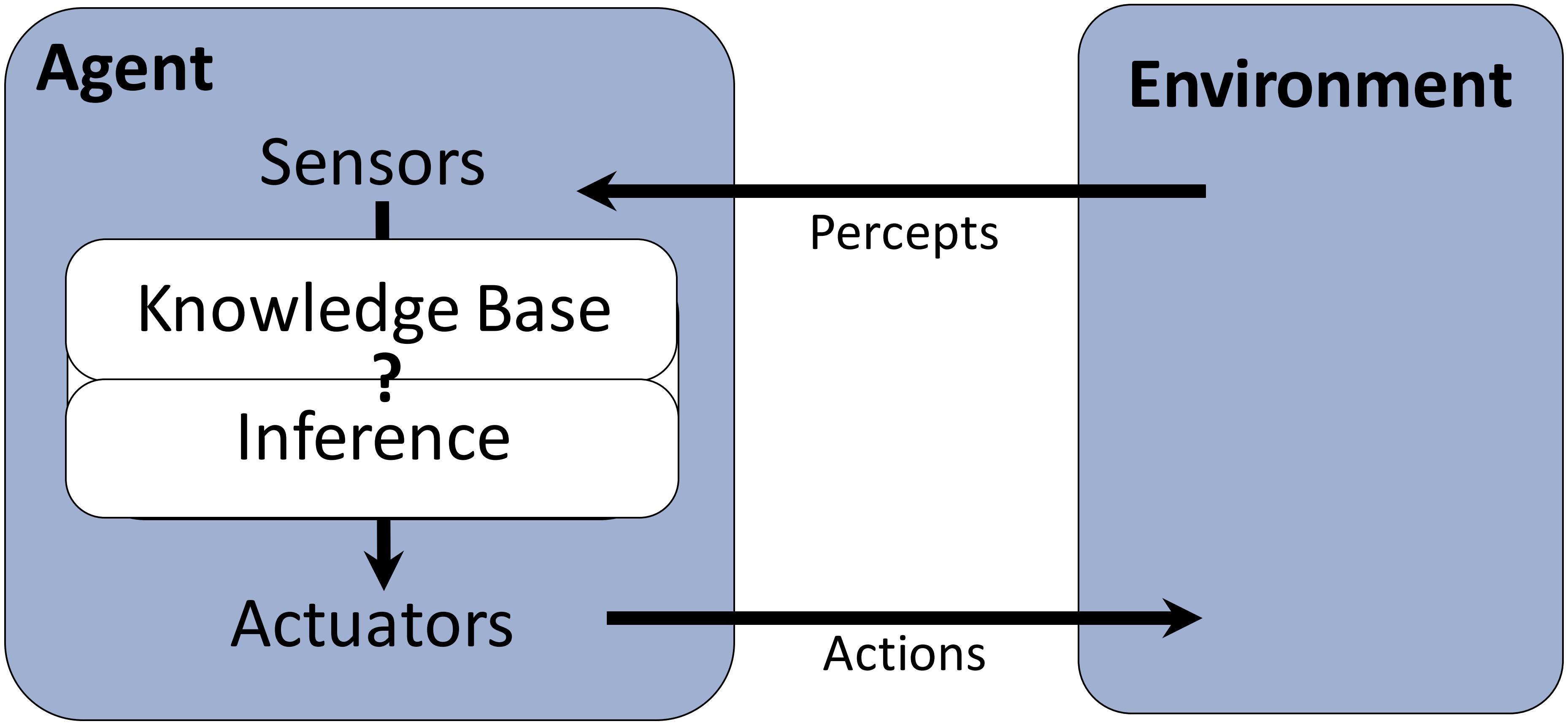
TFIP-AI - Machine Learning

Unit 3 Logical and Reasoning Systems

Part 1 Propositional Logic

Logical Agents

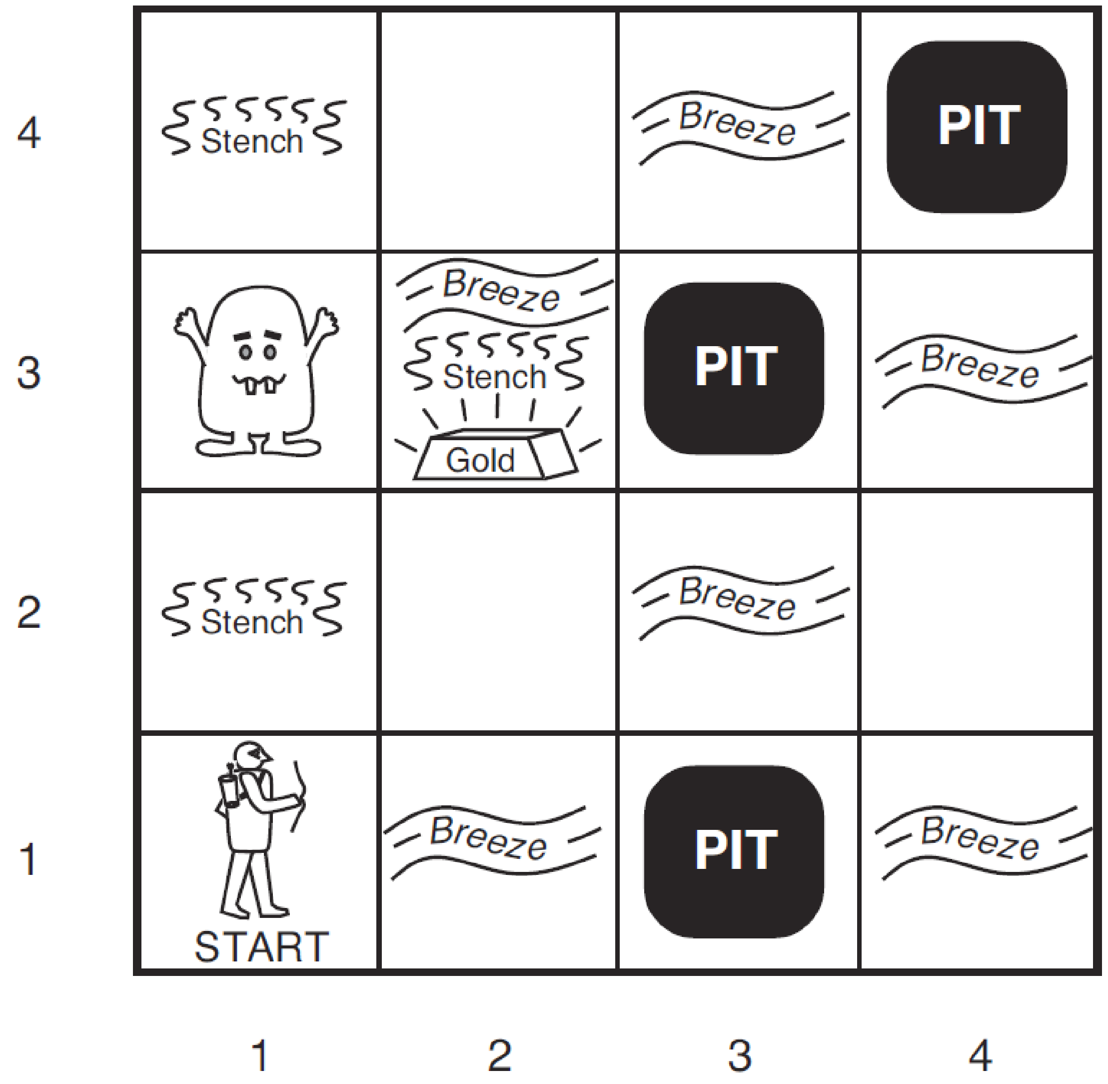
Logical agents and environments



Wumpus World

Logical Reasoning as a CSP

- B_{ij} = breeze felt
- S_{ij} = stench smelt
- P_{ij} = pit here
- W_{ij} = wumpus here
- G = gold



http://thiagodnf.github.io/wumpus-world-simulator/

A Knowledge-based Agent

```
function KB-AGENT(percept) returns an action
  persistent: KB, a knowledge base
              t, an integer, initially 0

  TELL(KB, PROCESS-PERCEPT(percept, t))
  action ← ASK(KB, PROCESS-QUERY(t))
  TELL(KB, PROCESS-RESULT(action, t))
  t←t+1
  return action
```

Logical Agents

So what do we TELL our knowledge base (KB)?

- Facts (sentences)
  - The grass is green
  - The sky is blue
- Rules (sentences)
  - Eating too much candy makes you sick
  - When you’re sick you don’t go to school
- Percepts and Actions (sentences)
  - Pat ate too much candy today

What happens when we ASK the agent?

- Inference – new sentences created from old
  - Pat is not going to school today

Logical Agents

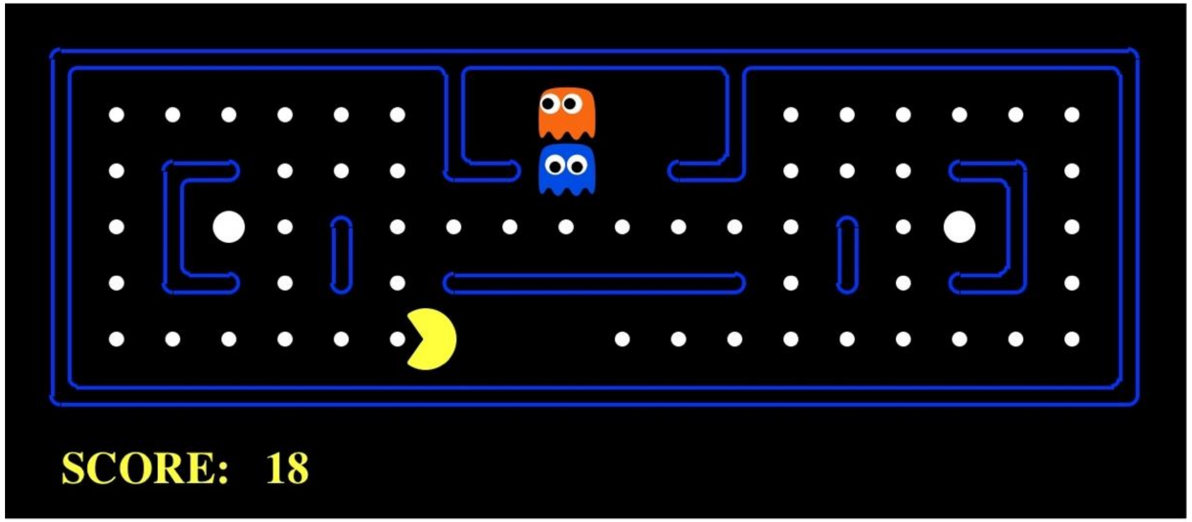
Sherlock Agent

- Really good knowledge base
  - Evidence
  - Understanding of how the world works (physics, chemistry, sociology)
- Really good inference
  - Skills of deduction
  - “It’s elementary my dear Watson”

Dr. Strange?
Alan Turing?
Kahn?

Worlds

What are we trying to figure out?



- Who, what, when, where, why
- Time: past, present, future

- Actions, strategy
- Partially observable? Ghosts, Walls

Which world are we living in?

Models

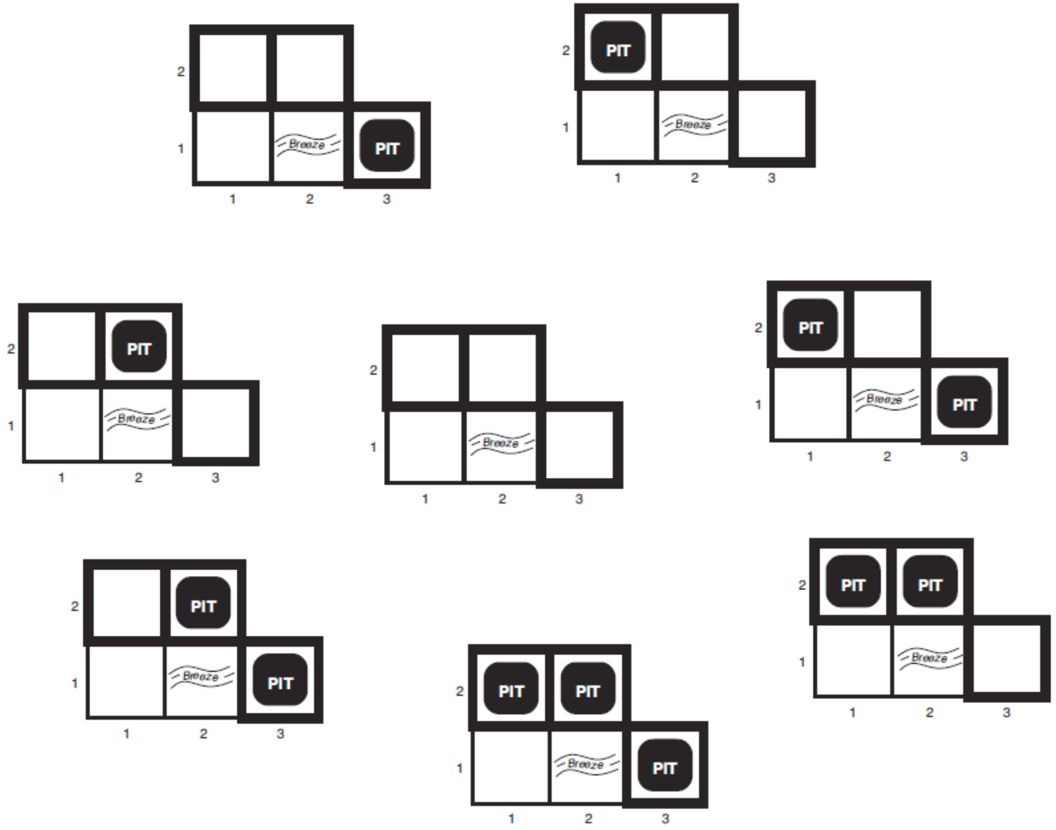
How do we represent possible worlds with models and knowledge bases?

How do we then do inference with these representations?

Wumpus World

Possible Models

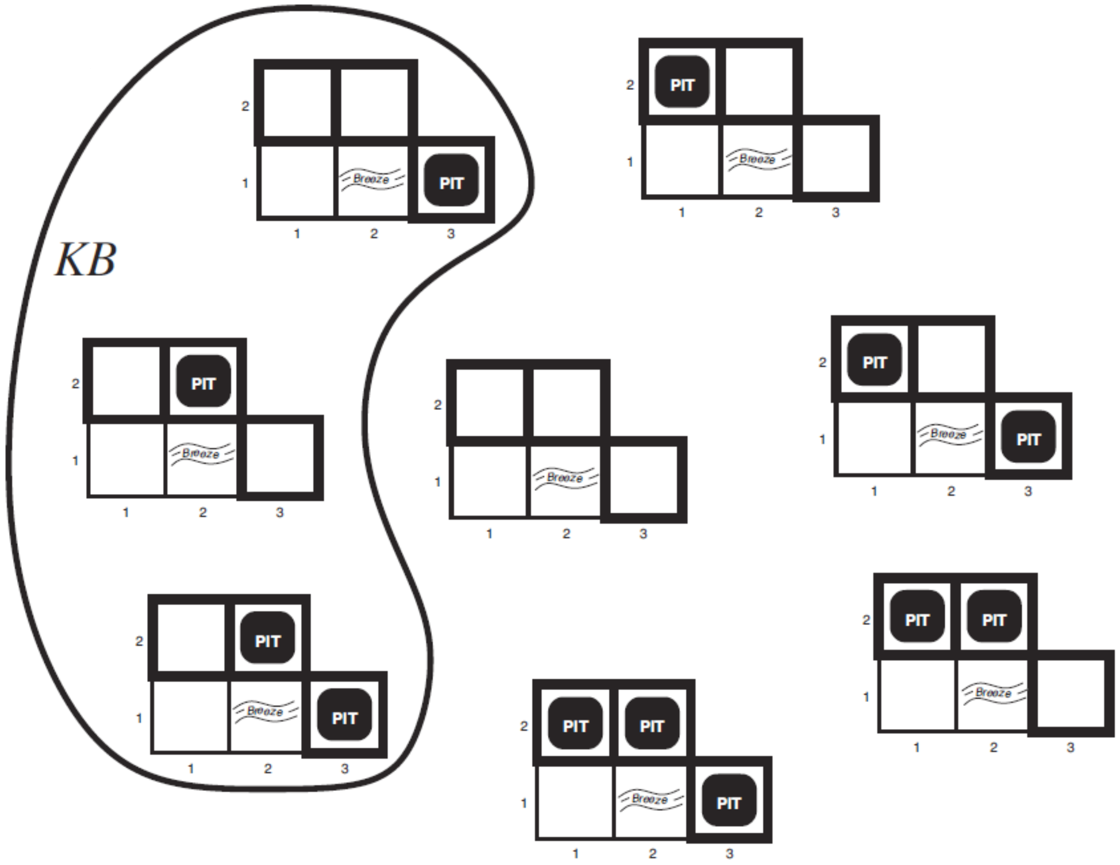
- $P_{1,2}$ $P_{2,2}$ $P_{3,1}$



Wumpus World

Possible Models

- $P_{1,2}$ $P_{2,2}$ $P_{3,1}$
- Knowledge base
  - Nothing in [1,1]
  - Breeze in [2,1]



Wumpus World

Possible Models

- $P_{1,2}$ $P_{2,2}$ $P_{3,1}$
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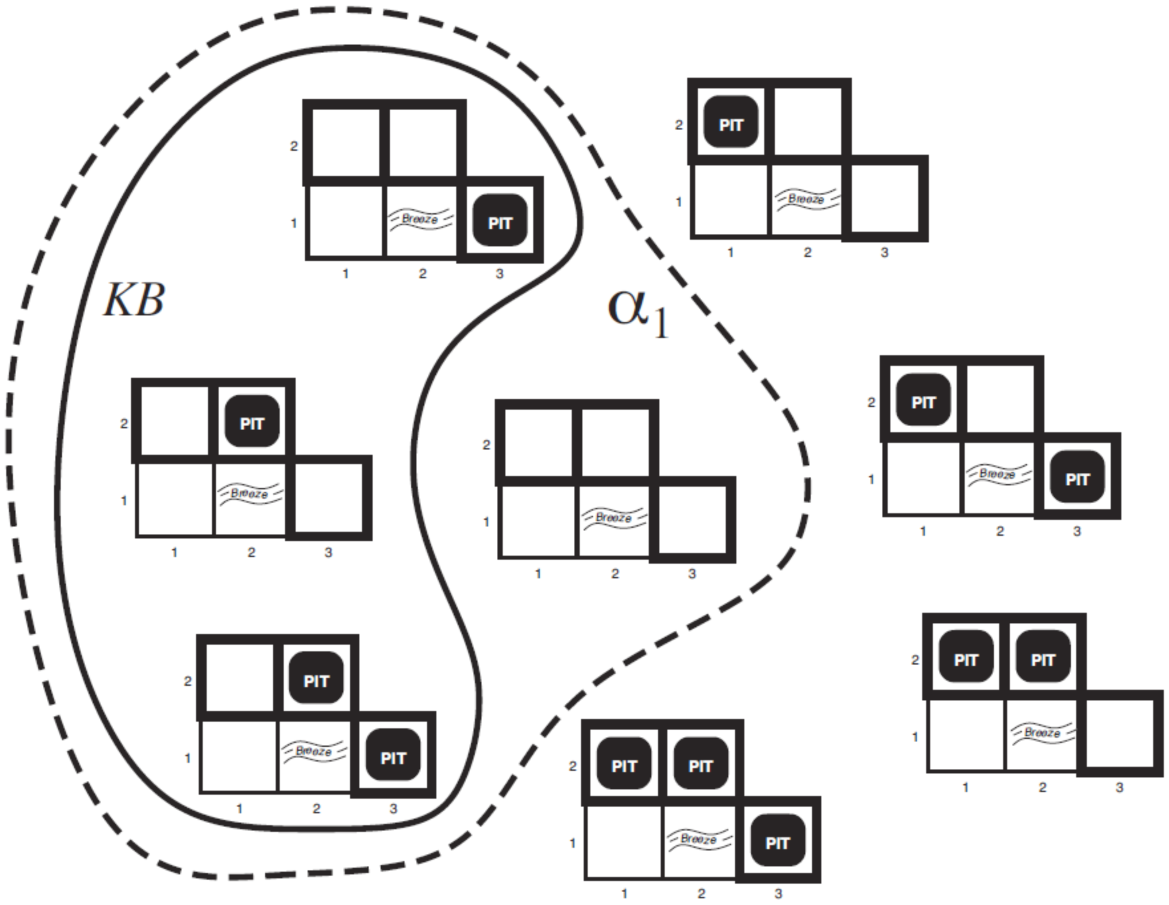
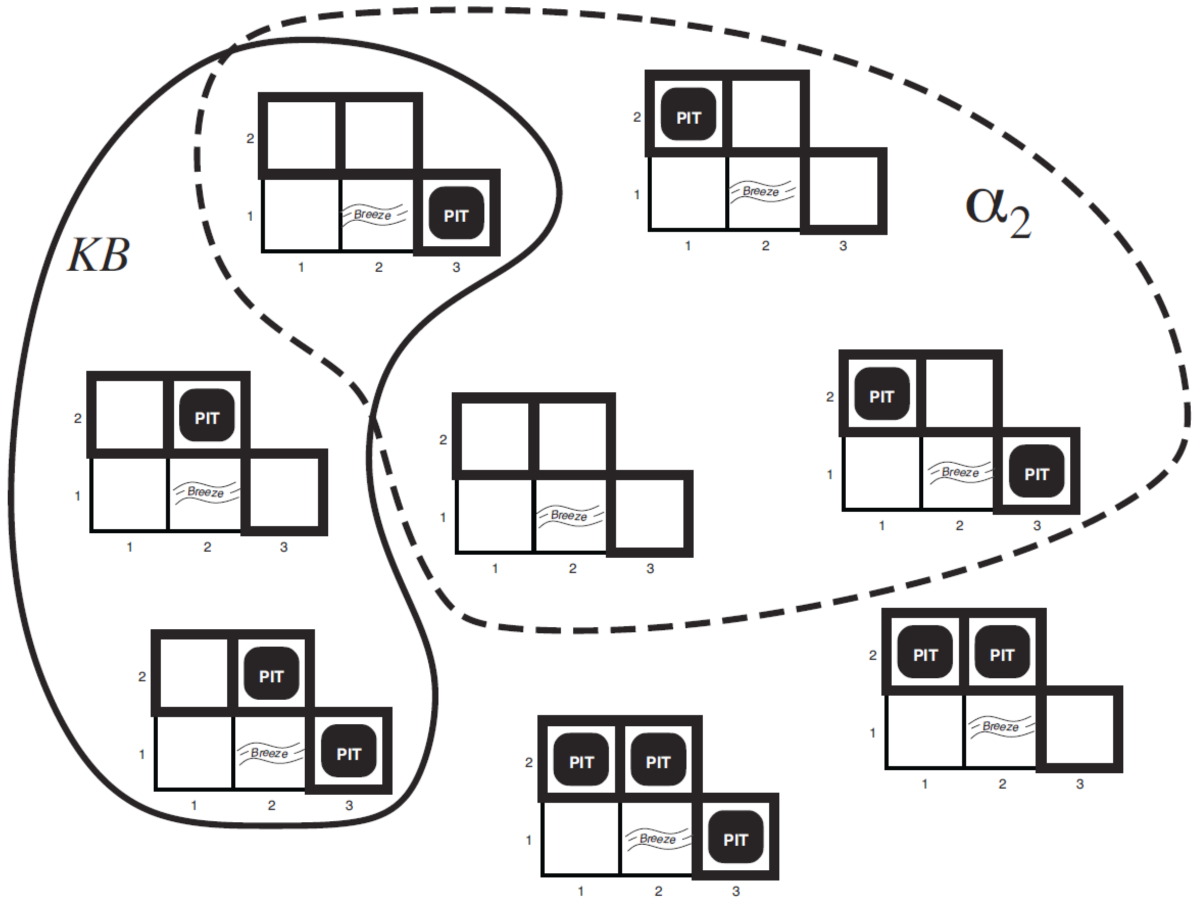
- Query α_1:
  - No pit in [1,2]



Wumpus World

Possible Models

- $P_{1,2}$ $P_{2,2}$ $P_{3,1}$
- Knowledge base
  - Nothing in [1,1]
  - Breeze in [2,1]
- Query α_2:
  - No pit in [2,2]



Logic Language

Natural language?

Propositional logic

- Syntax: $P \vee (\neg Q \wedge R)$; $X_1 \Leftrightarrow (\text{Raining} \Rightarrow \text{Sunny})$
- Possible world: {P=true, Q=true, R=false, S=true} or 1101
- Semantics: $\alpha \wedge \beta$ is true in a world iff is α true and β is true (etc.)

First-order logic

- Syntax: $\forall x\ \exists y\ P(x,y) \wedge \neg Q(Joe,f(x)) \Rightarrow f(x)=f(y)$
- Possible world: Objects o_1, o_2, o_3; P holds for <o_1,o_2>; Q holds for <o_3>; $f(o_1)=o_1$; $Joe=o_3$; etc.
- Semantics: $\phi(\sigma)$ is true in a world if $\sigma=o_j$ and ϕ holds for o_j; etc.

Propositional Logic

Propositional Logic

Symbol:

- Variable that can be true or false
- We'll try to use capital letters, e.g. A, B, $P_{1,2}$
- Often include True and False

Operators:

- $\neg$ A: not A
- A $\wedge$ B: A and B (conjunction)
- A $\vee$ B: A or B (disjunction) Note: this is not an "exclusive or"
- A $\Rightarrow$ B: A implies B (implication). If A then B
- A $\Leftrightarrow$ B: A if and only if B (biconditional)

Sentences

Propositional Logic Syntax

Given: a set of proposition symbols $\{X_1, X_2, \ldots, X_n\}$

- (we often add True and False for convenience)

X_i is a sentence

If α is a sentence then $\neg\alpha$ is a sentence

If α and β are sentences then $\alpha \wedge \beta$ is a sentence

If α and β are sentences then $\alpha \vee \beta$ is a sentence

If α and β are sentences then $\alpha \Rightarrow \beta$ is a sentence

If α and β are sentences then $\alpha \Leftrightarrow \beta$ is a sentence

And p.s. there are no other sentences!

Notes on Operators

$\boldsymbol{\alpha} \vee \boldsymbol{\beta}$ is <u>inclusive or</u>, not exclusive

Truth Tables

$\boldsymbol{\alpha} \vee \boldsymbol{\beta}$ is inclusive or, not exclusive

α	β	$\alpha \wedge \beta$
F	F	F
F	T	F
T	F	F
T	T	T

α	β	$\alpha \vee \beta$
F	F	F
F	T	T
T	F	T
T	T	T

Notes on Operators

$\boldsymbol{\alpha} \lor \boldsymbol{\beta}$ is inclusive or, not exclusive

$\boldsymbol{\alpha} \Rightarrow \boldsymbol{\beta}$ is equivalent to $\neg\boldsymbol{\alpha} \lor \boldsymbol{\beta}$

- Says who?

Truth Tables

$\alpha \Rightarrow \beta$ is equivalent to $\neg\alpha \vee \beta$

α	β	$\alpha \Rightarrow \beta$	$\neg\alpha$	$\neg\alpha \vee \beta$
F	F	T	T	T
F	T	T	T	T
T	F	F	F	F
T	T	T	F	T

Notes on Operators

$\alpha \lor \beta$ is <u>inclusive or</u>, not exclusive

$\alpha \Rightarrow \beta$ is equivalent to $\neg\alpha \lor \beta$

- Says who?

$\alpha \Leftrightarrow \beta$ is equivalent to $(\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)$

- Prove it!

Truth Tables

$\boldsymbol{\alpha} \Leftrightarrow \boldsymbol{\beta}$ is equivalent to $(\boldsymbol{\alpha} \Rightarrow \boldsymbol{\beta}) \wedge (\boldsymbol{\beta} \Rightarrow \boldsymbol{\alpha})$

$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	$\boldsymbol{\alpha} \Leftrightarrow \boldsymbol{\beta}$	$\boldsymbol{\alpha} \Rightarrow \boldsymbol{\beta}$	$\boldsymbol{\beta} \Rightarrow \boldsymbol{\alpha}$	$(\boldsymbol{\alpha} \Rightarrow \boldsymbol{\beta}) \wedge (\boldsymbol{\beta} \Rightarrow \boldsymbol{\alpha})$
F	F	T	T	T	T
F	T	F	T	F	F
T	F	F	F	T	F
T	T	T	T	T	T

Equivalence: it's true in all models. Expressed as a logical sentence:

$$(\boldsymbol{\alpha} \Leftrightarrow \boldsymbol{\beta}) \Leftrightarrow [(\boldsymbol{\alpha} \Rightarrow \boldsymbol{\beta}) \wedge (\boldsymbol{\beta} \Rightarrow \boldsymbol{\alpha})]$$

Literals

A *literal* is an atomic sentence:

- True
- False
- Symbol
- $\neg$ Symbol

Monty Python Inference

There are ways of telling whether she is a witch

Sentences as Constraints

Adding a sentence to our knowledge base constrains the number of possible models:

KB: Nothing

Possible Models

P	Q	R
false	false	false
false	false	true
false	true	false
false	true	true
true	false	false
true	false	true
true	true	false
true	true	true

Sentences as Constraints

Adding a sentence to our knowledge base constrains the number of possible models:

KB: Nothing

KB: $[(P \land \neg Q) \lor (Q \land \neg P)] \Rightarrow R$

Assign: {Q:1, P:0, R:0} ,i.e. row 3, then

[(0   ¬1)   (1   ¬0)]   0

[(0   0)   (1   1)]   0

[0   1]   0 (Contradiction!)

Therefore, this model is NOT possible.

Possible Models

P	Q	R
false	false	false
false	false	true
false	true	false
false	true	true
true	false	false
true	false	true
true	true	false
true	true	true

Sentences as Constraints

Adding a sentence to our knowledge base constrains the number of possible models:

KB: Nothing

KB: $[(P \land \neg Q) \lor (Q \land \neg P)] \Rightarrow R$

KB: R, $[(P \land \neg Q) \lor (Q \land \neg P)] \Rightarrow R$

R must be true because R is now included in KB.

Possible Models

P	Q	R
false	false	false
false	false	true
false	true	false
false	true	true
true	false	false
true	false	true
true	true	false
true	true	true

Sherlock Entailment

“When you have eliminated the impossible, whatever remains, however improbable, must be the truth” – *Sherlock Holmes via Sir Arthur Conan Doyle*

- Knowledge base and inference allow us to remove impossible models, helping us to see what is true in all of the remaining models

Entailment

Entailment: $\alpha \models \beta$ ("α entails β" or "β follows from α") iff in every world where α is true, β is also true

- I.e., the α-worlds are a subset of the β-worlds [*models*(α) $\subseteq$ *models*(β)]

Usually we want to know if *KB* $\models$ *query*

- *models*(*KB*) $\subseteq$ *models*(*query*)
- In other words
  - *KB* removes all impossible models (any model where *KB* is false)
  - If β is true in all of these remaining models, we conclude that β must be true

Entailment and implication are very much related

- However, entailment relates two sentences, while an implication is itself a sentence (usually derived via inference to show entailment)

Wumpus World

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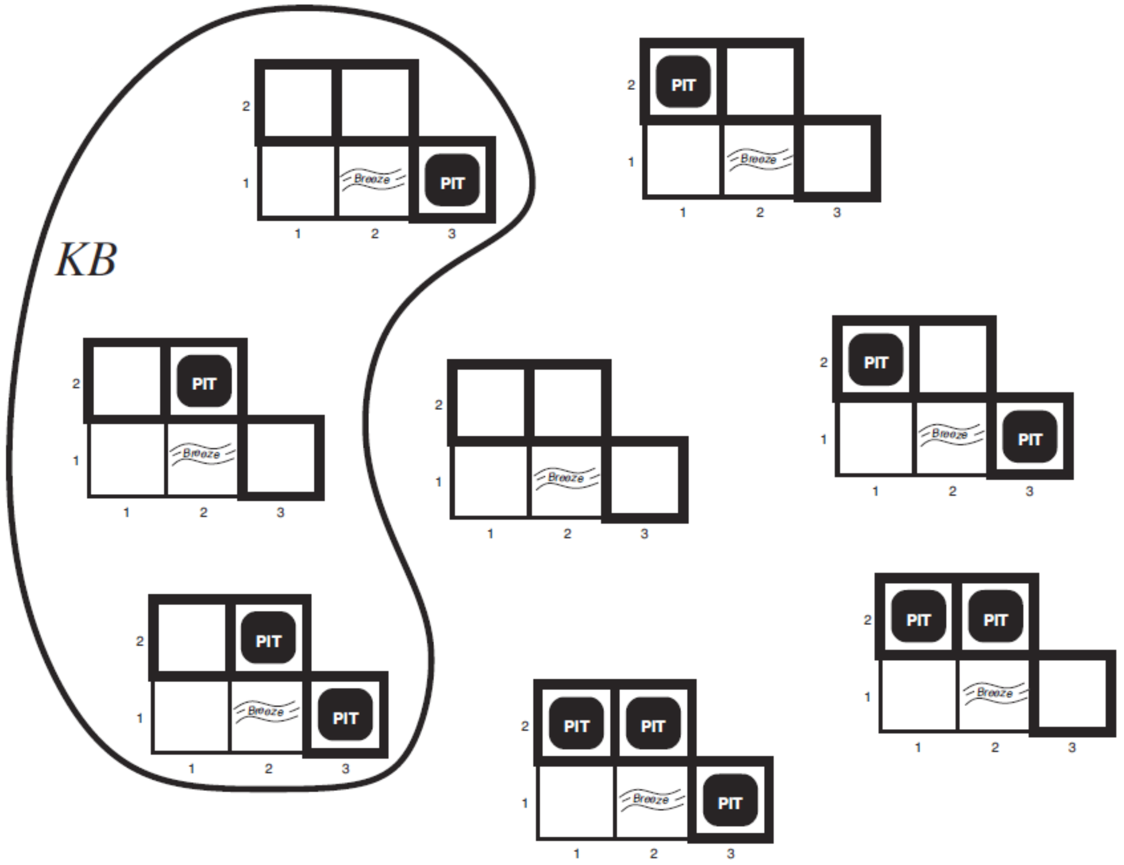

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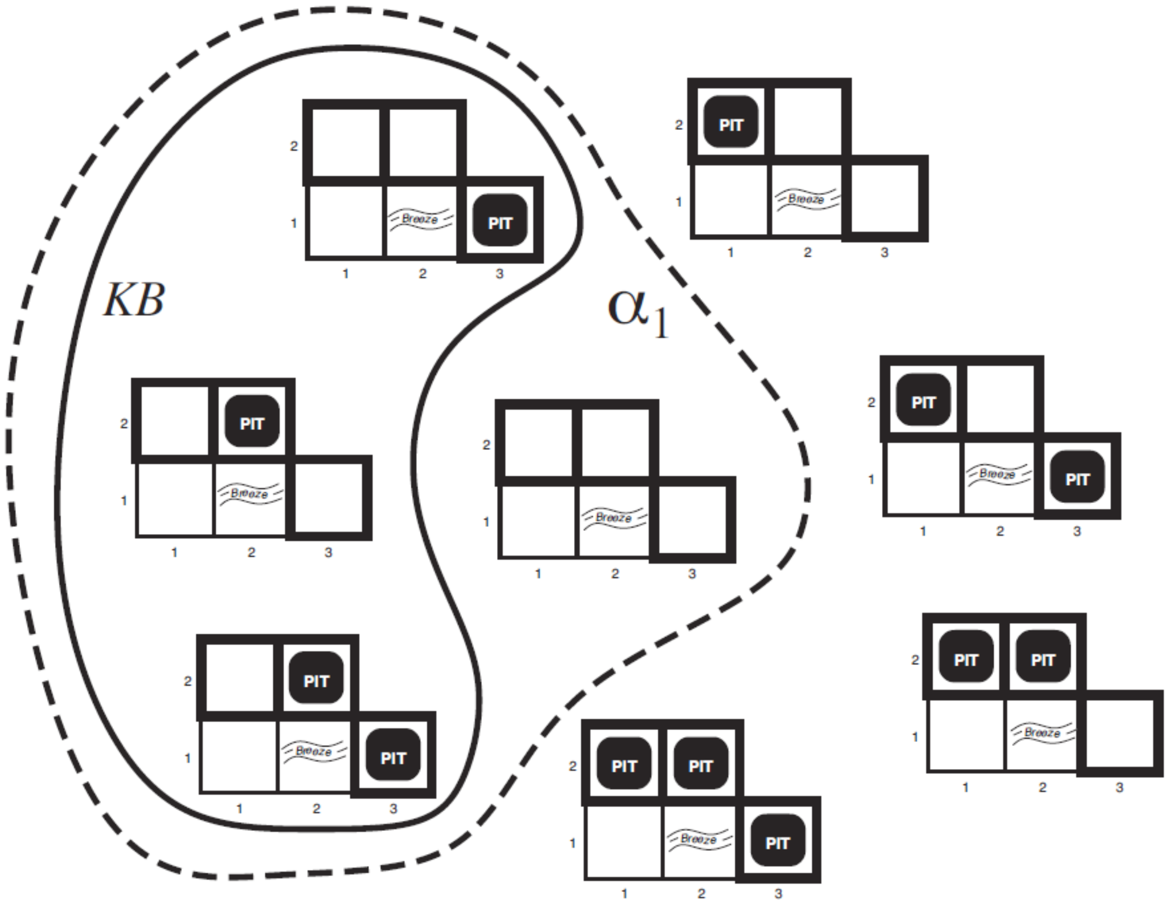

Wumpus World

Possible Models

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- Knowledge base
    - Nothing in [1,1]
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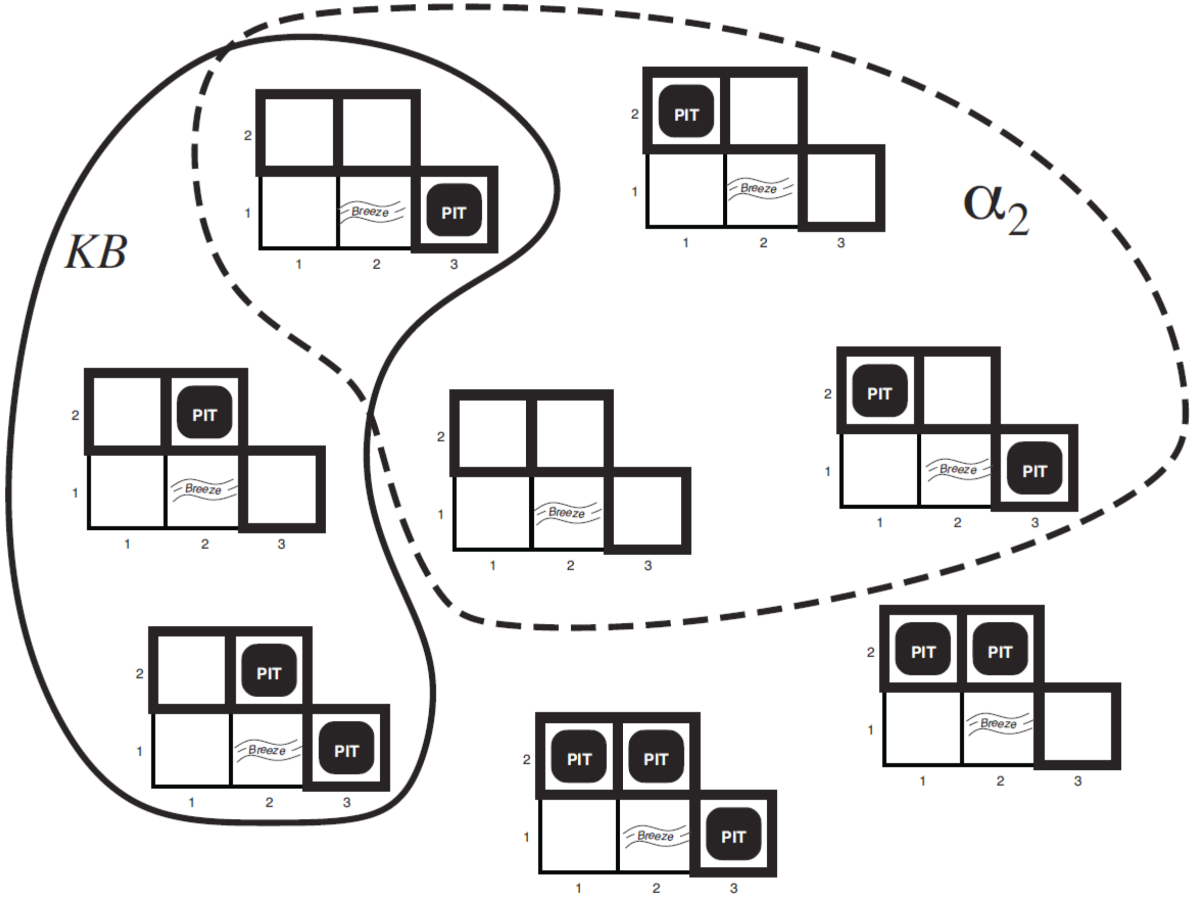

Propositional Logic Models

All Possible Models

Model Symbols

A	0	0	0	0	1	1	1	1
B	0	0	1	1	0	0	1	1
C	0	1	0	1	0	1	0	1

What is the answer for this?

Does the KB entail query C?

Entailment: $\alpha \models \beta$

"α entails β" iff in every world where α is true, β is also true

All Possible Models

Model Symbols	A	0	0	0	0	1	1	1	1
	B	0	0	1	1	0	0	1	1
	C	0	1	0	1	0	1	0	1
Knowledge Base	A	0	0	0	0	1	1	1	1
	$B \Rightarrow C$	1	1	0	1	1	1	0	1
	$A \Rightarrow B \vee C$	1	1	1	1	0	1	1	1
Query	C	0	1	0	1	0	1	0	1

Entailment

How do we implement a logical agent that proves entailment?

- Logic language
  - Propositional logic
  - First order logic
- Inference algorithms
  - Theorem proving
  - Model checking

Given KB: {A, B=>C, A=> B V C} [take as {first clause, second clause, third clause}]
Question, does KB entail C?

Solving strategy: Compute KB ¬C.
If KB ¬C = empty clause, it means KB entails C. Otherwise, the answer can be, 1) no, KB does NOT entail C OR 2) no idea (I don't know).

Write KB as a conjunctive: KB = A B -> C A=> B V C, then write KB ¬C, meaning
A B => C A=> B V C ¬C
= A A=> B V C B => C ¬C (swap second and third clause)
= A ¬A V B V C ¬B V C ¬C
= 0 V B V C ¬B V 0 ¬C (A ¬A is 0 (FALSE) same for C ¬C and C V C = C)
= B V C ¬B ¬C
= B ¬B V C ¬C
= 0 0 = 0

Propositional Logic

Check if sentence is true in given model

In other words, does the model *satisfy* the sentence?

function PL-TRUE?(α,model) returns true or false

  if α is a symbol then return Lookup(α, model)

  if Op(α) = $\neg$ then return not(PL-TRUE?(Arg1(α),model))

  if Op(α) = $\wedge$ then return and(PL-TRUE?(Arg1(α),model),

  PL-TRUE?(Arg2(α),model))

  etc.

(Sometimes called "recursion over syntax")

Simple Model Checking

```
function TT-ENTAILS?(KB, α) returns true or false
    return TT-CHECK-ALL(KB, α, symbols(KB) U symbols(α),{})

function TT-CHECK-ALL(KB, α, symbols,model) returns true or false
    if empty?(symbols) then
        if PL-TRUE?(KB, model) then return PL-TRUE?(α, model)
        else return true
    else
        P ← first(symbols)
        rest ← rest(symbols)
        return  and (TT-CHECK-ALL(KB, α, rest, model ∪ {P = true})
                    TT-CHECK-ALL(KB, α, rest, model ∪ {P = false }))
```

Simple Model Checking, contd.

Same recursion as backtracking

$O(2^n)$ time, linear space

We can do much better!