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투자론

- *R*과 Excel을 통한 금융데이터 분석 -

12주차
성과분석 및 자본(주식) 가치 평가

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Unit 01

Introduction to Performance Evaluation

Overview

- Performance Evaluation
  - Average portfolio return
    1. Time-weighted returns
    2. Dollar-weighted returns
  - Risk adjusted performance comparison
    1. Sharpe ratio
    2. Treynor measure
    3. Jensen's alpha
    4. Information ratio
    5. M^2 (Modigliani and Modigliani) measure

Why Evaluation?

- **If markets are efficient, investors must be able to measure asset management performance**
- **Two common ways to measure average portfolio return:**
  1. Time-weighted returns
  2. Dollar-weighted returns
- **Returns must be adjusted for risk**

Dollar- and Time-Weighted Returns

Time weighted returns

- The geometric average is a time-weighed average
- Each period's return has equal weight

$$(1+r_G)^n = (1+r_1)\times(1+r_2)\times\cdots\times(1+r_n)$$

$$r_G = [(1+r_1)\times(1+r_2)\times\cdots\times(1+r_n)]^{1/n} - 1$$

Dollar- and Time-Weighted Returns

Dollar-weighted returns

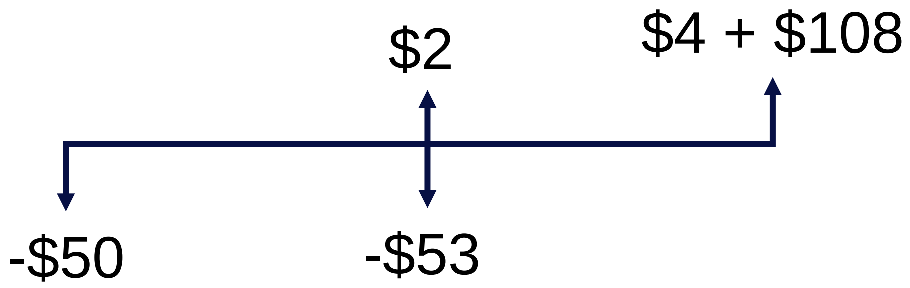
- Internal rate of return considering the cash flow from or to investment
- Returns are weighted by the amount invested in each period:

$$PV = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \cdots + \frac{C_n}{(1+r)^n}$$

Example of Multi-Period Returns

Dollar-weighted return

Time	Outlay
0	$50 to purchase first share
1	$53 to purchase second share a year later
	Proceeds
1	$2 dividend from initially purchased share
2	$4 dividend from the 2 shares held in the second year, plus
	$108 received from selling both shares at $54 each



- Dollar-weighted return (IRR): $50 = \frac{-51}{(1+r)^1} + \frac{112}{(1+r)^2} \Rightarrow r = 7.117\%$
  - Households should maintain a spreadsheet of time-dated cash flows (in and out) to determine the effective rate of return for any given period

Example of Multi-Period Returns

Time-weighted return

$$r_1 = \frac{53 - 50 + 2}{50} = 10\%$$

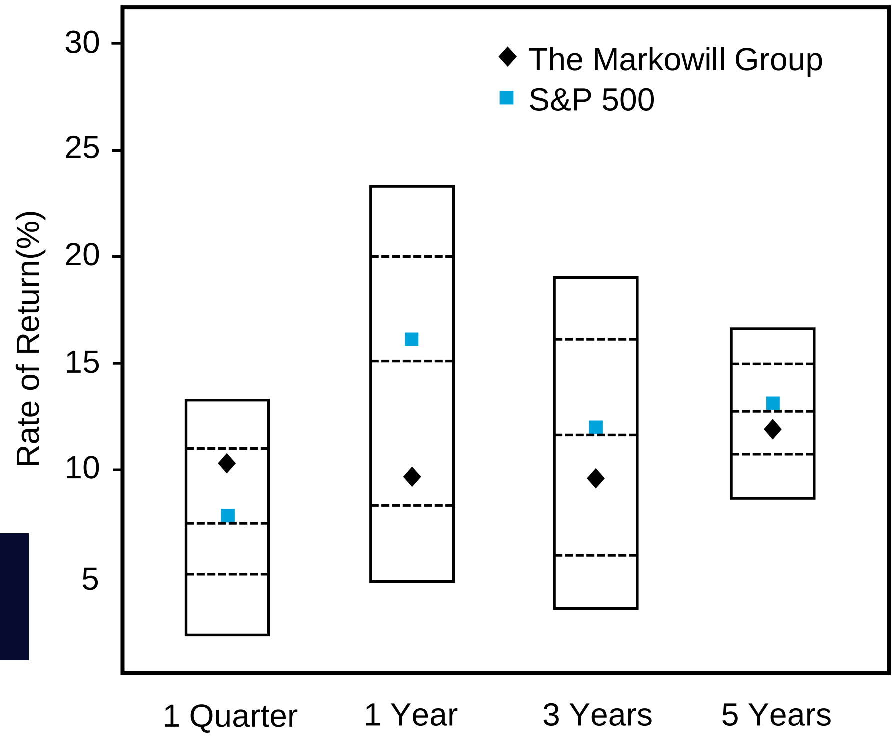
$$r_2 = \frac{54 - 53 + 2}{53} = 5.66\%$$

$$r_G = [(1.10) \times (1.0566)]^{1/2} - 1 = 7.81\%$$

Adjusting Returns for Risk

- **The simplest way to adjust for risk is to <u>compare the portfolio's return with the returns of a comparison universe</u>**
  - The comparison universe is called the benchmark
  - It is composed of a group of funds or portfolios with similar risk characteristics

More accurate means for risk adjustment is needed



Example Figure: Universe Comparison, periods ending 12/31/2022

Risk Adjusted Performance

Sharpe ratio

$$SR = \frac{\overline{r_p} - \overline{r_f}}{\sigma_p}$$

$\overline{r_p} = Average\ return\ on\ the\ portfolio$

$\overline{r_f} = Average\ risk\ free\ rate$

$\sigma_p = Standard\ deviation\ of\ portfolio\ return$

Risk Adjusted Performance

Treynor Measure

$$TR = \frac{\overline{r_p} - \overline{r_f}}{\beta_p}$$

$\overline{r_p}$ = *Average return on the portfolio*

$\overline{r_f}$ = *Average risk free rate*

β_p = *Weighted average beta for portfolio*

Risk Adjusted Performance

Jensen's Alpha

$$\alpha_p = \overline{r_p} - [\overline{r_f} + \beta_p(\overline{r_M} - \overline{r_f})]$$

α_p = Alpha for the portfolio

$\overline{r_p}$ = Average return on the portfolio

β_p = Weighted average beta

$\overline{r_f}$ = Average risk free rate

$\overline{r_M}$ = Average return on the market index portfolio

Risk Adjusted Performance

Information Ratio

$$IR = \frac{\alpha_p}{\sigma(e_p)}$$

- The information ratio divides the alpha of the portfolio by the nonsystematic (idiosyncratic) risk
- Nonsystematic risk could, in theory, be eliminated by diversification

Residual: $e_p \equiv r_p - \left[r_f + \hat{\beta}_p(\bar{r}_M - \bar{r}_f)\right]$

Concept Check

Consider the following data for a particular sample period:

	Portfolio P	Market M
Average ret	35%	28%
Beta	1.20	1.00
Standard deviation	42%	30%
Tracking error (nonsystematic risk), $\sigma(e)$	18%	0

Calculate the following performance measures for portfolio P and the market: Sharpe, Jensen (alpha), Treynor, information ratio. The T-bill rate during the period was 6%. By which measures did portfolio P outperform the market?

Concept Check

$Sharpe: S_P = (35 - 6)/42 = 0.69 , S_M = (28 - 6)/30 = 0.733$

$Alpha: \alpha_P = 35 - [6 + 1.2(28 - 6)] = 2.6 , \alpha_M = 0$

$Treynor: T_P = (35 - 6)/1.2 = 24.2 , T_M = (28 - 6)/1 = 22$

$Information\ Ratio: IR_P = 2.6/18 = 0.144 , I_M = 0$

Conclusion: P outperformed the market (Jensen & Treynor), but had an Sharpe measure

M^2 measure

- Developed by Modigliani and Modigliani
- Create an adjusted portfolio P* that combines P with Treasury Bills
- Set P* to have the same standard deviation as the market index
- Now compare market and P* returns: $M^2 = r_{P*} - r_M$

M² measure -- Example

Market Portfoilo P: $r_P = 35\%$ $\sigma_P = 42\%$

Market Portfoilo: $r_M = 28\%$ $\sigma_M = 30\%$

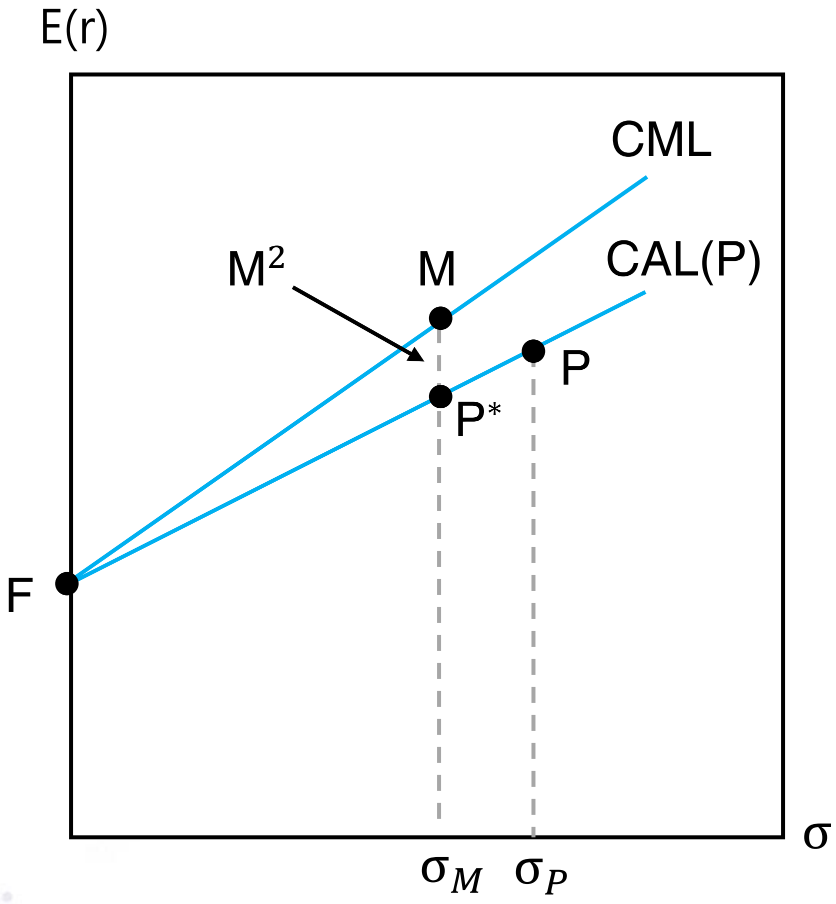
T-bill return = 6%

P*Prorfoil: 30/42=.714 in P and .286 T-bills

$\boldsymbol{r_{P*}} = (\mathbf{.714}) \times (\mathbf{.35}) + (\mathbf{.286}) \times (\mathbf{.06}) = \mathbf{26.7\%}$

$\boldsymbol{r_{P*}} < \boldsymbol{r_M}$ → ***the managed portfolio underperformed***

M^2 measure – Graphical Expression



M^2 of Portfolio P

Example: Comparison of Two Diversified Funds

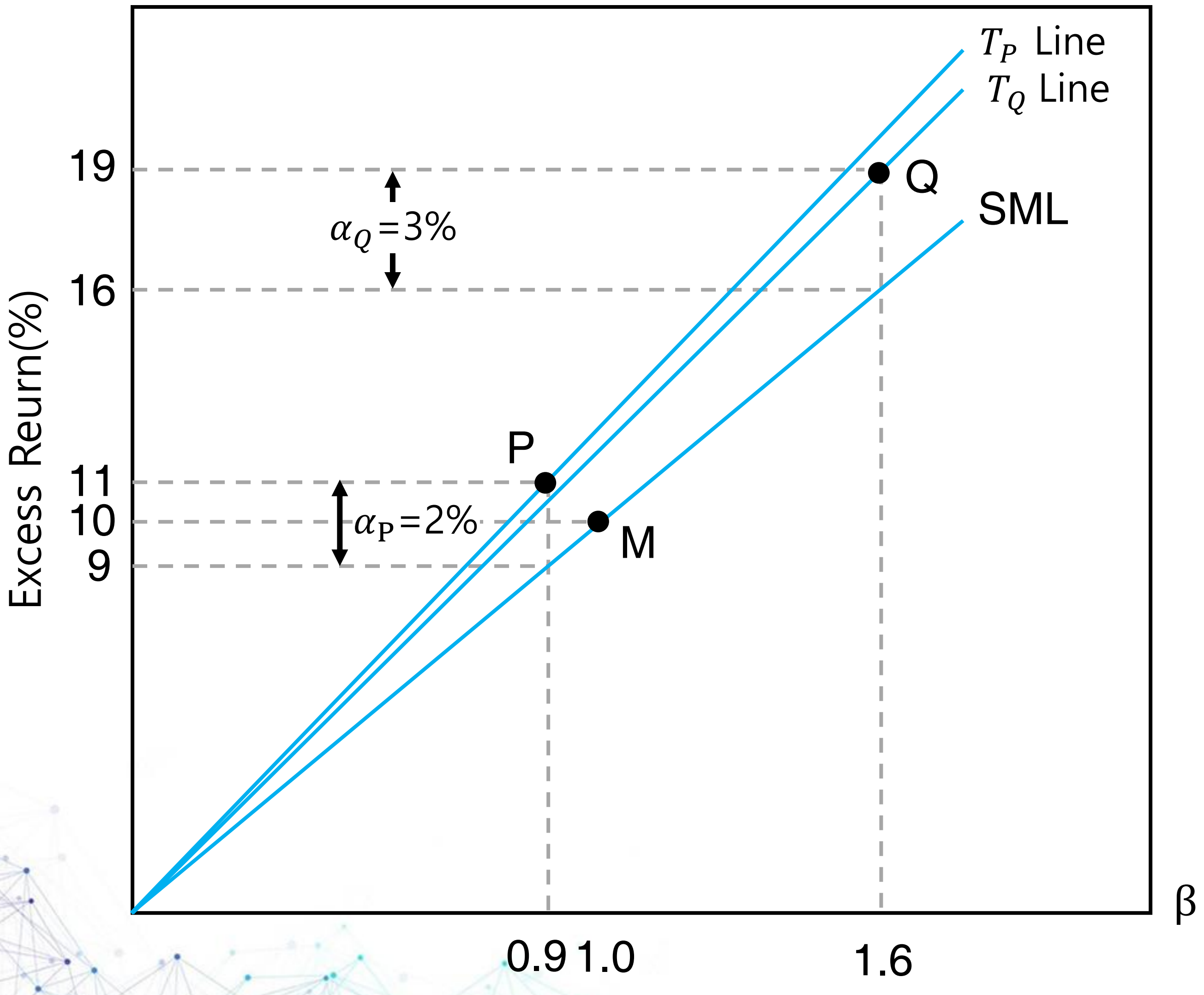
	Portfolio *P*	Portfolio *Q*	Market
Beta	0.90	1.60	1.0
Excess return ($\bar{r} - \bar{r}_f$)	11%	19%	10%
Alpha*	2%	3%	0

*Alpha = Excess return – (Beta×Market excess return)

$$=(\bar{r} - \bar{r}_f)\text{-}\beta(\bar{r}_M - \bar{r}_f)=\bar{r} - [\bar{r}_f+\beta(\bar{r}_M - \bar{r}_f)]$$

Is Q better than P?

Example: Comparison of Two Funds



The Use of Information Ratio

When we add a stock with positive alpha to the already managed portfolio, then the Sharpe ratio of the new fund will be determined by

$SR_{p*} = SR_p + (\frac{a_n}{\sigma(e_n)})$, where p^* is the newly composed fund, p is the existing portfolio, and n is a new security.

The Role of Alpha

	Treynor(T_p)	Sharpe*(S_p)	Information Raio
Relation to alpha	$\frac{E(r_p)-r_f}{\beta_p}=\frac{\alpha_p}{\beta_p}+T_M$	$\frac{E(r_p)-r_f}{\alpha_p}=\frac{\alpha_p}{\sigma_p}+\rho S_M$	$\frac{\alpha_p}{\sigma(e_p)}$
Improvement compared to market index	$T_p-T_M=\frac{\alpha_p}{\beta_p}$	$S_p-S_M=\frac{\alpha_p}{\sigma_p}-(1-\rho)S_M$	$\frac{\alpha_p}{\sigma(e_p)}$

p* denotes the correlation coefficient between portfolio $P$and the market, and is less than 1.

- Impossible to outperform the passive market index unless the fund is expected to generate a positive alpha.

- While positive alpha is necessary, it is not sufficient to guarantee that a portfolio will outperform the index: Taking advantage of mispricing means departing from full diversification, which entails a cost in terms of nonsystematic risk. A fund can achieve a positive alpha, yet, at the same time, increase its volatility enough that its Sharpe ratio will actually fall.

Which Measure is Appropriate?

It depends on investment assumptions

1. If P is not diversified, then use the Sharpe measure as it measures reward to risk.
2. If the P is diversified, nonsystematic risk is negligibleand the appropriate metric is Treynor's, measuring excess return to beta

Performance Measure	***Definition***	***Application***
Sharp	$\frac{\text{Excess return}}{\text{Standard deviation}}$	When choosing among portfolios competing for the overall risky portfolio
Treynor	$\frac{\text{Excess return}}{\text{Beta}}$	When ranking many portfolios that will be mixed to form the overall risky portfolio
Information ration	$\frac{\text{Alpha}}{\text{Residual standard deviation}}$	When evaluating a portfolio to be mixed with the benchmark portfolio

Case Study

1. Examine the performance measures of the funds included in the spreadsheet.Rank performance and determine.
2. Which fund would you choose if you were considering investing the entire risky portion of your portfolio? What if you were considering adding a small position in one of these funds to a portfolio currently invested in the market index?

Month	*Portfolio P*	*Alternative Q*	*Index M*
1	3.58%	2.81%	2.20%
2	-4.91	-1.15	8-41
3	6.51	2.53	3.27
4	11.13	37.09	14.41
5	8.78	12.88	7.71
6	9.38	39.08	14.36
7	-3.66	-8.84	-6.15
8	5.56	0.83	2.74
9	-7.72	0.85	-15.27
10	7.76	12.09	6.49
11	-4.01	-5.68	-3.13
12	0.78	-1.77	1.41

Case Study

1. If P or Q represents the entire investment fund, Q would be preferable on the basis of its higher Sharpe measure (.49 vs. .43) and better M^2 (2.66% vs. 2.16%). If P and Q are competing for a role as one of a number of subportfolios, Q also dominates because its Treynor measure is higher (5.38 vs. 3.97).
2. As an active portfolio to be mixed with the index portfolio, P is preferred because its information ratio [IR = a/o(e)] is higher (.81 vs. .54). Thus, the example illustrates that the right way to evaluate a portfolio depends in large part on how the portfolio fits into the investor's overall investment plan.

	Portfolio P	*Portfolio Q*	*Portfolio M*
Sharpe ration	0.43	0.49	0.19
M^2	2.16	2.66	0.00
SCL regression statistics			
Alpha	1.63	5.26	0.00
Beta	0.70	1.40	1.00
Treynor	3.97	5.38	1.64
T^2	2.34	3.74	0.00
$\sigma(e)$	2.02	9.81	0.00
Informaion ratio	0.81	0.54	0.00
R-square	0.91	0.64	1.00

Exercise Problem 1

Consider the rate of return of stocks ABC and XYZ.

Year	r_{ABC}	r_{XYZ}
1	20%	30%
2	12%	12%
3	14%	18%
4	3%	0%
5	1%	-10%

a. Calculate the arithmetic average return on these stocks over the sample period
b. Which stock has greater dispersion around the mean return?
c. Calculate the geometric average returns of each stock. What do you conclude?

Exercise Problem 1

Consider the rate of return of stocks ABC and XYZ.

Year	r_{ABC}	r_{XYZ}
1	20%	30%
2	12%	12%
3	14%	18%
4	3%	0%
5	1%	-10%

d. If you were equally likely to earn a return of 20%, 12%, 14%, 3%, or 1% in each year (these are the five annual returns for stock ABC), what would be your expected rate of return?

e. What if the five possible outcomes were those of stock XYZ?

f. Given your answers to parts (d) and (e), which measure of average return, arithmetic or geometric, appears more useful for predicting future performance?

Exercise Problem 2

Based on current dividend yields and expected capital gains, the expected rates of return on portfolios A and B are 12% and 16%, respectively. The beta of A is .7, while that of B is 1.4. The T-bill rate is currently 5%, whereas the expected rate of return of the S&P 500 index is 13%. The standard deviation of portfolio A is 12% annually, that of B is 31%, and that of the S&P 500 index is 18%.

a. If you currently hold a market-index portfolio, would you choose to add either of these portfolios to your holdings? Explain.
b. If instead you could invest only in T-bills and one of these portfolios, which would you choose?

Exercise Problem 3

An analyst wants to evaluate portfolio X, consisting entirely of U.S. common stocks, using both the Treynor and Sharpe measures of portfolio performance. The following table provides the average annual rate of return for portfolio X, the market portfolio (as measured by the S&P 500), and U.S. Treasury bills during the past 8 years:

	Average Annual Rate of Return	Standard Deviation of Return	Beta
Portfolio X	10%	18%	0.60
S&P 500	12%	13%	1.00
T-bills	6%	N/A	N/A

a. Calculate the Treynor and Sharpe measures for both portfolio X and the S&P 500. Briefly explain whether portfolio X underperformed, equaled, or outperformed the S&P 500 on a risk-adjusted basis using both the Treynor measure and the Sharpe ratio.

b. On the basis of the performance of portfolio X relative to the S&P 500 calculated in part a), briefly explain the reason for the conflicting results when using the Treynor measure versus the Sharpe ratio.