Quiz #3

x_1 – hours studied
x_2 – GPA
Y – receive A

$\hat{\beta}_0 = -6$
$\hat{\beta}_1 = 0.05$
$\hat{\beta}_2 = 1$

Test $\Rightarrow$ hours studied: 40 h – GPA = 3.5

(a)

$$P(\text{getting } A) = \frac{1}{1+e^{-z}} = \frac{1}{1+e^{-(\hat{\beta}_0+\hat{\beta}_1 x_1+\hat{\beta}_2 x_2)}}$$

$$= \frac{1}{1+e^{-(-6+0.05 \times 40+3.5)}}$$

$$= 37\%$$

(b) $P(A) = \frac{1}{2} = \frac{1}{1+e^{-z}} \rightarrow z = 0$

$$\hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 = 0$$

$$-6 + 0.05h + 3.5 = 0 \rightarrow 0.05h = 2.5 \rightarrow h = \frac{2.5}{0.05} = \boxed{50h}$$

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2 $X \rightarrow$ weight

$$\underline{Pr(Y = \text{orange} \mid X)} = \boxed{\frac{e^{\beta_0 + \beta_1 X}}{1 + e^{\beta_0 + \beta_1 X}}} \leftarrow$$

$$\ln\left(\frac{P(\text{orange})}{P(\text{apple})}\right) = \ln\left(\frac{P(\text{orange})}{1 - P(\text{orange})}\right) =$$

$$\ln\left(\frac{\dfrac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}}}{1-\dfrac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}}}\right) = \ln\left(\frac{\dfrac{e^{\beta_0+\beta_1 x}}{\cancel{1+e^{\beta_0+\beta_1 x}}}}{\dfrac{\cancel{1+e^{\beta_0+\beta_1 x}}-\cancel{e^{\beta_0+\beta_1 x}}}{\cancel{1+e^{\beta_0+\beta_1 x}}}}\right)$$

$$\ln\left(e^{\beta_0+\beta_1 x}\right) = \boxed{\beta_0+\beta_1 x}$$

(b)

$\hat{\beta}_0 = 2$

$\hat{\beta}_1 = -1$

Test : $x = 16$

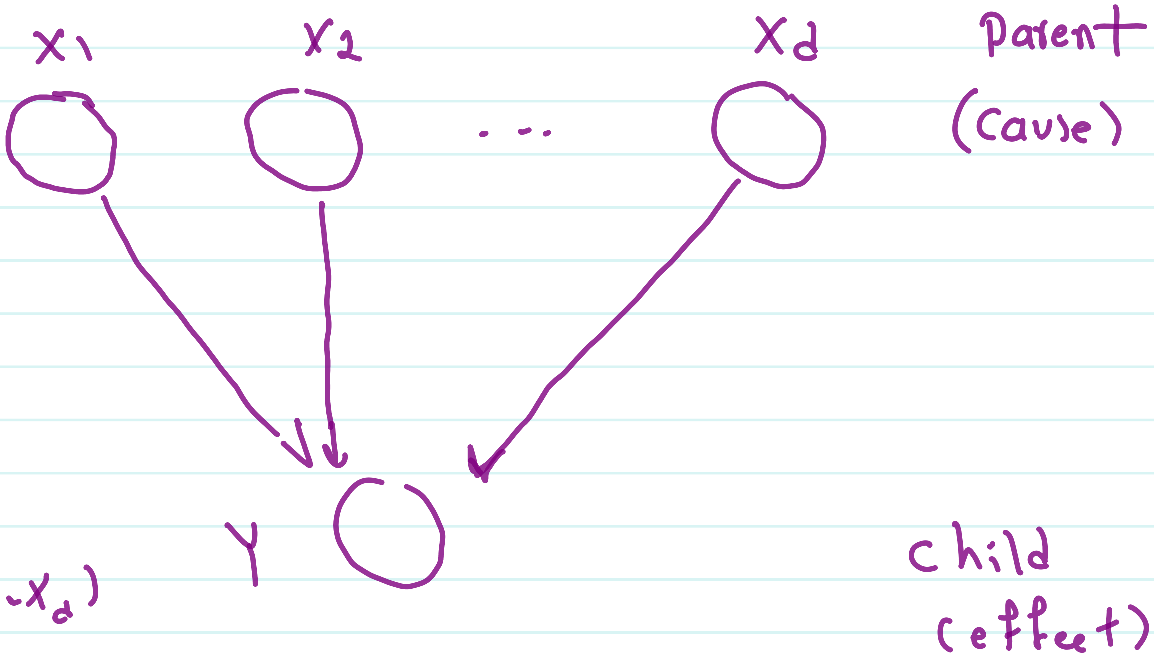
<u>$P(\text{orange} \mid x)$</u>

$P(\text{apple} \mid x) = 1 - P(\text{orange} \mid x)$

$$P(\text{orange} \mid x = 1\,\text{lb}) = \frac{e^{2+(-1)(1)}}{1+e^{2+(-1)(1)}} = \frac{e^{1}}{1+e^{1}} = 73\%$$

$$P(\text{orange} \mid x) > P(\text{appl} \mid x) \rightarrow \boxed{\text{orange}}$$

<u>PGM</u> : Probalistic Graphical Model



generative model

$P(Y, X_1, X_2, \ldots X_d)$

$P(120K \mid Yes)$

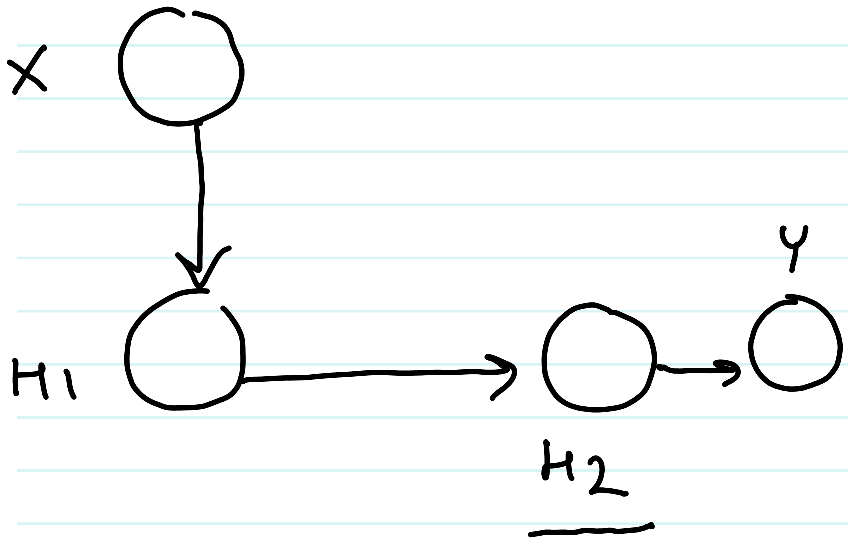
$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}$$

$$\underline{\bar{x}_{yes}} = \frac{95+85+90}{3} = 90$$

$$\underline{\sigma^2_{yes}} = \frac{(95-90)^2 + (85-90)^2 + (90-90)^2}{3}$$

$$P(120K \mid Yes) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(120-90)^2}{2\sigma^2}}$$

Tid	Refund	Marital Status	Taxable Income	Evade
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



$$P(H_1|X) = \frac{P(H_1, X)}{P(X)}$$

$$\boxed{P(H_1, X)} = P(H_1|X) \cdot P(X)$$

$$P(X, H_1, H_2) = P(H_2|H_1) \cdot P(X, H_1)$$

$$= P(H_2|H_1) \cdot P(H_1|X) \cdot P(X)$$

$$\boxed{P(Y, H_2, H_1, X)} = P(Y|H_2) \cdot P(H_2|H_1) \cdot P(H_1|X) \cdot P(X)$$

To Find

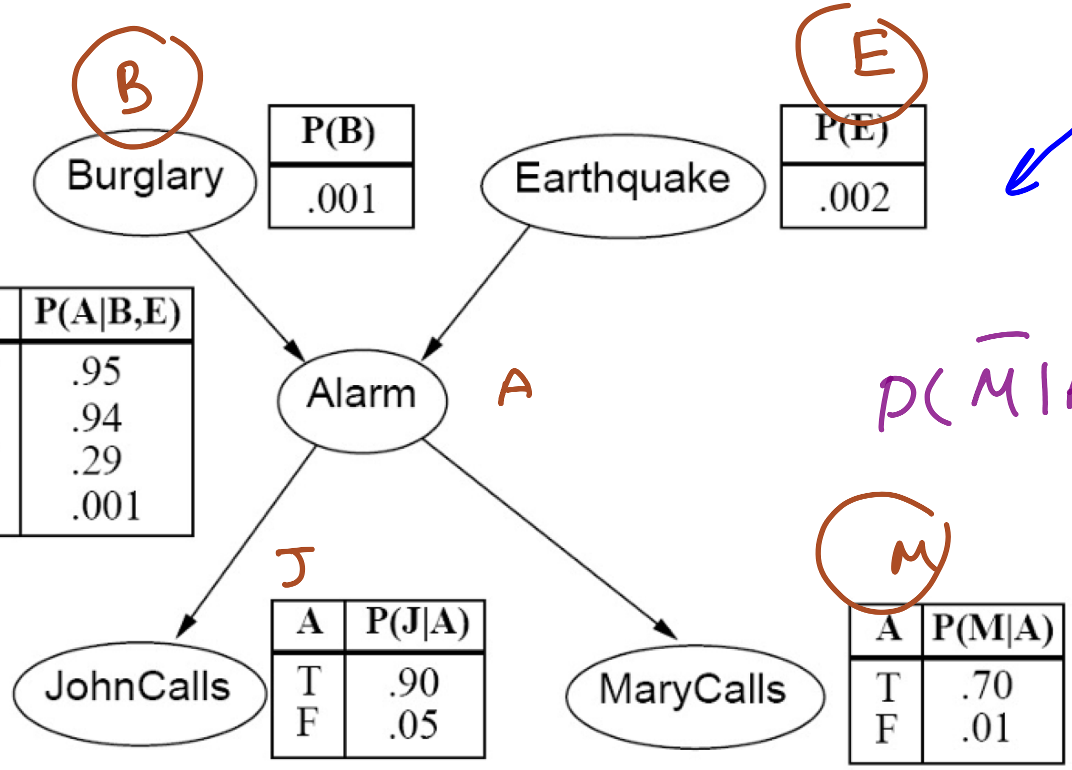
Marginalize $P(Y, H_1, H_2, X)$

$$P(Y|X) = \frac{\boxed{P(X, Y)}}{P(X)}$$

$$P(X, Y) = \sum_{h_2} \sum_{h_1} (Y, H_1 = h_1, H_2 = h_2, X)$$

$B \rightarrow T$

$\bar{B} \rightarrow F$



P(B)
.001

P(E)
.002

B	E	P(A|B,E)
T	T	.95
T	F	.94
F	T	.29
F	F	.001

A	P(J|A)
T	.90
F	.05

A	P(M|A)
T	.70
F	.01

$P(\bar{M} | A)$

Let suppose Mary Called.

$$P(B | M) = \frac{\boxed{P(B, M)}}{P(M)} = \,?$$

$$P(\bar{B} | M) =$$

$$P(\bar{M} | \bar{A}) = 1 - 0.01$$

$$P(\bar{A} | \bar{B}, \bar{E}) = 1 - 0.001$$

$$P(\bar{B}) = 1 - 0.001$$

$$P(\bar{E}) = 1 - 0.002$$

$$P(B, A, E, M) \longrightarrow \boxed{P(B, M)}$$

$$P(B, A, E, M) = P(M|A) * P(A|E, B) * P(B) * P(E)$$

$$= .70 * .95 * 0.001 * 0.002 = 1.3 \times 10^{-6}$$

$\boxed{P(B, E, A, M)}$	$\bar{M}, \bar{A}$	$\bar{M}, A$	$M, \bar{A}$	M, A
$\bar{B}, \bar{E}$	0.9860	2.99×10^{-4}	9.96×10^{-3}	6.9×10^{-4}
$\bar{B}, E$	1.4×10^{-3}	1.7×10^{-4}	1.4×10^{-5}	4.06×10^{-4}
$B, \bar{E}$	5.93×10^{-5}	2.8×10^{-4}	5.99×10^{-4}	6.57×10^{-4}
B, E	9.9×10^{-8}	5.7×10^{-7}	10^{-9}	1.3×10^{-6}

$$P(\bar{B}, \bar{E}, \bar{A}, \bar{M}) = P(\bar{M} \mid \bar{A}) \cdot P(\bar{A} \mid \bar{E}, \bar{B}) \cdot P(\bar{B}) \cdot P(\bar{E})$$

$$= (1 - 0.01)(1 - 0.001)(1 - 0.001)(1 - 0.002)$$

$$= 0.9860$$

$$P(\bar{B}, \bar{E}, \bar{M}, A) = \underbrace{P(\bar{M} \mid A)} \cdot P(A \mid \bar{E}, \bar{B}) \cdot P(\bar{B}) \cdot P(\bar{E})$$

$$0.3 \times 0.001 \times (1 - 0.001)(1 - 0.002)$$

$P(B, M)$	$\bar{M}$	M
$\bar{B}$	0.9878	0.01107
B	0.00034	0.000658

	M
$\bar{B}$	0.01107
B	0.000658

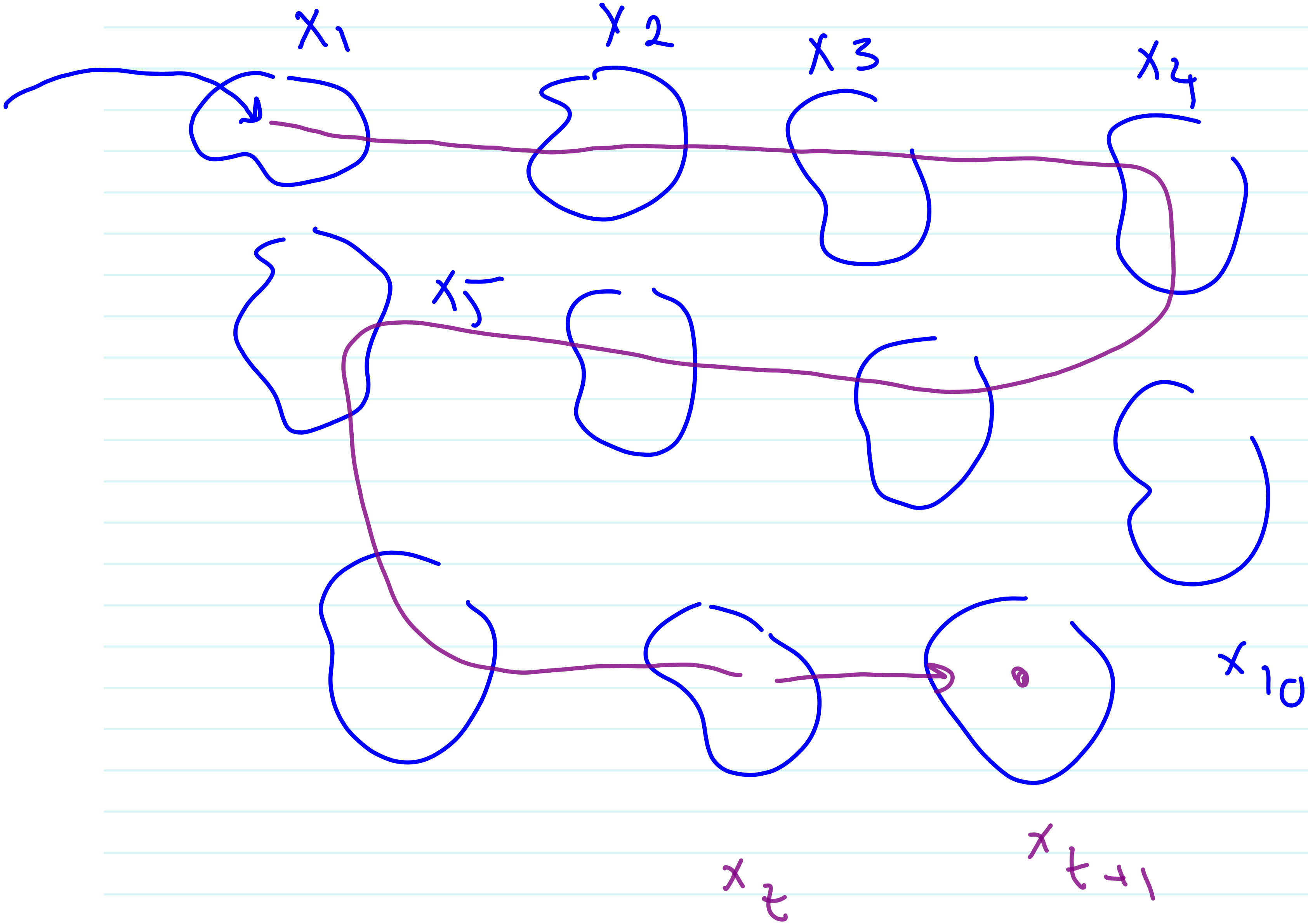
$$P(B|M) = \frac{P(B,M)}{P(M)} = \frac{0.000658}{0.000658 + 0.01107}$$

$$= 5\%$$

$$P(\bar{B}|M) = \frac{P(\bar{B},M)}{P(M)} = \frac{0.01107}{0.01107 + 0.000658}$$

$$= 94\%$$

$P(\bar{B}|M) > P(B|M) \rightarrow$ Burglary not likely.



$$y(t) + a_1 y(t-1) = e(t) \rightarrow e(t) \sim WN(\sigma, 1)$$

$$y(t) = -a_1 y(t-1) + e(t)$$

$t=1 \rightarrow y(1) = -a_1 y(0) + e(1)$

$t=2 \rightarrow y(2) = -a_1 y(1) + e(2)$

$$= -a_1 \left[-a_1 y(0) + e(1)\right] + e(2)$$

$\vdots$

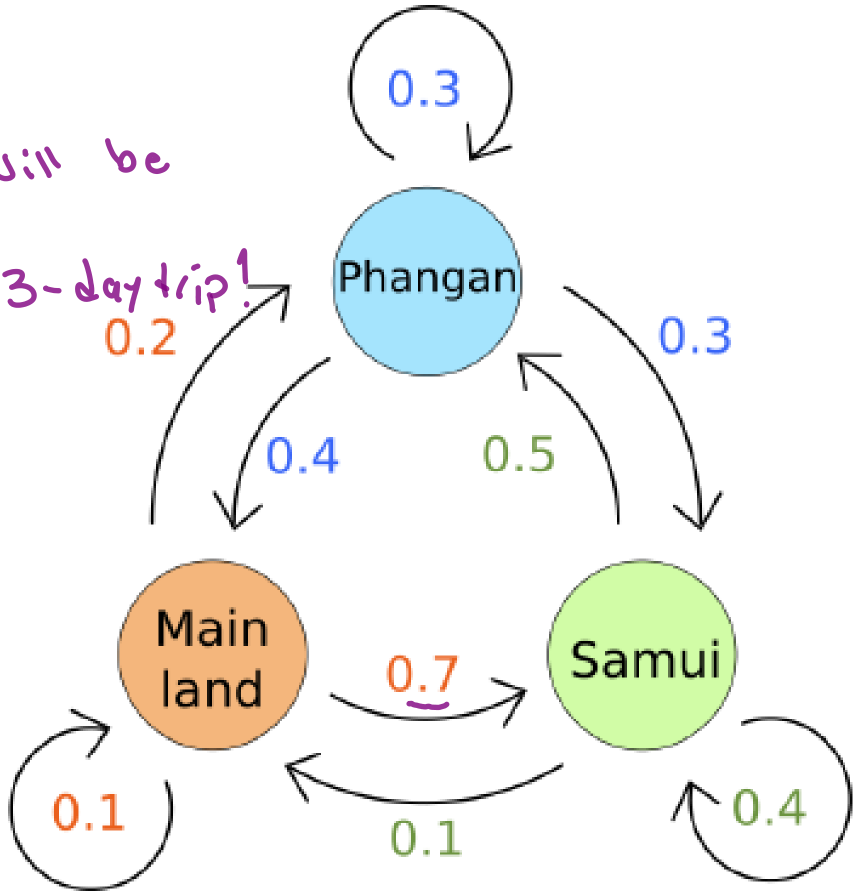
Starting from main land

Find % that traveleres will be

on the main land at the 3-day trip!

$\boxed{n=3}$

22.9%



$$A = \begin{bmatrix} 0.1 & 0.7 & 0.2 \\ 0.1 & 0.4 & 0.5 \\ 0.4 & 0.3 & 0.3 \end{bmatrix} \rightarrow A^3 = A \cdot A \cdot A = \begin{bmatrix} \boxed{0.229} & 0.405 & 0.366 \\ 0.211 & 0.438 & 0.351 \\ 0.196 & 0.425 & 0.379 \end{bmatrix}$$