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투자론

- R과 Excel을 통한 금융데이터 분석 -

5주차
포트폴리오 이론 3

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Unit 02

CAPM

CAPM

Capital Asset Pricing Model

- CAPM is the equilibrium model that underlies all modern financial theory
- Derived using principles of diversification with simplified assumptions
- Markowitz, Sharpe, Lintner, and Mossin contributed to its development

Assumptions of the CAP

Individuals

- Mean-Variance optimizers
- Homogeneous expectations
- All assets are publicly traded

Markets

- All assets are publicly held
- All information is available
- No taxes
- No transaction cost

Assumptions of the CAP

Implications of the assumptions

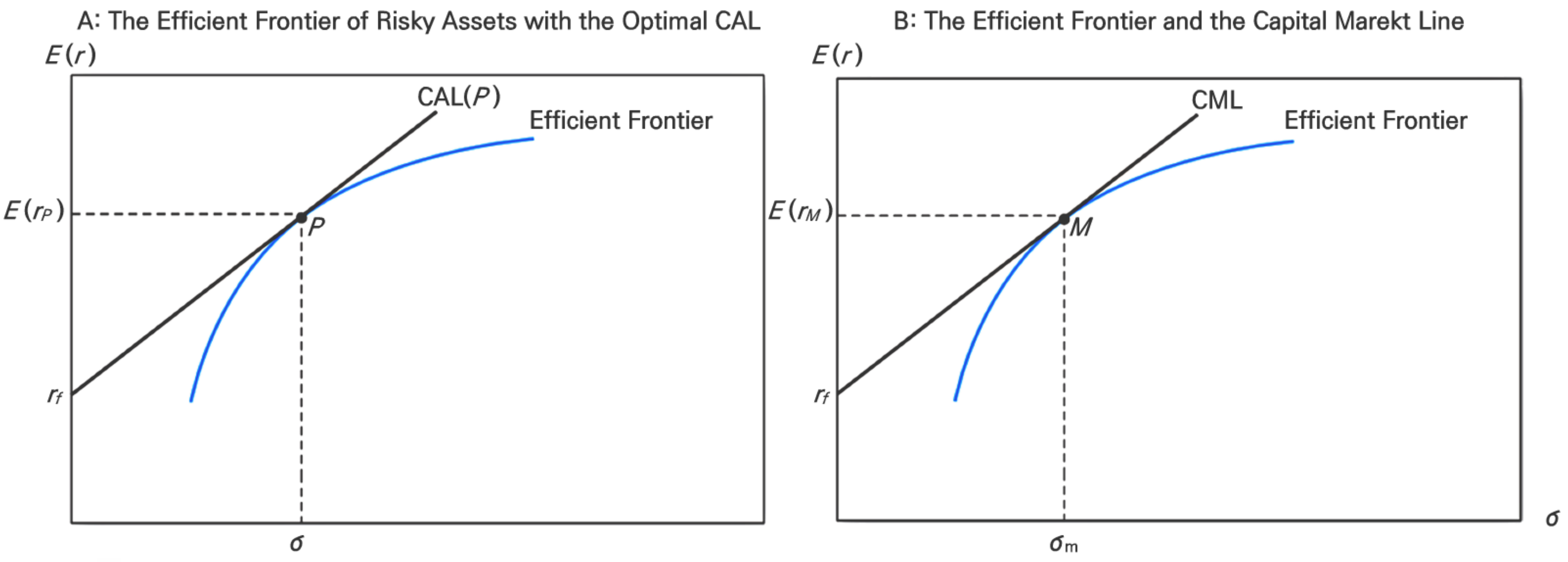
- Investors optimize portfolios a la Markowitz
- Investors use identical input list for efficient frontier
- Same risk-free rate, tangent CAL and risky portfolio are used
- Market portfolio is aggregation of all risky portfolios and has same weights

Resulting Equilibrium Conditions

- All investors will hold the same portfolio for risky assets
  — market portfolio

- Market portfolio contains all the securities and the proportion of each security is its market value as a percentage of total market value

Resulting Equilibrium Conditions



Market Risk Premium

- The risk premium on the market portfolio will be proportional to its risk and the degree of risk aversion of the investor

$$E(R_M) = \bar{A}\sigma_M^2$$

- where σ_M^2 is the variance of the market portfolio and $\bar{A}$ is the average degree of risk aversion across investors

Return and Risk for Individual Securities

- The risk premium on individual securities is a function of the individual security's contribution to the risk of the market portfolio

- An individual security's risk premium is a function of the covariance of returns with the assets that make up the market portfolio

Example

- Covariance of GE return with the market portfolio

$$Cov(R_M, R_{GE}) = Cov\left(\sum_{i=1}^{n} w_i R_i, R_{GE}\right) = \sum_{i=1}^{n} Cov(w_i R_i, R_{GE})$$

- Therefore, the reward-to-risk ratio for investment in GE would be

$$\frac{\text{GE's contribution to risk premium}}{\text{GE's contribution to variance}} = \frac{w_{GE} E(R_{GE})}{w_{GE} Cov(R_{GE}, R_M)} = \frac{E(R_{GE})}{Cov(R_{GE}, R_M)}$$

- Risk Premium

$$E(R_M) = E\left(\sum_{i=1}^{n} w_i R_i\right) = E\left(\sum_{i \neq GE} w_i R_i\right) + w_{GE} R_{GE}$$

Example

- Reward-to-Risk ratio for investment in market portfolio

$$\frac{\text{Market Risk Premium}}{\text{Market Variance}} = \frac{E(R_M)}{\sigma_M^2}$$

- Reward-to-Risk ratios of GE and the market portfolio should be equal

$$\frac{E(R_{GE})}{Cov(R_{GE}, R_M)} = \frac{E(R_M)}{\sigma_M^2}$$

Example

- The risk premium for GE

$$E(R_{GE}) = Cov(R_{GE}, R_M) \times \frac{E(R_M)}{\sigma_M^2}$$

- Restating, we obtain

$$\Rightarrow E(r_{GE}) - r_f = \beta_{GE} \times (E(r_M) - r_f)$$

$$\Rightarrow E(r_{GE}) = r_f + \beta_{GE} \times (E(r_M) - r_f)$$

Expected Return-β Relationship

- CAPM holds for the overall portfolio because

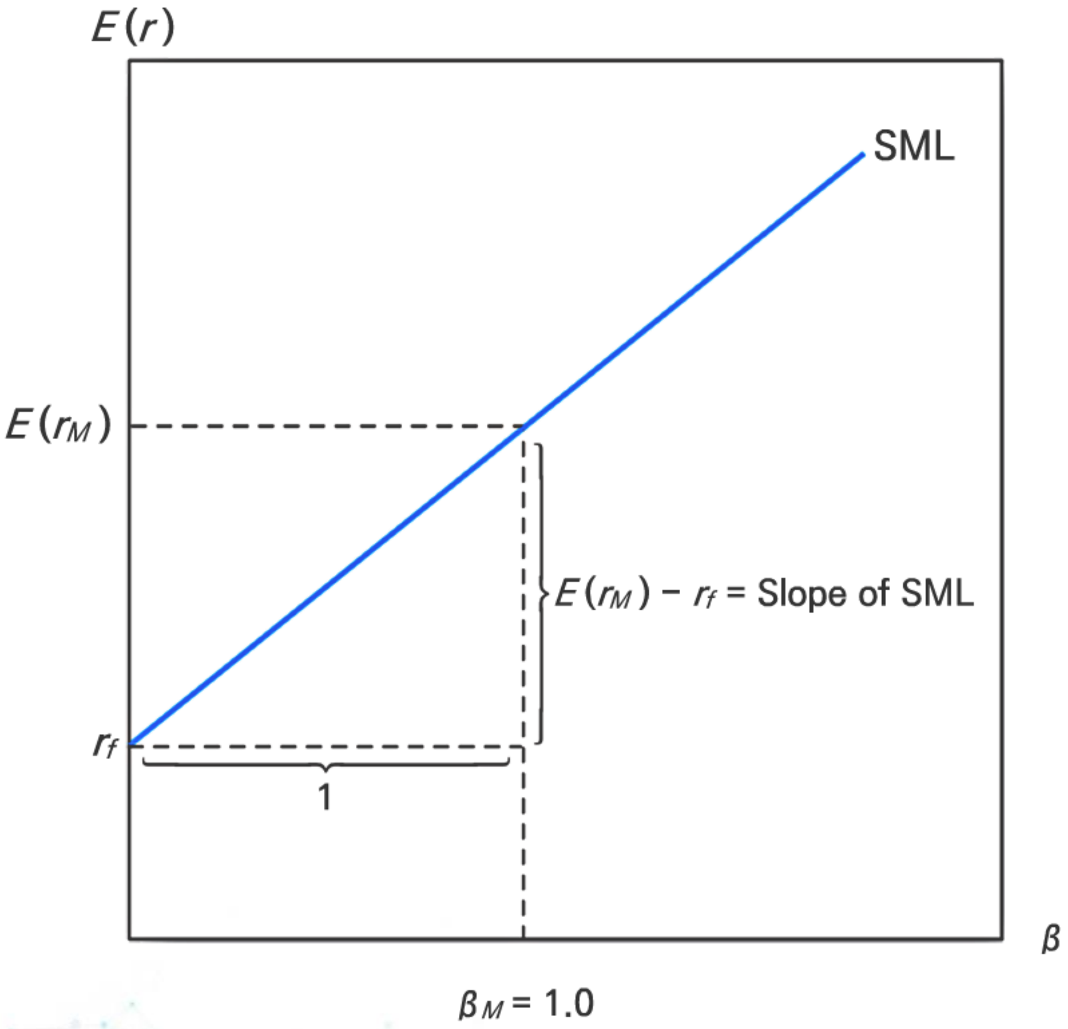
$$E(r_p) = \sum_k w_k E(r_k) \text{ and}$$

$$\beta_p = \sum_k w_k \beta_k$$

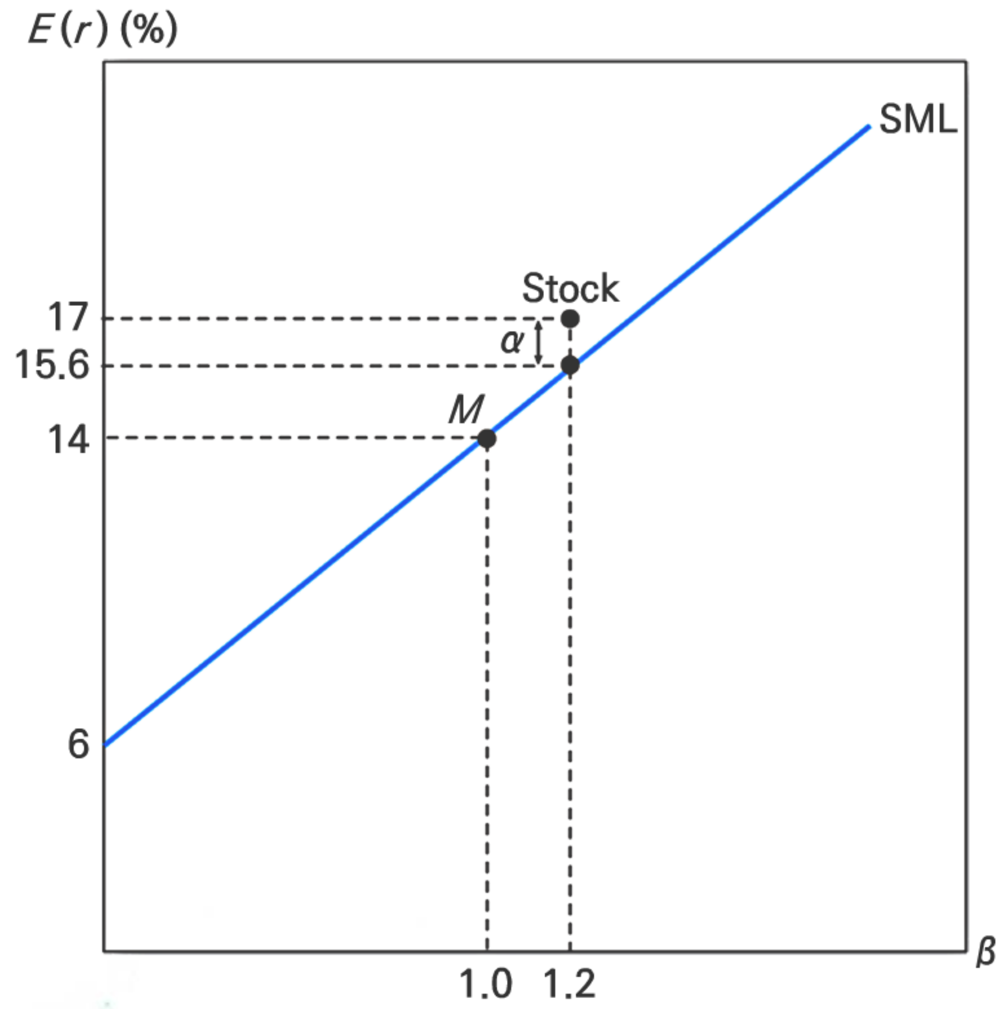
- This also holds for the market portfolio

$$E(r_M) = r_f + \beta_M \left[E(r_M) - r_f \right]$$

The Security Market Line



The SML and a (+) Alpha Stock



CAPM and the World

Academic World

- Cannot observe all tradable assets
- Impossible to pin down market portfolio
- Attempts to validate using regression analysis

Investment Industry

- Relies on the single-index CAPM model
- Most investors don't beat the index portfolio

Exercise Problem 1

Using 10 past years of annual data, the market model has been estimated for stocks A and B with the following results:

$R_A = 0.01 + 0.8R_M + e_A,$

$R_B = 0.02 + 1.2 + e_B,$

$\sigma_M = 0.20, \sigma(e_A) = 0.2, \sigma(e_B) = 0.10$, where R is the excess return.

1) What does the market model predict for the values of σ_A, σ_{B}, $\sigma_{A,\mathrm{B}}$, $\rho_{A,B}$?

2) Suppose we construct a portfolio that has weights: $w_A = \frac{1}{2}, w_B = \frac{1}{4}, w_f = \frac{1}{4}$.

   What is the risk of this portfolio?

Exercise Problem 1

Suppose that the current risk-free rate is 5% and that as an estimate for the expected market risk premium $R_M = r_M - r_f$, we use a long-run historical average of 8%.

3) Using the market model's estimate of β_A, what does the capital asset pricing model predict $E(r_A)$ should be?

4) According to the market model, what is the implied value of E(TA)?

5) If you find that the two values of $E(r_A)$ agree, comment on the reasons for this agreement. If you find that the two values of $E(r_A)$ disagree, comment on the reasons for this disagreement.