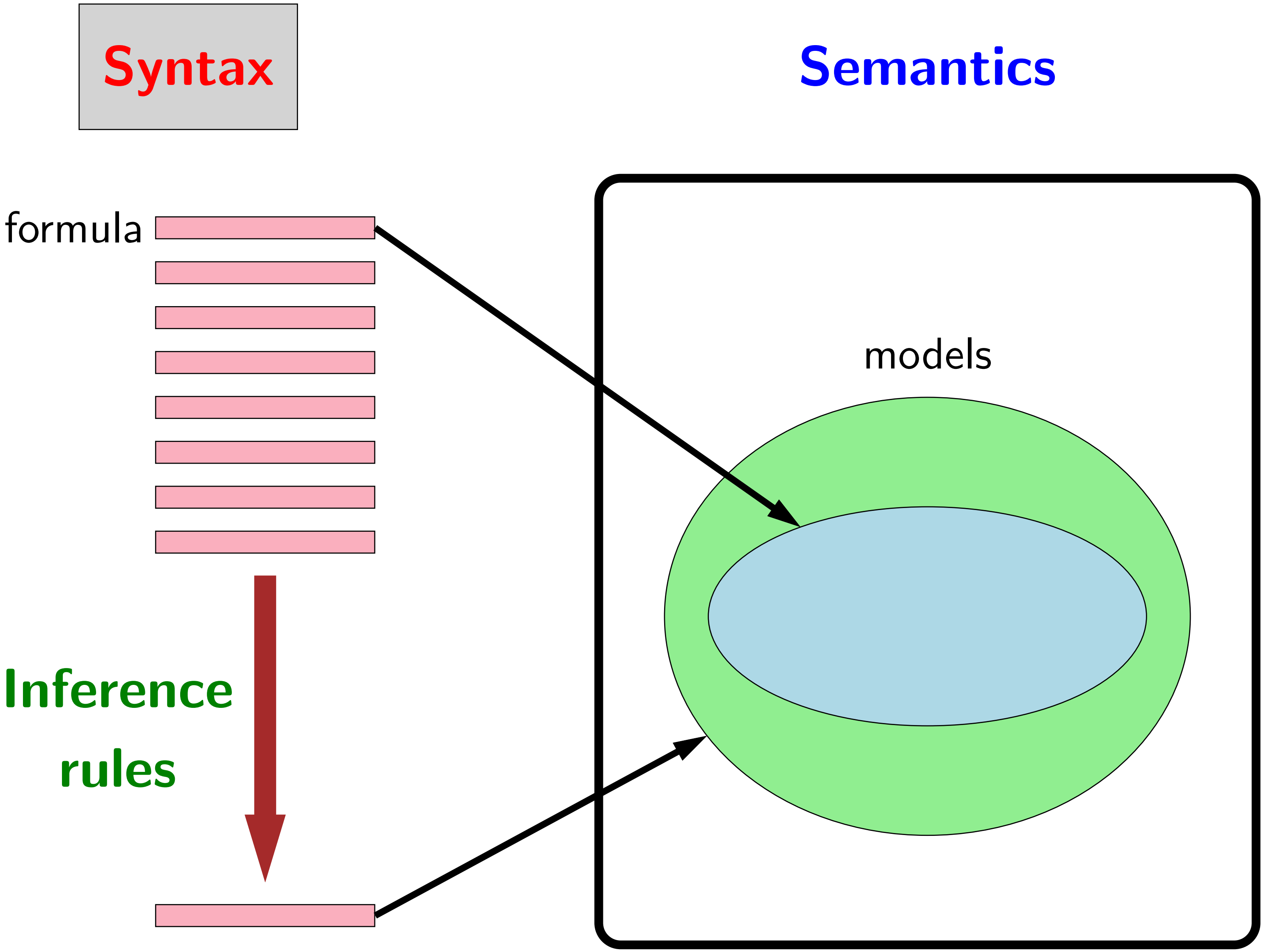
Logic: propositional logic syntax

Propositional logic



- We begin with the syntax of propositional logic: what are the allowable formulas?

Syntax of propositional logic

Propositional symbols (atomic formulas): A, B, C

Logical connectives: $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$

Build up formulas recursively—if f and g are formulas, so are the following:

- Negation: $\neg f$
- Conjunction: $f \wedge g$
- Disjunction: $f \vee g$
- Implication: $f \rightarrow g$
- Biconditional: $f \leftrightarrow g$

- The building blocks of the syntax are the propositional symbols and connectives. The set of propositional symbols can be anything (e.g., A, Wet, etc.), but the set of connectives is fixed to these five.
- All the propositional symbols are **atomic formulas** (also called atoms). We can **recursively** create larger formulas by combining smaller formulas using connectives.

Syntax of propositional logic

- Formula: A
- Formula: $\neg A$
- Formula: $\neg B \rightarrow C$
- Formula: $\neg A \wedge (\neg B \rightarrow C) \vee (\neg B \vee D)$
- Formula: $\neg\neg A$
- Non-formula: $A\neg B$
- Non-formula: $A + B$

- Here are some examples of valid and invalid propositional formulas.

Syntax of propositional logic

Key idea: syntax provides symbols

Formulas by themselves are just symbols (syntax).

No meaning yet (semantics)!

- It's important to remember that whenever we talk about syntax, we're just talking about symbols; we're not actually talking about what they mean — that's the role of semantics. Of course it will be difficult to ignore the semantics for propositional logic completely because you already have a working knowledge of what the symbols mean.