Computability Theory V

Turing Machine

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Oct. 19, 2015

Assignment 2 is announced! (deadline Nov. 7)

Turing Machine

Alan Turing

Alan Turing (23Jun.1912-7Jun.1954), an English student of Church, introduced a machine model for effective calculation in

“On Computable Numbers, with an Application to the Entsheidungs problem”,

Proc. of the London Mathematical Society, **42**:230-265, 1936.

Turing Machine, Halting Problem, Turing Test

British Prime Minister Gordon Brown:

"…I am pleased to have the chance to say how deeply sorry I and we all are for what happened to him ... So on behalf of the British government, and all those who live freely thanks to Alan's work, I am very proud to say: we're sorry, you deserved so much better."

Motivation

What are necessary for a machine to calculate a function?

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- The machine should be able to interpret numbers;
- The machine must be able to operate and manipulate numbers according to a set of predefined instructions;

and

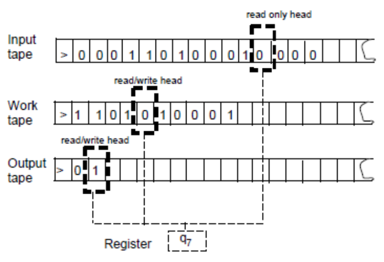
- The input number has to be stored in an accessible place;
- There should be an accessible place for the machine to store the intermediate results;
- The output number has to be put in an accessible place.

Turing Machine

A k-tape Turing Machine $\mathbb{M}$ has k-tapes such that

- The first tape is the read-only input tape.
- The other $k-1$ tapes are the read/write work tapes.
- The k-th tape is also used as the output tape.

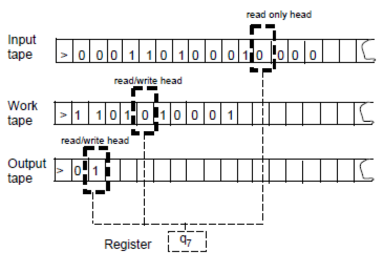
Every tape comes with a read/write head.



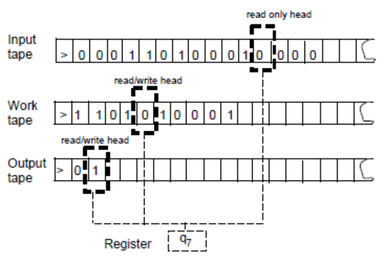
Turing Machine

The machine is described by a tuple (Γ, Q, δ) containing

- A finite set Γ, called alphabet, of symbols. It contains a blank symbol $\Box$, a start symbol $\triangleright$, and the digits 0 and 1.
- A finite set Q of states. It contains a start state q_{s} and a halting state q_{h}.
- A transition function $\delta : Q \times \Gamma^k \to Q \times \Gamma^{k-1} \times \{\leftarrow, -, \to\}^k$, describing the rules of each computation step.



Computation and Configuration



Configuration, initial configuration, final configuration, computation step

An Example

$Q = \{q_s, q_h, q_1, q_2, q_3\}$, $\Gamma = \{0, 1, \square, \triangleright\}$,
and δ is as follows:

$p \in Q$	$\sigma \in \Gamma$	$\delta(p, \sigma)$
q_s	0	$(q_s, 0, \rightarrow)$
q_s	1	$(q_s, 1, \rightarrow)$
q_s	$\square$	$(q_1, \square, \leftarrow)$
q_s	$\triangleright$	$(q_s, \triangleright, \rightarrow)$
q_1	0	$(q_2, \square, \rightarrow)$
q_1	1	$(q_3, \square, \rightarrow)$
q_1	$\square$	$(q_1, \square, -)$
q_1	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_2	0	$(q_s, 0, \leftarrow)$
q_2	1	$(q_s, 0, \leftarrow)$
q_2	$\square$	$(q_s, 0, \leftarrow)$
q_2	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_3	0	$(q_s, 1, \leftarrow)$
q_3	1	$(q_s, 1, \leftarrow)$
q_3	$\square$	$(q_s, 1, \leftarrow)$
q_3	$\triangleright$	$(q_h, \triangleright, \rightarrow)$

An Example

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$p \in Q$	$\sigma \in \Gamma$	$\delta(p, \sigma)$
q_s	0	$(q_s, 0, \rightarrow)$
q_s	1	$(q_s, 1, \rightarrow)$
q_s	$\square$	$(q_1, \square, \leftarrow)$
q_s	$\triangleright$	$(q_s, \triangleright, \rightarrow)$
q_1	0	$(q_2, \square, \rightarrow)$
q_1	1	$(q_3, \square, \rightarrow)$
q_1	$\square$	$(q_1, \square, -)$
q_1	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_2	0	$(q_s, 0, \leftarrow)$
q_2	1	$(q_s, 0, \leftarrow)$
q_2	$\square$	$(q_s, 0, \leftarrow)$
q_2	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_3	0	$(q_s, 1, \leftarrow)$
q_3	1	$(q_s, 1, \leftarrow)$
q_3	$\square$	$(q_s, 1, \leftarrow)$
q_3	$\triangleright$	$(q_h, \triangleright, \rightarrow)$

Start the machine with input 010

An Example

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q_s	$\square$	$(q_1, \square, \leftarrow)$
q_s	$\triangleright$	$(q_s, \triangleright, \rightarrow)$
q_1	0	$(q_2, \square, \rightarrow)$
q_1	1	$(q_3, \square, \rightarrow)$
q_1	$\square$	$(q_1, \square, -)$
q_1	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_2	0	$(q_s, 0, \leftarrow)$
q_2	1	$(q_s, 0, \leftarrow)$
q_2	$\square$	$(q_s, 0, \leftarrow)$
q_2	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_3	0	$(q_s, 1, \leftarrow)$
q_3	1	$(q_s, 1, \leftarrow)$
q_3	$\square$	$(q_s, 1, \leftarrow)$
q_3	$\triangleright$	$(q_h, \triangleright, \rightarrow)$

Start the machine with input 010

- $q_s, \underline{\triangleright}010$

An Example

$Q = \{q_s, q_h, q_1, q_2, q_3\}$, $\Gamma = \{0, 1, \Box, \rhd\}$,
and δ is as follows:

$p \in Q$	$\sigma \in \Gamma$	$\delta(p, \sigma)$
q_s	0	$(q_s, 0, \rightarrow)$
q_s	1	$(q_s, 1, \rightarrow)$
q_s	$\Box$	$(q_1, \Box, \leftarrow)$
q_s	$\rhd$	$(q_s, \rhd, \rightarrow)$
q_1	0	$(q_2, \Box, \rightarrow)$
q_1	1	$(q_3, \Box, \rightarrow)$
q_1	$\Box$	$(q_1, \Box, -)$
q_1	$\rhd$	$(q_h, \rhd, \rightarrow)$
q_2	0	$(q_s, 0, \leftarrow)$
q_2	1	$(q_s, 0, \leftarrow)$
q_2	$\Box$	$(q_s, 0, \leftarrow)$
q_2	$\rhd$	$(q_h, \rhd, \rightarrow)$
q_3	0	$(q_s, 1, \leftarrow)$
q_3	1	$(q_s, 1, \leftarrow)$
q_3	$\Box$	$(q_s, 1, \leftarrow)$
q_3	$\rhd$	$(q_h, \rhd, \rightarrow)$

Start the machine with input 010

- $q_s, \underline{\rhd}010$
- $q_s, \rhd\underline{0}10$

An Example

$Q = \{q_s, q_h, q_1, q_2, q_3\}$, $\Gamma = \{0, 1, \Box, \rhd\}$,
and δ is as follows:

$p \in Q$	$\sigma \in \Gamma$	$\delta(p, \sigma)$
q_s	0	$(q_s, 0, \rightarrow)$
q_s	1	$(q_s, 1, \rightarrow)$
q_s	$\Box$	$(q_1, \Box, \leftarrow)$
q_s	$\rhd$	$(q_s, \rhd, \rightarrow)$
q_1	0	$(q_2, \Box, \rightarrow)$
q_1	1	$(q_3, \Box, \rightarrow)$
q_1	$\Box$	$(q_1, \Box, -)$
q_1	$\rhd$	$(q_h, \rhd, \rightarrow)$
q_2	0	$(q_s, 0, \leftarrow)$
q_2	1	$(q_s, 0, \leftarrow)$
q_2	$\Box$	$(q_s, 0, \leftarrow)$
q_2	$\rhd$	$(q_h, \rhd, \rightarrow)$
q_3	0	$(q_s, 1, \leftarrow)$
q_3	1	$(q_s, 1, \leftarrow)$
q_3	$\Box$	$(q_s, 1, \leftarrow)$
q_3	$\rhd$	$(q_h, \rhd, \rightarrow)$

Start the machine with input 010

- $q_s, \underline{\rhd}010$
- $q_s, \rhd\underline{0}10$
- $q_s, \rhd0\underline{1}0$

An Example

$Q = \{q_s, q_h, q_1, q_2, q_3\}$, $\Gamma = \{0, 1, \Box, \triangleright\}$,
and δ is as follows:

$p \in Q$	$\sigma \in \Gamma$	$\delta(p, \sigma)$
q_s	0	$(q_s, 0, \rightarrow)$
q_s	1	$(q_s, 1, \rightarrow)$
q_s	$\Box$	$(q_1, \Box, \leftarrow)$
q_s	$\triangleright$	$(q_s, \triangleright, \rightarrow)$
q_1	0	$(q_2, \Box, \rightarrow)$
q_1	1	$(q_3, \Box, \rightarrow)$
q_1	$\Box$	$(q_1, \Box, -)$
q_1	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_2	0	$(q_s, 0, \leftarrow)$
q_2	1	$(q_s, 0, \leftarrow)$
q_2	$\Box$	$(q_s, 0, \leftarrow)$
q_2	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_3	0	$(q_s, 1, \leftarrow)$
q_3	1	$(q_s, 1, \leftarrow)$
q_3	$\Box$	$(q_s, 1, \leftarrow)$
q_3	$\triangleright$	$(q_h, \triangleright, \rightarrow)$

Start the machine with input 010

- q_s, $\underline{\triangleright}010$
- q_s, $\triangleright\underline{0}10$
- q_s, $\triangleright0\underline{1}0$
- q_s, $\triangleright01\underline{0}$

An Example

$Q = \{q_s, q_h, q_1, q_2, q_3\}$, $\Gamma = \{0, 1, \square, \triangleright\}$,
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q_s	$\square$	$(q_1, \square, \leftarrow)$
q_s	$\triangleright$	$(q_s, \triangleright, \rightarrow)$
q_1	0	$(q_2, \square, \rightarrow)$
q_1	1	$(q_3, \square, \rightarrow)$
q_1	$\square$	$(q_1, \square, -)$
q_1	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_2	0	$(q_s, 0, \leftarrow)$
q_2	1	$(q_s, 0, \leftarrow)$
q_2	$\square$	$(q_s, 0, \leftarrow)$
q_2	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_3	0	$(q_s, 1, \leftarrow)$
q_3	1	$(q_s, 1, \leftarrow)$
q_3	$\square$	$(q_s, 1, \leftarrow)$
q_3	$\triangleright$	$(q_h, \triangleright, \rightarrow)$

Start the machine with input 010

- $q_s, \underline{\triangleright}010$
- $q_s, \triangleright\underline{0}10$
- $q_s, \triangleright0\underline{1}0$
- $q_s, \triangleright01\underline{0}$
- $q_s, \triangleright010\underline{\square}$

An Example

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q_s	$\triangleright$	$(q_s, \triangleright, \rightarrow)$
q_1	0	$(q_2, \square, \rightarrow)$
q_1	1	$(q_3, \square, \rightarrow)$
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q_2	0	$(q_s, 0, \leftarrow)$
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q_2	$\square$	$(q_s, 0, \leftarrow)$
q_2	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_3	0	$(q_s, 1, \leftarrow)$
q_3	1	$(q_s, 1, \leftarrow)$
q_3	$\square$	$(q_s, 1, \leftarrow)$
q_3	$\triangleright$	$(q_h, \triangleright, \rightarrow)$

Start the machine with input 010

- $q_s, \underline{\triangleright}010$
- $q_s, \triangleright\underline{0}10$
- $q_s, \triangleright0\underline{1}0$
- $q_s, \triangleright01\underline{0}$
- $q_s, \triangleright010\underline{\square}$
- $q_1, \triangleright01\underline{0}\square$

An Example

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q_1	0	$(q_2, \square, \rightarrow)$
q_1	1	$(q_3, \square, \rightarrow)$
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q_1	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_2	0	$(q_s, 0, \leftarrow)$
q_2	1	$(q_s, 0, \leftarrow)$
q_2	$\square$	$(q_s, 0, \leftarrow)$
q_2	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_3	0	$(q_s, 1, \leftarrow)$
q_3	1	$(q_s, 1, \leftarrow)$
q_3	$\square$	$(q_s, 1, \leftarrow)$
q_3	$\triangleright$	$(q_h, \triangleright, \rightarrow)$

Start the machine with input 010

- $q_s, \underline{\triangleright}010$
- $q_s, \triangleright\underline{0}10$
- $q_s, \triangleright0\underline{1}0$
- $q_s, \triangleright01\underline{0}$
- $q_s, \triangleright010\underline{\square}$
- $q_1, \triangleright01\underline{0}\square$
- $q_2, \triangleright01\square\underline{\square}$

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q_s	$\triangleright$	$(q_s, \triangleright, \rightarrow)$
q_1	0	$(q_2, \square, \rightarrow)$
q_1	1	$(q_3, \square, \rightarrow)$
q_1	$\square$	$(q_1, \square, -)$
q_1	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_2	0	$(q_s, 0, \leftarrow)$
q_2	1	$(q_s, 0, \leftarrow)$
q_2	$\square$	$(q_s, 0, \leftarrow)$
q_2	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_3	0	$(q_s, 1, \leftarrow)$
q_3	1	$(q_s, 1, \leftarrow)$
q_3	$\square$	$(q_s, 1, \leftarrow)$
q_3	$\triangleright$	$(q_h, \triangleright, \rightarrow)$

Start the machine with input 010

- $q_s, \underline{\triangleright}010$
- $q_s, \triangleright\underline{0}10$
- $q_s, \triangleright 0\underline{1}0$
- $q_s, \triangleright 01\underline{0}$
- $q_s, \triangleright 010\underline{\square}$
- $q_1, \triangleright 01\underline{0}\square$
- $q_2, \triangleright 01\square\underline{\square}$
- $q_s, \triangleright 01\underline{\square}0$

An Example

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q_s	$\Box$	$(q_1, \Box, \leftarrow)$
q_s	$\triangleright$	$(q_s, \triangleright, \rightarrow)$
q_1	0	$(q_2, \Box, \rightarrow)$
q_1	1	$(q_3, \Box, \rightarrow)$
q_1	$\Box$	$(q_1, \Box, -)$
q_1	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_2	0	$(q_s, 0, \leftarrow)$
q_2	1	$(q_s, 0, \leftarrow)$
q_2	$\Box$	$(q_s, 0, \leftarrow)$
q_2	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_3	0	$(q_s, 1, \leftarrow)$
q_3	1	$(q_s, 1, \leftarrow)$
q_3	$\Box$	$(q_s, 1, \leftarrow)$
q_3	$\triangleright$	$(q_h, \triangleright, \rightarrow)$

Start the machine with input 010

- q_s, $\underline{\triangleright}010$
- q_s, $\triangleright\underline{0}10$
- q_s, $\triangleright0\underline{1}0$
- q_s, $\triangleright01\underline{0}$
- q_s, $\triangleright010\underline{\Box}$
- q_1, $\triangleright01\underline{0}\Box$
- q_2, $\triangleright01\Box\underline{\Box}$
- q_s, $\triangleright01\underline{\Box}0$
- q_1, $\triangleright0\underline{1}\Box0$

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q_s	$\square$	$(q_1, \square, \leftarrow)$
q_s	$\triangleright$	$(q_s, \triangleright, \rightarrow)$
q_1	0	$(q_2, \square, \rightarrow)$
q_1	1	$(q_3, \square, \rightarrow)$
q_1	$\square$	$(q_1, \square, -)$
q_1	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_2	0	$(q_s, 0, \leftarrow)$
q_2	1	$(q_s, 0, \leftarrow)$
q_2	$\square$	$(q_s, 0, \leftarrow)$
q_2	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_3	0	$(q_s, 1, \leftarrow)$
q_3	1	$(q_s, 1, \leftarrow)$
q_3	$\square$	$(q_s, 1, \leftarrow)$
q_3	$\triangleright$	$(q_h, \triangleright, \rightarrow)$

Start the machine with input 010

- $q_s, \underline{\triangleright}010$
- $q_s, \triangleright\underline{0}10$
- $q_s, \triangleright0\underline{1}0$
- $q_s, \triangleright01\underline{0}$
- $q_s, \triangleright010\underline{\square}$
- $q_1, \triangleright01\underline{0}\square$
- $q_2, \triangleright01\square\underline{\square}$
- $q_s, \triangleright01\underline{\square}0$
- $q_1, \triangleright0\underline{1}\square0$
- $q_3, \triangleright0\square\underline{\square}0$

An Example

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q_s	$\square$	$(q_1, \square, \leftarrow)$
q_s	$\triangleright$	$(q_s, \triangleright, \rightarrow)$
q_1	0	$(q_2, \square, \rightarrow)$
q_1	1	$(q_3, \square, \rightarrow)$
q_1	$\square$	$(q_1, \square, -)$
q_1	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_2	0	$(q_s, 0, \leftarrow)$
q_2	1	$(q_s, 0, \leftarrow)$
q_2	$\square$	$(q_s, 0, \leftarrow)$
q_2	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_3	0	$(q_s, 1, \leftarrow)$
q_3	1	$(q_s, 1, \leftarrow)$
q_3	$\square$	$(q_s, 1, \leftarrow)$
q_3	$\triangleright$	$(q_h, \triangleright, \rightarrow)$

Start the machine with input 010

- $q_s, \underline{\triangleright}010$
- $q_s, \triangleright\underline{0}10$
- $q_s, \triangleright0\underline{1}0$
- $q_s, \triangleright01\underline{0}$
- $q_s, \triangleright010\underline{\square}$
- $q_1, \triangleright01\underline{0}\square$
- $q_2, \triangleright01\square\underline{\square}$
- $q_s, \triangleright01\underline{\square}0$
- $q_1, \triangleright0\underline{1}\square0$
- $q_3, \triangleright0\square\underline{\square}0$
- $q_s, \triangleright0\underline{\square}10$

An Example

$Q = \{q_s, q_h, q_1, q_2, q_3\}$, $\Gamma = \{0, 1, \square, \triangleright\}$,
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q_s	1	$(q_s, 1, \rightarrow)$
q_s	$\square$	$(q_1, \square, \leftarrow)$
q_s	$\triangleright$	$(q_s, \triangleright, \rightarrow)$
q_1	0	$(q_2, \square, \rightarrow)$
q_1	1	$(q_3, \square, \rightarrow)$
q_1	$\square$	$(q_1, \square, -)$
q_1	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_2	0	$(q_s, 0, \leftarrow)$
q_2	1	$(q_s, 0, \leftarrow)$
q_2	$\square$	$(q_s, 0, \leftarrow)$
q_2	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_3	0	$(q_s, 1, \leftarrow)$
q_3	1	$(q_s, 1, \leftarrow)$
q_3	$\square$	$(q_s, 1, \leftarrow)$
q_3	$\triangleright$	$(q_h, \triangleright, \rightarrow)$

Start the machine with input 010

- $q_s, \underline{\triangleright}010$
- $q_s, \triangleright\underline{0}10$
- $q_s, \triangleright0\underline{1}0$
- $q_s, \triangleright01\underline{0}$
- $q_s, \triangleright010\underline{\square}$
- $q_1, \triangleright01\underline{0}\square$
- $q_2, \triangleright01\square\underline{\square}$
- $q_s, \triangleright01\underline{\square}0$
- $q_1, \triangleright0\underline{1}\square0$
- $q_3, \triangleright0\square\underline{\square}0$
- $q_s, \triangleright0\underline{\square}10$
- $q_1, \triangleright\underline{0}\square10$

An Example

$Q = \{q_s, q_h, q_1, q_2, q_3\}$, $\Gamma = \{0, 1, \Box, \triangleright\}$,
and δ is as follows:

$p \in Q$	$\sigma \in \Gamma$	$\delta(p, \sigma)$
q_s	0	$(q_s, 0, \rightarrow)$
q_s	1	$(q_s, 1, \rightarrow)$
q_s	$\Box$	$(q_1, \Box, \leftarrow)$
q_s	$\triangleright$	$(q_s, \triangleright, \rightarrow)$
q_1	0	$(q_2, \Box, \rightarrow)$
q_1	1	$(q_3, \Box, \rightarrow)$
q_1	$\Box$	$(q_1, \Box, -)$
q_1	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_2	0	$(q_s, 0, \leftarrow)$
q_2	1	$(q_s, 0, \leftarrow)$
q_2	$\Box$	$(q_s, 0, \leftarrow)$
q_2	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_3	0	$(q_s, 1, \leftarrow)$
q_3	1	$(q_s, 1, \leftarrow)$
q_3	$\Box$	$(q_s, 1, \leftarrow)$
q_3	$\triangleright$	$(q_h, \triangleright, \rightarrow)$

Start the machine with input 010

- $q_s, \underline{\triangleright}010$
- $q_s, \triangleright\underline{0}10$
- $q_s, \triangleright0\underline{1}0$
- $q_s, \triangleright01\underline{0}$
- $q_s, \triangleright010\underline{\Box}$
- $q_1, \triangleright01\underline{0}\Box$
- $q_2, \triangleright01\Box\underline{\Box}$
- $q_s, \triangleright01\underline{\Box}0$
- $q_1, \triangleright0\underline{1}\Box0$
- $q_3, \triangleright0\Box\underline{\Box}0$
- $q_s, \triangleright0\underline{\Box}10$
- $q_1, \triangleright\underline{0}\Box10$
- $q_2, \triangleright\Box\underline{\Box}10$

An Example

$Q = \{q_s, q_h, q_1, q_2, q_3\}$, $\Gamma = \{0, 1, \square, \triangleright\}$,
and δ is as follows:

$p \in Q$	$\sigma \in \Gamma$	$\delta(p, \sigma)$
q_s	0	$(q_s, 0, \rightarrow)$
q_s	1	$(q_s, 1, \rightarrow)$
q_s	$\square$	$(q_1, \square, \leftarrow)$
q_s	$\triangleright$	$(q_s, \triangleright, \rightarrow)$
q_1	0	$(q_2, \square, \rightarrow)$
q_1	1	$(q_3, \square, \rightarrow)$
q_1	$\square$	$(q_1, \square, -)$
q_1	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_2	0	$(q_s, 0, \leftarrow)$
q_2	1	$(q_s, 0, \leftarrow)$
q_2	$\square$	$(q_s, 0, \leftarrow)$
q_2	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_3	0	$(q_s, 1, \leftarrow)$
q_3	1	$(q_s, 1, \leftarrow)$
q_3	$\square$	$(q_s, 1, \leftarrow)$
q_3	$\triangleright$	$(q_h, \triangleright, \rightarrow)$

Start the machine with input 010

- q_s, $\underline{\triangleright}010$
- q_s, $\triangleright\underline{0}10$
- q_s, $\triangleright 0\underline{1}0$
- q_s, $\triangleright 01\underline{0}$
- q_s, $\triangleright 010\underline{\square}$
- q_1, $\triangleright 01\underline{0}\square$
- q_2, $\triangleright 01\square\underline{\square}$
- q_s, $\triangleright 01\underline{\square}0$
- q_1, $\triangleright 0\underline{1}\square 0$
- q_3, $\triangleright 0\square\underline{\square}0$
- q_s, $\triangleright 0\underline{\square}10$
- q_1, $\triangleright\underline{0}\square 10$
- q_2, $\triangleright\square\underline{\square}10$
- q_0, $\triangleright\underline{\square}010$

An Example

$Q = \{q_s, q_h, q_1, q_2, q_3\}$, $\Gamma = \{0, 1, \Box, \triangleright\}$,
and δ is as follows:

$p \in Q$	$\sigma \in \Gamma$	$\delta(p, \sigma)$
q_s	0	$(q_s, 0, \rightarrow)$
q_s	1	$(q_s, 1, \rightarrow)$
q_s	$\Box$	$(q_1, \Box, \leftarrow)$
q_s	$\triangleright$	$(q_s, \triangleright, \rightarrow)$
q_1	0	$(q_2, \Box, \rightarrow)$
q_1	1	$(q_3, \Box, \rightarrow)$
q_1	$\Box$	$(q_1, \Box, -)$
q_1	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_2	0	$(q_s, 0, \leftarrow)$
q_2	1	$(q_s, 0, \leftarrow)$
q_2	$\Box$	$(q_s, 0, \leftarrow)$
q_2	$\triangleright$	$(q_h, \triangleright, \rightarrow)$
q_3	0	$(q_s, 1, \leftarrow)$
q_3	1	$(q_s, 1, \leftarrow)$
q_3	$\Box$	$(q_s, 1, \leftarrow)$
q_3	$\triangleright$	$(q_h, \triangleright, \rightarrow)$

Start the machine with input 010

- $q_s, \underline{\triangleright}010$
- $q_s, \triangleright\underline{0}10$
- $q_s, \triangleright 0\underline{1}0$
- $q_s, \triangleright 01\underline{0}$
- $q_s, \triangleright 010\underline{\Box}$
- $q_1, \triangleright 01\underline{0}\Box$
- $q_2, \triangleright 01\Box\underline{\Box}$
- $q_s, \triangleright 01\underline{\Box}0$
- $q_1, \triangleright 0\underline{1}\Box 0$
- $q_3, \triangleright 0\Box\underline{\Box}0$
- $q_s, \triangleright 0\underline{\Box}10$
- $q_1, \triangleright\underline{0}\Box 10$
- $q_2, \triangleright\Box\underline{\Box}10$
- $q_0, \triangleright\underline{\Box}010$
- $q_1, \underline{\triangleright}\Box 010$

An Example

$Q = \{q_s, q_h, q_1, q_2, q_3\}$, $\Gamma = \{0, 1, \Box, \rhd\}$,
and δ is as follows:

$p \in Q$	$\sigma \in \Gamma$	$\delta(p, \sigma)$
q_s	0	$(q_s, 0, \rightarrow)$
q_s	1	$(q_s, 1, \rightarrow)$
q_s	$\Box$	$(q_1, \Box, \leftarrow)$
q_s	$\rhd$	$(q_s, \rhd, \rightarrow)$
q_1	0	$(q_2, \Box, \rightarrow)$
q_1	1	$(q_3, \Box, \rightarrow)$
q_1	$\Box$	$(q_1, \Box, -)$
q_1	$\rhd$	$(q_h, \rhd, \rightarrow)$
q_2	0	$(q_s, 0, \leftarrow)$
q_2	1	$(q_s, 0, \leftarrow)$
q_2	$\Box$	$(q_s, 0, \leftarrow)$
q_2	$\rhd$	$(q_h, \rhd, \rightarrow)$
q_3	0	$(q_s, 1, \leftarrow)$
q_3	1	$(q_s, 1, \leftarrow)$
q_3	$\Box$	$(q_s, 1, \leftarrow)$
q_3	$\rhd$	$(q_h, \rhd, \rightarrow)$

Start the machine with input 010

- $q_s, \underline{\rhd}010$
- $q_s, \rhd\underline{0}10$
- $q_s, \rhd0\underline{1}0$
- $q_s, \rhd01\underline{0}$
- $q_s, \rhd010\underline{\Box}$
- $q_1, \rhd01\underline{0}\Box$
- $q_2, \rhd01\Box\underline{\Box}$
- $q_s, \rhd01\underline{\Box}0$
- $q_1, \rhd0\underline{1}\Box0$
- $q_3, \rhd0\Box\underline{\Box}0$
- $q_s, \rhd0\underline{\Box}10$
- $q_1, \rhd\underline{0}\Box10$
- $q_2, \rhd\Box\underline{\Box}10$
- $q_0, \rhd\underline{\Box}010$
- $q_1, \underline{\rhd}\Box010$
- $q_h, \rhd\underline{\Box}010$

The Second Example

$Q = \{q_s, q_h, q_1\}$, $\Gamma = \{0, 1, \Box, \rhd\}$, and δ is as follows:

$p \in Q$	$\sigma \in \Gamma$	$\delta(p, \sigma)$
q_s	0	$(q_s, 0, \rightarrow)$
q_s	1	$(q_s, 1, \rightarrow)$
q_s	$\Box$	$(q_1, \Box, \leftarrow)$
q_s	$\rhd$	$(q_s, \rhd, \rightarrow)$
q_1	0	$(q_h, 1, -)$
q_1	1	$(q_1, 0, \leftarrow)$
q_1	$\rhd$	$(q_h, \rhd, \rightarrow)$

The Third Example

$Q = \{q_s, q_h, q_c, q_l, q_t\}$; $\Gamma = \{\Box, \rhd, 0, 1\}$; two work tapes.

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$Q = \{q_s, q_h, q_c, q_l, q_t\}$; $\Gamma = \{\Box, \rhd, 0, 1\}$; two work tapes.

$\langle q_s, \rhd, \rhd, \rhd \rangle \to \langle q_c, \rhd, \rhd, \to, \to, \to \rangle$

$\langle q_c, 0, \Box, \Box \rangle \to \langle q_c, 0, \Box, \to, \to, - \rangle$
$\langle q_c, 1, \Box, \Box \rangle \to \langle q_c, 1, \Box, \to, \to, - \rangle$
$\langle q_c, \Box, \Box, \Box \rangle \to \langle q_l, \Box, \Box, \leftarrow, -, - \rangle$

$\langle q_l, 0, \Box, \Box \rangle \to \langle q_l, \Box, \Box, \leftarrow, -, - \rangle$
$\langle q_l, 1, \Box, \Box \rangle \to \langle q_l, \Box, \Box, \leftarrow, -, - \rangle$
$\langle q_l, \rhd, \Box, \Box \rangle \to \langle q_t, \Box, \Box, \to, \leftarrow, - \rangle$

$\langle q_t, \Box, \rhd, \Box \rangle \to \langle q_h, \rhd, 1, -, -, - \rangle$
$\langle q_t, 0, 1, \Box \rangle \to \langle q_h, 1, 0, -, -, - \rangle$
$\langle q_t, 1, 0, \Box \rangle \to \langle q_h, 0, 0, -, -, - \rangle$
$\langle q_t, 0, 0, \Box \rangle \to \langle q_t, 0, \Box, \to, \leftarrow, - \rangle$
$\langle q_t, 1, 1, \Box \rangle \to \langle q_t, 1, \Box, \to, \leftarrow, - \rangle$

$\{0, 1, \square, \triangleright\}$ vs. Larger Alphabets

Suppose $\mathbb{M}$ has k tapes with the alphabet Γ.

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A symbol of $\mathbb{M}$ is encoded by a string $\sigma \in \{0, 1\}^*$ of length $\log |\Gamma|$.

$\{0, 1, \Box, \rhd\}$ vs. Larger Alphabets

Suppose $\mathbb{M}$ has k tapes with the alphabet Γ.

A symbol of $\mathbb{M}$ is encoded by a string $\sigma \in \{0,1\}^*$ of length $\log|\Gamma|$.

States: A state q is turned into states q, $\langle q, \sigma_1^1, \ldots, \sigma_1^k \rangle$ where $|\sigma_1^1| = \ldots = |\sigma_1^k| = 1$, …, $\langle q, \sigma_{\log|\Gamma|}^1, \ldots, \sigma_{\log|\Gamma|}^k \rangle$ where $|\sigma_{\log|\Gamma|}^1| = \ldots = |\sigma_{\log|\Gamma|}^k| = \log|\Gamma|$.

$\{0, 1, \square, \triangleright\}$ vs. Larger Alphabets

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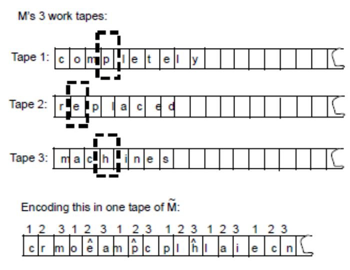
States: A state q is turned into states q, $\langle q, \sigma_1^1, \ldots, \sigma_1^k \rangle$ where $|\sigma_1^1| = \ldots = |\sigma_1^k| = 1$, ..., $\langle q, \sigma_{\log|\Gamma|}^1, \ldots, \sigma_{\log|\Gamma|}^k \rangle$ where $|\sigma_{\log|\Gamma|}^1| = \ldots = |\sigma_{\log|\Gamma|}^k| = \log|\Gamma|$.

A computation step of $\mathbb{M}$ is simulated in $\widetilde{\mathbb{M}}$ by $\log|\Gamma|$ steps to read, $\log|\Gamma|$ steps to write, and $\log|\Gamma|$ steps to relocate the heads.

One Tape vs. Many Tapes

One Tape vs. Many Tapes

The basic idea is to interleave k tapes into one tape.
The first $n + 1$ cells are reserved for the input.



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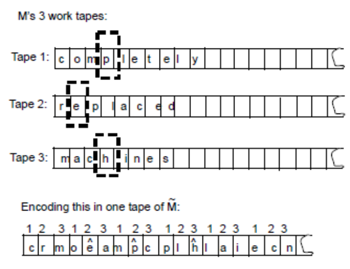

Every symbol a of $\mathbb{M}$ is turned into two symbols $a, \widehat{a}$ in $\widetilde{\mathbb{M}}$, with $\widehat{a}$ used to indicate head position.

One Tape vs. Many Tapes

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The machine $\widetilde{\mathbb{M}}$ copies the input bits to the first imaginary tape. The head then moves left to the $(n+2)$-th cell.

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Sweeping the tape cells from left to right. Record in the register the k symbols marked with the hat $\widehat{\_}$.

One Tape vs. Many Tapes

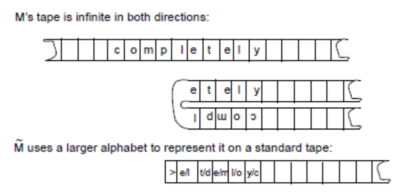
The machine $\widetilde{\mathbb{M}}$ copies the input bits to the first imaginary tape. The head then moves left to the $(n+2)$-th cell.

Sweeping the tape cells from left to right. Record in the register the k symbols marked with the hat $\widehat{\_}$.

Sweeping the tape cells from right to left to update using the transitions of $\mathbb{M}$.

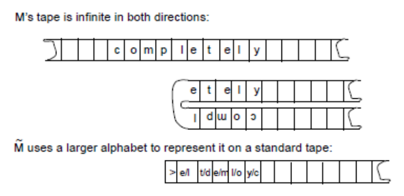
One Unidirectional vs. Bidirectional Tape

The idea is that $\widetilde{\mathbb{M}}$ makes use of the alphabet $\Gamma \times \Gamma$.



One Unidirectional vs. Bidirectional Tape

The idea is that $\widetilde{\mathbb{M}}$ makes use of the alphabet $\Gamma \times \Gamma$.



Every state q of $\mathbb{M}$ is turned into $\overline{q}$ and $\underline{q}$.

Simulation of TM by URM

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Suppose $\mathbb{M}$ is a 3-tape TM with the alphabet $\{0, 1, \Box, \triangleright\}$.

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Suppose $\mathbb{M}$ is a 3-tape TM with the alphabet $\{0, 1, \Box, \triangleright\}$.

The URM that simulates $\mathbb{M}$ can be designed as follows:

- Suppose that R_m is the right most register that is used by a program calculating $x \dot{-} 1$.
- The head positions are stored in $R_{m+1}, R_{m+2}, R_{m+3}$.
- The three binary strings in the tapes are stored respectively in
  $R_{m+4}, R_{m+7}, R_{m+10}, \ldots,$
  $R_{m+5}, R_{m+8}, R_{m+11}, \ldots,$
  $R_{m+6}, R_{m+9}, R_{m+12}, \ldots.$
- The states of $\mathbb{M}$ are encoded by the states of the URM.
- The transition function of $\mathbb{M}$ can be easily simulated by the program of the URM.

Homework

Encode the addition function by k-tape Turing machine two nature number partitioned by $\sharp$ on the input tape, for example, $11010\#1001$, and then try to encode the function by 1-tape Turing machine.