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ORIGINAL ARTICLE

Suppression of chip load variations by real-time spindle speed modulation

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Abstract

In machining a fixed depth of cut at a constant feedrate to generate a desired curvilinear shape, substantial variations in chip load can occur whenever the smallest concave radius of curvature is comparable to the tool radius. These chip load variations may result in a poor quality of the machined surface or premature tool wear. Conversely, attempting to suppress chip load variations by modulating the feedrate may incur high rates of feed acceleration, that may tax the machine drive systems or induce large contour errors. To address these conflicting influences, the feasibility of minimizing variations in chip load through real-time spindle speed modulation is examined herein. A second-order model is employed to determine the tool angular speeds and accelerations that are required to achieve a specified constant chip load for a given depth of cut along a desired part shape defined by a parametric curve, using a constant feedrate for a tool with a given radius and number of cutting edges. For a spindle driven by a DC motor, the motor voltage variation required to generate the modulated spindle speed is also determined. These analyses facilitate an *a priori* assessment of the part geometries and process parameters for which spindle speed modulation is a viable approach to the suppression of chip load variations.

Keywords CNC machining · Offset tool paths · Chip load variation · Spindle speed modulation · Angular acceleration · DC motor regulation

1 Introduction

In any machining operation, the specification of process parameters to achieve *chip loads* that guarantee accurate and efficient part fabrication is a primary consideration. The chip load is a measure of the average thickness of material chips sheared off the workpiece: chip loads that are too high, or that fluctuate substantially, can incur poor part accuracy, surface finish, or tool wear.

In milling operations, chip loads depend not only on feedrate and spindle speed but also on tool path curvature. In the context of machining a constant depth of cut at fixed feedrate from a part shape with curvature κ using a tool of radius d, substantial variations in the chip load can arise unless $d \ll |\kappa|^{-1}$ at every point. Consequently, machined parts may require extensive finishing (grinding/polishing) operations to achieve a desired surface quality, and tools may require frequent re-grinding or replacement. One approach to reducing chip load variations is through continuous feedrate modulation, but this can result in large acceleration/deceleration rates when d is comparable to $|\kappa|^{-1}$.

The present study explores the possibility of reducing chip load variations, at a constant feedrate, through the continuous modulation of spindle speed. The main application context is in finish-machining a fixed depth of cut from strongly curved parts, for which high accuracy and smooth surface finish are desired. However, the methodology can also be adapted to other machining applications when *a priori* knowledge of depth-of-cut variation is available. A key requirement is that the part shape be precisely communicated to the machine controller as a parametric curve $\mathbf{r}(\xi)$, to permit exact computation of the part curvature and its derivatives, and the offset tool path—in keeping with the STEP–NC philosophy [3, 9, 19] of more directly employing precise CAD geometry data in CNC machining, instead of relying upon piecewise-linear/circular G

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code tool path approximations. This theme has also been pursued in several prior studies [4, 8, 13, 14].

A further requirement for implementation of the proposed method is the availability of a spindle/controller system that permits real-time variation of spindle rotation speeds. For suitable spindles, this can be achieved through appropriate modifications to the algorithms of open-architecture controllers, or through a user-specified customization of commercial CNC machines, and continuous spindle speed variation has been extensively studied [1, 2, 10–12, 15–17, 21–24] in the context of tool chatter suppression. Since the spindle currently available to the authors only permits constant pre-set speeds, the focus of this study will be on developing the underlying analysis, and illustrating it through computed examples.

The paper is organized as follows. Section 2 gives a brief survey of prior work on spindle speed modulation, which has focused mainly on tool chatter suppression. Section 3 then elucidates the conflicting demands of minimizing variations in the chip load and feedrate when a tool of radius d, operating at constant spindle speed, is driven along an offset path to machine a depth of cut δ from a part geometry with concave radii of curvature comparable to d. To maintain both a constant chip load and constant feedrate, a methodology for real-time spindle speed modulation is then developed in Section 4, based on a local osculating-circle approximation of the part geometry at each point. Section 5 analyzes the required voltage regulation of a DC motor to achieve the desired spindle speed variation. The method is illustrated in Section 6 by a computed example, and the analysis is used to make *a priori* assessments of feasible tools and feedrates under known limits on spindle angular velocity and acceleration. Finally, Section 7 summarizes the key contributions of the present study, and identifies further possible developments of them.

2 Real-time spindle speed modulation

Commercial machine tool spindles are capable of speeds up to 100,000 rpm and acceleration rates of a few thousand rpm/sec. Factors that limit spindle performance include the basic design and construction, the type of bearings and lubrication used, thermal management, the intended application context (milling, grinding, drilling, etc.), and anticipated loads. Spindle acceleration rates may be constrained by factors such as the drive motor torque, spindle rotational inertia, frictional losses, bearing preload, etc. For high speeds, safe operation can require acceleration/deceleration times of several seconds.

Spindle speed modulation has mainly been used as a means of suppressing tool chatter, a critical consideration in ensuring high-quality machined parts and avoiding premature tool wear or breakage. The main focus of this study is not on tool chatter *per se*, but rather the use of spindle speed modulation to suppress the large chip load variations that occur when machining a constant depth of cut at a constant feedrate along strongly curved tool paths.

An online chatter avoidance scheme was proposed by Weck et al. [22] to stabilize a face-milling operation by selecting the spindle speed to lie within a stable region of the stability lobe diagram. Smith and Tlusty [15] developed a method to adaptively modify spindle speeds based on using sensors to detect the onset of tool chatter. The sinusoidal spindle speed variation scheme has been employed in turning and milling operations [1, 2, 16, 23, 24] to disrupt the regenerative effects of tool chatter vibrations by continuously varying the spindle speed. Further approaches to real-time detection and suppression of tool chatter, based upon analyzing signals from accelerometer, dynamometer, or microphone sensors, are described in [10–12, 17, 21].

For a planar part geometry described by one or more parametric curves $\mathbf{r}(\xi)$, the methodology developed herein achieves a constant chip load c for a given depth of cut δ, tool radius d, number of cutting edges m, and feedrate V_d of the tool center along the offset path, through continuous modulation of the spindle speed n. A simple expression for the required variation of n in terms of δ, d, the curvature $\kappa(\xi)$ of $\mathbf{r}(\xi)$, and a nominal speed n_0 (corresponding to $\kappa = 0$) is developed. This expression facilitates analysis of the feasibility of spindle speed modulation as a means of suppressing chip load variations for given machining parameters (depth of cut, tool radius, feedrate, etc.), or the selection of parameters to ensure feasibility for a given part geometry.

The spindle speed modulation approach may be regarded as a counterpart to the feedrate modulation method described in [6]. For a given depth of cut and tool radius, the latter varies the feedrate along the offset path to achieve a constant material removal rate, but this incurs a varying chip load. For the spinde speed modulation method, the feedrate and chip load are constant, but the material removal rate is varying. In practice, it may be preferable to adopt a combination of these two methodologies.

3 Chip load, feedrate, and offset paths

For a cylindrical tool with m flutes and rotational speed n rev/min executing a linear path at a constant feedrate V in/min, the *chip load* is conventionally defined [20] as

$$c = \frac{V}{m\,n}, \tag{1}$$

and is a measure of the average thickness of material chips sheared from the workpiece. With a fixed depth of cut δ

and width of cut w, the chip volume[1] removed by the engagement of a single cutting edge with the workpiece is

$$v = c\delta w. \tag{2}$$

The energy expended in removing this chip volume may be expressed as

$$W = E\, v\,,$$

where the *specific cutting energy* E is an intrinsic property of the workpiece material and machining process, defined as the work done in removing unit chip volume. The overall material removal rate of the machining process may be characterized as

$$M = V\delta w, \tag{3}$$

and the total power consumption is $P = E\,M$. Although these macroscopic process measures do not depend on c, large magnitudes or strong variations of the chip load can incur poor surface quality and reduced tool life.

The preceding analysis applies to a fixed spindle speed and feedrate along a linear path with a constant depth and width of cut. For a part with variable curvature, however, the constant chip load estimate (1) and material removal rate (3) are no longer valid when the tool radius is comparable to the radius of curvature (see Section 4 below). Consequently, the instantaneous machining force and power consumption are also no longer constant even if the feedrate, spindle speed, depth of cut, and width of cut are all constant.

Consider a part shape defined by a plane parametric curve $\mathbf{r}(\xi)$, $\xi \in [\,0, 1\,]$ with parametric speed

$$\sigma(\xi) = |\mathbf{r}'(\xi)|, \tag{4}$$

which defines the derivative $\mathrm{d}s/\mathrm{d}\xi$ of arc length s with respect to the curve parameter ξ along $\mathbf{r}(\xi)$. The cumulative arc length function is specified by

$$s(\xi) = \int_0^{\xi} \sigma(u)\mathrm{d}u. \tag{5}$$

The tangent and normal vectors and curvature of $\mathbf{r}(\xi)$ are defined [18] by the following:

$$\mathbf{t}(\xi) = \frac{\mathbf{r}'(\xi)}{\sigma(\xi)}, \quad \mathbf{n}(\xi) = \mathbf{t}(\xi) \times \mathbf{z}, \quad \kappa(\xi) = \frac{[\mathbf{r}'(\xi) \times \mathbf{r}''(\xi)] \cdot \mathbf{z}}{\sigma^3(\xi)}, \tag{6}$$

where $\mathbf{z}$ is a unit vector orthogonal to the plane of $\mathbf{r}(\xi)$. Note here that $\mathbf{n}(\xi)$ points locally to the *right* of $\mathbf{t}(\xi)$, and $\kappa(\xi)$ is negative or positive according to whether $\mathbf{n}(\xi)$ points toward or away from the center of curvature.

[1] Note that c is measured in the local feedrate direction; δ is orthogonal to the feedrate direction in the plane of the tool path (or in the osculating plane for a spatial path); and w is measured orthogonal to that plane.

To cut the shape $\mathbf{r}(\xi)$ with a tool of radius d, the tool center must follow the *offset path* at distance d from $\mathbf{r}(\xi)$, defined by the following:

$$\mathbf{r}_d(\xi) = \mathbf{r}(\xi) + \mathrm{d}\,\mathbf{n}(\xi). \tag{7}$$

Differentiating (7) twice yields [7] the relations

$$\mathbf{r}_d'(\xi) = [1 + \kappa(\xi)d]\mathbf{r}'(\xi),$$
$$\mathbf{r}_d''(\xi) = [1 + \kappa'(\xi)d]\mathbf{r}'(\xi) + \kappa(\xi)d\;\mathbf{r}''(\xi).$$

The parametric speed, tangent and normal, and curvature of the offset $\mathbf{r}_d(\xi)$ are therefore related [7] to those of $\mathbf{r}(\xi)$ by

$$\begin{aligned}
\sigma_d(\xi) &= |\mathbf{r}_d'(\xi)| = |1 + \kappa(\xi)d|\sigma(\xi),\\
\mathbf{t}_d(\xi) &= \frac{\mathbf{r}_d'(\xi)}{\sigma_d(\xi)} = \frac{1 + \kappa(\xi)d}{|1 + \kappa(\xi)d|}\mathbf{t}(\xi),\\
\mathbf{n}_d(\xi) &= \mathbf{t}_d(\xi) \times \mathbf{z} = \frac{1 + \kappa(\xi)d}{|1 + \kappa(\xi)d|}\mathbf{n}(\xi),\\
\kappa_d(\xi) &= \frac{[\mathbf{r}_d'(\xi) \times \mathbf{r}_d''(\xi)] \cdot \mathbf{z}}{\sigma_d^3(\xi)} = \frac{\kappa(\xi)}{|1 + \kappa(\xi)d|}.
\end{aligned} \tag{8}$$

Hence, we have $|\kappa_d(\xi)| > |\kappa(\xi)|$ when $-1 < \kappa(\xi)d < 0$, and $|\kappa_d(\xi)| < |\kappa(\xi)|$ when $\kappa(\xi)d > 0$. These cases identify *concave* and *convex* segments of $\mathbf{r}(\xi)$, respectively. In the singular case $\kappa(\xi)d = -1$, the point $\mathbf{r}_d(\xi)$ is a *cusp* on the offset, at which $\mathbf{t}_d(\xi)$ and $\mathbf{n}_d(\xi)$ exhibit sudden reversals. Note that concave segments with $\kappa(\xi)d < -1$ incur self-intersection loops on $\mathbf{r}_d(\xi)$, that must be trimmed off [7] to avoid gouging (over-cut) of the part shape.

Since the trimmed offset is always used as a tool path, we may henceforth assume that $\kappa(\xi)d \geq -1$ and replace $|1 + \kappa(\xi)d|$ by $1 + \kappa(\xi)d$. Consequently, the offset tangent and normal $\mathbf{t}_d(\xi)$ and $\mathbf{n}_d(\xi)$ always have the same sense as $\mathbf{t}(\xi)$ and $\mathbf{n}(\xi)$, and the curvature $\kappa_d(\xi)$ has the same sign as $\kappa(\xi)$.

Now the speeds of corresponding points that move along $\mathbf{r}(\xi)$ and $\mathbf{r}_d(\xi)$ are defined by the time derivatives

$$V = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \text{and} \quad V_d = \frac{\mathrm{d}s_d}{\mathrm{d}t},$$

where $s(\xi)$ and $s_d(\xi)$ denote arc lengths measured along $\mathbf{r}(\xi)$ and $\mathbf{r}_d(\xi)$, and we have

$$\sigma(\xi) = \frac{\mathrm{d}s}{\mathrm{d}\xi} \quad \text{and} \quad \sigma_d(\xi) = \frac{\mathrm{d}s_d}{\mathrm{d}\xi}.$$

Hence, we observe that

$$\frac{V_d}{V} = \frac{\mathrm{d}s_d}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}s} = \frac{\mathrm{d}s_d}{\mathrm{d}\xi}\frac{\mathrm{d}\xi}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}\xi}\frac{\mathrm{d}\xi}{\mathrm{d}s} = \frac{\sigma_d(\xi)}{\sigma(\xi)} = 1 + \kappa(\xi)d.$$

Here, V should be interpreted as the speed of the tool/part contact point on $\mathbf{r}(\xi)$, while V_d represents the speed of the tool center moving along $\mathbf{r}_d(\xi)$.

With V in in/min and n in rev/min, consecutive engagements of the tool cutting edges with $\mathbf{r}(\xi)$ occur at time intervals $\delta t = 1/mn$ min, and during these intervals the tool/part contact point travels distance $\delta s = V\delta t$ in along $\mathbf{r}(\xi)$. Since the chip load c correlates directly with the distance δs, it varies (approximately) in proportion to

$$V(\xi) = \frac{V_d}{1+\kappa(\xi)d} \tag{9}$$

when the tool rotational speed n is constant, and its center moves along $\mathbf{r}_d(\xi)$ with the constant speed V_d. For $\kappa(\xi) = 0$ we have $V = V_d$, but when $\mathbf{r}(\xi)$ has strong negative and positive curvatures, the denominator $1+\kappa(\xi)d$ of (9) induces large deviations of V about the nominal value V_d, and consequently large chip load variations. On the other hand, if we specify a constant speed V of the tool/part contact point to achieve an (approximately) constant chip load, for a fixed tool rotational speed n, the resulting tool center speed

$$V_d(\xi) = [1+\kappa(\xi)d]V \tag{10}$$

may experience strong variations due to the factor $1+\kappa(\xi)d$, that cause large acceleration/deceleration rates of the tool along the offset path.

Clearly, the suppression of chip load variations and maintenance of smooth tool motions impose conflicting demands on the specification of feedrate when machining complex shapes with radii of curvature comparable in magnitude to the tool radius. To address this, we consider the possibility of a real-time modulation of the spindle speed n as a mean to suppress chip load variations, when the tool center moves with constant speed V_d along the offset path.

The methodology developed here is primarily focused on finish machining strongly curved part shapes at constant depth and width of cut, but can be extended to accommodate other machining processes if *a priori* information about the variation of depth and width of cut is available.

From (1), we observe that, for a constant speed V of the tool/part contact point, the chip load c is inversely proportional to the rotational speed n of the spindle. Hence, using (9), to achieve an approximately constant chip load c for a fixed feedrate V_d of the tool center, the spindle speed should be expressed in terms of the curve parameter ξ as

$$n(\xi) = \frac{V_d}{mc[1+\kappa(\xi)d]}. \tag{11}$$

Defining for fixed V_d, m, c the nominal spindle speed

$$n_0 = \frac{V_d}{mc}, \tag{12}$$

corresponding to the case $\kappa\xi) = 0$, we have $n(\xi) > n_0$ when $-1/d < \kappa(\xi) < 0$ and $n(\xi) < n_0$ when $\kappa(\xi) > 0$ (the case $\kappa(\xi) < -1/d$ corresponds to segments of the offset curve that must be trimmed off, to prevent gouging).

The spindle speed formula (11) realizing an (approximately) constant chip load c when a tool of radius d moves at constant speed V_d on the offset path $\mathbf{r}_d(\xi)$ to the part shape $\mathbf{r}(\xi)$ is based on a first-order (tangent) approximation to estimate the spacing δs of successive engagements of tool cutting edges with $\mathbf{r}(\xi)$, and is valid for $|\kappa\delta| \ll 1$. A more accurate model can be developed through a local second-order (i.e., osculating circle) analysis of the chip load at each point of $\mathbf{r}(\xi)$, as described in the following section.

4 Second-order spindle speed model

The first-order spindle speed model (11) is independent of the depth of cut δ, and is valid only when $|\kappa\delta| \ll 1$. We now present a more precise model, based on a second-order approximation of $\mathbf{r}(\xi)$ by its osculating circle (or circle of curvature) at each point. The analysis follows the approach developed in [6], except that the focus is now on modulating the spindle speed rather than the feedrate, and the results are expressed in terms of the curvature variation of the part geometry, rather than the curvature of the offset tool path.

Consider the removal of a fixed depth of cut δ by a tool of radius d that moves with constant feedrate V_d along offset tool paths that correspond to (i) the interior of a circle of radius r; (ii) a straight line; and (iii) the exterior of a circle of radius r (see Fig. 1). With the convention that the material strip to be removed lies to the *left* of the path tangent, relative to the direction of motion, the part shape in cases (i) and (iii) has the parameterizations

$$\mathbf{r}(\theta) = r(\cos\theta, -\sin\theta) \quad \text{and} \quad \mathbf{r}(\theta) = r(\cos\theta, \sin\theta),$$

with the constant curvatures $\kappa(\theta) = -1/r$ and $\kappa(\theta) = 1/r$, respectively.

For case (i), the total tool path length and traversal time are $L = 2\pi(r-d)$ and $T = L/V_d$, and the total volume of material removed is

$$w\left[\pi r^2 - \pi(r-\delta)^2\right] = 2\pi\delta w\big(r - \tfrac{1}{2}\delta\big).$$

Thus, noting that $\kappa = -1/r$ in this case, the corresponding material removal rate can be expressed as

$$M = V_d\delta w\frac{r-\frac{1}{2}\delta}{r-d} = V_d\delta w\frac{1+\frac{1}{2}\kappa\delta}{1+\kappa d}. \tag{13}$$

Since the number of chips removed during time T is $N = mnT$, the volume per chip is given by

$$v = \frac{V_d\delta w}{mn}\frac{1+\frac{1}{2}\kappa\delta}{1+\kappa d}.$$

Hence, defining the chip load c through the relation (2), we obtain

$$c = \frac{V_d}{mn}\frac{1+\frac{1}{2}\kappa\delta}{1+\kappa d}. \tag{14}$$

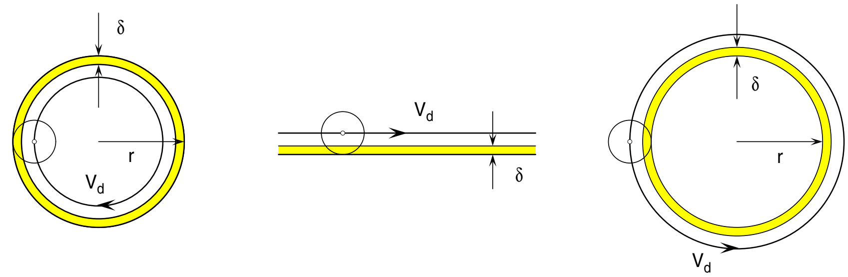

Fig. 1 Estimation of the chip load in machining a depth of cut δ with a tool of radius d moving with feedrate V_d for (left) an interior cut on a circle of radius r; (center) a linear cut; and (right) an exterior cut on a circle of radius r. The shaded areas identify the material strip removed from the workpiece

Conversely, for case (iii), the path length is $L = 2\pi(r+d)$, the traversal time is $T = L/V_d$, and the total volume of material removed is

$$w\left[\pi(r+\delta)^2 - \pi r^2\right] = 2\pi\delta w\left(r+\tfrac{1}{2}\delta\right).$$

With $\kappa = 1/r$ in this case, the corresponding material removal rate is

$$M = V_d\delta w\frac{r+\frac{1}{2}\delta}{r+d} = V_d\delta w\frac{1+\frac{1}{2}\kappa\delta}{1+\kappa d}. \tag{15}$$

Again, the total number of chips is $N = mnT$, so the volume per chip is

$$v = \frac{V_d\delta w}{mn}\frac{1+\frac{1}{2}\kappa\delta}{1+\kappa d}.$$

Hence, defining the chip load c through (2) gives

$$c = \frac{V_d}{mn}\frac{1+\frac{1}{2}\kappa\delta}{1+\kappa d}. \tag{16}$$

Note that the interior and exterior circle paths yield identical expressions (13) and (15) for the material removal rate M, and (14) and (16) for the chip load c, being distinguished only by the sign of the part curvature κ. These expressions also agree with the simple formulae (1) and (3) for a linear part shape with $\kappa = 0$ (since we have $V_d = V$ in that case).

When machining with a tool of radius d, a part shape specified by a curve $\mathbf{r}(\xi)$ with non-constant curvature $\kappa(\xi)$ is well-approximated by its osculating circle at each point if [6] the condition

$$\frac{1}{6}\frac{d\kappa}{ds}d^2 \ll 1 \tag{17}$$

holds everywhere. Here, the arc-length derivative $d\kappa/ds$ of the curvature is equal to $\kappa'(\xi)/\sigma(\xi)$, where

$$\kappa'(\xi) = \frac{[\mathbf{r}'(\xi)\times\mathbf{r}'''(\xi)]\cdot\mathbf{z} - 3\sigma^2(\xi)\sigma'(\xi)\kappa(\xi)}{\sigma^3(\xi)},$$

$$\sigma'(\xi) = \frac{\mathbf{r}'(\xi)\cdot\mathbf{r}''(\xi)}{\sigma(\xi)}. \tag{18}$$

Thus, when the condition (17) holds along $\mathbf{r}(\xi)$, the formula

$$c(\xi) = \frac{V_d}{mn}\frac{1+\frac{1}{2}\kappa(\xi)\delta}{1+\kappa(\xi)d}, \tag{19}$$

obtained by generalizing (1), (14), and (16) to non-constant curvature $\kappa(\xi)$, yields an accurate indication of chip load variation along the part shape, assuming a constant spindle speed n, depth of cut δ, tool radius d, number of cutting edges m, and feedrate V_d along the offset path.

From (19), we see that a *constant* chip load c may be achieved through a modulation of the spindle rotational speed in accordance with the curvature $\kappa(\xi)$ through the expression

$$n(\xi) = n_0\frac{1+\frac{1}{2}\kappa(\xi)\delta}{1+\kappa(\xi)d}, \tag{20}$$

where δ, d, m, V_d are all constant, and n_0 is specified by (12). Note that the material removal rate has the same dependence on the curvature, namely

$$M(\xi) = M_0\frac{1+\frac{1}{2}\kappa(\xi)\delta}{1+\kappa(\xi)d}, \tag{21}$$

where $M_0 = V_d\delta w$ is the nominal value corresponding to $\kappa = 0$. The factor $(1+\frac{1}{2}\kappa(\xi)\delta)/(1+\kappa(\xi)d)$ in (20) and (21) is non-constant whenever $\kappa(\xi)$ is non-constant, except when $\delta = 2d$ (i.e., the tool is fully immersed, since the depth of cut is equal to the tool diameter). For $\delta < 2d$, this factor is greater than or less than 1 according to whether $\kappa(\xi)$ is negative or positive.

Although the spindle speed variation (20) is mainly intended for planar machining applications, it is also applicable to spatial tool/part contact paths $\mathbf{r}(\xi)$, with a second-order (osculating plane) approximation being invoked at each point of $\mathbf{r}(\xi)$, and V_d, δ, d, m are all assumed to be constant.

To analyze whether the variable speed (20) is physically realizable for any given spindle, tool radius, depth of cut, part geometry, and feedrate V_d, the maximum speeds and angular acceleration/deceleration rates $a = dn/dt$ it incurs must be determined. The time derivative of $n(\xi)$ may be obtained as

$$a = \frac{dn}{dt} = \frac{ds_d}{dt}\frac{d\xi}{ds_d}\frac{dn}{d\xi} = \frac{V_d}{\sigma_d(\xi)}\frac{dn}{d\xi}, \tag{22}$$

where differentiating (20) yields

$$\frac{\mathrm{d}n}{\mathrm{d}\xi} = -n_0 \frac{d - \frac{1}{2}\delta}{[1 + \kappa(\xi)d]^2} \kappa'(\xi), \tag{23}$$

the derivative of the curvature being specified by (18).

Finally, substituting (8) and (23) into (22), and setting $\kappa'/\sigma = \mathrm{d}\kappa/\mathrm{d}s$, we obtain

$$a(\xi) = -n_0 V_d \frac{d - \frac{1}{2}\delta}{[1 + \kappa(\xi)d]^3} \frac{\mathrm{d}\kappa}{\mathrm{d}s}(\xi). \tag{24}$$

With units rev/min for n, in/min for V_d, in for d and δ, in^{-1} for κ, and in^{-2} for $\mathrm{d}\kappa/\mathrm{d}s$, the spindle angular acceleration (24) has units rev/min^2 (divide by 60 to convert to rpm/sec). Note here that $d - \frac{1}{2}\delta \geq 0$, since the case $\delta = 2d$ corresponds to immersion of the full tool diameter in the workpiece. Hence, a is negative or positive according to whether κ is increasing or decreasing, and it is zero when κ is locally stationary, i.e., $\mathrm{d}\kappa/\mathrm{d}s = 0$.

The preceding analysis can be used to ensure that known limits on spindle speed and angular acceleration are satisfied. Assuming that $\delta < 2d$ (i.e., the depth of cut is less than the tool diameter), the expression (20) is monotone-decreasing with increasing κ, and therefore attains its maximum value,

$$n_{\max} = n_0 \frac{1 + \frac{1}{2}\kappa_{\min}\delta}{1 + \kappa_{\min}d},$$

at the point of $\mathbf{r}(\xi)$ where $\kappa(\xi)$ attains its minimum (signed) value $\kappa_{\min}$, which may be identified by computing the roots of $\kappa'(\xi)$ on $\xi \in (0, 1)$. Note that the angular speed (20) and acceleration (24) are unbounded as $\kappa(\xi) \to -1/d$. To guarantee gouge-free machining of the part, the tool radius must be chosen such that $1 + \kappa_{\min}d > 0$, and d should be appreciably smaller than $-1/\kappa_{\min}$ when $\kappa_{\min} < 0$, to avoid excessive angular speeds and accelerations.

For fixed feedrate and depth of cut and a known negative $\kappa_{\min}$, the bound

$$d < \frac{h - 1 + \frac{1}{2}|\kappa_{\min}|\delta}{h|\kappa_{\min}|}$$

on the tool radius ensures that $n_{\max} < hn_0$ for any $h > 0$. Alternatively, for a prescribed tool radius d with $1+\kappa_{\min}d > 0$ and $\kappa_{\min}$ negative, a constant feedrate along the offset path satisfying

$$V_d < mc\, \frac{1 - |\kappa_{\min}|d}{1 - \frac{1}{2}|\kappa_{\min}|\delta} n_{\lim}$$

ensures that $n < n_{\lim}$ for any specified upper limit $n_{\lim}$.

The identification of constraints to ensure satisfaction of bounds on the angular acceleration (24) is more involved. Differentiating (24) with respect to s, we find that the extremal values of a (which may be negative or positive) are identified by satisfaction of the condition

$$\frac{\mathrm{d}^2\kappa}{\mathrm{d}s^2} = \frac{3d}{1 + \kappa d} \left(\frac{\mathrm{d}\kappa}{\mathrm{d}s}\right)^2.$$

For Pythagorean-hodograph (PH) curves [5], κ and its arc-length derivatives are rational in the curve parameter, and this condition can be reduced to a polynomial equation in the Bernstein basis on the interval $\xi \in [0, 1]$, whose roots can be accurately computed by standard methods.

5 Analysis of spindle motor dynamics

For brevity and clarity, we employ a simplified model of a spindle driven by a (brushed or brushless) DC motor, without consideration of frictional power losses and current or voltage limits—these refinements can be incorporated without undue difficulty into the basic model described below, based on the known characteristics of particular motors and controllers.

The total power consumed by the machining process is the product of the specific cutting energy and the material removal rate, and has the dependence

$$P(\xi) = EM(\xi) \tag{25}$$

on position along the tool path, the local material removal rate $M(\xi)$ being defined by (21). This power is expended on both the chip shearing mechanics and associated frictional dissipation, and is supplied by the drive systems of both the machine axes and the tool spindle (P does not account for frictional losses in the axis and spindle drive systems, or other sources of machine power consumption not directly related to the chip shearing mechanism).

For simplicity, we assume here that the spindle consumes a fixed fraction f of the total power (25)—more detailed knowledge of the division of the power P between the axis drives and the spindle, if available, can be incorporated into the following analysis by formulating f as a non-constant function.

In analyzing the motor dynamics, it is convenient to employ the angular speed $\omega = 2\pi n/60$ in rad/sec instead of n in rev/min. Neglecting dissipation, the operation of a DC motor driven by an applied voltage $\mathcal{V}$ is governed by two basic relations,

$$T = kI \quad \text{and} \quad \mathcal{V}_E = k\omega, \tag{26}$$

the *motor equation* and the *generator equation*. T and ω are the motor torque and angular speed, I is the armature current, $\mathcal{V}_E$ is the "back emf" generated by the motor (opposing the applied voltage), and k is a known constant[2] for a particular motor. By Ohm's law, we have

$$\mathcal{V} = \mathcal{V}_E + IR, \tag{27}$$

where R is the armature resistance. Combining (26) and (27) then yields the relation

$$T = T_s \left(1 - \frac{\omega}{\omega_*}\right), \tag{28}$$

known as the motor characteristic, where $T_s = k\mathcal{V}/R$ and $\omega_* = \mathcal{V}/k$ are the *stall torque* and *no-load speed* for a given applied voltage $\mathcal{V}$.

For fixed $\mathcal{V}$, the torque T and angular velocity ω are linearly dependent, with T decreasing from its maximum value T_s to zero as ω increases from 0 to ω_*. Thus, T and ω can be independently varied only by varying the motor voltage $\mathcal{V}$. By re-writing (28) as

$$T = \frac{k}{R}(\mathcal{V} - k\omega), \tag{29}$$

it is evident that any desired torque T can (within certain limits) be achieved at a given angular speed ω by adjusting the motor voltage $\mathcal{V}$.

Now if fP is the spindle power associated with the material removal rate (21), the spindle torque T_c required to achieve the cutting action at constant angular speed ω is

$$T_c = \frac{fP}{\omega}. \tag{30}$$

However, if the spindle accelerates, the torque T must overcome the spindle inertia J in addition to cutting the workpiece material, i.e., we must have

$$T = T_c + J\frac{\mathrm{d}\omega}{\mathrm{d}t}.$$

On substituting from (29) and (30) and solving for $\mathcal{V}$, we obtain

$$\mathcal{V} = k\omega + \frac{R}{k}\left[\frac{fP}{\omega} + J\frac{\mathrm{d}\omega}{\mathrm{d}t}\right]. \tag{31}$$

With $\omega = 2\pi n/60$ and $\mathrm{d}\omega/\mathrm{d}t = 2\pi a/3600$, where n and a are given in terms of the part curvature κ and its arc-length derivative $\mathrm{d}\kappa/\mathrm{d}s$ through (20) and (24), and P is given by (21) and (25), (31) determines the variation of the motor voltage $\mathcal{V}$ with the curve parameter ξ necessary to sustain the machining process for a constant feedrate V_d, depth of cut δ, width of cut w, and a constant chip load c achieved through spindle speed modulation.

[2]In the absence of dissipation, the same constant k, measured in Nm/A or V/(rad/sec), appears in both the relations (26).

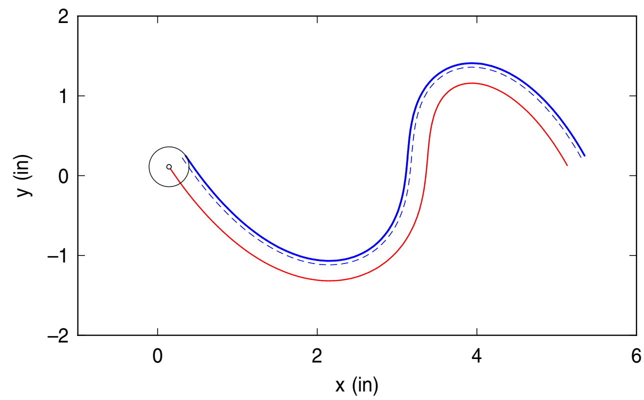

Fig. 2 The desired part shape $\mathbf{r}(\xi)$ is indicated by the blue curve, while the dashed blue curve indicates the depth of cut $\delta = 0.05$ in to be removed. The red curve indicates the offset path (7) for a tool of radius $d = 0.25$ in

6 Illustrative example

The example shown in Fig. 2 is used to illustrate the principles developed in the preceding section. The part shape is specified by a quintic PH curve $\mathbf{r}(\xi)$. Although the analysis holds for any parametric curve with C^3 continuity, PH curves offer some key advantages in the present context [5]: the parametric speed (4) and arc length (5) are both polynomial functions of the curve parameter; the offset (7) is a rational curve; and real-time interpolator algorithms are available for constant and variable feedrates. In Fig. 2, the curve dimensions are in inches, and the tool radius and depth of cut are specified as $d = 0.25$ in and $\delta = 0.05$ in.

Figure 3 shows the variation of the chip load estimate (19) with arc length along the offset path $\mathbf{r}_d(\xi)$ for a constant spindle speed $n = 4000$ rpm, and constant feedrates $V_d = 20$, 40, 60 in/min. Note that the overall magnitude of the chip load, and its range of variation, are

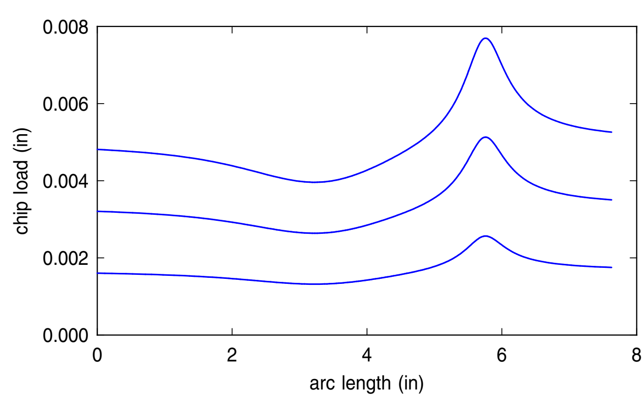

Fig. 3 Variation of the chip load (19) for a depth of cut $\delta = 0.05$ in along the curve in Fig. 2, using a tool with radius $d = 0.25$ in and $m = 3$ flutes at constant spindle speed $n = 4000$ rpm, and constant feedrates $V_d = 20$ in/min (lower), 40 in/min (middle), and 60 in/min (upper) along the offset path

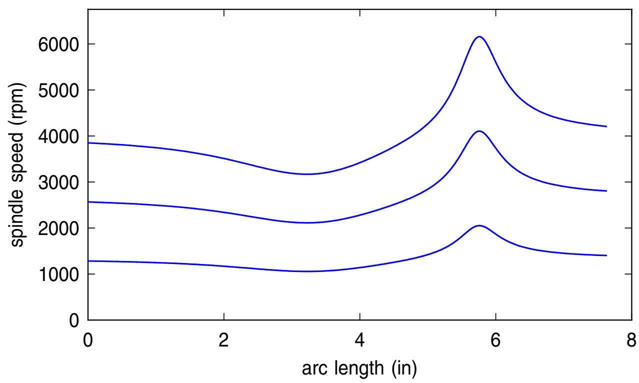

Fig. 4 Variation of the angular velocity (20) for a depth of cut $\delta = 0.05$ in along the curve in Fig. 2, using a tool with radius $d = 0.25$ in and $m = 3$ flutes, for a constant chip load $c = 0.005$ in and feedrates $V_d = 20$ in/min (lower), 40 in/min (middle), and 60 in/min (upper) along the offset path

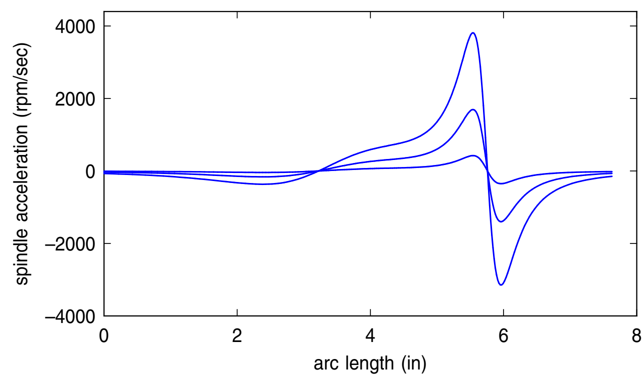

Fig. 5 The spindle angular acceleration (24) for a depth of cut $\delta = 0.05$ in along the curve in Fig. 2, using a tool with radius $d = 0.25$ in and $m = 3$ flutes, for a constant chip load $c = 0.005$ in and feedrates $V_d = 20$ in/min, 40 in/min, and 60 in/min along the offset path — the increasing amplitudes of the angular acceleration correspond to the increasing feedrate values

proportional to V_d. Although the depth of cut, feedrate, and spindle speed are all constant, the minimum and maximum chip loads are observed to differ by a factor ~ 2.

Figure 4 shows the spindle speed variation (20) with arc length along the offset path $\mathbf{r}_d(\xi)$ so as to achieve a constant chip load $c = 0.005$ in for constant feedrates $V_d = 20$, 40, 60 in/min. These spindle speed variations are in direct proportion to the chip load variations seen in Fig. 3, so the minimum and maximum spindle speeds also differ by ~ 2. The highest spindle speeds are correlated with the strongest concavities of $\mathbf{r}(\xi)$, which incur the highest chip loads when a constant spindle speed n and feedrate V_d are used.

In Fig. 5, we illustrate the variation of the spindle angular acceleration defined by (24), associated with the spindle speeds specified by (20), for the constant feedrates $V_d = 20$, 40, 60 in/min along the offset path $\mathbf{r}_d(\xi)$. The much stronger (quadratic) dependence on V_d is clearly evident in this plot.

For an assumed feedrate $V_d = 60$ in/min $= 25.4$ mm/sec, a depth of cut $\delta = 0.05$ in $= 1.27$ mm, and a width of cut $w = 0.2$ in $= 5.08$ mm, the nominal material removal rate is $M_0 = 163.87$ mm^3/sec, and for specific cutting energy $E = 2$ J/mm^3, the nominal power consumption is $P_0 = EM_0 = 327.74$ W. We assume that $f = 0.5$, i.e., half of this power is consumed by the spindle rather than the axis drive system. Selecting values $k = 0.0125$ Nm/A, $R = 0.5\ \Omega$, and $J = 4 \times 10^{-5}$ kg m^2 for the motor constant, the armature resistance,[3] and the spindle inertia, the expression (31) can be evaluated by using $\omega = 2\pi n/60$ and $\mathrm{d}\omega/\mathrm{d}t = 2\pi a/3600$, with n and a defined by (20) and (24) and P given by (21) and (25), to determine the required variation of motor voltage $\mathcal{V}$ for a constant chip load $c = 0.005$ in at a fixed feedrate V_d of the tool center.

[3]The value $R = 0.5\ \Omega$ represents the resistance under normal operating conditions: at start–up, before the back emf develops, R must be higher to avoid excessive currents.

Figure 6 illustrates, for the example shown in Fig. 2 and the parameters specified above, the motor voltage $\mathcal{V}$ computed from (31). Notwithstanding the strong variations of spindle speed and acceleration evident in Figs. 4 and 5 for the case $V_d = 60$ in/min, the required modulation of the motor voltage $\mathcal{V}$ seen in Fig. 6 is relatively mild. Among the three terms in (31) that contribute to $\mathcal{V}$, as shown in Fig. 7, the first (back emf) term shows the strongest variation, proportional to the angular speed. The second (cutting torque) term is dominant, and is actually constant, since P and ω are both proportional to the curvature-modulation factor $(1 + \frac{1}{2}\kappa\delta)/(1 + \kappa d)$. Finally, the third (spindle inertia) term is relatively minor.

Figure 6 also illustrates the variation of the motor current $I = (\mathcal{V} - k\omega)/R$. Since $k\omega$ makes a relatively modest contribution to $\mathcal{V}$ in (31), the variation of the current is also relatively subdued. Consequently, the total motor power $I\mathcal{V}$

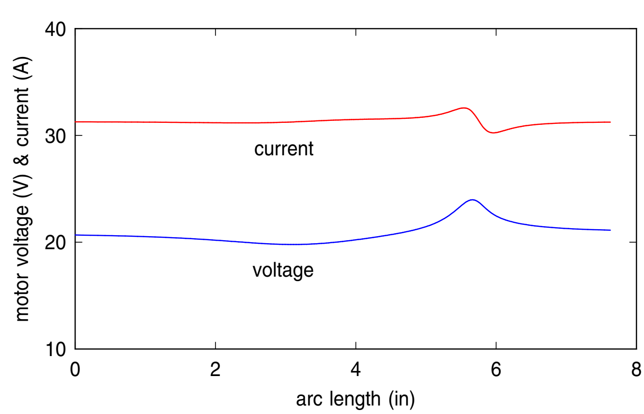

Fig. 6 Variation of the motor voltage $\mathcal{V}$ and current I to achieve a constant chip load c, for a fixed feedrate and depth of cut along the path in Fig. 2. The motor specifications and other parameters are as prescribed in the text

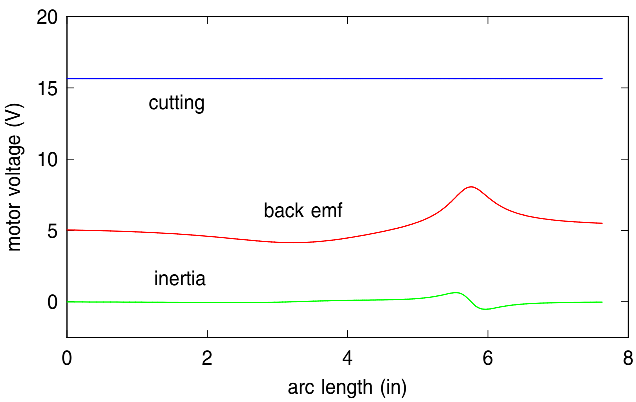

Fig. 7 The three terms in (31) that determine the motor voltage $\mathcal{V}$ — the back emf term $k\omega$, cutting term $RfP/k\omega$, and inertia term $(RJ/k)\mathrm{d}\omega/\mathrm{d}t$

varies by no more than $\sim 25\%$, despite the spindle speed modulation.

Although the results presented in Figs. 6 and 7 are specific to the part geometry and DC motor parameters enumerated above, and can be expected to vary according to each particular machining problem under consideration, the basic analysis of the methodology for suppressing chip load variations by spindle speed modulation at constant feedrate is broadly applicable.

7 Closure

Chip load variations in machining operations have undesirable consequences, including poor machined surface quality and premature tool wear or damage. One approach to suppressing chip load variations in the context of finish-machining a fixed depth of cut from a curved part shape is through feedrate modulation, but this can incur high feed acceleration/deceleration rates near concave radii of curvature comparable to the tool radius.

As an alternative, the present study has explored the feasibility of real-time spindle speed modulation as a means to suppress chip load variations. Using a second-order model, closed-form expressions for the required spindle angular velocities and accelerations were derived, and their realization by a spindle driven by a DC motor was analyzed. Representative results from a computed example indicate that spindle speed modulation should prove to be an effective means of subduing chip load variations and their undesirable consequences for typical ranges of machining process parameters.

The present study is preliminary and exploratory, and can be profitably extended in several directions. First, a generalization beyond the context of finish-machining a fixed depth of cut from planar shapes would be desirable. This will require knowledge of depth and width of cut variations that can be exploited in real time. Second, strategies that combine spindle speed and feedrate modulation may be advantageous in terms of broadening the range of machining operations that can be accommodated. Finally, implementation of the methodology developed herein on a CNC machine equipped with an open-architecture spindle controller will prove informative in its verification, and in identifying points of concern and possible enhancements.

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References

1. al–Regib E, Ni J, Lee SH (2003) Programming spindle speed variation for machine tool chatter suppression. Int J Mach Tools Manuf 43:1229–1240
2. Altintas Y, Chan PK (1992) In-process detection and suppression of chatter in milling. Int J Mach Tools Manuf 32:329–347
3. Campos JG, Hardwick M (2006) A traceability information model for CNC manufacturing. Comput Aided Design 38:540–551
4. Conway JR, Ernesto CA, Farouki RT, Zhang M (2012) Performance analysis of cross-coupled controllers for CNC machines based upon precise real-time contour error measurement. Int J Mach Tools Manuf 52:30–39
5. Farouki RT (2008) Pythagorean-hodograph curves: algebra and geometry inseparable. Springer, Berlin
6. Farouki RT, Manjunathaiah J, Nicholas D, Yuan G-F, Jee S (1998) Variable–feedrate CNC interpolators for constant material removal rates along Pythagorean-hodograph curves. Comput Aided Design 30:631–640
7. Farouki RT, Neff CA (1990) Analytic properties of plane offset curves. Comput Aided Geom Design 7:83–99
8. Farouki RT, Srinathu J (2017) A real-time CNC interpolator algorithm for trimming and filling planar offset curves. Comput Aided Design 86:1–11
9. Hardwick M, Loffredo D (2006) Lessons learned implementing STEP–NC AP–238. Int J Comput Integ Manuf 19:523–532
10. Kim H, Cho M-H, Koo J-Y, Lee J-W, Kim J-S (2015) Real time chatter vibration control system in high speed milling. J Mater Sci Eng 5:228–236
11. Liao YS, Young YC (1996) A new on-line spindle speed regulation strategy for chatter control. Int J Mach Tools Manuf 36:651–660
12. Lin SC, Hu MR (1992) Low vibration control system in turning. Int J Mach Tools Manuf 32:629–640
13. Nittler KM, Farouki RT (2016) A real-time surface interpolator methodology for precision CNC machining of swept surfaces. Int J Adv Manuf Technol 83:561–574
14. Schraeder TF, Farouki RT (2014) Experimental performance analysis of an inverse dynamics CNC compensation scheme for high-speed execution of curved toolpaths. Int J Adv Manuf Technol 73:195–208
15. Smith S, Tlusty J (1992) Stabilizing chatter by automatic spindle speed regulation. CIRP Ann 41:433–436
16. Soliman E, Ismail F (1997) Chatter suppression by adaptive speed modulation. Int J Mach Tools Manuf 37:355–369

17. Soliman E, Ismail F (1998) A control system for chatter avoidance by ramping the spindle speed. ASME J Manuf Sci Eng 120:674–683
18. Struik DJ (1961) Lectures on classical differential geometry. Dover Publications (reprint), New York
19. Suh S-H, Cho J-H, Hong H-D (2002) On the architecture of intelligent STEP–compliant CNC. Int J Comput Integ Manuf 15:168–177
20. Tlusty J (2000) Manufacturing processes and equipment. Prentice–Hall, Upper Saddle River
21. van Dijk NJM, van de Wouw N, Nijmeijer H, Faassen RPH, Doppenberg EJJ, Oosterling JAJ (2008) Real-time detection and control of machine tool chatter in high-speed milling, Intercut 2008, 2nd International Conference on Innovating Cutting Processes & Smart Machining, Cluny, October 22–23, 2008
22. Weck M, Verhaag E, Gather M (1975) Adaptive control for face–milling operations with strategies for avoiding chatter vibrations and for automatic cut distribution. CIRP Ann 24:405–409
23. Wu D, Chen K (2010) Chatter suppression in fast tool servo-assisted turning by spindle speed variation. Int J Mach Tools Manuf 50:1038–1047
24. Zhang HH, Jackson MJ, Ni J (2009) Spindle speed variation method for regenerative machining chatter control. Int J Nanomanuf 3:73–99