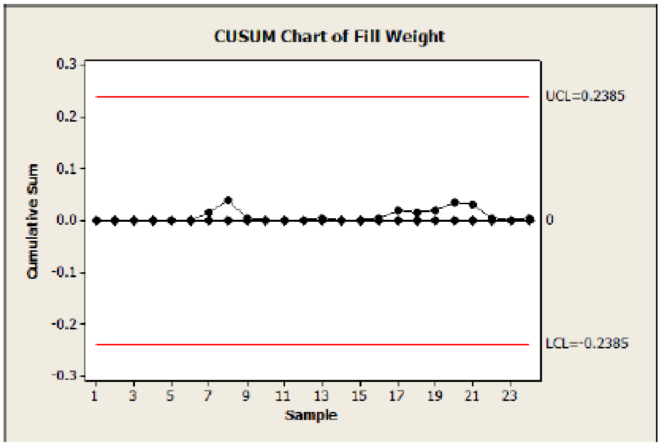
A machine is used to fill cans with motor oil additive. A single sample can is obtained. Since the filling process is automated, it has very stable variability, and long experience indicates that σ = 0.05 oz. The individual observations for 24 hours of operation are shown in Table 9E.2.

TABLE 9E.2
Fill Data for Exercise 9.4

Sample Number	x	Sample Number	x
1	8.00	13	8.05
2	8.01	14	8.04
3	8.02	15	8.03
4	8.01	16	8.05
5	8.00	17	8.06
6	8.01	18	8.04
7	8.06	19	8.05
8	8.07	20	8.06
9	8.01	21	8.04
10	8.04	22	8.02
11	8.02	23	8.03
12	8.01	24	8.05

(a) Assuming that the process target is 8.02 oz, set up a tabular CUSUM for this process. Design the CUSUM using the standardized values h = 4.77 and k = ½.

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM
Enter Subgroup size = 1 and Target = 8.02
In CUSUM Options, enter Standard deviation = 0.05, and on the Plan/Type tab, select One-sided type of CUSUM, and set h = 4.77 and k = 0.5



There are no out-of-control signals.

(b) Does the value of σ = 0.05 seem reasonable for this process?

$\hat{\sigma} = \overline{MR2}/1.128 = 0.0186957/1.128 = 0.0166$, so $\sigma = 0.05$ is probably not reasonable.

9.6.

Reconsider the data in Exercise 9.4. Suppose the data there represent observations taken immediately after a process adjustment that was intended to reset the process to a target of $\mu_0 = 8.00$. Set up and apply an FIR CUSUM to monitor this process.

$\mu_0 = 8.00$, $\sigma = 0.05$, $k = 0.5$, $h = 4.77$

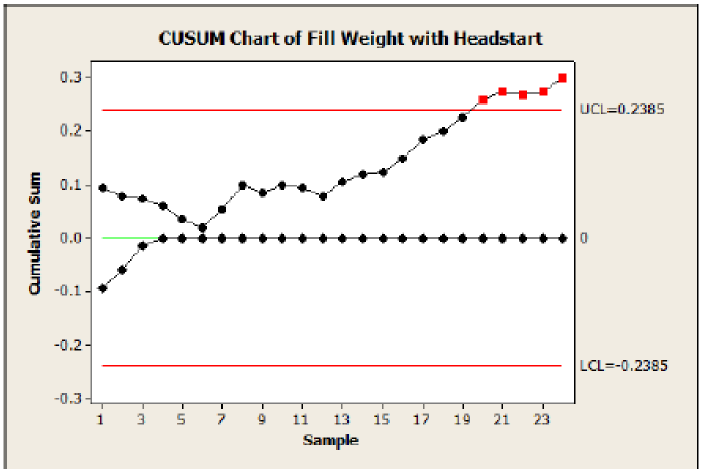
$H = h\ \sigma = 4.77\ (0.05) = 0.2385$

FIR = $H/2$, FIR in # of standard deviations = $h/2 = 4.77/2 = 2.385$

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM

Enter Subgroup size = 1 and Target = 8.00

In CUSUM Options, enter Standard deviation = 0.05; and on the Plan/Type tab, select One-sided type of CUSUM, enter FIR = 2.385, and set h = 4.77 and k = 0.5



Test Results for CUSUM Chart of Ex9-4can

```
TEST. One point beyond control limits.
Test Failed at points:  20, 21, 22, 23, 24
```

The process signals out of control at observation 20. Process was out of control at process start-up.

9.13. ☺

Consider the hospital emergency room waiting time data in Exercise 8.16. Set up a CUSUM chart for monitoring this process. Does the process seem to be in statistical control?

From 8.16:

■ **TABLE 8E.4**

Waiting Time Data for Exercise 8.16

9	1	4	1	2
8	8	11	2	4
6	2	2	2	1
3	3	7	3	6
2	5	10	1	3
5	7	3	2	7
8	8	3	3	5
1	8	4	5	7

$\overline{x} \approx p_{50} = 4.55; \quad p_{84} = 7.34; \quad \hat{\sigma} = p_{84} - p_{50} = 7.34 - 4.55 = 2.79$

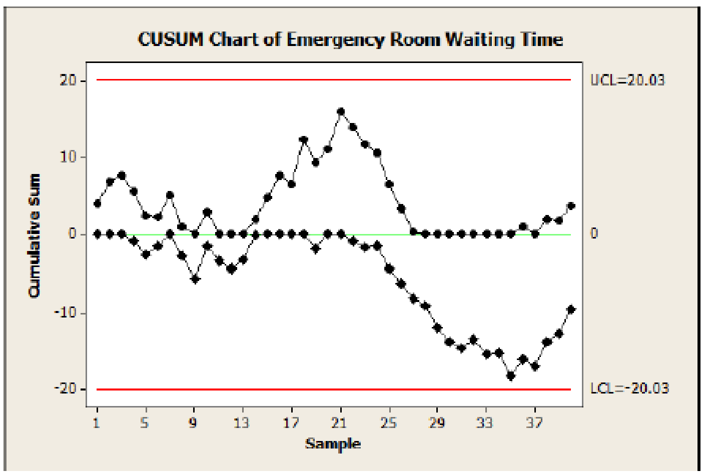
(read data down, then right)

Design to detect 0.5σ shift in process mean. For $\delta = 0.5$, $k = ½\,\delta = 0.25$. Select $h = 8.01$ to obtain $ARL_0 = 370$ (Table 9.4). Set target as $\mu_0 = 4.5$ min (USL = 10 min).

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM

Enter Subgroup size = 1 and Target = 4.5

In CUSUM Options, Plan/Type tab, select One-sided type of CUSUM, and set h = 8.01 and k = 0.25



The process is in statistical control, with no out-of-control signals.

9.14. ☺

Consider the hospital emergency room waiting time data in Exercise 8.16. Set up an EWMA control chart for monitoring this process using $\lambda = 0.2$. Does the process seem to be in statistical control?

From 8.16:

TABLE 8E.4

Waiting Time Data for Exercise 8.16

9	1	4	1	2
8	8	11	2	4
6	2	2	2	1
3	3	7	3	6
2	5	10	1	3
5	7	3	2	7
8	8	3	3	5
1	8	4	5	7

$\overline{x} \approx p_{50} = 4.55$; $p_{84} = 7.34$; $\hat{\sigma} = p_{84} - p_{50} = 7.34 - 4.55 = 2.79$

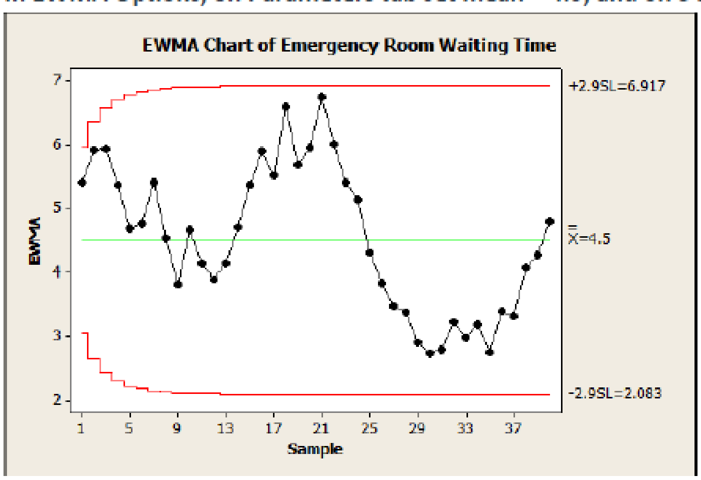
(read data down, then right)

Design to detect 0.5σ shift in process mean. Set $\lambda = 0.2$ and select $L = 2.9$ (Table 9.11). Set target as $\mu_0 = 4.5$ min (USL = 10 min).

MTB > Stat > Control Charts > Time-Weighted Charts > EWMA

Enter Subgroup size = 1 and Weight = 0.2

In EWMA Options, on Parameters tab set mean = 4.5, and on S Limits tab set limits = 2.9



The process is in statistical control, with no out-of-control signals.

9.21.

Consider a standardized two-sided CUSUM with k = 0.2 and h=8. Use Siegmund's procedure to evaluate the in-control ARL performance of this scheme. Find ARL1 for δ* = 0.5.

In control ARL performance:

$\delta^* = 0$

$\Delta^+ = \delta^* - k = 0 - 0.2 = -0.2$

$\Delta^- = -\delta^* - k = -0 - 0.2 = -0.2$

$b = h + 1.166 = 8 + 1.166 = 9.166$

$$\text{ARL}_0^+ = \text{ARL}_0^- \cong \frac{\exp[-2(-0.2)(9.166)] + 2(-0.2)(9.166) - 1}{2(-0.2)^2} = 430.556$$

$$\frac{1}{\text{ARL}_0} = \frac{1}{\text{ARL}_0^+} + \frac{1}{\text{ARL}_0^-} = \frac{2}{430.556} = 0.005$$

$\text{ARL}_0 = 1/0.005 = 215.23$

Out of control ARL Performance:

$\delta^* = 0.5$

$\Delta^+ = \delta^* - k = 0.5 - 0.2 = 0.3$

$\Delta^- = -\delta^* - k = -0.5 - 0.2 = -0.7$

$b = h + 1.166 = 8 + 1.166 = 9.166$

$$\text{ARL}_1^+ = \frac{\exp[-2(0.3)(9.166)] + 2(0.3)(9.166) - 1}{2(0.3)^2} = 25.023$$

$$\text{ARL}_1^- = \frac{\exp[-2(-0.7)(9.166)] + 2(-0.7)(9.166) - 1}{2(-0.7)^2} = 381,767$$

$$\frac{1}{\text{ARL}_1} = \frac{1}{\text{ARL}_1^+} + \frac{1}{\text{ARL}_1^-} = \frac{1}{25.023} + \frac{1}{381,767} = 0.040$$

$\text{ARL}_1 = 1/0.040 = 25.02$