투자론

- *R*과 Excel을 통한 금융데이터 분석 -

2주차
수익률과 위험

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Unit 01

Review of Statistics

Probability

Terminologies

- Ω: Sample Space = a set of all possible outcomes
- $\omega \in \Omega$: Sample Outcome = element of the sample space
- $A \subseteq \Omega$: Event = a set of ω that satisfies certain condition
- $X: \Omega \rightarrow \mathcal{R}$: Real-Valued Random Variable
  = a function that maps from the sample space to the real line
  (or Euclidean space)

Probability

- A probability is a set function that satisfies the following properties

1. $0 \leq P(A) \leq 1$ for each event A
2. $P(\Omega)=1$
3. For each event sequence $A_1,\ A_2,\cdots,A_n,\cdots$ of mutually exclusive (disjoint) events $P(\cup_{i=1}^{\infty} A_i)=\sum_{i=1}^{\infty} P(A_i)$

Random Variable

Discrete Random Variables

$X : \Omega \rightarrow \mathcal{R}$ takes on values in $S=\{x_1,\ x_2,\cdots\}$,

its probability mass function is defined by $P_X(x_i)=P(X=\ x_i),\ i\geq 1$

Given a collection $X_1, X_2, \cdots X_n$ of real-valued random variables,

its joint probability mass function (pmf) is defined as

$$P_{(X_1,\cdots,X_n)}(x_1,\cdots,x_n) = P(X_1 = x_1, \cdots, X_n = x_n)$$

The conditional pmf of X given $Y=y$ is then given by $P_{X|Y} = \frac{P_{(X,Y)}(x,y)}{P_X(x)}$

Random Variable

The collection of random variables

The collection of random variables $X_1, \cdots, X_n$ are independent

$$P_{(X_1,\cdots,X_n)}(x_1, \cdots, x_n) = P_{X_1}(x_1) \times P_{X_2}(x_2) \times \cdots \times P_{X_n}(x_n)$$

for all $(x_1, \cdots, x_n) \in S^n$

Expected(Mean) Returns

Expected(Mean) Returns

- Let r be a return of an asset
- Obviously, r is a random variable
- Think of n different possibilities (or scenarios)
- Technically speaking, we call this situation n states
- Return will take n different values with n different probabilities
- Expected return is defined by

$$E(r) = \sum_{s=1}^{n} p(s)r(s)$$

Expected(Mean) Returns

	A	B	C	D	E
1					
2			Scenario Rates of Return		
3	Scenario	Probability	$r_D(i)$	$r_D(i)$+0.03	0.4*$r_D(i)$
4	1	0.14	−0.10	−0.007	−0.040
5	2	0.36	0.00	0.03	0.000
6	3	0.30	0.10	0.13	0.040
7	4	0.20	0.32	0.35	0.128
8		Mean	0.080	0.110	0.032
9		Cell C8	=SUMPRODUCT($B$4:$B$7,C4:C7)		
10					
11					
12					

Expected Return of a Portfolio

- Portfolio is a collection of investment weights that are invested in assets

$$(w\_1, \cdots, w\_N)$$

- Return of a portfolio is given by

$$r_P = w_1 \times r_1 + w_2 \times r_2 + \cdots + w_N \times r_N$$

- Note that each $\boldsymbol{r_i, i = 1, \cdots, N}$ is a random variable
  Also, the portfolio is a random variable as it is the weighted sum of random variables

Expected Return of a Portfolio

- Expected return of a portfolio is just the linear sum of expected values of individual assets weighted by their investment weights

$$E(r_P) = \sum_{i=1}^{N} w_i \times E(r_i)$$

Variance

- Variance is the expected value that measures how typically a random variable moves around its mean

$$Var(X) = \sigma_X^2 = \sum_{\forall x}(x - \mu_X)^2 \times P(X = x)$$
$$= \sum_{i=1}^{N}(r_i - \mu_r)^2 \times P(r = r_i)$$

Excel 7.B.2

Scenario analysis for bonds and stocks

	H	I	J	K	L
1					
2			Scenario Rates of Return		Portfoilo Return
3	Scenario	Probability	$r_D(i)$	$r_E(i)$	$0.4{*}r_D(i)+0.6{*}r_E(i)$
4	1	0.14	−0.10	−0.35	−0.2500
5	2	0.36	0.00	0.20	0.1200
6	3	0.30	0.10	0.45	0.3100
7	4	0.20	0.32	−0.19	0.0140
8		Mean	0.080	0.12	0.1040
9		Cell L4	=0.4*J4+0.6*K4		
10		Cell L8	=SUMPRODUCT($I$4:$I$7,L4:L7)		
11					

Excel 7.B.3

Scenario analysis for bonds and stocks

	A	B	C	D	E
13					
14			Scenario Rates of Return		
15	Scenario	Probability	$r_D(i)$	$r_D(i)$+0.03	0.4*$r_D(i)$
16	1	0.14	−0.10	−0.007	−0.040
17	2	0.36	0.00	0.03	0.000
18	3	0.30	0.10	0.13	0.040
19	4	0.20	0.32	0.35	0.128
20		Mean	0.080	0.110	0.032
21		Variance	0.0185	0.0185	0.0034
22		SD	0.1359	0.1359	0.0584
23	Cell C21 =SUMPRODUCT($B$16:$B$19,C16:C19, C16:C19)−C20^2				
24	Cell C22 =C21^0.5				

Excel 7B.8

Variance of a portfolio

$$\sigma_P^2 = \sum_{i=1}^{n} P(i)[\mathrm{r}_p - \mathrm{E}(\mathrm{r}_p)]^2 = \sum_{i=1}^{n} [w_D d(i) + w_E e(i)]^2$$

$$= \sum_{i=1}^{n} P(i)[w_D^2 d(i)^2 + w_E^2 e(i)^2 + 2 w_D w_E d(i) e(i)]$$

$$= w_D^2 \sum_{i=1}^{n} P(i) d(i)^2 + w_E^2 \sum_{i=1}^{n} P(i) e(i)^2 + 2 w_D w_E \sum_{i=1}^{n} P(i) d(i) e(i)$$

$$= w_D^2 \sigma_D^2 + w_E^2 \sigma_E^2 + 2 w_D w_E \sum_{i=1}^{n} P(i) d(i) e(i)$$

Excel 7B.6

Computation of the portfolio variance

	H	I	J	K
13				
14			Scenario Rates of Return	
15	Scenario	Probability	$r_D(i)$	$r_E(i)$
16	1	0.14	−0.10	−0.35
17	2	0.36	0.00	0.20
18	3	0.30	0.10	0.45
19	4	0.20	0.32	−0.19
20		Mean	0.08	0.12
21		SD	0.1359	0.2918
22		Covariance	−0.0034	
23		Correlation	-0.0847	
24	Cell J22	=SUMPRODUCT(I16:I19,J16:J19.K16:K19)−J20*K20		
25	Cell J23	=J22/(J21*K21)		

Covariance

The covariance between two variables are defined as

$$Cov(r_D, r_E) = E\left[\left(r_D - E(r_D)\right) \times \left(r_E - E(r_E)\right)\right]$$
$$= E(r_D r_E) - E(r_D)E(r_E)$$

- The covariance quantifies the covariation of two random variables
- In particular, it measures how linearly two random variables move together

Excel 7B.4

Computation of the covariance

	A	B	C	D	E	F	G	H
1		Rates of Return			Deviation from Mean			Product of
2	Probability	Bonds	Stocks		Bonds	Stocks		Deviation
3	0.25	−2	30		−8	20		−160
4	0.50	6	10		0	0		0
5	0.25	14	−10		8	−20		−160
6	Mean:	6	10		0	0		−80

Correlation Coefficient

- Though the covariance measures how two random variables move together linearly, it is subject to the size issue; the larger the random variables, the bigger the covariance is

- We need to remove the size issue. How? Normalize the covariance by the standard measure of the variability of random variables, i.e., standard deviation

$$Corr(r_D, r_E) = \frac{Cov(r_D, r_E)}{\sigma_D, \sigma_E}$$

Correlation Coefficient

Note

- A correlation coefficient falls between −1 and +1

$$Cov(r_D, r_E) = \sigma_D, \sigma_E \; Corr(r_D, r_E)$$

Excel 7B.5

Computation of the correlation coefficient

	H	I	J	K
13				
14			Scenario Rates of Return	
15	Scenario	Probability	$r_D(i)$	$r_E(i)$
16	1	0.14	-0.10	-0.35
17	2	0.36	0.00	0.20
18	3	0.30	0.10	0.45
19	4	0.20	0.32	-0.19
20		Mean	0.08	0.12
21		SD	0.1359	0.2918
22		Covariance	-0.0034	
23		Correlation	-0.0847	
24	Cell J22	=SUMPRODUCT(I16:I19,J16:J19.K16:K19)-J20*K20		
25	Cell J23	=J22/(J21*K21)		