IBF TFIP Calculus Day 6 Exercise. Uploaded.
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Q1) $f'(x) = 1+2x$, $f''(x) = 2$, $a=1$

$f(x) = f(1) + \frac{f'(1)}{1!}(x-1) + \frac{f''(1)}{2!}(x-1)^2$

$= 3 + \frac{1+2}{1!}(x-1) + \frac{2}{2!}(x-1)^2 = 3 + 3(x-1) + (x-1)^2$. ✓

Q2) $f(x) = f(a) + f'(a)\frac{(x-a)}{1!} + f''(a)\frac{(x-a)^2}{2!} + f'''(a)\frac{(x-a)^3}{3!} + f^{(4)}(a)\frac{(x-a)^4}{4!}$.

$f(x) = \cos 2x$; $f'(x) = -2\sin 2x$; $f''(x) = -4\cos 2x$; $f'''(x) = 8\sin 2x$

$f^{(4)}(x) = 16\cos 2x$; $f(\pi) = \cos(2\pi) = 1$; $f'(\pi) = -2\sin 2\pi = 0$; $f''(\pi) = -4$; $f^{(3)}(\pi) = 0$

$f^{(4)}(\pi) = 16$. $\frac{4}{2!} = 2$ $\frac{4}{2!} = 2$ $f(x) = 1 - 2(x-\pi)^2 + \frac{2}{3}(x-\pi)^4$

$f(x) = 1 - 4(x-\pi)^2 + \frac{16}{4!}(x-\pi)^4 = 1 - 4(x-\pi)^2 + \frac{2}{3}(x-\pi)^4$

Q3) $f(x,y) = f(a,b) + \frac{\partial f}{\partial x}\Big|_{a,b}(x-a) + \frac{\partial f}{\partial y}\Big|_{a,b}(y-b) + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}\Big|_{a,b}(x-a)^2$
$+ \frac{1}{2}\frac{\partial^2 f}{\partial y^2}\Big|_{a,b}(y-b)^2 + (y-b)(x-a)\frac{\partial^2 f}{\partial x \partial y}\Big|_{a,b}$ at $(x,y) = (a,b)$.

$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x\sqrt{y}) = \sqrt{y}$; $\frac{\partial f}{\partial x}\Big|_{1,4} = \sqrt{4} = 2$; $\frac{\partial f}{\partial y} = \frac{1}{2}xy^{-\frac{1}{2}}$; $\frac{\partial f}{\partial y}\Big|_{1,4} = \frac{1}{2}(1)\frac{1}{\sqrt{4}} = \frac{1}{4}$

$\frac{\partial^2 f}{\partial x^2} = 0$; $\frac{\partial^2 f}{\partial y^2} = -\frac{1}{4}xy^{-\frac{3}{2}}$; $\frac{\partial^2 f}{\partial y^2}\Big|_{1,4} = -\frac{1}{4}(1)(4)^{-3/2} = -\frac{1}{32}$.

$\frac{\partial^2 f}{\partial x \partial y} = \frac{1}{2}y^{-\frac{1}{2}}$; $\frac{\partial^2 f}{\partial x \partial y}\Big|_{1,4} = \frac{1}{2}(4)^{-\frac{1}{2}} = \frac{1}{4}$.

$f(x,y) = 2 + 2(x-1) + \frac{1}{4}(y-4) + \frac{1}{2}\left(-\frac{1}{32}\right)(y-4)^2 + (x-1)(y-4)\left(\frac{1}{4}\right)$

$= 2 + 2(x-1) + \frac{1}{4}(y-4) - \frac{1}{64}(y-4)^2 + \frac{1}{4}(x-1)(y-4)$. ✓

Q4) $\frac{\partial f}{\partial x} = 12x^2y^2 - 2\frac{z^3}{x^3} - 16x^{15}$ ✓; $\frac{\partial f}{\partial z} = -e^z y^4 + \frac{3}{x^2}z^2$ ✓

$\frac{\partial f}{\partial y} = 8x^3y - 4e^z y^3 + 4$ ✓

Q5) $\frac{\partial f}{\partial x} = \left[\frac{\partial f}{\partial u} \ \frac{\partial f}{\partial v}\right]$; $H(f) = \begin{bmatrix} \frac{\partial^2 f}{\partial u^2} & \frac{\partial^2 f}{\partial u \partial v} \\ \frac{\partial^2 f}{\partial v \partial u} & \frac{\partial^2 f}{\partial v^2} \end{bmatrix}$

$\frac{\partial f}{\partial u} = 2u\sin(u+v^3) + u^2\cos(u+v^3) - \tan^{-1}(2v)\sec(4u)\tan(4u)\cdot(4)$.

$\frac{\partial f}{\partial v} = 3v^2u^2\cos(u+v^3) - \sec(4u)\frac{1}{1+(2v)^2}$ $\boxed{\frac{d}{dv}(2v) = 2}$

$= 3v^2u^2\cos(u+v^3) - \sec(4u)\left(\frac{1}{1+4v^2}\right)$ ✗ $\cdot \frac{2}{1+4v^2}$.

Used Sympy for rest.

Q6) $f(x) = (x+10)^{\frac{1}{2}}$; $f'(x) = \frac{1}{2}(x+10)^{-\frac{1}{2}}$

$f(x) \simeq 1 + f'(x)\cdot x = 1 + \frac{1}{2}(x+10)^{-\frac{1}{2}}\cdot x$ ✗ LP: $f(x) \simeq f(a) + \frac{df}{dx}\Big|_{x=a}(x-a)$.

TA: $f(x) \simeq f(a) + f'(a)(x-a) = \sqrt{11} + \frac{1}{2\sqrt{11}}(x-1)$. ✓

$f(0.04) = \sqrt{11} + \frac{1}{2\sqrt{11}}(0.04-1) = 3.172$; $f(-0.03) = \sqrt{11} + \frac{1}{2\sqrt{11}}(-0.03-1) = 3.163$.
✓? ✓?

Q7) $f(x,y) = x^2 y$, point$(3,8)$. ~~f(x)=f~~

$f(x,y) \cong f(a,b) + \frac{\partial f}{\partial x}\Big|_{x=a}(x-a) + \frac{\partial f}{\partial y}\Big|_{y=b}(y-b)$.

$= (3)^2(8) + 2xy\big|_{(3,8)}(x-3) + x^2\big|_{(3,8)}(y-8)$

$= 72 + 48(x-3) + 9(y-8)$.

Q8) $A = \begin{bmatrix} x_1 x_2 & x_3 & x_2+x_3 \\ x_1 & x_2 & x_3 \\ x_1+x_3 & x_1 & x_1^2 x_2 \end{bmatrix}$

$\frac{dA}{dx} = \left[\frac{dA}{dx}\right]_j$; $\left(\frac{dA}{dx}\right)_1 = \frac{dA}{dx_1} = \begin{bmatrix} x_2 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 2x_1x_2 \end{bmatrix}$

$\frac{dA}{dx_2} = \begin{bmatrix} x_1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & x_1^2 \end{bmatrix}$; $\frac{dA}{dx_3} = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$.

Q9) $R(x,y) = x^2(y^2+1)^{-1} - y^2(x^2+y)^{-1}$

$\frac{\partial R}{\partial x} = 2x(y^2+1)^{-1} - y^2(x^2+y)^{-2}(-1)(2x)$

$= \frac{2x}{y^2+1} + \frac{2xy^2}{(x^2+y)^2}$

$\frac{\partial R}{\partial x} = -x^2(y^2+1)^{-2}(2y) - [2y(x^2+y)^{-1} + y^2(-1)(x^2+y)^{-2}]$

$= \frac{-x^2(2y)}{(y^2+1)^2} - \frac{2y}{x^2+y} + \frac{y^2}{(x^2+y)^2}$.

Q10) $\Delta y = \frac{dy}{dx}\Delta x$

$= -400 + 0.4p$

At $p = 300$, $\Delta x = 0.1$, $\Delta y = [-400 + 0.4(300)](0.1)$

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