투자론

- R과 Excel을 통한 금융데이터 분석 -

4주차
포트폴리오 이론 2

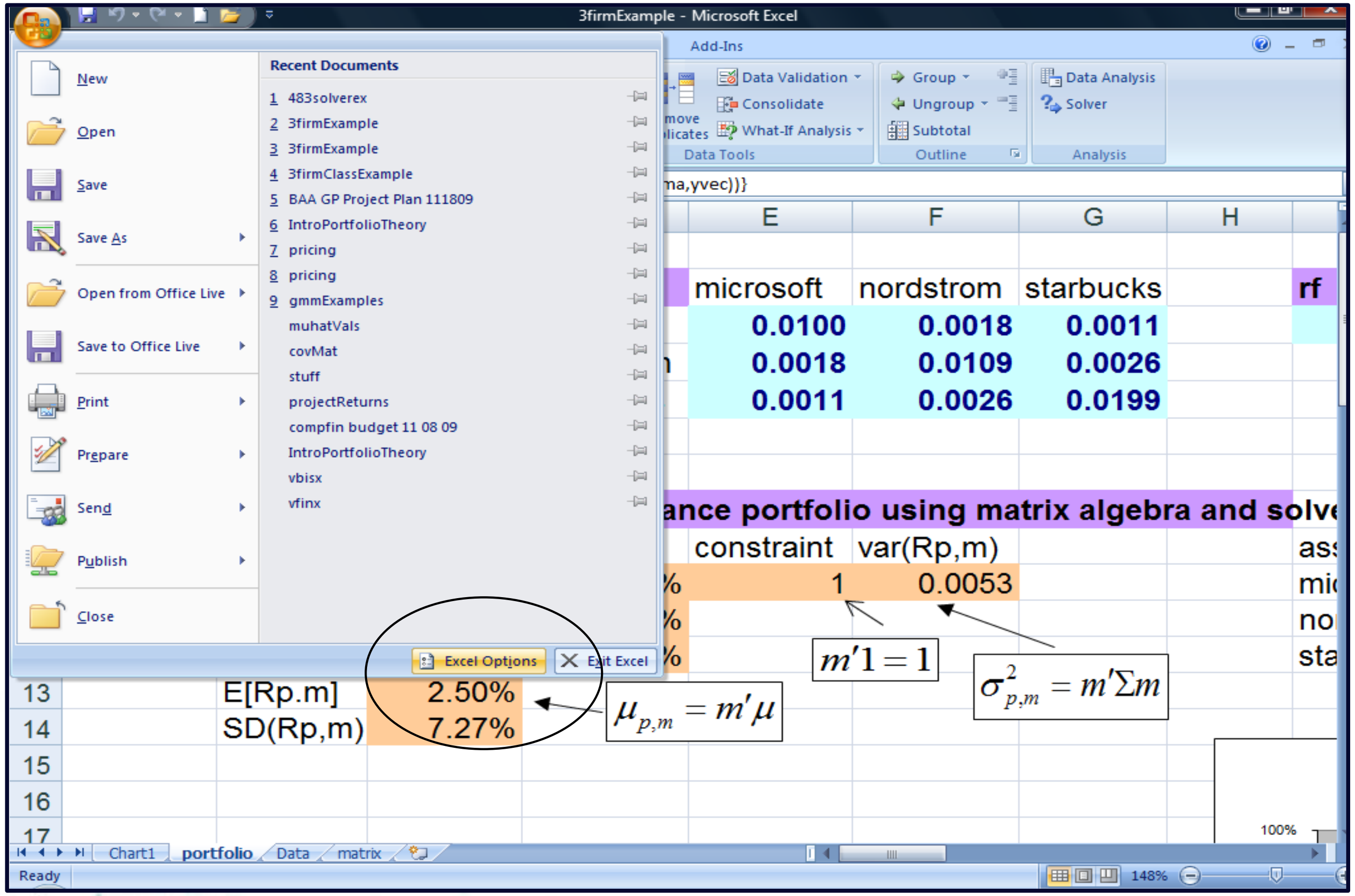
충남대학교
장호규 교수

Unit 02

Excel Workout for MV Analysis

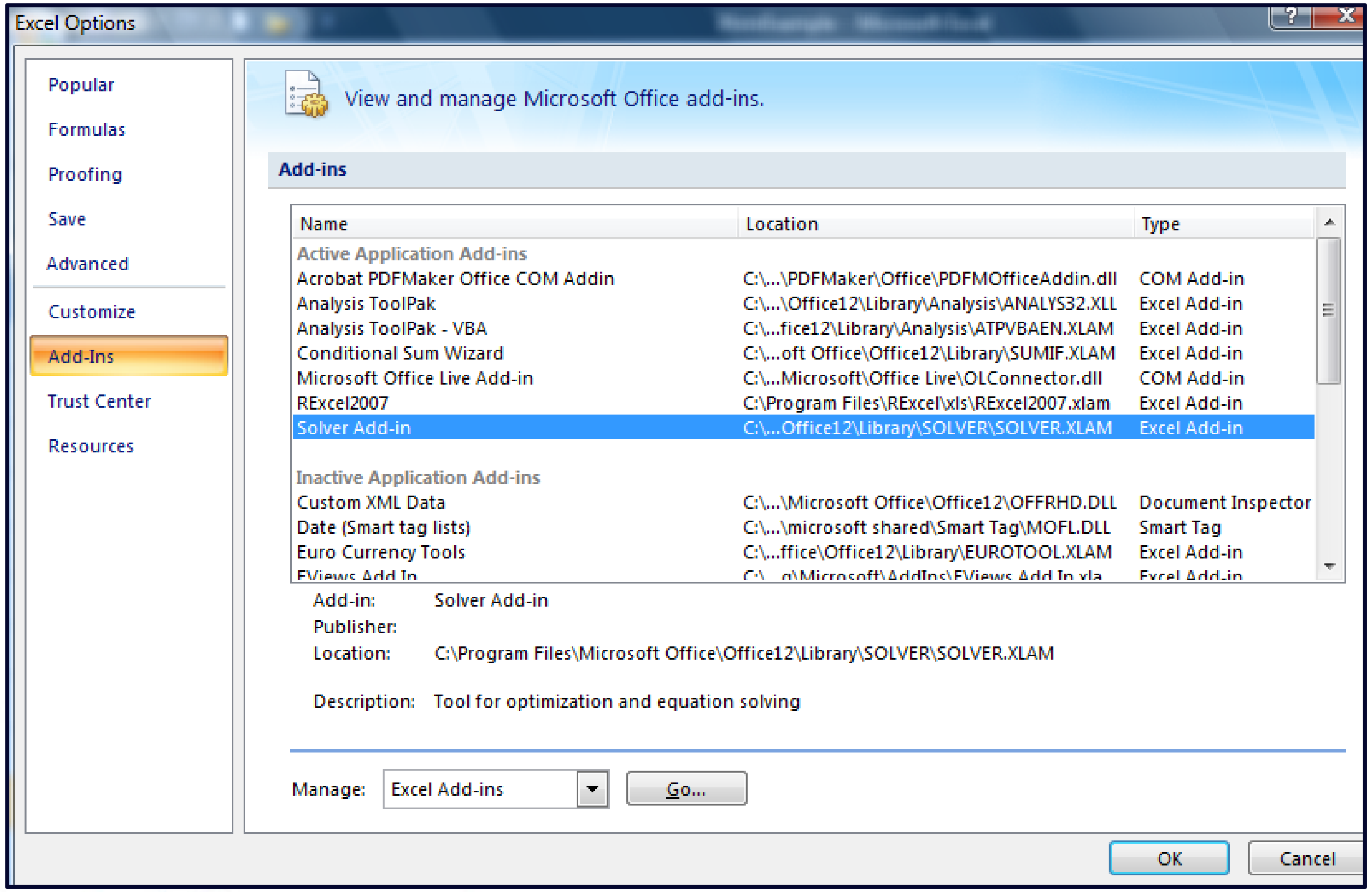
Efficient Portfolios

- The solver add-in must be activated before it can be used within Excel



Efficient Portfolios

- The solver add-in must be activated before it can be used within Excel



Matrix Algebra in Excel

- Excel has several built-in array formulas that can perform basic matrix algebra operations. The main functions are listed in table below

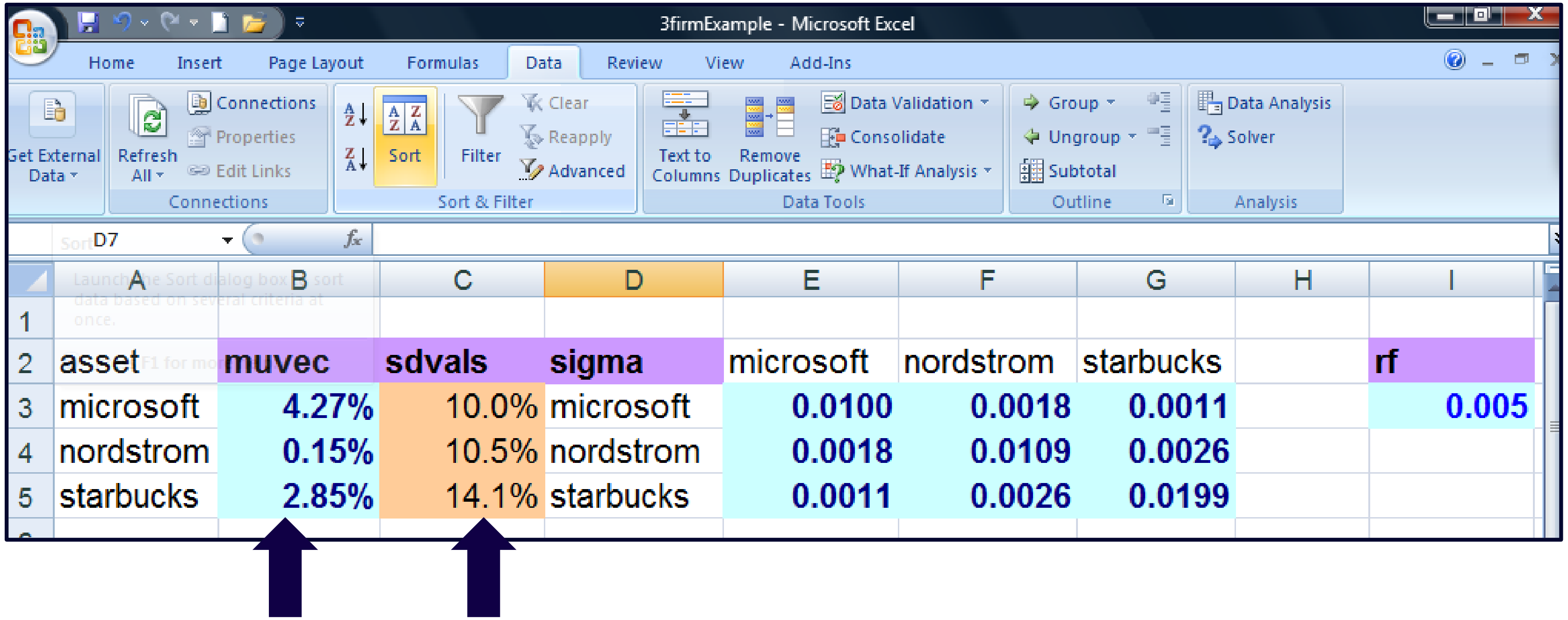
Array Function	Description
MINVERSE	Compure inverse of matrix
MMULT	Matix multiplication
TRANSPOSE	Compute transpose of matrix

Example

- In the Data tab of the spreadsheet 3firmExample.xls is the example monthly return data on three assets: Microsoft, Nordstrom and Starbucks

  The monthly means and covariance matrix of the returns are computed and these are referenced as the input data on the portfolio tab as illustrated in the screen shot below

Example



	A	B	C	D	E	F	G	H	I
1									
2	asset	muvec	sdvals	sigma	microsoft	nordstrom	starbucks		rf
3	microsoft	4.27%	10.0%	microsoft	0.0100	0.0018	0.0011		0.005
4	nordstrom	0.15%	10.5%	nordstrom	0.0018	0.0109	0.0026		
5	starbucks	2.85%	14.1%	starbucks	0.0011	0.0026	0.0199		

Input Output

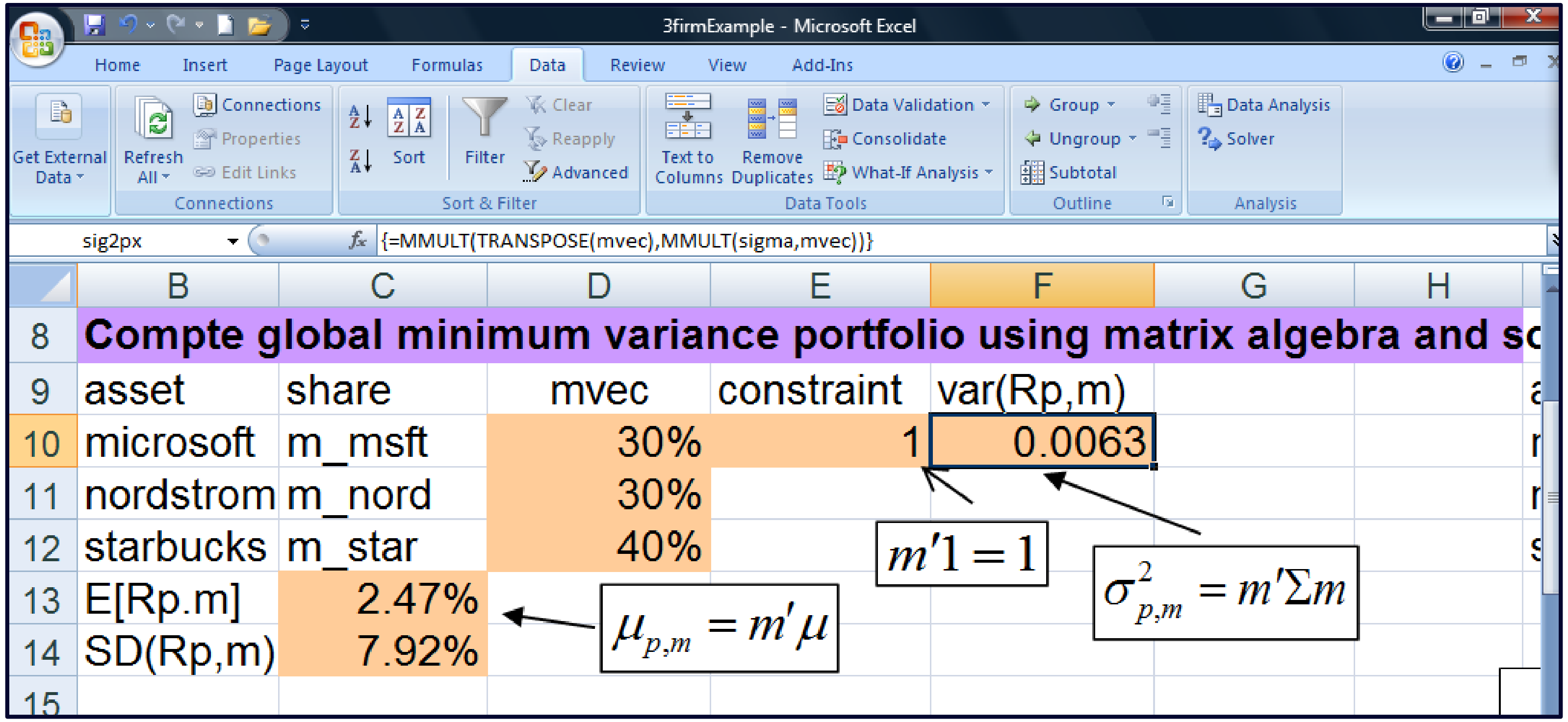
Global Minimum Variance Portfolio (GMVP)

- The global minimum variance portfolio solves the following optimization problem with the portfolio $m = (m_1, \cdots, m_N)$

$$\min_{m} \sigma_{p,m}^2 \ (\equiv m'\Sigma m) \ \ s.t. \ m'\mathbf{1} = 1, \text{ where } \mathbf{1} = (1,1,\cdots,1)'$$

- This optimization problem can be solved easily using the solver with matrix algebra functions. The screen shot of the portfolio tab below shows how to set-up this optimization problem in Excel

Global Minimum Variance Portfolio (GMVP)



The range of cells D10:D12 is called mvec and will contain the weights in the minimum variance portfolio once the solver is run and the solution to the optimization problem is found

Global Minimum Variance Portfolio (GMVP)

- Before the solver is to be run, these cells should contain an initial guess of the minimum variance portfolio. A simple guess for this vector whose weights sum to one is.

$$m\_msft = 0.3, m\_nord = 0.3, m\_sbux = 0.4.$$

- To use the solver, a cell containing the function to be maximized or minimized must be specified. Here, this cell is F10 which contains the array formula.

$$\{= MMULT(TRANSPOSE(mvec), MMULT(sigma, mvec))\}$$

Global Minimum Variance Portfolio (GMVP)

- **which evaluates the matrix algebra formula for the variance of a portfolio**

$$\sigma_{p,m}^2 = m'\Sigma \mathrm{m}$$

– Notice that the formula is surrounded by curly braces {}
  This indicates that 〈CTRL〉-〈Shift〉-〈Enter〉 was used to evaluate the formula so that it is to be interpreted as an array formula
  If you don't see the curly braces then the formula will not be evaluate correctly

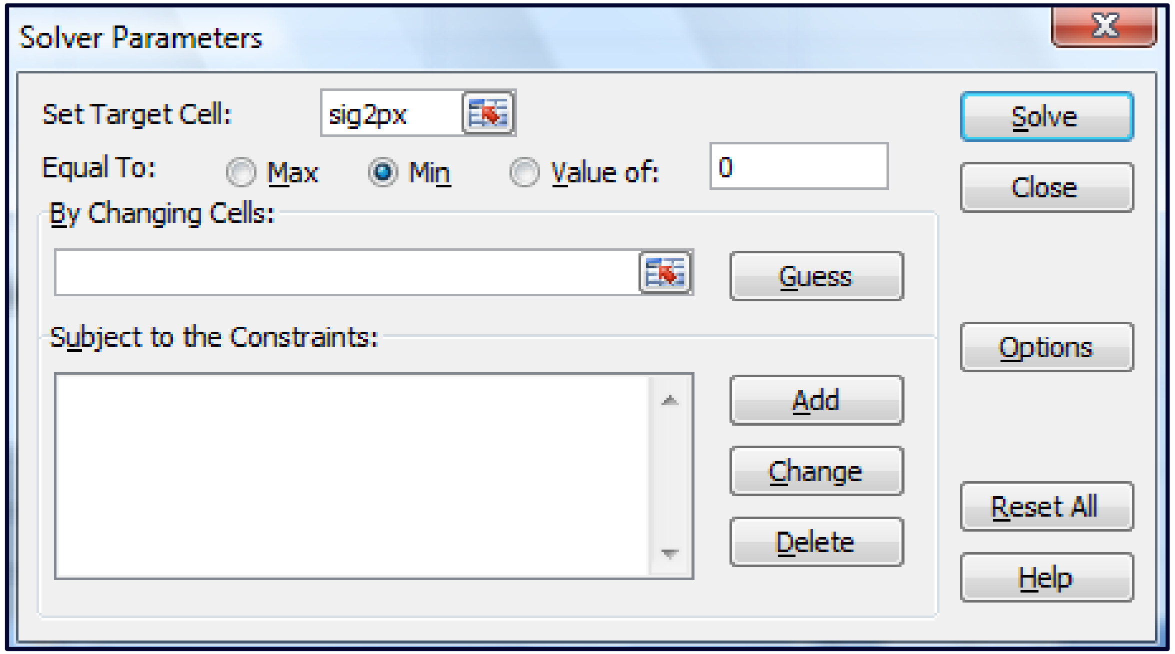
Global Minimum Variance Portfolio (GMVP)

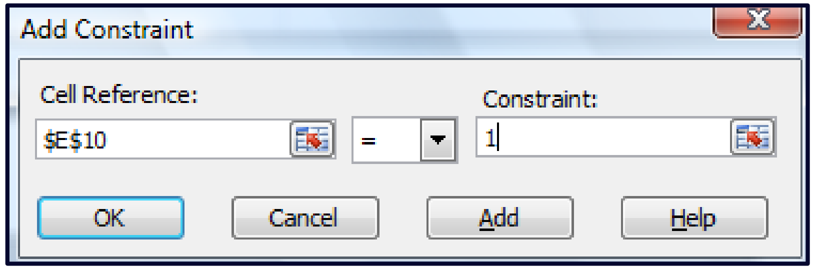
- We also need a cell to contain a formula that will be used to impose the constraint that the portfolio weights sum to one

$$m'1 = m_{msfd} + m_{nord} + m_{sbux} = 1 \rightarrow \text{ Cell E10 (=sum(mvec))}$$

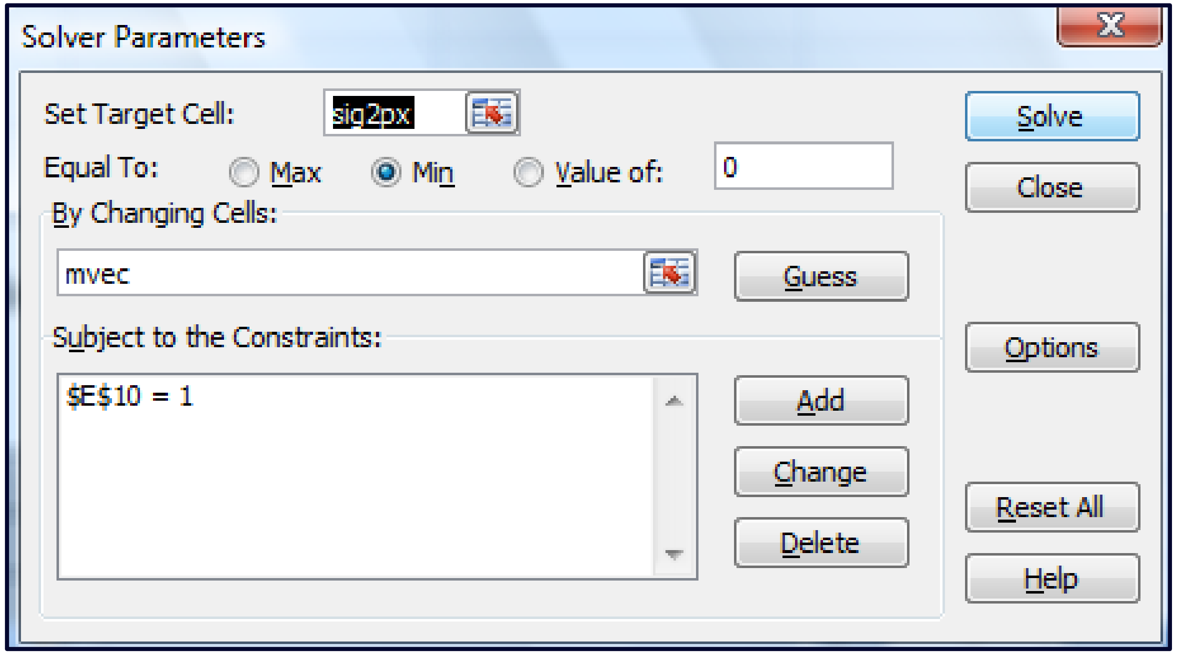
- To run the solver, click the cell containing the formula you want to optimize (cell F10, and named sig2px) and then click on the solver button.
  This will open up the solver dialogue box as shown below.

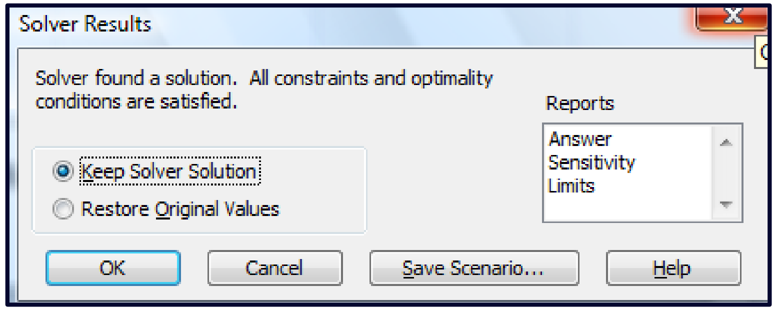
Global Minimum Variance Portfolio (GMVP)



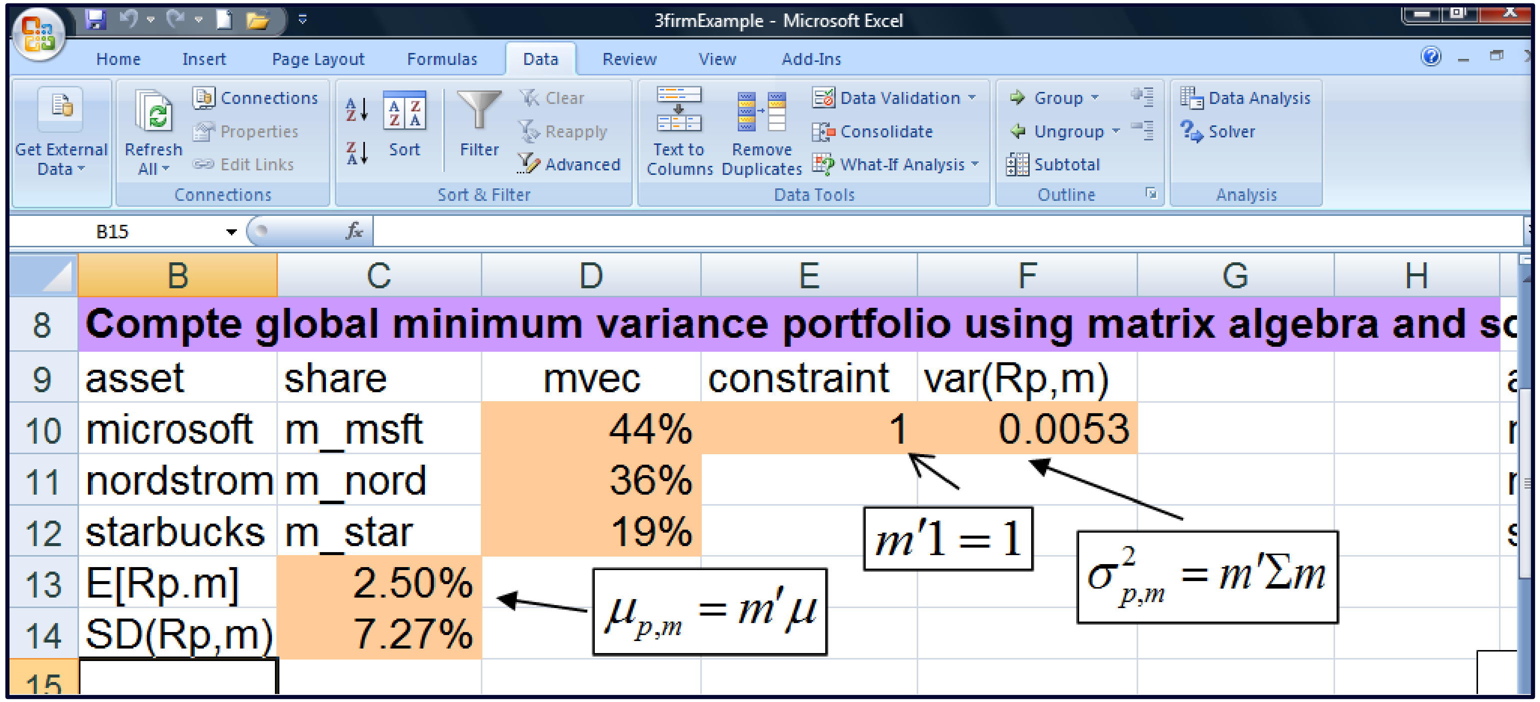


Global Minimum Variance Portfolio (GMVP)





Global Minimum Variance Portfolio (GMVP)


3firmExample - Microsoft Excel

	B	C	D	E	F
8	Compte global minimum variance portfolio using matrix algebra and sc				
9	asset	share	mvec	constraint	var(Rp,m)
10	microsoft	m_msft	44%	1	0.0053
11	nordstrom	m_nord	36%		
12	starbucks	m_star	19%		
13	E[Rp.m]	2.50%			
14	SD(Rp,m)	7.27%			

$m'1 = 1$

$\sigma^2_{p,m} = m'\Sigma m$

$\mu_{p,m} = m'\mu$

Global Minimum Variance Portfolio (GMVP)

- The global minimum variance portfolio has 44% in Microsoft, 36% in Nordstrom and 19% in Starbucks

  The expected return on this portfolio is given in cell C13 (called mupx) and is computed using the formula $\mu_{p,m} = m'\mu$

  The Excel array function is {=MMULT(TRANSPOSE(mvec),muvec)}

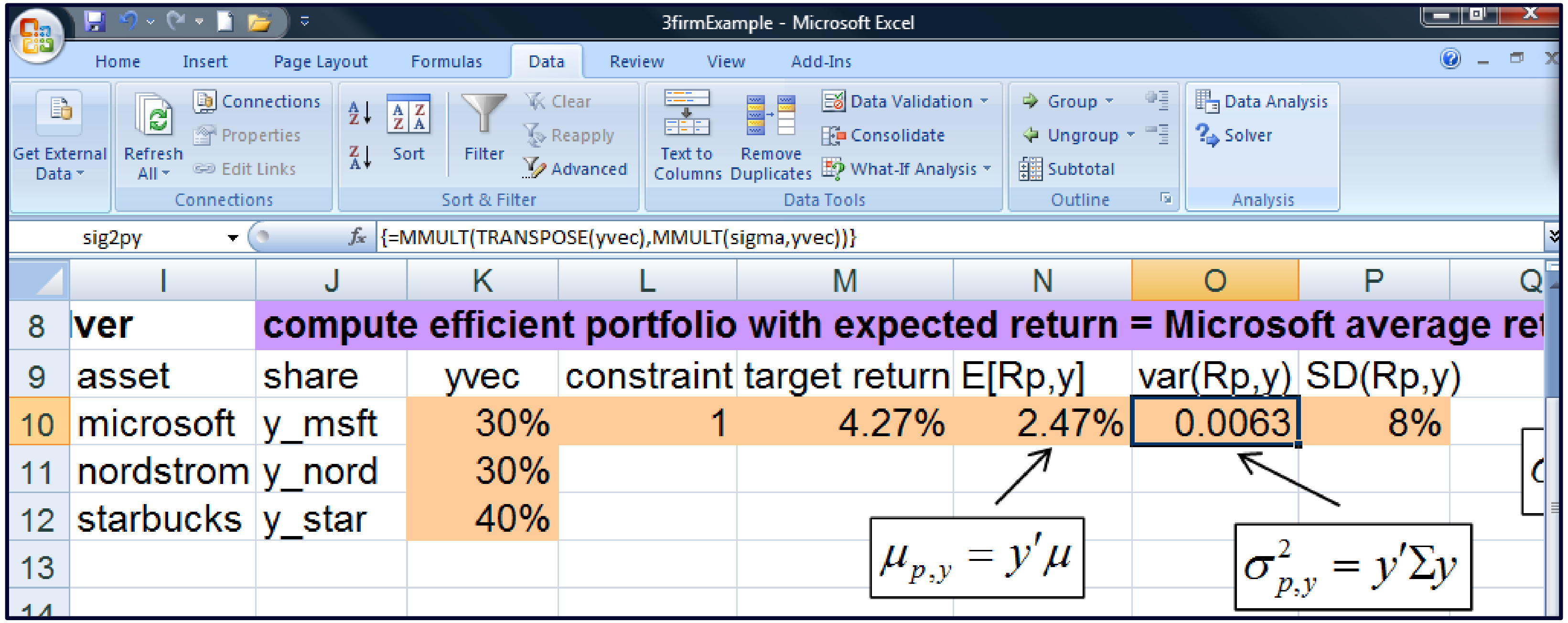
A MVP with target expected return

- A minimum variance portfolio with target expected return equal to μ_0 solves the optimization problem.

$$\min_{y} \sigma_{p,y}^2 = y'\Sigma y \ \text{s.t.}\ y'\mu = \mu_0 \ and \ y'\mathbf{1} = 1.$$

- This optimization problem can also be easily solved using the solver with matrix algebra functions
  The screenshot below shows how to set-up this optimization problem in Excel where the target expected return is the expected return on Microsoft (4.27%)

A MVP with target expected return



	I	J	K	L	M	N	O	P
8	ver	compute efficient portfolio with expected return = Microsoft average ret						
9	asset	share	yvec	constraint	target return	E[Rp,y]	var(Rp,y)	SD(Rp,y)
10	microsoft	y_msft	30%	1	4.27%	2.47%	0.0063	8%
11	nordstrom	y_nord	30%					
12	starbucks	y_star	40%					

$$\mu_{p,y} = y'\mu$$

$$\sigma^2_{p,y} = y'\Sigma y$$

A MVP with target expected return

- The range of cells K10:K12 is called yvec and will contain the weights in the efficient portfolio once the solver is run and the solution to the optimization problem is found. Before the solver is to be run, these cells should contain an initial guess of the minimum variance portfolio. A simple guess for this vector whose weights sum to one is $y_{msft} = 0.3, y_{nord} = 0.3, y_{sbux} = 0.4$.

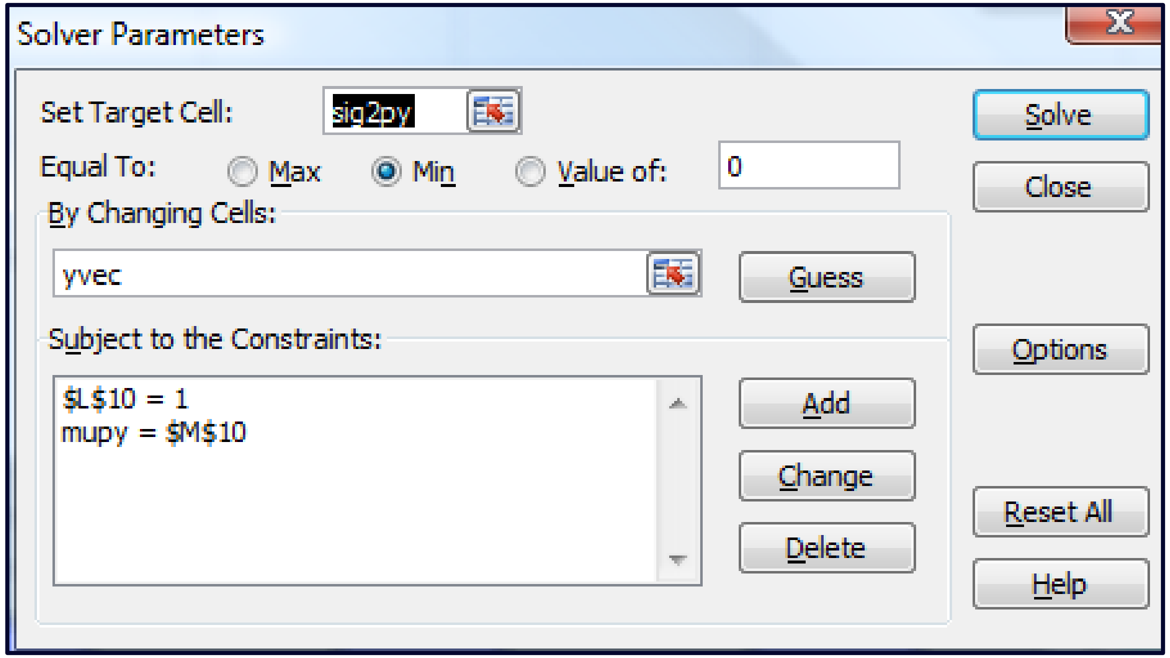
- The cell containing the formula for portfolio variance, $\sigma^2_{p,y} = y'\Sigma \text{y}$, is in cell O10 which contains the array formula {=MMULT(TRANSPOSE(yvec),MMULT(sigma,yvec))}.

A MVP with target expected return

- We also need two additional cells to contain formulas that will be used to impose the constraints that the portfolio expected return is equal to the target return, , $\mu_{p,y} = y'\mu = \mu_0$, and that the portfolio weights sum to one, $y'\mathbf{1} = y_{msft} + y_{nord} + y_{sbux} = 1.$

  These formulas are specified in cells L10 and N10, which contain the Excel formulas =SUM(yvec) and {=MMULT(TRANSPOSE(yvec),muvec)}, respectively.

A MVP with target expected return



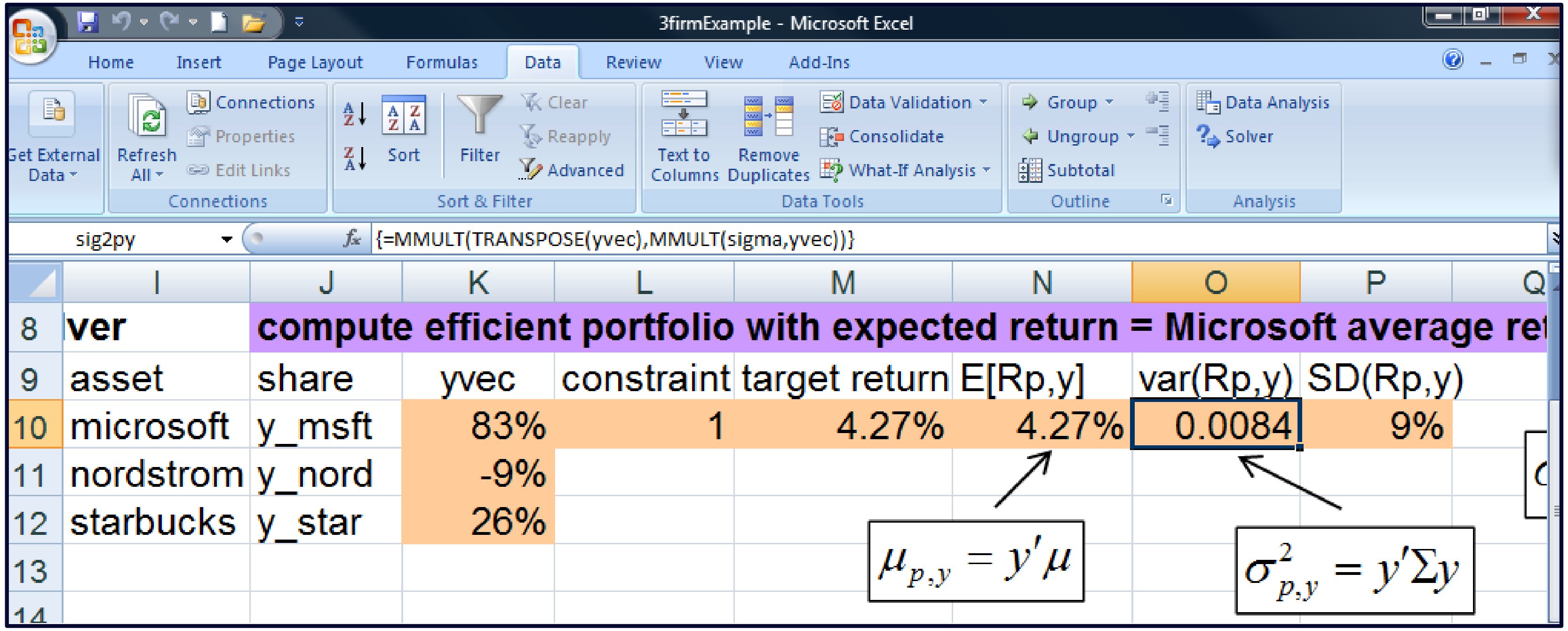
Notice that there are now two constraints specified.

The first one imposes $y'1 = y_{msft} + y_{nord} + y_{sbux} = 1$,

and the second one imposes $y_{py} = y'\mu = \mu_0 = \mu_{msft} = 0.0475$.

To run the solver, click the Solver button. You should see a dialogue box that says that the solver found a solution and that all optimality conditions are satisfied. Keep the solution and click OK. Your spreadsheet should look like the one below.

A MVP with target expected return


3firmExample - Microsoft Excel

sig2py | {=MMULT(TRANSPOSE(yvec),MMULT(sigma,yvec))}

	I	J	K	L	M	N	O	P
8	ver	compute efficient portfolio with expected return = Microsoft average ret						
9	asset	share	yvec	constraint	target return	E[Rp,y]	var(Rp,y)	SD(Rp,y)
10	microsoft	y_msft	83%	1	4.27%	4.27%	0.0084	9%
11	nordstrom	y_nord	-9%					
12	starbucks	y_star	26%					
13								

$\mu_{p,y} = y'\mu$

$\sigma^2_{p,y} = y'\Sigma y$

The efficient portfolio has weights $y_{msft} = 0.83$, $y_{nord} = -0.09$, $y_{sbux} = 0.26$.

Computing the Efficient Frontier with Risky Assets

- The efficient frontier of risky assets can be constructed from any two efficient portfolios. A natural question to ask is which two efficient portfolios should be used?
  I find that the following two efficient portfolios leads to the easy creation of the efficient frontier

1. Efficient portfolio 1

   : global minimum variance portfolio

2. Efficient portfolio 2

   : efficient portfolio with target expected return equal to the highest average return among the assets under consideration.

An important result from Black (1972)

- Given any two efficient portfolios with weight vectors m and y the convex combination $\boldsymbol{z} = \boldsymbol{\alpha} \cdot \boldsymbol{m} + (\mathbf{1} - \boldsymbol{\alpha}) \cdot \boldsymbol{y}$ for any constant α is also an efficient portfolio. The expected return and variance of this portfolio are

$$\mu_{p,z} = \alpha\mu_{p,m} + (1-\alpha)\mu_{p,y},$$

$$\sigma^2_{p,z} = \alpha^2\sigma^2_{p,m} + (1-\alpha)^2\sigma^2_{p,y} + 2\alpha(1-\alpha)\sigma_{m,y},$$

where the covariance between the returns on portfolios m and y is computed using $\boldsymbol{\sigma}_{\boldsymbol{m},\boldsymbol{y}} = \boldsymbol{m}'\boldsymbol{\Sigma}\mathbf{y}$.

- To create the efficient frontier, create a grid of α values starting at 1 and decrease increments of 0.1.

An important result from Black (1972)

3firmExample - Microsoft Excel

P20 =N20*mupx+O20*mupy

$$z = \alpha \cdot m + (1-\alpha) \cdot y$$

	N	O	P	Q	R	S	T	U
17						Frontier Portfolios		
18						microsof	nordstron	starbucks
19	alpha	1-alpha	E[Rp,z]	var(Rp,z)	SD(Rp,z)	z_msft	z_nord	z_sbux
20	1	0	2.50%	0.00528	7.27%	44%	36%	19%
21	0.9	0.1	2.68%	0.00531	7.29%	48%	32%	20%
22	0.8	0.2	2.85%	0.00541	7.35%	52%	27%	21%
23	0.7	0.3	3.03%	0.00556	7.46%	56%	23%	21%
24	0.6	0.4	3.21%	0.00578	7.60%	60%	18%	22%
25	0.5	0.5	3.38%	0.00605	7.78%	64%	14%	23%
26	0.4	0.6	3.56%	0.0064	8.00%	67%	9%	23%
27	0.3	0.7	3.74%	0.0068	8.24%	71%	5%	24%
28	0.2	0.8	3.92%	0.00726	8.52%	75%	0%	25%
29	0.1	0.9	4.09%	0.00779	8.82%	79%	-5%	26%
30	0	1	4.27%	0.00837	9.15%	83%	-9%	26%
31	-0.1	1.1	4.45%	0.00902	9.50%	87%	-14%	27%

Chart1 | portfolio | Data | matrix

An important result from Black (1972)

- The cell P20 contains the formula =N20*mupx+O20*mupy for the expected portfolio return

- The cell Q20 contains the formula
  = N20^{2}*sig2px+O20^{2} 2*sig2py+2*N20*O20* sigmaxy for the portfolio variance

- The covariance term sigmaxy is computed in the cell R9 (not shown) which contains the array formula {=MMULT(MMULT(TRANSPOSE(mvec),sigma),yvec)}

- The cells S20:U20 give the weights in the convex combination computed using the array formula {=TRANSPOSE(N20*D10:D12+O20*K10:K12)}

- The efficient frontier can be plotted by making a scatter plot with the expected return values (cells P20:P50) on the y-axis and the standard deviation values (cells R20:R50) on the horizontal axis

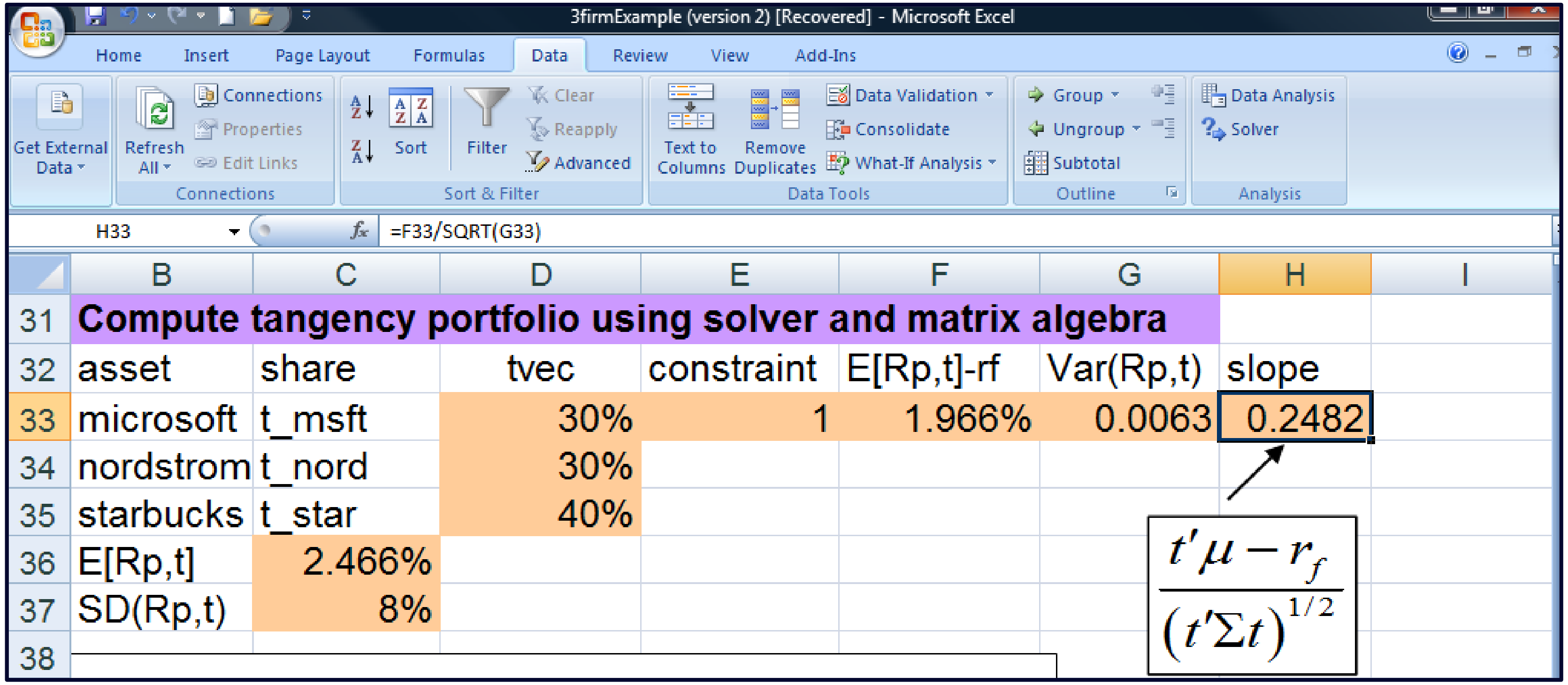
Computing Tangency Portfolio

- The tangency portfolio is the portfolio of risky assets that has the highest Sharpe's slope. This portfolio can be found by solving the optimization problem

$$\max_{\boldsymbol{t}} \frac{\boldsymbol{t}'\mu - r_f}{(\boldsymbol{t}'\Sigma\mathbf{t})^{1/2}} \quad s.t. \quad \boldsymbol{t}'\mathbf{1} = 1.$$

- This optimization problem can also be easily solved using the solver with matrix algebra functions. The screenshot below shows how to set-up this optimization problem in Excel.

Computing Tangency Portfolio



	B	C	D	E	F	G	H
31	Compute tangency portfolio using solver and matrix algebra						
32	asset	share	tvec	constraint	E[Rp,t]-rf	Var(Rp,t)	slope
33	microsoft	t_msft	30%	1	1.966%	0.0063	0.2482
34	nordstrom	t_nord	30%				
35	starbucks	t_star	40%				
36	E[Rp,t]	2.466%					
37	SD(Rp,t)	8%					
38							

$$\frac{t'\mu - r_f}{(t'\Sigma t)^{1/2}}$$

An important result from Black (1972)

- The range of cells D33:D35 is called tvec and will contain the weights in the tangency portfolio once the solver is run and the solution to the optimization problem is found. Before the solver is to be run, these cells should contain an initial guess of the minimum variance portfolio.

- A simple guess for this vector whose weights sum to one is $t_{msft} = 0.3,\ t_{nord} = 0.3,\ t_{sbux} = 0.4.$

- The computation of Sharpe's slope is broken down into two pieces. The first piece is the numerator of Sharpe's slope $\mu_{p,t} - r_f = \boldsymbol{t}'\mu - r_f$, and is computed in cell F33 using the array formula {=MMULT(TRANSPOSE(tvec),muvec)-rf}.

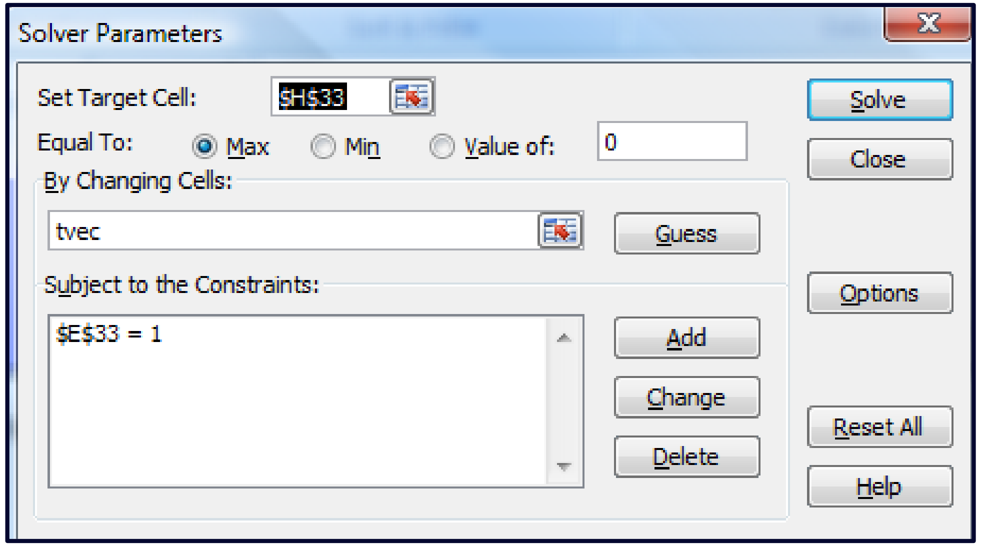
An important result from Black (1972)

- The second piece is the square of the denominator of Sharpe's slope, $\sigma^2_{p,t} = \boldsymbol{t}'\Sigma\mathbf{t}$, and is computed in cell G33 using the array formula {=MMULT(TRANSPOSE(tvec),MMULT(sigma,tvec))}.

- Finally, Sharpe's slope is evaluated in cell H33 using the formula =F33/SQRT(G33). This is the cell that is passed to the solver.

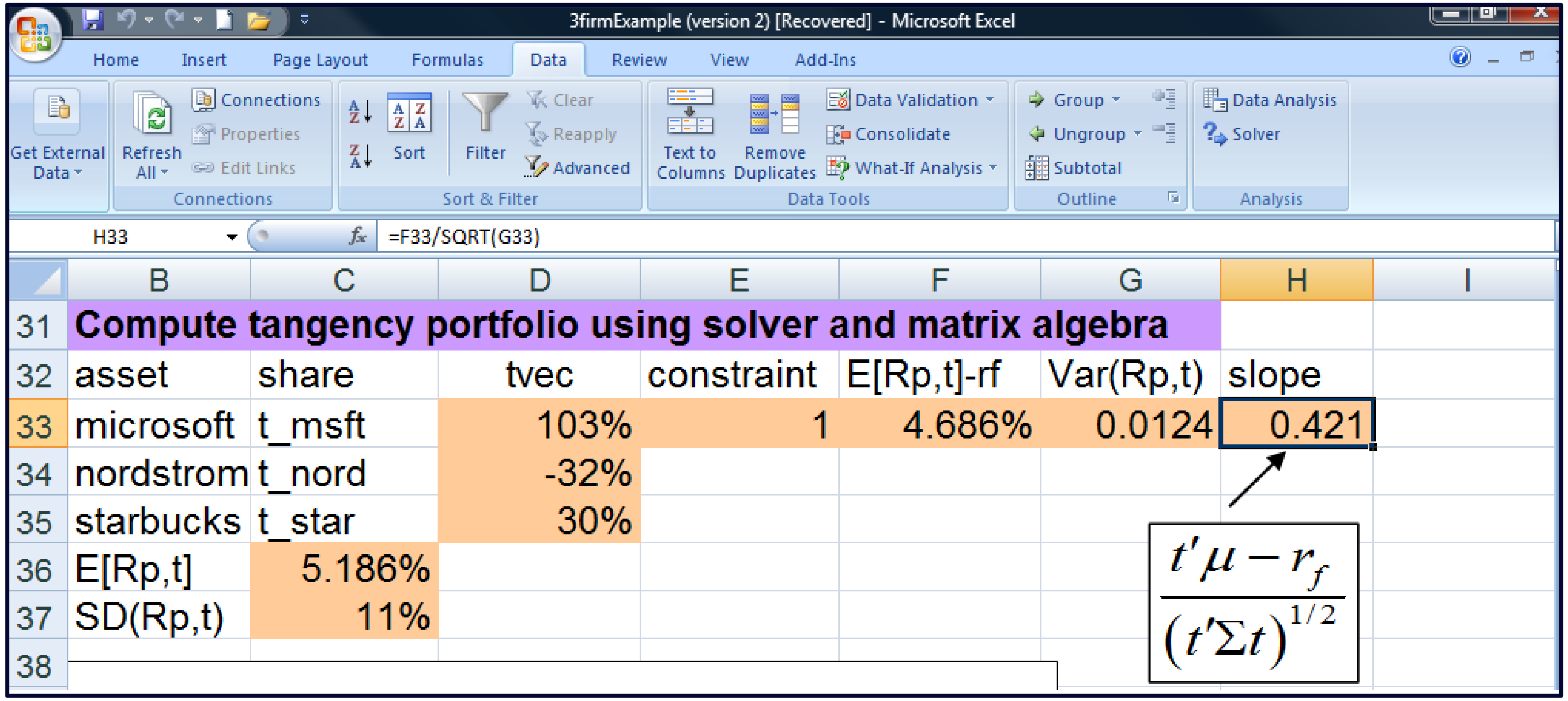
An important result from Black (1972)

- To run the solver, click cell H33 and then click on the solver button. Make sure the solver dialogue box is filled out to look like the one below.



- Make sure that the Max button is selected because we want to maximize the Sharpe's slope. To run the solver, click the Solve button. You should see a dialogue box that says that the solver found a solution and that all optimality conditions are satisfied. Keep the solution and click OK. Your spreadsheet should look like the one below.

An important result from Black (1972)


3firmExample (version 2) [Recovered] - Microsoft Excel

H33 =F33/SQRT(G33)

	B	C	D	E	F	G	H
31	Compute tangency portfolio using solver and matrix algebra						
32	asset	share	tvec	constraint	E[Rp,t]-rf	Var(Rp,t)	slope
33	microsoft	t_msft	103%	1	4.686%	0.0124	0.421
34	nordstrom	t_nord	-32%				
35	starbucks	t_star	30%				
36	E[Rp,t]	5.186%					
37	SD(Rp,t)	11%					
38							

$$\frac{t'\mu - r_f}{(t'\Sigma t)^{1/2}}$$

The tangency portfolio has weights $t_{msft} = 1.03,\ t_{nord} = -0.32,\ t_{sbux} = 0.30$.
The expected return on this portfolio, $\mu_{p,t} = \boldsymbol{t}'\mu$, is given in C36 (called mut)
and is computed using the array formula {=MMULT(TRANSPOSE(tvec),muvec)}.

Computing Efficient Portfolios of T-Bills and Risky Assets

- Mutual Fund Separation Theorem: Investors will choose their optimal complete portfolio using T-bills and the tangency portfolio. (2-fund separation theorem)

- Thus, the efficient portfolios of T-bills and risky assets are combinations of T-bills and the tangency portfolio.

- The expected return and standard deviation values of these portfolios are computed using

$$\mu_p^e = r_f + x_{tan}(\mu_{tan} - r_f),\ \sigma_p^2 = x_{tan}\sigma_{tan}.$$

- A screenshot of the spreadsheet where these portfolios are computed is given below.