투자론

- R과 Excel을 통한 금융데이터 분석 -

2주차
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Unit 03

Risk Aversion and Capital Allocation

Allocation to Risky Assets

- Investors will avoid risk unless there is a reward

- The utility model gives the optimal allocation between a risky portfolio and a risk-free asset

Risk Aversion and Utility Values

- Investors are willing to consider
  - risk-free assets
  - speculative positions with positive risk premiums
- Portfolio attractiveness increases with expected return and decreases with risk
- What happens when return increases with risk?

Available Risky Portfolios with r_f = 5%

Available Risky Portfolios (Risk-free rate =5%)

Portfolio	Risky Premium	Expected Return	Risk(SD)
L (low risk)	2%	7%	5%
M (medium risk)	4	9	10
H (high risk)	8	13	20

- Each portfolios receives a utility score to assess the investor's risk/return trade off

Utility Function

$$U = \text{Utility} = E(r) - \frac{1}{2}A\sigma^2$$

- where $E(r)$ = expected return on the asset or portfolio,
  A = coefficient of risk aversion, σ^2 = variance of returns,
  and $\frac{1}{2}$ = a scaling factor

Investor Risk Aversion(A)	Utility Score of Portfoilo L [$E(r)$= .07; σ= .05]	Utility Score of Portfoilo L [$E(r)$=.09; σ= .10]	Utility Score of Portfoilo L [$E(r)$= .13; σ= .20]
2.0	$.07-1/2\times2\times.05^2 = .0675$	$.09-1/2\times2\times.1^2 = .0800$	**$.13-1/2\times2\times.2^2 = .09$**
3.5	$.07-1/2\times3.5\times.05^2 = .0656$	**$.09-1/2\times3.5\times.1^2 = .0725$**	$.13-1/2\times3.5\times.2^2 = .06$
5.0	$.07-1/2\times5\times.05^2 = .0638$	**$.09-1/2\times5\times.1^2 = .0650$**	$.13-1/2\times5\times.2^2 = .03$

Utility Score of alternative portfoilos for investors with varying degree of risk aversion

Mean-Variance (MV) Criterion

Protfoil A dominates portfolio B

if $E(r_A) \geq E(r_B)$, and$\sigma_A \leq \sigma_B$

Estimating Risk Aversion

- Use questionnaires
- Observe individual's decisions when confronted with risk
- Observe how much people are willing to pay to avoid risk

Capital Allocation across Risky and Risk-Free Portfolios

Asset Allocation

- is a very important part of portfolio construction.
- refers to the choice among broad asset classes.

Controlling Risk

- Simplest way
  - manipulate the fraction of the portfolio invested in risk-free assets versus the portion invested in the risky assets

Basic Asset Allocation

Total market value = \$300,000

Risk-free money market fund = \$90,000

Equities = \$113,400

Bonds (long-term) = \$96,600

Total risk assets = \$210,000

$$W_E = \frac{\$114{,}400}{\$210{,}000} = 0.54 \ , W_B = \frac{\$96{,}600}{\$210{,}000} = 0.46$$

Basic Asset Allocation

- Let y be a weight of the risky portfolio, P, in the complete portfolio; $(1-y)$ be a weight of risk-free assets

$$y = \frac{\$210{,}000}{\$300{,}000} = 0.7 \qquad 1 - y = \frac{\$90{,}000}{\$300{,}000} = 0.3$$

$$E\colon \frac{\$113{,}400}{\$300{,}000} = 0.378 \qquad B\colon \frac{\$96{,}600}{300{,}000} = 0.322$$

The Risk-Free Asset

- Only the government can issue default-free bonds
  - Risk-free in real terms only if price indexed and maturity equal to investor's holding period
- T-bills viewed as "the" risk-free asset
- Money market funds also considered risk-free in practice

Spread between 3-month CD and T-bill rates

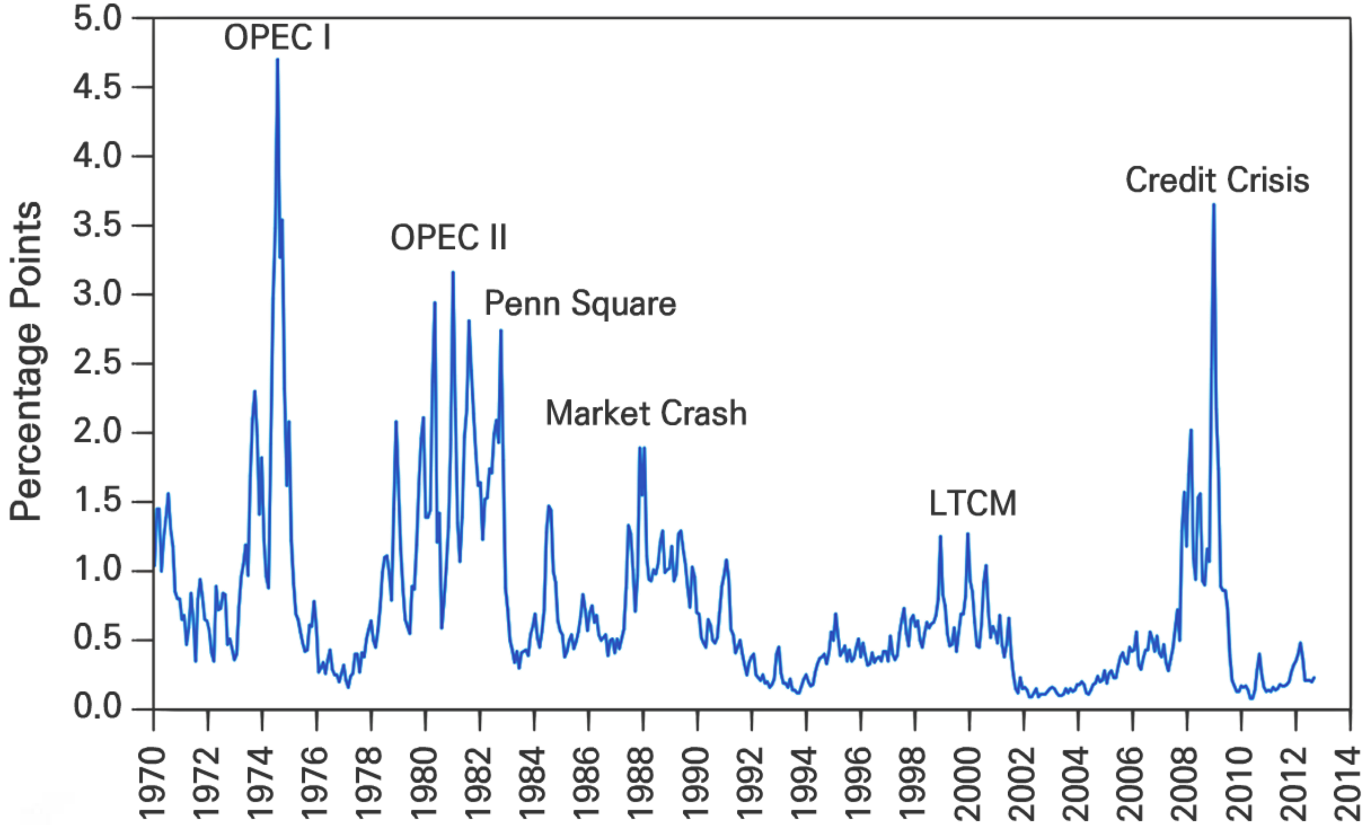

Figure: Spread between 3-month CD and T-bill rates

Portfolios of One Risky Asset and a Risk-Free Asset

- **It is possible to create a complete portfolio by splitting investment funds between safe and risky assets**
  - Let y = portion allocated to the risky portfolio, P
  - $(1-y)$ = portion to be invested in risk-free asset, F

Example using Ch.6.4 Numbers

$$r_f = 7\% \,, \sigma_{r_f} = 0\% \,, E(r_p) = 15\% \,, \sigma_p = 22\% \,, y = \% in p, \ (1-y) = \% in r_f$$

- The expected return on the complete portfolio is the risk−free rate plus the weight of P times the risk premium of P

$$E(r_c) = r_f + y\left[E(r_p) - r_f\right] = 7 + y(15-7) = (7+8y)\%$$

- The risk of the complete portfolio is the weight of P times the risk of P

$$\sigma_c = y\sigma_p = 22y\%$$

- Rearrange and substitute $y = \sigma_c/\sigma_p$

$$E(r_c) = r_f + \frac{\sigma_c}{\sigma_p}\left[E(r_p) - r_f\right] = 7 + \frac{8}{22}\sigma_c, \quad \text{Slope} = \frac{E(r_p) - r_f}{\sigma_p} = \frac{8}{22}$$

Investment Opportunity Set

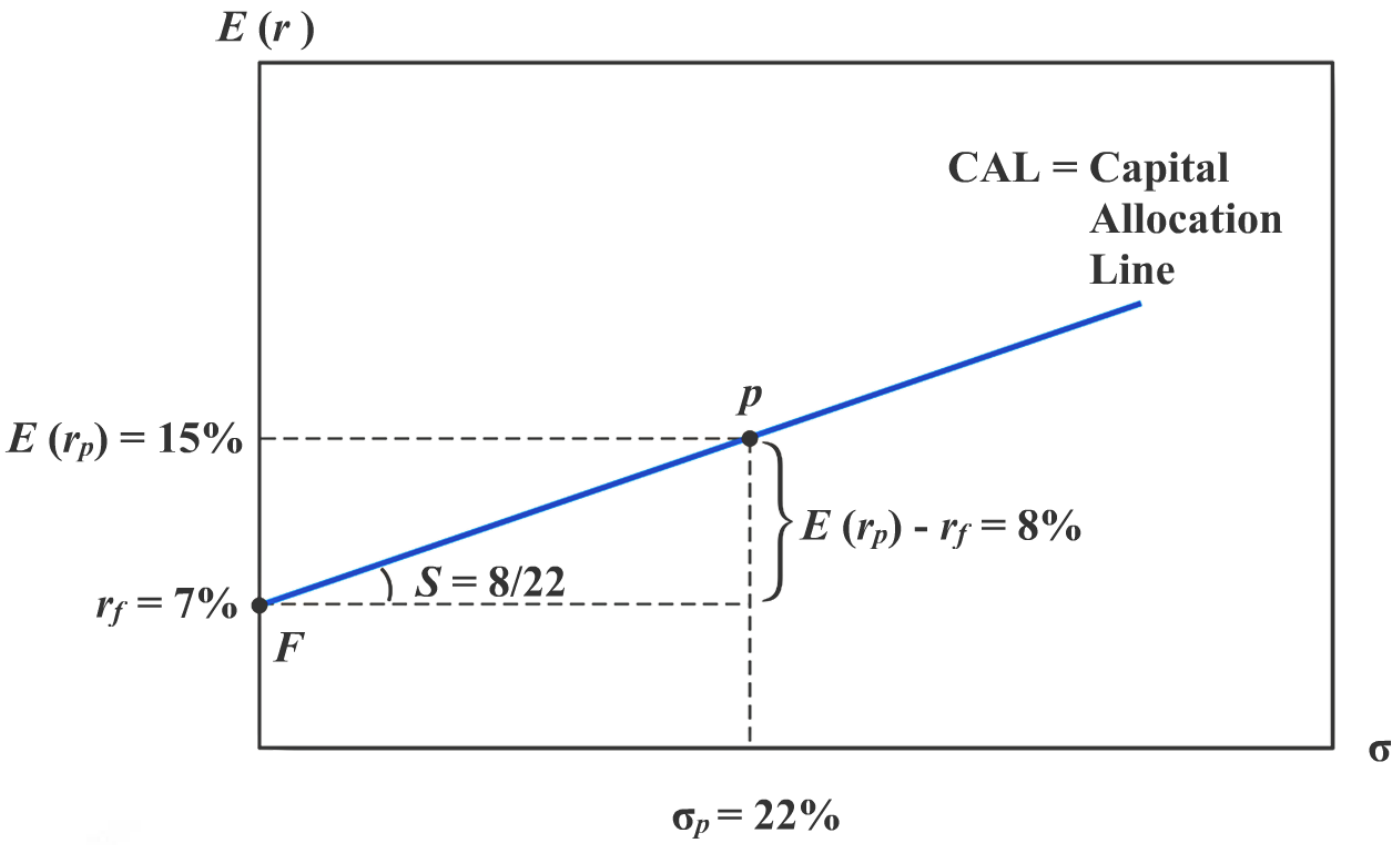

Figure : The investment opportunity set with a risky asset and a risk-free asset in the expected return-standard deviation plane

Application: Capital Allocation Line with Leverage

- **Lend at r_f =7% and borrow at r_f =9%**
  - Lending range slope = 8/22 = 0.36
  - Borrowing range slope = 6/22 = 0.27
- **CAL kinks at P**

Opportunity Set with Differential Borrowing and Lending Rates

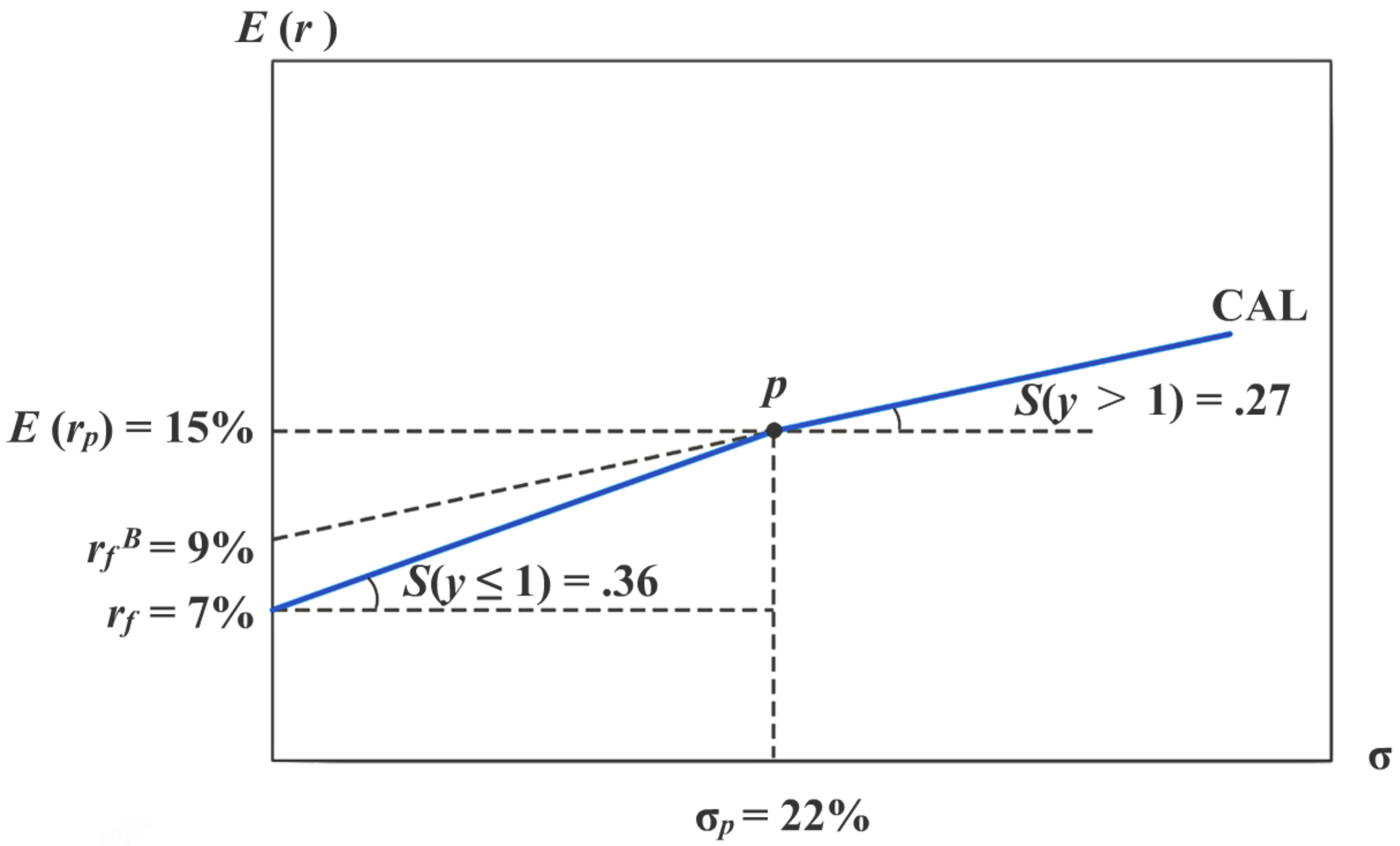

Figure: The opportunity set with differential borrowing and lending rates

Risk Tolerance and Asset Allocation

- The investor must choose one optimal portfolio, C, from the set of feasible choices.
  - Expected return of the complete portfolio

$$E(r_c) = r_f + y\left[E(r_p) - r_f\right]$$

  - Variance

$$\sigma_c^2 = y^2 \sigma_p^2$$

Utility Levels for Various Positions in Risky Assets (y) for an Investor with Risk Aversion A=4

(1)	(2)	(3)	(4)
y	$E(r_c)$	σ_c	U=E(r)-1/2Aσ ^{2}
0	.070	0	.0700
0.1	.078	.022	.0700
0.2	.086	.044	.0821
0.3	.094	.066	.0853
0.4	0.102	.088	.0865
0.5	.110	.110	.0858
0.6	.118	.132	.0832
0.7	.126	.154	.0786
0.8	.134	.176	.0720
0.9	.142	.198	.0636
1.0	.150	.220	.0532

Utility as a Function of Allocation to the Risky Asset, y

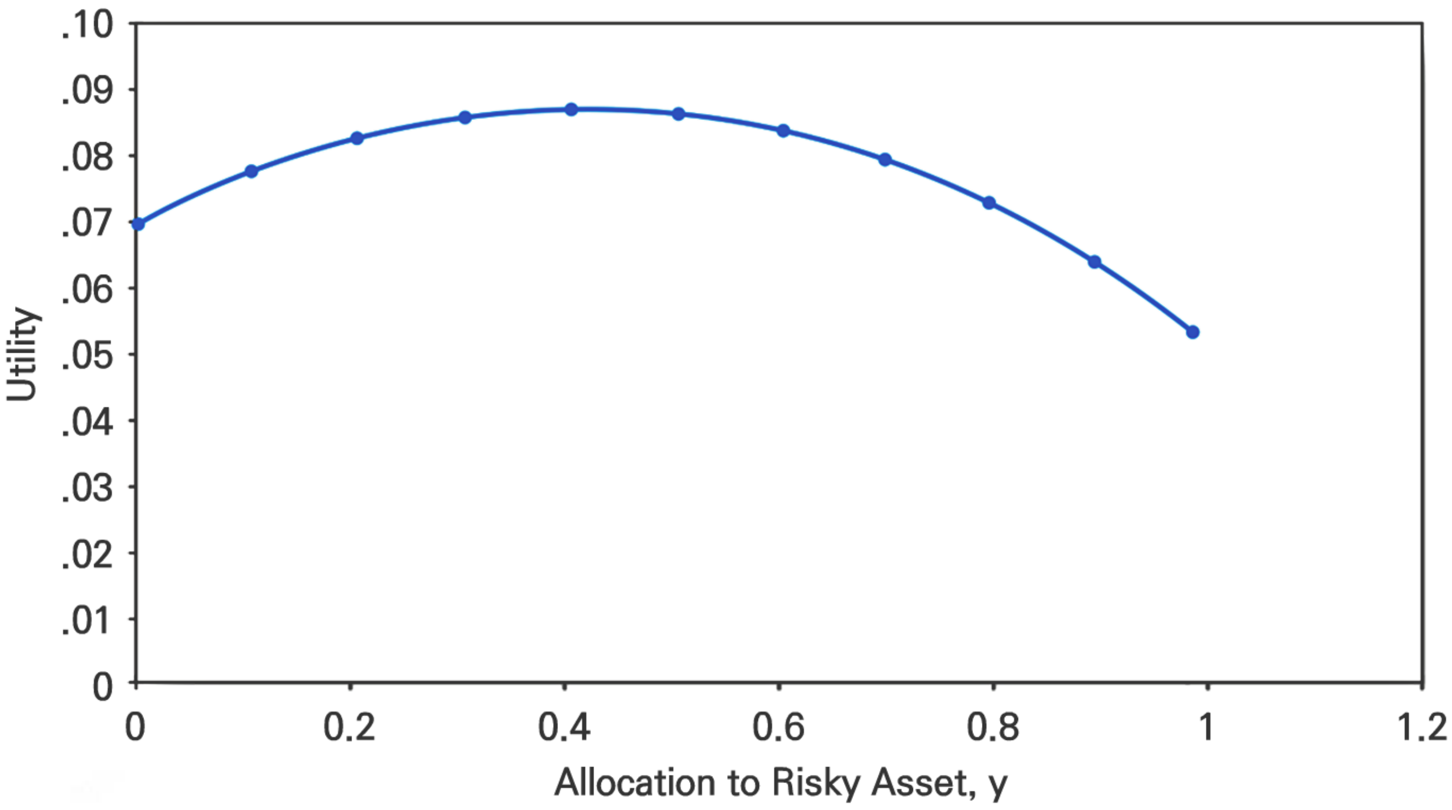

Figure : Utility as as function of allocation to the risky asset, y

Utility as a function of allocation to the risky asset, y

Problem

$$\max_y U = \max_y \left\{ r_f + y\left[E(r_p) - r_f\right] - \frac{1}{2} A y^2 \sigma_p^2 \right\}$$

- To find max, take derivative w.r.t. y and set equal to 0

$$\left[E(r_p) - r_f\right] - A y \sigma_p^2 = 0$$

- Solve for y

$$y^* = \frac{E(r_p) - r_f}{A\sigma_p^2}$$

Calculations of Utility Indifference Curves

	A=2		A=4	
σ	U=.05	U=.09	U=.05	U=.09
0	.500	.900	.050	.090
.05	.0525	.0925	.055	.095
.10	.0600	.1000	.070	.110
.15	.0725	.1125	.095	.135
.20	.0900	.1300	.130	.170
.25	.1125	.1525	.185	.215
.30	.1400	.1880	.230	.270
.35	.1725	.2125	295	.335
.40	.2100	.2500	.370	.410
.45	.2525	.2925	.455	.495
.50	.3000	.3400	.550	.590

Indifference Curves for U=0.05 and U=0.09 with A=2 and A=4

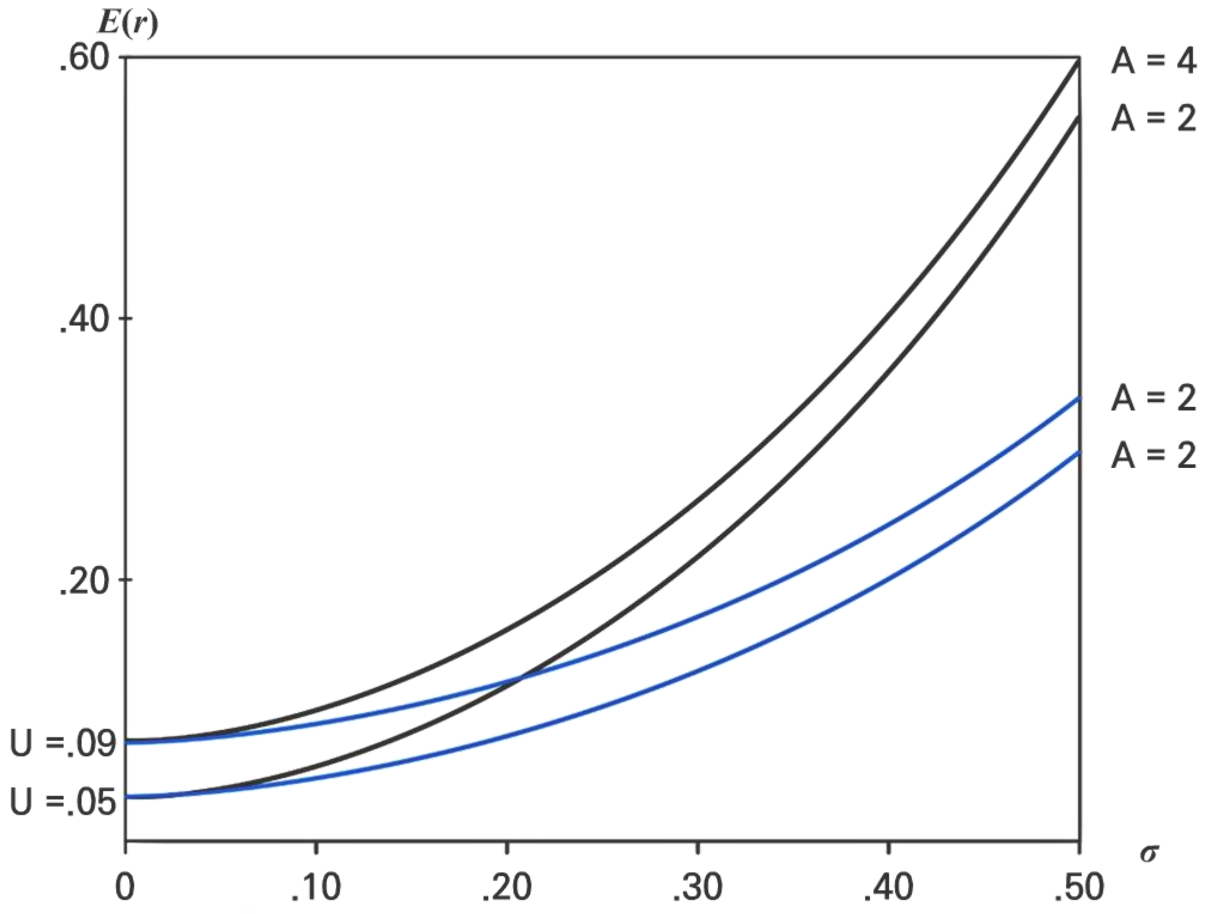

Figure : Indifference curves for U = .05 and U = .09 with A=2 and A=4

Indifference Curves for U=0.05 and U=0.09 with A=2 and A=4

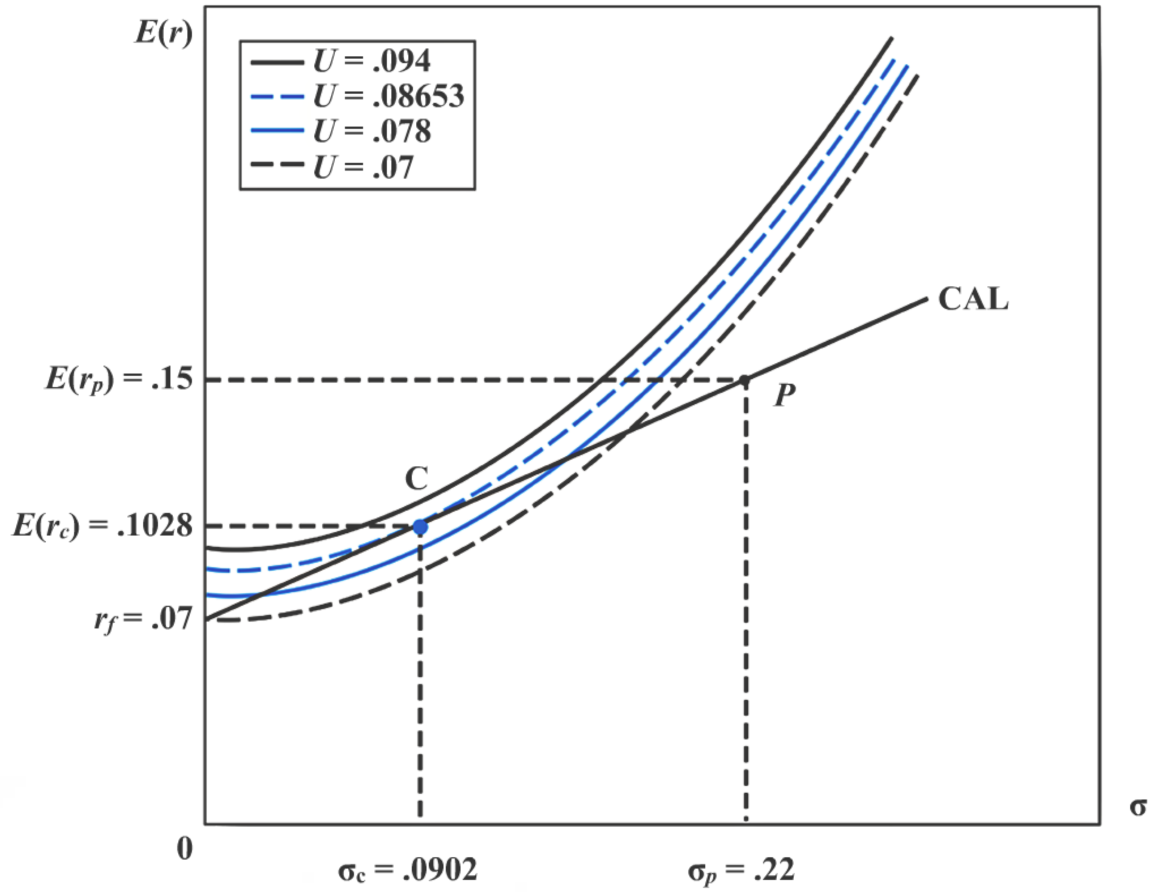

Figure: Finding the optinal complete portfolio by using indifference curves

Expected Returns on 4 Indifference Curves and the CAL

σ	U=.07	U=.078	U=.08653	U=.094	CAL
0	.0700	.0780	.0865	.0940	.0700
.02	.0708	.0788	.0873	.0948	.0773
.04	.0732	.0812	.0897	.0972	.0845
.06	.0772	.0852	.0937	.1012	.0918
.08	.0828	.0908	.0993	.1068	0991
.0902	.0863	.0943	.1028	.1103	.1028
.10	.0900	.0980	.1065	.1140	.1064
.12	.0988	.1068	.1153	.1228	.1136
.14	.1092	.1172	.1257	.1332	.1209
.18	.1348	.1428	.1513	.1588	.1355
.22	.1668	.1745	.1833	.1908	.1500
.26	.2052	.2132	.2217	.2292	.1645
.30	.2500	.2580	.2665	.2710	.1791

Caveat

- Above analysis implicitly assumes normality of the return
- VaR and ES asses exposure to extreme losses
- “Black Swan” events should concern investors (COVID-19 Pandemic, etc.)

Passive Strategies

The Capital Market Line

- The passive strategy avoids any direct or indirect security analysis
- Supply and demand forces may make such a strategy a reasonable choice for many investors
- A natural candidate for a passively held risky asset would be a well-diversified portfolio of common stocks such as the S&P 500
- The capital market line (CML) is the capital allocation line formed from 1-month T-bills and a broad index of common stocks (e.g. the S&P 500)