# 투자론

- R과 Excel을 통한 금융데이터 분석 -

## 4주차
## 포트폴리오 이론 2

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Unit 01

# Simple Matrix Algebra and Excel Computation

## Definitions and Notations

- There are $N$ risky assets whose expected returns are $E(r_i)'s$
- The matrix $E(r)$ is the column vector of expected returns of these assets

$$E(r) = \begin{pmatrix} E(r_1) \\ E(r_2) \\ \vdots \\ E(r_N) \end{pmatrix}$$

## Definitions and Notations

- $\Omega$ is the $N \times N$ variance–covariance matrix:

$$\Omega = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ \sigma_{1N} & \sigma_{2N} & \cdots & \sigma_{NN} \end{bmatrix} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ \sigma_{1N} & \sigma_{2N} & \cdots & \sigma_N^2 \end{bmatrix}$$

# Definitions and Notations

- A portfolio of risky assets is a column vector $\boldsymbol{x}$ whose coordinates sum to 1

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}, \sum_{i=1}^{N} x_1 = 1$$

 – where $x_i$ represents investment weight on $i$−th asset

## Definitions and Notations

- The expected portfolio return $E(r_x)$ of a portfolio $x$ is given by the product of $x$ and return vector $R$

$$E(r_x) = \sum_{i=1}^{N} x_i E(r_i) = x'R \text{, where } R = E(r) = \begin{bmatrix} E(r_1) \\ E(r_2) \\ \vdots \\ E(r_N) \end{bmatrix}$$

## Definitions and Notations

- The variance of portfolio $x$'s return, $\sigma_x^2 = \sigma_{xx}$, is given bye the product

$$x'\Omega x = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij}$$

- The covariance between the return of two portfolios $x$ and $y$, $Cov(r_x, r_y)$, is defined by the product

$$\sigma_{xy} = x'\Omega y = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij}. \text{ (Note: } \sigma_{ij} = \sigma_{ji}\text{)}$$

## Computation with Excel

- For a given portfolio vector $\boldsymbol{x} = (\boldsymbol{x}_1, \cdots, \boldsymbol{x}_n)'$ with return vector $\boldsymbol{r} = (\boldsymbol{r}_1, \cdots, \boldsymbol{r}_n)'$

  - $E(x) \Rightarrow$ MMult(tranpose(x),$E(r)$)
  - $Var(x) \Rightarrow$ MMult(MMult(transpose(x),Ω),x)
  - $SD(x) \Rightarrow sqrt(Var(x))$
  - $Cov(x, y) \Rightarrow$ MMult(MMult(transpose(x),Ω),y)
  - $Corr(x, y) \Rightarrow Cov(x, y)/(sqrt(Var(x)) * sqrt(Var(y)))$