Homework#10 P and NP

Textbook:

10.3. Design a polynomial time algorithm for the problem 2-COLORING defined on page 282. (Hint: Color the first vertex white, all adjacent vertices black, etc).

10.5. Let I be an instance of the problem COLORING, and let s be a claimed solution to I. Describe a deterministic algorithm to test whether s is a solution to I.

10.9. Let Π_1 and Π_2 be two problems such that $\Pi_1 \propto_{poly} \Pi_2$. Suppose that problem Π_2 can be solved in $O(n^k)$ time and the reduction can be done in $O(n^j)$ time. Show that problem Π_1 can be solved in $O(n^{jk})$ time.

10.19. In Chapter 7 it was shown that the problem KNAPSACK can be solved in time $\Theta(nC)$, where n is the number of items and C is the knapsack capacity. However, it was mentioned in this chapter that it is NP-complete. Is there any contradiction? Explain.

10.22. Prove that NP = P if and only if for some NP-complete problem Π, $\Pi \in$ P.

Optimization problems. Given an undirected graph with positive integer edge weights, the traveling salesperson problem is to find a simple cycle that visits every vertex and has minimum total weight. The search problem version of the problem is, given a parameter L, find a tour of length at most L. Prove that the optimization version of the problem polynomial-time reduces to the search version of the problem.

Hitting set problem. In the HITTING SET problem, we are given a family of sets {S1, S2, . . . , Sn} and a budget b, and we wish to find a set H of size ≤ b which intersects every Si, if such an H exists. In other words, we want H∩Si =/=∅for all i. Show that HITTING SET is **NP**-complete.