DigiPen Institute of Technology Singapore

IBF– Day 2 Exericise
Linear Algebra

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Name: ____

This exercise contains 7 pages (including this cover page) and 10 questions. Total of points is 100.
Good luck and Happy reading work!

Distribution of Marks

Question:	1	2	3	4	5	6
Points:	5	10	20	10	10	5
Score:						

Question:	7	8	9	10		Total
Points:	10	5	5	20		100
Score:						

1. (5 points) For the following pairs of matrices, determine if the sum $A+B$ is defined. If so, find the sum.

   - $$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 & 3 \\ 0 & 1 & 4 \end{bmatrix}$$

   - $$A = \begin{bmatrix} 1 & 0 \\ -2 & 3 \\ 4 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & 7 & -1 \\ 0 & 3 & 4 \end{bmatrix}$$

2. (10 points) For matrix A, find the product $(-2)A, 0A, 3A$.

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & -1 & 3 \\ 4 & 2 & 0 \end{bmatrix}$$

3. (20 points) Consider the matrices $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ -3 & 2 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$, $D = \begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix}$, $E = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$. Find the following if possible, If it is not possible explain why.

   1. $-3A$
   2. $3B - A$
   3. AC
   4. CB
   5. AE
   6. EA

4. (10 points) Let $X = [-1 - 11]$ and $Y = [012]$. Find $X^T Y$ and XY^T if possible.

5. (10 points) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & k \end{bmatrix}$. Is it possible to choose k such that $AB = BA$? If so what should k equal?

6. (5 points) Find 2×2 matrices, A, B, Csuch that $A \neq 0$, $C \neq B$, but $AC = AB$.

7. (10 points) Write the system

$$x_1 - x_2 + 2x_3$$

$$2x_3 + x_1$$

$$3x_3$$

$$3x_4 + 3x_2 + x_1$$

In the form $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ where A is an appropriate matrix.

8. (5 points) A matrix A is called idempotent if $A^2 = A$. Let

$$A = \begin{bmatrix} 2 & 0 & 2 \\ 1 & 1 & 2 \\ -1 & 0 & -1 \end{bmatrix}$$

and show that A is idempotent.

9. (5 points) Find the $(1,2)$-entry and $(2,3)$ - entry of the product AB given

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 4 & 0 \\ 2 & 5 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & 6 & -2 \\ 7 & 2 & 1 \\ -1 & 0 & 0 \end{bmatrix}$$

10. (20 points) Consider the matrices $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \\ 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -5 & 2 \\ -3 & 2 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 \\ 5 & 0 \end{bmatrix}$, $D = \begin{bmatrix} -1 & 1 \\ 4 & -3 \end{bmatrix}$, $E = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$. Find the following if possible. If it is not possible explain why.

1. $-3A^T$
2. $3B - A^T$
3. $E^T B$
4. EE^T
5. $B^T B$
6. CA^T
7. $D^T BE$

This page is intentionally left blank to accommodate work that wouldn't fit elsewhere and/or scratch work.