투자론

- *R*과 Excel을 통한 금융데이터 분석 -

3주차
포트폴리오 이론 1

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Unit 01

Optimal Portfolio 1

Overview

- The Investment Decision
- Optimal Risky Portfolio - Part 1

The Investment Decision

- Top-down process with 3 steps

1. Capital allocation: risky portfolio and risk-free asset
2. Asset allocation: across broad asset classes
3. Security selection: individual assets within asset class

Diversification and Portfolio Risk

Market Risk

- Market-wide risk sources
- Remains even after the diversification
- Also called: Systematic or Nondiversifiable

Firm-Specific Risk

- Risk that can be eliminated by diversification
- Also called: Diversifiable or Nonsystematic

Portfolio Risk and the Number of Stocks in the Portfolio

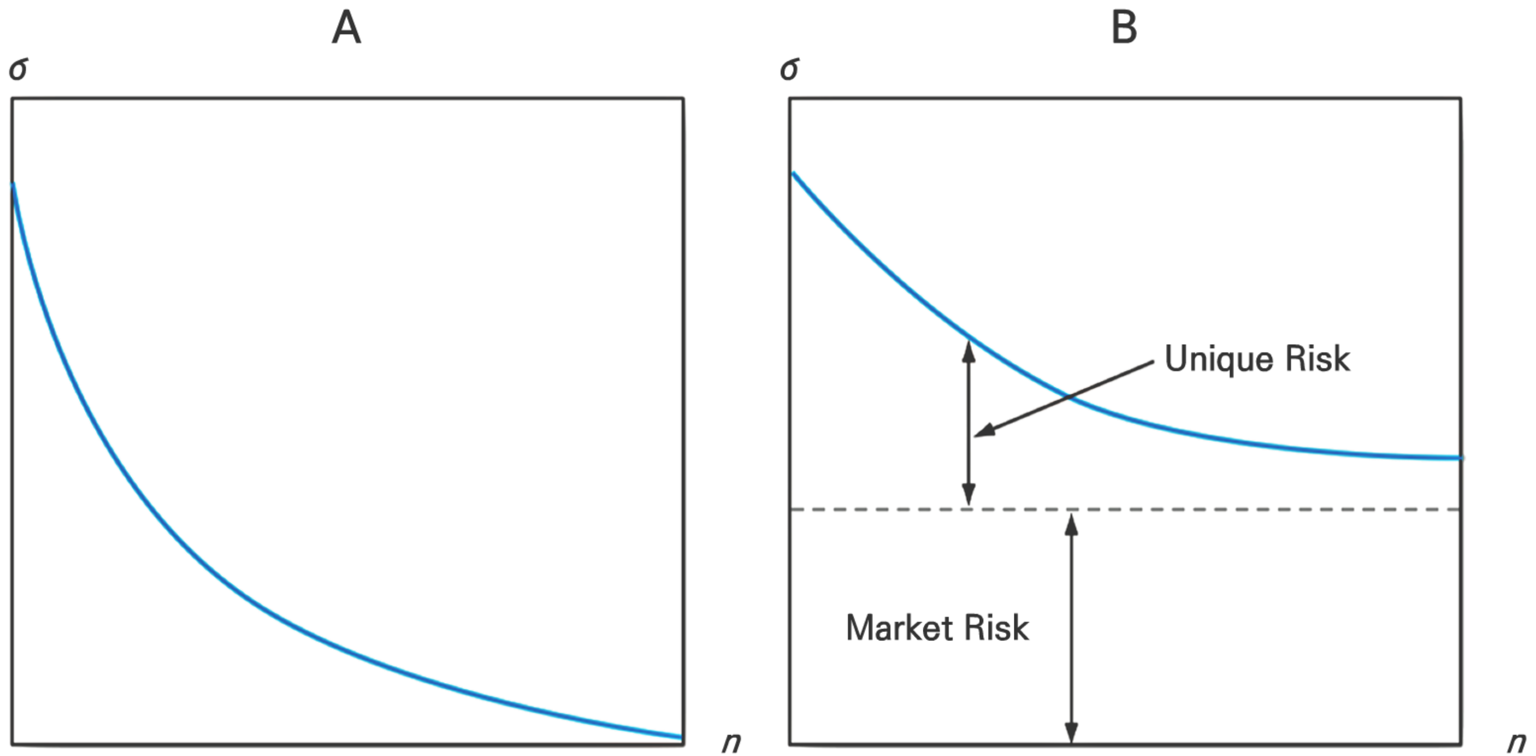


Figure 7.1 Portfolio risk as a function if the number of stocks in the portfolio
Panel A : All risk is firm specific Pnael B : Some risk is systematic, or marketwide

Portfolio Risk and the Number of Stocks in the Portfolio

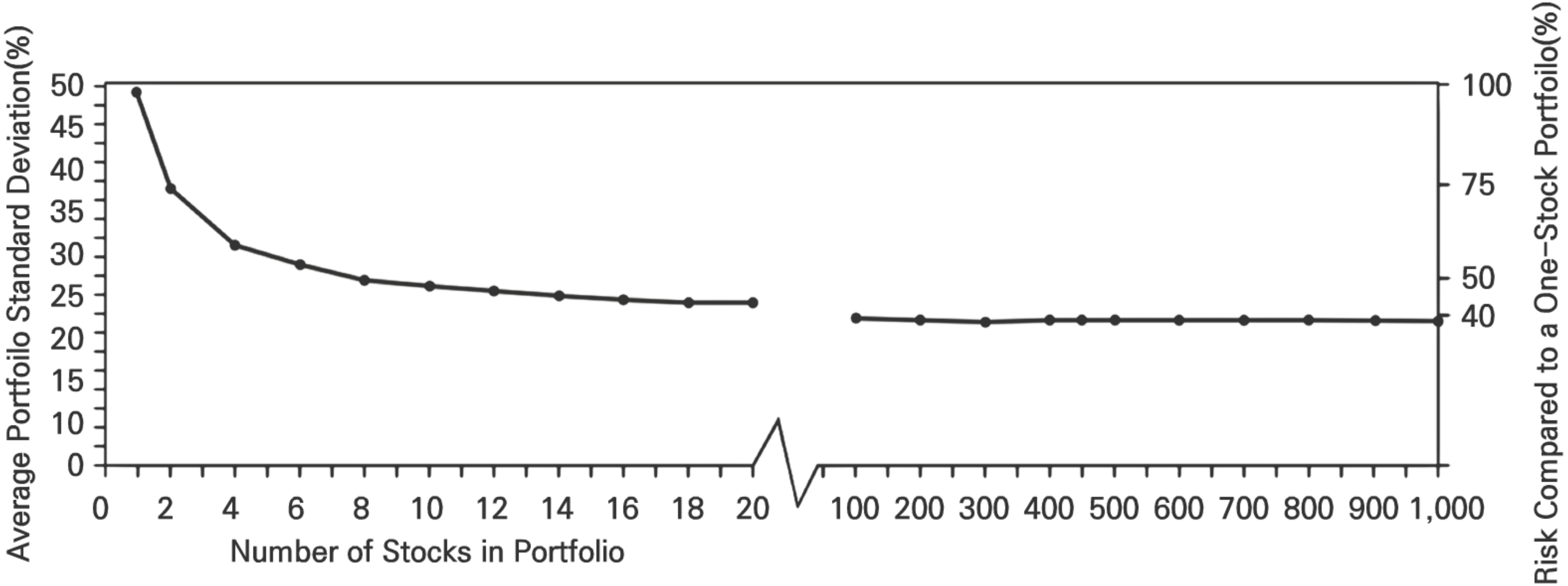


Figure 7.2 Portfolio diversification. The average standard deviation of returns of portfolios composed of only one stock was 49.2%. The average portfolio risk fell rapidly as the number of stocks included in the portfolio increased. In the limit, portfolio risk could be reduced to only 19.2%

출처 : Meir Statman, "How Many Stocks Make a Diversified Portfolio? " Journal of Financial and Quantitative Analysis 22 (septemper 1987)

Portfolio with Two Risky Assets

			Portfolio Standard Deviation for Given Correlation			
w_D	w_E	E(r_p)	ρ=-1	ρ=0	ρ=0.30	ρ=1
0.00	1.00	13.00	20.00	20.00	20.00	20.00
0.10	0.90	12.50	16.80	18.04	18.40	19.20
0.20	0.80	12.00	13.60	16.18	16.88	18.40
0.30	0.70	1150	10.40	14.46	15.47	17.60
0.40	0.60	11.00	7.20	12.92	14.20	16.80
0.50	0.50	10.50	4.00	11.68	13.11	16.00
0.60	0.40	10.00	0.80	10.76	12.26	15.20
0.70	0.30	9.50	2.40	10.32	11.70	14.40
0.80	0.20	9.00	5.60	10.40	11.45	13.60
0.90	0.10	8.50	8.80	10.98	11.56	12.80
1.00	0.00	8.00	12.00	12.00	12.00	12.00
			Minimum Variance Portfolio			
		w_D	0.6250	0.7353	0.8200	–
		w_E	0.3750	0.2647	0.1800	–
		E(r_p)	9.8750	9.3235	8.9000	–
		σ_p	0.0000	10.2899	11.4473	–

Example

- Descriptive statistics for two (one debt, another equity) mutual funds

$$E(r_D) = 8\%\ , E(r_E) = 13\%\ , \sigma_D = 12\%\ ,$$
$$\sigma_E = 20\%\ , \sigma_{D,E} = 72\ , \rho_{D,E} = 0.3$$

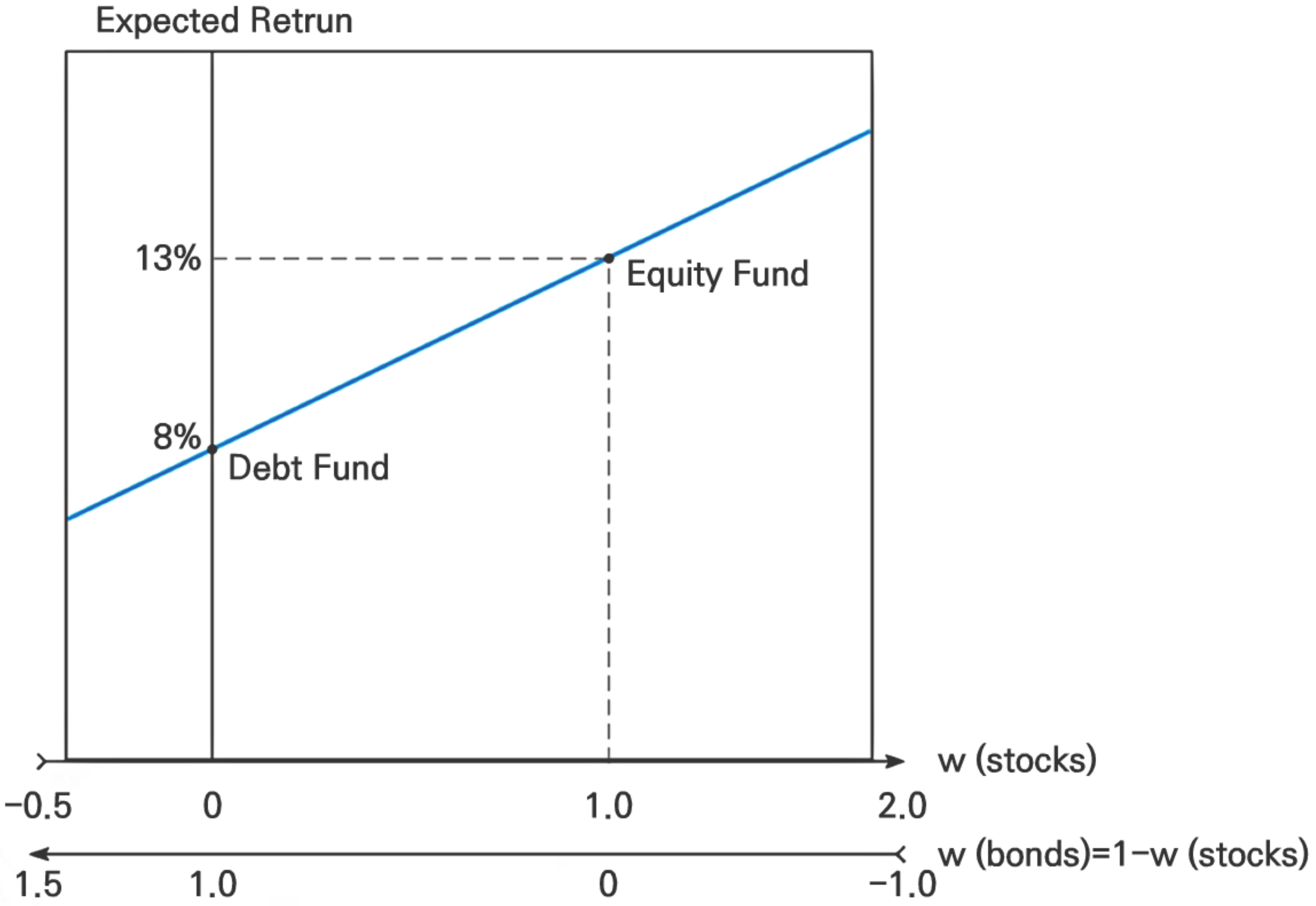
- Portfolio: 50% in Debt Fund, 50% in Equity Fund

  - $\mu_p = w_D E(r_D) + w_E E(r_E) = 0.5 \times 8\% + 0.5 \times 13\% = 10.5\%$
  - $\sigma_p^2 = w_D^2 \sigma_D^2 + w_E \sigma_E^2 + 2 w_D w_E Cov(r_D, r_E)$

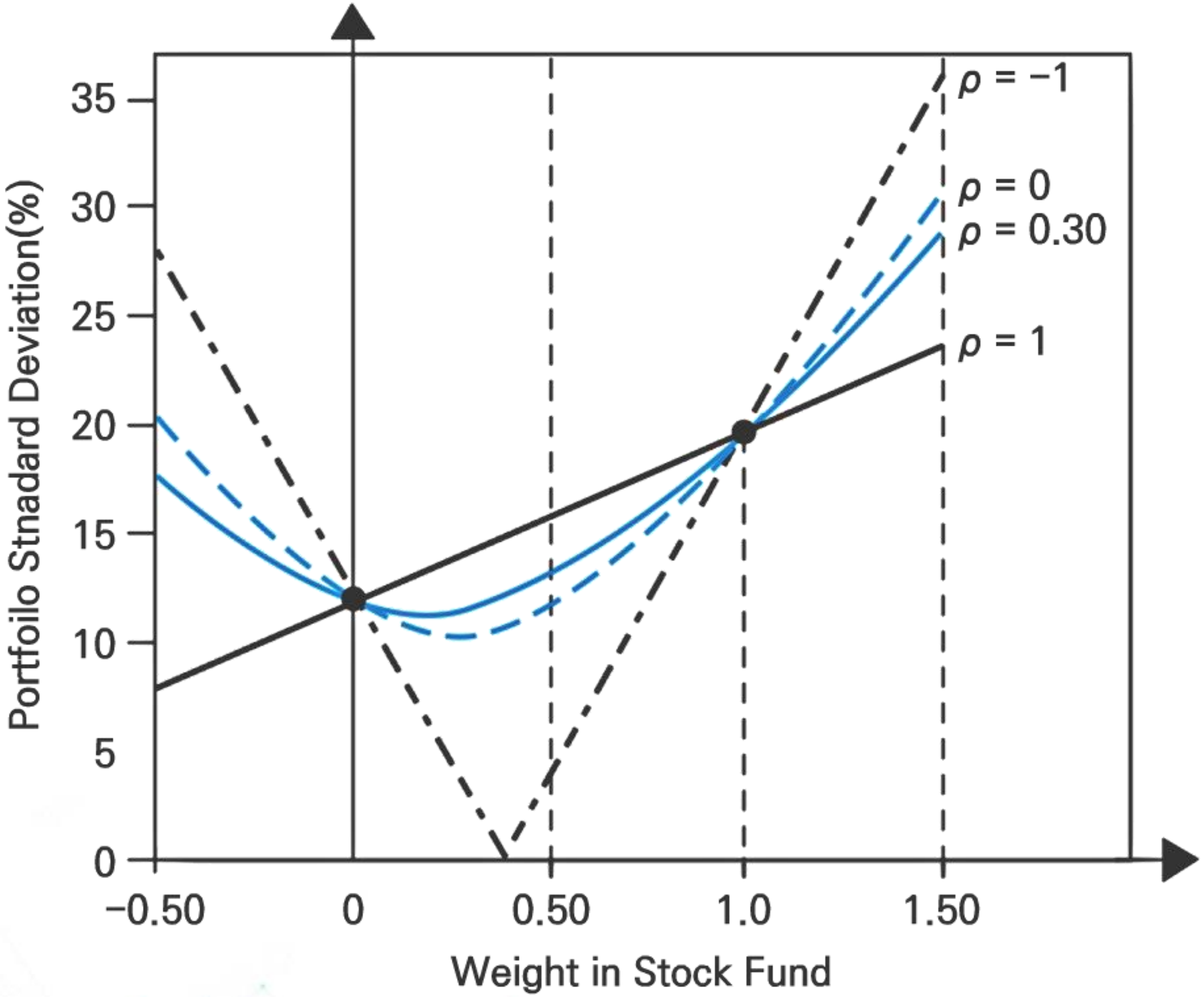
    $= .5^2 \times 12^2 + .5^2 \times 20^2 + 2 \times 0.5 \times 0.5 \times 72 = 172$

  $\Longrightarrow \sigma_p = \sqrt{172} = 13.32\%$

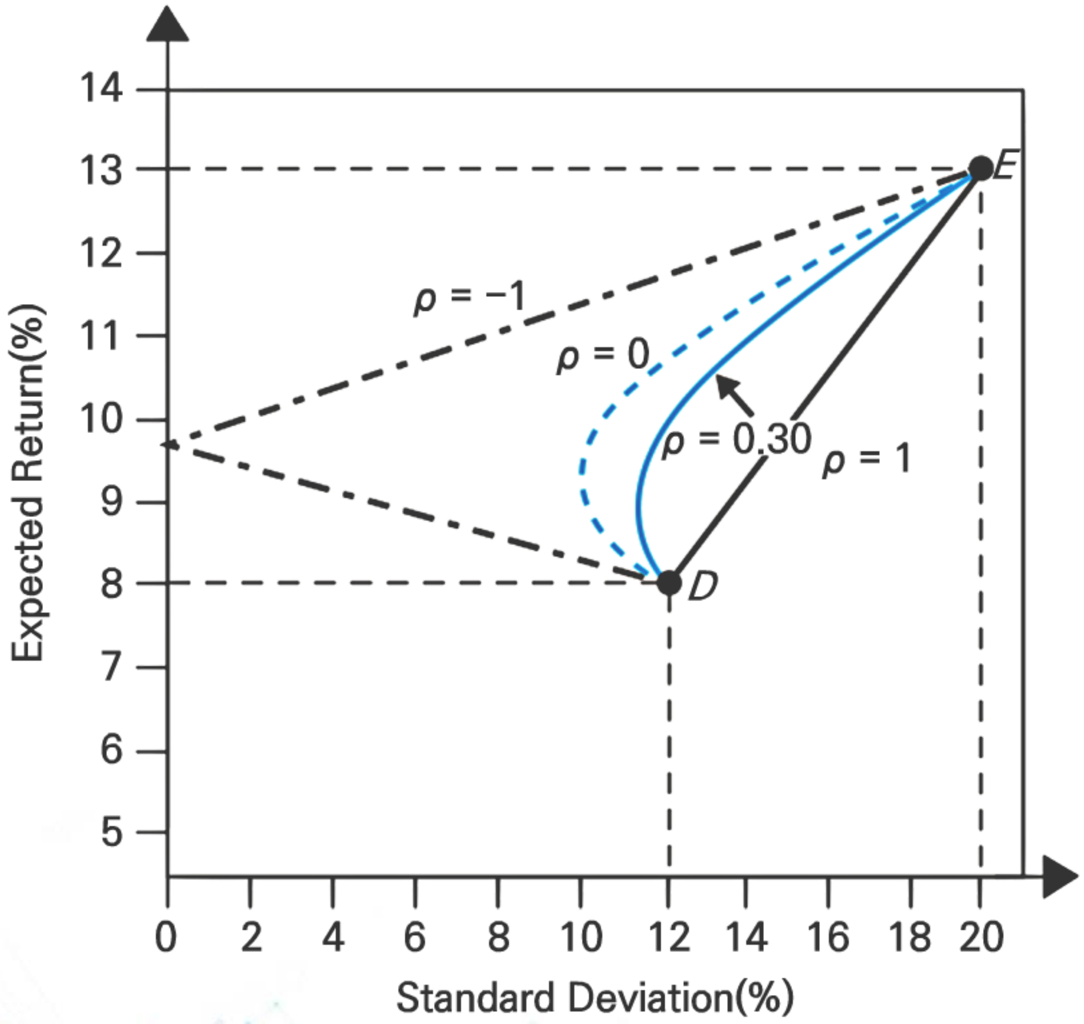
Portfolio Expected Return



Portfolio Expected Return



Expected Return as a Function of Standard Deviation



Minimum Variance Portfolio (MVP)

- The minimum variance portfolio: the portfolio composed of risky assets with smallest standard deviation

- Risk reduction depends on the correlation
  - If $\rho = +1.0$, no risk reduction
  - If $\rho = 0, \sigma_p$ may be less than the STD of either component asset
  - If $\rho = -1$, a riskless hedge is possible

MVP

$$Min_{w_D,w_E}\sigma_p^2$$
$$= w_D^2\sigma_D^2 + w_E^2\sigma_E^2 + 2w_D w_E Cov(r_D, r_E), \text{ where } w_D + w_E = 1$$

Solution

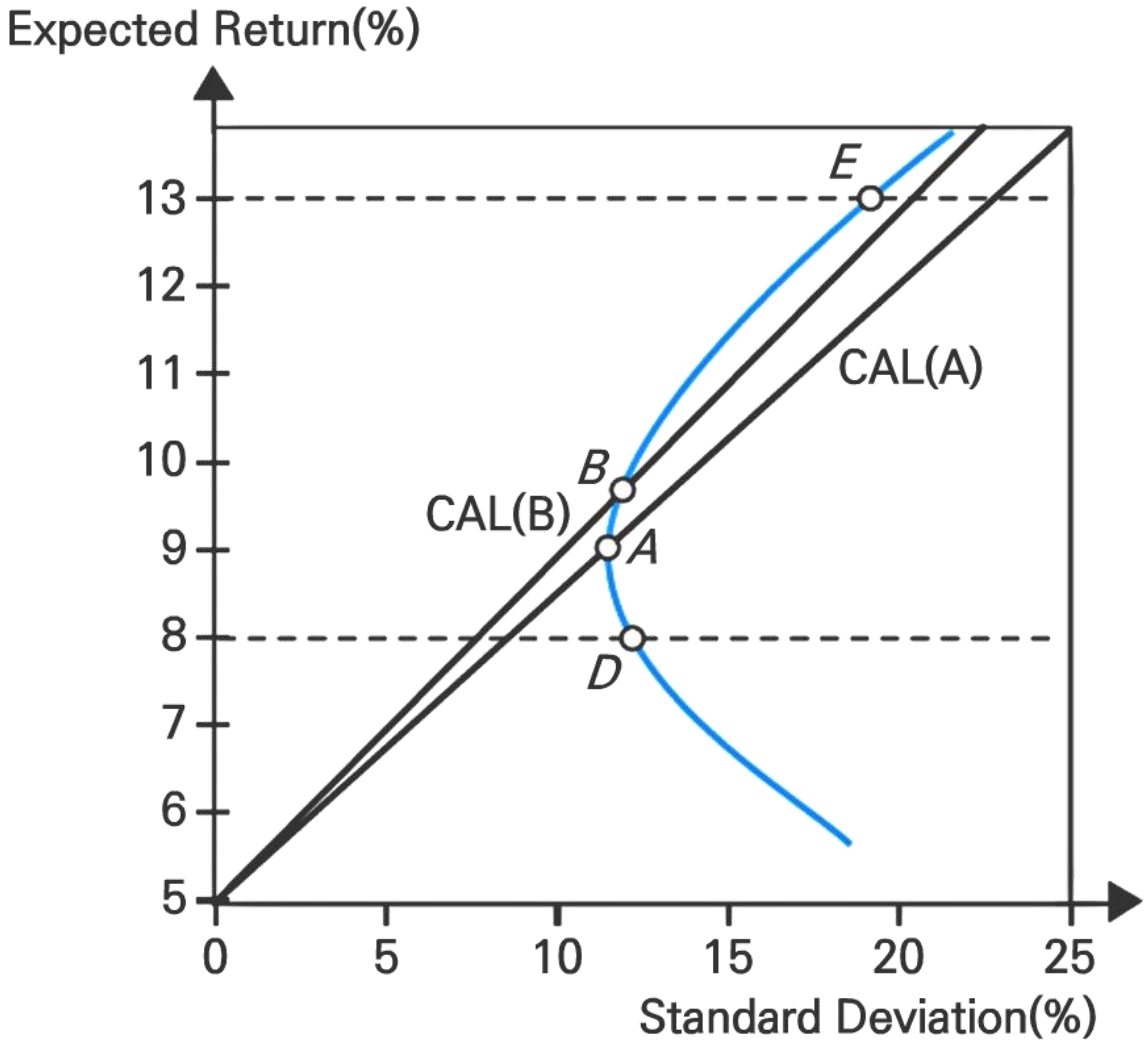
$$w_D^* = \frac{\sigma_E^2 - Cov(r_D, r_E)}{\sigma_D^2 + \sigma_E^2 - 2Cov(r_D, r_E)}$$

The Opportunity Set of the Debt and Equity Funds and Two Feasible CALs

- Two risky assets: bond and equity
- One riskless asset: T-bill
- Data

$E(r_E) = 13\%, E(r_D) = 8\%$
$SD(r_E) = 20\%, SD(r_D) = 12\%$
$Cov(r_E, r_B) = 72, \rho(r_E, r_B) = 0.3$
$r_f = 5\%$

- CAL(A): MVP
- CAL(B): 70% in B, 30% in E
- Goal: A → B→ where?



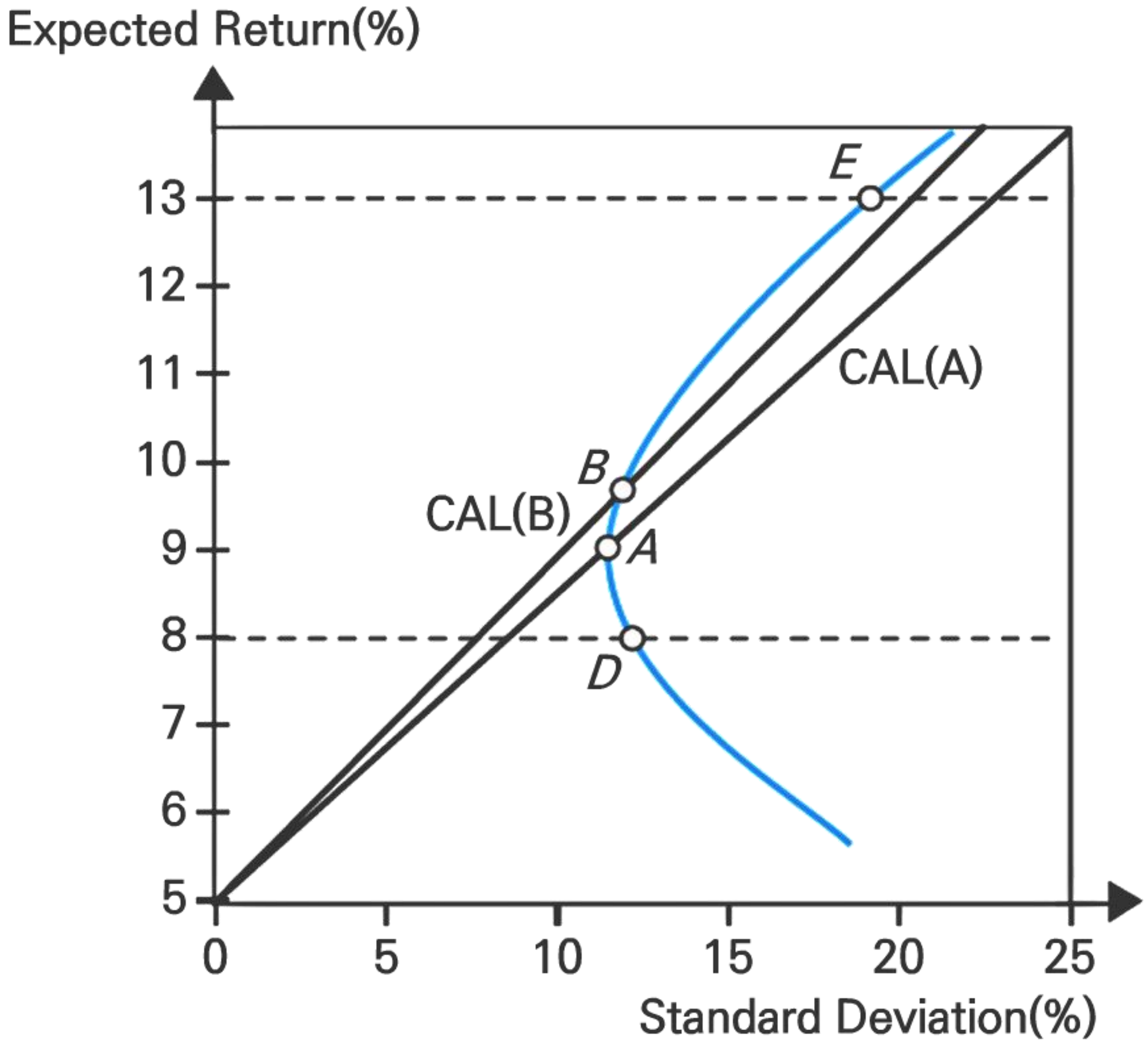
General Criteria: Sharpe Ratio

The Informational Role

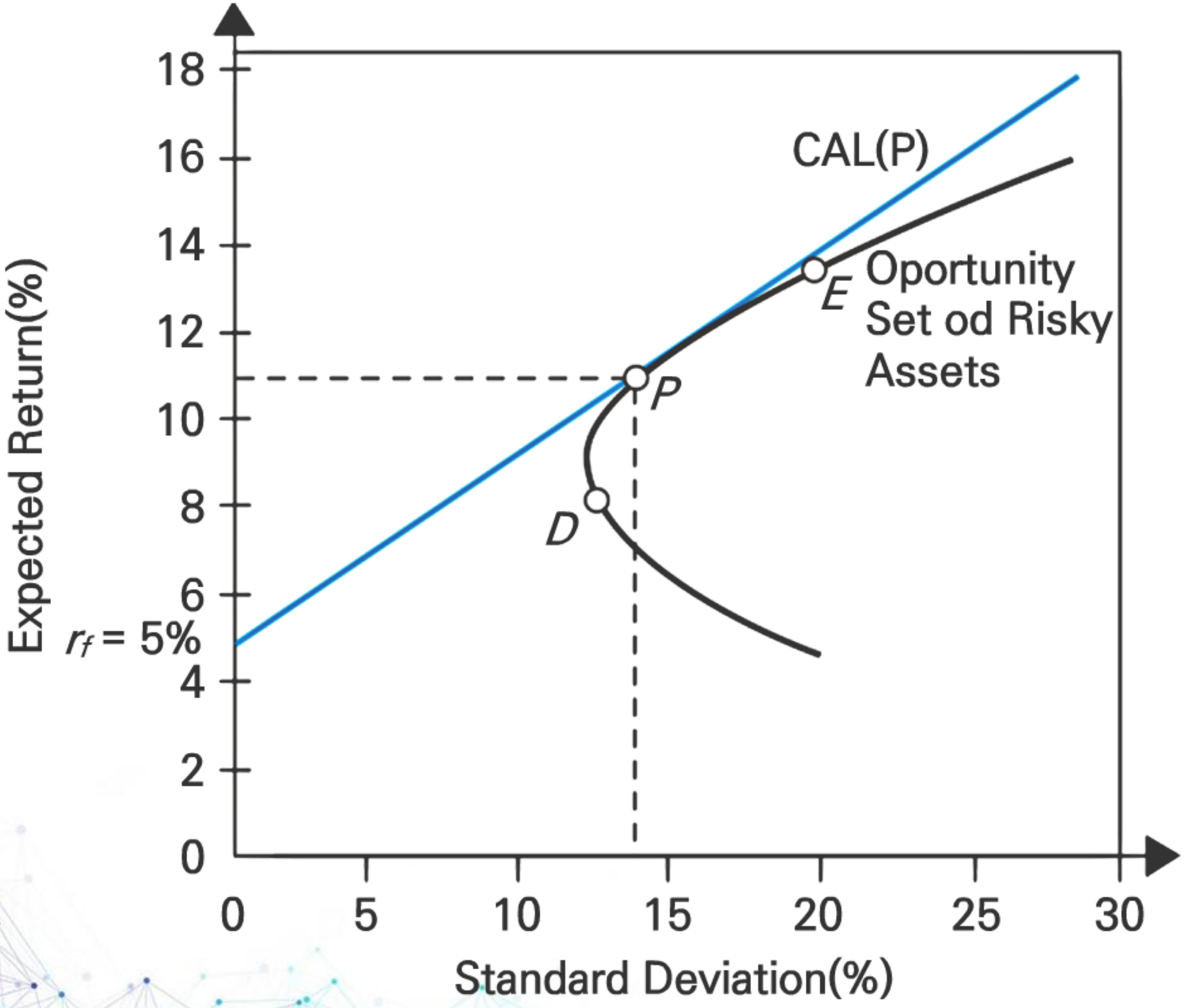
$$S_A = \frac{E(r_A) - r_f}{\sigma_A} = \frac{8.9 - 5}{11.45} = .34$$

$$S_B = \frac{E(r_B) - r_f}{\sigma_B} = \frac{9.5 - 5}{11.7} = .38$$

- On what point would the Sharpe ratio be maximized?
- Answer: Tangent Portfolio



General Criteria: Sharpe Ratio



Tangent Portfolio

$$Max_{w_i} S_p = \frac{E(r_p) - r_f}{\sigma_p}, \text{ subject to } \sum_{i=1}^{n} w_i = 1$$

- In the case of two risky assets, the optimal risky portfolio, $\boldsymbol{P}$, is given by

$$w_D = \frac{E(r_D - r_f)\sigma_E^2 - E(r_E - r_f)Cov(r_D, r_E)}{E(r_D - r_f)\sigma_E^2 + E(r_E - r_f)\sigma_B^2 - [E(r_D) - r_f + E(r_E) - r_f]Cov(r_D, r_E)},$$

$$w_E = 1 - w_D$$

Computation

In the previous example

$$w_D = \frac{(8-5)400-(13-5)72}{(8-5)400+(13-5)144-(8-5+13-5)72} = 0.4$$

$$w_E = 1 - w_D = 0.6$$

Optimal Complete Portfolio

- What is optimal?
- Optimality means the maximality of an investor's utility
- A simple form of an investor's utility is expressed as $U = E(r) - \frac{1}{2}A\sigma^2$
- GOAL

  $Max_i U = E(r_i) - \frac{1}{2}A\sigma_i^2$, where r_i is determined by the tangent portfolio and riskless asset (r_i is a general case of the portfolio of an investor i)

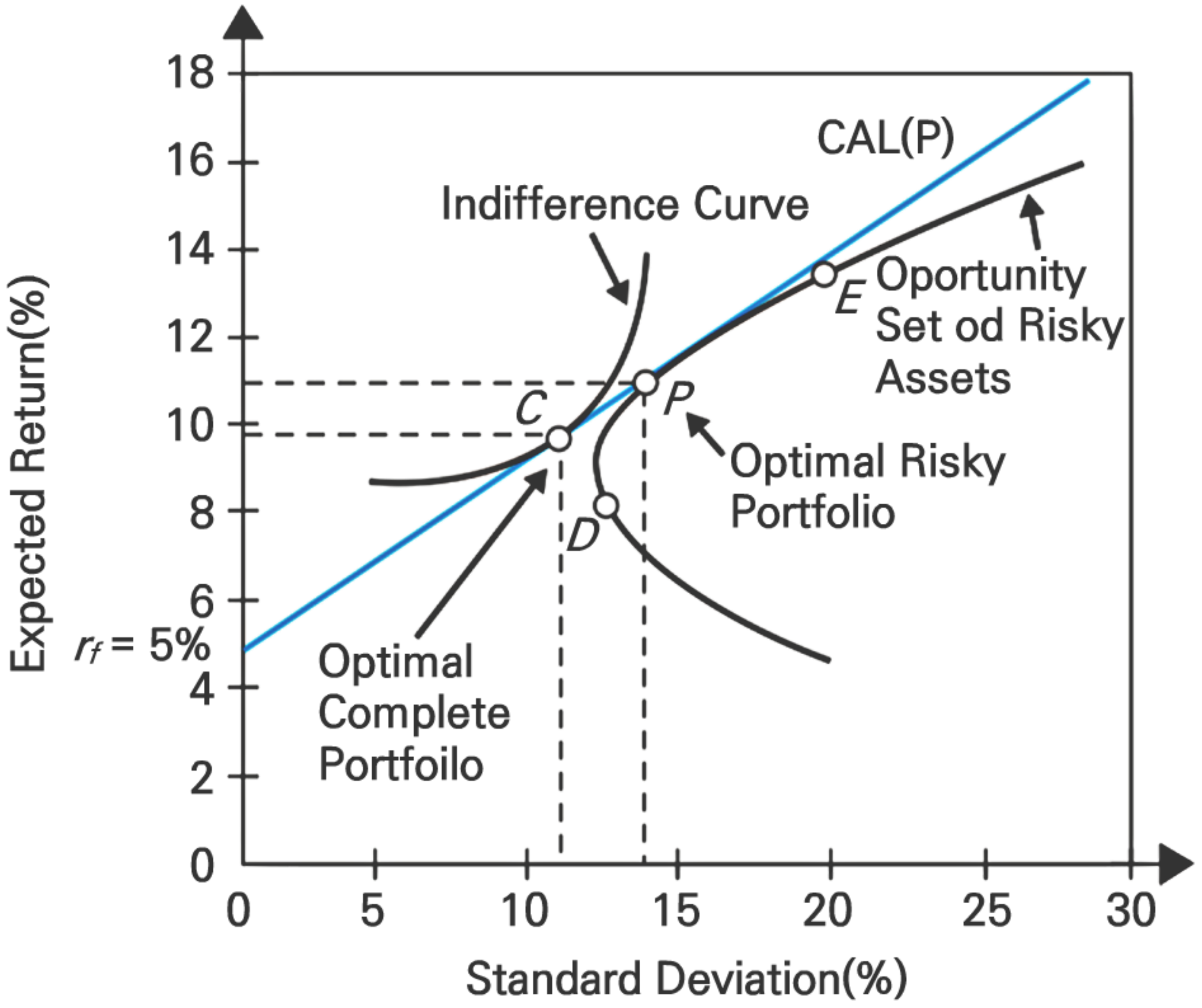
- From the previous result, we know that the optimal % investment in the risky portfolio is given by $y^* = \frac{E(r_p)-r_f}{A\sigma_p^2}$, where r_p is the return of the tangent portfolio

- Warning: one has to use decimals for y^*

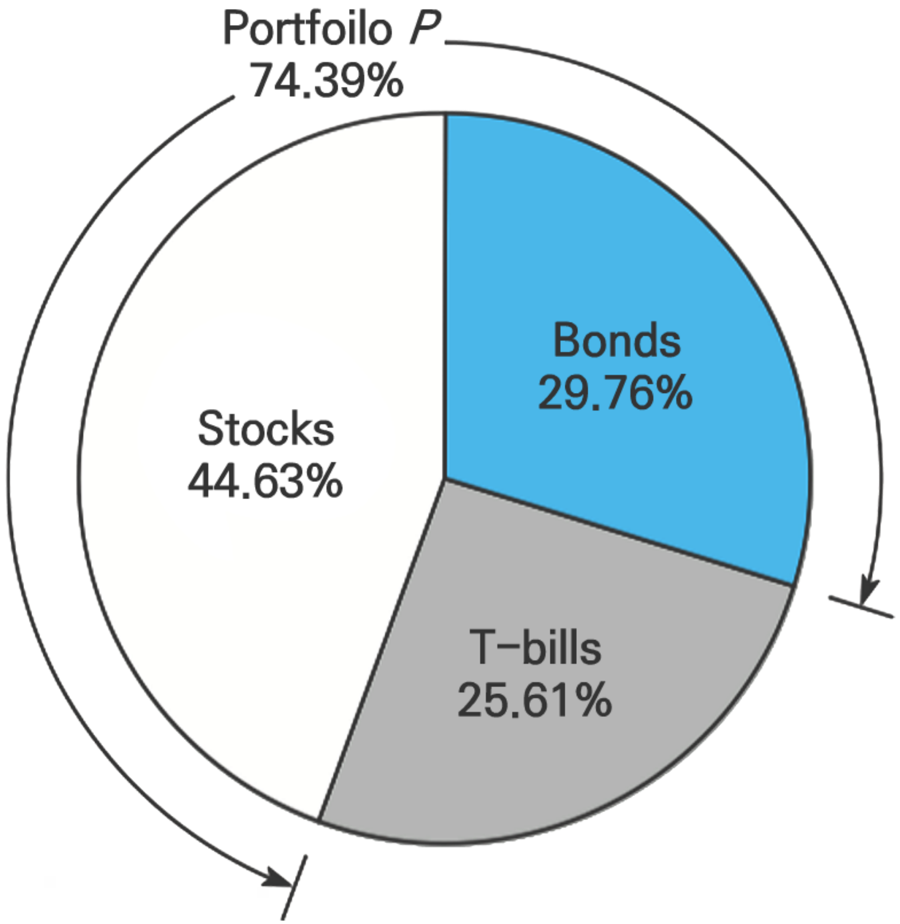
Optimal Complete Portfolio

Optimal Allocation to P with A=4:

$$y = \frac{E(r_p) - r_f}{A\sigma_p^2} = \frac{0.11 - 0.05}{4 \times (0.0142)^2} = 0.7439$$



The Proportions of the Optimal Complete Portfolio



Exercise Problem 1

Stocks offer an expected rate of return of 18%, with a standard deviation of 22%. Gold offers an expected return of 10% with a standard deviation of 30%.

1. In light of the apparent inferiority of gold with respect to both mean return and volatility, would anyone hold gold? If so, demonstrate graphically why one would do so.

2. Given the data above, reanswer (a) with the additional assumption that the correlation coefficient between gold and stocks equals 1. Draw a graph illustrating why one would or would not hold gold in one's portfolio.

3. Could the set of assumptions in part (b) for expected returns, standard deviations, and correlation represent an equilibrium for the security market?