Solution for Problem Set 6

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October 24, 2020

1 Problem 1

First randomly choose a bucket from the m buckets. Suppose the length of the chosen bucket is k, we use $RANDOM(1, L)$ and suppose the return value is a. If $a \leq k$, return the a^{th} element in that bucket. Otherwise, we continue this process until $a \leq k$. By this way, each element is chosen with probability $\frac{1}{mK}$.

The probability that we succeed chosing an element in a particular bucket is $\frac{k}{L}$, so the expected chosing times are $\frac{L}{k}$. Together with a times for retriving the element, total time is $O(a + \frac{L}{k}) = O(L \cdot (a/L + 1/k))$, so the excepted time is $O(L \cdot (1 + 1/\alpha))$(excepted value of k is α and $a/L \leq 1$).

2 Problem 2

(a) Suppose string x of length $l + 1$: $x_l x_{l-1} \cdots x_0$, x has key value $x_l \times (m + 1)^l + \cdots + x_0 \times (m + 1)^0$. We have

$$\begin{aligned} h(x) &= x \ mod \ m \\ &= ((x_l \times (m+1)^l) \ mod \ m + \cdots + (x_0 \times (m+1)^0) \ mod \ m) \ modm \\ &= (x_l \ mod \ m + \cdots + x_0 \ mod \ m) \ mod \ m \end{aligned}$$

We can find the hash value of a string is determined by all its characters but is independent with the order of them. Hence, x and y has the same value.