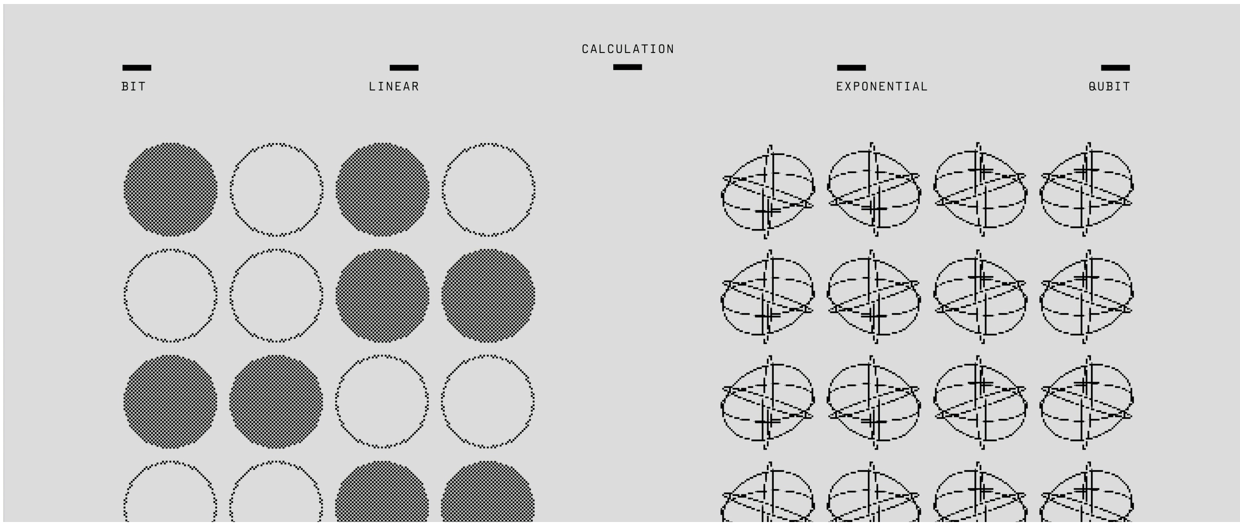

Draft 03: Understanding Qubits

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The entire framework of Classical computation is designed around bits. Either 0s or 1s, i.e. Current flowing and NOT flowing. The solid framework behind Quantum computation is what we call a 'Qubit', an analogous concept to the classical Bit. Just a caveat before we begin with Qubits, we in a general sense treat Qubits as abstract mathematical objects for the length of this draft, and that doesn't translate to qubits are just theoretical concepts with no physical actuality, we are doing so here for the length of our draft- to devise general quantum principles, algorithms and universal frameworks independent of the actual hardware and physical systems required to implement them, to essentially streamline the learning and absorption part of Qubits.

To represent/visualize a Qubit `| ψ ⟩` on what we call a **Bloch sphere**
[In quantum mechanics and computing, the Bloch sphere is a geometrical representation of the pure state space of a two-level quantum mechanical system (qubit), named after the physicist Felix Bloch]
we can represent it vertically as;

- The top as `|0>` [0th State]
- The bottom as `|1>` [1st State]

and Horizontally,

- `|->` [Minus state]
- `|+>` [Plus State]
- `|-i>` [Negative i state]
- `|i>` [Positive i state]

Notation like- `| >` is called the Dirac notation, it's the standard notation for states in quantum mechanics. The essential difference between *bits* and *qubits* is that a qubit can be in a state other than |0> or |1>. It is also possible to form linear combinations of states, often called superpositions:

`|ψ> = α |0> + β |1>`

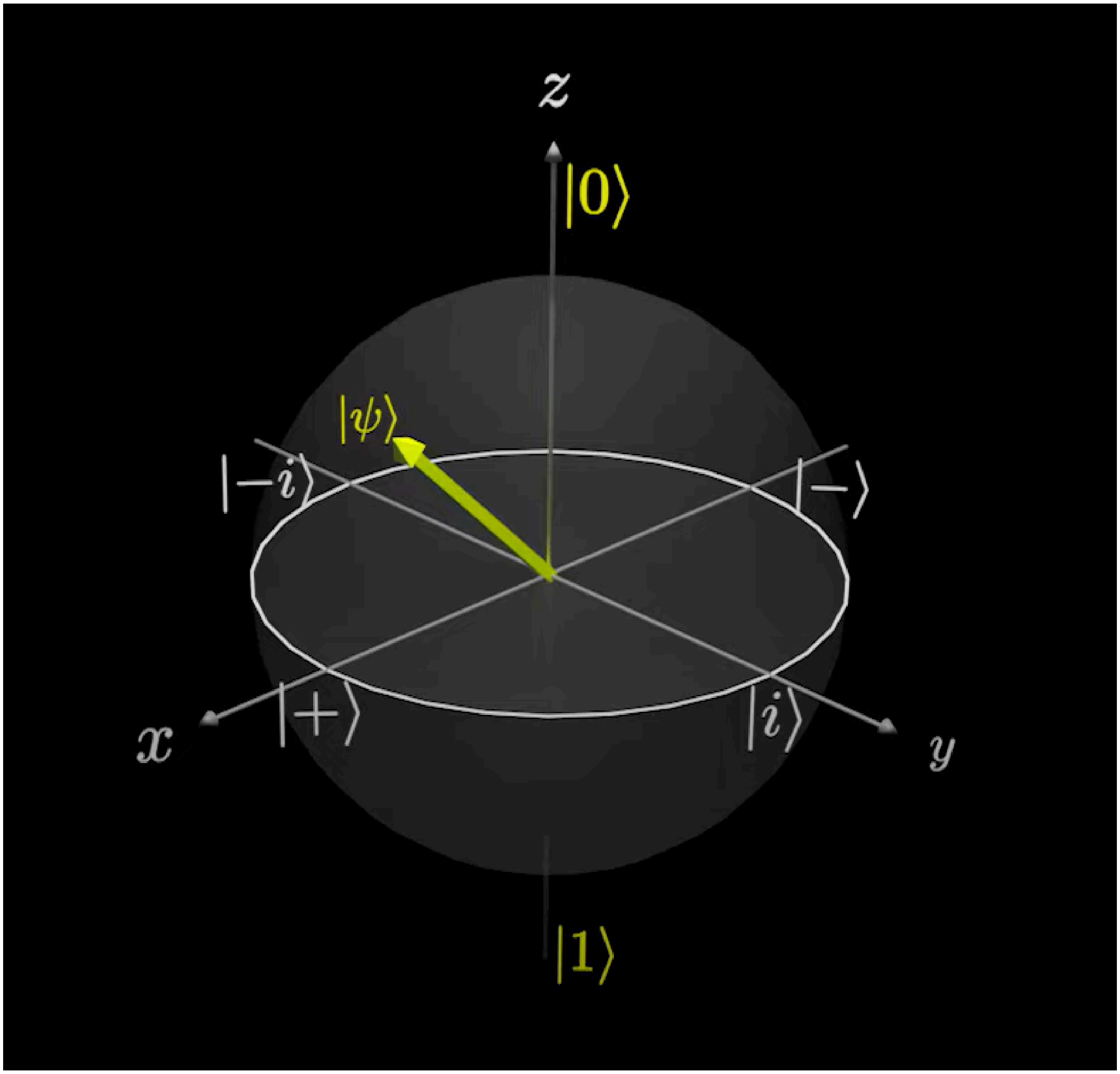
> α and β are complex numbers called probability amplitudes, although for valid and reasonable purposes not much would be lost, even if thought of them as real numbers. Can also be illustrated as the qubit state as a vector in a two-dimensional complex vector state.

Dirac notation (also called **bra–ket notation**) is a compact mathematical language introduced by Paul Dirac in 1939 to describe quantum states and operators. It's built on the mathematics of **linear algebra** and **Hilbert spaces**.

Dirac notation is used because it:

1. Unifies abstract and concrete representations.
2. Handles infinite-dimensional spaces elegantly.
3. Encodes physical meaning directly.
4. Simplifies operator algebra.

Dirac notation is used because it gives a clean, basis-independent, and physically transparent way of representing quantum states, their overlaps, and operators acting on them. The logic is linear algebra applied to Hilbert spaces, abstracted into a notation that aligns with physical intuition (states, measurements, probabilities).



> Higher the Qubit `| ψ ⟩` value i.e. its position towards the north, higher probability of the qubit value as `|0>`. Lower the Qubit `| ψ ⟩` value i.e. its position towards the south, higher probability of the qubit value as `|1>`.

Let’s reemphasize, when a Qubit is measured it can only give the result as a 0 or 1 probabilistically (not always logically true in the sense we conventionally would like to believe)

> 💡 Let’s consider a question.
> → Are apples red?
> Classical Computing: True/Yes
> Quantum Computing: 8 times YES/TRUE and 2 times NO/FALSE.
> We can presume that Apples are RED.

The question then arises, we might understand what a Qubit roughly seems to be like but then how can we actually realize them?
Classical computers use voltage levels or the flow of current to determine the instructions as 0s or 1s. But Quantum computing? And here’s where it actually gets very interesting:
There is no one way to achieve this in fact there are quite some ways you can do it, for instance;

1. **Superconducting Qubits** (IBM, Google, Rigetti)
2. **Trapped Ion Qubits** (IonQ, Honeywell/Quantinuum)
3. **Neutral Atom Qubits** (QuEra, Pasqual, Atom Computing)
4. **Spin Qubits** (Intel, others) and so on...

Despite these wildly different implementations, they all share:

1. **Two-level quantum systems** - they have at least two distinct quantum states that can be in superposition
2. **Controllability** - we can manipulate them with external fields/pulses
3. **Isolation** - they must be protected from environmental noise (the enemy of quantum states)
4. **Readout** - we can measure which state they're in

The "current flowing or not" analogy for classical bits is actually a simplification too - voltage levels, magnetic domains, or electrical charge can all represent bits. Similarly, **a qubit is fundamentally just any two-level quantum system we can control and measure**.

The physical substrate matters a lot for engineering (cooling requirements, speed, error rates), but mathematically and algorithmically, all these qubit types behave the same way - that's the beauty of quantum computing as an abstraction.

Now let's measure our qubit?

We can examine a bit to determine whether it is in the state 0 or 1. However, we cannot similarly read or check the qubit value directly check its quantum value, instead quantum mechanics reveals a rather limited form of resulting information, a result as either 0 with probability as $|\alpha|^2$ or as 1 with probability $|\beta|^2$, thus naturally;

$$|\alpha|^2 + |\beta|^2 = 1$$

The essential paradox between the unobserved state of a qubit and what we can observe is essentially the hear of Quantum computing/computation. In most real-world models we create—like architectural blueprints that directly map to actual buildings—there's a straightforward relationship between our abstractions and reality. Quantum mechanics breaks this pattern, making quantum systems counterintuitive and hard to grasp. However, even though we can't directly "see" quantum states, we can still manipulate them deliberately to produce specific measurement patterns that reveal their properties. These states have real, measurable effects that we can verify experimentally, and exploiting these effects is precisely what gives quantum computers their power.

The Qubit's ability to be in ***superposition*** is counter-intuitive to the common understanding of the ***real-world*** in the physical matters as we do. Although on measuring the state of a qubit it always returns 0 or 1 -probabilistically, for instance;

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

which, when measured, gives the result 0 fifty percent $\left(\frac{1}{\sqrt{2}}\right)^2$ of the time, and the result 1 fifty percent of the time. We will return often to this state, which is sometimes denoted `|+>`.

This oddness notwithstanding, qubits are concrete physical phenomena backed by extensive experimental validation, with multiple distinct physical platforms capable of realizing them.

To grasp concrete qubit implementations, consider these realizations: photon polarization orientations, nuclear spin alignment relative to a magnetic field, or discrete electron orbital states in isolated atoms (Figure 1.2 shows this). For the atomic implementation, electrons occupy either a ground state or excited state, labeled $|0\rangle$ and $|1\rangle$ respectively. Precisely calibrated optical pulses—with specific photon energy and exposure time—enable transitions between these states. Crucially, shining light on the atom, with appropriate energy, for an appropriate length of time, it is possible to move the electron from the |0> state to the |1> state and vice versa. But more interestingly, by reducing the time we shine the light, an electron initially in the state |0> can be moved 'halfway' between |0> and |1>, into the `|+>` state.

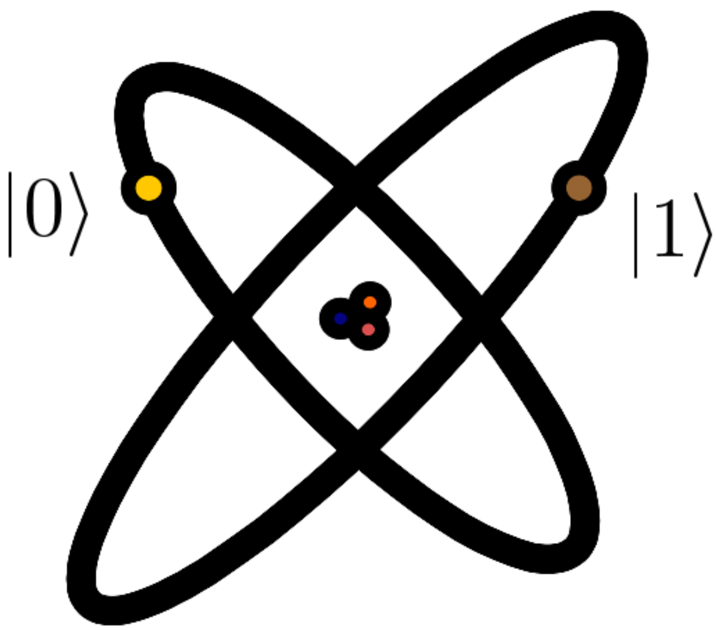


Figure 1.2. Qubit represented by two electronic levels in an atom.

Now let's circle back to our Bloch Sphere again, let's try to develop mathematical and conceptual clarity of this picture that are in some sense and form predictive in nature.

Because $|\alpha|^2 + |\beta|^2 = 1$ we can also represent it as:

$$|\psi\rangle = e^{i\gamma}\left(\cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle\right)$$

This is the one of those mathematical equations that *looks* scary in math form but is actually intuitive once you see what and how it's trying to achieve. We already know a qubit can be a blend (superposition) of two states `|0⟩` and `|1⟩` which are like the **0 and 1** of a classical bit — but the qubit can be in both at once, with certain "weights" `α` and `β`.
This means; if $|\alpha|^2$ = 0.6 and $|\beta|^2$ = 0.4 they must sum up to 1, because you'll always get *either* 0 or 1 when you measure a qubit.

Symbols	Explanation
$(e^{i\gamma})$	A **global phase factor** – it's like rotating the entire wave function by some angle γ. Physically irrelevant — doesn't change measurement outcomes (it has "no observable effect").
$(\cos\frac{\theta}{2})$	Amplitude for $\vert 0\rangle$ state.
$(\sin\frac{\theta}{2})$	Amplitude for $\vert 1\rangle$ state.
$(e^{i\varphi})$	Adds a **relative phase** — a kind of angle that changes the interference pattern between

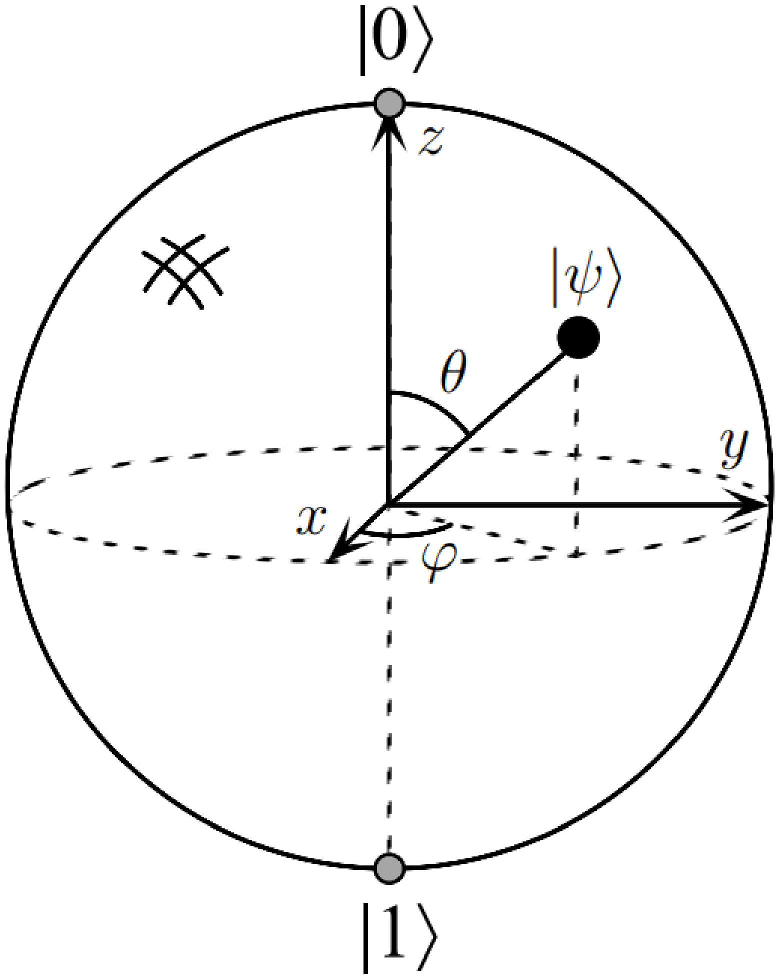


Figure 1.3. Bloch sphere representation of a qubit.

Why $(\cos\frac{\theta}{2})$ and $(\sin\frac{\theta}{2})$ guarantee normalization?

Remember that probabilities must sum to 1. Using the fundamental trigonometric identity:
$\cos^2(\theta/2) + \sin^2(\theta/2) = 1$
This **automatically ensures** that no matter what value θ takes, your probabilities always sum to 1. It's a built-in guarantee of validity.

Why θ/2 Instead of Just θ?

This is subtle but important, using θ/2 (half-angles) instead of θ directly gives us the full range we need:

> When **θ = 0**: cos(0) = 1, sin(0) = 0 → |ψ⟩ = |0⟩ (pure |0⟩ state)
> When **θ = π**: cos(π/2) = 0, sin(π/2) = 1 → |ψ⟩ = |1⟩ (pure |1⟩ state)
> When **θ = π/2**: cos(π/4) = 1/√2, sin(π/4) = 1/√2 → equal superposition

So **θ ranges from 0 to π** to cover all possible mixtures of |0⟩ and |1⟩.

θ and φ are just like the **latitude and longitude** that tell us *where on the sphere* the qubit lies, so:

- θ decides *how far down* you are from the north pole (the mix between 0 and 1)
- φ decides *which direction* you're rotated around the sphere (the phase difference between them)

Let's restate all of it once:

> A qubit can always be written as a mix of |0⟩ and |1⟩.
> How much of each you get is set by an angle θ (theta),
> and how their waves are twisted relative to each other is set by another angle φ (phi).

The overall $e^{i\gamma}$ (gamma) is just a spin of the entire sphere — since it doesn't change anything measurable, we usually drop it. That's why they later write it as:

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

and say "we can ignore the factor of $e^{i\gamma}$".

All this math is just saying:

> "Any qubit can be described by two angles — how much of |0⟩ and how much of |1⟩ — and an extra rotation we can ignore."

At first glance, a qubit seems capable of storing infinite information—since the unit sphere contains infinitely many points, you could theoretically encode all of Cervantes's works in θ's endless decimal expansion. But this reasoning is deceptive due to how qubits behave under observation. Measuring a qubit yields only a binary outcome: 0 or 1. Moreover, measurement fundamentally alters the qubit's state, forcing it from its superposition into whichever definite state that matches the measurement result. For instance, measuring the |+⟩ state and obtaining 0 means the qubit afterward exists in the $|0\rangle$ state. The reason for this collapse phenomenon remains unknown, it is also defined as one of fundamental postulates of quantum mechanics.

But consider this intriguing question:

What information does an unmeasured qubit contain?

This is somewhat paradoxical—how can we quantify information that's inherently inaccessible to measurement? Yet there's a conceptually significant point here: when Nature evolves an isolated quantum system without performing measurements, she evidently tracks all the continuous parameters like α and β that define the state. In this sense, a qubit's state harbors substantial 'hidden information' that Nature preserves. Even more remarkably, this concealed information scales exponentially as you add more qubits. Grasping the nature of this hidden quantum information is central to understanding why quantum mechanics enables such powerful computation.

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