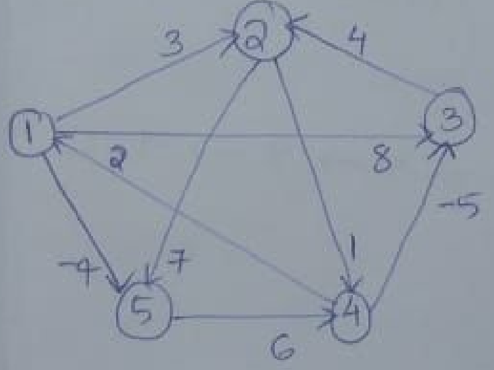
(1)

All Pairs Shortest Path

Algorithm

Cost $[i,i] = 0.0$ for $1 \le i \le n$

{
for $i = 1$ to n do
for $j = 1$ to n do
$A[i,j] = \text{Cost}[i,j]$;
for $k = 1$ to n do
for $i = 1$ to n do
for $j = 1$ to n do
$A[i,j] = \min(A[i,j] \cdot A[i,k] + A[k,j])$;



A^0	1	2	3	4	5
1	0	3	8	∞	-4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	∞	-5	0	∞
5	∞	∞	∞	6	0

Consider '1' as the intermediate vertex. when we consider vertex 1 . vertex 1 will remain unchanged so directly take the values. And also there is no loops.

A^1	1	2	3	4	5
1	0	3	8	∞	-4
2	∞	0			
3	∞		0		
4	2			0	
5	∞				0

$A[i,j] = \min\{A^0[i,j], A^0[i,k] + A^0[k,j])$

$k = 1, i = 2, j = 3$

②

$A^1(2,3) = \min\{A^0(2,3), A^0(2,1) + A^0(1,3)\}$

$= \min\{\infty, \infty+8\}$

$= \infty$

$A^1(2,4) = \min\{A^0(2,4), A^0(2,1) + A^0(1,4)$

$= \min\{1, \infty+\infty = \underline{\underline{1}}$

$A^1(2,5) = \min\{A^0(2,5), A^0(2,1) + A^0(1,5)$

$= \min(7, \infty + -4 = \underline{\underline{7}}$

$k=1, i=3, j=2$

$A^1(3,2) = \min\{A^0(3,2), A^0(3,1) + A^0(1,2)$

$= \min\{4, \infty+3 = \underline{\underline{4}}$

$A^1(3,4) = \min\{A^0(3,4), A^0(3,1) + A^0(1,4)\}$

$= \min\{\infty, \infty+\infty$

$= \underline{\underline{\infty}}$

$A^1(3,5) = \min\{A^0(3,5), A^0(3,1) + A^0(1,5)$

$= \min\{\infty, \infty + -4 = \underline{\underline{\infty}}$

$k=1, i=4, j=2$

$A^1(4,2) = \min\{A^0(4,2), A^0(4,1) + A^0(1,2)$

$= \min\{\infty, 2+3 = \underline{\underline{5}}$

$A^1(4,3) = \min\{A^0(4,3), A^0(4,1) + A^0(1,3)$

$= \min\{-5, 2+8 = \underline{\underline{-5}}$

$A^1(4,5) = \min\{A^0(4,5), A^0(4,1) + A^0(1,5)$

$= \min\{\infty, 2+-4 = \underline{\underline{-2}}$

$k=1, i=5, j=2$

$A^1(5,2) = \min\{A^0(5,2), A^0(5,1) + A^0(1,2)$

$= \min\{\infty, \infty+3$

$= \underline{\underline{\infty}}$

(3)

$A^1(5,3) = \min\{A^0(5,3), A^0(5,1) + A^0(1,3)\}$

$= \min\{\infty, \infty + 8\} = \underline{\underline{\infty}}$

$A^1(5,4) = \min\{A^0(5,4), A^0(5,1) + A^0(1,4)\}$

$= \min\{6, \infty + \infty\} = \underline{\underline{6}}$

A^1	1	2	3	4	5
1	0	3	8	∞	-4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

Consider 2 as intermediate vertex. Then the 2nd vertex will remain unchanged so directly take the values and also no loops.

A^2	1	2	3	4	5
1	0	3			
2	∞	0	∞	1	7
3		4	0		
4		5		0	
5		∞			0

$k=2, i=1, j=3$

$A^2(1,3) = \min\{A^1(1,3), A^1(1,2) + A^1(2,3)\}$

$= \min\{8, 3 + \infty\} = \underline{\underline{8}}$

$A^2(1,4) = \min\{A^1(1,4), A^1(1,2) + A^1(2,4)\}$

$= \min\{\infty, 3 + 1\} = \underline{\underline{4}}$

$A^2(1,5) = \min\{A^1(1,5), A^1(1,2) + A^1(2,5)\}$

$= \min\{-4, 3 + 7\} = \underline{\underline{-4}}$

$k=2, i=3, j=1$

$A^2(3,1) = \min\{A^1(3,1), A^1(3,2) + A^1(2,1)\}$

(4)

$= \min\{\infty, 4+\infty = \underline{\underline{\infty}}$

$A^2(3,4) = \min\{A^1(3,4), A^1(3,2)+A^1(2,4)\}$

$= \min\{\infty, 4+1 = \underline{\underline{5}}$

$A^2(3,5) = \min\{A^1(3,5), A^1(3,2)+A^1(2,5)\}$

$= \min\{\infty, 4+7 = \underline{\underline{11}}$

$k=2, i=4, j=1$

$A^2(4,1) = \min\{A^1(4,1), A^1(4,2)+A^1(2,1)\}$

$= \min\{2, 5+\infty = \underline{\underline{2}}$

$A^2(4,3) = \min\{A^1(4,3), A^1(4,2)+A^1(2,3)\}$

$= \min\{-5, 5+\infty = \underline{\underline{-5}}$

$A^2(4,5) = \min\{A^1(4,5), A^1(4,2)+A^1(2,5)\}$

$= \min\{-2, 5+7 = \underline{\underline{-2}}$

$k=2, i=5, j=1$

$A^2(5,1) = \min\{A^1(5,1), A^1(5,2)+A^1(2,1)\}$

$= \min\{\infty, \infty+\infty = \underline{\underline{\infty}}$

$A^2(5,3) = \min\{A^1(5,3), A^1(5,2)+A^1(2,3)\}$

$= \min\{\infty, \infty+\infty = \underline{\underline{\infty}}$

$A^2(5,4) = \min\{A^1(5,4), A^1(5,2)+A^1(2,4)\}$

$= \min\{6, \infty+1 = 6$

A^2	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

Consider 3 as intermediate vertex. Then the 3rd vertex will remain unchanged. So directly take the values

5

A^3	1	2	3	4	5
1	0		8		
2		0	∞		
3	∞	4	0	5	11
4			-5	0	
5			∞		0

$k=3,\ i=1,\ j=2$

$A^3(1,2) = \min\{A^2(1,2),\ A^2(1,3)+A^2(3,2)\}$

$= \min\{3,\ 8+4 = \underline{3}$

$A^3(1,4) = \min\{A^2(1,4),\ A^2(1,3)+A^2(3,4)\}$

$= \min\{4,\ 8+5 = \underline{4}$

$A^3(1,5) = \min\{A^2(1,5), A^2(1,3)+A^2(3,5)\} = \min\{-4,\ 8+11 = \underline{-4}$

$k=3,\ i=2,\ j=1$

$A^3(2,1) = \min\{A^2(2,1), A^2(2,3)+A^2(3,1)\} = \min\{\infty,\ \infty+\infty = \underline{\infty}$

$A^3(2,4) = \min\{A^2(2,4), A^2(2,3)+A^2(3,4)\} = \min\{1,\ \infty+5 = \underline{1}$

$A^3(2,5) = \min\{A^2(2,5), A^2(2,3)+A^2(3,5)\} = \min\{7,\ \infty+11 = \underline{7}$

$k=3,\ i=4,\ j=1$

$A^3(4,1) = \min\{A^2(4,1), A^2(4,3)+A^2(3,1)\} = \min\{2,\ -5+\infty = \underline{2}$

$A^3(4,2) = \min\{A^2(4,2), A^2(4,3)+A^2(3,2)\} = \min\{5,\ -5+4 = \underline{-1}$

$A^3(4,5) = \min\{A^2(4,5), A^2(4,3)+A^2(3,5)\} = \min\{-2,\ -5+11 = \underline{-2}$

$k=3,\ i=5,\ j=1$

$A^3(5,1) = \min\{A^2(5,1), A^2(5,3)+A^2(3,1)\} = \min\{\infty,\ \infty+\infty = \underline{\infty}$

$A^3(5,2) = \min\{A^2(5,2), A^2(5,3)+A^2(3,2)\} = \min\{\infty,\ \infty+4 = \underline{\infty}$

$A^3(5,4) = \min\{A^2(5,4), A^2(5,3)+A^2(3,4)\} = \min\{6,\ \infty+5 = \underline{6}$

A^3	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	-1	-5	0	-2
5	∞	∞	∞	6	0

(6) Consider 4 as intermediate vertex. Then the 4th vertex will remain unchanged so directly take the values

A^4	1	2	3	4	5
1	0			4	
2		0		1	
3			0	5	
4	2	-1	-5	0	-2
5				6	0

$k=4,\ i=1,\ j=2$

$A^4(1,2) = \min\{A^3(1,2), A^3(1,4)+A^3(4,2)\} = \{\min\{3, 4+-1$

$= \underline{\underline{3}}$

$A^4(1,3) = \min\{A^3(1,3), A^3(1,4)+A^3(4,3)\} = \min\{8, 4+-5$

$= \underline{\underline{-1}}$

$A^4(1,5) = \min\{A^3(1,5), A^3(1,4)+A^3(4,5)\} = \min\{-4, 4+-2$

$= \underline{\underline{-4}}$

$k=4,\ i=2,\ j=1$

$A^4(2,1) = \min\{A^3(2,1), A^3(2,4)+A^3(4,1)\}$

$= \min\{\infty, 1+2 = \underline{\underline{3}}$

$A^4(2,3) = \min\{A^3(2,3), A^3(2,4)+A^3(4,3)\}$

$= \infty, 1+-5 = \underline{\underline{-4}}$

$A^4(2,5) = \min\{A^3(2,5), A^3(2,4)+A^3(4,5)\}$

$= \min\{7, 1+-2$

$= \underline{\underline{-1}}$

$k=4,\ i=3,\ j=1$

$A^4(3,1) = \min\{A^3(3,1),\ A^3(3,4)+A^3(4,1)$

$= \min\{\infty,\ 5+2$

$= \underline{7}$

$A^4(3,2) = \min\{A^3(3,2),\ A^3(3,4)+A^3(4,2)$

$= \min\{4,\ 5+-1 = \underline{4}$

$A^4(3,5) = \min\{A^3(3,5),\ A^3(3,4)+A^3(4,5)$

$= \min\{11,\ 5+-2 = \underline{3}$

$k=4,\ i=5,\ j=1$

$A^4(5,1) = \min\{A^3(5,1),\ A^3(5,4)+A^3(4,1)$

$= \min\{\infty,\ 6+2 = \underline{8}$

$A^4(5,2) = \min\{A^3(5,2),\ A^3(5,4)+A^3(4,2)$

$= \min\{\infty,\ 6+-1 = \underline{5}$

$A^4(5,3) = \min\{A^3(5,3),\ A^3(5,4)+A^3(4,3)$

$= \min\{\infty,\ 6+-5 = \underline{1}$

A^4	1	2	3	4	5
1	0	3	-1	4	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0

⑧ Consider 5 as intermediate vertex. Then the 5^{th} vertex will remain unchanged. So directly take that values

A^5	1	2	3	4	5
1	0				-4
2		0			-1
3			0		3
4				0	-2
5	8	5	1	6	0

$k=5, i=1, j=2$

$A^5(1,2) = \min\{A^4(1,2), A^4(1,5)+A^4(5,2)\}$

$= \min\{3, -4+5\} = \underline{\underline{1}}$

$A^5(1,3) = \min\{A^4(1,3), A^4(1,5)+A^4(5,3)\}$

$= \min\{-1, -4+1\} = \underline{\underline{-3}}$

$A^5(1,4) = \min\{A^4(1,4), A^4(1,5)+A^4(5,4)\}$

$= \min\{4, -4+6\} = \underline{\underline{2}}$

$k=5, i=2, j=1$

$A^5(2,1) = \min\{A^4(2,1), A^4(2,5)+A^4(5,1)\}$

$= \min\{3, -1+8\} = \underline{\underline{3}}$

$A^5(2,3) = \min\{A^4(2,3), A^4(2,5)+A^4(5,3)\}$

$= \min\{-4, -1+1\} = \underline{\underline{-4}}$

$A^5(2,4) = \min\{A^4(2,4), A^4(2,5)+A^4(5,4)\}$

$= \min\{1, -1+6\} = \underline{\underline{1}}$

(9)

$k=5, i=3, j=1$

$A^5(3,1) = \min\{A^4(3,1), A^4(3,5) + A^4(5,1)$

$= \min\{7, 3+8 = \underline{\underline{7}}$

$A^5(3,2) = \min\{A^4(3,2), A^4(3,5) + A^4(5,2)$

$= \min\{4, 3+5 = \underline{\underline{4}}$

$A^5(3,4) = \min\{A^4(3,4), A^4(3,5) + A^4(5,4)$

$= \min\{5, 3+6 = \underline{\underline{5}}$

$k=5, i=4, j=1$

$A^5(4,1) = \min\{A^4(4,1), A^4(4,5) + A^4(5,1)$

$= \min\{2, -2+8 = \underline{\underline{2}}$

$A^5(4,2) = \min\{A^4(4,2), A^4(4,5) + A^4(5,2)$

$= \min\{-1, -2+5 = \underline{\underline{-1}}$

$A^5(4,3) = \min\{A^4(4,3), A^4(4,5) + A^4(5,3)$

$= \min\{-5, -2+1 = \underline{\underline{-5}}$

A^5	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0