Finite Element Analysis of Various Spur Gear Tooth Profiles for Contact and Fatigue Stress Distribution

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***Abstract*—Despite extensive research into the profile modification and performance enhancement of spur gear design metrics, a complete comparative and parametric study of different spur gear tooth profiles and the values of performance matrices is still to be clearly understood because of the great complexity in the gear tooth profile design. In this paper an attempt has been made to determine and compare the performance metrics of spur gears of various tooth profiles. The effect of major performance metrics of different spur gear tooth profiles are studied and compared with involute tooth profile of spur gear pair. The teeth of involute, cycloidal, circular and asymmetric spur gear pair are modelled for the same magnitude. The results of three dimensional finite element analyses (FEM) from ANSYS are presented. Contact and fatigue analysis are also performed in order to investigate the performance metrics of different spur gear tooth profiles. In addition in order to justify the FEM analysis of various spur gear tooth profiles a detailed study of analytical analysis also performed and compared with involute tooth profile.**

NOMENCLATURE

TB	Tooth bending stress
SF	Allowable surface fatigue stress
CS	Contact stress
SFS	Surface fatigue strength
BS	Allowable bending stress
SFG	Tooth surface strength of gear
SFP	Tooth surface strength of pinion
FEA	Finite element analysis
AA	Analytical analysis

I. INTRODUCTION

THE main purpose of gear mechanisms is to transmit rotation and torque between shaft axes. The gear wheel is a machine element that has intrigued many engineers because of numerous technological problems arising in a complete mesh cycle. In order to achieve high load carrying capacity with reduced weight of gear drives but with increased strength in gear transmission, gear design on the basis of tooth stress analysis, tooth modifications and optimum design of gear drives are becoming major research areas. Gears with involute teeth have widely been used in industry because of the low cost of manufacturing. Critical evaluation of spur gear design performance therefore plays a crucial role in estimating the degree of success of such gear systems in terms of stresses and deformation developed in spur gears.

In the evaluation of spur gear designs, certain basic gear design performance metrics such as tooth bending stress, allowable bending stress, contact stress, surface fatigue strength, allowable surface fatigue stress, tooth surface strength of gear and pinion etc. are to be carefully considered. The effectiveness of the spur gear design can be improved only when all these metrics are controlled properly. Gear designers are constantly looking for ways to improve effectiveness through various techniques. Despite such attempts, the control of all these metrics and achieving the desired performance is a very complicated task. Therefore, there is great need for detailed study of the intricacies of spur gear design especially for different types of gear profiles. In this paper, an attempt is made to study the performance of a spur gear system for four different tooth profiles namely involute, cycloidal, circular and asymmetric profiles.

II. ADVANCEMENTS IN GEAR TOOTH PROFILE DESIGN

Gear analysis is one of the most significant issues in the machine elements theory particularly in the field of gear design and gear manufacturing. Many of the researchers have proposed several concepts for gear design optimization to enhance the performance of gear systems. Cavdar et al. [1] has developed tooth model of involute spur gears with asymmetric teeth to improve the performance of gears such as increasing the load capacity or reducing noise and vibration. In this study, a computer program was developed for asymmetric gears with greater drive side pressure angle than coast side pressure angle to determine bending load carrying capacities and contact conditions of asymmetric gear drives. Kapelevich and Shekhtman [2] proposed a method called direct gear design using FEA for bending stress evaluation based on bending stress balance allowing equalizing of tooth strength and durability for the pinion and the gear. This method also describes the optimization of the fillet profile by reducing the maximum bending stress in the gear tooth root area by 10–30% for both symmetric and asymmetric gear tooth profiles. Huang and Liu [3] proposed a dynamic stiffness based method to calculate the dynamic response of a gear tooth subject to meshing force on equations of motion for a Timoshenko beam model. Li [4] has developed a loaded tooth contact analysis program to calculate all of the three-dimensional, thin-rimmed gear structures with all of the gear parameters. Lewicki [5] has performed analysis for a variety of gear tooth and rim configurations using the finite element method with principles of linear elastic fracture mechanics to predict crack propagation paths. Kapelevich [6] has developed a basic geometric theory of the gears with asymmetric teeth profile that allows for an increase in load capacity while reducing weight and dimensions for some types of gears to research and design gears independently from generating rack parameters. It also provides wide variety of solutions for a particular couple of gears that are included in the area of existence.

Kahramanet al. [7] has developed a surface wear for helical gear pairs to study the influence of tooth modifications on helical gear wear. The model uses a finite element based gear contact mechanics model to predict the contact pressures at a number of discrete rotational gear positions and a computational procedure for determining relative sliding distances of mating points on each gear for each rotational increment. In this method a simplified design formula was also proposed that links modification parameters directly to initial wear rates. Zhang et al. [8] proposed a mathematical model of parametric tooth profile of spur gears where the line of action is given. The line of action usually comprises a simple curve. The proposed mathematical model was aimed at enhancing the freedom of tooth profile design by combining the simple curves into the line of action. The curvature, sliding velocity, contact ratio and the limitation of undercutting can be derived directly from the equation of line of action.

Chen and Tsay [9] proposed a mathematical model of the modified helical gear with small number of teeth. This was developed by tooth-profile shifting and basic geometry modification to investigate the condition of tooth undercutting for the involute profile gears using the developed mathematical models. Ognyan [10] conducted research related to the geometric design of spur gear drives of symmetric and asymmetric teeth and proposed realized potential method for geometric design of involute gear drives of symmetric and asymmetric meshing. In addition, for the realization of gear drive potential, the introduction of different parameters exerts a decisive role for the determination of bottom clearances and depths of fillet curves of the rack-cutters.

Imrek and Duzcukoglu [11] conducted experimental study on width modification of a spur gear to fix instantaneous pressure changes along single meshing area on the gear profile. In this gear, variable pressure distribution caused by the single and double teeth meshing and the radius of curvature along the active gear profile was approximately kept constant by maintaining a constant ratio of applied load to the tooth width on every point. The amount of wear in the teeth profiles between the modified and unmodified gears was compared. Costopoulos and Spitas [12] proposed several tooth designs alternative to the standard involute for increasing the load carrying capacity of geared power transmissions and to combine the good meshing properties of the driving involute and the increased strength of non-involute curves to provide constant direction of rotation although they can be used in a limited way for reverse rotation. Ali and Mohammad [13] had done research on the effect of contact ratio change on the stresses generated on meshing involute profile of gear teeth using FEA for stress analysis on gear model.

All of the above works have attempted to enhance effectiveness of gear systems through weight reduction, wear reduction, vibration and noise reduction. Studies have also been performed mostly using involute and asymmetric gear tooth profiles. In addition, most of these works have estimated tooth bending stress and contact stress. The estimation of allowable surface fatigue stress, contact stress, surface fatigue strength, tooth surface strength of gear and pinion and permissible bending stress have not received much attention. The performance of alternative tooth profiles such as circular and cycloidal, generally not in use on account of manufacturing difficulties or reduced strength at root, have also not received much attention.

III. Modeling Of Tooth Profiles

A. Basis for Comparative Study

In this paper, an attempt has been made to study four different tooth profiles namely involute, cycloidal, circular and asymmetric profiles in terms of tooth bending stress and contact stress as studied by most of the researchers as well as other critical stresses such as allowable surface fatigue stress, contact stress, surface fatigue strength, tooth surface strength of gear and pinion and permissible bending stress. An attempt is thus made to identify the best suited tooth profile for a given application in terms of all these stresses. This would give a complete picture of the load bearing performance of a given profile. The refined form of the Lewis equation for tooth bending stress is adopted. Relationships for permissible tooth bending stress, tooth surface strength of the pinion and gear, dynamic contact stress, surface fatigue strength, allowable surface fatigue stress as per AGMA standards are adopted. Based on these relationships, the performance metrics were computed for the design specifications mentioned in Table 1.

B. Modeling of Spur gear tooth profiles

Using the specifications listed in Table 1, analytical analysis was carried out for each of the tooth profiles listed above in order to compute the various stress values. In addition, each of the above tooth geometries were first modelled using Pro/E and then later analysed in ANSYS. In order to perform parametric study on the distribution of various bending stresses, contact stresses and fatigue stresses developed in the spur gear pair tooth profile, a finite element analysis approach was adopted for the different spur gear tooth profiles and the results of the analysis were validated with that obtained from the analytical approach.

The different tooth profiles taken into consideration for modelling and FEA were involute, cycloidal, circular and asymmetric profiles. Finite element analysis of the spur gear pair assembly has been performed in three steps in order to evaluate the performance metrics of spur gear pair.

Table I: Specifications considered for comparative study.

GEAR PARAMETERS	SPEC.
GEAR RATIO	6-11
FACE WIDTH IN METERS	0.1
TYPE OF GEAR TEETH SYSTEM	20
TORQUE IN NEWTON-METER	100
CENTRE DISTANCE BETWEEN GEAR AND PINION SHAFT IN METERS	2.5
ANGULAR VELOCITY OF PINION IN RAD/SEC	100
MATERIAL FOR GEAR AND PINION	STRUCTURAL STEEL
SOURCE OF POWER	UNIFORM
TYPE OF DRIVEN MACHINERY	UNIFORM
TYPE OF LOAD	CONTINUOUS
FACTOR OF SAFETY	1.1
POISSON'S RATIO	0.3
YOUNGS MODULUS IN GPa	207
MODULE IN MM	9

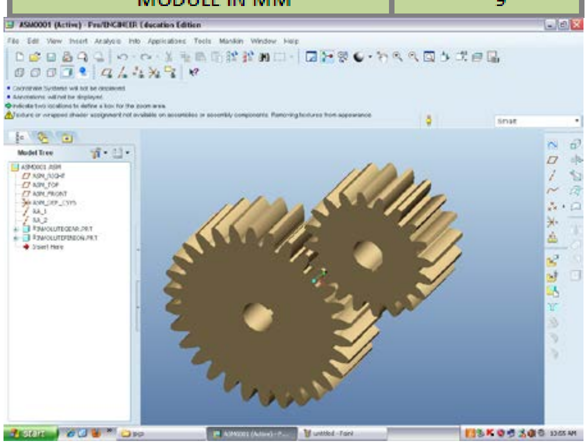

Fig. 1. Modeling of involute tooth profile model in Pro/E

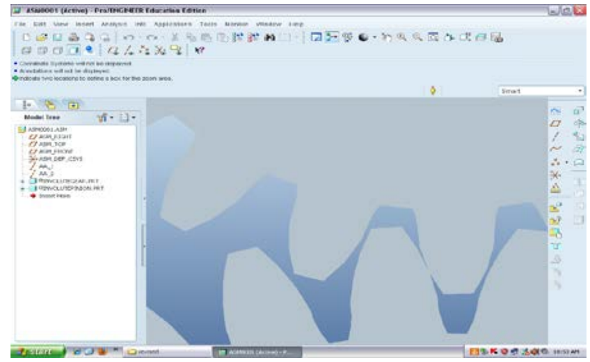

Fig. 2. Mating of spur gear pair with involute tooth profile

Initially, modelling of the gear was carried out in Pro/E. The model is shown in Fig. 1 and was done by sketching the base circle, adding stock to the generated circular profile and creating involute profile and extruding the profile using pattern feature. In the same way modelling of the pinion was also accomplished. Finally, assembling of both gear and pinion was done to obtain the gear pair. The spur gear train model showing the involute tooth profile with two-dimensional contact developed in Pro/E is illustrated in Fig. 2 Similarly modeling of spur gear train models with cycloidal, circular and asymmetric, tooth profiles has also been accomplished in Pro/E.

IV. FINITE ELEMENT ANALYSIS OF GEAR TOOTH PROFILES

Finite element analysis of the developed spur gear pair was executed in ANSYS. The first step is to perform structural analysis in order to calculate tooth bending stress and permissible bending stress, tooth surafce strength of gear and pinion. The second step in the finite element analysis approach is to perform contact stress analysis in order to calculate contact stress. The final step involved is to perform fatigue stress analysis in order to calculate allowable surface fatigue stress, surface fatigue strength. Each of these steps were executed and is described below.

The structural analysis of the spur gear train was performed in six stages namely input of engineering data, definition of geometry, development of model, setup and generation of solution and results. Structural steel was used in this problem having material properties of elastic modulus 207 GPa and Poisson's ratio 0.3. After input of these data, the model created in Pro/E was imported. After the model was imported, meshing operation was performed on the model to divide the model into several elements or nodes. The type of node element considered was tetrahedron and The torque, angular velocity of required range as specified in Table 1 were applied on the spur gear pair entities after the meshing operation. Two coordinate systems were taken for spur gear pair one is global coordinate system for gear and another is normal coordinate system for pinion. Torque was applied on the pinion by considering normal coordinate system means torque will be applied on pinion about pinion central axis and angular velocity of pinion is considered by considering the coordinating system for pinion about pinion central axis. After completion of preprocessing steps post processing steps were accomplished in ANSYS. In order to execute this several tools were imported such as fatigue tool, contact tool etc. In addition vonmises stresses, principal stresses were also given for analysis in order to calculate the performance metrics of spur gear pair. Based on these input details, the solution was generated by ANSYS. This structural analysis was executed for all the four tooth profiles listed earlier. The tooth bending stress distribution for the various tooth profiles are in Fig. 3 (for involute profile), Fig. 4 (for cycloidal profile), Fig. 5 (for circular profile) and Fig. 6 (for asymmetric profile).To examine the contact stresses in the gear pair, the spur gear train with two-dimensional contact developed in Pro/E was analysed in ANSYS.

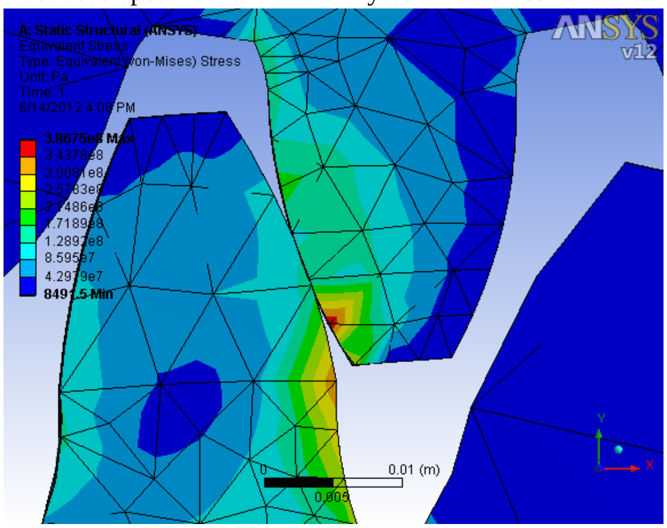

Fig.3. Tooth bending stress distribution for involute tooth profile

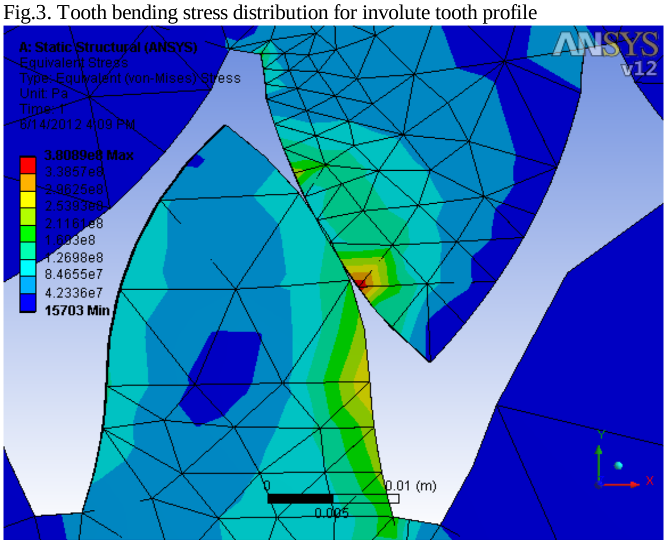

Fig. 4. Tooth bending stress distribution for cycloidal tooth profile.

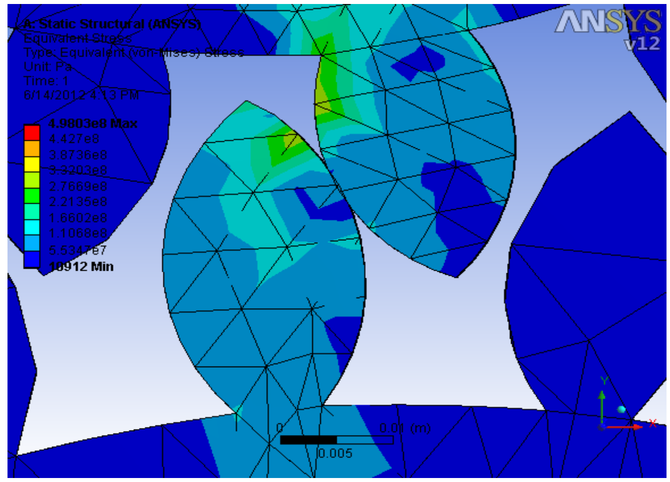

Fig. 5. Tooth bending stress distribution for circular tooth profile.

The numerical solutions obtained in ANSYS were compared with that of the Hertz theory contact stress through analytical analysis for involute tooth profile. Similarly contact stress analysis was also carried out for the remaining tooth profiles in ANSYS and compared with the analytical results. The results of the stress distribution are shown in Fig. 7 for the asymmetric tooth profile as a case in point.To examine the fatigue stresses in gear pair, the spur gear train with two-dimensional contact developed in Pro/E was analysed in ANSYS. The maximum principal stress at the root on the tensile side of the tooth was used for evaluating the tooth bending strength of a gear and pinion [14].

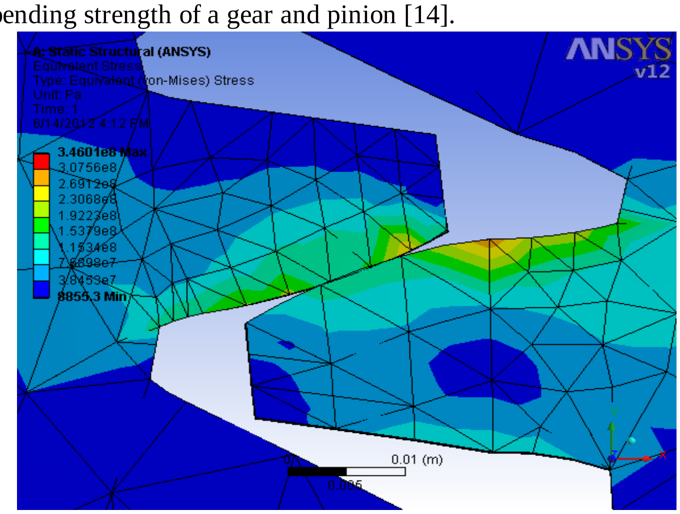

Fig. 6. Tooth bending stress distribution for asymmetric tooth profile.

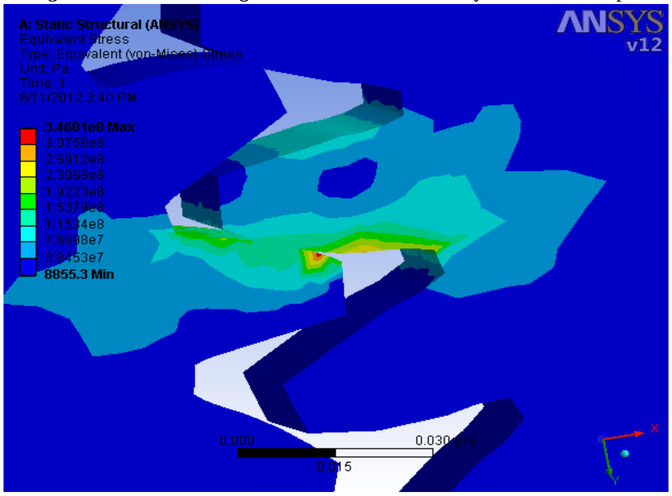

Fig. 7 Contact stress analysis with asymmetric tooth profile.

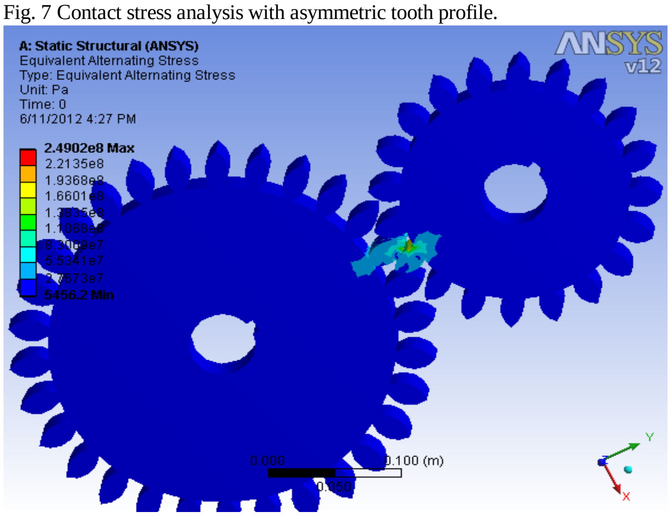

Fig. 8. Fatigue analysis with circular tooth profile.

Surface fatigue strength of the tooth profile is calculated by multiplying allowable surface fatigue stress with factor of safety. The numerical solutions are compared with that of the analytical analysis for involute tooth profile. Similarly fatigue stress analysis of remaining tooth profiles has been accomplished in ANSYS. The solution is generated automatically by ANSYS. The stress and strength results have been obtained as shown in Fig. 8 for circular tooth profile as a case in point. It can be seen from all of these that the maximum tooth bending stress was obtained at the tensile side of tooth of gear. In addition it can also be seen from the figures that the stress distribution is maximum at the contact side and minimum stress distribution obtained at the flank of gear and pinion. The comparison of various performance metrics for different tooth profiles will be illustrated in the next section.

V. Comparative Study of Gear Tooth Profiles

In this section, a comparative study of the results obtained through analytical analysis and finite element analysis is presented for the specifications listed in Table 1. The results are as given in Table II.

Table II. Comparison of various spur gear design metrics for different tooth profiles.

	INVOLUTE		CYCLOIDAL	CIRCULAR	ASSYMMETRIC
	AA	FEA	FEA	FEA	FEA
TBS	36	42.97	42.33	55.34	38.45
SFS	48.4	47.267	46.55	60.88	42.284
SF	44	42.97	42.32	55.347	38.44
CS	108	128	126	166	115.34
SFG	200.89	269	175.7	399	209.9
SFP	187.98	209.7	126.3	200	167
BS	50.94	50.94	50.94	50.94	50.94

It is observed that the predicted values from FEA are very close to the values obtained through the analytical analysis for involute tooth profile.

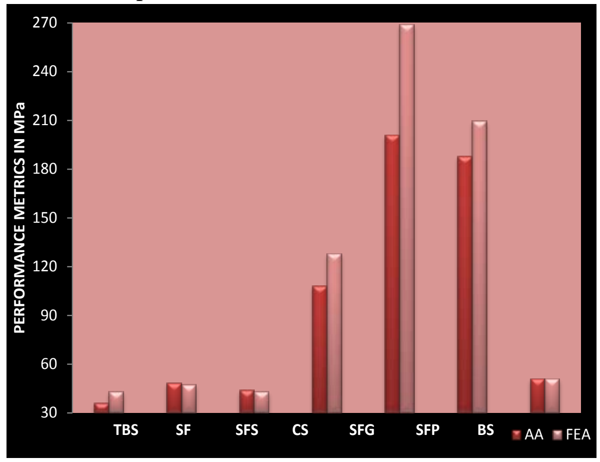

Fig. 9 Variation of different performance metrics for different analysis

In the case of tooth bending stress, it can be observed from Fig. 9 that the FEA values and the values obtained from analytical analysis and are fairly close. Out of the 7 performance metrics of the spur gear model, four performance metrics predicted by the FEA show an error less than 1% in comparison with the analytical analysis results and for remaining 3 performance metrics the FEA show an error less than 5% in comparison with the analytical analysis results. It can be concluded thus that the developed FEA model is an accurate representation of the stress distribution pattern. Fig. 10 shows the variation of tooth bending stress for different tooth profiles. It is found from the graph that the maximum value of tooth bending stress (55 MPa) is obtained for circular tooth profile and minimum value of 36 MPa is obtained for asymmetric tooth profile. For cycloidal and involute tooth profile the values were same. Overall increase in percentage of tooth bending stress is 35% in case of circular profile. Fig. 11 shows the variation of surface fatigue strength for different tooth profiles. It is found from the graph that the maximum value of surface fatigue strength (60 MPa) is obtained for circular tooth profile and minimum value of 42 Mpa is obtained for asymmetric tooth profile. For cycloidal tooth profile and involute tooth profiles the variation is very less and is about 2%. Overall increase in percentage of surface fatigue strength is 30% in case of circular tooth profile.

Fig. 12 shows the variation of allowable surface fatigue stress for different tooth profiles. It is found from the graph that the maximum value of allowable surface fatigue stress (55 MPa) is obtained for circular tooth profile and minimum value of 38 MPa is obtained for asymmetric tooth profile. For cycloidal tooth profile and involute tooth profiles the variation is very less and is about 2%. Overall increase in percentage of surface fatigue strength is 30% in case of circular tooth profile. Fig. 13 shows the variation of contact stress for different tooth profiles. It is found from the graph that the maximum value of contact stress (166 MPa) is obtained for circular tooth profile and minimum value of 115 MPa obtained for asymmetric tooth profile. For cycloidal tooth profile and involute tooth profiles the variation is very less and is about 2%. Overall increase in percentage of surface fatigue strength is 30% in case of circular tooth profile.

Fig. 14 shows the variation of tooth surface strength of gear for different tooth profiles. It is found from the graph that the maximum value of tooth surface strength of gear is obtained for circular tooth profile (399 MPa) and minimum value of 175 MPa is obtained for cycloidal tooth profile. Fig. 15 shows the variation of tooth surface strength of pinion for different tooth profiles. It is found from the graph that the maximum value of tooth surface strength (210 MPa) of pinion is obtained for involute tooth profile and minimum value of 126 MPa is obtained for cycloidal tooth profile. It can also be seen from the charts that the values corresponding to tooth bending stress, allowable surface fatigue stress, surface fatigue strength and permissible bending stress are within a range of 30-60 MPa for the same loading condition for different tooth profiles and for remaining metrics such as tooth surface strength of gear and pinion the range of strength varies from 200-270 MPa and for contact stress the range of stress varies from 90-120 MPa.

It is also observed from the figures that the specific performance metrics such as tooth bending stress, allowable surface fatigue stress, surface fatigue strength, contact stress, tooth surface strength of gear are higher for circular tooth profile and the values are least for asymmetric tooth profile. For the remaining performance metric namely tooth surface strength of pinion, the value is higher for involute tooth profile and least for cycloidal tooth profile.

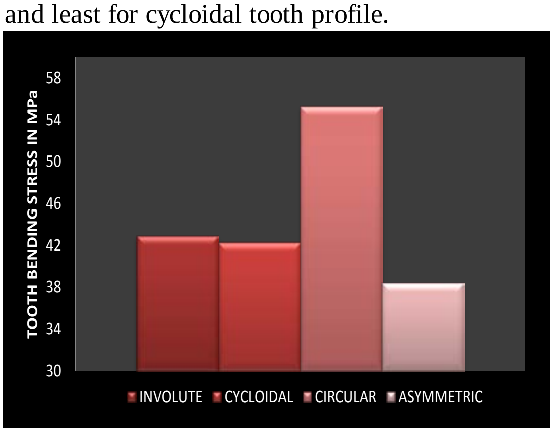

Fig.10 Variation of tooth bending stress for different profiles

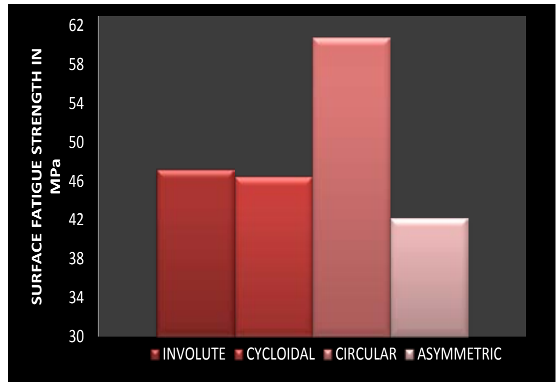

Fig.11 Variation of surface fatigue strength for different profiles

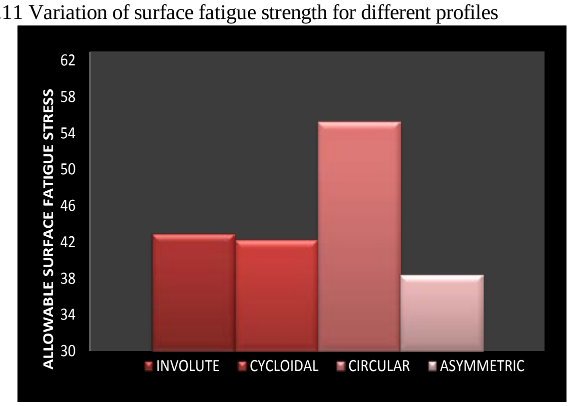

Fig.12 Variation of allowable surface fatigue stress for different profiles

It can thus be concluded that circular tooth profile is not at all suitable because all the stresses induced in the gear with such a profile are the highest in comparison with other tooth profiles while the strength corresponding to this tooth profile is the least. This explains why circular tooth profile is not used in gear design. In addition, the manufacture of gears using circular profiles poses specific difficulties rendering the use of circular profiles completely unviable. The same is the case with cycloidal profile along with the fact that the gear weight using these two profiles is more than that with the involute profile.

The overall performance of asymmetric tooth profile was found to be the best among those compared on account of the least stress values induced in the tooth. The strength of this tooth is also the highest among the compared options. The performance of this tooth is very close to that of the involute profile which is the most commonly used profile in gear design and understandably so. The specific benefit of using an asymmetric involute profile is seen from the above graphs in terms of its higher strength and lower stress development. The weight of the gear with asymmetric involute profile is lower than that with a conventional symmetric involute profile.

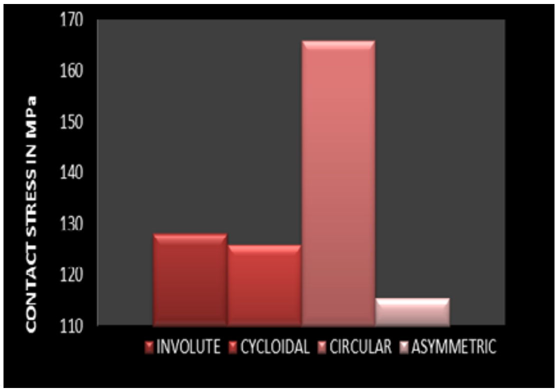

Fig.13 Variation of contact stress for different profiles

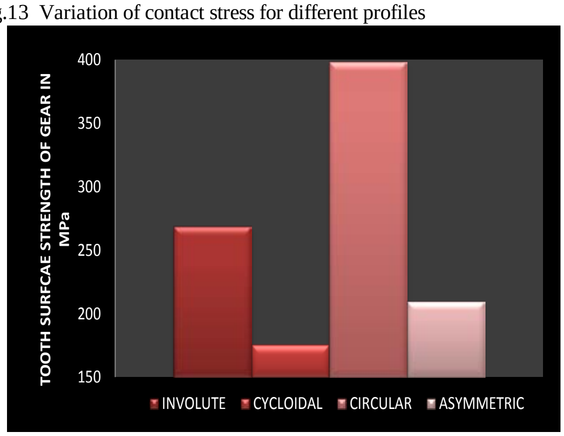

Fig.14 Variation of tooth surface strength of gear for different profiles

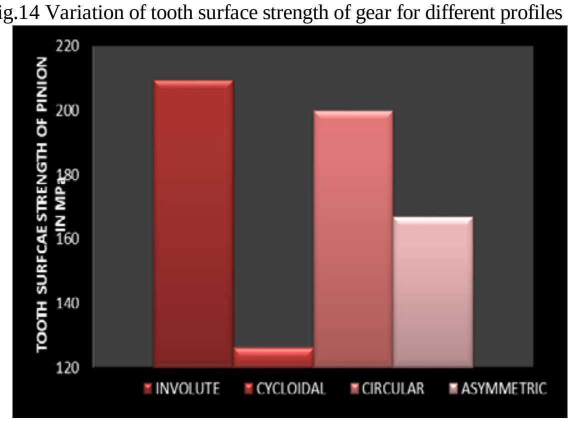

Fig.15 Variation of tooth surface strength of pinion for different profiles

On account of this, the wear in these gears is heavily reduced. The frictional stresses developed in the gear with asymmetric tooth profile are at least 10 Mpa lower than that with involute profile. Especially in applications that call for minimising weight of the gear and for smoother operation, asymmetric involute tooth profiles are recommended over the conventional use of symmetric involute gear tooth profiles.

VI. Conclusion

In this paper, an attempt has been made to compare the performance of various spur gear tooth profiles for a given set of specification through an analytical approach based on AGMA standards as well as a finite element analysis approach. Four different profiles namely symmetric involute, asymmetric involute, cycloidal and circular profiles were evaluated. The developed FEA model was validated against the analytical approach and was found to be very close. Further stress analysis was carried out using FEA. It was found that the overall performance of asymmetric involute tooth profile was found to be the best in terms of stress as well as tooth strength. The performance of the symmetric tooth profile is very close to that of the asymmetric involute profile. However, in cases where overall gear weight as well as wear are critical considerations, the performance of the asymmetric involute profile outweighs that of the symmetric involute profile. It is also seen that both the cycloidal and circular tooth profiles develop at least 30% higher stresses for the same conditions and are certainly not viable options.

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