Basic Algebra Review

1 Real Numbers

Anything that can be quantified or measured can be described by a **Real Number**. Two non overlapping subsets of **Real Number** are the **rational numbers** and the **irrational numbers**. All rational numbers can be expressed as either a terminating decimal or as a repeating decimal. Any real number that is equivalent to a nonterminating, nonrepeating decimal is an **irrational number**.
Natural numbers are made up of the counting numbers starting as 1,2,3,....
The set includes the natural number and 0 is called the **Whole numbers**.
The set of numbers called the **integers** is the subset of the real numbers that includes all positive whole numbers, negative whole numbers, and 0.

2 Integer Exponents

If a is a nonzero real number and n is an integral, then $a^n = a*a*a*a*a...*a$, where a ias a factor n times. In the exponential expression a^n, a is the **base**, and n is the **exponent**

Product of Powers Property: If a is a nonzero real number and m and n are integers, then

$$a^m \cdot a^n = a^{m+n}$$

.

Power of a Power Property: If a is a nonzero real number and m and n are integers, then

$$(a^m)^n = a^{mn}$$

.

Power of a Product Property: If a is a nonzero real number and n is an integer, then

$$(ab)^n = a^n b^n$$

.

Quotient of Powers Property: If a is a nonzero real number and m and n are integers, then

$$\frac{a^m}{a^n} = a^{m-n}$$

.

Negative Exponent Property: If a is a nonzero real number and m is an integer, then

$$a^{-m} = \frac{1}{a^m}$$

.

Zero Exponent Property: If a is a nonzero real number, then

$$a^0 = 1$$

.

Power of a Quotient Property: If a and b are a nonzero real numbers and n is an integer, then

$$(\frac{a}{b})^n = \frac{a^n}{b^n}$$

.

3 Rational Exponents and Radicals

Definition: If a and b are real numbers and n is an integer greater than or equal to 2 such that $a = b^n$, then b is an nth root of a.

Principal nth Root of a Number: The principal nth root of a, denoted by the **radical** $\sqrt[n]{a}$, is the nth root of a with the same sign as a. The positive integer n is the radical's **index**, and the number a is the radicand.

Properties of Radicals:

$$\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Rational Exponents:

$$a^{1/n} = \sqrt[n]{a}$$

$$a^{m/n} = (\sqrt[n]{a})^m$$

$$a^{n/m} = (\sqrt[n]{a^m})$$

4 Polynomials

Definition: A **Polynomial** in terms of x is an expression of the form $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, where $a_0, a_1, \cdots, a_n$ are real numbers and n is a non negative integers.

Formulas for Special Products of Binomials

Sum and Difference of Same Terms	$(x-y)(x+y) = x^2 - y^2$
Square of a Sum	$(x+y)^2 = x^2 + 2xy + y^2$
Square of a Difference	$(x-y)^2 = x^2 - 2xy + y^2$
Cube of a Sum	$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
Cube of a difference	$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

5 Factoring

Factoring $ax^2 + bx + c$ when $a = 1$: If a, b, c, m and n are numbers such that $a = 1$, $mn = c$, and $m + n = b$, then

$$ax^2 + bx + c = (x + m)(x + n)$$

Factoring $ax^2 + bx + c$ when $a \neq 1$: If a, b, c, m and n are numbers such that $pq = a$, $mn = c$, and $qm + pn = b$, then

$$ax^2 + bx + c = (px + m)(qx + n)$$

6 Rational Expressions

Definition: A fractional expression where the numerator and denominator are polynomials is called a **rational expression**.

Steps for Simplifying a Rational Expression

- Factor the polynomials in the numerator and denominator completely.
- Remove all factors common to the numerator and denominator.
- Write the remaining factors and multiply.
- State the domain of the original expression.

Simplifying Complex Fractions

- **Method 1: Multiply by the Reciprocal**: Write the fraction as division, then multiply the dividend by the reciprocal of the divisor.
- **Method 2: Use the LCM**: Find the LCM of the denominators, then multiply the expression by the $\frac{LCM}{LCM}$.

7 Solving Linear and Quadratic Equations

Cross- Product Property: If A, B, C, and D are algebraic expressions such that $B \neq 0$ and $D \neq 0$ and $\frac{A}{B} = \frac{C}{D}$, then $AD = BC$.

Steps for Solving a Quadratic Equation by Factoring

- Write the equation in general form, $ax^2 + bx + c = 0$
- Factor the quadratic expression
- Set each factor equal to 0, and solve each equation

The Quadratic Formula: The solution of a quadratic equation of the form $ax^2 + bx + c = 0$ are given by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Steps for Solving a Quadratic Equation by Using the Quadratic Formula

- Write the equation in general form, $ax^2 + bx + c = 0$
- Identify the values of a,b, and c from the general form equation.
- Substitute the values of a,b, and c into the Quadratic Formula, and simplify starting with the radicand.

8 Other Types of Equations

Steps for Solving a Radical equation with One Radical Expression

- Isolate the radical expression on one side of the equation
- Apply a power equal to the radical's index to each side of the equation

- Solve the resulting equation
- Check for extraneous solution

Steps for Solving a Radical equation with two Radical Expression and Additional Terms

- Isolate one of the radical expressions on one side of the equation
- Square both sides of the equation and simplify
- Isolate the remaining radical expression on one side of the equation
- Square both sides of the equation and simplify
- Solve the resulting equation
- Check for extraneous solutions

Steps for Solving an Absolute Value Equation

- Isolate an absolute value expression on one side of the equation: $|A| = B$
- Write the equation as the two equivalent equations $A = B$ and $A = -B$
- Solve each equation
- Check for extraneous solution

9 Inequalities

Absolute Value Inequalities: If A and B are some numbers or algebraic expression such that

- $|A| < B$, then $-B < A < B$
- $|A| \leq B$, then $-B \leq A \leq B$
- $|A| > B$, then $A > B$ or $A < -B$
- $|A| < B$, then $A \geq B$ or $A \leq -B$

Steps for Solving a Quadratic Inequality

- Write the inequality in general form and then find the solutions of the related equation.
- Begin a sign chart: divide a number line into test intervals where the endpoints are the solutions of the related equation
- Complete the sign chart: Substitute a value from each test interval into the quadratic expression, and note the sign of the resulting value
- Identity the test intervals where the inequality is satisfied. State the solution intervals, including the solutions of the related equation when the inequality symbols is $\leq$ or $\geq$