DigiPen Institute of Technology Singapore

IBF– Day 7 Exericise
Linear Algebra

Instructor: Yilin Wu

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Name: _____

This exercise contains 7 pages (including this cover page) and 10 questions. Total of points is 110. Good luck and Happy reading work!

Distribution of Marks

Question:	1	2	3	4	5	6
Points:	10	10	10	10	10	10
Score:						
Question:	7	8	9	10		Total
Points:	20	10	10	10		110
Score:						

1. (10 points) Find the number of positive integers not exceeding 1000 that are neither the square nor the cube of a positive integer.

2. (10 points) How many ordered, nonnegative integer triples (x, y, z) satisfy the equation $x + y + z = 15$?

3. (10 points) A circular table has exactly 60 chairs around it. There are N people seated around this table in such a way that the next person to be seated must sit next to someone. What is smallest possible value of N?

4. (10 points) A bag contains a number of marbles of which 78 are red, 24 are blue, and the rest are green. If the probability of selecting a green marble is 1/3, what is the probability of selecting a red marble?

5. (10 points) An experiment consists of rolling a 20 sided die three times. The number on top of each die is recorded. The numbers are written down in the order in which they are observed. How many possible ordered triples of numbers can result from the experiment? (Note the triple $(17, 10, 3)$ is not the same result as the triple $(3, 10, 17)$.)

6. (10 points) When you buy a Powerball ticket, you select 5 different white numbers from among the numbers 1 through 59 (order of selection does not matter), and one red number from among the numbers 1 through 35. How many different Powerball tickets can you buy?

7. (20 points) Recall that a standard deck of cards has 52 cards. The cards can be classified according to suits or denominations. There are 4 suits, hearts, diamonds, spades and clubs. There are 13 cards in each suit. There are 13 denominations, Aces, Kings, Queens, ... ,Twos, with 4 cards in each denomination. A poker hand consists of a sample of size 5 drawn from the deck. Poker problems are often like urn problems, with a hitch or two.

    1. How many poker hands consist of 2 Aces and 3 Kings?
    2. How many poker hands consist of 2 Aces, 2 Kings and a card of a different denomination?
    3. How many Poker hands have three cards from one denomination and two from another (a full house)?
    4. A royal flush is a hand consisting of an Ace, King, Queen, Jack and Ten, where all cards are from the same suit. How many royal flushes are possible?
    5. A flush is a hand consisting of five cards from the same suit. How many different flushes are possible?

8. (10 points) Show the **sample space** for tossing one penny and rolling one die. (H = heads, T = tails)

9. (10 points) How many license-plates with 3 letters followed by 3 digits exist?

10. (10 points) How many numbers in the range 1000 - 9999 have no repeated digits?

This page is intentionally left blank to accommodate work that wouldn't fit elsewhere and/or scratch work.