DigiPen Institute of Technology Singapore

IBF– Exam
Vector Calculus

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Name: _____

This quiz contains 7 pages (including this cover page) and 10 questions. Total of points is 100. Good luck and Happy reading work!

Distribution of Marks

Question:	1	2	3	4	5	6
Points:	10	10	10	10	10	10
Score:						
Question:	7	8	9	10		Total
Points:	10	10	10	10		100
Score:						

1. (10 points) Please give an example of one application of Calculus in Machine Learning (including Deep Learning). What's the advantage of using such technology.

2. (10 points) Find the derivative of the single variable functions below and list all the rules that are applied here.

   1. $y = g(x)^3 \sqrt[3]{f(x)}$
   2. $y = e^{g(f(x))}$

3. (10 points) Find the derivative of the single variable functions below and show all your work.

    1. $y = \dfrac{e^{\cos^2(x)}}{\ln(x) + \sin x}$

    2. $y = e^{\tan^3 x \cos^2 x}$

4. (10 points) Find the Taylor polynomial of degree 4 for

$$f(x) = \ln(3 + 4x) \qquad \text{about} \qquad x = 0$$

5. (10 points) Consider the quadratic function defined by $Q(x, y, z) = x^2 + 5y^2 + 4xy - 2yz$. Compute the Hessian matrix.

6. (10 points) Describe the Backropagation algorithm, how does it work?

7. (10 points) Find all the 1st order partial derivatives for

$$f(x, y, z) = \frac{1}{(x^2 + y^2 + z^2)^n}$$

8. (10 points) Let $f(x, y) = e^{xy}$, with $x = r \cos \theta$ and $y = r \sin \theta$. Compute the partial derivatives $\frac{\partial f}{\partial r}$ and $\frac{\partial f}{\partial \theta}$ in two ways:

1. Use the chain rule on $f = f(x(r, \theta), y(r, \theta))$;

2. Eliminate x and y in favor of r and θ and compute the partial derivative directly.

9. (10 points) Let $f(x) = \sqrt{1 + 2x}$ and use the linearization to approximate $f(4.01)$ and $f(3.99)$.

10. (10 points) Find the linearization $L(x, y)$ for the function defined by

$$f(x, y) = \frac{x}{x^2 + y^2}$$

at the point $(1, 2)$. Then use the linearization to estimate the value of $f(0.8, 2.3)$

This page is intentionally left blank to accommodate work that wouldn't fit elsewhere and/or scratch work.