Functions

1 Functions

1.1 Functions and the Vertical Line Test

Definition: A **relation** that assign each element in a set A to exactly one element in a set B is a **function** from A to B. The set A is the **domain**, or set of allowable input values, and the set B is the **range**, or set of corresponding output values.

Vertical Line Test: If any vertical line intersects a graph more than once, then the graph is not a function.

Function Notation: **Function notation** can be used to write the equation of a function where the output value, y, is replaced with $f(x)$, which is read as "f of x". This notation

- names the function with a letter
- shows the input value
- shows the rule described by the function

Zeros: The **zeros** of a function $f(x)$ are the x-values where $f(x) = 0$. If the graph of f has an x-intercept at $(a, 0)$, then a is a zero of f.

2 Combining Functions

2.1 Using Operation with Functions

- Sum of Functions: $(f + g)(x) = f(x) + g(x)$
- Difference of Functions: $(f - g)(x) = f(x) - g(x)$
- Product of Functions: $(fg)(x) = f(x)g(x)$
- Quotient of Functions: $(\frac{f}{g})(x) = \frac{f(x)}{g(x)}$

2.2 Composition Functions

Definition: The **composition** of the function f with the function g, $f(g(x))$, is the result of evaluating f for g.

$$f(g(x)) = (f \circ g)(x)$$

The domain of $f \circ g$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f.

3 Inverse Function

Definition: If two functions f and g are **inverse functions** then

- for every x in the domain of f, $g(f(x)) = x$ and
- for every x in the domain of g, $f(g(x)) = x$.

The domain of f must be equal to the range of g, and the domain of g must be equal to the range of f.

Note: Not all the functions have an inverse function.

Horizontal Line test: A function f has an inverse function if any possible horizontal line can intersect the graph of f at most once. ($f(x) = x^2$ fails the Horizontal line test)

Steps for Find the Inverse Function of a Function f

1. Use the Horizontal Line Test to verify that the inverse function of f exists.
2. If the equation is in function notation, replace $f(x)$ with y.
3. Interchange x and y in the equation, and then solve for y.
4. Replace y with f^{-1} in the equation from step (3).