# Homework#8 Greedy Approach

Textbook:

**8.5.** Suppose in the money change problem of Exercise 8.3 the coin values are: $1, 2, 4, 8, 16, \ldots, 2^k$, for some positive integer $k$. Give an $O(\log n)$ algorithm to solve the problem if the value to be paid is $n < 2^{k+1}$.

**8.7.** Let $G = (V, E)$ be an undirected graph. A vertex cover for $G$ is a subset $S \subseteq V$ such that every edge in $E$ is incident to at least one vertex in $S$. Consider the following algorithm for finding a vertex cover for $G$. First, order the vertices in $V$ by decreasing order of degree. Next execute the following step until all edges are covered. Pick a vertex of highest degree that is incident to at least one edge in the remaining graph, add it to the cover, and delete all edges incident to that vertex. Show that this greedy approach does not always result in a vertex cover of minimum size.

**8.8.** Let $G = (V, E)$ be an undirected graph. A clique $C$ in $G$ is a subgraph of $G$ that is a complete graph by itself. A clique $C$ is maximum if there is no other clique $C'$ in $G$ such that the size of $C'$ is greater than the size of $C$. Consider the following method that attempts to find a maximum clique in $G$. Initially, let $C = G$. Repeat the following step until $C$ is a clique. Delete from $C$ a vertex that is not connected to every other vertex in $C$. Show that this greedy approach does not always result in a maximum clique.

**8.27.** Does Algorithm PRIM work correctly if the graph has negative weights? Prove your answer.

**8.28.** Let $G$ be an undirected weighted graph such that no two edges have the same weight. Prove that $G$ has a unique minimum cost spanning tree.

**Maximum Spanning Tree.** Show how to find the maximum spanning tree of a graph, that is, the spanning tree of largest total weight.