Nonlinear PDEs

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Generalized and potential symmetries of the Rudenko–Robsman equation

Research Article

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Abstract: The paper presents a concrete study of the existence of generalized and potential symmetries for the 1+1 dimensional version of the Rudenko–Robsman equation, an interesting fourth-order partial differential equation that describes the evolution of nonlinear waves in a dispersive medium. As the main results, the existence of a two-parameter algebra of generalized symmetries and of an infinite-dimensional algebra when potential symmetries are taken into account is proven.

Rudenko-Robsman Equation

- The RR equation

$$u \cdot ux + ut + uxxxx$$

  was introduced in 2002 for describing the nonlinear wave propagation in scattering media.
- The first physical interpretation of the RR equation was that it describes shock waves in liquids containing gas bubbles.
- Very similar to 2nd order Burgers equation: Stationary solutions in the form of a shock wave with unusual oscillations around shock front.
- The RR equation also represents a good toy model of a different fourth-order differential equation that could be investigated with these techniques.

Paper's Overview

- They present the general method that can be used for finding generalized symmetries to integrated related equation of RR,

$$vt - (1/2) \cdot vx^2 + vxxxx$$

, where

$$vx = u$$

.

- Symmetry groups can be used to understand the dynamics of physical systems. By finding conservation laws and constructing exact solutions.

Boldea Results of potential RR

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$$U1 = \frac{\partial}{\partial x}$$

$$U2 = -a(t)\frac{\partial}{\partial x} - xa'(t)\frac{\partial}{v}$$

$$U3 = b(t)\frac{\partial}{\partial v}$$

RR equation

Compatability of:

$$u \cdot ux + ut + uxxxx$$

$$ut + X(t, x, u) \cdot ux - U(t, x, u)$$

$$u\left(\frac{\partial}{\partial x}U(t,x,u)\right)+4\left(\frac{\partial}{\partial x}X(t,x,u)\right)U(t,x,u)+\frac{\partial}{\partial t}U(t,x,u)+\frac{\partial^4}{\partial x^4}U(t,x,u)$$

$$4\left(\frac{\partial}{\partial u}X(t,x,u)\right)u-4\left(\frac{\partial}{\partial u}X(t,x,u)\right)X(t,x,u)+6\left(\frac{\partial^4}{\partial x^2\,\partial u^2}U(t,x,u)\right)-4\left(\frac{\partial^4}{\partial x^3\,\partial u}X(t,x,u)\right)$$

$$-10\left(\frac{\partial}{\partial u}X(t,x,u)\right)$$

$$3\left(\frac{\partial^2}{\partial u^2}U(t,x,u)\right)-12\left(\frac{\partial^2}{\partial x\,\partial u}X(t,x,u)\right)$$

Our Results RR equation

$$X(t,x,u) = \frac{-x + c2t + c3}{3c1 - 4t}$$

.

$$U(t,x,u) = \frac{3u + c2}{3c1 - 4t}$$

Potential RR

Compatability:

$$ut - (1/2) \cdot ux^2 + uxxxx$$

.

$$ut + X(t, x, u) \cdot ux - U(t, x, u)$$

$$4\left(\frac{\partial}{\partial x}X(t,x,u)\right)U(t,x,u)+\frac{\partial}{\partial t}U(t,x,u)+\frac{\partial^4}{\partial x^4}U(t,x,u)$$

$$-10\left(\frac{\partial^3}{\partial u^3}X(t,x,u)\right)$$

$$-\frac{1}{2}\frac{\partial}{\partial u}U(t,x,u)-\left(\frac{\partial}{\partial x}X(t,x,u)\right)-4\left(\frac{\partial}{\partial u}X(t,x,u)\right)X(t,x,u)-4\left(\frac{\partial^4}{\partial x^3\,\partial u}X(t,x,u)\right)+6\left(\frac{\partial^4}{\partial x^2\,\partial u^2}U(t,x,u)\right)$$

$$-\left(\frac{\partial}{\partial x}U(t,x,u)\right)+4\left(\frac{\partial}{\partial u}X(t,x,u)\right)U(t,x,u)-4\left(\frac{\partial}{\partial x}X(t,x,u)\right)X(t,x,u)-\left(\frac{\partial}{\partial t}X(t,x,u)\right)+4\left(\frac{\partial^4}{\partial x^3\,\partial u}U(t,x,u)\right)-\left(\frac{\partial^4}{\partial x^4}X(t,x,u)\right)$$

$$4\left(\frac{\partial^2}{\partial u^2}U(t,x,u)\right)-16\left(\frac{\partial^2}{\partial x\,\partial u}X(t,x,u)\right)$$

Potential Results

$$X(t,x,u) = \frac{1}{2}\frac{2c1 \cdot c3 - 4c3 \cdot t - 4c4 - x}{-2t + c1}$$

.

$$U(t,x,u) = \frac{2c3 \cdot x + c2 + u}{-2t + c1}$$

Potential Results

$$X(t, x, u) = \frac{1}{2}\frac{4c3 \cdot t - x - c5}{-2t + c1}$$

.

$$U(t, x, u) = \frac{2c3 \cdot x + c2 + u}{-2t + c1}$$

Our Generators

$$u_t + X(t, x, u) \cdot u_x = U(t, x, u)$$

$$2(c1 - 2t) \cdot u_t + (4c3t - x - c5) \cdot u_x = 4c3x + 2c2 + 2u$$

Set All constants to 0.

$$-4tu_t - xu_x = 2u$$

$\Longrightarrow \Gamma_1 = -4t\frac{\partial}{\partial t} - x\frac{\partial}{\partial x} + 2\frac{\partial}{\partial u}$

Generators Continued

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Setting c1=1 and all others to 0.
$\Longrightarrow \Gamma_2 = -(2-2t)\frac{\partial}{\partial t} - x\frac{\partial}{\partial x} + 2\frac{\partial}{\partial u}$

Setting c2=1 and all others to 0.
$\Longrightarrow \Gamma_3 = -4t\frac{\partial}{\partial t} - x\frac{\partial}{\partial x} + 2 + 2\frac{\partial}{\partial u}$

Boldea Results of potential RR

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$$U1 = \frac{\partial}{\partial x}$$

$$U2 = -a(t)\frac{\partial}{\partial x} - xa'(t)\frac{\partial}{v}$$

$$U3 = b(t)\frac{\partial}{\partial v}$$

Boldea Results of potential RR

$$\log \frac{p}{1-p} = \beta_0 + \beta_1 Recency + \beta_2 Frequency + \beta_3 Amount + \beta_4(FirstDonation)$$

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Comparing Results

The paper went through the process of classical symmetries. We used nonclassical symmetries to arrive at our answers. Thus their symmetry groups should be a subset of our symmetry groups. Which we see is not the case. Using compatibility, we found more infinitesimal generators and show there are no arbitrary functions.