FINANCE 601

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Chapter 7: Answers and Solutions to Exercises

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Question 1 (C50, Ch. 7, #1)

As the price of a put option on a non-dividend paying stock is no lower than the present value of the exercise price and the underlying stock price, we have

$$P \geq Xe^{-rT} - S.$$

For $X = \$40$, $r = 5\% = 0.05$, $T = 0.5$ years, and $S = \$37$, we have

$$P \geq 40 \times e^{-5\% \times 0.5} - 37 = \$2.0124.$$

As the put is selling for only \$1, it is under-priced.

An arbitrage strategy is as follows:

	$t = 0$	Outcome at $t = 0.5$ years	
		if $S \geq 40$	if $S < 40$
Borrow at the risk-free interest rate	\$38	$-$ \$38.962	$-$ \$38.962
		(The above amount is for loan repayment.)	
Buy put	$-$\$1	0	\$40 $- S$
Buy stock	$-$\$37	S	S
	\$0	\$$S - 38.962$ $\geq$ \$1.038	\$1.038

This is an investment requiring no outlays. In 6 months from now, the arbitrage profit will be at least \$1.038.

Question 2 (C51, Ch. 7, #2)

The Black-Scholes option pricing formula is

$$C = S \cdot \mathbf{N}(d_1) - X \cdot e^{-rT} \cdot \mathbf{N}(d_2)$$

where

$$\begin{aligned} d_1 &= \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T\right] \\ \text{and } d_2 &= d_1 - \sigma\sqrt{T}. \end{aligned}$$

For $X = \$40$, $T = 0.5$ years, $S = \$28$, $\sigma = \sqrt{0.5}$, and $r = 6\% = 0.06$, we have

$$\begin{aligned} d_1 &= \frac{1}{\sqrt{0.5}\times\sqrt{0.5}}\left[\ln\left(\frac{28}{40}\right) + \left(0.06 + \frac{0.5}{2}\right)\times 0.5\right] \\ &= \frac{1}{0.5}[-0.356675 + 0.155] \\ &= -0.40335 \\ \text{and } d_2 &= -0.40335 - \sqrt{0.5}\times\sqrt{0.5} = -0.90335. \end{aligned}$$

By linear interpolation of available information from a table of the standard normal distribution, we have

$$\begin{aligned} \frac{0.40335 - 0.40}{0.41 - 0.40} &= \frac{x - 0.1554}{0.1591 - 0.1554}, \\ x &= 0.1566, \end{aligned}$$

and then

$$\boldsymbol{N}(-0.40335) = 0.5 - 0.1566 = 0.3434.$$

Likewise, we also have

$$\begin{aligned} \frac{0.90335 - 0.90}{0.91 - 0.90} &= \frac{y - 0.3159}{0.3186 - 0.3159}, \\ y &= 0.3168, \end{aligned}$$

and then

$$\boldsymbol{N}(-0.90335) = 0.5 - 0.3168 = 0.1832.$$

It follows that

$$\begin{aligned} C &= \$28 \times 0.3434 - \$40 \times e^{(-0.06\times 0.5)} \times 0.1832 \\ &= \$9.6152 - \$7.1114 = \$2.5038. \end{aligned}$$

Question 3 (C52, Ch. 7, #3)

According to the put-call parity, we have

$$C - P = S - X \cdot e^{-rT}$$

If we can get C from Black-Scholes option pricing formula, we can compute P via the put-call parity.

For $X = \$20$, $T = 0.5$ years, $S = \$20$, $\sigma = 60\% = 0.6$, and $r = 8\% = 0.08$, we use

$$C = S \cdot \boldsymbol{N}(d_1) - X \cdot e^{-rT} \cdot \boldsymbol{N}(d_2) = \$20 \cdot \boldsymbol{N}(d_1) - \$20 \cdot e^{-0.08\times0.5} \cdot \boldsymbol{N}(d_2)$$

where

$$\begin{aligned} d_1 &= \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T\right] \\ &= \frac{1}{0.6 \times \sqrt{0.5}}\left[\ln\left(\frac{20}{20}\right) + \left(8\% + \frac{0.6^2}{2}\right) \times 0.5\right] \\ &= 0.3064 \\ \text{and} \quad d_2 &= d_1 - \sigma\sqrt{T} = 0.3064 - 0.6\sqrt{0.5} = -0.1179 \end{aligned}$$

to compute C.

By linear interpolation of available information from a table of the standard normal distribution, we have

$$\begin{aligned} \frac{0.3064 - 0.30}{0.31 - 0.30} &= \frac{x - 0.1179}{0.1217 - 0.1179}, \\ x &= 0.1203, \end{aligned}$$

and then

$$\boldsymbol{N}(0.3064) = 0.5 + 0.1203 = 0.6203.$$

Likewise, we have

$$\begin{aligned} \frac{0.1179 - 0.11}{0.12 - 0.11} &= \frac{y - 0.0438}{0.0478 - 0.0438}, \\ y &= 0.04696, \end{aligned}$$

and then

$$\boldsymbol{N}(-0.1179) = 0.5 - 0.04696 = 0.45304.$$

It follows that

$$\begin{aligned} C &= \$20 \times 0.6203 - \$20 \times e^{(-0.08\times 0.5)} \times 0.45304 \\ &= 12.406 - 8.7055 = \$3.7005 \end{aligned}$$

and then

$$\begin{aligned} P &= C - S + X \cdot e^{-rT} \\ &= \$3.7005 - \$20 + \$20 \times e^{(-0.08\times 0.5)} = \$2.9163. \end{aligned}$$

- Alternatively, the put price can be computed by using the Black-Scholes option pricing formula for put options. The end result will be the same.

Question 4 (X19b, Ch. 7, #4)

Two alternative methods are provided below; find C first and use put-call parity to deduce P, or find P directly. Regardless the method involved, the first step is find $\boldsymbol{N}(d_1)$ and $\boldsymbol{N}(d_2)$. Linear interpolation for each leads to

$$\begin{aligned} \frac{0.0367-0.03}{0.04-0.03} &= \frac{x-0.0120}{0.0160-0.0120} \\ \frac{0.0067}{0.01} &= \frac{x-0.012}{0.004} \\ x &= 0.01468; \\ \frac{0.0633-0.06}{0.07-0.06} &= \frac{y-0.0239}{0.0279-0.0239} \\ \frac{0.0033}{0.01} &= \frac{y-0.0239}{0.004} \\ y &= 0.02522. \end{aligned}$$

$$\begin{aligned} \boldsymbol{N}(d_1) &= 0.5 + 0.01468 = 0.51468 \\ \boldsymbol{N}(d_2) &= 0.5 - 0.02522 = 0.47478. \end{aligned}$$

Then, we have

$$\begin{aligned} \boldsymbol{N}(-d_1) &= 0.5 - 0.01468 = 0.48532; \\ \boldsymbol{N}(-d_2) &= 0.5 + 0.02522 = 0.52522; \end{aligned}$$

$$
\begin{aligned}
C &= S\mathbf{N}(d_1) - X\exp(-rT)\mathbf{N}(d_2) \\
&= \$19.75 \times 0.51468 - \$20 \times [\exp(-0.045 \times 0.25)] \times 0.47478 \\
&= \$10.1649 - \$20 \times 0.988813 \times 0.47478 = \$0.7756;
\end{aligned}
$$

$$
\begin{aligned}
\text{and } P &= C - S + X\exp(-rT) \\
&= \$0.7756 - \$19.75 + \$20 \times 0.988813 \\
&= \$0.8018.
\end{aligned}
$$

Alternatively, we have directly

$$
\begin{aligned}
P &= C - S + X\exp(-rT) \\
&= S\mathbf{N}(d_1) - X\exp(-rT)\mathbf{N}(d_2) - S + X\exp(-rT) \\
&= X\exp(-rT)[1 - \mathbf{N}(d_2)] - S[1 - \mathbf{N}(d_1)] \\
&= X\exp(-rT)\mathbf{N}(-d_2) - S\mathbf{N}(-d_1) \\
&= \$20 \times [\exp(-0.045 \times 0.25)] \times 0.52522 - \$19.75 \times 0.48532 \\
&= \$20 \times 0.988813 \times 0.52522 - \$9.5851 \\
&= \$0.8018.
\end{aligned}
$$

Question 5 (X23b, Ch. 7, #5)

To satisfy the put-call parity holds requires that

$$C - P = S - Xe^{-rT}.$$

For $S = \$52$, $X = \$50$, $r = 0.04$, $T = 0.25$ years, $C = \$3.100$, and $P = \$0.585$, we have

$$C - P = \$3.100 - \$0.585 = \$2.515$$

and

$$S - Xe^{-rT} = \$52 - \$50 \times e^{-0.04 \times 0.25} = 2.4975,$$

indicating that

$$C - P > S - Xe^{-rT}.$$

This inequality is equivalent to

$$P - C + S < Xe^{-rT}.$$

As

$$S \geq C \geq 0 \ \text{ and } \ P \geq 0,$$

we must have

$$P - C + S \geq 0.$$

An investor buys the put option with $P = \$0.585$, writes the corresponding call option to receive $C = \$3.100$, and buys the underlying stock with $S = \$52.000$. The required cash is $P - C + S = \$49.485$. To finance this portfolio, the investor also borrows less than $Xe^{-rT} = \$49.5025$, at a continuously compounded risk-free interest rate $r = 0.04$.

On the expiry date of the two options, which is $T = 0.25$ years afterwards, the repayment of the loan requires less than

$$(Xe^{-rT})e^{rT} = X = \$50.$$

Let us label, on the expiry date of the two options, the stock price as S_T, the call option price as C_T, and the put option price as P_T. We can have $S_T > X = \$50$, $S_T = X = \$50$, or $S_T < X = \$50$.

If $S_T > X = \$50$, only the call option is exercised. With

$$C_T = S_T - X \ \text{ and } P_T = 0,$$

the investor's portfolio is worth

$$P_T - C_T + S_T = 0 - (S_T - X) + S_T = X = \$50.$$

If $S_T = X = \$50$, neither option is exercised. With

$$C_T = 0 \ \text{ and } P_T = 0,$$

the investor's portfolio is worth

$$P_T - C_T + S_T = 0 - 0 + S_T = S_T = X = \$50.$$

If $S_T < X = \$50$ instead, only the put option is exercised. With

$$C_T = 0 \ \text{ and } P_T = X - S_T,$$

the investor's portfolio is worth

$$P_T - C_T + S_T = (X - S_T) - 0 + S_T = X = \$50.$$

As the loan repayment requires less than \$50, the investor has an arbitrage profit in each of the three cases.

Question 6 (X23a, Ch. 7, #6)

For a European put option on a stock that pays no dividend, the following inequality holds:

$$P \geq Xe^{-rT} - S.$$

For $T = 0.25$ years, $S = \$38$, $X = \$40$, and $r = 0.05$, we have

$$Xe^{-rT} - S = \$40e^{-0.05\times 0.25} - \$38 = \$1.503,$$

which is greater than $P = \$1.45$. This is a case where

$$P < Xe^{-rT} - S$$

or, equivalently,

$$P + S < Xe^{-rT}.$$

We show below that an investor will earn an arbitrage profit by buying both the put option and the underlying stock, entirely with borrowed money, which is

$$P + S = \$39.45.$$

A crucial point here is that the investor's immediate cash outlay is intended to be zero.
With the dollar amount of the loan being less than

$$Xe^{-rT} = \$39.503,$$

the repayment of the loan on the expiry date of the option is less than

$$(Xe^{-rT})e^{rT} = X = \$40.$$

On the expiry date of the option, if $S < X = \$40$, the investor will exercise the option by selling the stock (that the investor has acquired at the beginning) to the writer of the option and will receive

$X = \$40$ for it. As the repayment of the loan is less than $X = \$40$, the investor will make an arbitrage profit.

On the expiry date of the option, if $S = X = \$40$ instead, there is no incentive for the investor to exercise the option. The investor will sell the stock in the market for a price that is equal to $X = \$40$. As the amount is more than what is required to repay the loan, the arbitrage profit will still be the same as that for the case of $S < X = \$40$.

On the expiry date of the option, if $S > X = \$40$ instead, the option will not be exercised. The investor will sell the stock in the market for a price that is greater than $X = \$40$. As the repayment for the loan requires an amount less than X, the arbitrage profit is even higher.

Question 7 (X16a23c, Ch. 7, #7)

(a) The connection between the two models pertaining to the time-value factors is via

$$(1 + r_f)^N = \exp(rT),$$

which can be expressed equivalently as

$$1 + r_f = [\exp(rT)]^{1/N}.$$

For $N = 10$, $r = 6.5\% = 0.065$, and $T = 2/12$ years, we have

$$\exp(rT) = \exp\left(0.065 \times \frac{2}{12}\right) = 1.010892$$

and then

$$r_f = (1.010892)^{1/10} - 1 = 0.001084 = 0.1084\%.$$

(b) There are different ways to set parameters u and d in an N-period setting to match the volatility parameter σ in the Black-Scholes model. One of them is

$$u = \frac{1}{d} = \exp\left(\sigma\sqrt{\frac{T}{N}}\right).$$

For $u = 0.00160$, $T = 0.25$ years, and $N = 10,000$, we have

$$\begin{aligned} \sigma\sqrt{\frac{T}{N}} &= \ln(u); \\ \sigma &= \ln(u)\sqrt{\frac{N}{T}} = 0.0015987\sqrt{\frac{10,000}{0.25}} = 0.31974 = 31.974\%. \end{aligned}$$