Trigonometric Identity

1 Basic Identities

An **identity** is an equation that is always **true**.

Quotient Identity:
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Cofunction Identity:

$\sin(\pi/2 - \theta) = \cos\theta$ $\qquad$ $\cos(\pi/2 - \theta) = \sin\theta$ $\qquad$ $\tan(\pi/2 - \theta) = \cot\theta$

$\csc(\pi/2 - \theta) = \sec\theta$ $\qquad$ $\sec(\pi/2 - \theta) = \csc\theta$ $\qquad$ $\cot(\pi/2 - \theta) = \tan\theta$

2 Solving Trig Equations

Steps for Solving trig equations:

- Isolate the trig function on one side of the equation

- Identity all solutions within one period

- Add multiple of the period to each solutions to write the general form of all solutions

3 More Identities

Sum and Difference Formula:

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Double Angle Formula:

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta = 1 - 2 \sin^2 \theta = 2 \cos^2 \theta - 1$$

$$\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Power Reducing Formula:

$$\sin^2 \theta = \frac{1 - \cos(2\theta)}{2}$$

$$\cos^2 \theta = \frac{1 + \cos(2\theta)}{2}$$

$$\tan^2 \theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

Power Reducing Formula:

$$\sin x \sin y = 1/2(\cos(x - y) - \cos(x + y))$$

$$\cos x \cos y = 1/2(\cos(x - y) + \cos(x + y))$$

$$\sin x \cos y = 1/2(\sin(x + y) + \sin(x - y))$$

$$\cos x \sin y = 1/2(\sin(x + y) - \sin(x - y))$$

Sum Product Formula:

$$\sin x + \sin y = 2\sin(\frac{x + y}{2})\cos(\frac{x - y}{2})$$

$$\sin x - \sin y = 2\cos(\frac{x + y}{2})\sin(\frac{x - y}{2})$$

$$\cos x + \cos y = 2\cos(\frac{x + y}{2})\cos(\frac{x - y}{2})$$

$$\cos x - \cos y = -2\sin(\frac{x + y}{2})\sin(\frac{x - y}{2})$$