충남대학교
Chungnam National University

투자론

- *R*과 Excel을 통한 금융데이터 분석 -

11주차
이자율 기간구조 및 채권펀드 위험관리

충남대학교
장호규 교수

충남대학교
Chungnam National University

Unit 02

Bond Portfolio Management

Overview

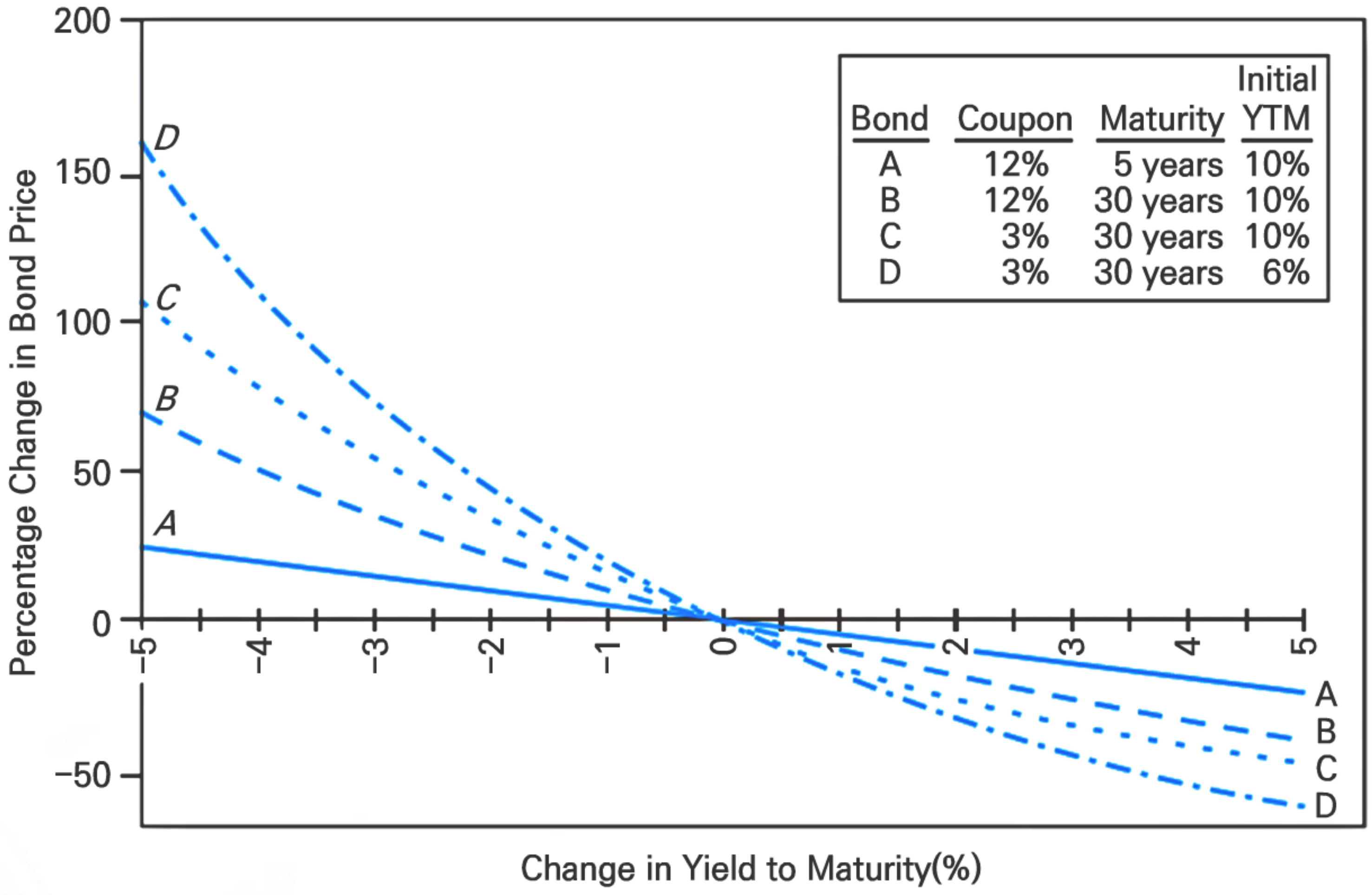
- Interest rate risk
  - Interest rate sensitivity of bond prices
  - Duration and its determinants
- Convexity
- Passive management strategies

Interest Rate Risk

Interest Rate Sensitivity

1. Bond prices and yields are inversely related
2. An increase in a bond’s yield to maturity results in a smaller price change than a decrease of equal magnitude
3. Long-term bonds tend to be more price sensitive than short-term bonds
4. As maturity increases, price sensitivity increase at a decreasing rate
5. Interest rate risk is inversely related to the bond’s coupon rate
6. Price sensitivity is inversely related to the yield to maturity at which the bond is selling

Figure -Change in bond price as a function of change in YTM



Example

Prices of 8% coupon bond (coupon paid semiannually)

Yield to Maturity(APR)	$T = 1$ Year	$T = 10$ Year	$T = 20$ Year
8%	1,000.00	1,000.00	1,000.00
9%	990.64	934.96	907.99
Fall in Price (%)*	0.94%	6.50%	9.20%

*Equal value of bond at a 9% yield to maturity divided by value bond of at (the original) 8% yield, minus 1.

Example

Prices of zero-coupon bond (semiannual compounding)

Yield to Maturity(APR)	$T = 1$ Year	$T = 10$ Year	$T =$20 Year
8%	924.56	456.39	208.29
9%	915.73	414.64	171.93
Fall in Price (%)*	0.96%	9.15%	17.46%

*Equal value of bond at a 9% yield to maturity divided by value bond of at (the original) 8% yield, minus 1.

Duration

- A measure of the effective maturity of a bond
- The weighted average of the times until each payment is received, with the weights proportional to the present value of the payment
- It is shorter than maturity for all bonds, and is equal to maturity for zero coupon bonds

Duration Calculation

(Macaulay) Duration calculation

$$D = \sum_{t=1}^{T} t \times w_t$$

$$w_t \equiv \frac{CF_t/(1+y)^t}{Price}, \; CF = Cash\ Flow\ at\ time\ t$$

- In the case of traditional duration measure, we use Macaulay duration using yield to maturity to obtain the weight. In general, however, we use yield curve (spot rates) to compute the weights.

$w_t \equiv \frac{CF_t/(1+r_t)^t}{Price}$,where r_t = spot rate for t-year zero

Duration-Price Relationship

- **Price change is proportional to duration and not to maturity:**

$$\frac{\Delta P}{P} = -D\left[\frac{\Delta(1+y)}{1+y}\right]$$

- **D^* = modified duration**

$$\frac{\Delta P}{P} = -D^*\Delta y \text{ since } \Delta(1+y) = \Delta y$$

What determines Duration?

Rule 1

- The duration of a zero-coupon bond equals its time to maturity

Rule 2

- Holding maturity constant, a bond's duration is higher when the coupon rate is lower

Rule 3

- Holding the coupon rate constant, a bond's duration generally increases with its time to maturity

What determines Duration?

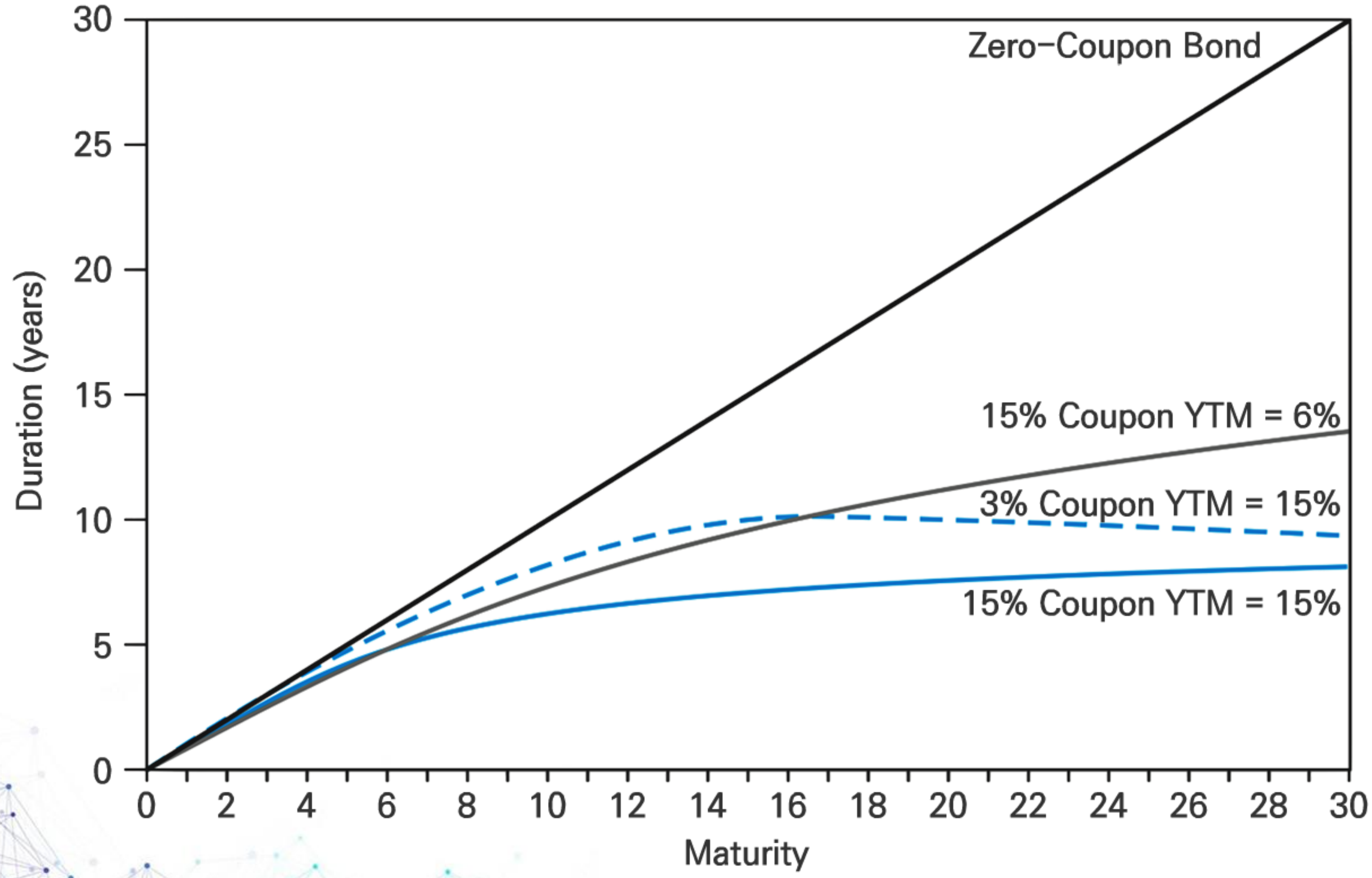
Rule 4

- Holding other factors constant, the duration of a coupon bond is higher when the bond's yield to maturity is lower

Rule 5

- The duration of a level perpetuity is equal to $\frac{1+y}{y}$

Figure - Bond duration vs. Bond maturity



Example

Bond durations (YTM = 8% APR, Semiannual coupons)

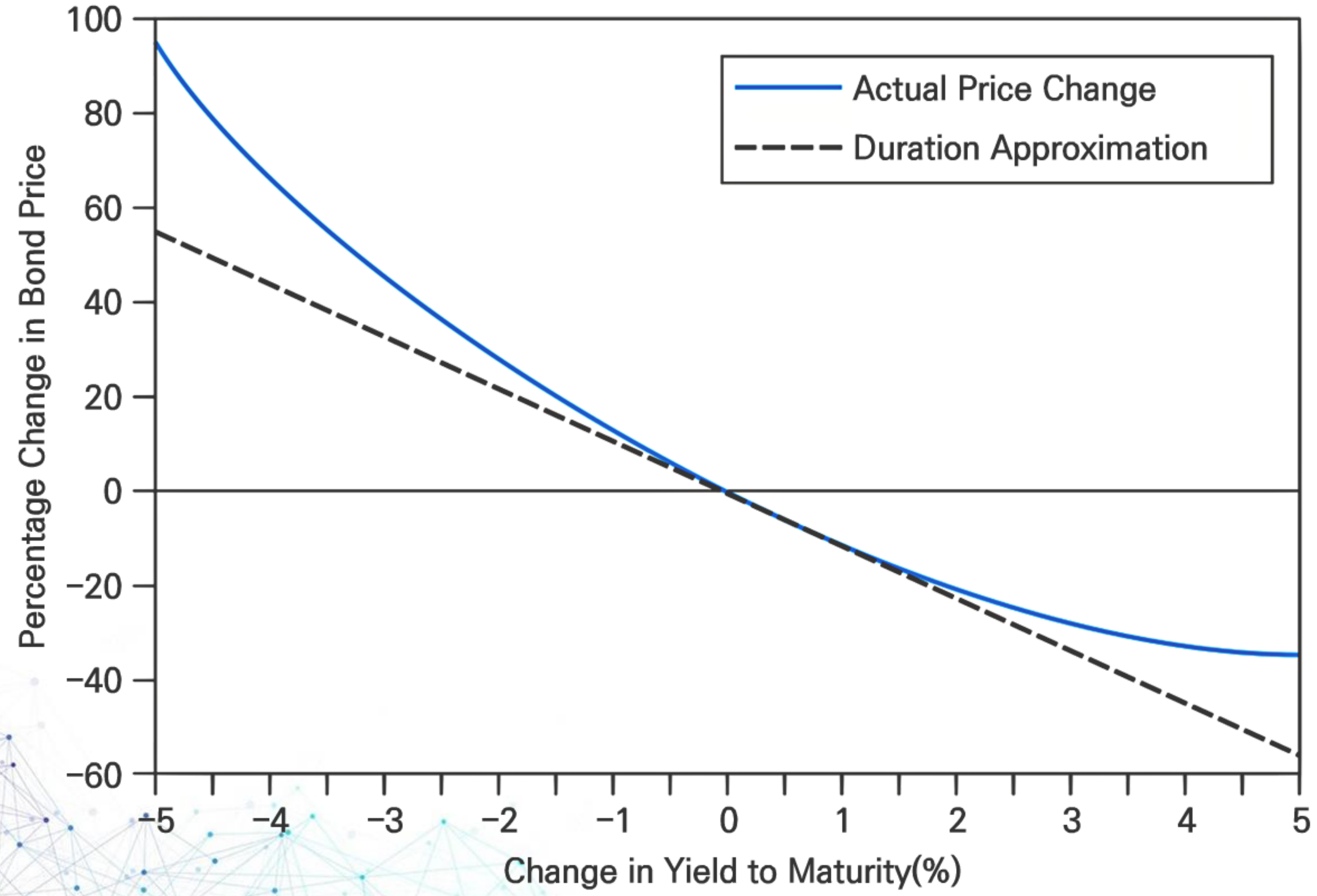
Maturity (years)	Coupon Rates (per Year)				
	2%	4%	6%	8%	10%
1	0.995	0.990	0.985	0.981	0.976
5	4.742	4.533	4.361	4.218	4.095
10	8.762	7.986	7.454	7.067	6.772
20	14.026	11.966	10.922	10.292	9.870
Infinite(perpetuity)	13.000	13.000	13.000	13.000	13.000

Convexity

- The relationship between bond prices and yields is not linear
- Duration rule is a good approximation for only small changes in bond yields
- Bonds with greater convexity have more curvature in the price-yield relationship

Figure

- **Bond price convexity: 30-yr 8% coupon, initial YTM = 8%**



Figure

Definition of Convexity

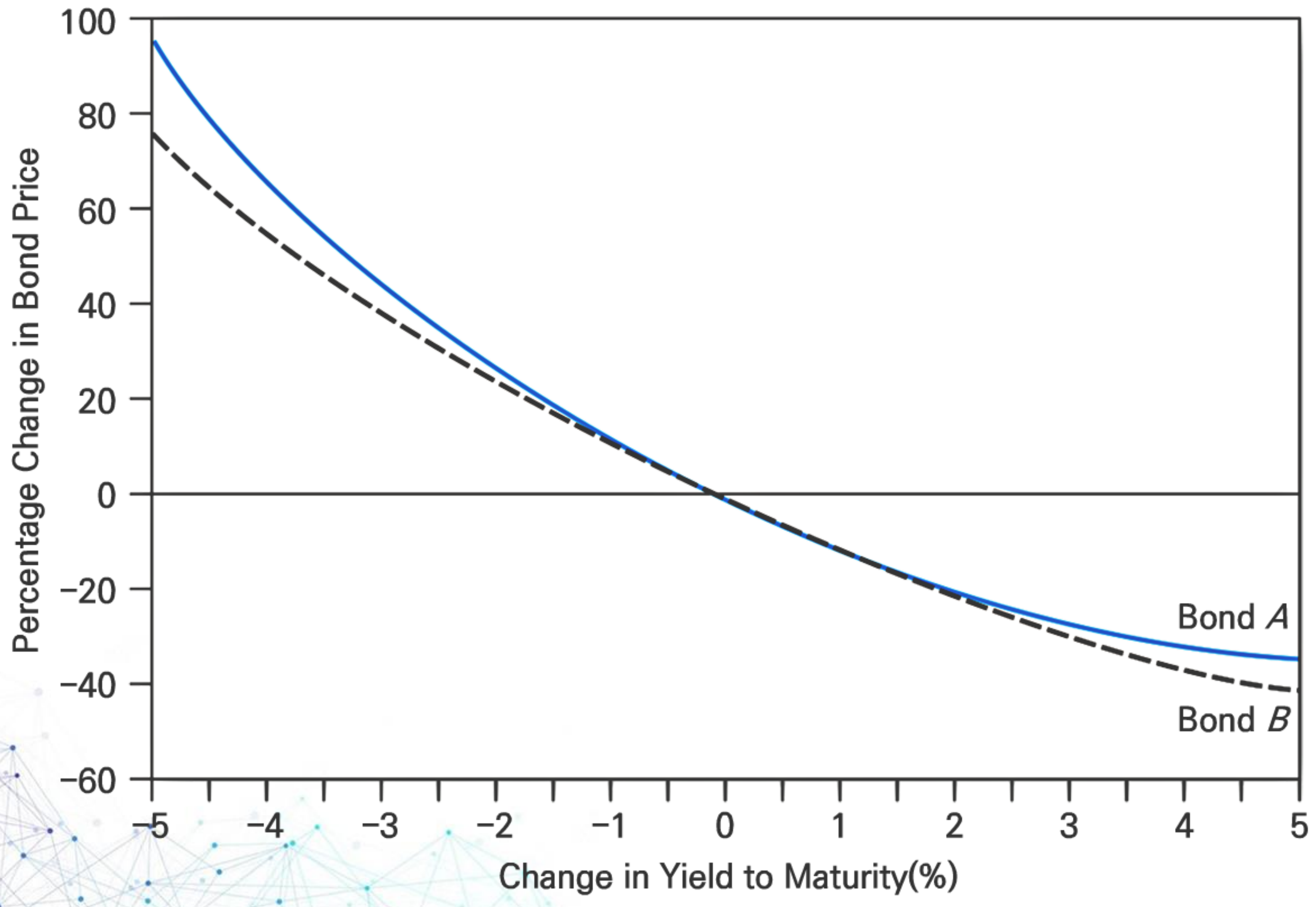
$$\text{Convexity} = \frac{1}{P\times(1+y)^2}\sum_{t=1}^{n}\left[\frac{CF_t}{(1+y)^t}(t^2+t)\right]$$

Correction for Convexity:

$$\frac{\Delta P}{P} = -D^*\Delta y + \frac{1}{2}[\text{Convexity} \times (\Delta y)^2]$$

Figure

Convexity of Two Bonds



Why Do Investors Like Convexity?

- Bonds with greater curvature gain more in price when yields fall than they lose when yields rise
- The more volatile interest rates, the more attractive this asymmetry
- Bonds with greater convexity tend to have higher prices and/or lower yields, all else equal

Bond Fund Management - Immunization

- **Control interest rate risk**
- **Widely used by pension funds, insurance companies, and banks**
- **The interest rate exposure of assets and liabilities are matched in the portfolio**
  - Match the duration of the assets and liabilities
  - Price risk and reinvestment rate risk exactly cancel out
  - Value of assets match liabilities whether rates rise/fall

Bond Fund Management - Immunization

Example: An insurance company sold a guaranteed investment contract. This is the same as the zero-coupon bond. It has 5-year maturity and a guaranteed interest rate of 8%. So, the insurance company promised to pay $\$10,000 \times 1.08^5$ $= \$14,693.28$ in 5 years.

Suppose that the insurance company chooses to fund its obligation with $10,000 of 8% annual coupon bond, selling at par with 6-year maturity. As long as the market interest rate stays at 8%, the company has fully funded the obligation, as the present value of the obligation exactly equals the value of the bonds.

Terminal Value of a Bond Portfolio after 5 years

Payment Number	Years Remaining until Obligation	Accumulated Value of Invested Payment		
A. Rates remain at 8%				
1	4	$800 \times (1.08)^4$	=	1,088.39
2	3	$800 \times (1.08)^3$	=	1,007.77
3	2	$800 \times (1.08)^2$	=	933.12
4	1	$800 \times (1.08)^1$	=	864.00
5	0	$800 \times (1.08)^0$	=	800.00
Sale of bond	0	10,800/1.08	=	10,000.00
				14,693.28
B. Rates fall to 7%				
1	4	$800 \times (1.07)^4$	=	1,048.64
2	3	$800 \times (1.07)^3$	=	980.03
3	2	$800 \times (1.07)^2$	=	915.92
4	1	$800 \times (1.07)^1$	=	856.00
5	0	$800 \times (1.07)^0$	=	800.00
Sale of bond	0	10,800/1.07	=	10,093.46
				14,694.05
C. Rates increase to 9%				
1	4	$800 \times (1.09)^4$	=	1,129.27
2	3	$800 \times (1.09)^3$	=	1,036.02
3	2	$800 \times (1.09)^2$	=	950.48
4	1	$800 \times (1.09)^1$	=	872.00
5	0	$800 \times (1.09)^0$	=	800.00
Sale of bond	0	10,800/1.09	=	9,908.26
				14,696.02

출처 : 교수자 제공

Growth of Invested Funds

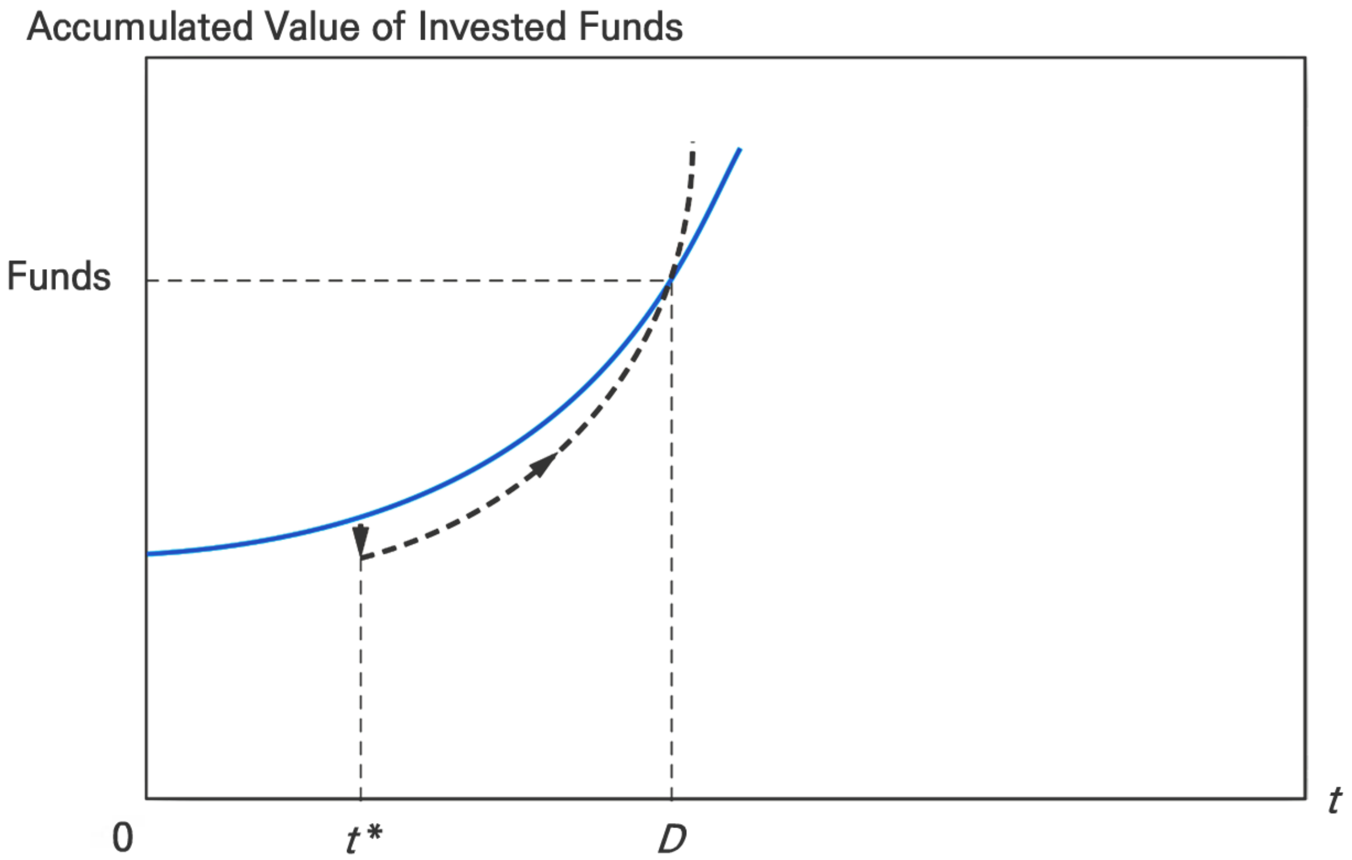


Figure -- Growth of invested funds. The solid colored curve represents the growth of portfolio value at the original interest rate. If interest rates increase at time t*, the portfolio value initially falls but increases thereafter at the faster rate represented by the broken curve. At time D (duration), the curve cross.

Immunization Example

Constructing an immunized portfolio

An insurance company must make a payment of $ 19,487 in seven years. The market interest rate is 10%, so the present value of the obligation is $ 10,000. The company's portfolio manager wishes to fund the obligation using 3-year zero-coupon bonds and perpetuities paying annual coupons. (We focus on zeros and perpetuities to keep the algebra simple.) How can the manager immunize the obligation?

Immunization Example

Constructing an immunized portfolio

Immunization requires that the duration of the portfolio of assets equal the duration of the liability. We can proceed in four steps:

1. Calculate the duration of the liability. In this case, the liability duration is simple to compute. It is a single-payment obligation with duration of seven years.

Immunization Example

Constructing an immunized portfolio

2. Calculate the duration of the asset portfolio. The portfolio duration is the weighted average of duration of each component asset, with weights proportional to the funds placed in each asset. The duration of the zero-coupon bond is simply its maturity, three years. The duration of the perpetuity is 1.10/.10 = 11 years. Therefore, if the fraction of the portfolio invested in the zero is called w and the fraction invested in the perpetuity is (1 - w), the portfolio duration will be

$$\text{Asset duration} = w \; x \; 3 \; years \; + \; (1 \; - \; w) \; x \; 11 \; years$$

Immunization Example

Constructing an immunized portfolio

3. Find the asset mix that sets the duration of assets equal to the 7-year duration of liabilities. This requires us to solve for w in the following equation:

$$w\ x\ 3\ years\ +\ (1\ -\ w)\ x\ 11\ years\ =\ 7\ years$$

This implies that $w\ =\ 1/2$. The manager should invest half the portfolio in the zero and half in the perpetuity. This will result in an asset duration of seven years.

Immunization Example

Constructing an immunized portfolio

4. Fully fund the obligation. Because the obligation has a present value of $ 10,000, and the fund will be invested equally in the zero and the perpetuity, the manager must purchase $5,000 of the zero-coupon bond and $5,000 of the perpetuity. (The face value of the zero will be $\$5{,}000\ x\ 1. = 1.10^3\ = \$6{,}655$.)

Example

Rebalancing

Suppose that one year has passed, and the interest rate remains at 10%. The portfolio manager of Example 16.4 needs to reexamine her position. Is the position still fully funded? Is it still immunized? If not, what actions are required?

Example

Rebalancing

First, examine funding. The present value of the obligation will have grown to $11,000, as it is one year closer to maturity. The manager's funds also have grown to $1 1,000: The zero-coupon bonds have increased in value from $5,000 to $5,500 with the passage of time, while the perpetuity has paid its annual $500 coupon and remains worth $5,000. Therefore, the obligation is still fully funded.

The portfolio weights must be changed, however. The zero-coupon bond now has a duration of two years, while the perpetuity's duration remains at 11 years. The obligation is now due in six years. The weights must now satisfy the equation

$$w \times 2 + (1 - w) \times 11 = 6$$

Example

Rebalancing

which implies that w = 5/9. To rebalance the portfolio and maintain the duration match, the manager now must invest a total of $\$11{,}000\ x\ 5/9 = \$6{,}111.11$ in the zero-coupon bond. This requires that the entire $500 coupon payment be invested in the zero, with an additional $\$111.11$ of the perpetuity sold and invested in the zero-coupon bond.

Cash Flow Matching

Cash flow matching and dedication

- Cash flow matching = automatic immunization
- Cash flow matching is a dedication strategy
- Not widely used because of constraints associated with bond choices

Exercise Problem 1

A newly issued bond has a maturity of 10 years and pays a 7% coupon rate (with coupon payments coming once annually). The bond sells at par value.

a. What are the convexity and the duration of the bond?
b. Find the actual price of the bond assuming that its yield to maturity immediately increases from 7% to 8% (with maturity still 10 years).
c. What price would be predicted by the modified duration rule? What is the percentage error of that rule?
d. What price would be predicted by the modified duration-with-convexity rule? What is the percentage error of that rule?

Exercise Problem 2

A 12.75- year maturity zero-coupon bond selling at a yield to maturity of 8% has convexity of 150.3 and modified duration of 11.81 years. A 30-y ear maturity 6% coupon bond making annual coupon payments also selling at a yield to maturity of 8% has nearly identical duration-11.79 years-but considerably higher convexity of 231.2.

a. Suppose the yield to maturity on both bonds increases to 9%. (i) What will be the actual percentage capital loss on each bond? (ii) What percentage capital loss would be predicted by the duration-with-convexity rule?

Exercise Problem 2

b. Repeat part (a) , but this time assume the yield to maturity decreases to 7% .

c. Compare the performance of the two bonds in the two scenarios, one involving an increase in rates, the other a decrease. Based on the comparative investment performance, explain the attraction of convexity.

Exercise Problem 3

My pension plan will pay me $10,000 once a year for a 10-year period. The first come in exactly five years. The pension fund wants to immunize its position.

a. What is the duration of its obligation to me? The current interest rate is 10% per year.

b. If the plan uses 5-year and 20-year zero-coupon bonds to construct the immunized position, how much money ought to be placed in each bond?