Algorithmn HW7

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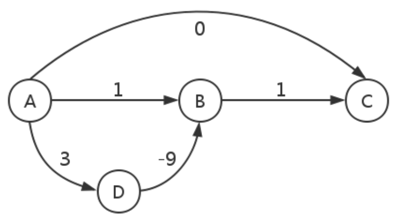
Problem 8.16

Just change the RELAX part to store the best predecessor of the vertex, and later we can find the shortest path iterately.

procedure RELAX(u, v)
    if $d[v] > d[u] + w(u, v)$ **then**
        $d[v] \leftarrow d[u] + w(u, v)$
        $pred[v] \leftarrow u$
    end if
end procedure

Problem 8.19

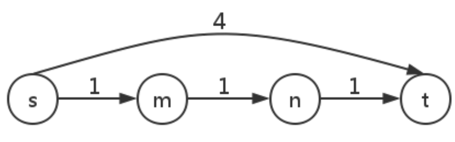


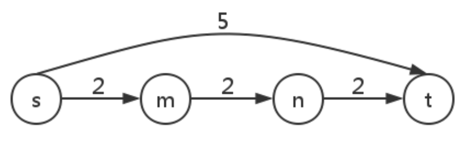
Considering this case, Dijkstra works like this:
(1) Starting from A, set $d(A) = 0$ and $d(others) = +\infty$;
(2) A out, set $d(B) = 1$, $d(C) = 0$ and $d(D) = 3$;
(3) C out, with no successor edge;
(4) B out, with no change $(1 + 1 > 0)$;
(5) D out, updating $d(B) = -6$, then terminate.

Obviously, $d(C)$ should be -5 instead of 0 in this case, which shows Dijkstra does not work in this case.

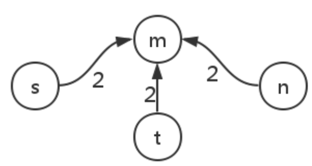
True or False

(a) **False**, orignally it's $s \rightarrow m \rightarrow n \rightarrow t$ but after modification it will be $s \rightarrow t$





(b)**False**, in the DAG on the right, Dijkstra starting at m definitely won't pick vertexes as any of the topological orders.



(c)**False**, because in that case, the short path would be either **(1)**modified to other path-that won't change-or **(2)**remain the same path-that will increase the short path by exactly x

(d)**True**, because Bellman-Ford will search every possible edge for every vertex regardless of the order, so it will update later if there is some negative value.

Paths in DAG

Require: a DAG $G = (V, S)$
Ensure: the number of paths in G

procedure CountPath(G)
  topologically sort $G.V$
  $p[s] \leftarrow 1$, $p[\text{other } v \in G.V] \leftarrow 0$ ▷ $p[]$ stores the number of paths
  for each vertex i in topological sorted sequence **do**
    for each vertex j connected with i **do**
      $p[j] \leftarrow p[j] + p[i]$ ▷ The last element will be the total number of paths
    end for
  end for
end procedure

We can have the relationship as follows:
$initial(u) = u$ is the last element in topo order? $1 : 1 + \sum_{(u,v)\in E}$
$path(u) = initial(u) + \sum_{(u,v)\in E} path(v)$
Since $path(u)$ and $initial(u)$ can be computed in linear time, the algorithm takes $O(V + E)$ time

Shortest path tree

Require: directed graph $G = (V, E)$, a tree $T = (V, E')$, $s \in V$ and $E' \in E$
Ensure: whether T is the shortest-path tree of G starting with s

$Q \leftarrow [s]$
while Q is not empty **do** ▷ travel the tree
  $u = Q.pop()$ ▷ Q is a priority queue
  for each edge $(u, v) \in E'$ **do**
    $d[v] = d[u] + w(u, v)$ ▷ $d[]$ keeps the shortest paths
    $Q.push(v)$
    remove (u, v) from E'
  end for
end while
for each edge $(u, v) \in E$ **do** ▷ check in the graph whether it is shortest
  $temp \leftarrow d[v]$
  Relax(u, v) ▷ compare the value before and after Relax
  if $temp \neq d[v]$ **then**
    return $False$
  end if
end for
return $True$