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투자론

- *R*과 Excel을 통한 금융데이터 분석 -

10주차
채권가격과 수익률 및 수익률 곡선과 선도 이자율

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Unit 01

Bond Pricing

Overview

- Fixed-Income securities characteristics
- Bond pricing
  - Prices and yields
  - Prices over time
- Impact of default and credit risk on bond pricing

Bond Characteristics

- **Bonds are debt obligations of issuers (borrowers) to bondholders (creditors)**
  - Face or par value is the principal repaid at maturity, typically $1,000
  - The coupon rate determines the interest payment ("coupon payments") paid semiannually
  - The indenture is the contract between the issuer and the bondholder that specifies the coupon rate, maturity date, and par value

Definition of Efficient Market Hypothesis

Bond Pricing

$$P_B = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t} + \frac{Par\ Value}{(1+r)^T}$$

- P_B= price of the bond
- C_t= interest or coupon payment at time t
- T = number of periods to maturity (= number of coupon payments)
- r = $YTM\ in\ APR/(\#\ of\ coupon\ payments\ in\ year)$

Exercise 1

- **Price of a 30 year, 8% coupon bond. Market rate of interest is 10%**

$$Price = \sum_{t=1}^{60} \frac{\$40}{1.05^t} + \frac{\$1{,}000}{1.05^{60}} = \$810.71$$

Bond Prices and Yields

- Prices and yields (required rates of return) have an inverse relationship
- The bond price curve is convex.
- The lower the current yield or interest rate, the more sensitive the bond's price to changes in market interest rates

Figure – Bond Price Curve

- **The inverse relationship b/w bond prices and yields**

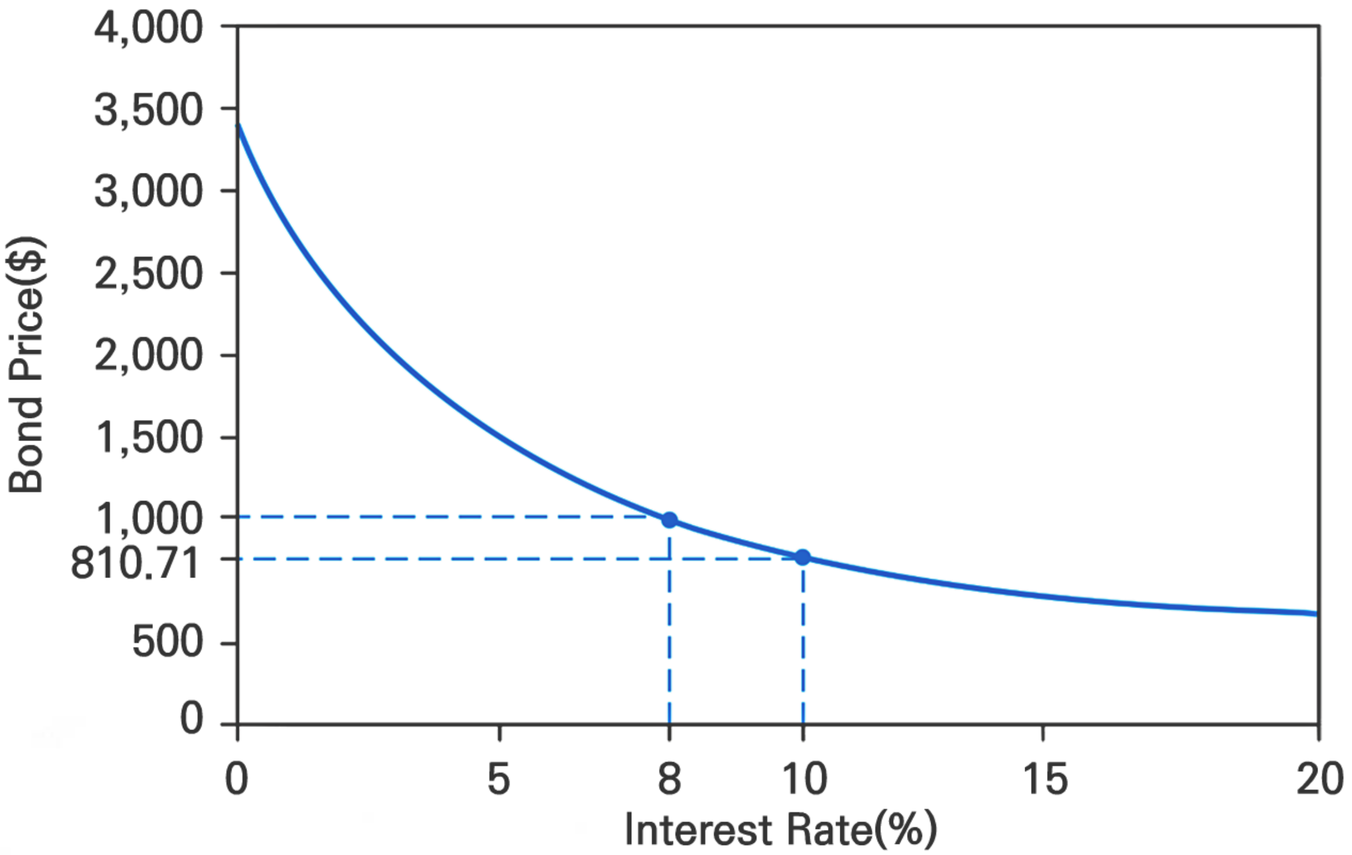


Figure -- The inverse relationship between bond prices and yields. Price of an 8% coupon with 30-year maturity making semiannual payments

Table – Bond prices at different interest rates

	Bond Pice at Given Market Interest Rate				
Time to Maturuty	2%	4%	6%	8%	10%
1 year	1,059.11	1,038.83	1,019.13	1,000.00	98.41
10 year	1,541.37	1,327.03	1,148.77	1,000.00	875.35
20 year	1,985.04	1,547.11	1,231.15	1,000.00	828.41
30 year	2,348.65	1,695.22	1,276.76	1,000.00	810.71

Bond Yields: yield to maturity (YTM)

- **Interest rate that makes the present value of the bond's payments equal to its price is the yield to maturity (YTM):**

  - Solve the bond formula for r

$$P_B = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t} + \frac{Par\ Value}{(1+r)^T}$$

Calculating YTM (y)

- **We need to solve for the constant interest rate (y) in the bond valuation formula:**

$$V = \sum_{t=1}^{n} \frac{CF_t}{(1+y)^t}$$

- **Given, V and CFs, we can solve for y**
  - Using Excel Solver, or '=YIELD()', or =RATE()
  - Using Financial Calculator

Calculationg YTM (y)

	A	B	C	D
1		**Semiannual coupons**		**Annual coupons**
2				
3	Settlement date	2000/01/01		2000/01/01
4	Maturity date	2030/01/01		2030/01/01
5	Annual coupon rate	0.08		0.08
6	Bond price (flat)	127.676		127.676
7	Redemption value (% of face value)	100		100
8	Coupon payments per year	2		1
9				
10	**Yield to maturity (decimal)**	**0.0600**		**0.0599**
11				
12	The formula entered here is =YIELD(B3,B4,B5,B6,B7,B8)			
13				
14				

Or, one can always use equation solver in Excel

출처 : 교수자 제공

Bond Pricing with Excel

- =PRICE(settlement date, maturity date, annual coupon rate, yield To maturity, redemption value as percent of par value, number of coupon payments per year)

	A	B	C	D	E	F
1		2.5% coupon bond,		2% coupon bond,		8% coupon bond,
2		maturing May 15, 2046	Formula in column B	maturing August 2025		30-year maturity
3						
4	Settlement date	2016/05/15	=DATE(2016,5,15)	2016/05/15		2000/01/01
5	Maturity date	2046/05/15	=DATE(2046,5,15)	2025/08/15		2030/01/01
6	Annual coupon rate	0.025		0.02		0.08
7	Yield to maturity	0.02595		0.0173		0.1
8	Redemption value (% of face value)	100		100		100
9	Coupon payments per year	2		2		2
10						
11						
12	**Flat price (% of par)**	**98.0282**	**=PRICE(B4,B5,B6,B7,B8,B9)**	**102.2977**		**81.0707**
13	Days since last coupon	0	=COUPDAYBS(B4,B5,2,1)	90		0
14	Days in coupon period	184	=COUPDAYS(B4,B5,2,1)	182		182
15	Accrued interest	0	=(B13/B14)*B6*100/2	0.495		0
16	**Invoice price**	**98.0282**	**=B12+B15**	**102.7922**		**81.0707**
17						

출처 : 교수자 제공

Example: YTM

- **Suppose an 8% coupon, 30 year bond is selling for $1276.76. What is its average rate of return?**

$$1{,}276.76 = \sum_{t=1}^{60} \frac{\$40}{(1+r)^t} + \frac{1{,}000}{(1+r)^{60}}$$

- r=3%per half year
- Bond equivalent yield = 6%
- EAR = $(1+0.03)^2$−1=6.09%

Yield to Call

- Callable Bonds can be repurchased before the maturity date
- If interest rates fall, price of straight bond can rise considerably
- The price of the callable bond is flat over a range of low interest rates because the risk of repurchase or call is high
- When interest rates are high, the risk of call is negligible and the values of the straight and the callable bond converge

Figure 14

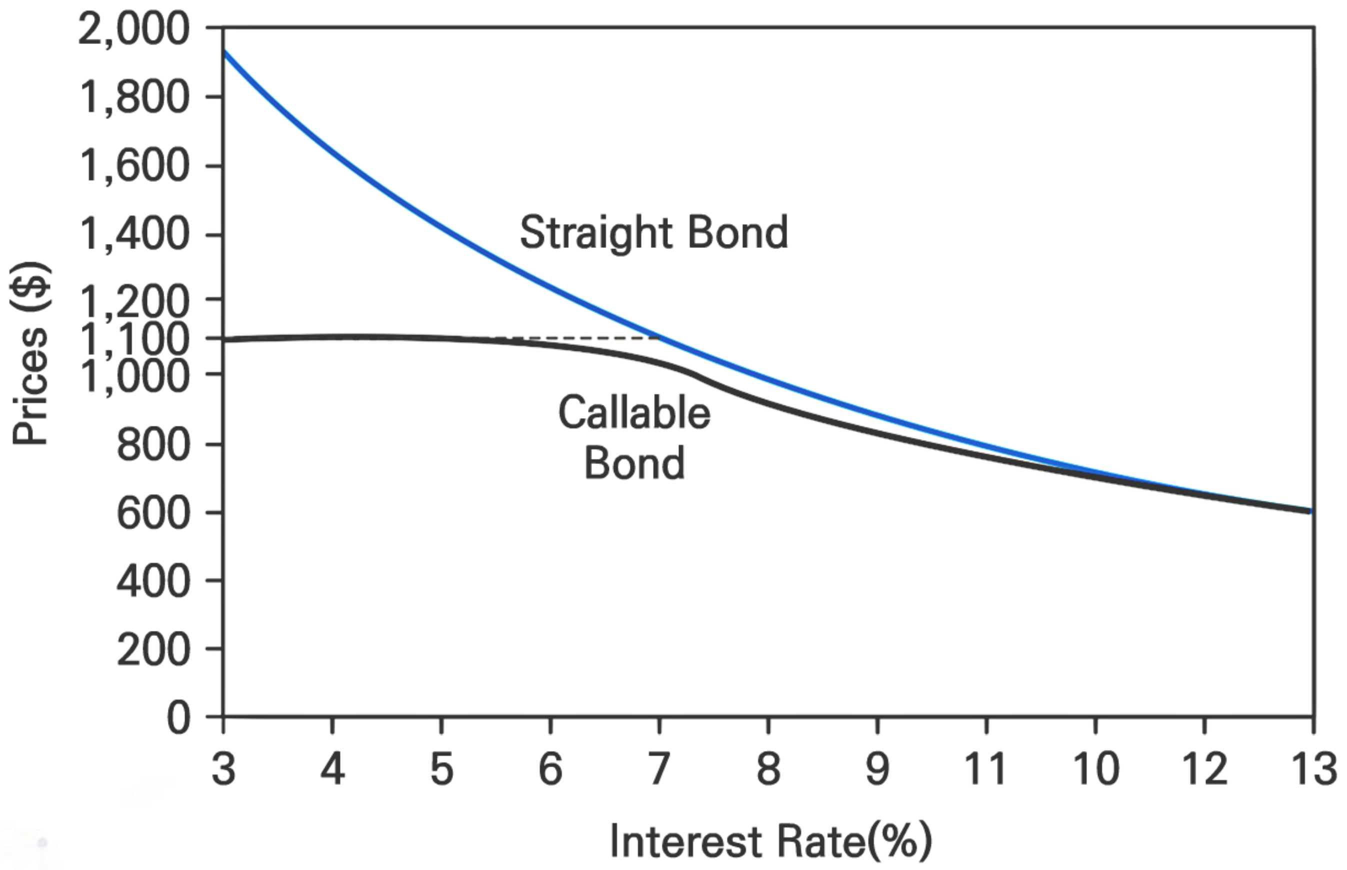


Figure -- Bond prices: Callable and straight debt (coupon=8%; maturity=30 years; semiannual payments)

Suppose the 8% coupon, 30-year maturity bond sells for $1,150 and is callable in 10 years at a call price of $1.100. Its yield to maturity and yield to call would be calculated using the following inputs:

	Yield to Call	Yield to Maturity
Coupon payment	$40	$40
Number of semiannual periods	20 periods	60 periods
Final payment	$1,100	$1,000
Price	$1,150	$1,150

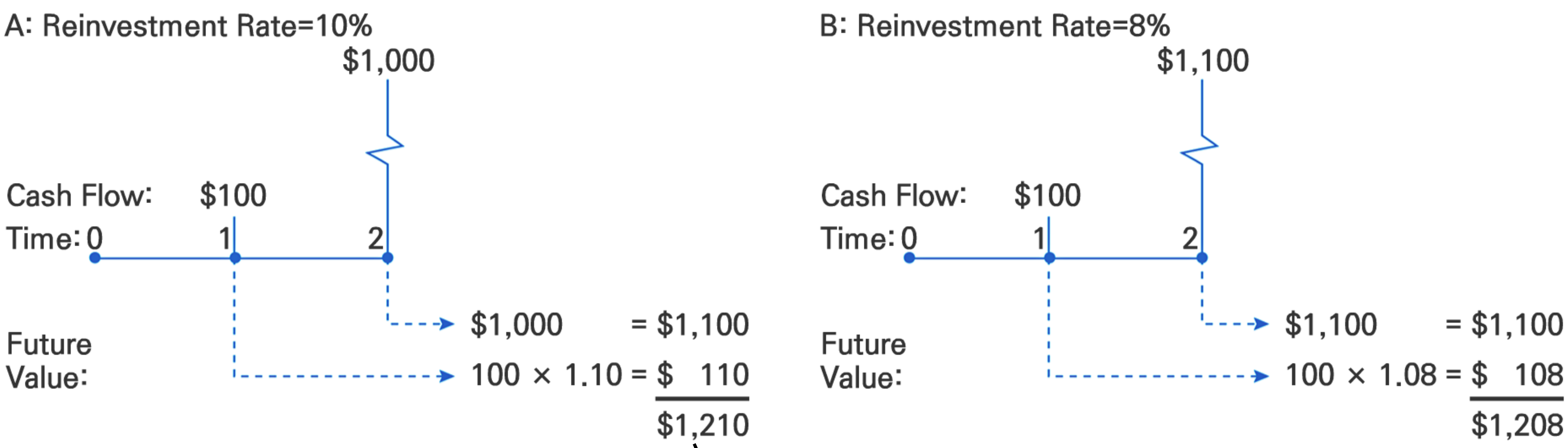
Yield to call is then 6.64%. [To confirm this on a calculator, input n = 20; PV = (-)1 150; FV = 1100; PMT = 40; compute / as 3.32%, or 6.64% bond equivalent yield.] Yield to maturity is 6.82%. [To confirm, input n = 60; PV = (-)1150; FV = 1000; PMT = 40; compute *i* as 3.41% or 6.82% bond equivalent yield.] In Excel, you can calculate yield to call as =YIELD(DATE(2000, 1,1), DATE(2010,1,1). .08, 115, 110, 2). Notice that redemption value is input as 110, that is, 110% of par value.

Bond Yields

Realized yield vs. YTM

- Reinvestment assumptions
- Holding period return
  - Changes in rates affect returns
  - Reinvestment of coupon payments
  - Change in price of the bond

Figure – Growth of Invested Funds



- YTM will equal the rate of return realized over the life of the bond <u>if all coupons are reinvested at an interest rate equal to the bond's yield to maturity.</u>

$$V_0(1+r)^2 = V_2$$
$$\$1{,}000(1+r)^2 = \$1{,}210$$
$$r = .10 = 10\%$$

$$V_0(1+r)^2 = V_2$$
$$\$1{,}000(1+r)^2 = \$1{,}208$$
$$r = .0991 = 9.91\%$$

Example -- Realized Compound Return

- The problem with conventional YTM occurs when reinvestment rates can change over time. Thus, conventional YTM $\neq$ realized compound return. However, reinvestment rate is known priori before we actually do it. Because of this, realized return measure is not much useful.

Bond Prices Over Time

YTM vs. HPR

YTM	HPR
It is the average return if the bond is held to maturity	It is the rate of return over a particular investment period
Depends on coupon rate, maturity, and par value	Depends on the bond's price at the end of the holding period, an unknown future value
All of these are readily observable	Can only be forecasted

Figure – Bond Price over Time

- Prices over time of 30-year maturity, 6.5% coupon bonds

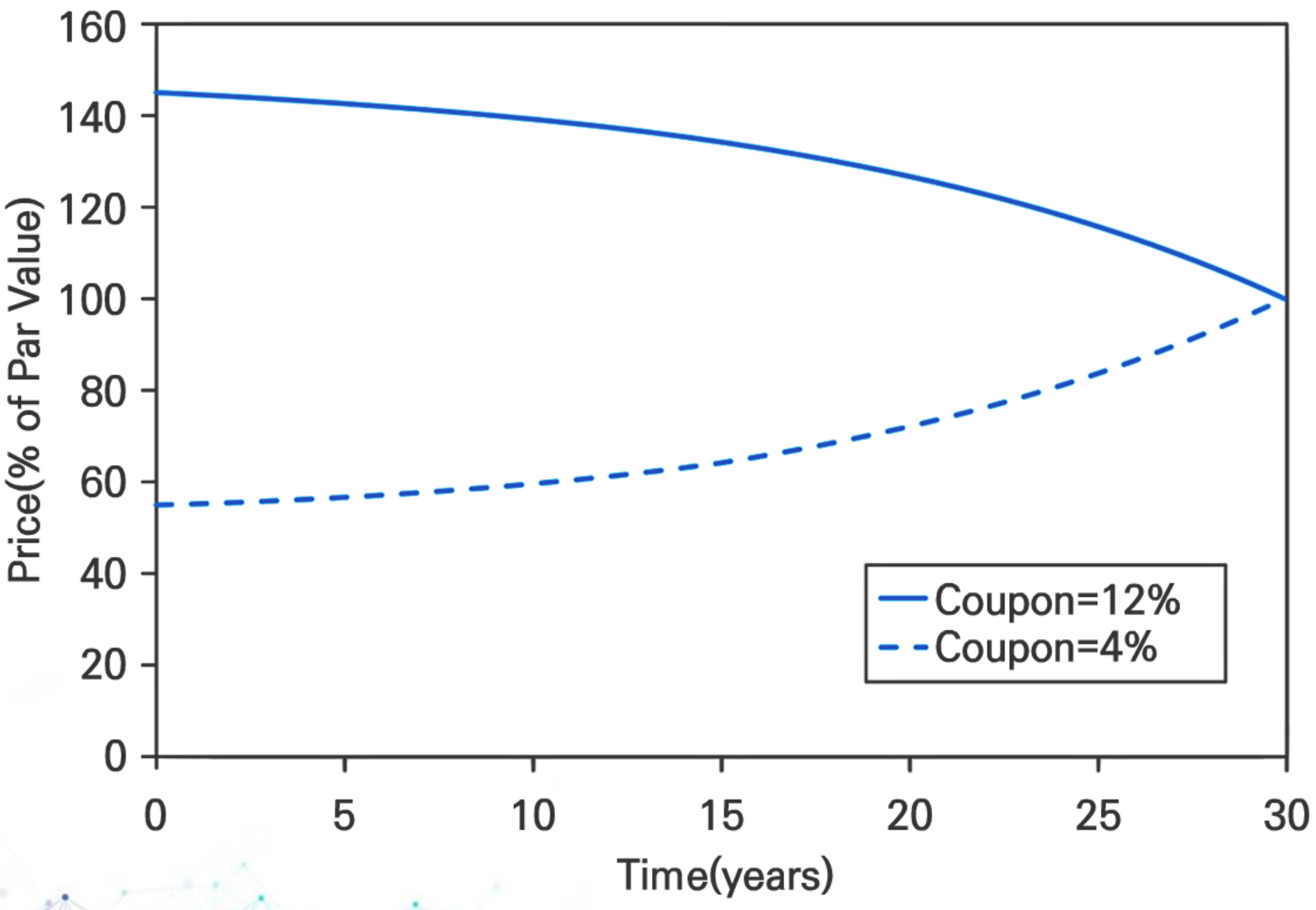
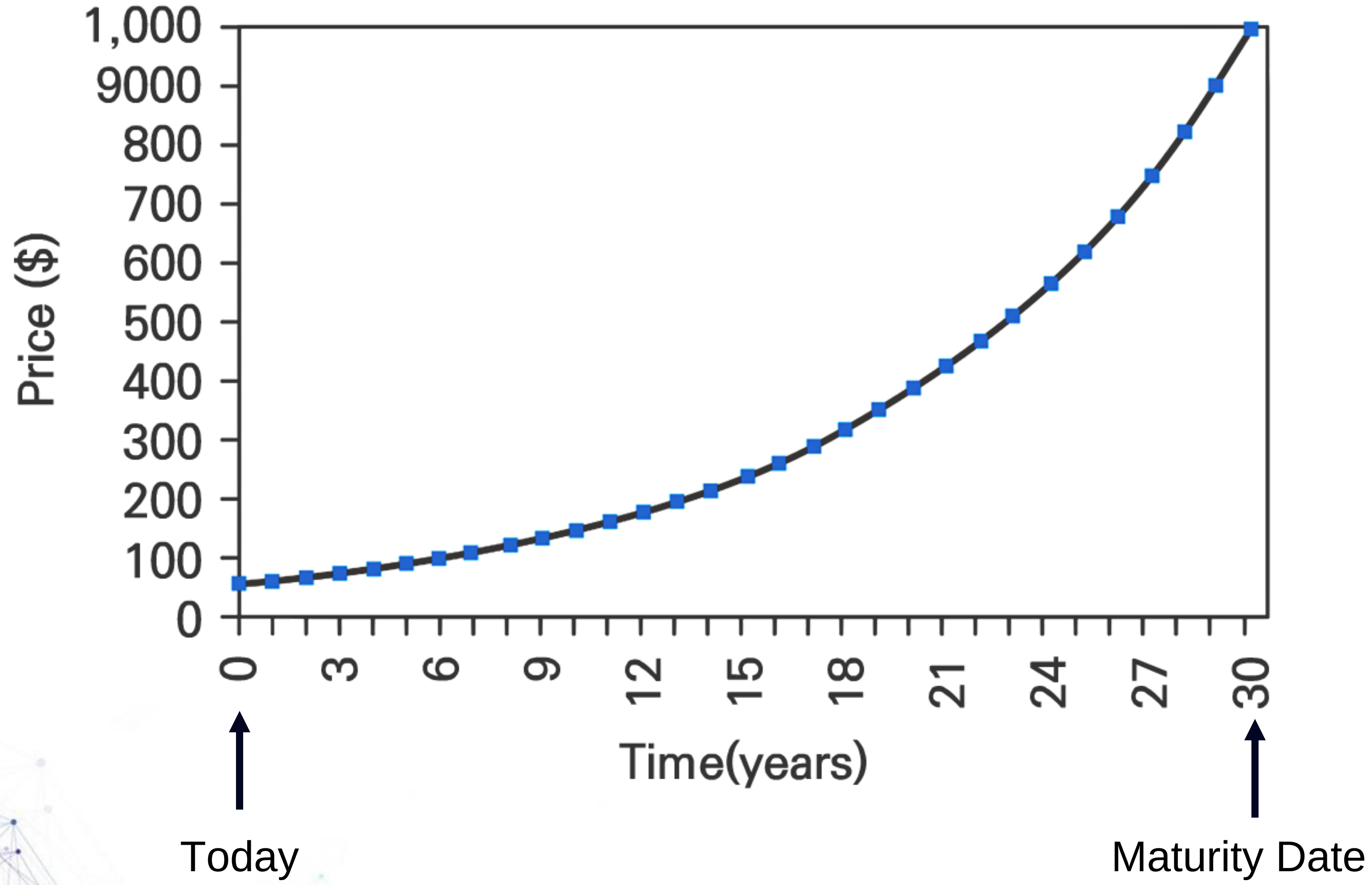


Figure -- The price of 30yr zero over time



Default Risk and Bond Pricing

Rating Companies

- Moody's Investor Service, Standard & Poor's, Fitch

Rating Categories

- Highest rating is AAA or Aaa
- Investment grade bonds are rated BBB or Baa and above
- Sepculative grade/junk bonds have ratings below BBB or Baa

Default Risk and Bond Pricing

Determinants of bond safety

- Coverage ratios
- Leverage ratios, debt-to-equity ratio
- Liquidity ratios
- Profitability ratios
- Cash flow-to-debt ratio

YTM and Default Risk

- **The risk structure of interest rates refers to the pattern of default premiums**
- **There is a difference between the yield based on expected cash flows and yield based on promised cash flows**
- **The difference between the expected YTM and the promised YTM is the default risk premium**

$$P' = \sum_{t=1}^{T} \frac{E(C_t)}{(1+y_D)^t} + \frac{E(Par)}{(1+y_D)^T}, P' = \sum_{t=1}^{T} \frac{C_t}{(1+y_P)^t} + \frac{Par}{(1+y_P)^T}$$

$$\Rightarrow y_P - y_D = default\ risk\ premium$$

YTM and Default Risk

Example : Expected versus Promised Yield to Maturity

Suppose a firm issued a 9% coupon bond 20 years ago. The bond now has 10 years left until its maturity date, but the firm is having financial difficulties. Investors believe that the firm will be able to make good on the remaining interest payments, but at the maturity date, the firm will be forced into bankruptcy, and bondholders will receive only 70% of par value. The bonds selling at $750.

Yield to maturity (YTM) would then be calculated using the following inputs:

	Expected YTM	Stated YTM
Coupon payment	$45	$45
Number of semiannual periods	20periods	20periods
Final payment	$700	$1,000
Price	$750	$750

The stated yield to maturity, which is based on promised payments, is 13.7%. Based on the expected payment of $700 at maturity, however, the yield to maturity is only 11.6%. The stated yield to maturity is greater than the yield investors actually expect to earn.

YTM and Default Risk

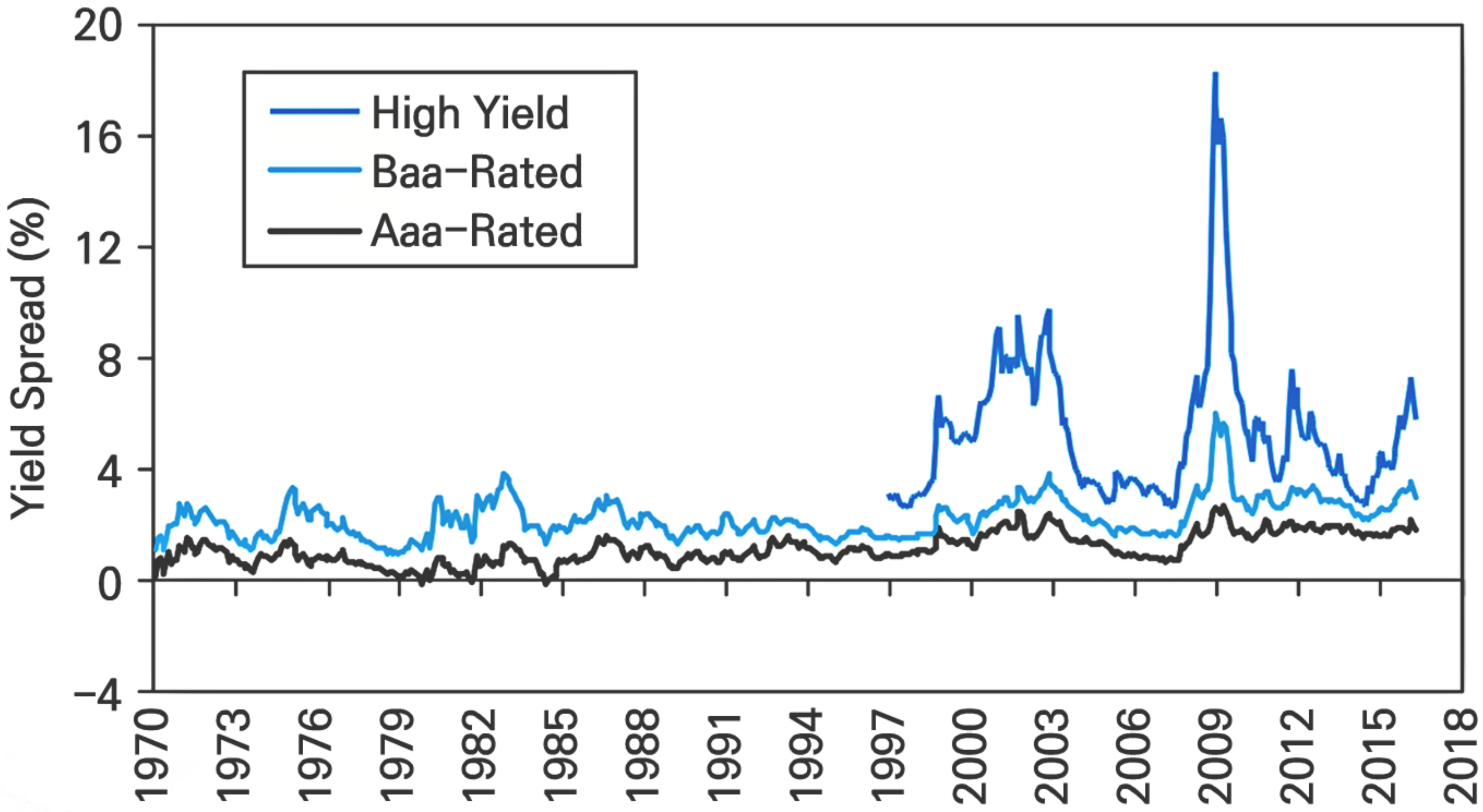


Figure -- Yield spreads between corporate and 10-year Treasury bonds

출처 : Federal Reserve Bank of St. Louis

Exercise Problem 1

Consider a bond with a 10% coupon and yield to maturity = 8%.
If the bond's yield to maturity remains constant, then in one year,
will the bond price be higher, lower, or unchanged? Why?

Exercise Problem 2

A 20-year maturity bond with par value of $1,000 makes semiannual coupon payments at a coupon rate of 8%. Find the bond equivalent and effective annual yield to maturity of the bond if the bond price is $950.

Exercise Problem 3

Consider a bond paying a coupon rate of 10% per year semiannually when the market interest rate is only 4% per half-year. The bond has three years until maturity.

a. Find the bond's price today and six months from now after the next coupon is paid.
b. What is the total (6-month) rate of return on the bond?