DigiPen Institute of Technology Singapore

IBF– Day 3 Exericise
Linear Algebra

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Name: ____

This exercise contains 10 pages (including this cover page) and 15 questions. Total of points is 110.
Good luck and Happy reading work!

Distribution of Marks

Question:	1	2	3	4	5	6	7	8
Points:	5	10	20	10	10	5	10	5
Score:								

Question:	9	10	11	12	13	14	15	Total
Points:	5	5	5	5	5	5	5	110
Score:								

1. (5 points) Show that the main diagonal of every skew symmetric matrix consists of only zeros.

$A = -A^T \Rightarrow a_{ij} = -a_{ji}$

For $i = j$, $a_{ii} = -a_{ii} \Rightarrow a_{ii} = 0 \ \forall i$

$\therefore A = -A^T \Rightarrow a_{ii} = 0 \ \forall i$

2. (10 points) Give an example of a matrix A such that $A^2 = I$ and yet $A \neq I$ and $A \neq -I$.

$AA = I \Rightarrow A^{-1}AA = A^{-1}I \Rightarrow IA = A^{-1}I \Rightarrow A = A^{-1}$

$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $A^{-1} = \frac{1}{-1}\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

3. (20 points) Let

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$$

Find A^{-1} if possible. If it doesn't exist, explain why. Verify your result with Python

$\det A = ad - bc = 2(3) - (-1)(1)$

$= 6 + 1 = 7 \neq 0$

$\therefore A^{-1}$ exists.

$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$

4. (10 points) Let

$$A = \begin{bmatrix} 0 & 1 \\ 5 & 3 \end{bmatrix}$$

Find A^{-1} if possible. If it doesn't exist, explain why. Verify your result with Python

$\det(A) = 0(3) - (1)(5) = -5 \neq 0$

$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{-1}{5} \begin{bmatrix} 3 & -1 \\ -5 & 0 \end{bmatrix}.$

5. (10 points) Let

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}$$

Find A^{-1} if possible. If it doesn't exist, explain why. Verify your result with Python

$\det(A) = -3$

$A^{-1} = \frac{-1}{3} \begin{bmatrix} 0 & -1 \\ -3 & 2 \end{bmatrix}.$

6. (5 points) Let

$$A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$$

Find A^{-1} if possible. If it doesn't exist, explain why. Verify your result with Python

$\det(A) = 2(2) - 4(1) = 0$

A^{-1} does not exist.

7. (10 points) Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 1 & 0 & 2 \end{bmatrix} \qquad \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

Find A^{-1} if possible. If it doesn't exist, explain why. Verify your result with Python

$$\det(A) = -2[2(2) - 3 \cdot 1] + [1 \cdot 2 - 1 \cdot 3] \quad 4 \quad (0)$$

$$= -2(4-3) + (2-3) = -2(1) + (-1) = -3 \qquad -1$$

$$\mathrm{Cof}(A) = \begin{bmatrix} 2 & 0 & -1 \\ 1 & -1 & -2 \\ 5 & 2 & -3 \end{bmatrix}, \quad \mathrm{Adj}(A) = \begin{bmatrix} 2 & 1 & 5 \\ 0 & -1 & 2 \\ -1 & -2 & -3 \end{bmatrix}$$

$-4, -1, 2$ $\qquad$ $-4, -1, 2$

$$A^{-1} = -\frac{1}{3} \begin{bmatrix} 2 & 4 & 5 \\ 0 & -1 & 3 \\ -1 & -2 & -3 \end{bmatrix} \qquad A^{-1} = -\begin{bmatrix} 2 & -4 & 5 \\ 0 & -1 & 2 \\ -1 & 2 & -3 \end{bmatrix}$$

8. (5 points) Let

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 3 & 4 \\ 1 & 0 & 2 \end{bmatrix} \qquad \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

Find A^{-1} if possible. If it doesn't exist, explain why. Verify your result with Python.

$$\det(A) = 3[(1)(2) - (3)(1)] = -3.$$

$$\mathrm{Cof}(A) = \begin{bmatrix} 6 & 0 & -3 \\ 1 & -3 & 0 \\ -9 & 2 & 3 \end{bmatrix} \qquad \mathrm{cof}(A) = \begin{bmatrix} 6 & 0 & -3 \\ 0 & -1 & 0 \\ -9 & 2 & 3 \end{bmatrix}$$

LP: $A^{-1} = \frac{1}{|A|} \mathrm{adj}(A)$

$\mathrm{adj}(A) = [\mathrm{Cof}(A)]^T$, transpose.

$$A^{-1} = \frac{1}{\det(A)} (\mathrm{Cof}(A)) = -\frac{1}{3} \begin{bmatrix} 6 & 0 & -3 \\ 1 & -3 & 0 \\ -9 & 2 & 3 \end{bmatrix}$$

TA: $\mathrm{adj}(A) = \begin{bmatrix} 6 & 0 & -9 \\ 0 & -1 & 2 \\ -3 & 0 & 3 \end{bmatrix}$

$$A^{-1} = \begin{bmatrix} -2 & 0 & 3 \\ 0 & -\frac{1}{3} & -\frac{2}{3} \\ 1 & 0 & -1 \end{bmatrix}.$$

9. (5 points) Let

$$A = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 1 & 1 & 2 & 0 \\ 2 & 1 & -3 & 2 \\ 1 & 2 & 1 & 2 \end{bmatrix}$$

Find A^{-1} with Python.

10. (5 points) Using the inverse of the matrix, find the solution to the systems:

$$\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Now give the solution in terms of a and b to

$$\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = -\frac{1}{2} \begin{bmatrix} 1 & -4 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = -\frac{1}{2} \begin{bmatrix} -7 \\ 3 \end{bmatrix}.$$

$$\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{2} \begin{bmatrix} 1 & -4 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

11. (5 points) Using the inverse of the matrix, find the solution to the systems:

$$\begin{bmatrix}1 & 0 & 3\\ 2 & 3 & 4\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}1\\ 0\\ 1\end{bmatrix}$$

Now give the solution in terms of a, b, c to

$$\begin{bmatrix}1 & 0 & 3\\ 2 & 3 & 4\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}a\\ b\\ c\end{bmatrix}$$

12. (5 points) Let $A = \begin{bmatrix}1 & 2 & 4\\ 0 & 1 & 3\\ -2 & 5 & 1\end{bmatrix}$ Find the following.

1. $\text{minor}(A)_1 1$
2. $\text{minor}(A)_2 1$
3. $\text{minor}(A)_3 2$
4. $\text{cof}(A)_1 1$
5. $\text{cof}(A)_2 1$
6. $\text{cof}(A)_3 2$

LP: $\begin{bmatrix}+ & - & +\\ - & + & -\\ + & - & +\end{bmatrix}$

21 32

1) $A_{\setminus 1 \setminus 1} = \begin{bmatrix}1 & 3\\ 5 & 1\end{bmatrix}$

2) $A_{\setminus 2 \setminus 1} = \begin{bmatrix}2 & 4\\ 5 & 1\end{bmatrix}$

3) $A_{\setminus 3 \setminus 2} = \begin{bmatrix}1 & 2\\ -2 & 5\end{bmatrix}$

4) $\text{Cof}(A_{11}) = 1 - 15 = -14$

5) $\text{Cof}(A_{21}) = 2 - 20 = -18$ x 18

6) $\text{Cof}(A_{32}) = 5 - (-2)(2) = 5 + 4 = 9$ x -9

13. (5 points) Find the determinants of the following matrices

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

1.
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 2 \\ 0 & 9 & 8 \end{bmatrix}$$

1) $\det(A) =$

$1[(2)(8)-(2)(9)]$

$-3[2(8)-3(9)]$

$= -2 - 3[16-27]$

$= -2 - 3(9)$ ~~X~~ -11

$= -2 - 27 = \cancel{-29}.$

$= -2 - 3(-11) = -2 + 33$

$= 31.$

2.
$$B = \begin{bmatrix} 4 & 3 & 2 \\ 1 & 7 & 8 \\ 3 & -9 & 3 \end{bmatrix}$$

3.
$$C = \begin{bmatrix} 5 & -1 & 3 \\ 4 & 1 & 1 \\ 0 & 4 & -2 \end{bmatrix}$$

14. (5 points) Construct a 2 by 2 matrices A and B to show that

$$det A det B = det(AB)$$

$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 0 & 7 \end{bmatrix}$, $AB = \begin{bmatrix} 8 & 0 \\ 0 & 21 \end{bmatrix}$

$\det(A) = 6$, $\det(B) = 28$, $\det(AB) = 8 \times 21$

15. (5 points) Is it true that $det(A+B) = det(A) + det(B)$? Give an counter example or explain why.

Not true.

$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$, $A+B = \begin{bmatrix} 3 & 0 \\ 0 & 7 \end{bmatrix}$

$det(A) = 6$, $det(B) = 4$, $det(A+B) = 21$

$21 \neq 6 \times 4$.

This page is intentionally left blank to accommodate work that wouldn't fit elsewhere and/or scratch work.