# Propositional Logic:
# Methods of Proof (Part II)

# You will be expected to know

- Basic definitions
  - Inference, derive, sound, complete
- Conjunctive Normal Form (CNF)
  - Convert a Boolean formula to CNF
- Do a short resolution proof
- Horn Clauses
- ~~Do a short forward-chaining proof~~
- ~~Do a short backward-chaining proof~~
- ~~Model checking with backtracking search~~
- ~~Model checking with local search~~

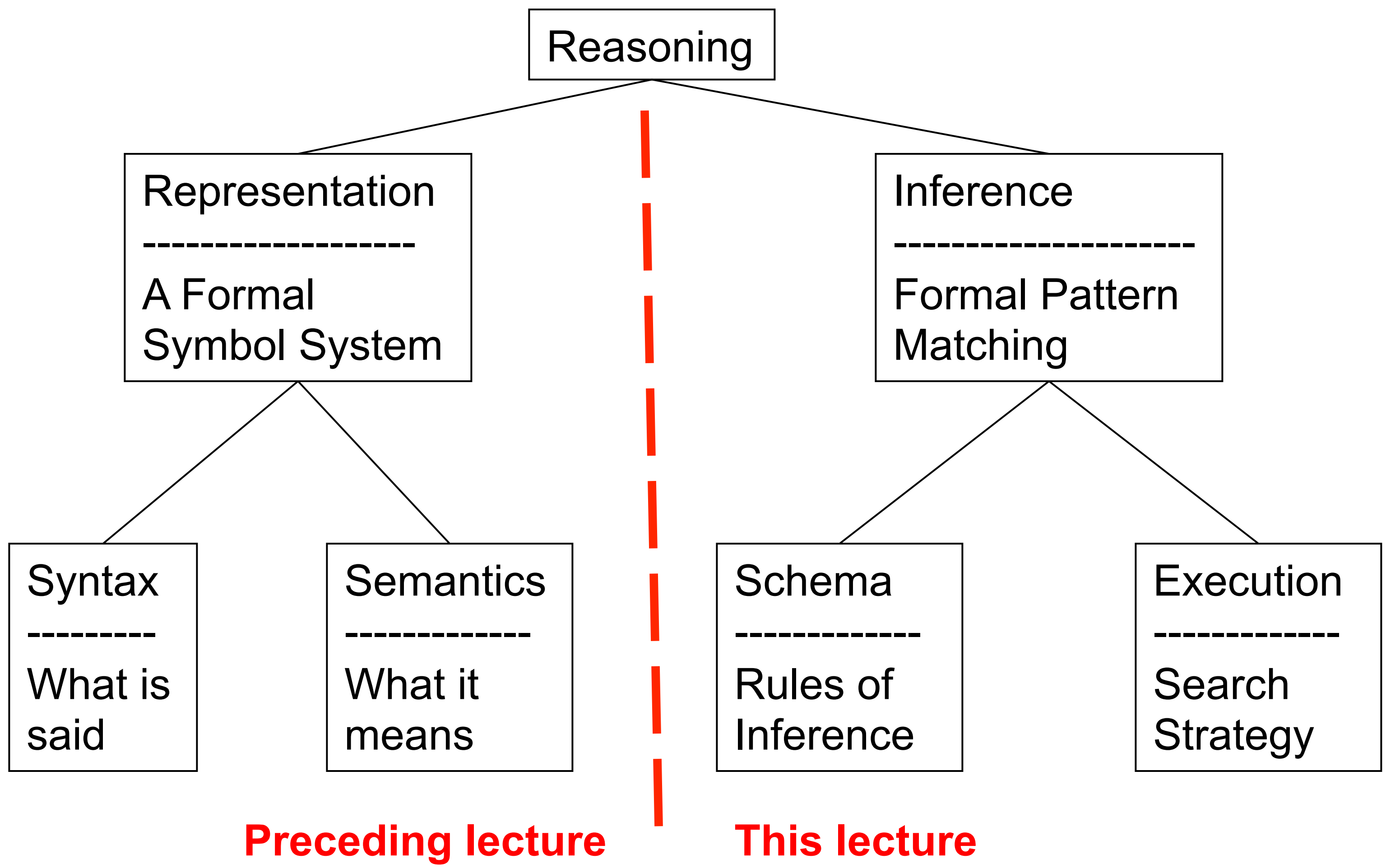
# Review: Inference in Formal Symbol Systems Ontology, Representation, Inference

- **Formal Symbol Systems**
  - **Symbols** correspond to **things/ideas** in the world
  - **Pattern matching & rewrite** corresponds to **inference**

- **Ontology:** What exists in the world?
  - What must be represented?
- **Representation:** Syntax vs. Semantics
  - What's Said vs. What's Meant
- **Inference:** Schema vs. Mechanism
  - Proof Steps vs. Search Strategy

**Ontology:**
What kind of things exist in the world?
What do we need to describe and reason about?

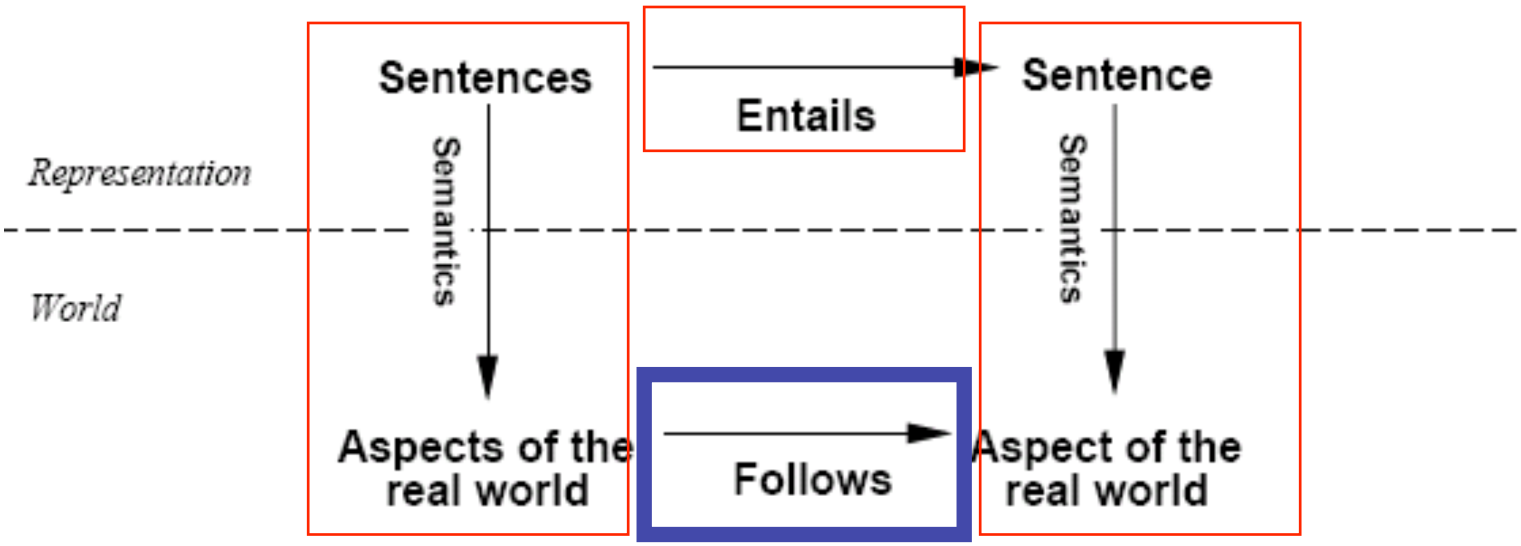
# Review



# Review

- Definitions:
  - Syntax, Semantics, Sentences, Propositions, Entails, Follows, Derives, Inference, Sound, Complete, Model, Satisfiable, Valid (or Tautology)
- Syntactic Transformations:
  - E.g., $(A \Rightarrow B) \Leftrightarrow (\neg A \vee B)$
- Semantic Transformations:
  - E.g., $(KB \models \alpha) \equiv (\models (KB \Rightarrow \alpha))$
- Truth Tables
  - Negation, Conjunction, Disjunction, Implication, Equivalence (Biconditional)
  - Inference by Model Enumeration

# Review: Schematic perspective




*If KB is true in the real world,*

*then any sentence* $\alpha$ ***entailed*** *by KB*

*is also true in the real world.*

# So --- how do we keep it from “Just making things up.” ?

**Is this inference correct?**

**How do you know?**
**How can you tell?**


How can we **make correct** inferences?
How can we **avoid incorrect** inferences?

“Einstein Simplified: Cartoons on Science” by Sydney Harris, 1992, Rutgers University Press

# So --- how do we keep it from "Just making things up." ?

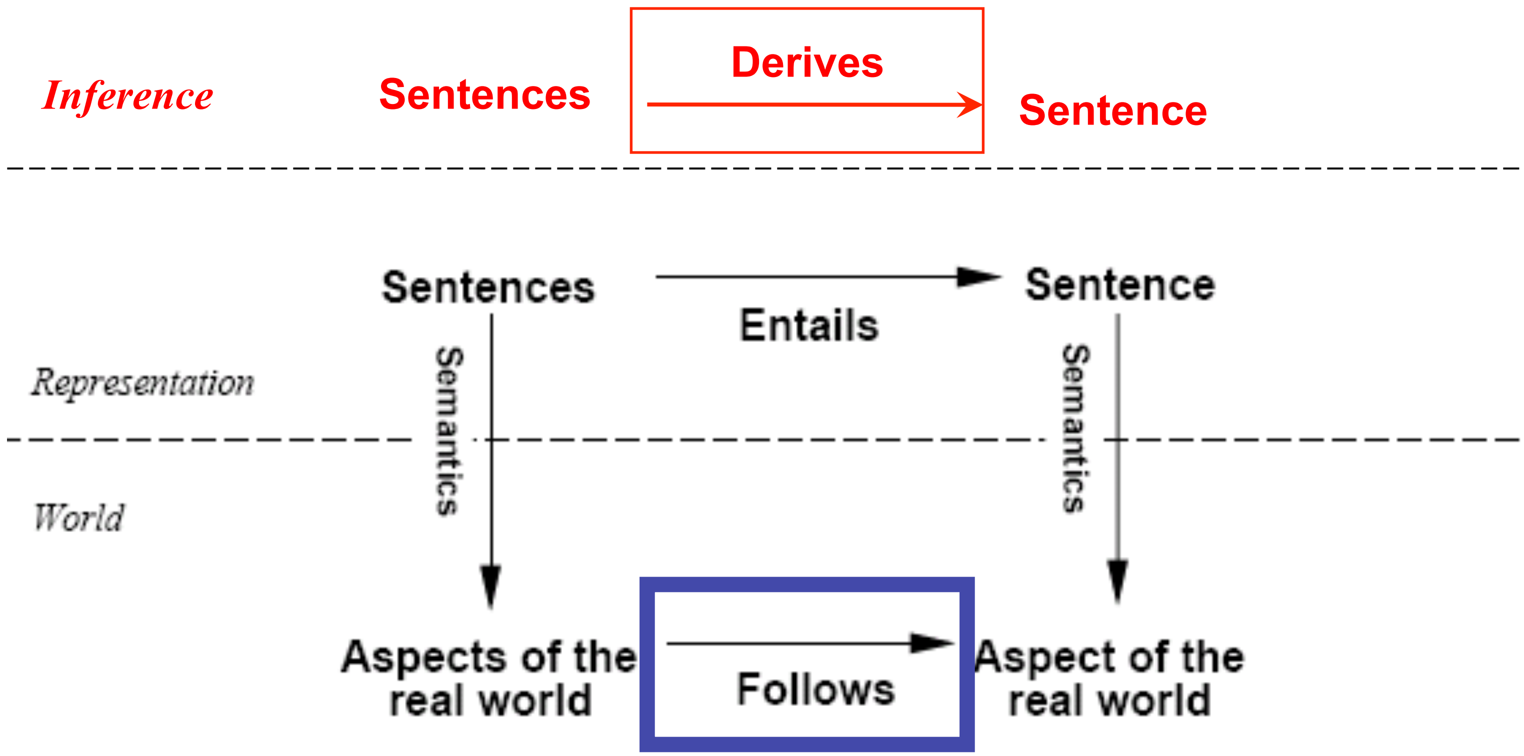
**Is this inference correct?**

**How do you know?**
**How can you tell?**

- All men are people;
  Half of all people are women;
  Therefore, half of all men are women.

- Penguins are black and white;
  Some old TV shows are black and white;
  Therefore, some penguins are old TV shows.

# Schematic perspective




*If KB is true in the real world,*

*then any sentence* $\alpha$ ***derived*** *from KB*

***by a sound inference procedure***

*is also true in the real world.*

# Logical inference

- The notion of entailment can be used for logic inference.
  - Model checking (see wumpus example):
    enumerate all possible models and check whether $\alpha$ is true.

- *Sound* (or *truth preserving*):
  The algorithm **only** derives entailed sentences.
  - Otherwise it just makes things up.
    *i is sound iff whenever KB* $\vdash_i \alpha$ *it is also true that KB* $\models \alpha$
  - *E.g., model-checking is sound*
  Refusing to infer any sentence is Sound; so, Sound is weak alone.

- *Complete*:
  The algorithm can derive **every** entailed sentence.
    *i is complete iff whenever KB* $\models \alpha$ *it is also true that KB* $\vdash_i \alpha$
  Deriving every sentence is Complete; so, Complete is weak alone.

# Proof methods

- Proof methods divide into (roughly) two kinds:

Application of inference rules:

Legitimate (sound) generation of new sentences from old.

– Resolution --- KB is in Conjunctive Normal Form (CNF)

– ~~Forward & Backward chaining~~

Model checking

Searching through truth assignments.

- ~~Improved backtracking: Davis--Putnam-Logemann-Loveland (DPLL)~~
- ~~Heuristic search in model space: Walksat.~~

# Examples of Sound Inference Patterns

**Classical Syllogism (due to Aristotle)**

| | |
|---|---|
| **All Ps are Qs** | **All Men are Mortal** |
| **X is a P** | **Socrates is a Man** |
| **Therefore, X is a Q** | **Therefore, Socrates is Mortal** |

Why is this different from:
All men are people
Half of people are women
So half of men are women

**Implication (Modus Ponens)**

| | |
|---|---|
| **P implies Q** | **Smoke implies Fire** |
| **P** | **Smoke** |
| **Therefore, Q** | **Therefore, Fire** |

**Contrapositive (Modus Tollens)**

| | |
|---|---|
| **P implies Q** | **Smoke implies Fire** |
| **Not Q** | **Not Fire** |
| **Therefore, Not P** | **Therefore, not Smoke** |

**Law of the Excluded Middle (due to Aristotle)**

| | |
|---|---|
| **A Or B** | **Alice is a Democrat or a Republican** |
| **Not A** | **Alice is not a Democrat** |
| **Therefore, B** | **Therefore, Alice is a Republican** |

# Inference by Resolution

- KB is represented in CNF
  - KB = AND of all the sentences in KB
  - KB sentence = clause = OR of literals
  - Literal = propositional symbol or its negation
- Find two clauses in KB, one of which contains a literal and the other its negation
- Cancel the literal and its negation
- Bundle everything else into a new clause
- Add the new clause to KB

# Conjunctive Normal Form (CNF)

- Boolean formulae are central to CS
  - Boolean logic is the way our discipline works
- Two canonical Boolean formulae representations:
  - CNF = Conjunctive Normal Form
    - A conjunct of disjuncts = (AND (OR …) (OR …) )
      Clause
    - "…" = a list of literals (= a variable or its negation)
    - CNF is used by Resolution Theorem Proving
  - DNF = Disjunctive Normal Form
    - A disjunct of conjuncts = (OR (AND …) (AND …) )
      Term
    - DNF is used by Decision Trees in Machine Learning
- Can convert any Boolean formula to CNF or DNF

# Conjunctive Normal Form (CNF)

We'd like to prove: KB |= $\alpha$
(This is equivalent to KB $\wedge\ \neg\ \alpha$ is unsatisfiable.)

We first rewrite $KB \wedge \neg \alpha$ into **conjunctive normal form (CNF).**

A "conjunction of disjunctions"

literals

$$\underbrace{(A \vee \neg B)}_{\text{Clause}} \wedge \underbrace{(B \vee \neg C \vee \neg D)}_{\text{Clause}}$$

- Any KB can be converted into CNF.
- In fact, any KB can be converted into CNF-3 using clauses with at most 3 literals.

# Example: Conversion to CNF

Example: $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

1. Eliminate $\Leftrightarrow$ by replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.
   $= (B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$

2. Eliminate $\Rightarrow$ by replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$ and simplify.
   $= (\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$

3. Move $\neg$ inwards using de Morgan's rules and simplify.

$$\neg(\alpha \vee \beta) = \neg\alpha \wedge \neg\beta$$

   $= (\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$

4. Apply distributive law ($\wedge$ over $\vee$) and simplify.
   $= (\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$

# Example: Conversion to CNF

Example: $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

From the previous slide we had:

$= (\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$

5. KB is the conjunction of all of its sentences (all are true), so write each clause (disjunct) as a sentence in KB:

KB =

...

$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \longrightarrow (\neg B_{1,1} \quad P_{1,2} \quad P_{2,1})$

$(\neg P_{1,2} \vee B_{1,1}) \longrightarrow (\neg P_{1,2} \quad B_{1,1})$

$(\neg P_{2,1} \vee B_{1,1}) \longrightarrow (\neg P_{2,1} \quad B_{1,1})$

...

**(same)**

Often, Won't Write "∨" or "∧"

(we know they are there)

# Inference by Resolution

- KB is represented in CNF
  - KB = AND of all the sentences in KB
  - KB sentence = clause = OR of literals
  - Literal = propositional symbol or its negation
- Find two clauses in KB, one of which contains a literal and the other its negation
- Cancel the literal and its negation
- Bundle everything else into a new clause
- Add the new clause to KB

# Resolution = Efficient Implication

Recall that (A => B) = ( (NOT A) OR B)
and so:

(Y OR X) = ( (NOT X) => Y)
( (NOT Y) OR Z) = (Y => Z)

which yields:

( (Y OR X) AND ( (NOT Y) OR Z) ) = ( (NOT X) => Z) = (X OR Z)

| | | |
|---|---|---|
| (OR A B C D) | ->Same -> | (NOT (OR B C D)) => A |
| (OR ¬A E F G) | ->Same -> | A => (OR E F G) |
| ---------------------------- | | -------------------------------------------------- |
| (OR B C D E F G) | | (NOT (OR B C D)) => (OR E F G) |
| | | -------------------------------------------------- |
| | | (OR B C D E F G) |

Recall: All clauses in KB are conjoined by an implicit AND (= CNF representation).

# Resolution Examples

- Resolution: inference rule for CNF: sound and complete! *

$(A \vee B \vee C)$

$(\neg A)$ "If A or B or C is true, but not A, then B or C must be true."

– – – – – – – – – – – – – – –

$\therefore (B \vee C)$

$(A \vee B \vee C)$

$(\neg A \vee D \vee E)$

"If A is false then B or C must be true, or if A is true then D or E must be true, hence since A is either true or false, B or C or D or E must be true."

– – – – – – – – – – – – – –

$\therefore (B \vee C \vee D \vee E)$

$(A \vee B)$

$(\neg A \vee B)$

"If A or B is true, and not A or B is true, then B must be true."

– – – – – – – – – – –

$\therefore (B \vee B) \equiv B \leftarrow$ Simplification is done always.

* Resolution is "refutation complete" in that it can prove the truth of any entailed sentence by refutation.
* You can start two resolution proofs in parallel, one for the sentence and one for its negation, and see which branch returns a correct proof.

# Only Resolve ONE Literal Pair!

## If more than one pair, result always = TRUE.

## **Useless!!** Always simplifies to TRUE!!

**No!**

(OR A B C D)
(OR ¬A ¬B F G)
-----------------------------
(OR C D F G)

**No!**

**No!**

(OR A B C D)
(OR ¬A ¬B ¬C )
-----------------------------
(OR D)

**No!**

**Yes! (but = TRUE)**

(OR A B C D)
(OR ¬A ¬B F G)
-----------------------------
(OR B ¬B C D F G)

**Yes! (but = TRUE)**

**Yes! (but = TRUE)**

(OR A B C D)
(OR ¬A ¬B ¬C )
-----------------------------
(OR A ¬A B ¬B D)

**Yes! (but = TRUE)**

# Resolution Algorithm

- The resolution algorithm tries to prove: $KB \models \alpha$ *equivalent to* $KB \wedge \neg\alpha$ *unsatisfiable*

- Generate all new sentences from KB and the (negated) query.
- One of two things can happen:

1. We find $P \wedge \neg P$ which is unsatisfiable. I.e. we can entail the query.

2. We find no contradiction: there is a model that satisfies the sentence $KB \wedge \neg\alpha$ (non-trivial) and hence we cannot entail the query.

# Resolution example

Stated in English

- "Laws of Physics" in the Wumpus World:
  - "A breeze in B11 is equivalent to a pit in P12 or a pit in P21."
- Particular facts about a specific instance:
  - "There is no breeze in B11."
- Goal or query sentence:
  - "Is it true that P12 does not have a pit?"

# Resolution example

## Stated in Propositional Logic

- “Laws of Physics” in the Wumpus World:
  - “A breeze in B11 is equivalent to a pit in P12 or a pit in P21.”

    $(B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1}))$

We converted this sentence to CNF in the CNF example we worked above.

- Particular facts about a specific instance:
  - “There is no breeze in B11.”

    $(\neg B_{1,1})$
- Goal or query sentence:
  - “Is it true that P12 does not have a pit?”

    $(\neg P_{1,2})$

# Resolution example

## Resulting Knowledge Base stated in CNF

- "Laws of Physics" in the Wumpus World:

  $(\neg B_{1,1} \quad P_{1,2} \quad P_{2,1})$
  $(\neg P_{1,2} \quad B_{1,1})$
  $(\neg P_{2,1} \quad B_{1,1})$

- Particular facts about a specific instance:

  $(\neg B_{1,1})$

- Negated goal or query sentence:

  $(P_{1,2})$

# Resolution example

## A Resolution proof ending in ( )

- Knowledge Base at start of proof:

  $(\neg B_{1,1} \quad P_{1,2} \quad P_{2,1})$
  $(\neg P_{1,2} \quad B_{1,1})$
  $(\neg P_{2,1} \quad B_{1,1})$
  $(\neg\ B_{1,1})$
  $(P_{1,2})$

**A resolution proof ending in ( ):**

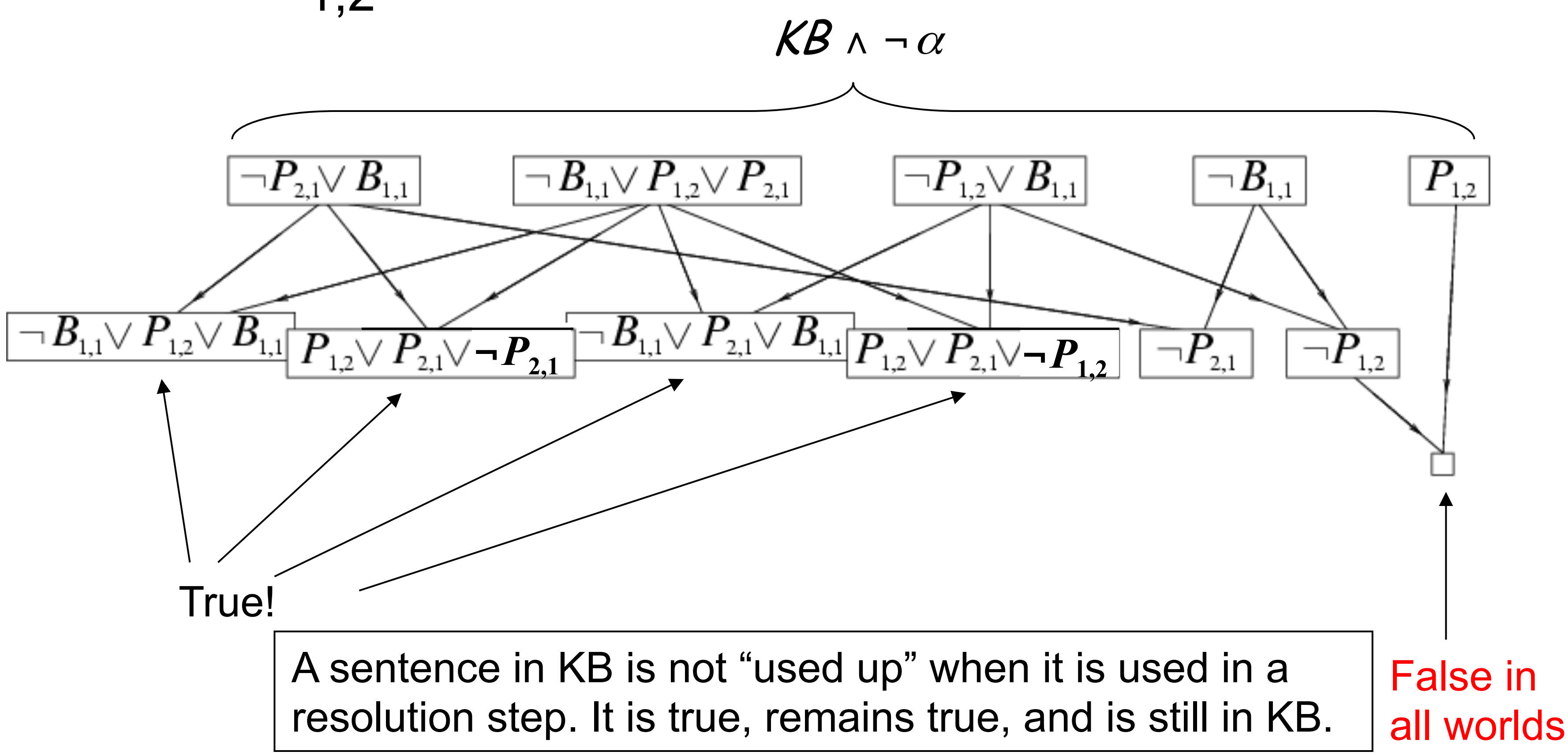
- Resolve $(\neg P_{1,2} \quad B_{1,1})$ and $(\neg\ B_{1,1})$ to give $(\neg P_{1,2})$
- Resolve $(\neg P_{1,2})$ and $(P_{1,2})$ to give ( )

- Consequently, the goal or query sentence is entailed by KB.
- Of course, there are many other proofs, which are OK iff correct.

# Resolution example

## Graphical view of the proof

- $KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1}$
- $\alpha = \neg P_{1,2}$




A sentence in KB is not "used up" when it is used in a resolution step. It is true, remains true, and is still in KB.

# Detailed Resolution Proof Example

- **In words:** *If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.*

  *<u>Prove that the unicorn is both magical and horned.</u>*

Problem 7.2, R&N page 280. (Adapted from Barwise and Etchemendy, 1993.)

Note for non-native-English speakers: immortal = not mortal

# Detailed Resolution Proof Example

- **In words:** *If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.*

  <u>*Prove that the unicorn is both magical and horned.*</u>

- **<u>First, Ontology</u>**<u>:</u> What do we need to describe and reason about?

- Use these propositional variables ("immortal" = "not mortal"):

  Y = unicorn is m<u>Y</u>thical    R = unicorn is mo<u>R</u>tal

  M = unicorn is a ma<u>M</u>mal    H = unicorn is <u>H</u>orned

  G = unicorn is ma<u>G</u>ical

# Detailed Resolution Proof Example

- **In words:** *If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.*

  *Prove that the unicorn is both magical and horned.*

  Y = unicorn is mYthical        R = unicorn is moRtal

  M = unicorn is a maMmal        H = unicorn is Horned

  G = unicorn is maGical

- **Second, translate to Propositional Logic, then to CNF:**
- Propositional logic (prefix form, aka Polish notation):
  - **(=> Y (NOT R) )**        ; same as ( Y => (NOT R) ) in infix form
- CNF (clausal form)        ; recall (A => B) = ( (NOT A) OR B)
  - **( (NOT Y) (NOT R) )**

Prefix form is often a better representation for a parser, since it looks at the first element of the list and dispatches to a handler for that operator token.

# Detailed Resolution Proof Example

- **In words:** *If the unicorn is mythical, then it is immortal, but <u>if it is not mythical, then it is a mortal mammal</u>. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.*

  <u>*Prove that the unicorn is both magical and horned.*</u>

  Y = unicorn is m<u>Y</u>thical R = unicorn is mo<u>R</u>tal

  M = unicorn is a ma<u>M</u>mal H = unicorn is <u>H</u>orned

  G = unicorn is ma<u>G</u>ical

- <u>**Second, translate to Propositional Logic, then to CNF**:</u>
- Propositional logic (prefix form):
  - **(=> (NOT Y) (AND R M) )** ;same as ( (NOT Y) => (R AND M) ) in infix form
- CNF (clausal form)
  - **(M Y)**
  - **(R Y)**

If you ever have to do this "for real" you will likely <u>invent a new domain language</u> that allows you to state important properties of the domain --- then parse that into propositional logic, and then CNF.

# Detailed Resolution Proof Example

- **In words:** *If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.*

  *Prove that the unicorn is both magical and horned.*

  Y = unicorn is mYthical — R = unicorn is moRtal

  M = unicorn is a maMmal — H = unicorn is Horned

  G = unicorn is maGical

- **Second, translate to Propositional Logic, then to CNF**:
- Propositional logic (prefix form):
  - **(=> (OR (NOT R) M) H)** ; same as ( (Not R) OR M) => H in infix form
- CNF (clausal form)
  - **(H (NOT M) )**
  - **(H R)**

# Detailed Resolution Proof Example

- **In words:** *If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.*

  *Prove that the unicorn is both magical and horned.*

  Y = unicorn is mYthical

  R = unicorn is moRtal

  M = unicorn is a maMmal

  H = unicorn is Horned

  G = unicorn is maGical

- **Second, translate to Propositional Logic, then to CNF**:
- Propositional logic (prefix form)
  - **(=> H G)** ; same as H => G in infix form
- CNF (clausal form)
  - **( (NOT H) G)**

# Detailed Resolution Proof Example

- **In words:** *If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.*

  <u>*Prove that the unicorn is both magical and horned.*</u>

  Y = unicorn is m<u>Y</u>thical

  R = unicorn is mo<u>R</u>tal

  M = unicorn is a ma<u>M</u>mal

  H = unicorn is <u>H</u>orned

  G = unicorn is ma<u>G</u>ical

- <u>**Current KB**</u> (in CNF clausal form) =

  **( (NOT Y) (NOT R) )**  **(M Y)**  **(R Y)**  **(H (NOT M) )**

  **(H R)**  **( (NOT H) G)**

# Detailed Resolution Proof Example

- **In words:** *If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.*

  *Prove that the unicorn is both magical and horned.*

  Y = unicorn is mYthical

  R = unicorn is moRtal

  M = unicorn is a maMmal

  H = unicorn is Horned

  G = unicorn is maGical

- **Third, negated goal to Propositional Logic, then to CNF:**
- Goal sentence in propositional logic (prefix form)
  - **(AND H G)** ; same as H AND G in infix form
- Negated goal sentence in propositional logic (prefix form)
  - **(NOT (AND H G) ) = (OR (NOT H) (NOT G) )**
- CNF (clausal form)
  - **( (NOT G) (NOT H) )**

# Detailed Resolution Proof Example

- **In words:** *If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.*

  *Prove that the unicorn is both magical and horned.*

  Y = unicorn is mYthical
  R = unicorn is moRtal
  M = unicorn is a maMmal
  H = unicorn is Horned
  G = unicorn is maGical

- **Current KB + negated goal** (in CNF clausal form) =

  **( (NOT Y) (NOT R) )** **(M Y)** **(R Y)** **(H (NOT M) )**
  **(H R)** **( (NOT H) G)** **( (NOT G) (NOT H) )**

# Detailed Resolution Proof Example

- **In words:** *If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.*

  <u>*Prove that the unicorn is both magical and horned.*</u>

| | | | |
|---|---|---|---|
| **( (NOT Y) (NOT R) )** | **(M Y)** | **(R Y)** | **(H (NOT M) )** |
| **(H R)** | **( (NOT H) G)** | **( (NOT G) (NOT H) )** | |

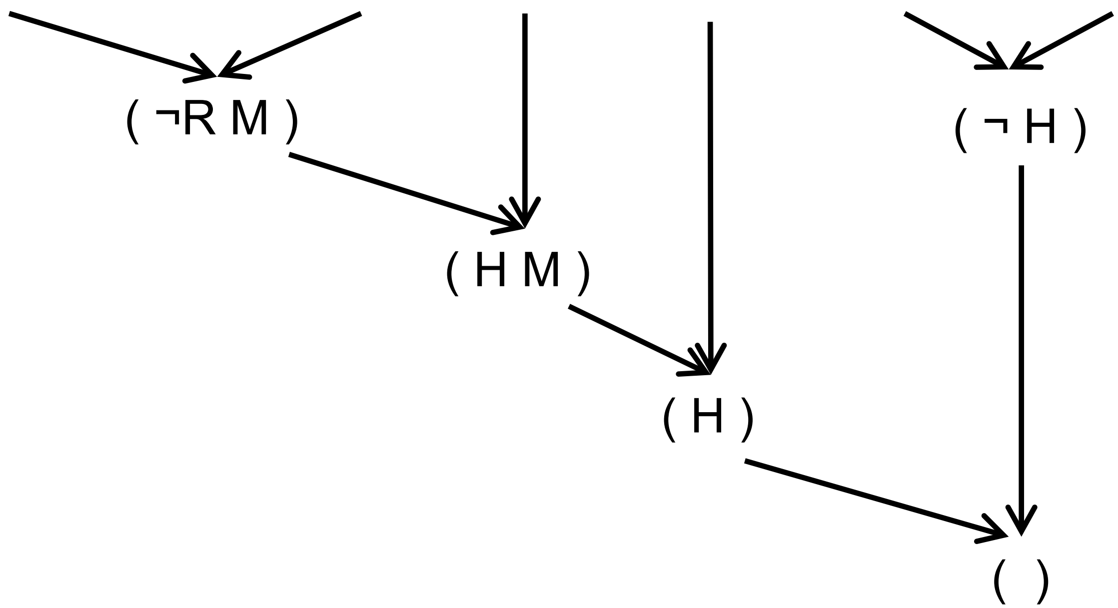
- **Fourth, produce a resolution proof ending in ( ):**
- Resolve (¬H ¬G) and (¬H G) to give (¬H)
- Resolve (¬Y ¬R) and (Y M) to give (¬R M)
- Resolve (¬R M) and (R H) to give (M H)
- Resolve (M H) and (¬M H) to give (H)
- Resolve (¬H) and (H) to give ( )

- Of course, there are many other proofs, which are OK iff correct.

# Detailed Resolution Proof Example

## Graph view of proof

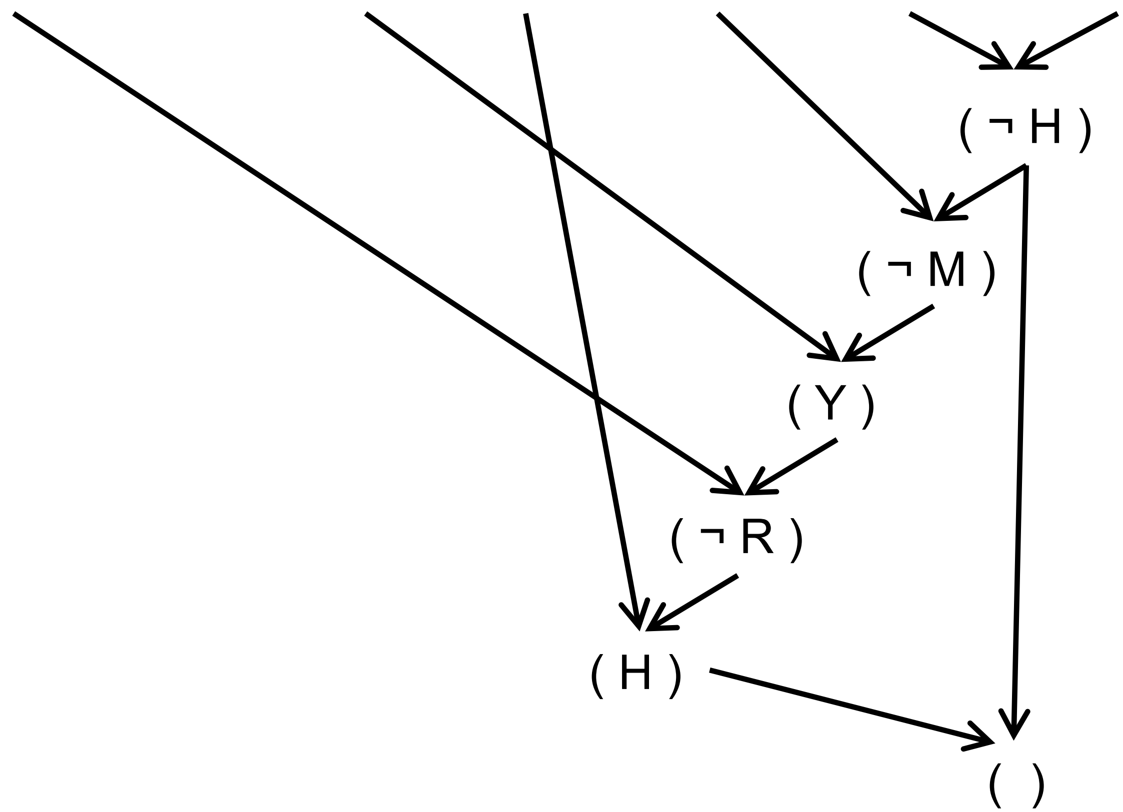
- ( ¬ Y ¬ R ) ( Y R ) ( Y M ) ( R H ) ( ¬ M H ) ( ¬ H G ) (¬ G ¬ H )



# Detailed Resolution Proof Example

## Graph view of a different proof

- ( ¬ Y ¬ R ) ( Y R ) ( Y M ) ( R H ) ( ¬ M H ) ( ¬ H G ) (¬ G ¬ H )



# Horn Clauses

- Resolution can be exponential in space and time.
- If we can reduce all clauses to "Horn clauses" inference is linear in space and time

A clause with at most 1 positive literal.
e.g. $A \vee \neg B \vee \neg C$

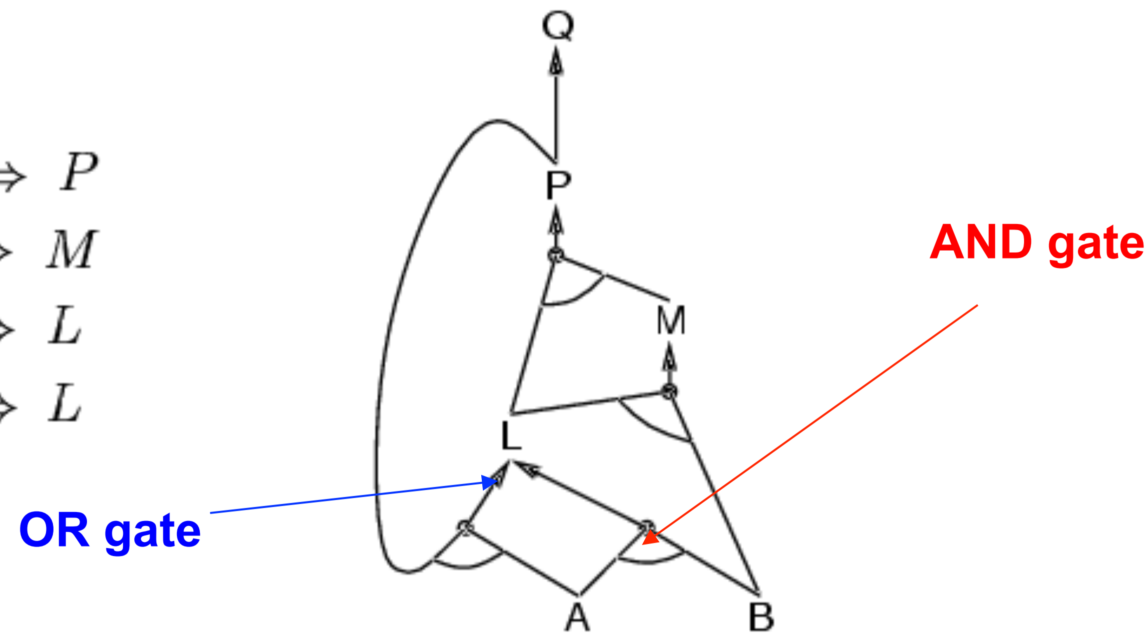
- Every Horn clause can be rewritten as an implication with a conjunction of positive literals in the premises and at most a single positive literal as a conclusion.
  e.g. $A \vee \neg B \vee \neg C \equiv B \wedge C \Rightarrow A$
- 1 positive literal and ≥ 1 negative literal: definite clause (e.g., above)
- 0 positive literals: integrity constraint or goal clause
  e.g. $(\neg A \vee \neg B) \equiv (A \wedge B \Rightarrow False)$ states that (A ∧ B) must be false
- 0 negative literals: fact
  e.g., (A) ≡ (True ⇒ A) states that A must be true.
- Forward Chaining and Backward chaining are sound and complete with Horn clauses and run linear in space and time.

# Forward chaining (FC)

- Idea: fire any rule whose premises are satisfied in the *KB*, add its conclusion to the *KB*, until query is found.

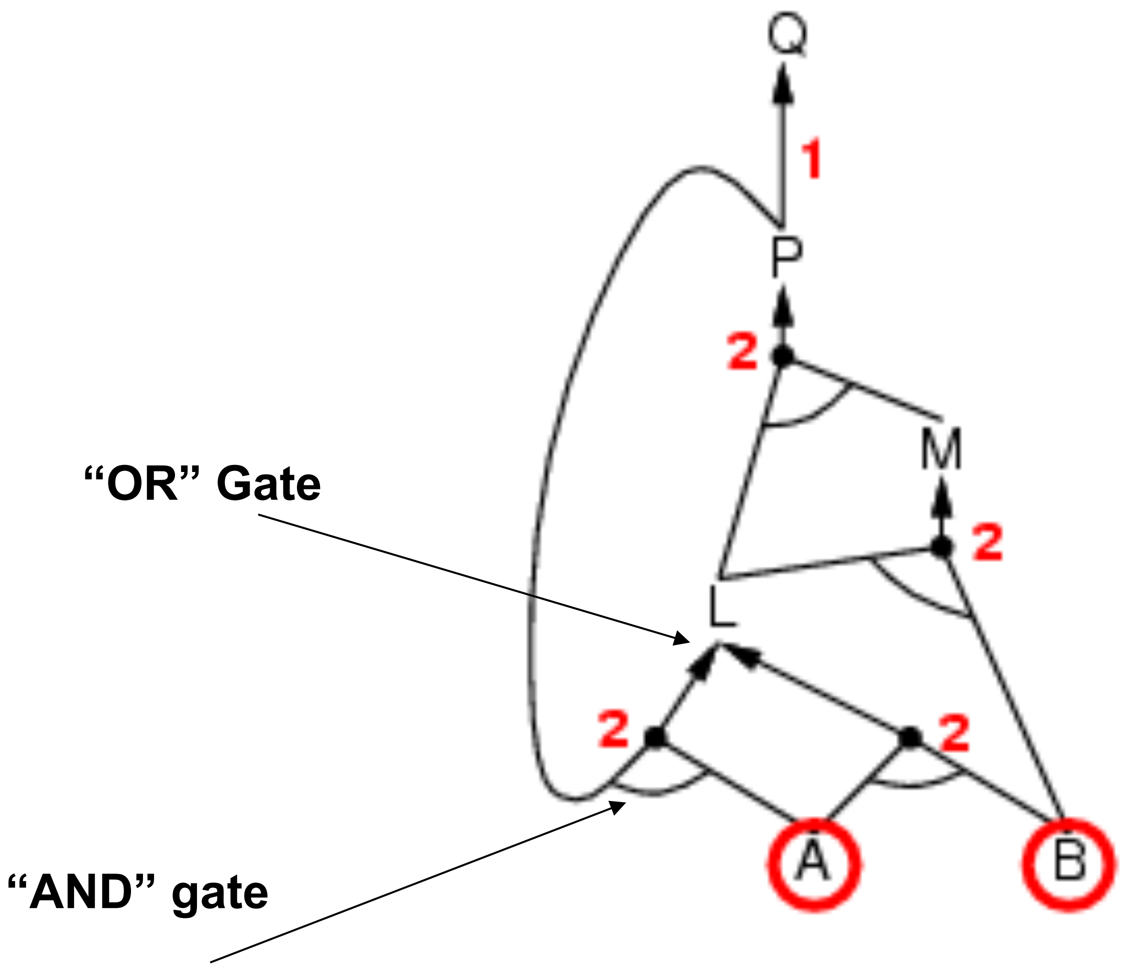
- This proves that $KB \Rightarrow Q$ is true in all possible worlds (i.e. trivial), and hence it proves entailment.

$$
\begin{aligned}
&P \Rightarrow Q \\
&L \wedge M \Rightarrow P \\
&B \wedge L \Rightarrow M \\
&A \wedge P \Rightarrow L \\
&A \wedge B \Rightarrow L \\
&A \\
&B
\end{aligned}
$$

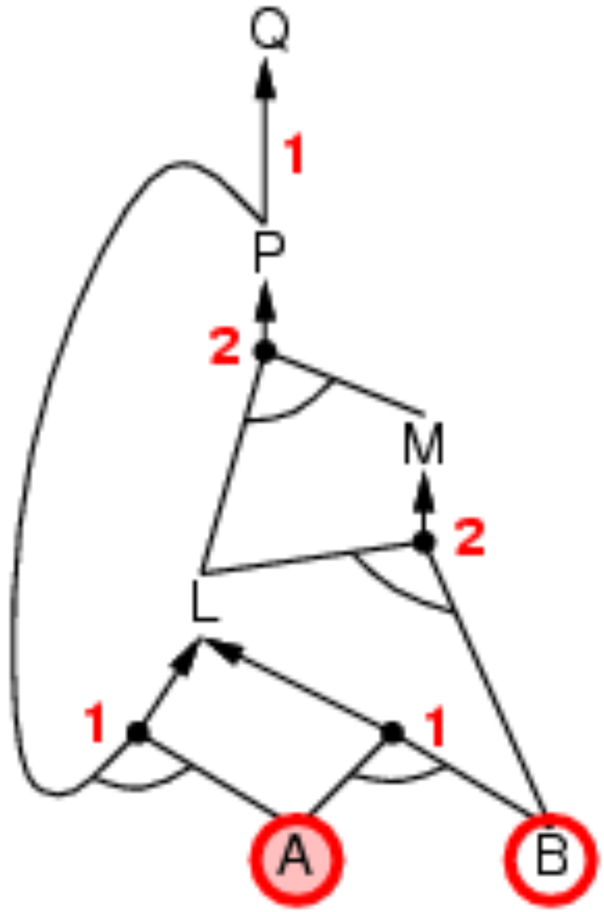



- Forward chaining is sound and complete for Horn KB

# Forward chaining example



# Forward chaining example



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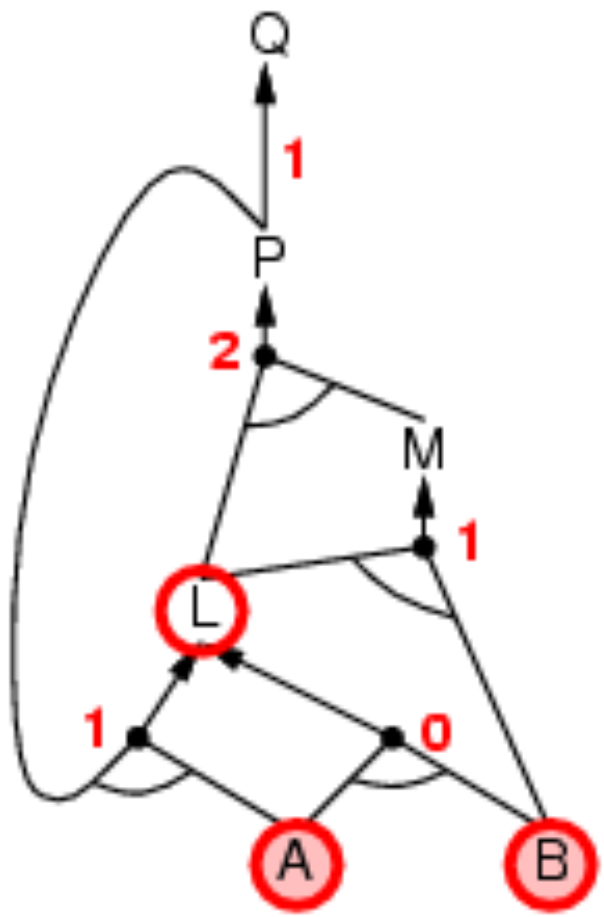

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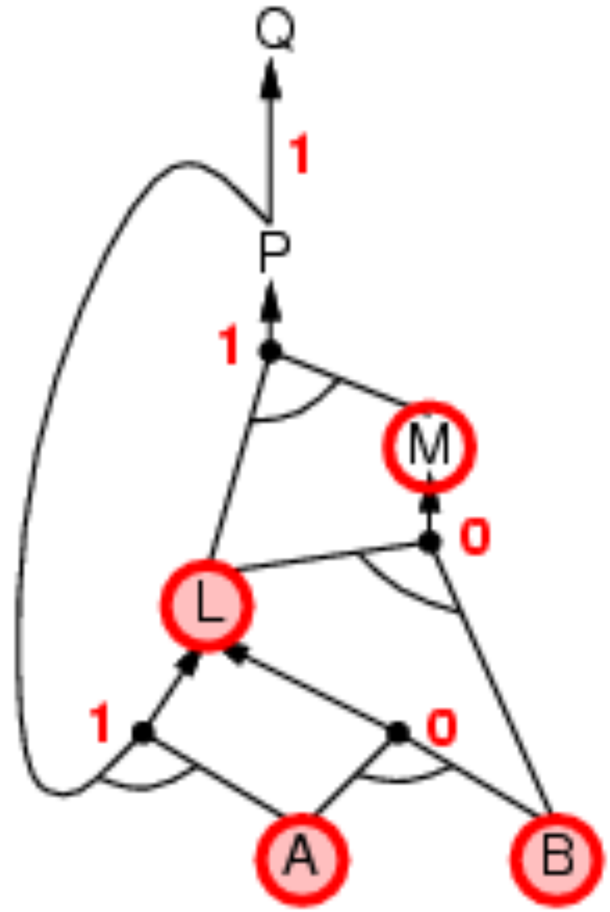

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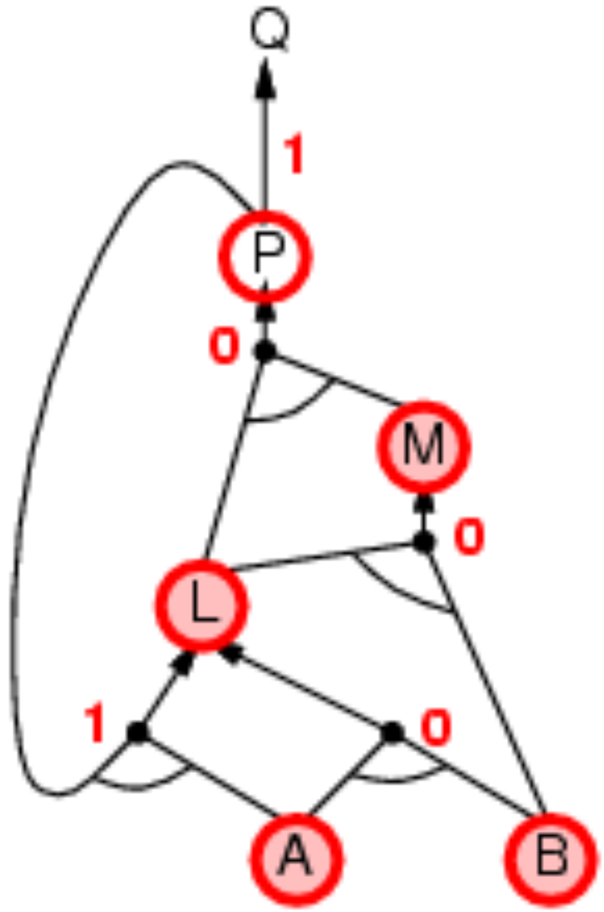
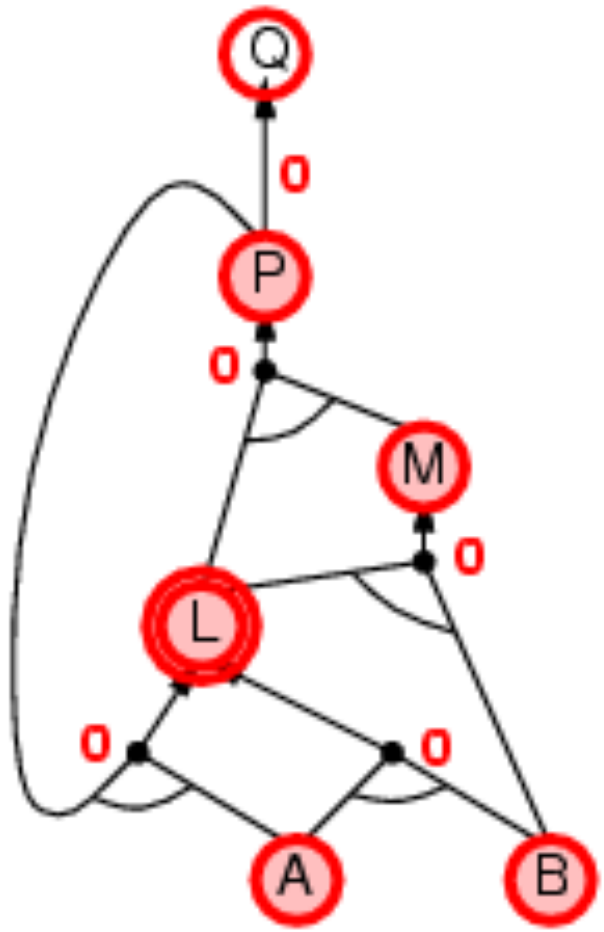


# Forward chaining example



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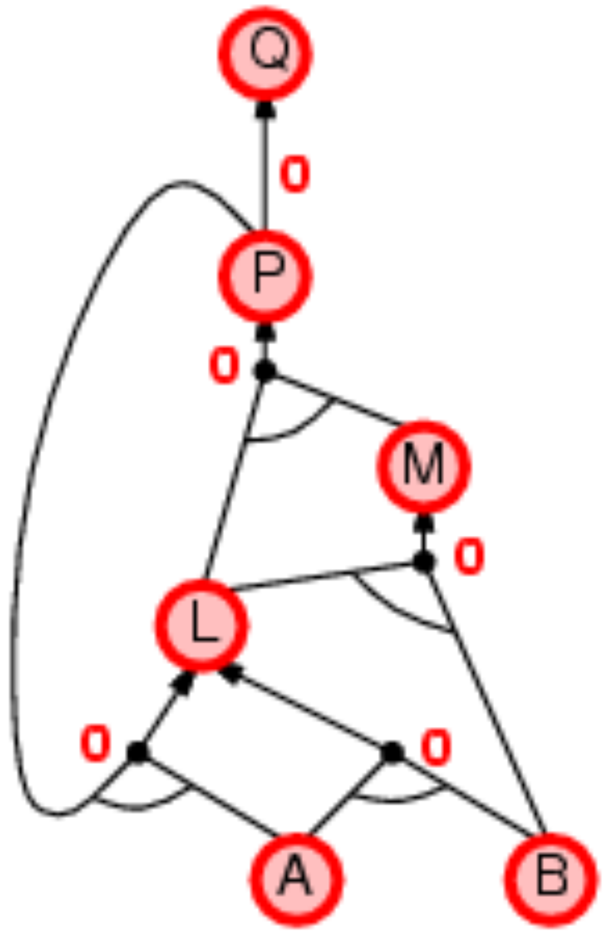


# Backward chaining (BC)

Idea: work backwards from the query *q*

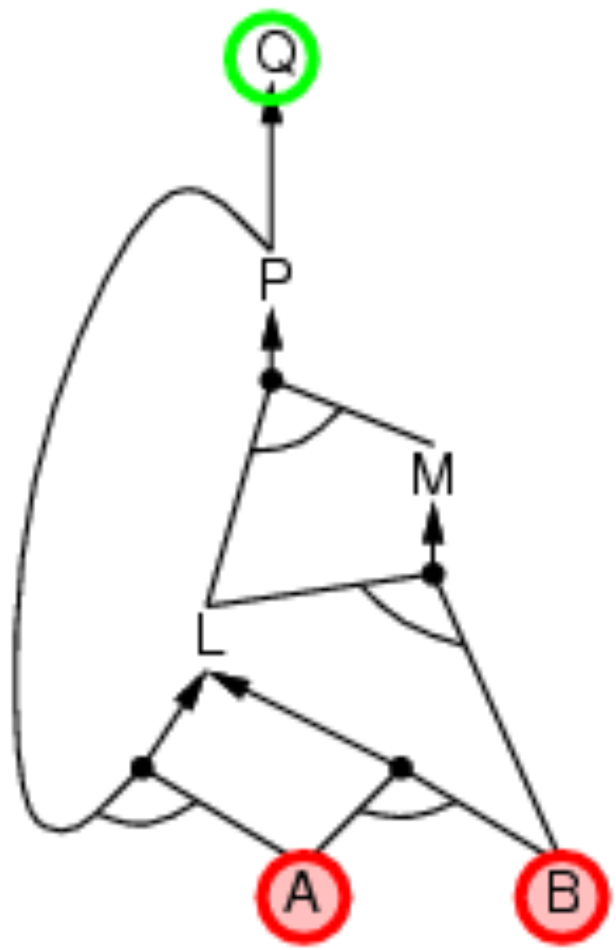
- check if *q* is known already, or
- prove by BC all premises of some rule concluding *q*
- Hence BC maintains a stack of sub-goals that need to be proved to get to q.

Avoid loops: check if new sub-goal is already on the goal stack

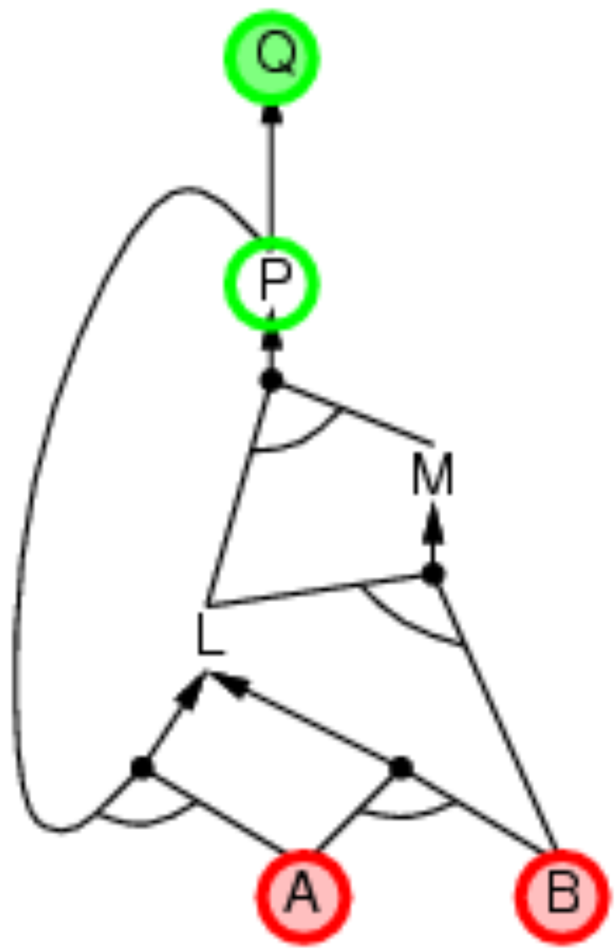
Avoid repeated work: check if new sub-goal

1. has already been proved true, or
2. has already failed

# Backward chaining example



# Backward chaining example



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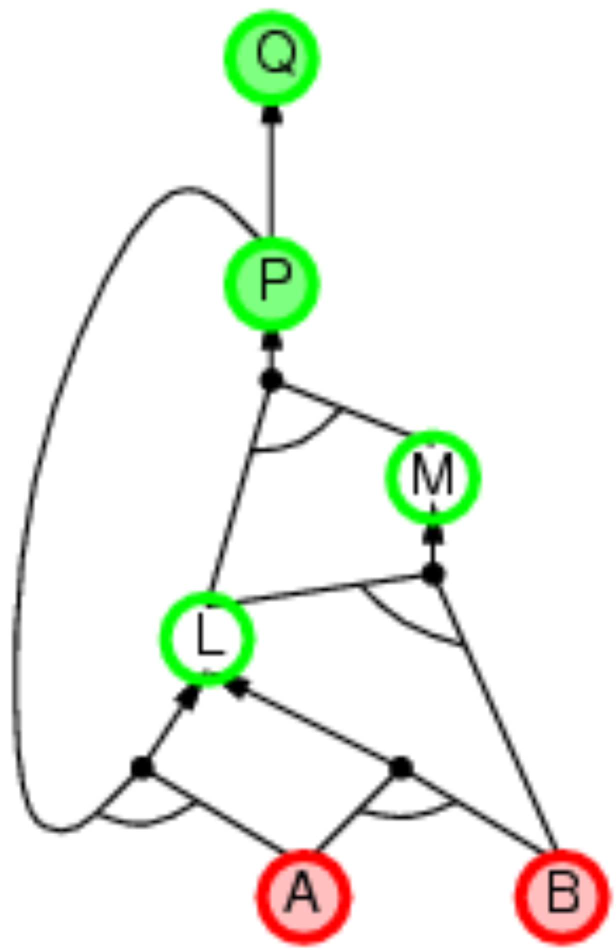

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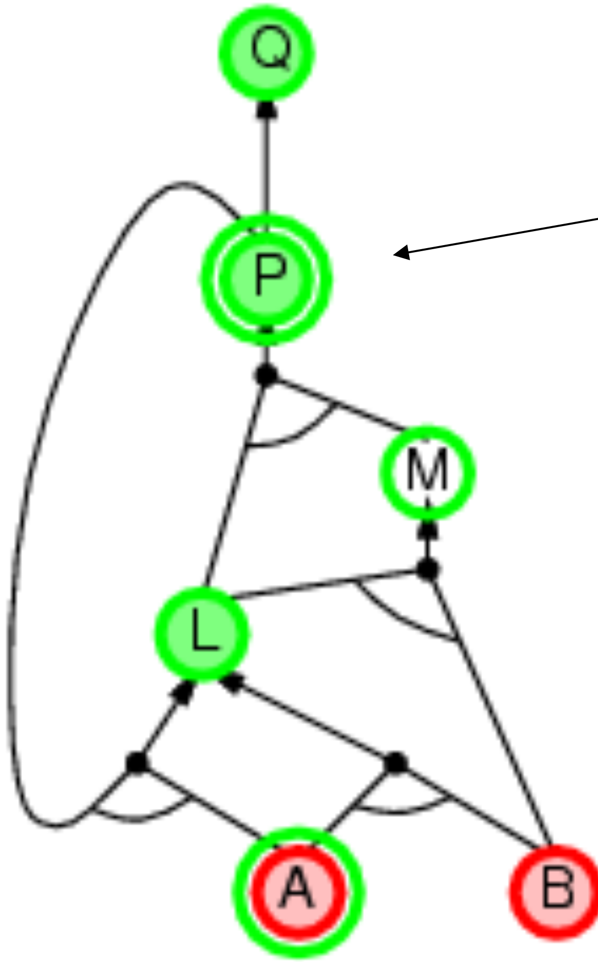
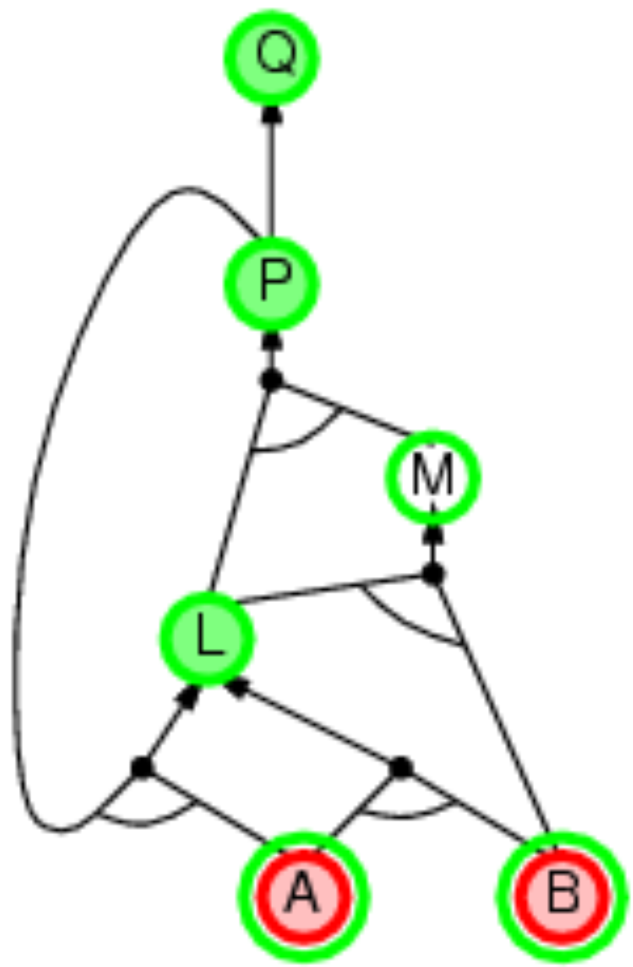


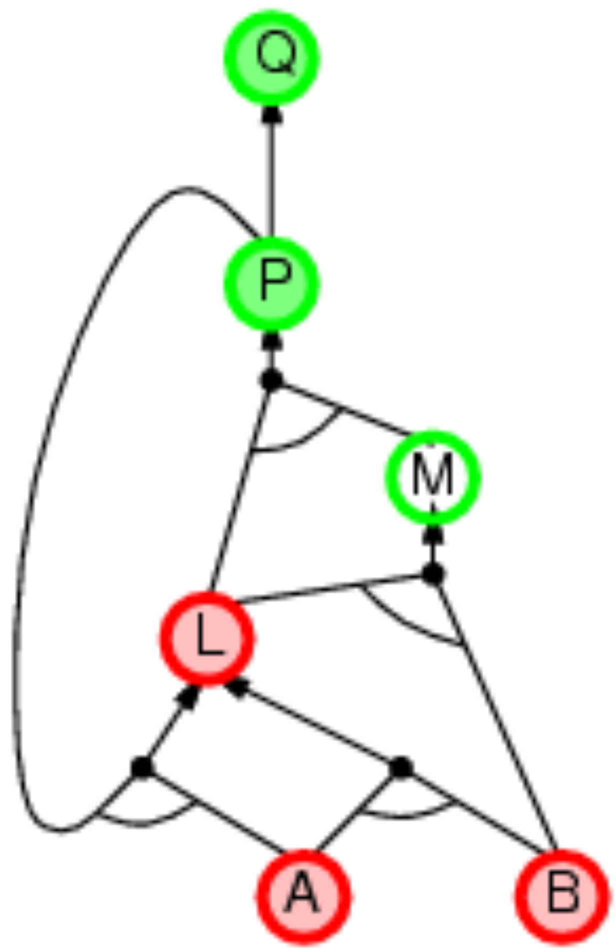

we need P to prove
L and L to prove P.

# Backward chaining example




As soon as you can move forward, do so.

# Backward chaining example



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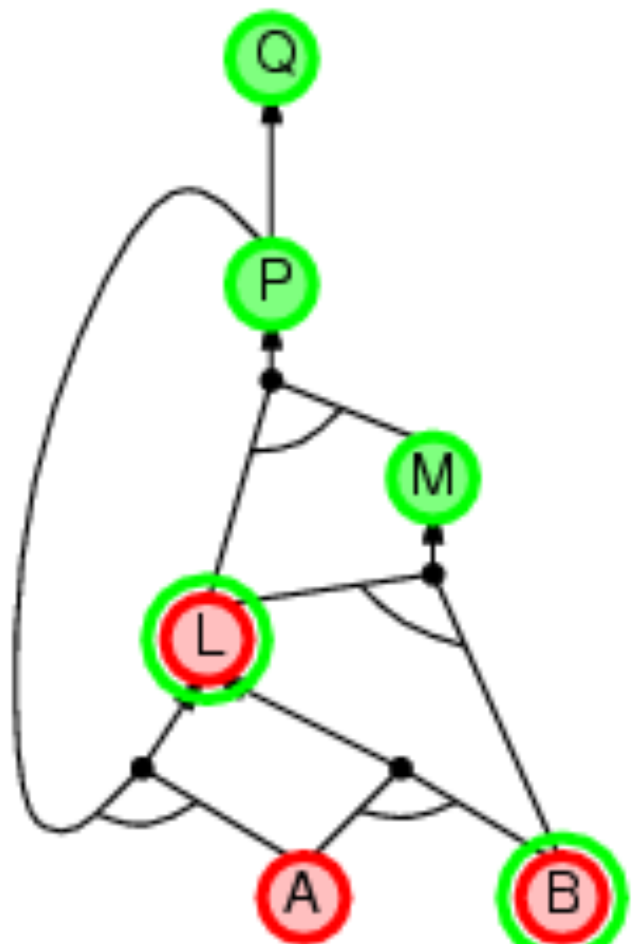

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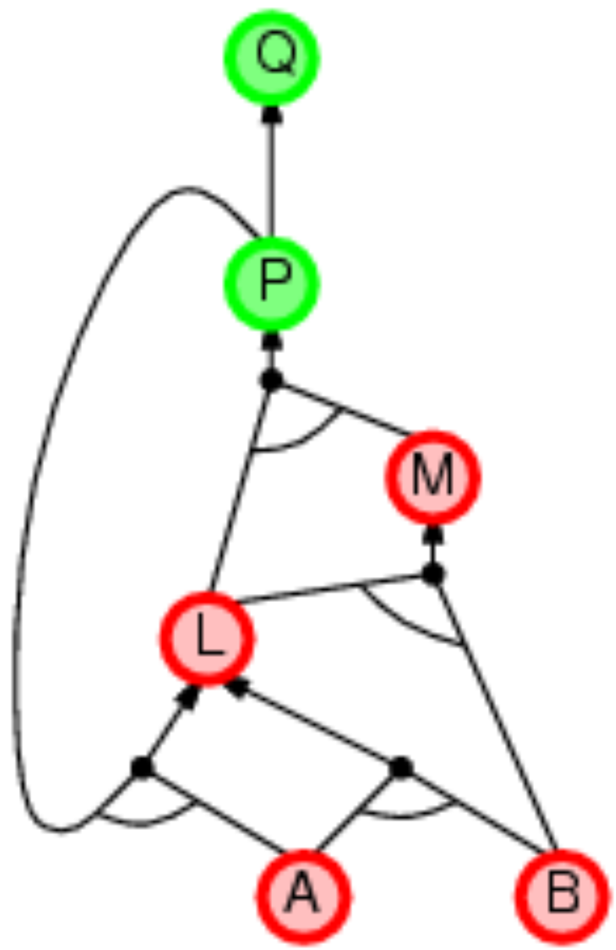

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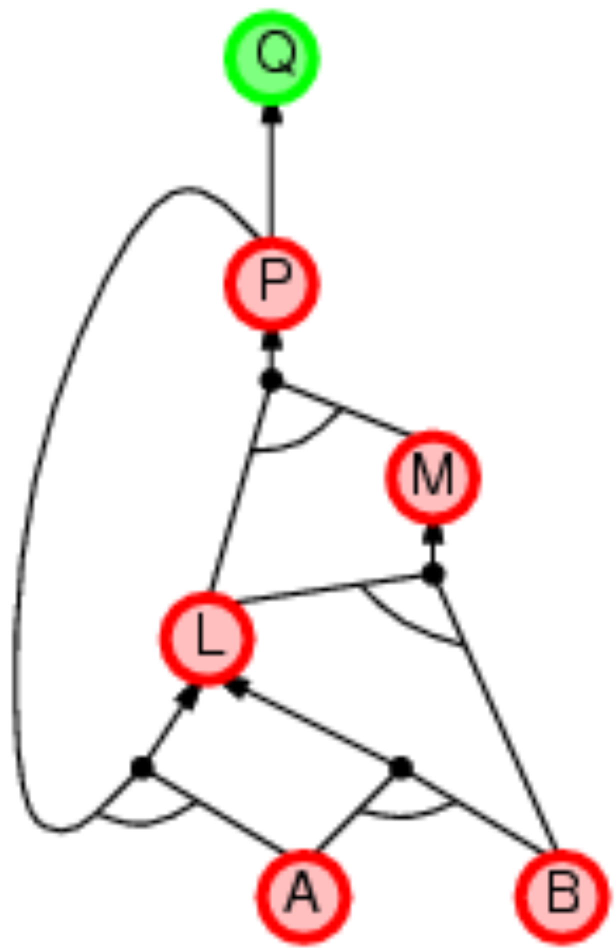

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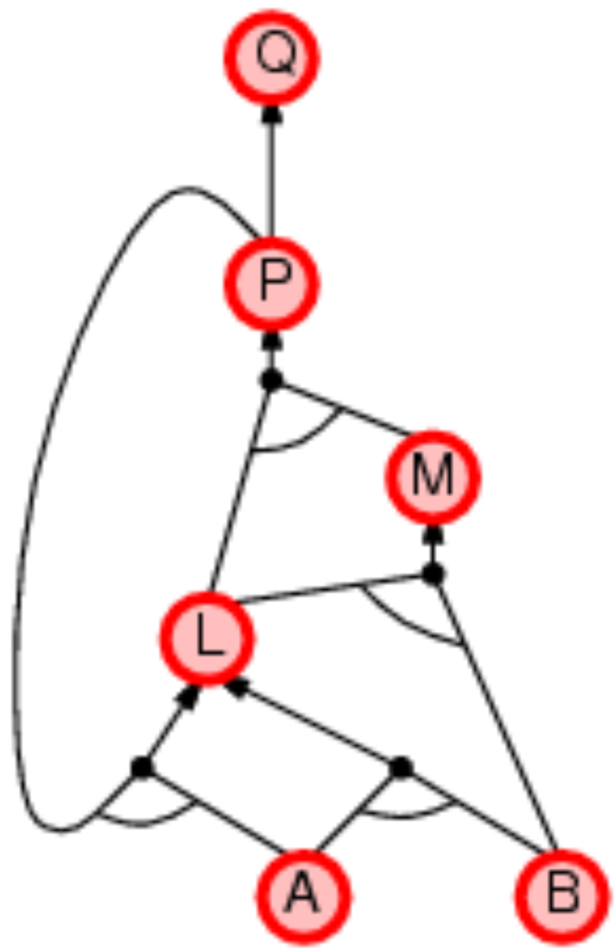


# Forward vs. backward chaining

- FC is data-driven, automatic, unconscious processing,
  - e.g., object recognition, routine decisions
- May do lots of work that is irrelevant to the goal
- BC is goal-driven, appropriate for problem-solving,
  - e.g., Where are my keys? How do I get into a PhD program?
- Complexity of BC can be much less than linear in size of KB

# Model Checking

Two families of efficient algorithms:

- Complete backtracking search algorithms:
  - E.g., DPLL algorithm
- Incomplete local search algorithms
  - E.g., `WalkSAT` algorithm

# The DPLL algorithm

Determine if an input propositional logic sentence (in CNF) is satisfiable. This is just backtracking search for a CSP.

Improvements:

1. Early termination

   A clause is true if any literal is true.

   A sentence is false if any clause is false.

2. Pure symbol heuristic

   Pure symbol: always appears with the same "sign" in all clauses.

   e.g., In the three clauses (A ∨ ¬B), (¬B ∨ ¬C), (C ∨ A), A and B are pure, C is impure.

   Make a pure symbol literal true. (if there is a model for S, then making a pure symbol true is also a model).

3 Unit clause heuristic

   Unit clause: only one literal in the clause

   The only literal in a unit clause must be true.

   Note: literals can become a pure symbol or a unit clause when other literals obtain truth values. e.g.

$(A \lor \cancel{True}) \land (\neg A \lor B)$

$A = pure$

# The `WalkSAT` algorithm

- Incomplete, local search algorithm
- Evaluation function: The min-conflict heuristic of minimizing the number of unsatisfied clauses
- Balance between greediness and randomness

**Walksat Procedure**

Start with random initial assignment.
Pick a random unsatisfied clause.
Select and flip a variable from that clause:
    With probability p, pick a random variable.
    With probability 1-p, pick greedily
        a variable that minimizes the number of unsatisfied clauses
Repeat to predefined maximum number flips;
  if no solution found, restart.

# Hard satisfiability problems

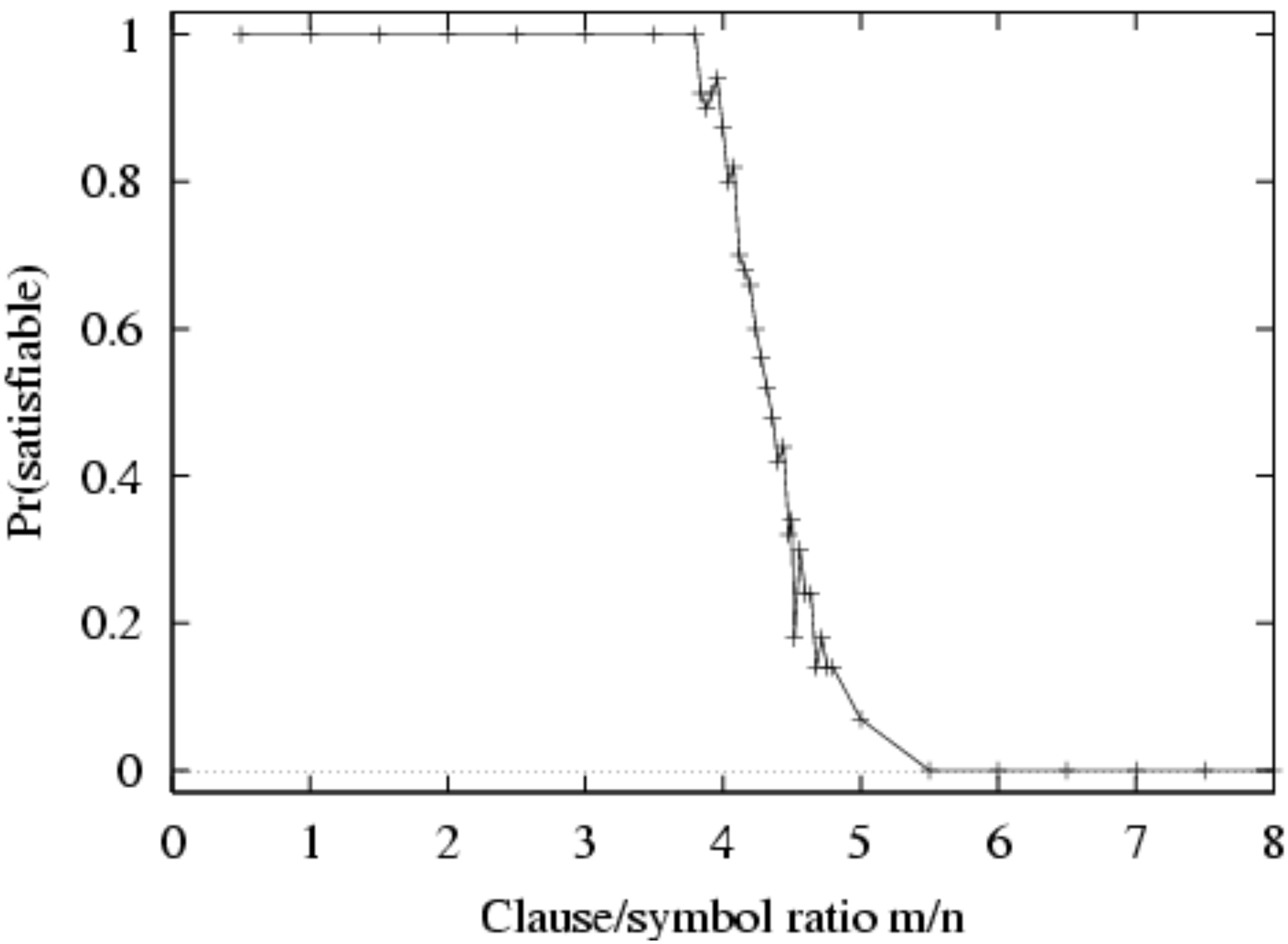
- Consider *random* 3-CNF sentences. e.g.,
  $(\neg D \lor \neg B \lor C) \land (B \lor \neg A \lor \neg C) \land (\neg C \lor \neg B \lor E) \land (E \lor \neg D \lor B) \land (B \lor E \lor \neg C)$

  *m* = number of clauses (5)

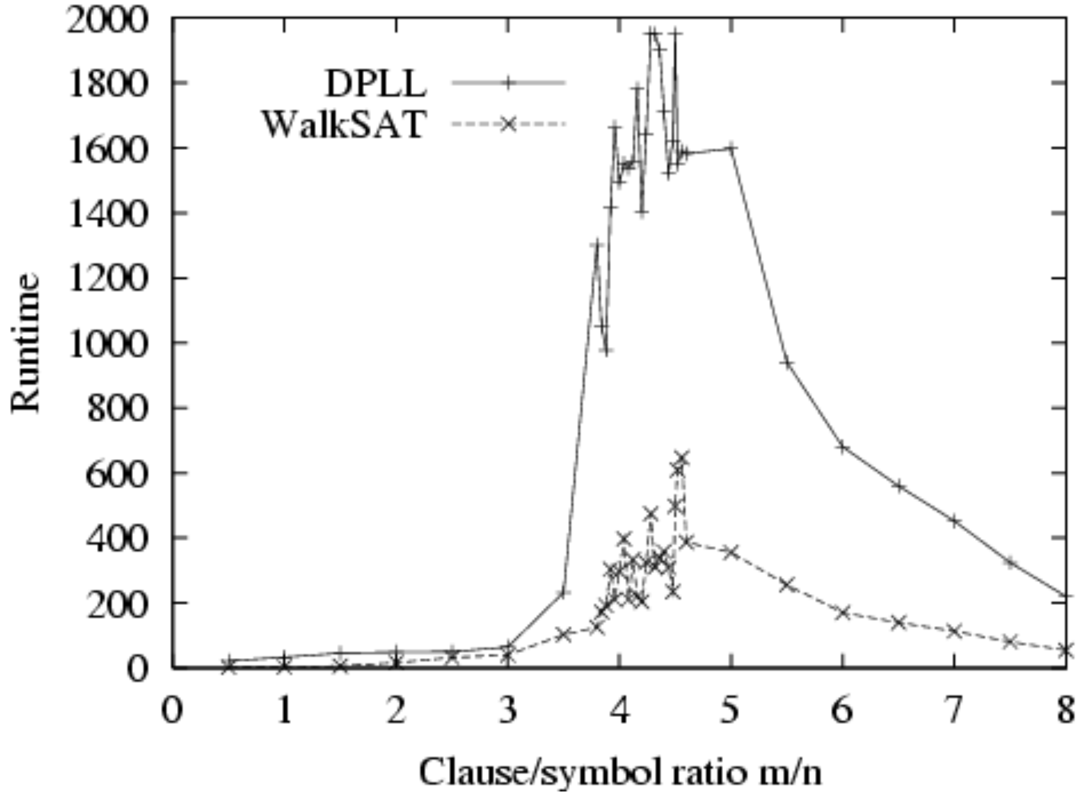
  *n* = number of symbols (5)

  – Hard problems seem to cluster near $m/n = 4.3$ (critical point)

# Hard satisfiability problems



# Hard satisfiability problems




- Median runtime for 100 satisfiable random 3-CNF sentences, *n* = 50

# Common Sense Reasoning

Example, adapted from Lenat

You are told: John drove to the grocery store and bought a pound of noodles, a pound of ground beef, and two pounds of tomatoes.

- Is John 3 years old?
- Is John a child?
- What will John do with the purchases?
- Did John have any money?
- Does John have less money after going to the store?
- Did John buy at least two tomatoes?
- Were the tomatoes made in the supermarket?
- Did John buy any meat?
- Is John a vegetarian?
- Will the tomatoes fit in John's car?

- Can Propositional Logic support these inferences?

# Summary

- Logical agents apply inference to a knowledge base to derive new information and make decisions

- Basic concepts of logic:
  - syntax: formal structure of sentences
  - semantics: truth of sentences wrt models
  - entailment: necessary truth of one sentence given another
  - inference: deriving sentences from other sentences
  - soundness: derivations produce only entailed sentences
  - completeness: derivations can produce all entailed sentences

- Resolution is complete for propositional logic.
  Forward and backward chaining are linear-time, complete for Horn clauses

- Propositional logic lacks expressive power