IBF Day 2 Exercise Linear A

Q1 i) A+B is defined.

$$A+B=\begin{bmatrix}1 & 1 & 5\\ 1 & 2 & 4\end{bmatrix}$$

Q1 ii) A+B is not defined.

Q2) Skipped.

Careless! 3B−A is possible.

Q3) 1 - Possible, 2 - Not possible ✗, 3 - Not ~~possi~~ possible,
4 - Possible, 5 - Not possible, 6 - Not possible.

Q4) $X^TY=\begin{bmatrix}-1\\-1\\1\end{bmatrix}\begin{bmatrix}0 & 1 & 2\end{bmatrix}=\begin{bmatrix}0 & -1 & -2\\ 0 & -1 & -2\\ 0 & 1 & 2\end{bmatrix}$

$XY^T=\begin{bmatrix}-1 & -1 & 1\end{bmatrix}\begin{bmatrix}0\\1\\2\end{bmatrix}=1$

got 3 cases NOT just 1.

Q5) $AB=BA$ if ($a=d$ and $bg=cf$) or ($f=g=0$) ✗

CTA behind.

where $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ and $B=\begin{bmatrix}e & f\\ g & h\end{bmatrix}$.

Since $f\neq g$ ~~and f~~ ~~then~~ and $a=d$, check if $bg=cf$.

$A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$, $B=\begin{bmatrix}1 & 2\\ 3 & k\end{bmatrix}$ => $2(3)=3(2)$ is satisfied,

then any value of k will make AB=BA.

Q6) $AB=\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}e & f\\ g & h\end{bmatrix}=\begin{bmatrix}ae+bg & af+bh\\ ce+dg & cf+dh\end{bmatrix}$, $AC=\begin{bmatrix}ax+by & ap+bq\\ cx+dy & cp+dq\end{bmatrix}$

$ax+by=ae+bg$ ~~constant~~.

Let $A=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$, $B=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$, $C=\begin{bmatrix}1 & 2\\ 5 & 6\end{bmatrix}$.

Q7) $A=\begin{bmatrix}1 & -1 & 2 & 0\\ 1 & 0 & 3 & 0\\ 0 & 0 & 3 & 0\\ 1 & 3 & 0 & 3\end{bmatrix}$

Q8) $AA = A \Rightarrow A^{-1}AA = A^{-1}A \Rightarrow A^{-1}AA = I \Rightarrow IA = I$

$A \Rightarrow A^{-1}$ does NOT exist unless $A = I$.

But since $A \neq I$, A^{-1} does not exist.

Multipy A by A to get A again.

Q9) $$C_{ik} = \sum_j A_{ij} B_{jk}$$

$$C_{12} = \sum_j A_{1j} B_{j2} = [1\ 2\ -1]\begin{bmatrix}6\\2\\0\end{bmatrix} = 10$$

$$C_{23} = \sum_j A_{2j} B_{j3} = [3\ 4\ 0]\begin{bmatrix}-2\\1\\0\end{bmatrix} = -2$$

Q10) 1- Possible ; 4- Possible ; 7- Not possible as BE is not possible.

2- Possible ; 5- Possible ;

3- Possible ; 6- Possible ;

CTA:

Q5) $A = \begin{bmatrix}1&2\\3&4\end{bmatrix}$, $B = \begin{bmatrix}1&2\\3&k\end{bmatrix}$. Let $A = \begin{bmatrix}a&b\\c&d\end{bmatrix}$, $B = \begin{bmatrix}e&f\\g&h\end{bmatrix}$, then $a = d$ but $e \neq h$.

Therefore, ~~condition~~ condition for $AB = BA$ is ($e = h$ and $bg = cf$) or ($b = c = 0$). Since $b \neq c$, ~~then~~ then $e = h$ and $bg = cf$ must be satisfied. X LP: Should go more general, then specific.

CTA: General Conditions $\Rightarrow$

I) $bg = cf$; II) $f(a-d) = b(e-h)$; III) $g(a-d) = c(e-h)$

Since, ~~a=e~~ $b = f$, $c = g$, then I is satisfied,

~~II: $0 = b(e-k)$ and III: $0 = c(e-k)$~~

~~but since $b \neq 0$ and $c \neq 0$, II and III combined yields:~~

~~$b(e-k) = c(e-k) \Rightarrow e-k = \frac{c}{b}(e-k)$~~

~~Since~~ ~~$\Rightarrow e-k = e-k \cdot \frac{c}{b} = 1 \Rightarrow$~~

~~II OR III implies: $b(e-k) = 0 \Rightarrow e-k = 0 \Rightarrow k = e = 1$.~~

Since $a \neq d$, then ~~$e \neq$~~ $e \neq h \Rightarrow k \neq 1$.

Then since $b = f$, by II: $a-d = e-h \Rightarrow -3 = 1-k \Rightarrow k = 4$.

and since $g = c$, by III: $a-d = e-h \Rightarrow k = 4$.