Exponential and Logarithmic Functions

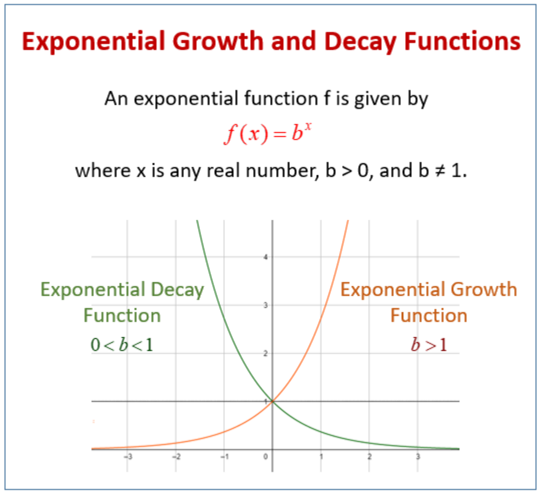
1 Exponential Functions

1.1 Exponential Functions

Definition: A function of the form $f(x) = b^x$ where b is a real number such that $b > 0$ and $b \neq 1$ is an **exponential function**.

1.2 Graphs of Exponential Functions

The graph of the exponential function $f(x) = b^x$ has a horizontal asymptote at $y = 0$, and passes through $(0, 1)$. The domain of the function is all real numbers, and the range is $(0, \infty)$.



1.3 Applications of the Exponential Function

Compound Interest Formula:

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

Compound interest is compounded m times per year.

- P = the principle amount in the account
- t = the length of time that the money is invested, in years
- A = the balance in the account after t years
- r = the annual interest rate
- m = the number of times per year that the interest is compounded

1.4 Natural Exponential Functions

Definition: The **natural exponential function** is the exponential function $f(x) = e^x$.

Note: The number $e = 2.7182818284...$, and e is a irrational number.

2 Logarithmic Functions

2.1 Logarithmic Functions

Definition: The function $f(x) = \log_b x$ is called the **logarithmic function with base b**. If b is a positive number such that $b \neq 1$, then $y = log_b x$ or $b^y = x$

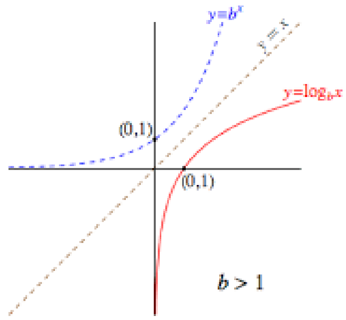
Exponential Form Equation	Logarithmic Form Equation
$b^y = x$	$y = \log_b x$

2.2 Properties for Logarithmic Functions

Property	Exponential Form
$\log_b 1 = 0$	$b^0 = 1$
$\log_b b = 1$	$b^1 = 1$
$\log_b b^x = x$	$b^x = b^x$
$b^{\log_b x} = x$	$y = \log_b x$ and $b^y = x$

2.3 Graph of Logarithmic Functions

Since $y = b^x$ and $y = \log_b x$ are inverse, their graphs are reflections over the line $y = x$.



2.4 Common Logs

Definition: A **common log** is a logarithm where the base is 10.

$$\log x = \log_1 0x$$

The common log function $y = \log x$ is the inverse function of $y = 10^x$

2.5 Natural Logs

Definition: A **natural log** is a logarithm where the base is the number 10.

$$\ln x = \log_e x$$

The natural log function $y = \ln x$ is the inverse function of $y = e^x$

Some special values:

- $\log 1 = 0$, $\ln 1 = 0$
- $\log 10 = 1$, $\ln e = 1$
- $\log 10^x = x$, $\ln e^x = x$

- $10^{\log x} = x$, $e^{\ln x} = x$

2.6 More Properties

	Product Property	Quotient Property	Power Property
Log with base b	$\log_b(xy) = \log_b x + \log_b y$	$\log_b(\frac{x}{y}) = \log_b x - \log_b y$	$\log_b x^y = y \log_b x$
Common Log	$\log(xy) = \log x + \log y$	$\log(\frac{x}{y}) = \log x - \log y$	$\log x^y = y \log x$
Natural Log	$\ln(xy) = \ln x + \ln y$	$\ln(\frac{x}{y}) = \ln x - \ln y$	$\ln x^y = y \ln x$

3 Exponential and Logarithmic Equations

3.1 One to One Property

One to One Property (Exponential): If $b > 0$ and $b \neq 1$, then

$$b^x = b^y \text{if and only if} x = y$$

One to One Property (Log): If $b > 0$ and $b \neq 1$, then for all x and y where $\log_b x$ and $\log_b y$ are defined,

$$\log_b x = \log_b y \text{if and only if} x = y$$

3.2 Steps for Solving an Exponential Equation by Using the One to One Property

- Write the equation in the form $b^x = b^y$.
- Set the exponents equal to each other.
- Solve the equation made form the exponents.

Example:

$$2^x = \frac{1}{32}$$

3.3 Steps for Solving an Exponential Equation by Using Logs

- Write the equation in the form $a^x = b^y$.
- Take the common log of each side
- Use the Power Property of logs to write the exponents as coefficients of the logs
- Use the Product or Quotient Properties of logs to expand the logs as needed
- Solve for x

Example:

$$8^x - 5^{x+9} = 0$$

3.4 Steps for Solving a Natural Exponential Equation by Using Natural Logs

- Isolate e^x on one side of the equation
- Take the natural log of each side
- Use the Power Property of logs to write the exponents as coefficients of the natural logs
- Use the fact that $\ln e = 1$ to simplify
- Solve for x

Example:

$$3 - 2e^{5-3x} = 2$$

3.5 Steps for Solving a Log Function

- Isolate the log on one side of the equation
- Write the equation in exponential form
- Solve the resulting equation

Example:

$$\ln x^3 = 15$$