Chapter-8 Remainder and Factor Theorm

Example: $89 \div 9$

Divisor ← $9)\,89\,(9$ → quotient

81

(8)

Remainder   Dividend

② $x^2+2x \div x$

$\frac{x^2}{x} + \frac{2x}{x} = x+2$

$\frac{x^2}{x} = x$

$\frac{2x}{x} = 2.$

$x)\ x^2+2x\ (x+2$

$-x^2$

$2x$

$-2x$

0

③ $x^2+2x+4 \div x+3$

$x+3)\ x^2+2x+4\ (x-1$

$-x^2 (-) 3x$

$-x+4$

$-x-3$

$(+)\ (+)$

(7)

Remainder

$\frac{x^2}{x} = x$

$\frac{-x}{x} = -1$

Remainder Theorm

Example 3: $x^2+2x+4 \div x+3$

let $f(x) = x^2+2x+4$

divisor $= x+3$

let $x+3=0$

$\boxed{x=-3}$

$f(-3) = (-3)^2+2(-3)+4$

$= 9-6+4$

$= 13-6$

$= 7$ → Remainder

Exercise - 8(A)

8A I Find, in each case, the remainder when

① $x^4 - 3x^2 + 2x + 1$ is divided by $(x-1)$

A) Given,

$f(x) = x^4 - 3x^2 + 2x + 1$

The divisor is $(x-1)$.

let $x - 1 = 0$

$\boxed{x = 1}$

$f(x) = (1)^4 - 3(1)^2 + \cancel{2} 2(1) + 1$

$= 1 - 3 + 2 + 1$

$f(x) = 1$

∴ Remainder is 1.

② $x^3 + 3x^2 - 12x + 4$ is divided by $x - 2$

A) Given,

$f(x) = x^3 + 3x^2 - 12x + 4$

The divisor is $(x-2)$

let $x - 2 = 0$

$\boxed{x = 2}$

$f(x) = (2)^3 + 3(2)^2 - 12(2) + 4$

$= 8 + 12 - \cancel{}^{24} + 4$

$= 24 - 24$

$= 0.$

Note: If the remainder is zero, then the divisor is called factor of $f(x)$

iii) $x^4 + 1$ is divided by $x + 1$

A) Given,

$f(x) = x^4 + 1$

The divisor is $x + 1$

let $x+1=0 \Rightarrow \boxed{x=-1}$

$f(x)=(-1)^4+1$

$=1+1$

$=2$

II Show that

i) $x-2$ is a factor of $5x^2+15x-50$

A) Given, $f(x)=5x^2+15x-50$

The divisor is $x-2$

$\boxed{x=2}$

$\therefore f(x)=5(2)^2+15(2)-50$

$=5(4)+30-50$

$=20+30-50$

$=50-50$

$f(2) \quad =0$

ii) $3x+2$ is a factor of $3x^2-x-2$

A) Given,

$f(x)=3x^2-x-2$

The divisor is $(3x+2)$

let $3x+2=0$

$3x=-2$

$x=-\frac{2}{3}$

$\therefore f\left(-\frac{2}{3}\right)=3\left(-\frac{2}{3}\right)^2-\left(-\frac{2}{3}\right)-2.$

$=\cancel{3}\times\frac{4}{\cancel{9}_3}+\frac{2}{3}-\frac{2}{1}$

$=$

$=\frac{4}{3}+\frac{2}{3}-\left(\frac{2}{1}\times\frac{3}{3}\right)$

$=\frac{4}{3}+\frac{2}{3}-\frac{6}{3}$

$=\frac{4+2-6}{3}=\frac{6-6}{3}=\frac{0}{3}=0$

$\therefore f\left(-\frac{2}{3}\right) = 0$ hence $3x+2$ is a factor of $f(x)$.

③ Use Remainder Theorm, to find which of the following is a factor of $2x^3+3x^2-5x-6$

i) $x+1$ ii) $2x-1$ iii) $x+2$

A) Given,

$$f(x) = 2x^3+3x^2-5x-6$$

The divisor is $x+1$.

let $x+1=0$

$$\boxed{x=-1}$$

$$\therefore f(x) = 2(-1)^3+3(-1)^2-5(-1)-6$$
$$= -2+3+5-6$$
$$= 8-8$$
$$= 0.$$

ii) $2x-1=0$

A) Given,

$$f(x) = 2x^3+3x^2-5x-6$$

The divisor is $2x-1$

let $2x-1=0$

$$2x = 1$$
$$x = \frac{1}{2}$$

$$\therefore f(x) = 2\left(\frac{1}{2}\right)^3+3\left(\frac{1}{2}\right)^2-5\left(\frac{1}{2}\right)-6$$

$$\therefore f\left(\frac{1}{2}\right) = 2\times\frac{1}{8}+3\times\frac{1}{4}-\frac{5}{2}-\frac{6}{1}$$

$= \frac{1}{4} + \frac{3}{4} - \frac{5}{2} - \frac{6}{1}$

$= \frac{1}{4} + \frac{3}{4} - \left(\frac{5}{2} \times \frac{2}{2}\right) - \left(\frac{6}{1} \times \frac{4}{4}\right)$

$= \frac{1}{4} + \frac{3}{4} - \frac{10}{4} - \frac{24}{4}$

$= \frac{1+3-10-24}{4}$

$= 4 - 34/4$

$= \frac{-\cancel{30}^{15}}{\cancel{4}_{2}}$

$= -\frac{15}{2}.$

(ii) $x+2$

Given;

$f(x) = 2x^3 + 3x^2 - 5x - 6$

The divisor is $x+2$

let $x+2=0$

$x = -2$

$f(-2) = 2(-2)^3 + 3(-2)^2 - 5\cancel{x}(-2) - 6$

$= 2(-8) + 12 + \cancel{5x}10 - 6$

$= -16 + 12 + 10 - 6$

$= -16 - 6 + 12 + 10$

$= \cancel{-10 + 12 + 10}\ -22 + 22$

$= 0,$

$\therefore f(-2) = 0$ hence $x+2$ is a factor of $f(x)$