투자론

- R과 Excel을 통한 금융데이터 분석 -

2주차
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Unit 02

Risk, Return, and Historical Record

Risk and Risk Premium

Rates of Return: Single Period

- $HPR = \dfrac{P_1 - P_0 + D_1}{P_0}$
- HPR = Holding Period Return
- P_0 = Begining Price
- P_1 = Ending Price
- D_1 = Dividend during period one

Example

Ending Price = 110

Beginning Price = 100

Dividend = 4

$$HPR = \frac{(110 - 100 + 4)}{(100)} = 0.14 = 14\%$$

Expected Return and Standard Deviation

Expected Returns

$$E(r) = \sum_{s} p(s)r(s)$$

- Where $p(s)$ = probability of a state s, $r(s)$ = return if a state occurs, and s = state

Scenario Returns: Example

State	Prob. Of Sate	r in State
Excellent	.25	0.3100
Good	.45	0.1400
Poor	.25	-0.0675
Crast	.05	-0.5200

$E(r) =$ (.25)(.31)+(.45)+(.14)+(.25)(-.0675)+0.05(-0.52)

$E(r) =$ .0976 or 9.76%

Variance and Standard Deviation

- Variance (Var)

$$Var(r) = \sigma_r^2 = \sum_s p(s)[r(s) - E(r)]^2$$

- Standard Deviation (STD)

$$STD = \sqrt{\sigma_r^2}$$

Scenario Var and STD

Example VAR calculation

$$\sigma^2 = .25(.31 - 0.0976)^2 + .45(.14 - .0976)^2 + .25(0.0675 - 0.0976)^2 + .05(.52 - .0976)^2 = 0.38$$

Example STD calculation

$$\sigma = \sqrt{0.38} = .1949$$

Time Series Analysis of Past Rates of Returns

The Arithmetic Average of Rate of Returns

$$E(r) = \sum_{s=1}^{n} p(s)r(s) = \frac{1}{n}\sum_{s=1}^{n} r(s)$$

- without considering relative importance, take every observation with equal weight

Geometric Average Return

- $TV_n = (1 + r_1)(1 + r_2) \cdots (1 + r_n)$
- TV = Terminal Value of the Investment
- $g = TV^{1/n} - 1$
- g = geometric average rate of return

Geometric Variance and Standard Deviation

- Estimated Variance = expected value of squared deviations

$$\sigma^2 = \frac{1}{n}\sum_{s=1}^{n}[r(s) - \bar{r}]^2$$

- When eliminating the bias, Variance and Standard Deviation become

$$\hat{\sigma} = \sqrt{\frac{1}{n}\sum_{s=1}^{n}[r(s) - \bar{r}]^2}$$

The Reward-to-Volatility(Sharpe) Ratio

Sharpe Ratio for Portfolios

$$SR = \frac{Risk\ Premium}{STD\ of\ Excess\ Returns} = \frac{E(r) - r_f}{\sigma_r}$$

- Then, why not for individual assets?

  Not a reliable measure

The Normal Distribution

- **Investment management is easier when returns are normal**
  - Standard deviation is a good measure of risk when returns are symmetric
  - If security returns are symmetric, portfolio returns will be, too
  - Future Scenarios can be estimated using only the mean and the standard deviation

The Normal Distribution

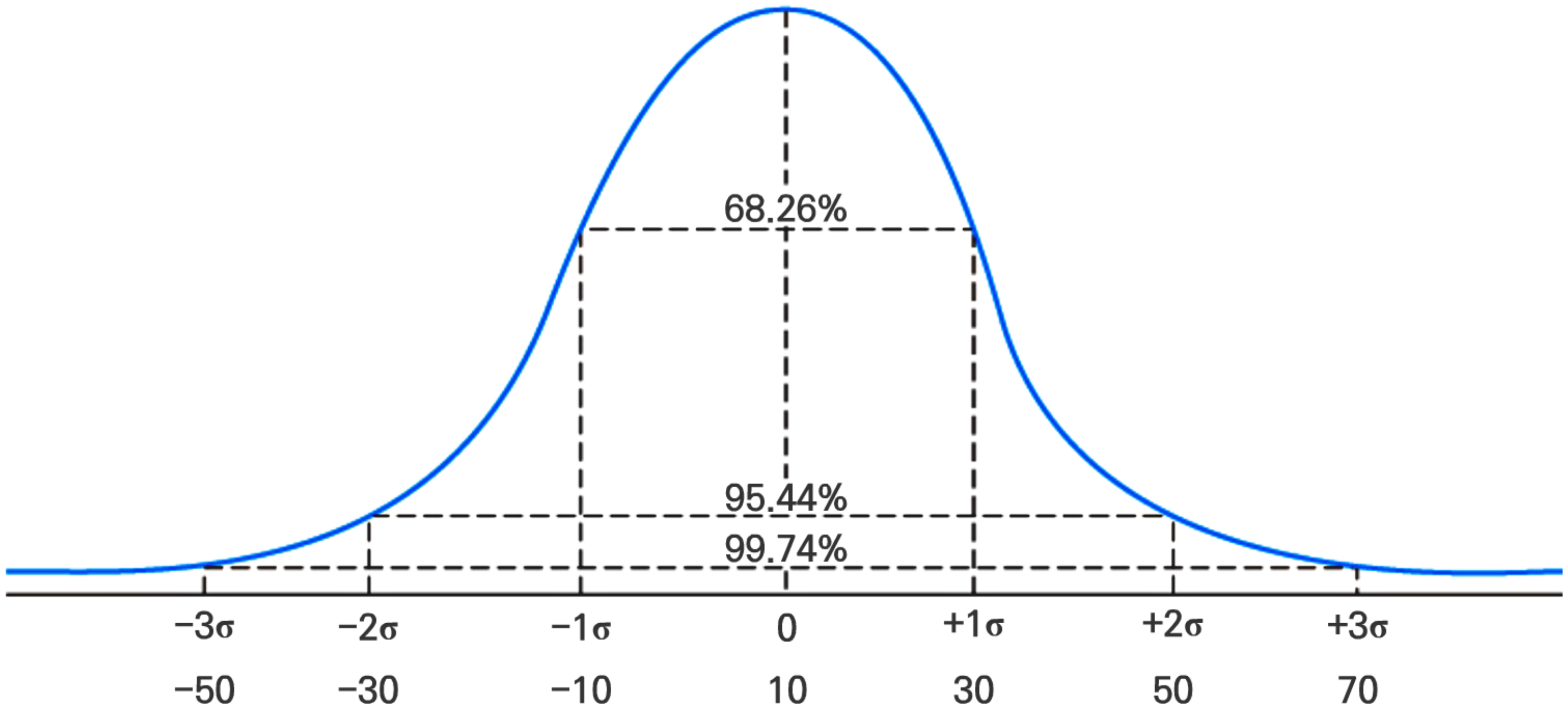

Figure : The normal distribution with mean 10% and standard deviation 20%

Normality and Risk Measures

- **What if excess returns are not normally distributed?**
  - Standard deviation is no longer a complete measure of risk
  - Sharpe ratio is not a complete measure of portfolio performance
  - Need to consider skew and kurtosis

Skew and Kurtosis

Skew

$$Skew = Average\left[\frac{(R-\bar{R})^3}{\hat{\sigma}^3}\right]^3$$

Kurtosis

$$Kurtosis = Average\left[\frac{(R-\bar{R})^4}{\hat{\sigma}^4}\right]^4$$

Normal and Skewed Distributions

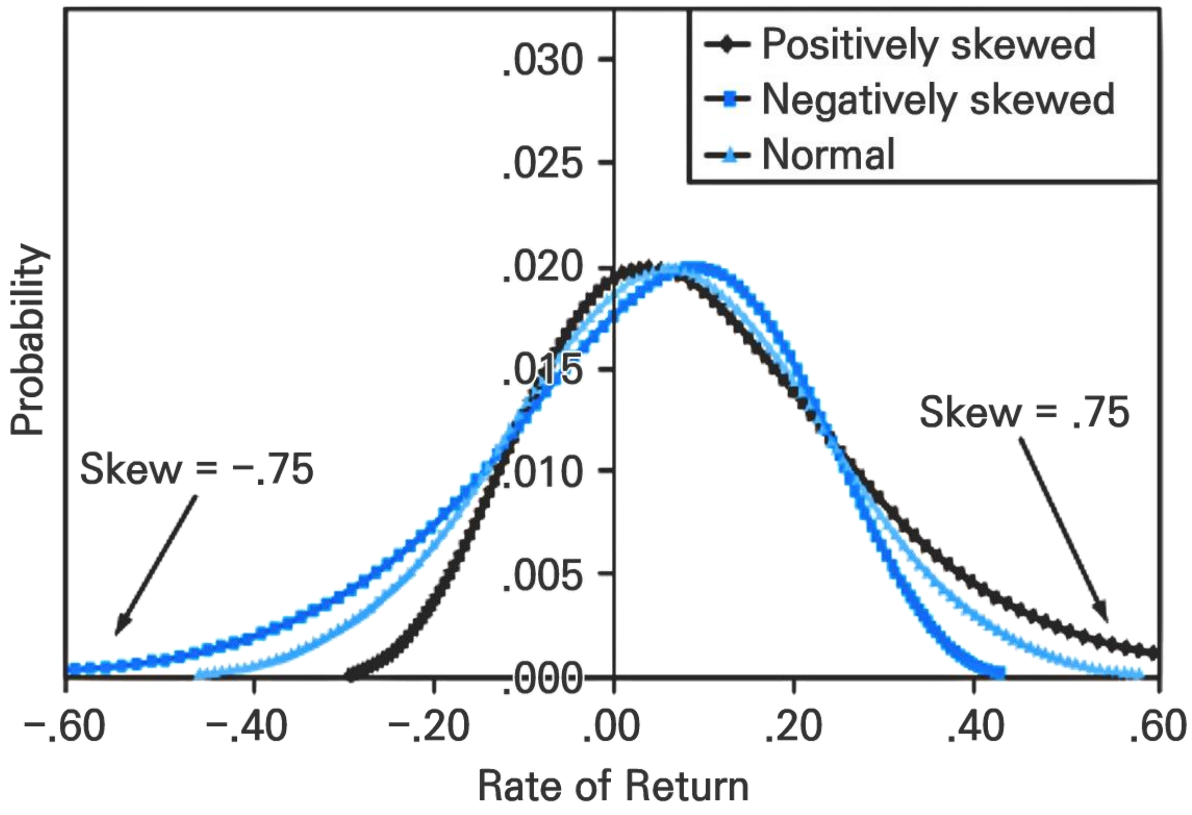

Figure: Normal and Skewed Distributions (mean=6%, SD=17%)

Normal and Fat-Tailed Distribution(mean=0.1, SD=0.2)

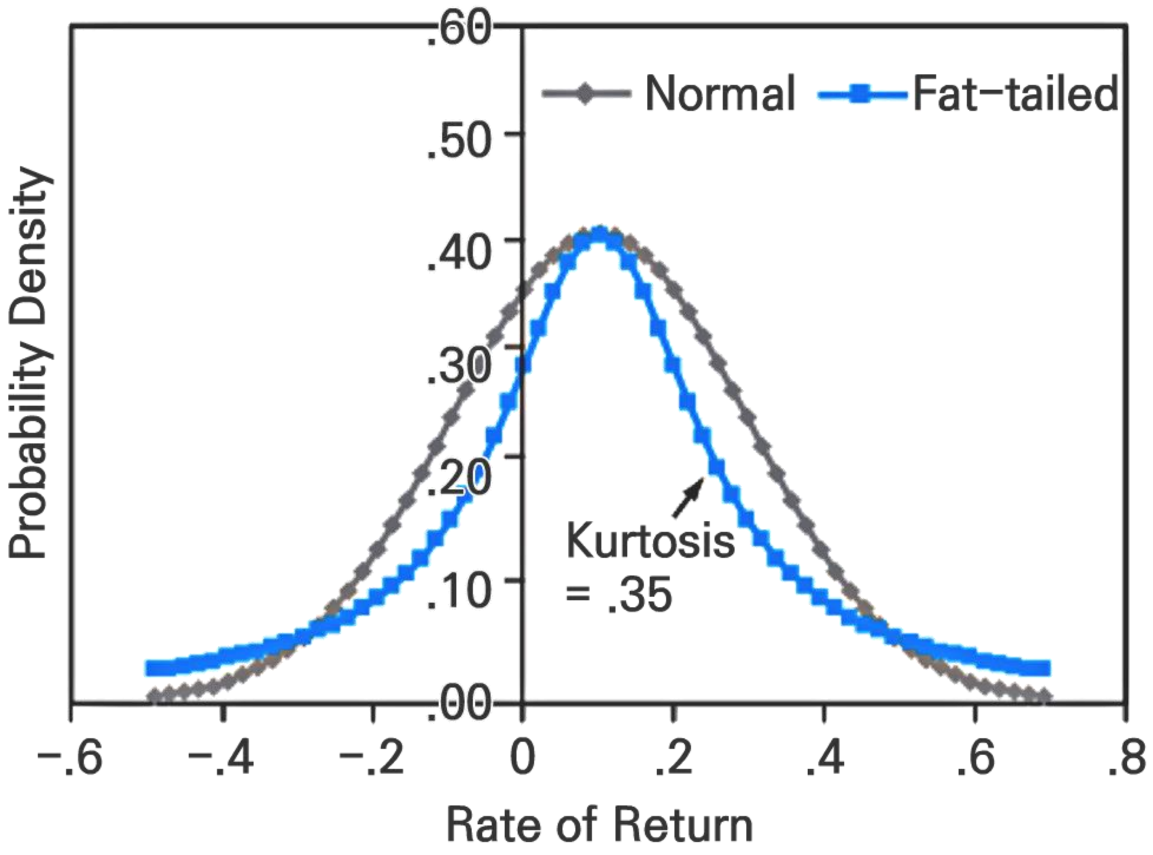

Figure: Normal and Fat-Tailed Distribution (mean=0.1, SD=0.2)

Historic Returns on Risky Portfolios

- Returns appear normally distributed
- Returns are lower over the most recent half of the period (1986-2009)
- SD for small stocks become smaller;
  SD for long-term bonds got bigger
- Better diversified portfolios have higher Sharpe Ratios
- Negative Skew

Norminal and Real Equity Returns around World

- 1900-2000

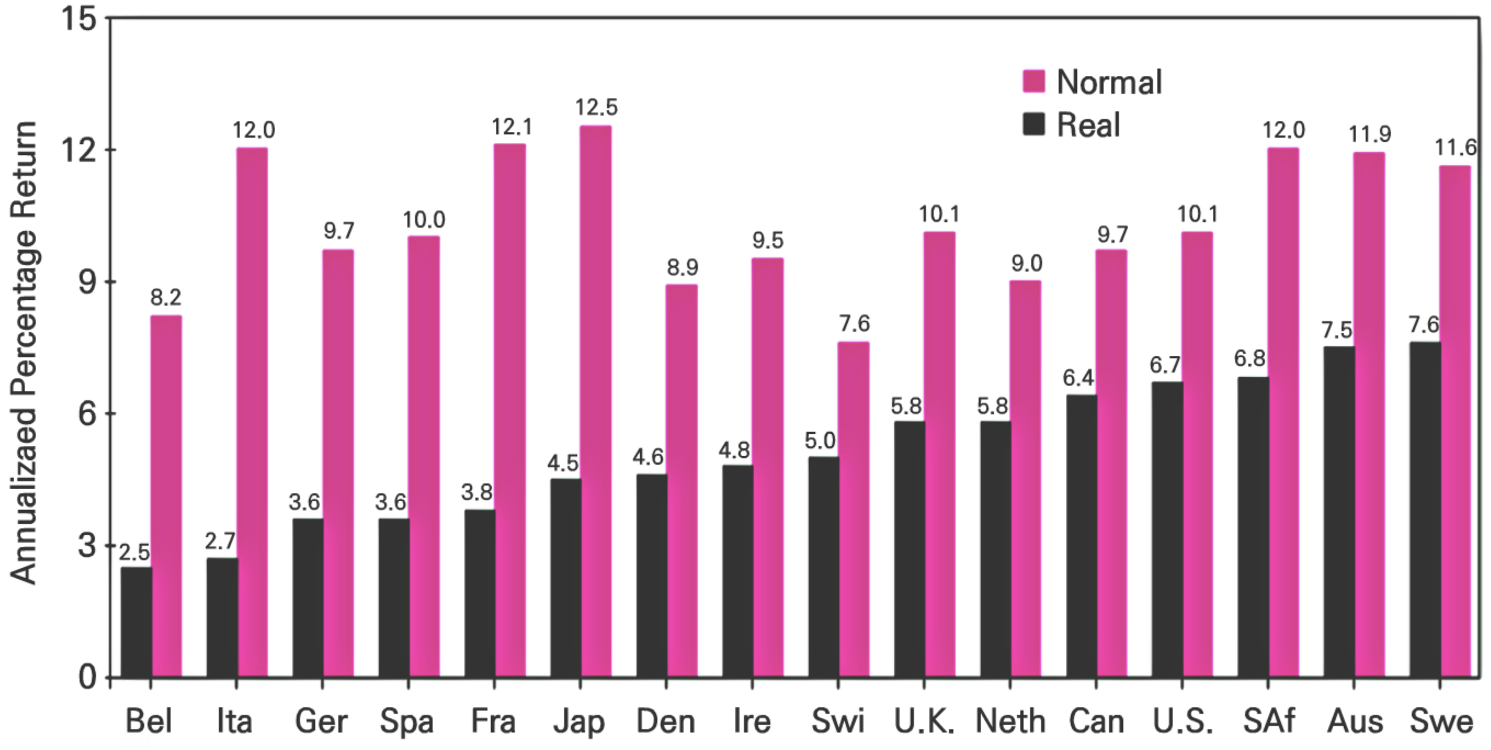

Figure: Norminal and Real Equity Returns around World, 1900-2000

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Standard Deviations of Real Equity and Bond Returns around the World

- 1900-2000

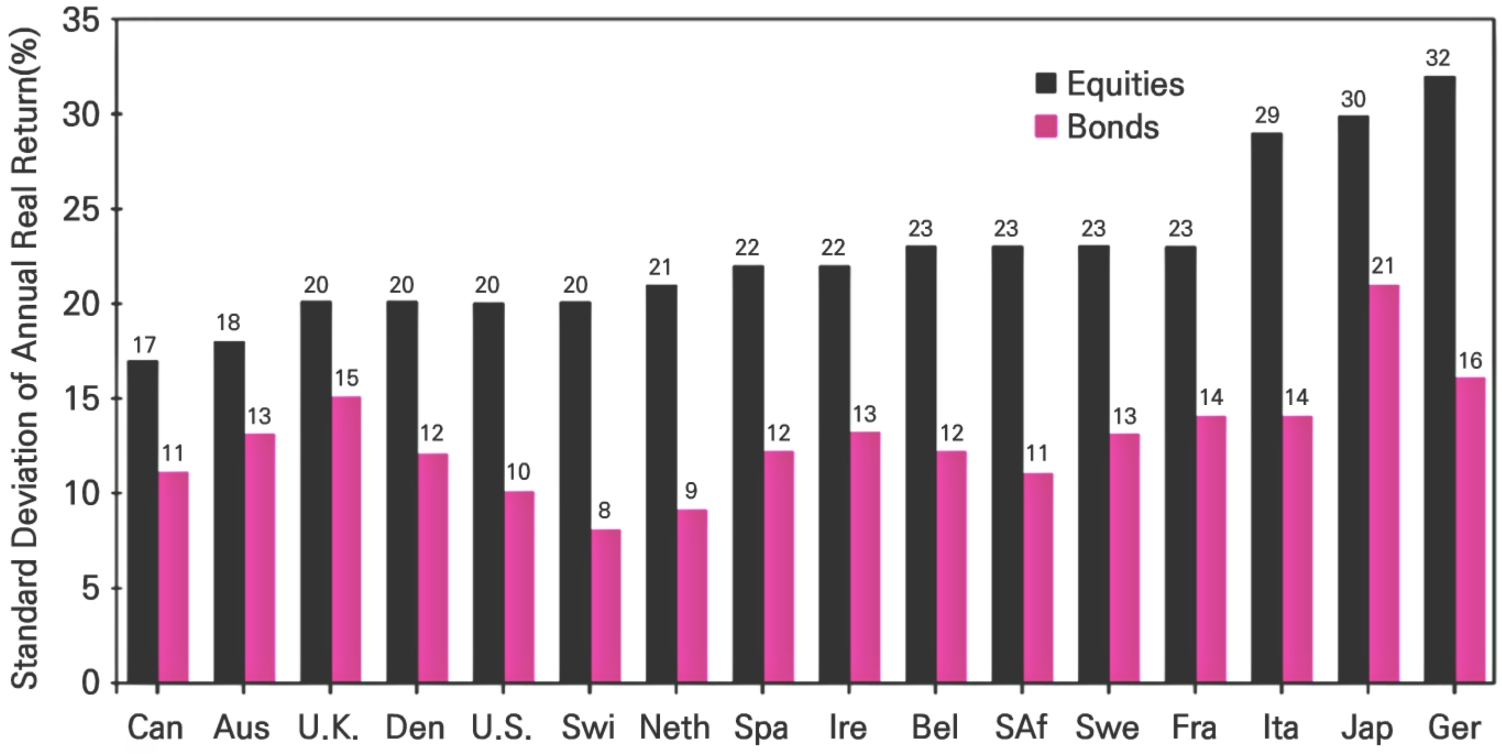

Figure :Standard Deviations of Real Equity and Bond Returns around the World ,1990-200

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