Polynomial and Rational Functions

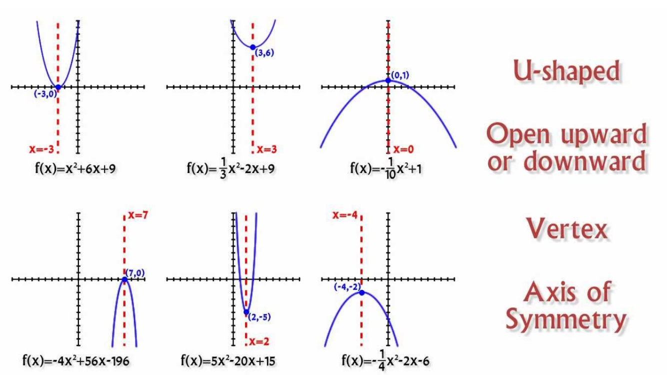
1 Quadratic Functions and Models

1.1 Quadratic Functions

Definition: A **quadratic function** in terms of x is a function of the form

$$f(x) = ax^2 + bx + c$$

where a, b, c are real numbers, and $a \neq 0$



1.2 Standard Form of a Quadratic Function

The equation of a parabola with vertex at (h, k), where the axis of symmetry is the vertical line $x = h$, can be written as $f(x) = a(x-h)^2 + k$, where $a \neq 0$.

- If $a > 0$, then the parabola opens upward and its vertex is a minimum
- If $a < 0$, then the parabola opens downward and its vertex is a maximum

The **vertex** of a quadratic function $f(x) = ax^2+bx+c$ is given by $(-\frac{b}{2a}, f(-\frac{b}{2a}))$, and the **axis of symmetry** is the vertical line $x = -\frac{b}{2a}$.

Discriminant: The discriminant D of $f(x) = ax^2 + bx + c$ is given by $D = b^2 - 4ac$

- If $D > 0$, then $f(x)$ has two x-intercepts
- If $D = 0$, then $f(x)$ has one x-intercepts
- If $D < 0$, then $f(x)$ has no x-intercepts

2 Rational Functions

Definition: A **rational function** $r(x)$ is a quotient of polynomials,

$$r(x) = \frac{f(x)}{g(x)}$$

, where f and g are polynomials such that $g(x) \neq 0$.

Definition: An **asymptote** is a line that a graph is heading toward and approaches more and more closely. A **horizontal asymptote** is an asymptote that is a horizontal line. A **vertical asymptote** is an asymptote that is a vertical line.

Steps for Writing the Equation of a Rational Function's Vertical Asymptote(s)

1. Write the function in simplest form
2. Set the polynomial in the denominator equal to 0 and solve.

Horizontal Asymptotes: If $r(x) = \frac{f(x)}{g(x)}$ is a rational function in simplest form where m is the degree of $f(x)$ and n is the degree of $g(x)$, then

- $m < n$: $r(x)$ has a horizontal asymptote at $y = 0$
- $m = n$: $r(x)$ has a horizontal asymptote at $y = \frac{\textit{leading coefficient of } f(x)}{\textit{leading coefficient of } g(x)}$
- $m > m$: $r(x)$ has no horizontal asymptote.