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투자론

- *R*과 Excel을 통한 금융데이터 분석 -

10주차
채권가격과 수익률 및 수익률 곡선과 선도 이지율

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Unit 02

Yield Curve and Forward Rate

Overview

- The yield curve
- Interest rates under certainty
- Interest rates under uncertainty
- Forward rates

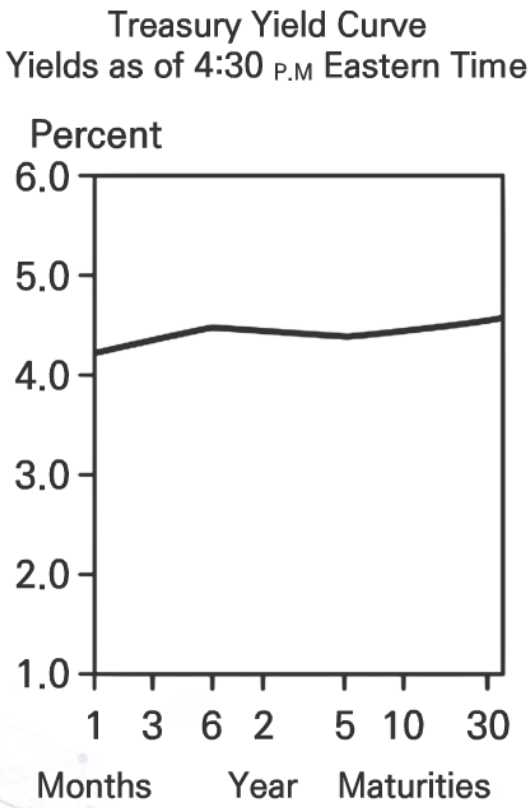
The Yield Curve

- The yield curve is a graph that displays the relationship between YTM and time to maturity

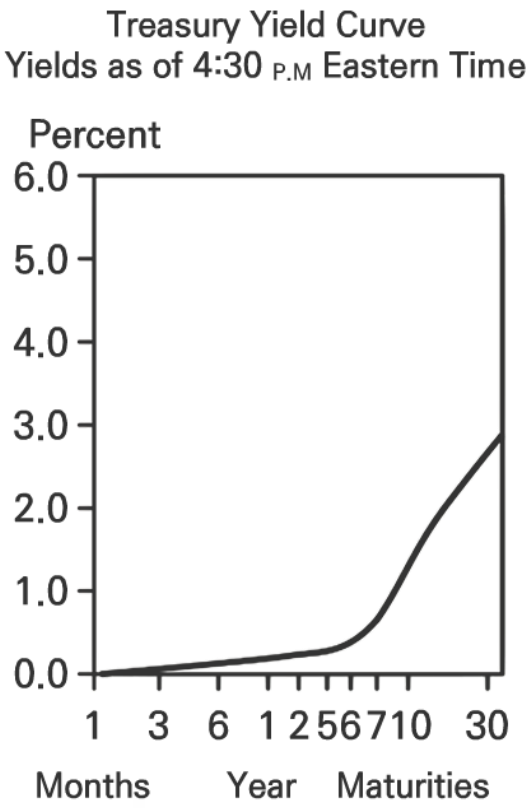
- Information on expected future short-term rates can be implied from the yield curve

Figure – Shape of Yield Curve and Its Implication

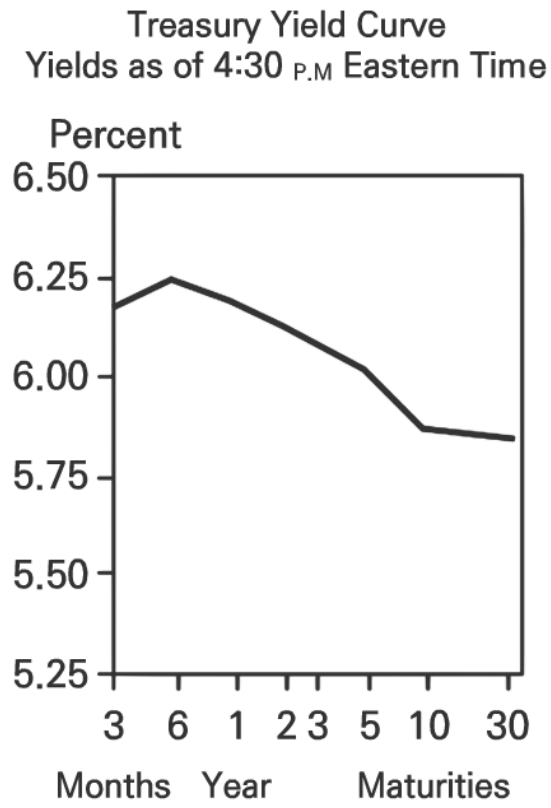
Treasury Yield Curve



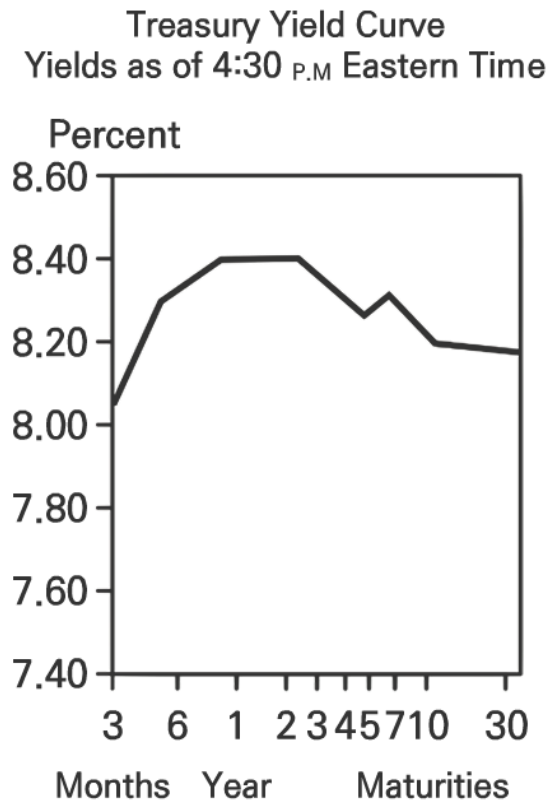
A. (January 2006)
Flat Yield Curve



B. (December 2012)
Rising Yield Curve



C. (September 11, 2000)
Inverted Yield Curve



D. (October 4, 1989)
Hump-shaped Yield Curve

Yield Curve: Bond Pricing

- Yields on different maturity bonds are not all equal
- We need to consider each bond cash flow as a stand-alone zero-coupon bond
- Bond stripping and bond reconstitution offer opportunities for arbitrage
- The value of the bond should be the sum of the values of its parts

Prices and YTMs on Zeros ($1,000 par value)

Maturity (years)	Yield to Maturity (%)	Price
1	5%	$952.38 = $1,000/1.05
2	6	$890.00 = $ 1,000/1.06^2
3	7	$816.30 = $1,000/$1.073^3$
4	8	$735.03 = $1,000/$1.084^4$

Example

Valuing coupon bonds

- Value a 3yr, 10% coupon (annual coupon payment) using discount rates from Table 15.1:

$$P = \frac{\$100}{1.05} + \frac{\$100}{1.06^2} + \frac{\$1{,}100}{1.07^3} = \$1082.1654$$

What about YTM?

$$\frac{\$100}{(1+r)} + \frac{\$100}{(1+r)^2} + \frac{\$1{,}100}{(1+r)^3} = \$1082.1654$$ (**use Excel**)

$\Rightarrow \; r = 6.88\%$ (less than 3yr rate of 7%, why?)

The Yield Curve and Future Interest Rates

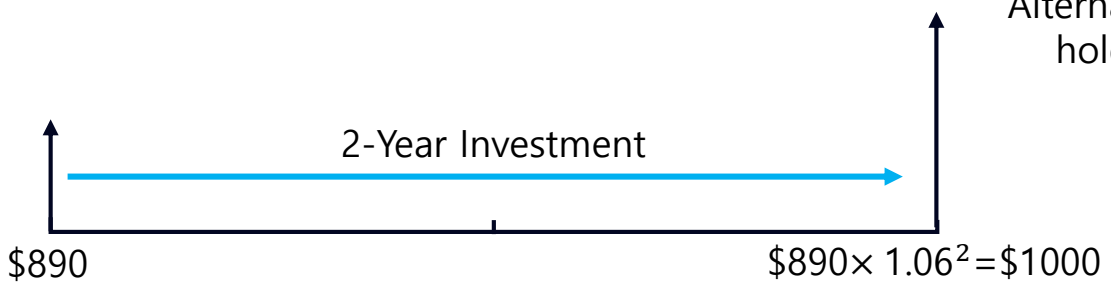
Yield Curve under Certainty

- Investment for 2 years:
  1. Buy and hold for 2 years
  2. Rollover a series of 1-year bonds

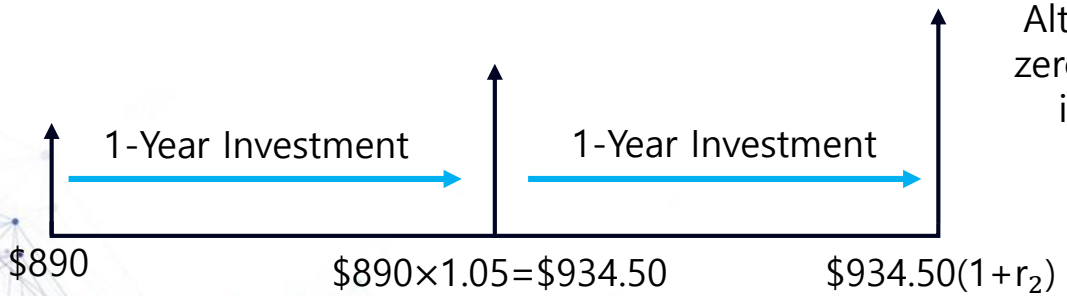
Equilibrium (via no-arbitrage) requires that both strategies provide the same return!

2-yr Investmetn Program

Time Line



Alternative 1: Buy and hold 2 year zero



Alternative 2: Buy a 1-year zero, and reinvest proceeds in another 1-year zero

2-yr Investmetn Program

- **Spot rate**
  - The rate that prevails today for a given maturity
- **Short rate**
  - The rate for a given maturity (e.g. one year) at different points in time
- **A spot rate is the geometric average of its component short rates**

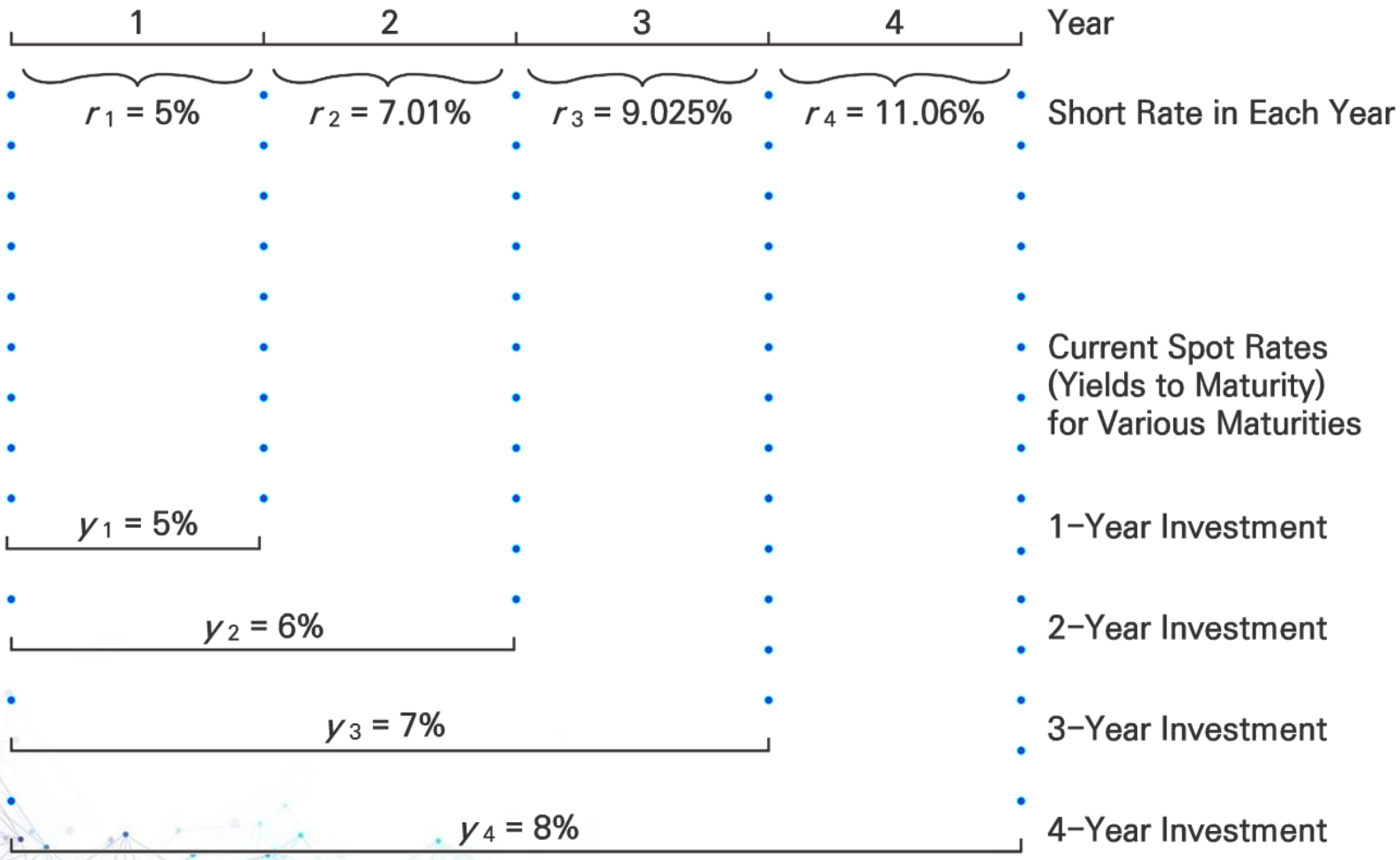
2-yr Investmetn Program

Short rates and yield curve slope

- When next year's short rate, r_2, is greater than this year's short rate, r_1, the yield curve slopes up
  - May indicate rates are expected to rise

- When next year's short rate, r_2, is less than this year's short rate, r_1, the yield curve slopes down
  - May indicate rates are expected to fall

Short rates vs. spot rates



Forward Rates

Definition of Forward Rates

$$(1 + f_n) = \frac{(1 + y_n)^n}{(1 + y_{n-1})^{n-1}}$$

- f_n = one-year forward rate for period n
- y_n = yield for a security with a maturity of n

$$\Rightarrow (1 + y_n)^n = (1 + y_{n-1})^{n-1}(1 + f_n)$$

Example

Forward rates

- The forward interest rate is a forecast of a future short rate
- Rate for 4-year maturity = 8%, rate for 3-year maturity = 7%

$$1 + f_4 = \frac{(1 + y_4)^4}{(1 + y_3)^3} = \frac{1.08^4}{1.07^3} = 1.1106$$

$$f_4 = 11.06\%$$

Interest Rate Uncertainty and Forward Rates

- Suppose that today's rate is 5% and the expected short rate for the following year is E(r_2) = 6%. The value of a 2-year zero is:

$$\frac{\$1{,}000}{(1.05)(1.06)} = \$898.47$$

- The value of a 1-year zero is:

$$\frac{\$1{,}000}{1.05} = \$952.38$$

Interest Rate Uncertainty and Forward Rates

The investor wants to invest for 1 year

- Buy the 2-year bond today and plan to sell it at the end of the first year for $1000/1.06 =$943.40

Or

- Buy the 1-year bond today and hold to maturity
- What if next year's interest rate is more (or less) than 6%?
- <u>The actual return on the 2-year bond is uncertain!</u>
  - The actual return on the 2-year bond is uncertain!

Interest Rate Uncertainty and Forward Rates

- Investors require a risk premium to hold a longer-term bond
- This liquidity premium compensates short-term investors for the uncertainty about future prices

Exercise Problem 1

Which of the following is true according to the pure expectations theory? Forward rates:

a. Exclusively represent expected future short rates.
b. Are biased estimates of market expectations.
c. Always overestimate future short rates.

Exercise Problem 2

The following is a list of prices for zero-coupon bonds of various maturities.

Maturitiy (years)	Price of Bond
1	$943.4
2	$898.47
3	$847.62
4	$792.16

a. Calculate the yield to maturity for a bond with a maturity of (i) one year; (ii) two years; (iii) three years; (iv) four years.

b. Calculate the forward rate for (i) the second year; (ii) the third year; (iii) the fourth year.

Exercise Problem 3

Prices of zero-coupon bonds reveal the following pattern of forward rates:

Year	Forward Rate
1	5%
2	7%
3	8%

In addition to the zero-coupon bond, investors also may purchase a 3-year bond making annual payments of $60 with par value $1,000.

a. What is the price of the coupon bond?
b. What is the yield to maturity of the coupon bond?
c. Under the expectations hypothesis, what is the expected realized compound yield of the coupon bond?
d. If you forecast that the yield curve in one year will be flat at 7%, what is your forecast for the expected rate of return on the coupon bond for the 1-year holding period?