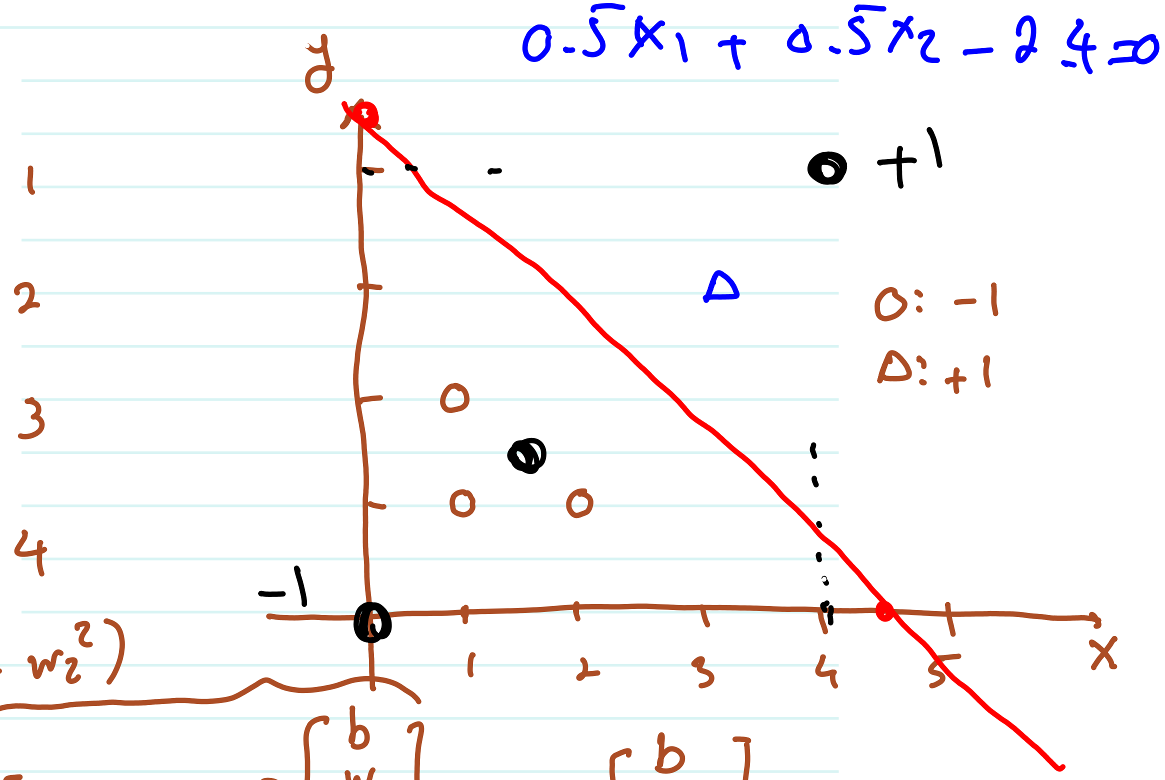
$$x_1 = [1, 1]^T, class\# - 1$$
$$x_2 = [2, 1]^T, class\# - 1$$
$$x_3 = [1, 2]^T, class\# - 1$$
$$x_4 = [3, 3]^T, class\# + 1$$



$$\underbrace{\frac{1}{2}[b \ \ w_1 \ \ w_2]\begin{bmatrix} b \\ w_1 \\ w_2 \end{bmatrix}}_{\frac{1}{2}(b^2 + w_1^2 + w_2^2)}$$

Min $\frac{1}{2}\|w\|^2 \to \frac{1}{2}[b \ \ w_1 \ \ w_2]\begin{bmatrix} b \\ w_1 \\ w_2 \end{bmatrix}$ $\qquad W = \begin{bmatrix} b \\ w_1 \\ w_2 \end{bmatrix}$

s.t $\quad y_i(b + w_1 x + w_2 x) \geq 1$

$y_1 \to -1$

$y_2 \to -1$

$y_3 \to -1$

..

$$\boxed{\begin{array}{ll} -1(b + w_1 + w_2) \geq 1 & i=1 \\ -1(b + 2w_1 + w_2) \geq 1 & i=2 \\ -1(b + w_1 + 2w_2) \geq 1 & i=3 \\ b + 3w_1 + 3w_2 \geq 1 & i=4 \end{array}}$$

$$x_5 = [4, 4]^T, class\# + 1$$
$$x_6 = [0, 0]^T, class\# - 1$$
$$x_7 = [1.5, 1.5]^T, class\# - 1$$

Lagrange function :

$$L_p = \frac{1}{2} b^2 + \frac{1}{2} w_1^2 + \frac{1}{2} w_2^2 - \sum_{i=1}^{4} \lambda_i \left(y_i (b + w_1 x_{i1} + w_2 x_{i2}) - 1 \right)$$

$$= \frac{1}{2} b^2 + \frac{1}{2} w_1^2 + \frac{1}{2} w_2^2 - \Big(\lambda_1 (-b - w_1 - w_2 - 1) + \lambda_2 (-b - 2w_1 - w_2 - 1)$$

$$+ \lambda_3 (-b - w_1 - 2w_2 - 1) + \lambda_4 (b + 3w_1 + 3w_2 - 1) \Big)$$

$$\frac{\partial L_p}{\partial w_1} = w_1 + \lambda_1 + 2\lambda_2 + \lambda_3 - 3\lambda_4 = 0 \quad (1)$$

$$\lambda_i \geq 0 \quad \forall i \in [1, 4]$$

$$\frac{\partial L_p}{\partial w_2} = w_2 + \lambda_1 + \lambda_2 + 2\lambda_3 - 3\lambda_4 = 0 \quad (2)$$

$$\frac{\partial L_p}{\partial b} = b + \lambda_1 + \lambda_2 + \lambda_3 - \lambda_4 = 0 \quad (3)$$

$$\frac{\partial L_p}{\partial \lambda_1} = b + w_1 + w_2 = -1 \quad (4)$$

$$\frac{\partial L_p}{\partial \lambda_2} = b + 2w_1 + w_2 = -1 \qquad (5)$$

$$\frac{\partial L_p}{\partial \lambda_3} = b + w_1 + 2w_2 = -1 \qquad (6)$$

$$\frac{\partial L_p}{\partial \lambda_4} = b + 3w_1 + 3w_2 = 1 \qquad (7)$$

Unknowns

w_1
w_2
b
λ_1
λ_2
λ_3
λ_4

$$\underset{7\times 7}{A} \times \underset{7\times 1}{x} = \underset{7\times 1}{b}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 2 & 1 & -3 \\ 0 & 1 & 0 & 1 & 1 & 2 & -3 \\ 0 & 0 & 1 & 1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 3 & 3 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \underset{dx}{\begin{bmatrix} w_1 \\ w_2 \\ b \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$

$$x = A^{-1} \cdot b = \begin{bmatrix} w_1 = 0.5 \\ w_2 = 0.5 \\ b = -2.4 \end{bmatrix}$$

Sude inverse

$$A^T \cdot A$$

$$w_1 x_1 + w_2 x_2 + b = 0$$

$$0.5 x_1 + 0.5 x_2 - 2.4 = 0$$

class

(a) $4,4 \rightarrow 0.5 * 4 + 0.5 * 4 - 2.4 = 1.6 > 0$ $+1$

(a) $0,0 \rightarrow 0 + 0 - 2.4 < 0$ class -1

$$Ax = b$$

$x \in \mathbb{R}^{d \times 1}$

$A \in \mathbb{R}^{n \times d}$

$b \in \mathbb{R}^{n \times 1}$

$$\underbrace{x}_{d \times 1} = \underbrace{A^{+}}_{d \times n} \underbrace{b}_{n \times 1} \quad , \quad A^{+} = \underbrace{(A^{T} \cdot A)}_{d \times d}^{-1} \cdot \underbrace{A^{T}}_{d \times n}$$

Class A: 1 TP and 1 FP

$$Pr = \frac{TP}{TP + FP}$$

class B : 10 TP 90 FP

class C : 1 TP 1 FP

class D : 1 TP 1 FP

$$\text{Macr} - \text{Average} - Pr = \frac{0.5 + 0.1 + 0.5 + 0.5}{4} = 0.4$$

$Pr_{-A} = 50\%$ $Pr_{-B} = 10\%$ $Pr_C = P_D = 50\% = \boxed{0.5}$

$$\text{Micro} - \text{Average} - Pr = \frac{1 + 10 + 1 + 1}{13 + 1 + 90 + 1 + 1} = \frac{13}{106} = \boxed{0.123}$$

Weighted Average

$$= 0.5 \times \frac{2}{106} + 0.1 \times \frac{100}{106} + 0.5 \times \frac{2}{106} + 0.5 \times \frac{2}{106}$$

$= \boxed{0.123}$